

# Novel String Field Theory Solving String Theory Liberating Left and Right Movers

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# Main Points to Remember:

- We develop a formalism for description of an arbitrary number of strings (String Field Theory) by means of a Hilbert space formally with “objects” that are more like particles ( they are “bits” in a technically a little different way from Thorns).
- In terms of our “objects” (~ bits) the second quantized string world get totally **static, scattering becomes a fake, the scattering amplitude becomes just the overlap of the initial with final state!**
- In terminology of “layers of degree of existence” the **scattering does not belong to the layer of truly existing.**
- We get after some technical trouble the S-matrix = the overlap between initial and final string state to be the Veneziano amplitude, **but with the little need...**
- **That the objects must be able to have energy of both signs.**
- So one piece of a cyclic chain can **cancel another piece completely!**
- And thus pairs of compensating pieces of chains of objects may be **phantazised where-ever it may be.**
- “String” comes ONLY in via the initial (and final state) conditions.

# Our SFT model Equivalent to String Theory

The final SFT-model of mine and Ninomiyas is described by the Fock space for massless non-interacting scalar bosons in 25 +1 dimensions. That is to say described by a Hilbert space which is generated by a series of creation  $a^+(\vec{p})$  and destruction  $a(\vec{p})$  operators for scalar particles with 25-momenta  $\vec{p}$ , which can act successively on a zero-particle state  $|0\rangle$ . That is to say that the typical states in the Fock space - or the Hilbert space describing the world state - are

$$a^+(\vec{p}_1)a^+(\vec{p}_2)\cdots a^+(\vec{p}_n)|0\rangle \quad (1)$$

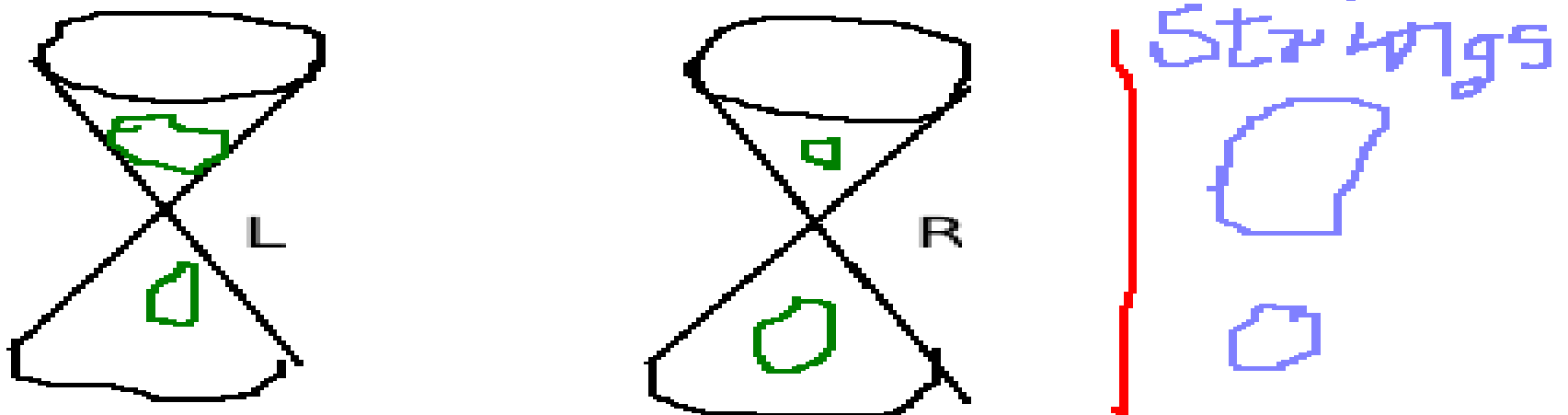
In the language, which we use, we call the scalar particles “even objects” and denote their momenta by  $J^\mu$  instead by  $p^\mu$  and their conjugate momenta by  $\Pi^\mu$  (well really we only consider the conjugate momenta for the transverse components  $i = 1, 2, \dots, 24$ , i.e.  $J^i$ )

# Relation To Charles Thorn's String Bits

- We deviate from the original Charles Thorn idea of describing the string as a chain of string-bits by Thorn using bits corresponding to small pieces of the string  $\sigma$ -variable, while we take the step into bits- which we then to distinguish can call "objects" - instead after, we have split the solution into right and left mover and thus rather put into bits or now to distinguish objects the right-mover variable " $\tau - \sigma$ " or the left mover one " $\tau + \sigma$ ".
- Actually Thorn has begun to do the same as we later at least for fermion modes.

# Detail of Open Versus Closed Strings in Our Model

For Theory with Only Closed Strings:

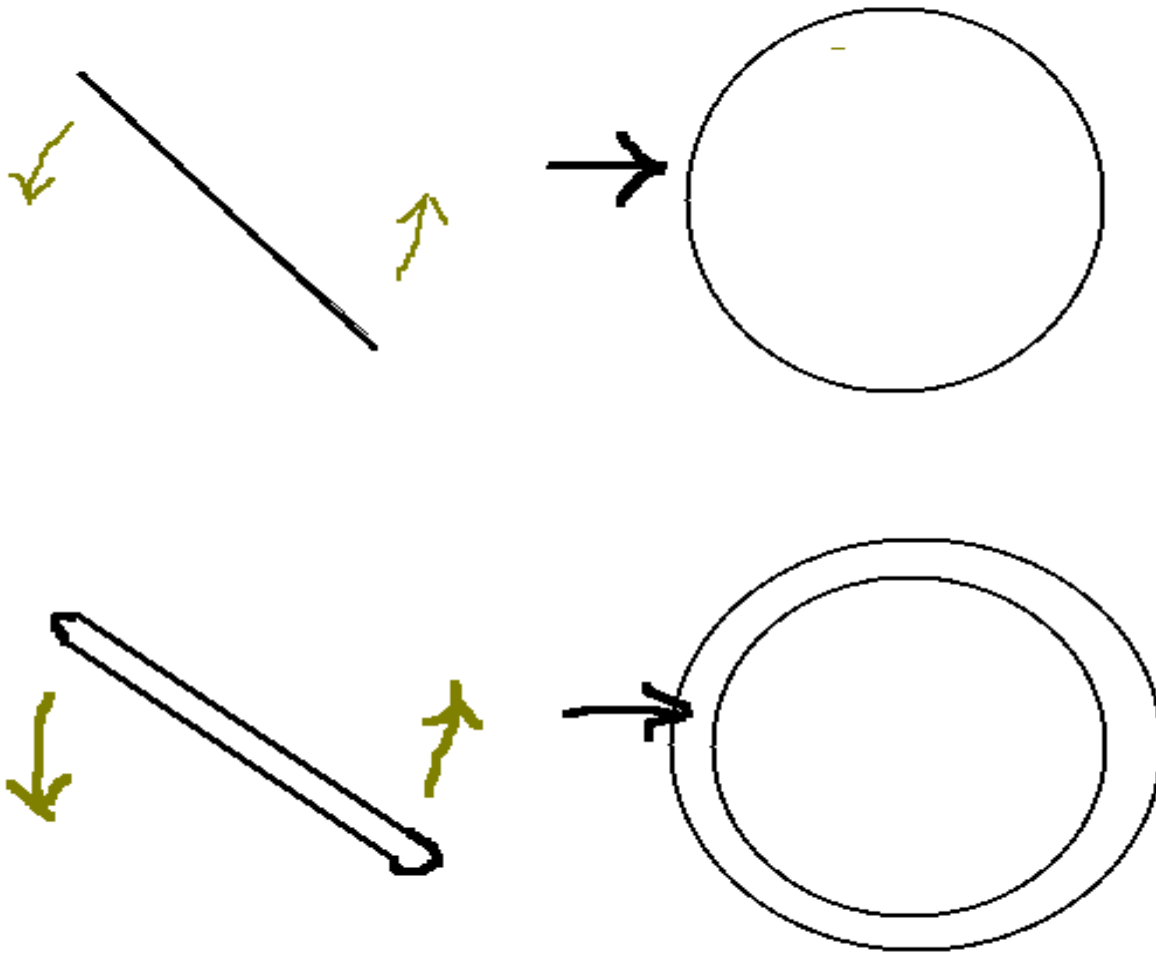


For Theories with also open strings:



Chains of Objects on the Lightcones

# Translation from strings to “Cyclic Chains”



# Translation from Strings to ``Cyclic Chains''

- For illustration of our formulation/model for our novel string field theory you shall imagine drawing in 25 or 26 dimensional perspective the right mover field  $X_R$  differentiated w.r.t. to its variable  $\tau - \sigma$ , thinking classically.
- To each open string right mover derivative is a 26-vector being a periodic function with the periode used for  $\sigma$ . The boundary condition at the edn ensures that right and left mover derivatives are equal for the closed string.

Thus we get to the open string a corresponding topologically circular figure the ``cyclic chain''.

- For a closed string one can both imagine drawing the right and the left mover and they become two in general different closed curve images (= two ``cyclic chains'').

