Super No-Scale Models

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16 September 2016

Based on works done in collaboration with Costas Kounnas
arXiv:1511.02709, 1607.01767

Corfu Summer Institute :
Recent Developments in Strings and Gravity
Ioanina, 13 – 17 September 2016

Outline

1. Main ideas
2. Example of $\mathcal{N} = 4 \to 0$ super no-scale model
3. Moduli deformations
4. $\mathcal{N} = 2 \to 0$ and $\mathcal{N} = 1 \to 0$ super no-scale models

Main ideas

- Strings unify gravity and gauge interactions at the quantum level.

- Particle physics : Start with classical 4D Minkowski space + implement perturbation theory to derive quantum physics.

- But from gravity point of view : Cosmological constant generated by quantum loops.
  - Except if susy : perturbative $\Lambda = 0$
  - If not susy at all : $\Lambda = \mathcal{O}(M_s^4)$

- Intermediate situation : **No-Scale Models** [Cremmer, Ferrara, Kounnas, Nanopoulos,'83]
  - At tree level : susy spontaneously broken + Minkowski space
  - Potential $\mathcal{V}_{\text{tree}} \geq 0$ and admits $m_{3/2}$ as a modulus : flat direction

• Magnitude of the 1-loop effective potential $\mathcal{V}_{\text{1-loop}}$ is dictated by $m_{3/2}$.
For small $m_{3/2}$, does $\mathcal{V}_{\text{1-loop}}$ admit a small expectation value ?

• Generically, **NO** : runaway behavior of $m_{3/2}$, tadpole for dilaton and other moduli, destabilized, magnitude of $\mathcal{V}_{\text{1-loop}}$ still too large,...

• To find a loophole, we consider a context where all computations can be done explicitly, in perturbation theory :

- **Heterotic string**
- **Coordinate Dependent Compactification**
  = "stringy Scherk-Schwarz mechanism", to break spontaneously susy and gauge symmetry : [Rhom,'84] [Kounnas, Porrati,'88] [Kounnas, Rostand,'90]

$$m_{3/2} = \frac{M_s}{R}$$

where R is the characteristic size of the compact space involved in the susy breaking

• Taking R large, to have $m_{3/2}$ and hopefully $|\mathcal{V}_{\text{1-loop}}|$ small,

$\Longrightarrow$ **Light towers of KK modes** : They **dominate in $\mathcal{V}_{\text{1-loop}}$**.

• In an $\mathcal{N} = 4, 2, 1 \to 0$ No-Scale Model,
choose a point in classical moduli space $\langle G_{IJ} \rangle$, $\langle B_{IJ} \rangle$, $\langle$Wilson lines$\rangle$,...

• **Suppose there are no scales between 0 and $m_{3/2}$.**

——— cM_s : large Higgs, GUT or string scale

——— $m_{3/2}$: towers of KK modes of mass $\propto m_{3/2}$

——— 0 : n_B massless bosons and n_F massless fermions

$$\mathcal{V}_{\text{1-loop}} = \xi(n_F - n_B) m_{3/2}^4 + \mathcal{O}(e^{-cM_s/m_{3/2}}) \,, \qquad \xi > 0$$

$$\mathcal{V}_{\text{1-loop}} = \xi(n_F - n_B)m_{3/2}^4 + \mathcal{O}(e^{-cM_s/m_{3/2}})$$

• **Other contexts:** In the No-Scale Model in Supergravity, $\mathcal{V}_{\text{1-loop}}$ contains

$$\mathrm{Str}\, M^0\, \Lambda^4_{\text{cut-off}} = 0 \qquad\qquad (\Lambda_{\text{cut-off}} = M_s)$$

$$\mathrm{Str}\, M^2\, \Lambda^2_{\text{cut-off}} = 2\, Q^2\, m^2_{3/2}\, \Lambda^2_{\text{cut-off}}$$

- $Q^2(z, \bar{z})$ can be nonzero if $\mathcal{N} = 2, 1 \to 0$.
- At low energy, keeping only relevant operators

$$Q^2\Lambda^2_{\text{cut-off}} \to cst\Lambda^2_{\text{cut-off}} \Longrightarrow \partial^2_\phi = 0 \ : \ \text{No gauge hierarchy problem}$$

$\mathcal{L}_{sugra} \to \mathcal{L}_{susy} + \mathcal{L}_{soft}$, MSSM-like [Barbieri, Ferrara, Savoy,'82] [Cremmer, Fayet, Girardello,'83]

Bottom-Up : In MSSM, impose $m_{3/2} \sim m_Z$ by hand, e.g. for the electroweak radiative breaking to take place.

