DIVERGENCE STRUCTURE OF $6D$, $\mathcal{N} = (1, 0)$ SUPERSYMMETRIC GAUGE THEORIES

I.L. Buchbinder

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Goals

- Developing the $\mathcal{N}=(1,0)$ harmonic superfield formulation for $6D$ vector multiplet coupled to $6D$ hypermultiplet
- Constructing the background field methods, to get the manifestly supersymmetric and gauge invariant effective action
- Studying the general structure of the divergences depending both on vector multiplet and on hypermultiplet
- Calculating the one-loop divergences

Based on:
I.L.B, N.G. Pletnev, Nucl.Phys. B896, 1, 2015, arXiv:1411.1848[hep-th];
I.L.B, N.G. Pletnev, Phys.Lett. B744, 125, 2015, arXiv:1502.03257[hep-th];
I.L.B, B.M. Merzlikin, N.G. Pletnev, Phys.Lett. B759, 626, 2016, arXiv:1604.06186[hep-th].
I.L.B, E.A. Ivanov, K.V. Stepanayantz, B.M. Merzlikin, arXiv:1609.?????[hep-th]; to be in arXiv tomorrow.

Motivations

The modern interest to $6D$ supersymmetric gauge theories is stipulated by the problem of field description of the interacting multiple $M5$-branes (see e.g. review J. Bagger, N. Lambert, S. Mikhu, C. Papageorgakis, Phys.Repts. 527 (2013) 1).

- Hypothetic M-theory is characterized by two extended objects: $M2$-brane and $M5$-brane.
- The field description of interacting multiple $M2$-branes is given by Bagger-Lambert-Gustavsson (BGL) theory which is $3D$, $\mathcal{N} = 8$ supersymmetric gauge theory.
- The field description of the interacting multiple $M5$-branes is not constructed.

Motivations

- Low-energy dynamics of a single $M5$-brane is described by the six-dimensional Abelian $\mathcal{N} = (2,0)$ tensor multiplet (see e.g. S. Ferrara, E. Sokachev, 2000).
- Field contents: 5 bosonic degrees of freedom, provided by five scalars, eight fermionic degrees of freedom and three bosonic degrees of freedom provided by Abelian antisymmetric tensor gauge field B_{ab} which has a self-dual field strength H_{abc}.
- Problem of Lagrangian description: kinetic term for field B_{ab} is identically zero because of the self-duality condition.
- Problem of non-Abelian formulation: non-Abelian generalization of tensor multiplet is unknown (e.g. X. Bekaert, M. Henneaux, A. Sevrin, 1999, 2001; I. Bandos, H. Samtleben, D. Sorokin, 2013)

- From M-theory side, the low-energy interacting multiple $M5$-brane theory should be six-dimensional conformal non-Abelian gauge model with $\mathcal{N} = (2,0)$ supersymmetry (e.g. review J. Teschner, arXiv:1412.7145).
- From field theory side such a model is not formulated so far.
- Recently it was proposed to construct the non-Abelian $\mathcal{N} = (2,0)$ supersymmetric theory using the non-Abelian $\mathcal{N} = (1,0)$ supersymmetric gauge theory (H. Samtleben, E. Sezgin, R. Wimmer, 2011, 2013) in the framework of non-Abelian hierarchy of p-forms (B. de Wit, H. Samtleben, 2005; E.A. Bergshoeff, J. Hartong, O. Holm, M. Huebscher, T. Ortin, 2009).
- Attempts to develop the superfield formulation of the tensor hierarchy have been given (I. Bandos, 2013; I.L.B, N.G. Pletnev, 2015).

- $6D$ supersymmetric gauge models are characterized by the miraculous cancellations of the divergences (P.S. Howe, K.S. Stelle, 1984, 2003; G. Bossard, P.S. Howe, K.S. Stelle, 2009).
- Constructing the systematic superfield procedure to explain an origin of these cancellations.
- Constructing the superfield background field method.
- Developing the manifestly gauge invariant and supersymmetric procedure to calculate the divergent and finite parts of the quantum effective action

Goal

Goal of talk: brief review a harmonic superspace formulation of the $6D$, $\mathcal{N} = (1,0)$ supersymmetric gauge models, construction of the quantum effective action and calculation of the one-loop divergences in Abelian theory.