For a single string in the "conformal gauge" we have the wellknown

$$(\partial_\tau^2 - \partial_\sigma^2)X^\mu(\sigma, \tau) = 0, \quad (1)$$

and solve it by the splitting

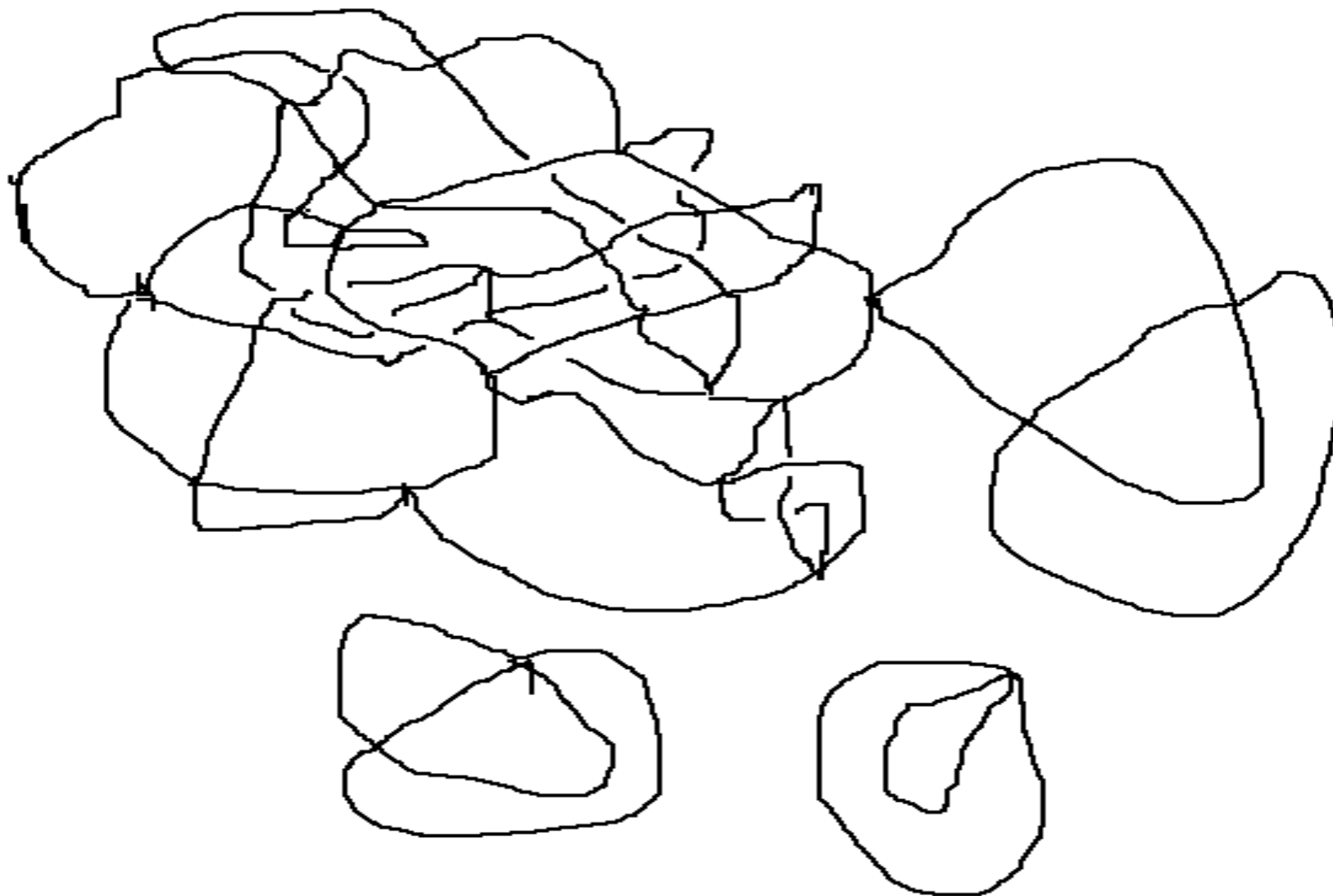
$$X^\mu(\sigma, \tau) = X_R^\mu(\tau - \sigma) + X_L^\mu(\tau + \sigma) = X_R^\mu(\tau_R) + X_L^\mu(\tau_L) \quad (2)$$



As introduction to our Novel String Field Theory we shall imagine - and let us first think classically - that for each string development in time - in Minkowski space - **draw** a to such a moving/oscillating string **corresponding** image of the  $\tau_R = \tau - \sigma$  respective  $\tau_L = \tau + \sigma$  **derivatives**  $\dot{X}_R^\mu(\tau_R)$  and  $\dot{X}_L^\mu(\tau_L)$  of these  $X_R^\mu(\tau_R)$  and  $X_L^\mu(\tau_L)$ .

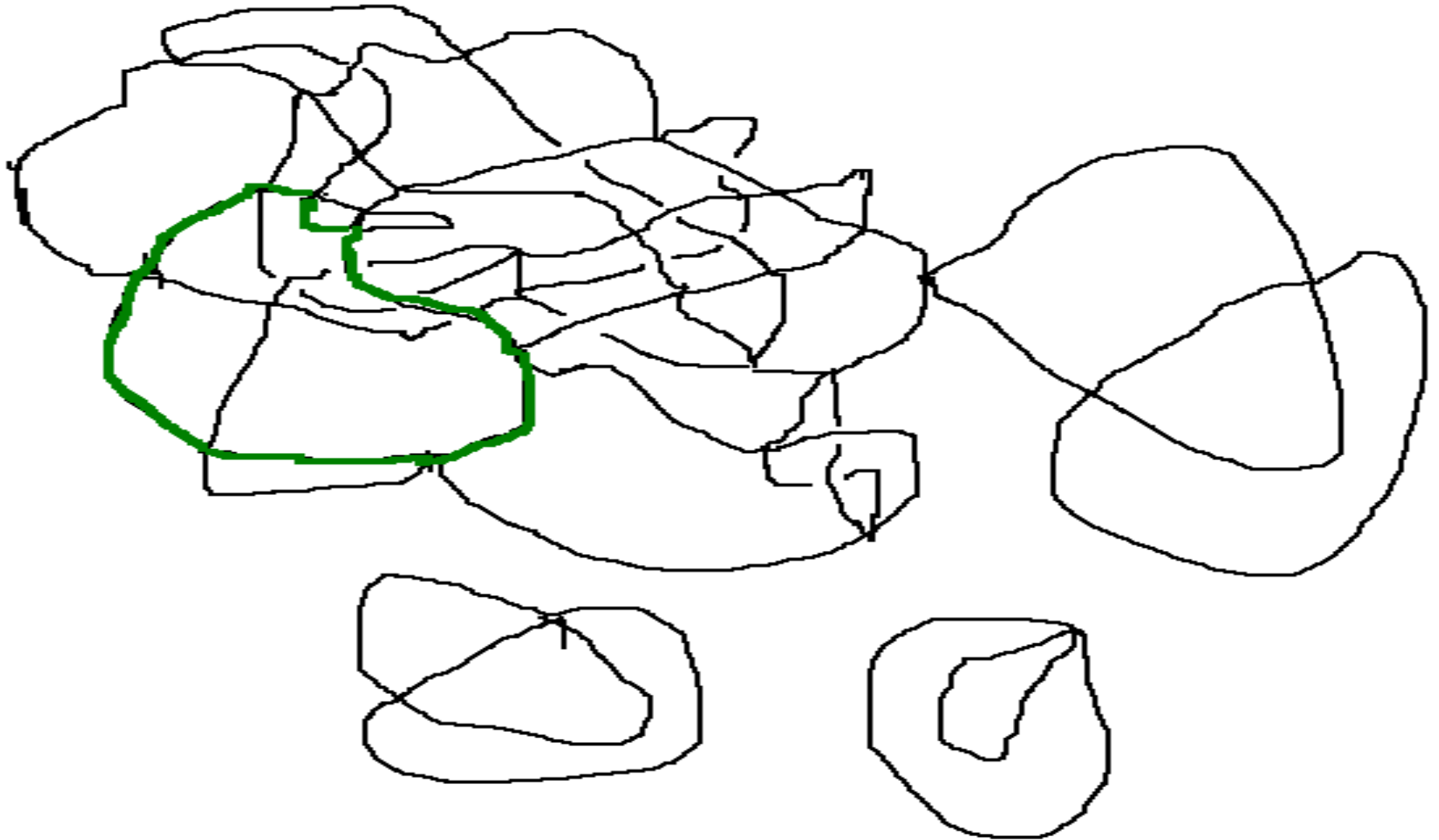
Because of the periodicity for finite size strings the two images of  $X_R$  and  $X_L$  will be closed curves, called "cyclic chains".

Image of all left and right mover  
X-derivatives for all strings:



The meaning of the foregoing figure with the net of curves is, that there is in a perspectively drawn Minkowski space of  $26 = 25 + 1$  dimensions (in bosonic string case) a point on the net each time **some one of all the strings present in Universe** for some value of  $\tau_R = \tau - \sigma$  or  $\tau_L = \tau + \sigma$  the respective derivatives  $\dot{X}_R^\mu(\tau_R)$  or  $\dot{X}_L^\mu(\tau_L)$  of the string space time position field  $X^\mu(\tau, \sigma)$  take their vectorial value equal to that point. This net of curves is thought classically at first: i.e.  $\dot{X}_R^\mu(\tau_R)$  and  $\dot{X}_L^\mu(\tau_L)$  have meaningful vectorial values once the last bit of gauge has been chosen.