Up to Bottom : In sugra, $m_{3/2}$ is a field $\Longrightarrow$ minimize $\mathcal{V}_{\text{1-loop}}$

If $\boldsymbol{Q \neq 0}$, with subdominant $\boldsymbol{m_{3/2}^4}$:

$$\langle m_{3/2} \rangle = 0 : \text{exact susy} \quad \text{or} \quad \langle m_{3/2} \rangle \sim \Lambda_{\text{cut-off}} : \text{ hard breaking}$$

$\Longrightarrow$ **L**arge **H**ierarchy **C**ompatible (LHC) no-scale models :

$$Q(z, \bar{z}) \equiv 0$$

[Kounnas, Ferrara, Zwirner,'94]

Compute gauge radiative corrections in MSSM, with dynamical $m_{3/2}$, in presence of leftover 1-loop cosmological term $m_{3/2}^4$

$\Longrightarrow$ Stabilization of $m_{3/2}$ and electroweak radiative breaking.

[Kounnas, Pavel, Zwirner,'94]

But: Cosmological constant still big.

• In our case, LHC automatically.

$$\mathcal{V}_{\text{1-loop}} = \xi(n_F - n_B)m_{3/2}^4 + \mathcal{O}(e^{-cM_s/m_{3/2}})$$

A large $\boldsymbol{M_s^2\, m_{3/2}^2}$ would arise if we would keep a finite number of states below M_s in the loop.

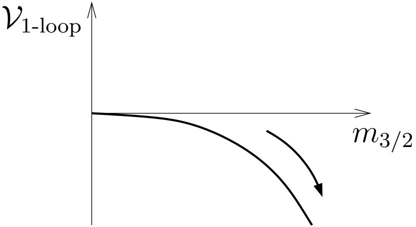
Here, we keep the contribution of **infinite towers of KK modes.**

This would work in a KK field theory (not string), where susy is spontaneously broken by the Sherk-Schwarz mechanism.

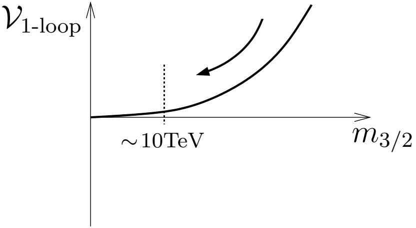
• We could apply the LHC stabilization picture, but we are worried about the large remaining cosmological term.

$$\mathcal{V}_{\text{1-loop}} = \xi(n_F - n_B)m_{3/2}^4 + \mathcal{O}(e^{-cM_s/m_{3/2}})$$

$n_F < n_B$



$n_F > n_B$



$$\langle m_{3/2}\rangle^4 \gg \Lambda_{obs}$$

$\Longrightarrow$ We cannot really decouple gravity to end up with an effective theory in flat space.

$\Longrightarrow$ Define **"Super No-Scale Models" in string theory** :

$\boldsymbol{n_F = n_B}$ when $m_{3/2} < cM_s$. But susy spontaneously broken !

[Abel, Dienes, Mavroudi,'15] [Kounnas, H.P.,'15] [Florakis, Rizos,'16]

$\Longrightarrow$ Standard Model needs hidden sector

• General case : **Some scales may be lower than $m_{3/2}$.**

• Switch on small deviations collectively denoted Y, to $\langle G_{IJ}\rangle$, $\langle B_{IJ}\rangle$, $\langle$Wilson lines$\rangle$,...

cM_s : large Higgs, GUT or string scale

$m_{3/2}$: towers of KK modes of mass $\propto m_{3/2}$

YM_s : some of the $n_B + n_F$ states get a Higgs mass YM_s

0

• **$n_B(Y)$ and $n_F(Y)$ interpolate between different integer values**, reached at distinct points in moduli space.

$\Longrightarrow$ **Expand in Y**

- In the particular case of $\boldsymbol{\mathcal{N}=4 \to \mathcal{N}=0}$

$$\mathcal{V}_{\text{1-loop}} = \xi(n_F - n_B)m_{3/2}^4 - b\,\tilde{\xi}\,m_{3/2}^2(YM_s)^2 + \cdots + \mathcal{O}(M_s^4\,e^{-cM_s/m_{3/2}})$$

- The Y's are **Wilson lines of the gauge group factors.**
- The b's are their **β-function coefficients.** $\tilde{\xi} > 0$.
  - $b < 0 \Longrightarrow$ **Y stabilized at 0**
  - $b > 0 \Longrightarrow$ **Instability**
  - $b = 0 \Longrightarrow$ **Massless** [Kounnas, H.P.,'15]

- The **Stable Super No-Scale Models** satisfy

$$\mathcal{V}_{\text{1-loop}} = \mathcal{O}(e^{-cM_s/m_{3/2}})$$

$$m_{3/2} \simeq 10 \text{ TeV} \quad \Longrightarrow \quad |\mathcal{V}_{\text{1-loop}}| \simeq e^{-10^{13}}$$

$$|\mathcal{V}_{\text{1-loop}}| \simeq 10^{-120} \quad \Longrightarrow \quad m_{3/2} \simeq 10^{-2} M_s$$