Reminder of the notions

- Supersymmetry is extension of special relativity symmetry by fermionic generators.
- Supersymmetry unifies some bosonic and fermionic fields into one supermultiplet.
- Supersymmetric field models can be formulated in terms of above bosonic and fermionic fields. Component approach.
- In some cases the supersymmetric field models can be formulated in terms of superfields. A superfield depends on space-time coordinates x and some number of anicommuting (Grassmann) coordinates θ. The coefficients of expansion of superfield in anticommuting coordinates are the conventional bosonic and fermionic fields of supermultiplet.
- Advantage of component formulation: close relation with conventional field theory, convenient in classical field theory and to calculate the quantum processes (S-matrix). Disadvantage: supersymmetry is not manifest.
- Advantage of superfield formulation: manifest supersymmetry, convenient in quantum field theory to study off-shell effects.
- Problem of unconstrained formulation

Plan

- Review part
  1. $6D$ supersymmetry
  2. $6D, \mathcal{N} = (1,0)$ harmonic superspace
  3. $6D, \mathcal{N} = (1,0)$ hypermultiplet
  4. $6D, \mathcal{N} = (1,0)$ vector multiplet
  5. Action of Abelain vector multiplet coupled to hypermultiplet
- Background field method
- Structure of one-loop counterterms
- Divergent part of one-loop effective action
- Summary

Basic reference: P.S. Howe, G. Sierra, P.K. Townsend, 1983.

$6D$ Minkowski space

- Coordinates x^a, $a = 0, 1, 2, 3, 4, 5$
- Metric $\eta_{ab} = diag(1, -1, -1, -1, -1, -1)$
- Proper Lorentz group $SO(1, 5)$

Two types of $6D$ Spinors

- Left $(1, 0)$ spinors ψ_α, $\alpha = 1, 2, 3, 4$
- Right $(0, 1)$ spinors ϕ^α, $\alpha = 1, 2, 3, 4$

Dirac matrices

- 8×8 Dirac matrices Γ^a,

$$\Gamma^a \Gamma^b + \Gamma^b \Gamma^a = 2\eta^{ab}$$

- Representation of the Dirac matrices

$$\Gamma^a = \begin{pmatrix} 0 & (\gamma^a)_{\alpha\beta} \\ (\tilde{\gamma}^a)^{\beta\alpha} & 0 \end{pmatrix},$$

  $\alpha, \beta = 1, 2, 3, 4$

- Pauli-type matrices γ^a and $\tilde{\gamma}^a$,
  $\gamma^a_{\alpha\beta} = -\gamma^a_{\beta\alpha}$, $\tilde{\gamma}^a_{\alpha\beta} = -\tilde{\gamma}^a_{\beta\alpha}$,
  $(\gamma^a \tilde{\gamma}^b + \gamma^b \tilde{\gamma}^a)_\alpha{}^\beta = 2\eta^{ab} \delta_\alpha{}^\beta$
- Spinor representation of the vectors, $V_{\alpha\beta} = \gamma^a_{\alpha\beta} V_a$; $V^{\alpha\beta} = \tilde{\gamma}^{\alpha\beta}_a V_a$

$6D$ superalgebra

- Two types of independent supercharges
  $Q^I_\alpha, Q^\alpha_J,\ I = 1, ..., 2m; J = 1, ..., 2n$
- Anticommutational relations for supercharges

$$\{Q^I_\alpha, Q^K_\beta\} = 2\Omega^{IK} P_{\alpha\beta}$$

$$\{Q^\alpha_J, Q^\beta_L\} = 2\Omega_{JL} P^{\alpha\beta}$$

  Matrix Ω_{IK} belongs to $USp(2n)$ group (R-symmetry group), $\Omega_{IK}\Omega^{KJ} = \delta^J_I$
- $\mathcal{N} = (m, n)$ supersymmetry
- $\mathcal{N} = (1, 0)$ superspace
  Coordinates $z = \{x^{a\beta}, \theta^\alpha_I\}, I = 1, 2$
- Basic spinor derivatives

$$D^I_\alpha = \frac{\partial}{\partial\theta^\alpha_I} - i\theta^{I\beta}\partial_{\alpha\beta}, \quad \{D^I_\alpha, D^J_\beta\} = -2i\Omega^{IJ}\partial_{\alpha\beta}$$

Harmonic superspace

Basic references:

$4D$

A.Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky, E. Sokatchev, 1985.
A.Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic Superspace, 2001.