One Open string would contribute  
say the green closed curve  
contained in the net



For an open string one has at both end-points (say  $\sigma = 0, 2\pi$ )

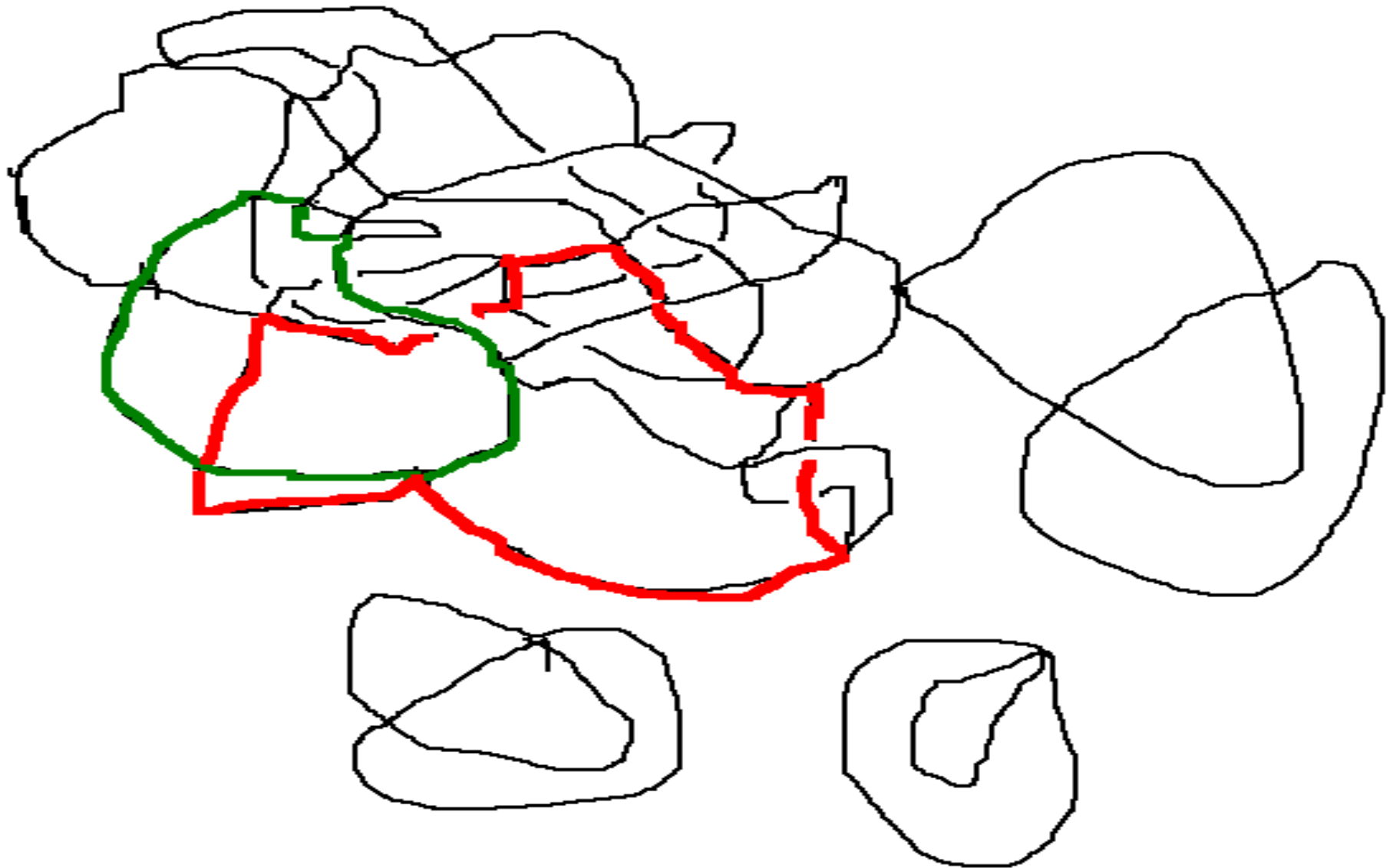
$$X'^{\mu} = -\dot{X}_R^{\mu} + \dot{X}_L^{\mu} = 0. \quad (3)$$

Since this must be true for all  $\tau$ , it is enough information to deduce that for the **open** string

$$\dot{X}_R^{\mu}(\tau) = \dot{X}_L^{\mu}(\tau). \quad (4)$$

Thus at any moment of  $\tau$  there is for each open string a closed circle, a "cyclic chain", of  $\dot{X}_R^{\mu}$  or  $\dot{X}_L^{\mu}$ , so that we get for each open string a closed circle (the cyclic chain) of image points in the (perspectively imagined)  $26 = 25 + 1$  dimensional space on the figure.

We Add one more – now red – Open string Contribution, a Cyclic Chain



# Whole Universe is Described by a Set of Strings in String (Field) Theory



# Comments on Our net of Cyclic Chains Representing String Via their Right and Left mover Derivatives

- The whole set of all the right or left mover derivatives symbolized by the thin-curved network, can of course be interpreted as coming from various cyclic chains associated with various open string, or some pairs of them could correspond to closed strings.
- We illustrated by various colors and thicknesses of the curves, how the image of all the right and left mover derivatives could be divided into contributions e.g. from different open strings. ( some pairs of cyclic chains potentially correspond to just same closed string instead of to two open ones).
- We stress that the division into contributions from different cyclic chains , different open strings say, is **not unique !**



# Further Comments Our Novel String Field Theory Model

- For seeing our model, our novel string field theory, it is the crucial point to imagine that the cyclic chains shown on our figures here in the  $25+1 = 26$  dimensional space(-time) should be thought of as **series of ``objects''** meaning a discretization of the cyclic chains into particle-like objects, that can be created or annihilated by **creation and annihilation operators**. So in this creation and annihilation we already now think quantum mechanically.
- Thus the net of cyclic chains is truly thought as represented by the effect of a lot of actions with creation or annihilation operators on a ``background state''(=vacuum)

# The Rough Dirac Sea

- At first one would be tempted to think of the vacuum or “background state” for the objects as a state in which objects with negative energy (if that makes any sense) were the only ones possible to produce, **but...**
- Even though energy of a single object can be given a meaning, we shall assume that the “back ground state” for the second quantized objects-theory **is not of the simple type that can only be modified to make the sum of the energies of the objects larger! Rather you can also add to it negative energy.**
- This is analogous to what we call the “rough Dirac sea”

# What is the Rough Dirac Sea?

- In a free theory of second quantized fermions it is wellknown that the negative energy single fermion states are filled while the positive energy single fermion states are empty.
- When there are interactions between such fermions or with other fields the ground state when also the interaction energy is included is no longer so simple. The vacuum is in this case rather **a superposition of a lot of states of the second quantized theory, a lot of which have empty single fermion negative energy states, or filled positive single fermion states.**
- This is analogous to that in a peaceful sea there is water for negative height and air for positive height, while....
- In a rough sea there is near height zero almost equal probability for finding water and air.

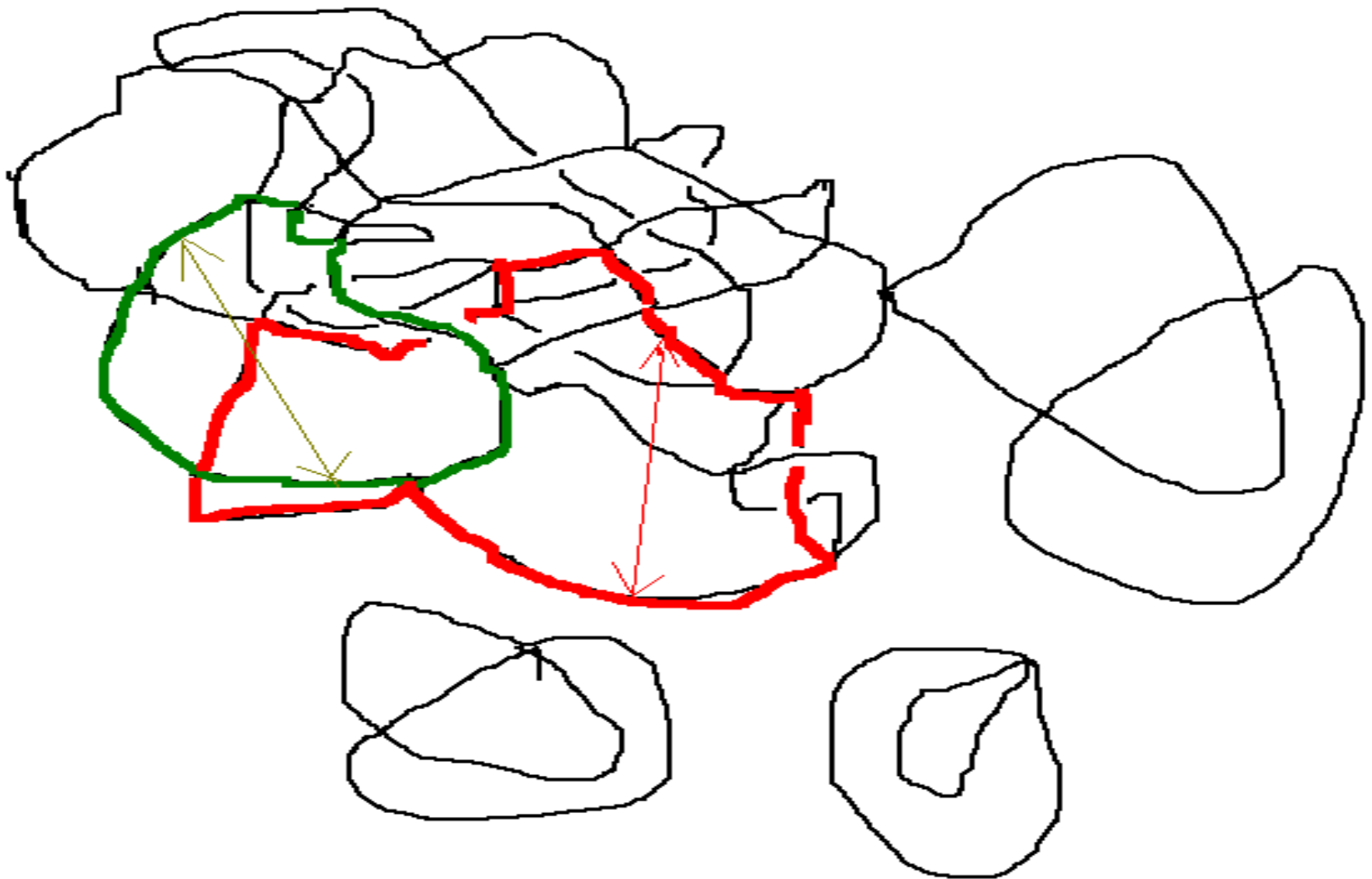
# What is the Rough Dirac Sea ? (Continued)

- So if you act with an annihilation operator for a positive energy single fermion state or with a negative single fermion creation operator on a vacuum **with interaction, then you obtain a state, in which the sum of the single fermion energies (ignoring the interaction) has been lowered.**
- The interacting vacuum is by definition the lowest energy state, when the interaction is included, but it is **not the lowest energy state for the free fermion energy, so the free fermion energy can easily be lowered by some annihilation of a positive energy fermion or creation of a negative energy one.**
- This is analogous to that you could remove a droplet of water from a positive height position from a rough sea; or you could add a droplet in a negative height place, with some slight amount of luck only needed.