They extend the notion of No-Scale Models at the 1-loop level :

$\boldsymbol{\mathcal{V}_{\text{1-loop}} \geq 0}$ and $\boldsymbol{m_{3/2}}$ **is a flat direction,**

as long as $m_{3/2}$ is small enough

• Note that in Type II and orientifold theories, there exist non-susy models with

$$\mathcal{V}_{\text{1-loop}} = 0 \qquad i.e. \qquad N_F = N_B \text{ are any mass level !}$$

[Kachru, Kumar, Silverstein,'98] [Harvey,'98]
[Shiu, Tye,'98] [Blumenhagen, Gorlich,'98]
[Angelantonj, Antoniadis, Forger,'99]
[Satoh, Sugawara, Wada,'15]

When obtained *via* spontaneous breaking of susy, they are super no-scale models in a "strong sense".

• However

- $\mathcal{V}_{\text{2-loops}}$ has no reason to vanish. [Aoki, D'Hoker, Phong,'03]
- When a perturbative heterotic dual is known, it is a conventional super no-scale models : $n_F = n_B$.
[Harvey,'98] [Angelantonj, Antoniadis, Forger,'99]

Outline

Example of $\mathcal{N}=4 \to 0$ super no-scale model

• Start from $\mathcal{N}=4$, $E_8 \times E_8$ heterotic string on $T^2 \times T^2 \times T^2$:

$$Z = \frac{1}{\tau_2 \eta^2 \bar{\eta}^2} \; \frac{\Gamma^{(1)}}{\eta^2 \bar{\eta}^2} \; \frac{\Gamma^{(2)}}{\eta^2 \bar{\eta}^2} \; \frac{\Gamma^{(3)}}{\eta^2 \bar{\eta}^2} \; (V_8 - S_8) \; \bar{E}_8 \; \bar{E}_8$$

where the left-moving worldsheet fermions contribute

$$V_8 - S_8 = \sum_{a,b=0}^{1} (-1)^{a+b+ab} \, \frac{\theta\begin{bmatrix} a \\ b \end{bmatrix}^4}{\eta^4}$$

and the lattice is modular invariant

$$\Gamma^{(1)} = \sum_{\substack{m_1, m_2 \\ n_1, n_2}} q^{\frac{1}{2}|p_L|^2} \bar{q}^{\frac{1}{2}|p_R|^2} = \frac{\sqrt{\det G}}{\tau_2} \sum_{\substack{\tilde{m}_1, \tilde{m}_2 \\ n_1, n_2}} e^{-\frac{\pi}{\tau_2}(\tilde{m}_i + n_i \tau)(G+B)_{ij}(\tilde{m}_j + n_j \bar{\tau})}$$

• To break susy, couple lattice to the spin structure *via* a modular invariant sign, *e.g.* 1^{st} directon of T^2

$$(-1)^{\tilde{m}_1 a + n_1 b + \tilde{m}_1 n_1}$$

$$(-1)^{\tilde{m}_1 a + n_1 b + \tilde{m}_1 n_1} \quad \Longrightarrow \quad m_1 + \frac{a+n_1}{2} \text{ : shifts the KK masses}$$

- When the first T^2 is large, all massless states have $n_1 = 0$.

$\Longrightarrow$ The massless fermions $(a=1)$ get a KK mass $m_{3/2}$

$\Longrightarrow$ $n_F = 0$. Cannot be super no-scale.

- We need to keep some fermions massless :

$$\bar{E}_8 \equiv \bar{O}_{16} + \bar{S}_{16} = \frac{1}{2} \sum_{\gamma,\delta=0}^{1} \frac{\bar{\theta}\left[{\gamma \atop \delta}\right]^8}{\bar{\eta}^8}, \quad \text{where } \gamma = 0 \Leftrightarrow \bar{O}_{16} \text{ and } \gamma = 1 \Leftrightarrow \bar{S}_{16}.$$

- Insert

$$(-1)^{\tilde{m}_1 (a+\gamma+\gamma') + n_1 (b+\delta+\delta') + \tilde{m}_1 n_1}$$

$$\Longrightarrow \begin{cases} \text{When } \gamma + \gamma' = 0 \text{ or } 2, \text{ nothing changes.} \\ \\ \text{When } \gamma + \gamma' = 1, \text{ roles of Bosons and Fermions reversed.} \end{cases}$$

$$Z = \frac{1}{\tau_2 \eta^2 \bar{\eta}^2} \; \frac{\Gamma^{(2)}}{\eta^2 \bar{\eta}^2} \; \frac{\Gamma^{(3)}}{\eta^2 \bar{\eta}^2} \; \frac{1}{\eta^2 \bar{\eta}^2} \quad \times$$