General purpose: to formulate $\mathcal{N}=2$ models in terms of unconstrained $\mathcal{N}=2$ superfields. General idea: to use the parameters $u^{\pm i}(i=1,2)$ (harmonics) related to $SU(2)$ automorphism group of the $\mathcal{N}=2$ superalgebra and parametrizing the 2-sphere,

$u^{+i}u_i^- = 1$

It allows to introduce the $\mathcal{N}=2$ superfields with the same number of anticommuting coordinates as in case of the $\mathcal{N}=1$ supersymmetry. Prices for this are the extra bosonic variables, harmonics $u^{\pm i}$.

$6D$

P.S. Howe, K.S. Stelle, P.C. West, 1985.
B.M. Zupnik, 1986; 1999.

$(1,0)$ harmonic superspace

- $USp(2) \sim SU(2), I = i$ The same harmonics $u^{\pm i}$ as in $4D, \mathcal{N} = 2$ supersymmetry
- Harmonic $6D, (1,0)$ superspace with coordinates $(x^a, \theta^{\alpha i}, u^{\pm i})$
- Analytic basis $(\zeta_A^M = \{x_A^a, \theta^{+\alpha}\}, u_i^{\pm}, \theta^{-\alpha})$,
  $x_A^a = x^a + i\theta^- \gamma^a \theta^+, \quad \theta^{\pm\alpha} = u_i^{\pm}\theta^{\alpha i}$
  The coordinates $\zeta_A^M, u_i^{\pm}$ form a subspace closed under $(1,0)$ supersymmetry
- The harmonic derivatives

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} + i\theta^+ \not{\partial}\theta^+ + \theta^{+\alpha}\frac{\partial}{\partial\theta^{-\alpha}},$$

$$D^{--} = u^{-i}\frac{\partial}{\partial u^{+i}} + i\theta^- \not{\partial}\theta^- + \theta^{-\alpha}\frac{\partial}{\partial\theta^{+\alpha}},$$

$$D^0 = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} + \theta^{+\alpha}\frac{\partial}{\partial\theta^{+\alpha}} - \theta^{-\alpha}\frac{\partial}{\partial\theta^{-\alpha}}$$

- Spinor derivatives in the analytic basis

$$D_\alpha^+ = \frac{\partial}{\partial\theta^{-\alpha}}, \quad D_\alpha^- = -\frac{\partial}{\partial\theta^{+\alpha}} - 2i\partial_{\alpha\beta}\theta^{-\beta}, \quad \{D_\alpha^+, D_\beta^-\} = -2i\partial_{\alpha\beta}$$

- Analytic superfields ϕ do not depend on $\theta^{-\alpha}$, $D_\alpha^+\phi = 0$

Hypermultiplet in conventional $6D$ superspace

- The $\mathcal{N}=(1,0)$ hypermultiplet is described in conventional $6D, \mathcal{N}=(1,0)$ superspace by the superfields $q^i(x,\theta)$ and their conjugate $\bar{q}_i(x,\theta)$, where $\bar{q}_i=(q^i)^+$ under the constraint

$$D_\alpha^{(i}q^{j)}(x,\theta)=0$$

- On-shell component form of the hypermultiplet

$$q^i(z)=f^i(x)+\theta^{\alpha i}\psi_\alpha(x)$$

  where the scalar field $f^i(x)$ and the spinor field $\psi_\alpha(x)$ satisfy the equations $\Box f^i=0, \partial^{\alpha\beta}\psi_\beta=0$

- The on-shell $\mathcal{N}=(1,0)$ hypermultiplet in six dimensions has 2 bosonic+2 fermionic complex degrees of freedom.