# Use of Rough Dirac sea Analogy for Our Novel String Field Theory Background State

- Although we do not have any genuine interaction between the objects in our model, we shall nevertheless imagine that the “background state” on which we act with object-creation and object-annihilation operators is a complicated state, so that it is **not the ground state for the sum of the single object states, so that it is not a problem to act with some annihilation or creation operator so as to add negative free energy, meaning make the sum of the object energies more negative.**

# Our Objects do NOT Describe Everything (?)



# Our Objects do NOT Describe Everything ???

- First the chains of objects – or equivalently (classical) images of the right and left mover derivatives – could have originated from several combinations of open strings say: We could have filled out the network of cyclical chains with cycles corresponding to individual strings in many ways, provided they cross.
- Secondly there could be several ways of declaring pairs of cyclic chains to together correspond to a closed string.  
(remember a closed string corresponds to two cyclic chains).

# Illustration of Open or Closed by “Arrows”

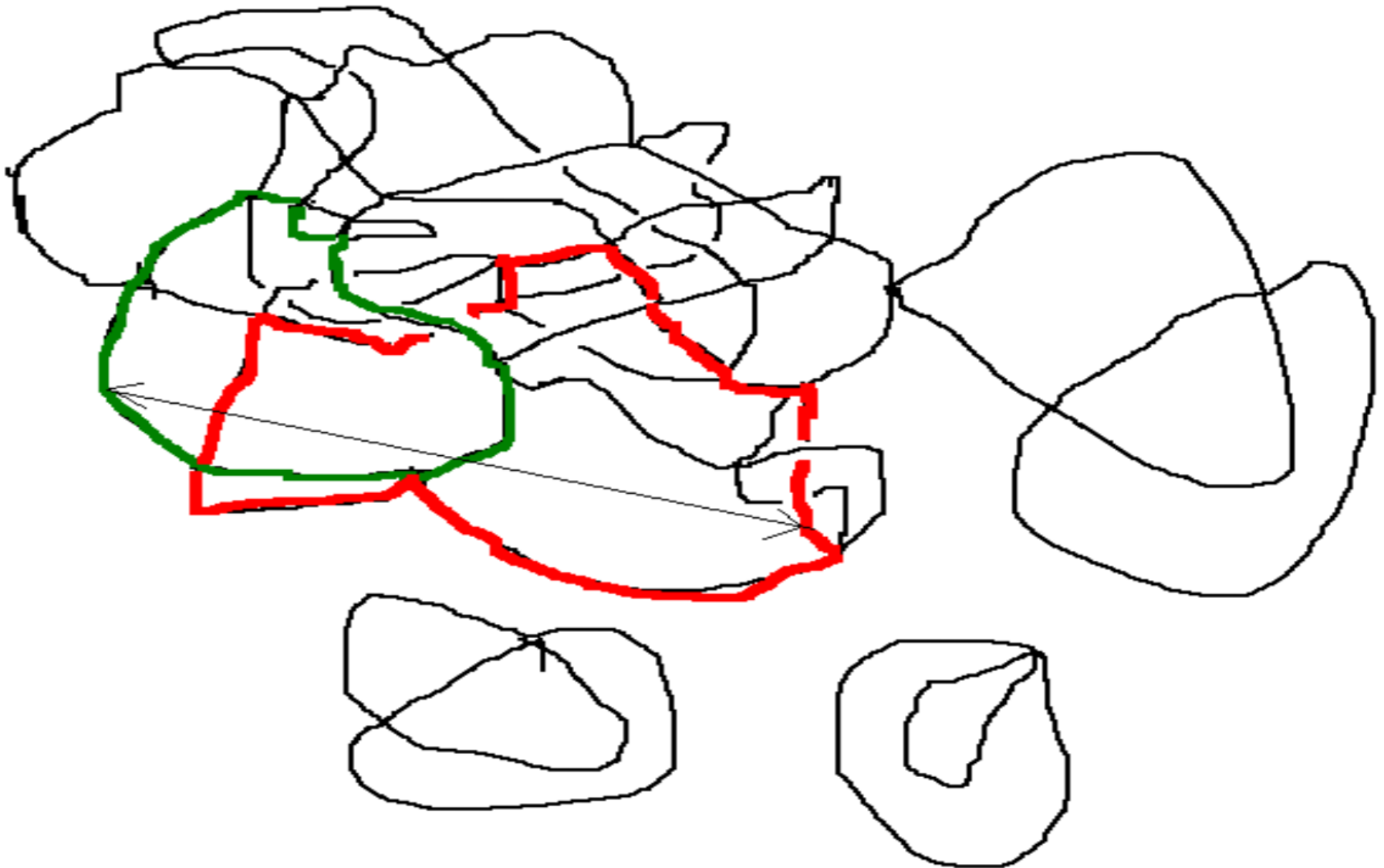
- On the last figure we show putting some arrows, how to construct the points or small pieces of the string time track – i.e. the surface in Minkowski space through which the string in question passes: To each pair of small bits on the cyclical chain corresponds a little area on the space-time track of the string.
- The string time track is a **two-dimensional** manifold and thus one needs **two** one-dimensional parameters to parametrize it. We use for the open string the same cyclic chain as being both parameters (two different point on the cyclic chain), while we for the closed string use two different cyclic chains.



# Illustration of Connection to the String

- We have put in on the picture of the curve a couple of arrows. Such an arrow corresponds to a point on the string time track, or rather one point for each period of the string motion, in the sense that a couple of tangent vectors spanning the tangent to the string time track at the point in question are given by the two points in  $25+1 = 26$  space(-time) at the two ends of the (double)arrow.
- To obtain all the time track points of an open string modulo periodicity you must take all the arrows that can connect two points on the cyclical chain describing the open string in question.
- To obtain those for a closed string you must use all the arrows connecting one point on one of the two cyclic chains to the other one.