$$\Big[\; \Gamma^{(1)}\big[{}^{e}_{e}\big] \Big(V_8 (\bar{O}_{16} \bar{O}_{16} + \bar{S}_{16} \bar{S}_{16}) - S_8 (\bar{O}_{16} \bar{S}_{16} + \bar{S}_{16} \bar{O}_{16}) \Big)$$

$$+ \Gamma^{(1)}\big[{}^{e}_{o}\big] \Big(V_8 (\bar{O}_{16} \bar{S}_{16} + \bar{S}_{16} \bar{O}_{16}) - S_8 (\bar{O}_{16} \bar{O}_{16} + \bar{S}_{16} \bar{S}_{16}) \Big)$$

$$+ \Gamma^{(1)}\big[{}^{o}_{e}\big] \Big(O_8 (\bar{V}_{16} \bar{C}_{16} + \bar{C}_{16} \bar{V}_{16}) - C_8 (\bar{V}_{16} \bar{V}_{16} + \bar{C}_{16} \bar{C}_{16}) \Big)$$

$$+ \Gamma^{(1)}\big[{}^{o}_{o}\big] \Big(O_8 (\bar{V}_{16} \bar{V}_{16} + \bar{C}_{16} \bar{C}_{16}) - C_8 (\bar{V}_{16} \bar{C}_{16} + \bar{C}_{16} \bar{V}_{16}) \Big) \Big]$$

where $\Gamma^{(1)}\Big[{}^{\text{parity of winding}}_{\text{parity of momentum}}\Big]$

- $m^2_{3/2} = \dfrac{|U_1|^2 M_s^2}{\mathrm{Im}\, T_1 \, \mathrm{Im}\, U_1}$

where $\boldsymbol{T_1, U_1}$ **are the Kähler and complex structure of the first $\boldsymbol{T^2}$.**

- When $\operatorname{Im} T_1$ is large, the gauge group is

$$U(1)^2 \times G^{(2)} \times G^{(3)} \times SO(16) \times SO(16)$$

- The massless spectrum satisfies

$$n_B = 8\Big(244 + \dim G^{(2)} + \dim G^{(3)}\Big) \,, \qquad n_F = 8 \times 256 \,.$$

12 missing bosons are obtained when T_2, U_2 and T_3, U_3 at the enhanced symmetry points

$$G^{(2)} \times G^{(3)} = SU(2)^4 \qquad \text{or} \qquad G^{(2)} \times G^{(3)} = SU(3) \times SU(2) \times U(1)$$

- At these points, the model develops a **super no-scale structure.**

Properties of the model

$$m_{3/2}^2 = \frac{|U_1|^2 M_s^2}{\operatorname{Im} T_1 \operatorname{Im} U_1}$$

- When $\operatorname{Im} T_1 > 1, \operatorname{Im} U_1 \sim 1$

$$m_{3/2} < M_s \ , \quad \mathcal{V}_{\text{1-loop}} = \mathcal{O}(e^{-cM_s/m_{3/2}}) \ : \quad \text{super no-scale regime}$$

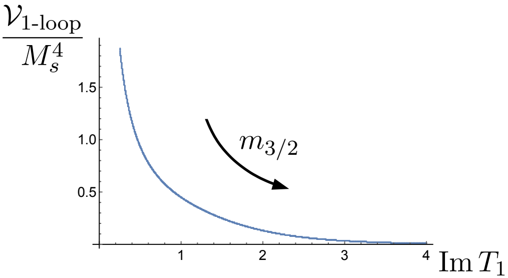
- When $\operatorname{Im} T_1$ decreases up to ~ 1

$$m_{3/2} \sim M_s \ , \qquad \mathcal{V}_{\text{1-loop}} \text{ is not small.}$$

Do we have an Hagedorn-like divergence of $\mathcal{V}_{\text{1-loop}}$?

- In $(-1)^{\tilde{m}_1 a}$ breaking, **YES :** $\quad O_8 \bar{O}_{16} \bar{O}_{16} \Longrightarrow$ Tachyons
- In $(-1)^{\tilde{m}_1 (a+\gamma+\gamma')}$ breaking, **NO:** $O_8 \bar{V}_{16} \bar{V}_{16} \Longrightarrow$ Non-level matched

• For $\mathrm{Re}\, T_1 = 0$ and
$U_1 = T_2 = U_2 = T_3 = U_3 = i$:



• When $\mathrm{Im}\, T_1 \to 0$, the first T^2 shrinks,
which is equivalent to a dual theory in 6D, explicitly non susy.

So, when $m_{3/2} > M_s$, the model is better interpreted as a compactification of this non-susy theory down to 4 dimensions.

• The model is self-dual under

$$(T_1, U_1) \longrightarrow \left(-\frac{1}{U_1}, -\frac{1}{T_1}\right)$$

So, evolving T_1 from 0 (non susy) to $i\infty$(super no-scale)
$\Longleftrightarrow$ evolving U_1 from $i\infty$ (super no-scale) to 0 (non susy).