Hypermultiplet in harmonic superspace: off-shell Lagrangian formulation

- Off-shell hypermultiplet is described by the analytic superfield $q^+_A(\zeta, u)$, $D^+_\alpha q^+_A(\zeta, u) = 0$, satisfying the reality condition $\widetilde{(q^{+A})} \equiv q^+_A = \varepsilon_{AB} q^{+B}$. Pauli-Gürsey indices $A, B = 1, 2$
- Off-shell hypermultiplet harmonic superfield contains infinite set of auxiliary fields which vanish on-shell due to the equations of motion

$$D^{++} q^+(\zeta, u) = 0$$

- The equations of motion follow from the action

$$S_q = -\frac{1}{2} \int d\zeta^{(-4)} du \; q^{+A} D^{++} q^+_A$$

Here $d\zeta^{(-4)} = d^6 x d^4 \theta^+$.

The $\mathcal{N}=(1,0)$ non-Abelian vector multiplet in $6D$ conventional superspace

- Gauge covariant derivatives

$$\nabla_{\mathcal{M}} = D_{\mathcal{M}} + \mathcal{A}_{\mathcal{M}}, \quad [\nabla_M, \nabla_N] = T_{MN}{}^{L}\nabla_L + F_{MN}$$

  with $D_{\mathcal{M}} = \{\partial_m, D^i_\alpha\}$ being the flat covariant derivatives and $\mathcal{A}_M$ being the gauge connection taking the values in the Lie algebra of the gauge group.

- The constraints

$$F^{ij}_{\alpha\beta} = 0, \quad \{\nabla^i_\alpha, \nabla^j_\beta\} = -2i\varepsilon^{ij}\nabla_{\alpha\beta}, \quad [\nabla^i_\gamma, \nabla_{\alpha\beta}] = -2i\varepsilon_{\alpha\beta\gamma\delta}W^{i\delta}$$

  Here $W^{i\alpha}$ is the superfield strength obeying the Bianchi identities.

The constraints are solved in the framework of the harmonic superspace

The $\mathcal{N}=(1,0)$ non-Abelian vector multiplet in $6D$, $\mathcal{N}=(1,0)$ harmonic superspace

- Harmonic covariant derivative

$$\nabla^{++} = D^{++} + V^{++}$$

Connection V^{++}, taking the values in the Lie algebra of the gauge group, this is an unconstrained analytic potential of the $6D, \mathcal{N}=(1,0)$ SYM theory.

- On-shell field content: vector $A_{\alpha\beta}$ and spinor $\lambda^{-\alpha}$
- The superfield action of $6D, \mathcal{N}=(1,0)$ SYM theory is written in the form

$$S_{SYM} = \frac{1}{f^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\mathrm{tr}\int d^{14}z du_1 \dots du_n \frac{V^{++}(z,u_1)\dots V^{++}(z,u_n)}{(u_1^+u_2^+)\dots(u_n^+u_1^+)}$$

Here f is the dimensional coupling constant ($[f]=-1$)

- Gauge transformations

$$V^{++\prime} = -ie^{i\lambda}D^{++}e^{-i\lambda} + e^{i\lambda}V^{++}e^{-i\lambda}, \qquad q^{+\prime} = e^{i\lambda}q^+$$

Theory of Abelian vector multiplet coupled to hypermultiplet

- Action

$$S[V^{++},q^{+}] = \frac{1}{4f^2}\int d^{14}z\,\frac{du_1du_2}{(u_1^+u_2^+)^2}V^{++}(z,u_1)V^{++}(z,u_2)$$

$$-\int d\zeta^{-4}du\tilde{q}^{+}\nabla^{++}q^{+}$$

- Dimensional coupling constant f $([f]=-1)$
- Harmonic covariant derivative

$$\nabla^{++} = D^{++} + iV^{++}$$

- Equations of motion

$$\frac{1}{2f^2}F^{++} - i\tilde{q}^{+}q^{+} = 0\,, \qquad \nabla^{++}q^{+} = 0\,.$$