# Illustration of Closed String from Cyclical Chains



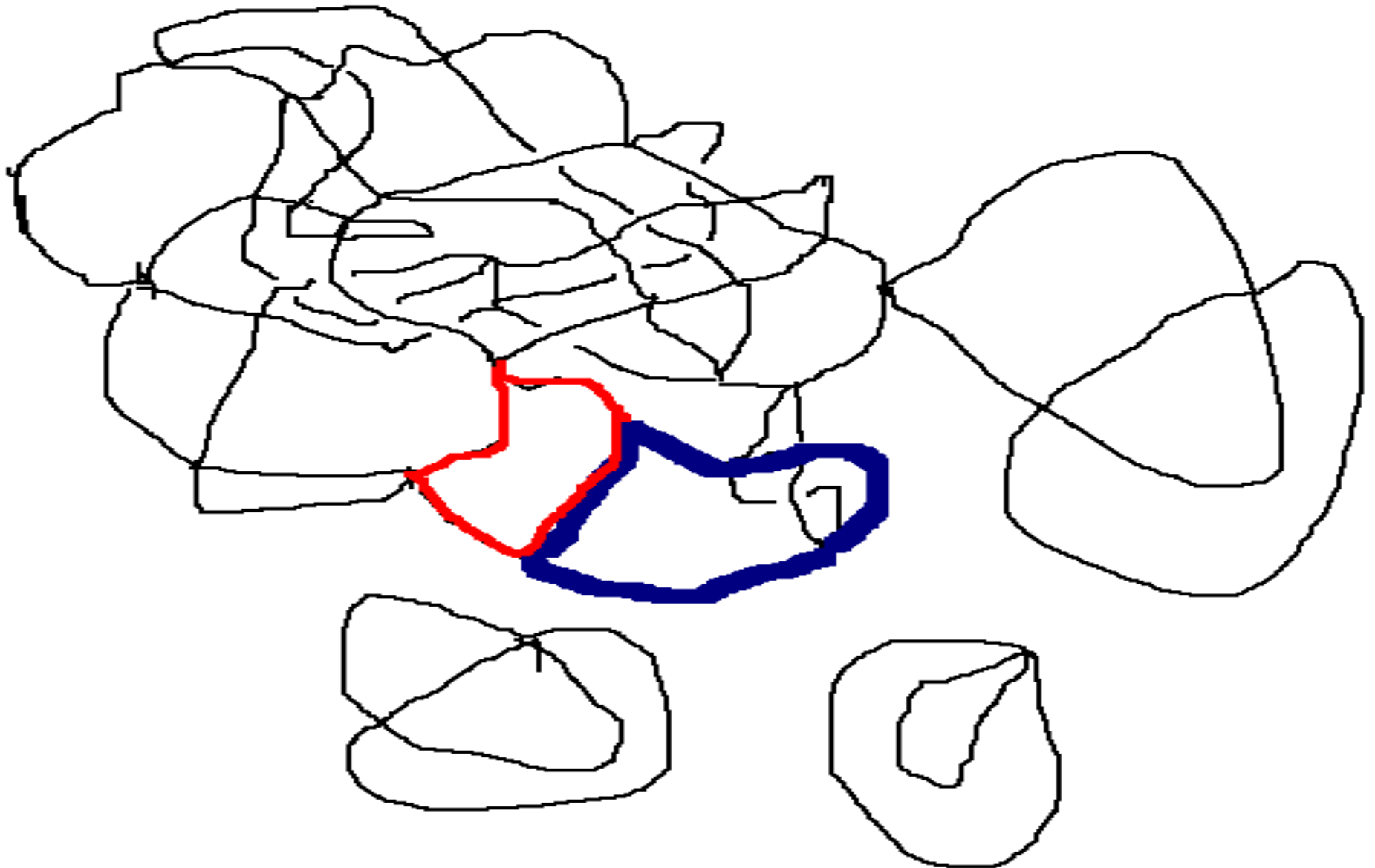
# Comment on Closed String Illustration

- On the figure illustrating the closed string in terms of two cyclical chains you obtain the tangent- basis vectors of the various infinitesimal pieces of the closed string time track by going through all possible arrows connecting one point on the cyclic chain, green, and one on the cyclic chain, red.
- Again we get a two-dimensional time-track of a string – now closed – by having it parametrized by two parameters running along cyclic chains. But for the closed string these two parameters run on two different cyclic chains.

# An Easy to Forget Possibility

- What happens if we first by creation or annihilation operator insert a piece of cyclic chain with one momentum distribution and then add the one with just the opposite one ? Actually they cancel and it becomes as if nothing had been done.
- This opens up a strange possibility for inserting the cyclic chains corresponding to a couple of say open strings: We could let a piece of the cyclic chain corresponding to one of the two strings happen to be just the “opposite” of a piece of the cyclic chain for the other open string.
- In that case inserting the cyclic chains corresponding to the two open strings leads to there being two pieces of cyclic chains canceling each other....
- And thus the final state in our  $25+1 = 26$  dimensional space for “objects” would have got zero objects along the piece of cancellation. And the latter would not be marked in our Hilbert space for second quantized object..
- One would only there “see” the pieces that were NOT canceled.

# Phantasy Curve of Cancellation



# Phantacy Curve of Cancellation

- Having in mind the possibility of canceling two opposite pieces of cyclic chain we can without any trace in our second quantized object-state have two (oppositely oriented but otherwise locally in same state) pieces of the to the blue and the red cyclic chains present.
- On the above figure these canceling pieces are illustrated with the curve where the blue and the red curves follow each other.
- That there along this piece is no sign of the cyclic chains in the object-description is illustrated by there being no thin black curve along this piece before we drew in the red and the blue circles (cyclic chains) elsewhere on top of the thin black curves illustrating the second quantized object-state.

# The Importance of this Cancellation Pieces of Cyclic Chains

- We believe that this inclusion of such pieces of cyclic chains canceling each other so that no track of their existence is left in our description by means of objects can be of help to solve a problem, which we met in our development of our string field theory based on such object description:
- The problem were the following: We sought to calculate from our novel string field theory the scattering of two particles into two others expecting to obtain a Veneziano model with **three terms corresponding to the usual three pairs among the channeles s, t, and u. But ...**
- We got only **one of these terms!**

# Expected Origin of Our Only One Term Problem

- We did the calculation, that turned out in this way unsuccessfully in an infinite momentum frame ``gauge''(=parametrization) choosing the right mover and the left mover coordinates ensuring a fixed amount of the ``longitudinal'' component of the 26-momentum for all objects.
- Thus there were in this ``gauge'' choice no way to have the ``longitudinal'' component of momentum made opposite.
- So there were no way in that ``gauge'' to realize the ``Phantasy or cancellation pieces of cyclic chain''.
- All cyclic chains corresponding to (open) strings would have to be ``visible'' in the second quantized object description.
- But then two strings cannot become one by partial annihilation in the cyclic chain description .



# The Goal is to Only Reformulate Second Quantized String Theory(String Field Theory)

In principle our “Novel String Field Theory” should just be a rewriting of a system of many strings interacting with each other. There should be nothing logically new, only reformulation! Whether we really have logical perfect correspondance (after quantization) depends, however, on how much information we count it that there is in our formalism. trictly speaking we could make “philosophically different versions” of our model, each including different amounts of information in them. Only the one with large amount of information would match usual string theory. But we suggest to take the version with minimal amount of information most serious as our novel string field heory.

# Classical Approximation Summary of “Layers of Existence Degrees”:

But in our formulation, the “Novel String Field Theory” it is we think pedagogical to consider several *layers of truth or existence* corresponding to different versions w.r.t. to including information into the formalism. Let us first describe these “layers of existence” in our *classical* (by classical formulation we have in mind that the single particle states are described classically with both  $\Pi$  and  $J$  having values simultaneously - in disagreement with Heisenberg uncertainty principle - while the objects are still second-quantized, so that a state in superposition of having different numbers of objects is in the picture) formalism, with which we started.

# The Layers of Existence Degrees:

We have a series of steps from truly existing in our novel theory to being more and more phantasy, not really existing:

- 1. Fully existing The system of objects that can have both negative and positive energy - because they sit on a background of the "rough dirac sea" (which is also fully existent, although we avoid having to go in detail formulating it).
- 2. Chaining of Objects into Cyclical Chains From the continuity of the strings and the boundary conditions we have the objects forming cyclically ordered chains with objects sitting with neighbor distances of the order of the "latticification cut off distance". Really we do not take it in our model that this chaining order has any physical existence in itself; but seeing a pattern of the "truly" existing objects with their  $J^\mu$ 's and  $\Pi^\mu$ 's we may let the nearness define for us a chaining. (A wrong way of chaining  $\rightarrow$  bad continuity)

# Resume and Continuation of Series of Layers of Existence

- 1. Fully existing
- 2. Chaining of Objects into Cycles
- 3. Pairing By this “pairing” we mean the information telling, which cyclically ordered chains together corresponds to a string. Open strings come from just one cyclically ordered chain each, while closed strings each need two, but here the total 26-momentum for the two shall be the same. So again knowing the cyclic chains there is some basis/restriction for guessing, which ones to combine.
- 4. Cancelling Pieces The phantasy pieces of opposite 26-momenta just invented.

# Technical Problems in Quatizing Single Particle

- As told: We first attempted a description with the single object – and also single string – being treated classically, but allowing quantum mechanics in the second quantization, so that we could a superposition of even different numbers of objects or strings.
- While in Thorns bits from pieces of sigma these bits have positions commuting with each other, the right-mover part of the position does NOT commute with itself. Rather there is for its derivative – which we want to work with a delta-prime function commutator.

## With Each Even-Object Associated a Set of Variables having Zero Poisson Brackets with Variables for other Even-Objects

If we shall have separate creation and annihilation operators objects in any state, it would at least be a very unwanted complication if the degrees of freedom for one object and another one did not commute. Thinking classically on the single object state we should thus have **zero Poisson bracket between the variables associated with two different objects**. This is, however, **impossible** if we want the objects to represent at least the  $\tau$ -derivative of say the right mover part of the string position field  $\dot{X}_R^\mu(\tau_R)$ , because these rightmover fields or their derivatives for different values of the argument  $\tau_R$  ( $\tau_R$  is usually replaced by a complex variable  $\bar{z}$ ) do **not commute**, do not have zero Poisson bracket.

# The No-Poisson Bracket Inter Objects Problem

Wanting

$$J_R^\mu(\tau_R) \propto \dot{X}_R^\mu, \quad (5)$$

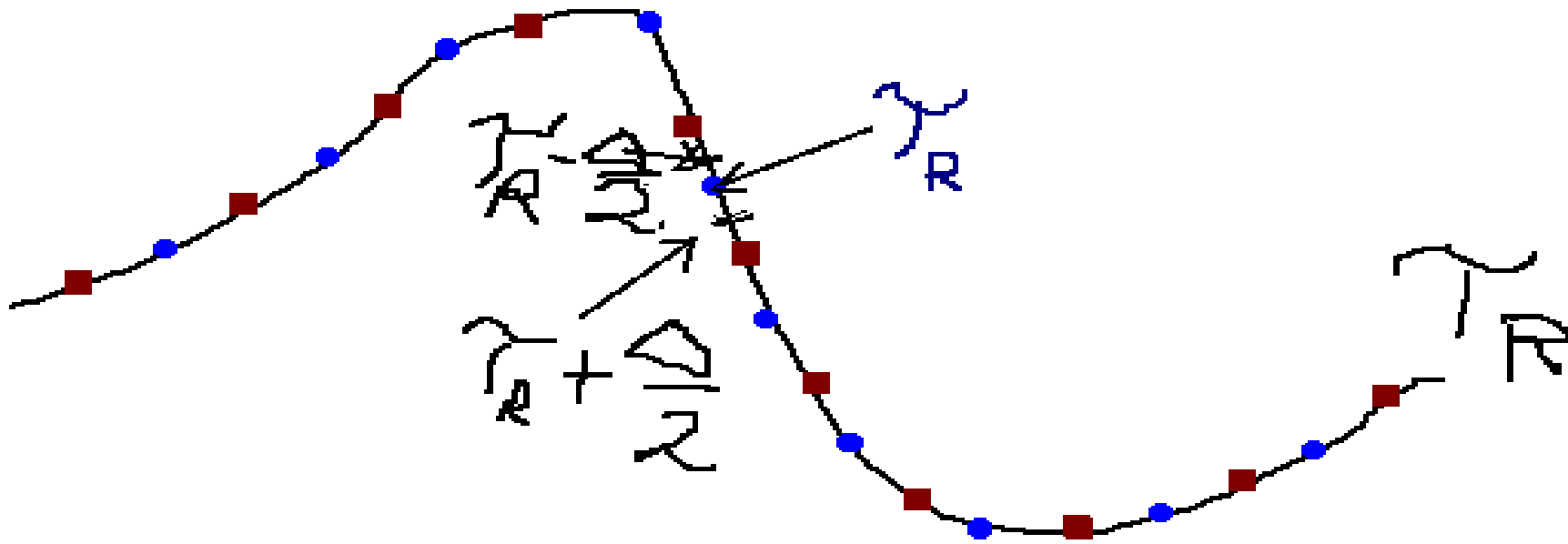
we define

$$J_R^\mu(\text{for interval bit } [\tau_R - \Delta/2, \tau_R + \Delta/2]) = X_R(\tau_R - \Delta/2) - X_R(\tau_R + \Delta/2), \quad (6)$$

where  $\Delta$  is our cut off "length" into "bits" or "objects".