Outline

Moduli deformations

- The $\mathcal{N}=4$ **models** have gauge groups of rank $6+16$.

- The moduli are the 6 scalars of the $\mathcal{N}=4$ vector multiplets associated to these Cartan $U(1)^{6+16}$

$$\frac{SO(6,6+16)}{SO(6)\times SO(6+16)}$$

- The $\mathcal{N}=4\to 0$ susy breaking does not reduce the rank. Thus, we give a mass to the fermionic part of the $\mathcal{N}=4$ vector multiplets.

- $\Longrightarrow$ The $\mathcal{N}=4\to 0$ **no-scale models have the same moduli space.**

- The $\mathcal{N}=4 \to 0$ **super no-scale model model we have presented is at a point** where
  - $T^2 \times T^2 \times T^2$
  - the first torus is large
  - the last two tori are at enhanced symmetry points where the model is **super no-scale**, *e.g.*

$$U(1)^2 \times SU(2)^4 \times SO(16)^2$$

We switch on arbitrary marginal deformations of the classical theory around this point and compute $\mathcal{V}_{\text{1-loop}}$ to study the local stability :

Is the super no-scale point a minimum, maximum or saddle ?

- Compute $$\mathcal{V}_{\text{1-loop}} = -\frac{M_s^4}{(2\pi)^4} \int_{\mathcal{F}} \frac{d^2\tau}{2\tau_2^2}\, Z(\text{moduli})$$

• **In the undeformed background,**
for any **massless state s_0 of fermion number F_0,**
Z gets contributions from

- the KK tower of modes with even momenta and fermion number F_0.

- the KK tower of their superpartners with odd momenta
($\Longrightarrow$ mass shift $m_{3/2}$).

$$(-1)^{F_0}\sum_{m_1,m_2}(-1)^{m_1}\, e^{-\pi\tau_2\frac{|U_1 m_1-m_2|^2}{\mathrm{Im}\,T_1\,\mathrm{Im}\,U_1}}=(-1)^{F_0}\,\frac{\mathrm{Im}\,T_1}{\tau_2}\sum_{\tilde m_1,\tilde m_2}e^{-\frac{\pi\,\mathrm{Im}\,T_1}{\tau_2\,\mathrm{Im}\,U_1}|\tilde m_1+\frac{1}{2}+U_1\tilde m_2|^2}$$

$$\Longrightarrow\quad \int_{\mathcal{F}} d^2\tau\,(\,\cdots\,)=\int_{-1/2}^{1/2}d\tau_1\int_0^{+\infty}d\tau_2\,(\,\cdots\,)+\mathcal{O}(e^{-\mathrm{Im}\,T_1})$$

- Only the level matched states contribute

- The super massive states (windings, oscillators) yield $\mathcal{O}(e^{-c\sqrt{\mathrm{Im}\,T_1}})$

• Only the **pure KK modes along the large $\boldsymbol{T^2}$** contribute substantially

$$\mathcal{V}_{\text{1-loop}} = -\frac{M_s^4}{(2\pi)^4} \sum_{s_0=1}^{n_B+n_F} (-1)^{F_0} \int_0^{+\infty} \frac{d\tau_2}{2\tau_2^3} \sum_{m_1,m_2} (-1)^{m_1}\, e^{-\pi\tau_2 M_L^2} + \mathcal{O}(e^{-c\sqrt{\operatorname{Im} T_1}})$$

- Same result in a pure KK field theory, when susy is spontaneously broken by the usual Sherk-Schwarz mechanism.

- **It is UV finite.** As is the case when finite temperature is switched on in a susy field theory. No $\Lambda^2_{\text{cut-off}}$ divergence.

• When we switch on small background deformations, only M_L varies.

- All worldsheet marginal operators are

$$Y_{IJ}\,\partial X^I \bar{\partial} X^J\,,\; I,J = 1,\ldots,6$$

i.e. Metric and antisymmetric tensor of T^6

$$Y_{I\alpha}\,\partial X^I \bar{\partial}\phi^\alpha\,,\; \alpha = 1,\ldots,16$$

i.e. Wilson lines of $SO(16)^2$ along T^6

- The deformed KK masses are

$$M_L^2 = P_I\,G^{IJ}P_J$$

where $$P_I = m_I + Y_{I\alpha}\,Q^\alpha + \tfrac{1}{2}Y_{I\alpha}\,Y_{J\alpha}\,n_J + (B+G)_{IJ}\,n_J$$

and $\boldsymbol{Q^\alpha}$ are the charges of the state s_0 under $SO(16)^2$:

In the Singlet, Adjoint or Spinorial representation.