$F^{++} = (D^{+})^4V^{--}$, $D^{++}V^{--} - D^{--}V^{++} = 0$

Aim: construction of gauge invariant effective action
Realization

- Splitting the superfields V^{++}, q^{+} into the sum of the background superfields V^{++}, Q^{+} and the quantum superfields v^{++}, q^{+}

$$V^{++} \to V^{++} + f v^{++}, \qquad q^{+} \to Q^{+} + q^{+}$$

- Expanding the action in a power series in quantum fields. As a result, we obtain the initial action $S[V^{++}, q^{+}]$ as a functional $\tilde{S}[v^{++}, q^{+}; V^{++}, Q^{+}]$ of background superfields and quantum superfields.
- Gauge fixing conditions are imposed only on quantum field v^{++}.
- Appropriate gauge fixing function

$$\mathcal{F}^{(+4)} = D^{++} v^{++}.$$

- Using the Faddev-Popov procedure. Ones obtain the effective action $\Gamma[V^{++}, Q^{+}]$ which is gauge invariant under the classical gauge tansformations.

One-loop approximation

- $\Gamma[V^{++},Q^+] = S[V^{++},Q^+] + \Gamma^{(1)}[V^{++},Q^+]$

$$e^{i\Gamma^{(1)}[V^{++},Q^+]} = \int \mathcal{D}v^{++}\mathcal{D}q^+\mathcal{D}\tilde{q}^+\, e^{iS_2[v^{++},q^+;V^{++},Q^+]}$$

- S_2 is a quadratic part of the action $\tilde{S}$
-

$$S_2 = \frac{1}{4}\int d\zeta^{(-4)}du v^{++}\,\widehat{\Box}\,v^{++}$$

$$-\int d\zeta^{(-4)}du\{\tilde{q}^+\nabla^{++}q^+ + f\tilde{Q}^+iv^{++}q^+ + f\tilde{q}^+iv^{++}Q^+\}$$

where

$$\widehat{\Box} = \Box + iW^{+a}\nabla^-_a + iF^{++}\nabla^{--} - \frac{i}{4}(D^-_\alpha W^{+\alpha})$$

$$\nabla^{--} = D^{--} + iV^{--}$$

One-loop approximation

Action S_2 mixes the quantum fields v^{++} and q^+. They can be decoupled by some change of variables in path integral and we can integrate over independent variables.

One-loop correction in effective action is expressed as follows

$$\Gamma^{(1)}[V^{++},Q^+] = \frac{i}{2}\mathrm{Tr}\,\ln\left\{\Box - 4f^2\widetilde{Q}^+G^{(1,1)}Q^+\right\} + i\mathrm{Tr}\,\ln\nabla^{++}$$

where $G^{(1,1)}$ is a hypermultiplet propagator in the background field V^{++} and the Tr is a functional traces in analytic subspace of harmonic superspace.

Problem of calculation of $\Gamma^{(1)}[V^{++},Q^+]$

Structure of the one-loop counterterms

Model

$$S[V^{++}, q^{+}] = \frac{1}{4f^2} \int d^{14}z \, \frac{du_1 du_2}{(u_1^+ u_2^+)^2} V^{++}(z, u_1) V^{++}(z, u_2) - \int d\zeta^{-4} du \tilde{q}^{+} \nabla^{++} q^{+}$$

Quantum perturbation theory for effective action can be completely constructed in superspace terms. Supergraphs.

- Propagators

  Vector multiplet propagator

$$G^{(2,2)}(1|2) = -2 \frac{(D_1^+)^4}{\Box_1} \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_2)$$

  Hypermultiplet propagator

$$G^{(1,1)}(1|2) = \frac{(D_1^+)^4 (D_2^+)^4}{\Box_1} \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}$$

- Vertex

$$-i\tilde{q}^{+} V^{++} q^{+}$$

Structure of the one-loop counterterms

$\delta^{14}(z_1 - z_2) = \delta^6(x_1 - x_2)\delta^8(\theta_1 - \theta_2)$

Superficial degree of divergence ω-total degree in momenta in loop integral.