But these  $\dot{X}_R^\mu$ 's do not commute, but rather

$$[\dot{X}_R^\mu(\tau_R), \dot{X}_R^\mu(\tau'_R)] = i\delta'(\tau'_R - \tau) \neq 0 \quad (7)$$



• EVEN

• ODD



# The Delta prime Function $\delta'(\tau'_R - \tau_R)$ Discretize to Only Neighbors Do Not Commute

Replacing the  $\tau_R$  parametrization of right movers - replacing  $\bar{z}$  which were complex - by a discrete counting  $l$  taking *integer* values our discretized object variables

$$J_R^\mu(l) = X_R^\mu(\tau_R(l) + \Delta/2) - X_R^\mu(\tau_R(l) - \Delta/2), \quad (8)$$

where say  $\tau_R(l) = \text{constant} + l * a$ , we have crudely at least

- The even  $l$  object variables  $J_R^\mu(l)$  commute with each other, and
- the odd ones commute with themselves, but..
- The even ones do NOT commute with their TWO ODD neighbors !

This is just describing a discretized delta-prime function.

# Solution to Getting Different Object Sets of Variables Commute(= having Zero Poisson Brackets)

We **only consider the even numbered objects as independent objects**, and let the variables - especially  $J_R$  for the ODD objects be written in terms of the neighboring even object variables:

$$J_R^\mu(l) (\text{for odd } l) = -\alpha' \pi (\Pi^\mu(l+1) - \Pi^\mu(l-1)), \quad (9)$$

the  $\Pi(l \pm 1)$  to be used here are numbered by the even numbers  $l \pm 1$  and thus can be the conjugate momenta of the to the even objects assigned  $J_R^\mu(l \pm 1)$  respectively. Of course the conjugate momentum  $\Pi^\mu(K)$  of  $J_R^\mu(K)$  must be assigned to the *same* object since they do not commute.

## Concept of Integrating Up, say the Odd $J_R^\mu(l)$ 's

We can consider this expression for the odd  $J_R$  as a kind of "integrating up" the odd  $J_r$  to construct / give us the  $\Pi_R$ 's:  
For an even value of the integer  $K$  we solve our proposed equation for the odd  $J_R$ 's

$$\Pi_R^\mu(K) = \frac{J_R^\mu(K-1) + J_R^\mu(K-3) + J_R^\mu(K-5) + \dots}{-\alpha' \pi}. \quad (10)$$

It looks that with this "integrating up" information on the original continuum string variable  $\dot{X}_R^\mu(\tau_R) \sim J_R^\mu$  has been moved away in a non-local way for odd discrete points and is stored as the  $\Pi_R^\mu$  for even argument.

It may be less serious though since  $\dot{X}_R^\mu(\tau_R) \sim J_R^\mu$  were already differentiated, so really the  $\Pi_R^\mu$  becomes essentially the right mover part of the position variable for the string.

# Where is the STRING?

- If our so called string field theory is only a theory of essentially free massless objects, then it is a mystery where is the string ?
- Do we even get the Veneziano model out of it? Yes we do. We actually can calculate to obtain Veneziano model – actually though for an OVERLAP between initial and final state rather than for a complicated S-matrix.

# Main idea in Calculating Veneziano Amplitude

- 1. Since nothing goes on the S-matrix can only be unity and the S-matrix element just an overlap of in and outgoing states  $\langle f|i\rangle$ .
- 2. We write these in or outgoing states by having for each particle a wave function in terms of objects.
- 3. These wave functions are written by means of an IMAGINARY TIME functional integral for a STRING extracting the ground state (of string) by it surviving long imaginary time development.

# Conclusion

- We – Ninomiya and I- develop a new description for an arbitrary number of strings, a string field theory.
- It is formulated in terms of a discretization into pieces – much like Thorn's string bits, but we do it right and left – called by us “objects”.
- These objects have dynamics like free massless particles. In momentum space they are static.
- So nothing happens, even if the strings scatter!

# Conclusion continued

- We have arguments that our model really is a transformation of theory for several strings.
- We derive the Veneziano model, first with some troubles, but having negative even energy for the objects will presumably help to get the full Veneziano model ( we missed two terms first)
- Also spectrum we got o.k., except for a species doubler problem .

# We Just Describe String Theory with Several Strings in Novel Way

- (Apart from null sets) our string field theory should be just a rewriting of usual say string field theory.
- The Hilbert space describing all the possible world states in a string world is the Fock space of -either one or two – theories of massless noninteracting scalars(for the bosonic 25+1 model).
- Two massless free scalar theories/species for purely closed string theory, while only one when there are open strings.
- But allowed states are restricted to obey – approximately?- some “chiral” invariant continuity condition: this means that the stringyness only



# Some Motivational Thoughts

- We – Ninomiya and I – think we have a new(novel) way of representing string theory, which because of being in some respects simpler could be helpful in understanding some aspects of string theory better.
- Even if string theory should not turn out to be the final truth – as can still be the case – its abilities for providing a cut off are so good that alone in looking for cut off it may have inspiration.
- It happens generally thinking of seeking a cut off you easily get in the direction of the string theory, especially the aspect of not having any true interaction as is a trademark for Ninomias and my model.

# String Field Theory Deviating from Earlier Ones by having Thrown Out Information

- Our – Ninomiyas and mine - novel field theory deviates from usual ones – Kaku Kikkawas or Wittens by including (a nul set of) of information less in its description of state of the world, i.e. of a set of strings present.
- We have rewritten the information – the kept part – on a state of several strings into a state of something (more like particles), which we call “objects”, to such a degree that one only sees the connection to genuine strings by quite a bit of complicated rewriting.

# Our Novel SFT has character of a Solution of String Theory

- Our formulation in terms of the “objects” has the character of being a system of parameters describing the development of a system of strings, since:
- These “object”-parameters essentially do not change at all.
- The reconstruction of the strings involves integrating the object parameters up and even contain ambiguities (corresponding to that we left out some information).

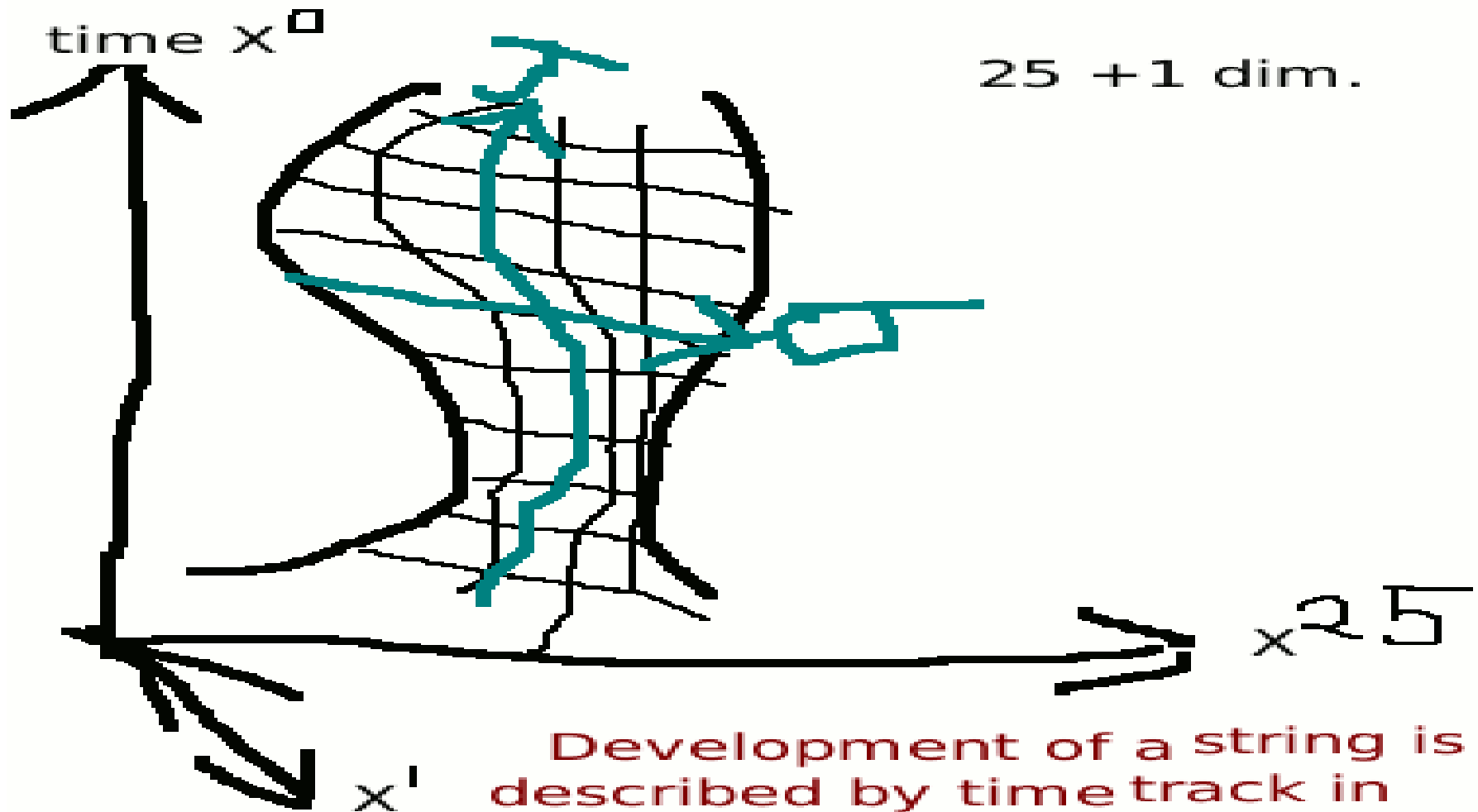
# Recent Victory: Veneziano Model comes out of our Novel String Field Theory