- Compute $\mathcal{V}_{\text{1-loop}}$ and expand at order Y^2, around the point

$$G = \prod_{i=1}^{8} G_i = U(1)^2 \times SU(2)^4 \times SO(16)^2$$

Redefine the Wilson lines Y's

$$Y^{(i)}_{IA} \qquad I = 1, \dots, 6\,, \quad A = 1, \dots, \text{rk}\, G_i$$

- For **any $\mathcal{N} = 4 \to 0$ no-scale model**, super or not :

$$\mathcal{V}_{\text{1-loop}} = \xi'(n_F - n_B) m'^4_{3/2} - \tilde{\xi}\, m^2_{3/2} \sum_{i=1}^{8} b_i \sum_{I=1}^{6} \sum_{A=1}^{\text{rk}\, G_i} (Y^{(i)}_{IA} M_s)^2 + \cdots + \mathcal{O}(M_s^4\, e^{-cM_s/m_{3/2}})$$

where $\quad m'^2_{3/2} = \dfrac{|U'_1|^2 M_s^2}{\text{Im}\, T'_1\, \text{Im}\, U'_1}\,, \qquad \xi' = \dfrac{(\text{Im}\, U'_1)^2}{16\pi^7\, |U'_1|^4}\, E_{(1,0)}(U'_1|3)$

$$E_{(1,0)}(U|s) = \sum_{\tilde{m}_1, \tilde{m}_2} \frac{(\text{Im}\, U)^s}{\left|\tilde{m}_1 + \frac{1}{2} + \tilde{m}_2 U\right|^{2s}}\,, \quad \tilde{\xi} = \frac{3\, \text{Im}\, U_1}{16\pi^5\, |U_1|^2}\, E_{(1,0)}(U_1|2)$$

$$\mathcal{V}_{\text{1-loop}} = \xi'(n_F - n_B)m'^4_{3/2} - \tilde{\xi}\, m^2_{3/2} \sum_{i=1}^{8} b_i \sum_{I=1}^{6} \sum_{A=1}^{\text{rk}\,G_i} (Y^{(i)}_{IA} M_s)^2 + \cdots + \mathcal{O}(M_s^4\, e^{-cM_s/m_{3/2}})$$

• T'_1, U'_1 and thus $m'_{3/2}$ involve the deformations of the metric that break the $T^2 \times T^4$ factorization.

• The β-function coefficients arise from the fact that at quadratic order, the Wilson lines are dressed by charges.

Sum over all states s_0 *i.e.* charges $\Longrightarrow b_i$.

- $b_{U(1)} = 0 \quad \Longrightarrow$ **Massless**
- $b_{SU(2)} = -\frac{8}{3} \times 2 < 0 \quad \Longrightarrow$ **Moduli stabilized at the origin.**
- $b_{SO(16)} = +\frac{8}{3} \times 2 > 0 \quad \Longrightarrow$ **Tachyonic :** They condense and break $SO(16)^2$ to subgroups with negative or vanishing b's and total rank 16.

Stabilization

• The super no-scale models can be considered in a **cosmological scenario.**

• They are all **stable at early times, if finite temperature effects are taken into account.**

• At finite T,

$$\mathcal{V}_{\text{1-loop}} \quad \longrightarrow \quad \text{free energy}$$

$$(\text{mass})^2 \quad \longrightarrow \quad T^2 + (\text{mass})^2 \quad \Longrightarrow \quad T^2 \pm m_{3/2}^2 \ \text{ for moduli}$$

• As the Universe expands, T decreases :

- **As long as $T > m_{3/2}$:** No tachyons $\Longrightarrow$ the models are **stable.**
- **When T crosses $m_{3/2}$: Higgs phase transitions** take place.

Outline

$\mathcal{N} = 2 \to 0$ super no-scale models

- Consider an $\boldsymbol{\mathcal{N} = 4 \to 0}$ **No-Scale Model** on $T^2 \times T^4$, where susy is broken by Coordinate dependent compactification (Scherk-Schwarz) along the 1^{st} direction of T^2.

- Implement $\mathbb{Z}_2$ **twist on** T^4.

$$\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=2\to 0} = -\frac{M_s^4}{(2\pi)^4} \int_{\mathcal{F}} \frac{d^2\tau}{2\tau_2^2} \left[\underset{untw}{\text{Str}} \frac{1+\mathbb{G}}{2} q^{\frac{1}{4}M_L^2} \bar{q}^{\frac{1}{4}M_R^2} + \underset{twist}{\text{Str}} \frac{1+\mathbb{G}}{2} q^{\frac{1}{4}M_L^2} \bar{q}^{\frac{1}{4}M_R^2} \right]$$

- The modes with $n_1 \neq 0$ are super massive $\Longrightarrow \mathcal{O}(e^{-\text{Im}\, T_1})$.