- Consider the L loop supergraph G with P propagators, V vertices, N_Q external hypermultiplet legs, and an arbitrary vector multiplet external legs.
- One can prove that due to the Grassmann delta-functions in the propagators, any supergraph for effective action contains only a single integral over $d^8\theta$ (non-renormalization theorem).
- Mass dimensions: $[x] = -1$, $[p] = 1$, $[\int d^6p] = 6$, $[\theta] = -\frac{1}{2}$, $[\int d^8\theta] = 4$, $[q^+] = 1$, $[V^{++}] = 0$.
- After summing all dimensions and using some identities, power counting gives

$$\omega(G) = 2L - N_Q$$

- A number of space-time derivatives in the counterterms increases with L. The theory is multiplicatively non-renormalizable.
- One loop approximation

$$\omega_{1-loop}(G) = 2 - N_Q$$

- The possible divergences correspond to $\omega_{1-loop} = 2$ and $\omega_{1-loop} = 0$

Expectable result on the based of dimensions, gauge invariance and $\mathcal{N}=(1,0)$ supersymmetry (G. Bossard, E. Ivanov, A. Smilga, 2015)

$$\Gamma^{(1)}_{div} = \int d\zeta^{(-4)} du \Big\{ c_1 (F^{++})^2 + i c_2 \widetilde{Q}^+ F^{++} Q^+ + c_3 (\widetilde{Q}^+ Q^+)^2 \Big\}$$

Analysis on the base of superficial degree of divergence ω.

- $\omega_{1-loop} = 2$

$$\Gamma^{(1)}_1 \sim \int d^{14}z \, du \, V^{--} \Box V^{++} = \int d\zeta^{(-4)} \, du \, (F^{++})^2$$

- $\omega_{1-loop} = 0$

$$\Gamma^{(1)}_2 \sim \int d^{14}z \, du \, \widetilde{Q}^+ V^{--} Q^+ = \int d\zeta^{(-4)} \, du \, \widetilde{Q}^+ F^{++} Q^+$$

Term $(\widetilde{Q}^+ Q^+)^2$ is prohibited in one-loop approximation

Explicit calculations of the one-loop divergences

$$\Gamma^{(1)}[V^{++}, Q^+] = \frac{i}{2}\mathrm{Tr}\,\ln\left\{\Box - 4f^2\widetilde{Q}^+ G^{(1,1)} Q^+\right\} + i\mathrm{Tr}\,\ln\nabla^{++}$$

$(F^{++})^2$ part of effective action is concentrated only in

$$\Gamma^{(1)}_{F^2}[V^{++}] = i\mathrm{Tr}\,\ln\nabla^{++} = -i\mathrm{Tr}\,\ln G^{(1,1)}$$

$G^{(1,1)}$ is a hypermultiplet propagator in the background field V^{++}.
Scheme of calculations

- The $\mathrm{Tr}\,\ln\nabla^{++}$ is ill defined because of chiral anomaly. However, its variation is well defined.
- This variation is calculated in the framework of superfield proper-time technique which preserve manifest supersymmetry and gauge invariance
- Restoration of divergent part of $\mathrm{Tr}\,\ln\nabla^{++}$ on the base of its variation. Result

$$\Gamma^{(1)}_{F^2}[V^{++}] = -\frac{1}{6(4\pi)^3\varepsilon}\int d\zeta^{(-4)} du\,(F^{++})^2$$

$\varepsilon = 6 - d$

Divergent part of one-loop effective action

The hypermultiplet dependent part of effective action is concentrated only in

$$\frac{i}{2}\mathrm{Tr}\ \ln\left\{\Box - 4f^2\widetilde{Q}^+G^{(1,1)}Q^+\right\}$$

Scheme of calculations

- Propagator in background field

$$G^{(1,1)}(1|2) = \frac{(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4}{\widehat{\Box}_1}\frac{\delta^{14}(z_1 - z_2)}{(u_1^+u_2^+)^3}$$

- Supercovariant d'Alembertian

$$\widehat{\Box} = \Box + iW^{+a}\nabla_a^- + iF^{++}\nabla^{--} - \frac{i}{4}(D_\alpha^- W^{+\alpha}) = \Box + iF^{++}\nabla^{--} + ...$$