- Veneziano Model is obtained by putting up - translated to our ``objects'' - the state for the incoming set of strings and analogously for the outgoing set. Then the S-matrix of the Veneziano model appears as the overlap of these two states in the Hilbert space (``Fock space'') for the ``objects''.
- The technical calculation quickly comes to remind strongly about a string scattering amplitude, but we should keep in mind that what I shall calculate in the seminar is only an overlap of two state-vectors in the ``object'' formalism.

# Our Throw Away of Information

- If two (open) strings cross, four pieces of string meet in one point.
- We throw away – before describing the situation in our model – the information about which pieces among these 4 are connected to form the strings with which of them.
- So we only keep the information as to where you find some string or the other (but do not keep which string it may be)
- The individuality of the separate strings is gone.

# Single String Description by Parametrized Surface in Minkowski Space



25 + 1 dim.

Development of a string is described by time track in Minkowski space 'time.

Running point on time track  $X(\sigma, \tau)$ .

# History , About Me and String

- The String Theory started with the Veneziano Model, but Veneziano knew nothing about that it were a string theory, he were about to make.
- It were then independently Nambu Susskind and myself (H.B. Nielsen) that found out that it were truly a theory of strings, that could deliver just the Veneziano model,
- Here included also the generalized Veneziano models for scattering of more than just four external particles – as several physicist had made, among which also Koba and myself.

# Important to Test the New Formalism for the Veneziano Model

$$B(-\alpha(u), -\alpha(t)) \quad (1)$$

where

$$\alpha(t) = \alpha' * t + \alpha(0) \quad (2)$$

(linear Regge trajectories), and where the Mandelstam variables as  $s = (p_1 + p_2)^2 = (p_3 + p_4)^2$ ,  $t = (p_1 - p_4)^2 = (p_2 - p_3)^2$ ,  $u = (p_1 - p_3)^2 = (p_2 - p_4)^2$  and obey

$$s + t + u = 4m^2 \quad (3)$$

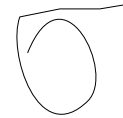
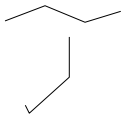
where  $m$  is the mass of the external particles. If as in our model in the present form one has no "quark"-marks on the end of the open strings, the 4-point scattering amplitude has actually three terms so that it is rather proportional to

$$B(-\alpha(s), -\alpha(t)) + B(-\alpha(u), -\alpha(t)) + B(-\alpha(u), -\alpha(s)). \quad (4)$$



# Reminding Single String Theory

- First fix the main part of the “gauge choice” in the sense of fixing parametrization in  $\tau$  and  $\sigma$



to obey the conformal gauge choice.

- Next one can solve the equations of motion for the point of string position variable  $X$  by writing it as a sum of right and left mover parts.
- This solves it because these left and right movers only depend on one component of the two coordinates each.

# Left and Right Mover Parts

Ninomiya and mine model might be thought of as “liberating” left and right movers obtained in the conformal gauge formulation of a single string in which the Lagrangian density is taken as  $\frac{\dot{X}^2 - X'^2}{2\pi\alpha'}$ , rather than as in the Nambu-action  $\frac{1}{2\pi\alpha'} * \sqrt{\dot{X}^2 X'^2 - (X' \cdot \dot{X})^2}$ ,

$$X^\mu(\sigma, \tau) = X_R^\mu(\tau - \sigma) + X_L^\mu(\tau + \sigma) \quad (5)$$

For our model the tau derivatives of left and right movers important

Our main variables considered are the time, i.e.  $\tau$  derivatives of the right and left movers:

$$\dot{X}_R^\mu(\tau_R) \text{ and } \dot{X}_L^\mu(\tau_L)$$

(where

$$\tau_R = \tau - \sigma$$

$$\tau_L = \tau + \sigma$$

# Important properties of the time/tau derivatives of left and right movers:

- The constraints just take the form of them being lightlike.
- We have a theorem about their images being conserved, even under scattering! (except for a nulset).
- They represent so many conserved degrees of freedom that we can say they “solved string theory”
- Except for perhaps some integration constants and nulsets one should be able to integrate them up and obtain almost the state of the string ( in fact we shall get it for several strings in our picture)

# “Object” J equal difference of Discrete X’s, So Derivative

In mine and Ninomias SFT our “objects” are not simply constituents - although it looks that we could essentially look at them as so -. Rather these “objects” are connected with the left and right mover parts of the position field on the string:

$$X^\mu(\sigma, \tau) = X_R^\mu(\tau - \sigma) + X_L^\mu(\tau + \sigma). \quad (10)$$

Here the string time track is parametrized with a parameter  $\sigma$  along the string, and essentially a time  $\tau$ . Such a parametrization of the string time track can be made in infinitely many ways. A class of parametrizations is selected by “conformal gauge choice”. With such a special choice you can solve for the position field  $X^\mu(\sigma, \tau)$  by the above equation in terms of “right mover”  $X_R^\mu$  and “left mover”  $X_L^\mu$ , each depending on only ONE variable.

# “Objects” are Differences of Right or Left Mover Position X Part

In Mine and Ninomiyas SFT we rather define our “objects”  $J^\mu$  in terms of the “right”  $X_R(\tau - \sigma)$  and “left mover” components  $X_L^\mu(\tau + \sigma)$  of the position field  $X^\mu(\sigma, \tau) = X_R^\mu(\tau - \sigma) + X_L(\tau + \sigma)$  in the conformal gauge. In fact we put for the  $J$ 's corresponding to a small (discretizing) intervals  $[\tau_R(l - 1/2), \tau_R(l + 1/2)]$  and  $[\tau_L(l - 1/2), \tau_L(l + 1/2)]$  for the  $l$ th discretized pieces

$$J_R^\mu(l) = X_R^\mu(\tau_R(l + 1/2)) - X_R(\tau_R(l - 1/2)) \quad (11)$$

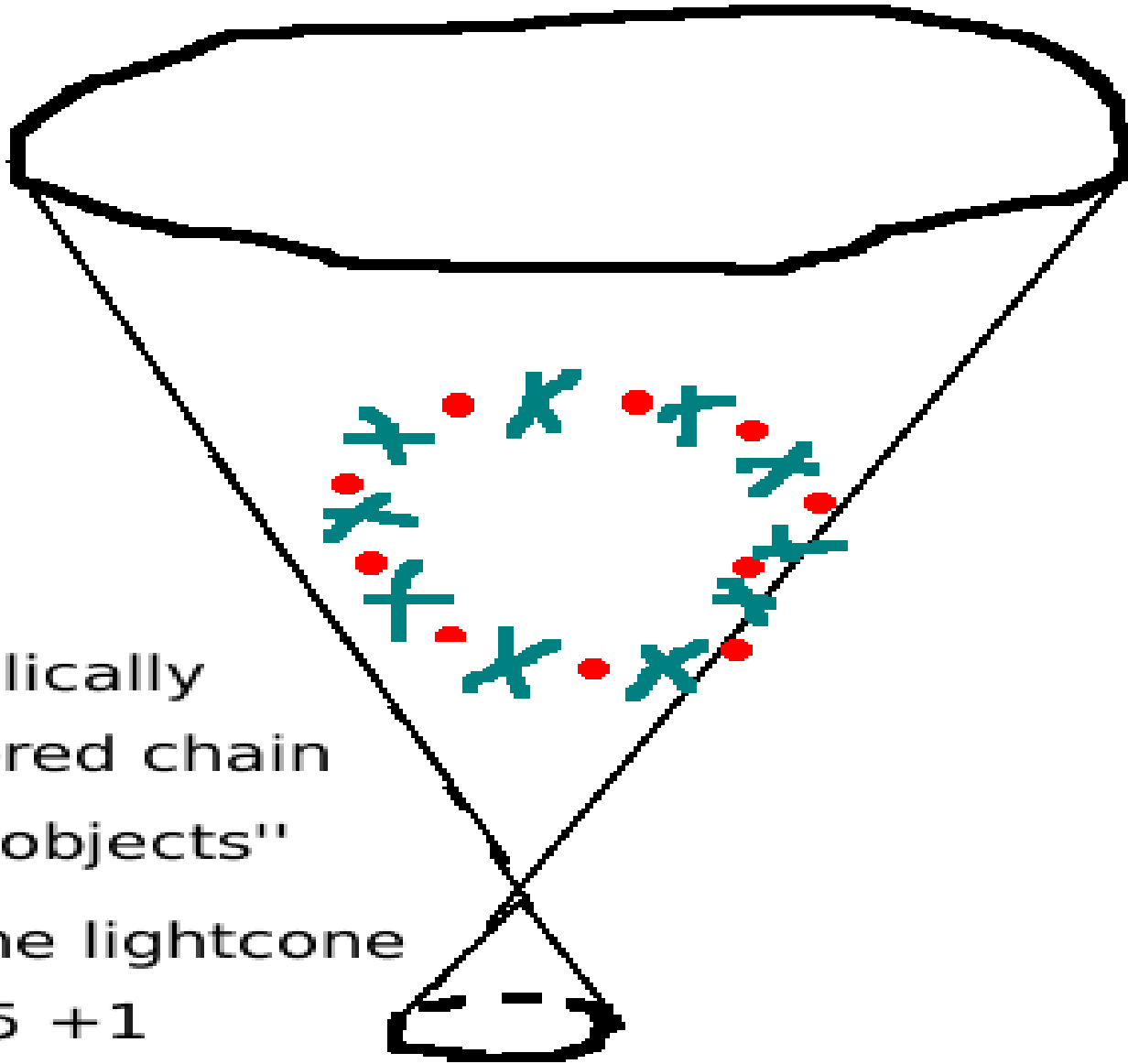
$$J_L^\mu(l) = X_L^\mu(\tau_L(l + 1/2)) - X_L(\tau_L(l - 1/2)). \quad (12)$$