  For $n_1 = 0$, $\underset{untw}{\text{Str}}\, \mathbb{G}(\cdots) = 0$

$$\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=2\to 0} = \frac{1}{2}\, \mathcal{V}_{\text{1-loop}}^{\mathcal{N}=4\to 0} + \mathcal{V}_{\text{1-loop}}^{\mathcal{N}=2\to 0}\Big|_{twist} + \mathcal{O}(e^{-\text{Im}\, T_1})$$

$\frac{1}{2}\, \mathcal{V}_{\text{1-loop}}^{\mathcal{N}=4\to 0}$ depends on Wilson lines that survive the $\mathbb{Z}_2$ projection *i.e.* the $T^2 \times T^4$ factorization.

$\mathcal{N} = 2 \to 0$ super no-scale models

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- The modes with $n_1 \neq 0$ are super massive $\Longrightarrow \mathcal{O}(e^{-\mathrm{Im}\,T_1})$.

  For $n_1 = 0$, $\underset{untw}{\mathrm{Str}}\,\mathbb{G}(\cdots) = 0$

$$\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=2\to 0} = \frac{1}{2}\,\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=4\to 0} + \mathcal{V}_{\text{1-loop}}^{\mathcal{N}=2\to 0}\Big|_{twist} + \mathcal{O}(e^{-\mathrm{Im}\,T_1})$$

$\frac{1}{2}\,\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=4\to 0}$ depends on Wilson lines that survive the $\mathbb{Z}_2$ projection *i.e.* the $T^2 \times T^4$ factorization.

$$\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=2\to 0}\Big|_{twist} = -\frac{M_s^4}{(2\pi)^4}\int_{\mathcal{F}}\frac{d^2\tau}{2\tau_2^2}\,\underset{twist}{\mathrm{Str}}\,\frac{1+\mathbb{G}}{2}\,q^{\frac{1}{4}M_L^2}\,\bar{q}^{\frac{1}{4}M_R^2}$$

Twisted Hypermultiplets, with a mass splitting $m_{3/2}$ between the bosons and fermions

- charged under gauge group

  $\Longrightarrow$ Their mass depends on untwisted Wilson lines

  $\Longrightarrow$ they modify the Wilson lines stability condition

- may contain at the massless level **twisted moduli**

  $\Longrightarrow$ **Quaternionic manifold** [Work in progress]

- Except if the twisted states are super massive, in which case

$$\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=2\to 0}\Big|_{twist} = \mathcal{O}(e^{-\operatorname{Im} T_1})$$

- Impose $\mathbb{Z}_2$ **twist of T^4** to also **shift 2^{nd} direction of T^2**, which is very large $\Longrightarrow$ $\mathbb{Z}_2^{free}$

$$(X^1, X^2, X^3, X^4, X^5, X^6) \longrightarrow (X^1, X^2 + \frac{1}{2}, -X^3, -X^4, -X^5, -X^6)$$

- The 2 ends of the string are separated by half a perimeter of X^2 $\Longrightarrow$ super massive. They decouple in the large T^2 limit.

$$\text{Spontaneous breaking}: \quad \mathcal{N}=4 \longrightarrow \mathcal{N}=2 \longrightarrow \mathcal{N}=0$$

$$\text{at scales} \qquad \frac{M_s^2}{\operatorname{Im} T_1 \operatorname{Im} U_1}\,, \qquad \frac{|U_1| M_s^2}{\operatorname{Im} T_1 \operatorname{Im} U_1}$$

$$\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=4\to 2\to 0} = \frac{1}{2}\,\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=4\to 0} + \mathcal{O}(e^{-\operatorname{Im} T_1})$$

- The descendent is super no-scale if the parent is.
- Its stability is dictated by the b's of the $\mathcal{N}=4\to 0$ parent theory.

$\mathcal{N} = 1 \to 0$ super no-scale models

- Consider an $\boldsymbol{\mathcal{N} = 2 \to 0}$ **No-Scale Model**
  - On $T^2 \times T^2 \times T^2$, with $\mathbb{Z}_2$ twists on $T^2 \times T^2$ (free or not)
  - susy is broken by Coordinate dependent compactification (Scherk-Schwarz) along the 1^{st} direction of T^2.
  - **Add** $\mathbb{Z}_2'$ **twists on** $T^2 \times T^2$

$$\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=1\to 0} = -\frac{M_s^4}{(2\pi)^4}\int_{\mathcal{F}} \frac{d^2\tau}{2\tau_2^2}\left[\underset{untw'}{\mathrm{Str}}\, \frac{1+\mathbb{G}'}{2} q^{\frac{1}{4}M_L^2}\bar{q}^{\frac{1}{4}M_R^2} + \underset{twist'}{\mathrm{Str}}\,(\cdots)\right]$$

- *untw'* is the $\mathcal{N} = 2 \to 0$ spectrum

- *twist'* contains states twisted along $T^2 \Longrightarrow$ Vanishing momentum and windings $\Longrightarrow$ masses independent of T_1, U_1 and $m_{3/2} \Longrightarrow$ SUSY