- Expansion of $\mathrm{Tr}\ \ln\left\{\Box - 4f^2\widetilde{Q}^+G^{(1,1)}Q^+\right\}$ up to first order.
- Expansion of $\widehat{\Box} = \Box + iF^{++}\nabla^{--} + ...$ in the denominator of propagator up to first order
- Result

$$\Gamma^{(1)}_{QFQ}[V^{++}, Q^+] = \frac{2if^2}{(4\pi)^3\varepsilon}\int d\zeta^{(-4)}du\,\widetilde{Q}^+F^{++}Q^+$$

Divergent part of one-loop effective action

Final result for one-loop divergences

$$\Gamma^{(1)}_{div}[V^{++}, Q^{+}] = -\frac{1}{6(4\pi)^3\varepsilon}\int d\zeta^{(-4)}du\left\{(F^{++})^2 - 12\,if^2\widetilde{Q}^{+}F^{++}Q^{+}\right\}$$

- In the vector multiplet sector, the divergent part of the effective action is proportional to the classical equation of motion $F^{++} = 0$. Therefore the divergence as a whole can be eliminated by a field redefinition $(\delta V^{++} \sim \frac{1}{\varepsilon}F^{++})$ in the classical action and the theory under consideration is one-loop finite on-shell.
- If the background hypermultiplet does not vanish, we obtain, after some field redefinition proportional to the equation of motion, the divergent part of effective action in the form $\Gamma^{(1)}_{div} \sim \frac{1}{\varepsilon}\int d\zeta^{(-4)}du(\widetilde{Q}^{+}Q^{+})^2$. Thus the divergence in the hypermultiplet sector cannot be eliminated and the full theory is not on-shell finite at the one-loop level.
- Situation is completely analogous to one in Einstein quantum gravity with matter

Summary

- The six-dimensional $\mathcal{N} = (1,0)$ supersymmetric theory of the Abelian vector multiplet coupled to hypermultiplet in the $6D$, $\mathcal{N} = (1,0)$ harmonic superspace was formulated
- Background field method in harmonic superspace was constructed .
- Manifestly supersymmetric and gauge invariant effective action, depending both on vector multiplet anf hypermultiplet superfields, was defined.
- Superficial degree of divergence is evaluated and structure of one-loop counterterms was calculated.
- An efficient manifestly gauge invariant and $\mathcal{N} = (1,0)$ supersymmetric technique to calculate the one-loop effective action was developed. As an application of this technique, we found the one-loop divergences of the theory under consideration.
- It was shown that, if the background hypermultiplet superfield does not vanish, the one-loop divergences cannot be eliminated by any field redefinition and the theory is not one-loop finite on-shell.

Open problems

- To study the structure of the effective action for the non-Abelian $6D$, $\mathcal{N} = (1, 0)$ SYM theories. All such theories admit a formulation in $6D$, $\mathcal{N} = (1, 0)$ harmonic superspace. The background field method can be developed quite analogously to the Abelian case and the one-loop divergences can be calculated. The basic difference from the Abelian theory considered here will be a self-interaction of vector multiplet and a non-trivial ghost contribution to the effective action, which can change the relative coefficient between the $(F^{++})^2$ and the $\widetilde{Q}^+ F^{++} Q^+$ terms in the one-loop divergent part. We expect that the purely hypermultiplet contribution to the divergent part of the one-loop effective action in non-Abelian theory will be absent as in the Abelian theory.
- To study the finite contributions to low-energy effective action, which have never been considered before. To construct the Heisenberg-Euler type effective Lagrangian.

Outlook

- To study the effective action in non-Abelian $6D, \mathcal{N} = (1,1)$ SYM theory. Such a theory can be formulated in $6D, \mathcal{N} = (1,0)$ harmonic superspace in terms of $\mathcal{N} = (1,0)$ analytic harmonic superfields, where the gauge connection V^{++} and the hypermultiplet $q^+, \tilde{q}^+$, both in the adjoint representation. This theory exhibits the manifest off-shell $\mathcal{N} = (1,0)$ supersymmetry and an additional hidden on-shell $\mathcal{N} = (0,1)$ supersymmetry, and in many aspects is analogous to $4D, \mathcal{N} = 4$ SYM theory. It is expected, that $\mathcal{N} = (1,1)$ SYM theory is completely one-loop finite.

THANK YOU VERY MUCH!