- *twist'* + the conformal block *untw'* with $\mathbb{G}'$ form an $SL(2,\mathbb{Z})$ modular orbit $\Longrightarrow$ *untw'* with $\mathbb{G}'$ vanishes

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$$\mathcal{V}_{\text{1-loop}}^{\mathcal{N}=1\to 0} = -\frac{M_s^4}{(2\pi)^4}\int_{\mathcal{F}}\frac{d^2\tau}{2\tau_2^2}\left[\underset{untw'}{\mathrm{Str}}\,\frac{1+\mathbb{G}'}{2}q^{\frac{1}{4}M_L^2}\bar{q}^{\frac{1}{4}M_R^2} + \cancel{\underset{twist'}{\mathrm{Str}}}(\cdots)\right]$$

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- $untw'$ is the $\mathcal{N} = 2 \to 0$ spectrum

- $twist'$ contains states twisted along $T^2 \implies$ Vanishing momentum and windings $\implies$ masses independent of T_1, U_1 and $m_{3/2} \implies$ SUSY

- $twist'$ + the conformal block $untw'$ with $\mathbb{G}'$ form an $SL(2,\mathbb{Z})$ modular orbit $\implies$ $untw'$ with $\mathbb{G}'$ vanishes

- For all these $\boldsymbol{\mathcal{N}=1\to 0}$ **No-Scale Models**

$$\mathcal{V}^{\mathcal{N}=1\to 0}_{\text{1-loop}} = \frac{1}{2}\,\mathcal{V}^{\mathcal{N}=2\to 0}_{\text{1-loop}}$$

  - It is **Super No-Scale if the parent $\boldsymbol{\mathcal{N}=2\to 0}$ is.**
  - The moduli (and instabilities) are those of the parent model that survive the $\mathbb{Z}_2'$ projection *i.e.* $T^2\times T^2\times T^2$ factorization.

- If $\mathbb{Z}_2^{free}\times\mathbb{Z}_2'$, we can see $\mathbb{Z}_2'$ as a hard breaking of $\mathcal{N}=4$ to $\mathcal{N}=2$ and $\mathbb{Z}_2^{free}$ as a spontaneous breaking of $\mathcal{N}=2\to\mathcal{N}=1$

$$\mathcal{V}^{\mathcal{N}=2\to\mathcal{N}=1\to 0}_{\text{1-loop}} = \frac{1}{4}\,\mathcal{V}^{\mathcal{N}=4\to 0}_{\text{1-loop}}$$

However, not chiral.

• **Morality :** **For $\mathcal{V}_{\text{1-loop}}$, nothing new in the $\mathcal{N} = 1 \to 0$ No-Scale Models ($\mathbb{Z}_2 \times \mathbb{Z}_2'$), compared to $\mathcal{N} = 2 \to 0$ case ($\mathbb{Z}_2$).**

- Because the twisted spectrum under $\mathbb{Z}_2'$ has tree level susy degeneracy $\implies$ contribution to Z at 1-loop vanishes.
- The classical moduli in this sector are still marginal at 1-loop.

• **At 2 loops,** interactions with non susy states (untwisted$'$) will lift the degeneracy

$\implies$ **The moduli in the twisted$'$ sector will have a non-trivial potential at 2-loops.**

Conclusion

• The **Super No-Scale Models** are those which induce an almost vanishing cosmological term at 1-loop

$$\mathcal{V}_{\text{1-loop}} = \xi \underbrace{(n_F - n_B)}_{0} m_{3/2}^4 + \mathcal{O}(e^{-cM_s/m_{3/2}})$$

$\Longrightarrow$ **Bosons - Fermions degeneracy at the massless level.**

• Up to $\mathcal{O}(e^{-cM_s/m_{3/2}})$ terms ($e.g.$ 10^{-120}, $10^{-10^{13}}$) :
No tadpoles for dilaton, $m_{3/2}$, other moduli.

• In the $\boldsymbol{\mathcal{N} = 4 \to 0}$ **case**, their **quantum stability** is guaranteed if there are no Non-Asymptotically Free gauge groups ($b > 0$).

- The $\mathcal{N} = 2 \to 0$ case requires the analysis of the twisted sector.

  [Work in progress]

- The $\mathcal{N} = 1 \to 0$ case requires the analysis at 2-loops.

Or : Quantum stability if finite T is greater than $m_{3/2}$.

- When such a model is stable, it makes sense to
  - decouple gravity to obtain a theory in flat space
  - It is susy + soft terms

- **Question :** Is the effective potential at genus $g \geq 2$ still flat ?

$$\Big(\xi \underbrace{(n_F - n_B)}_{0} + \kappa g_s^2\Big) m_{3/2}^4 \quad \Longrightarrow \quad \textbf{New constraint } \boldsymbol{\kappa = 0} \textbf{ ?}$$