Here  $\tau_R = \tau - \sigma$  and  $\tau_L = \tau + \sigma$  and e.g.  $\tau_R(l - 1/2)$  is the lower end of the discretization interval  $[\tau_R(l - 1/2), \tau_R(l + 1/2)]$  corresponding to the “object” number  $l$ .

# Important Technical Details

- The main point is that we LIBERATE the right and left movers so that the tau-derivatives of the right and left movers – depending only on one variable – are replaced by CHAINS of “objects” one for each value of a discretization of this one variable.
- But only the “objects” with an EVEN number in the discretization are considered fundamental “objects” in our formulation. The ODD ones are instead replaced by differences of the conjugate variables for the neighboring even “objects”.
- This is done in order that the “objects” taken as “fundamental”, i.e. the even ones, shall have their variables  $J$  commute with each other (otherwise we cannot make a Fock space with them)

# Picture of Chain of ``objects''



- A cyclically ordered chain of ``objects'' on the lightcone in  $25 + 1$



# Figure of Chain of ``Objects'' Illustrates

- That our ``objects'', when we count them as  $25 + 1$  dimensional 26-vectors  $J$ , are light-like, i.e. They lie on the light-cone.
- That we discretize the a priori continuous series into discrete points (regularization).
- That we distinguish even numbered and odd numbered: Actually we take the philosophy that only the even ones truly exist in our formalism. The odd ones are described as proportional to a difference between the conjugate variables for the neighboring even numbered  $J$ 's.
- There is a continuity in the sense that the ``objects'' in the chain lie crudely on a one-dimensional curve.
- Because of the construction formula for the odd numbered ``objects'' and the fact that even the odd ones are also on the curve the continuity is actually orientation dependent: The curve may be continuous with one orientation but not with the opposite orientation!

# Odd Numbered “Objects” Given in Terms of Canonically Conjugate of Even Ones

For the construction at the end of our Hilbert space for the world/the Fock space we shall only take the EVEN “objects” to be included as “fundamental objects” to be represented. The ODD ones instead are replaced by a construction in terms of the canonically conjugate variables/operators  $\Pi^i(l)$  of the even  $l$  J-variables  $J^i(l)$ , say for an ODD  $K$ ,

$$J^i(K) = -\pi\alpha'(\Pi^i(K+1) - \Pi^i(K-1)). \quad (1)$$

In this way it is arranged that neighboring “objects” in the chain do NOT commute, but rather simulates a delta prime commutation relation (in a discrete way)

# Orientation dependent Continuity Condition:

Continuity condition:

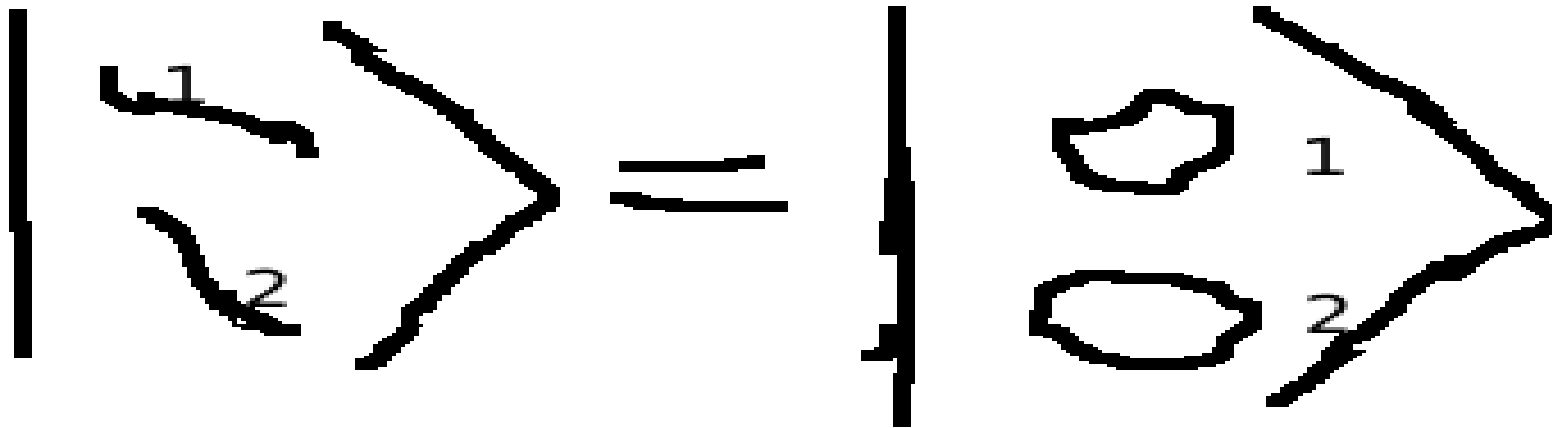
$$\begin{aligned}
 & J^i(I + D) \approx \\
 & \frac{z - \pi}{z + \pi} \alpha' (\pi^i (I + D) - \\
 & - \pi^i (I - D)) \approx \\
 & \approx J^i(I - D) \\
 & \pi^i(K) \text{ conjugate of } J^i(K).
 \end{aligned}$$

The continuity condition - which is not even invariant under inversion of the enumeration  $l$  - means approximately (but Heisenberg uncertainty prevents too accurate continuity) slow variation of even and odd objects as going along the cyclically ordered chain:

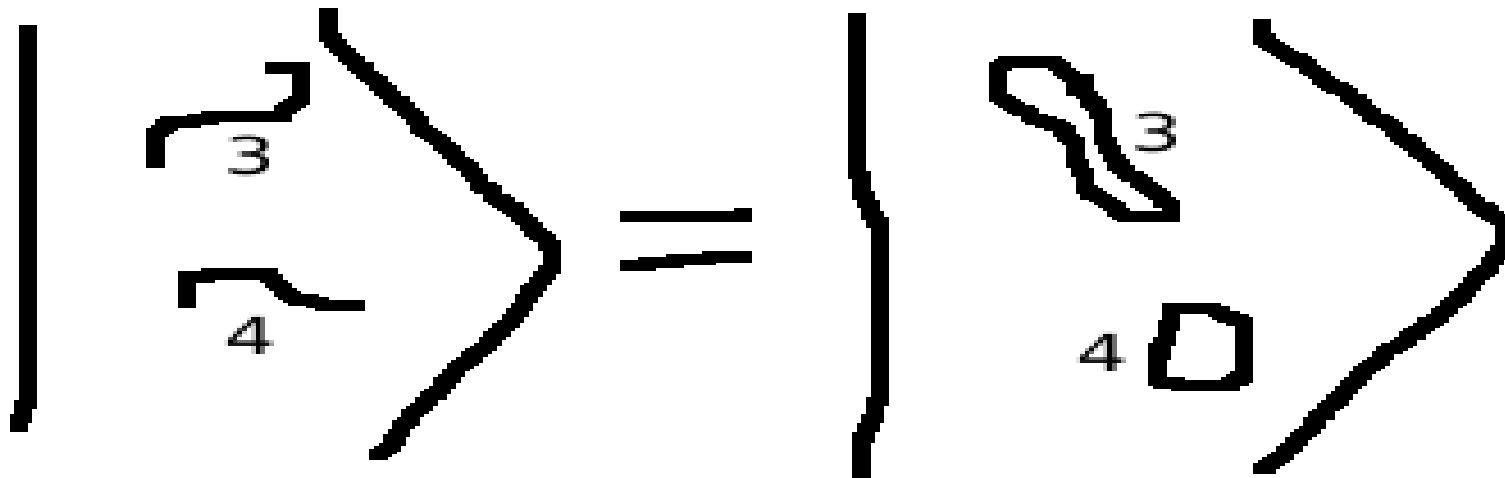
$$J^i(l-1) \approx -\pi\alpha'(\Pi^i(l+1) - \Pi^i(l-1)) \approx J^i(l+1), \quad (1)$$

where  $l$  is odd.

# Representing states of several strings as cyclic chains of objects

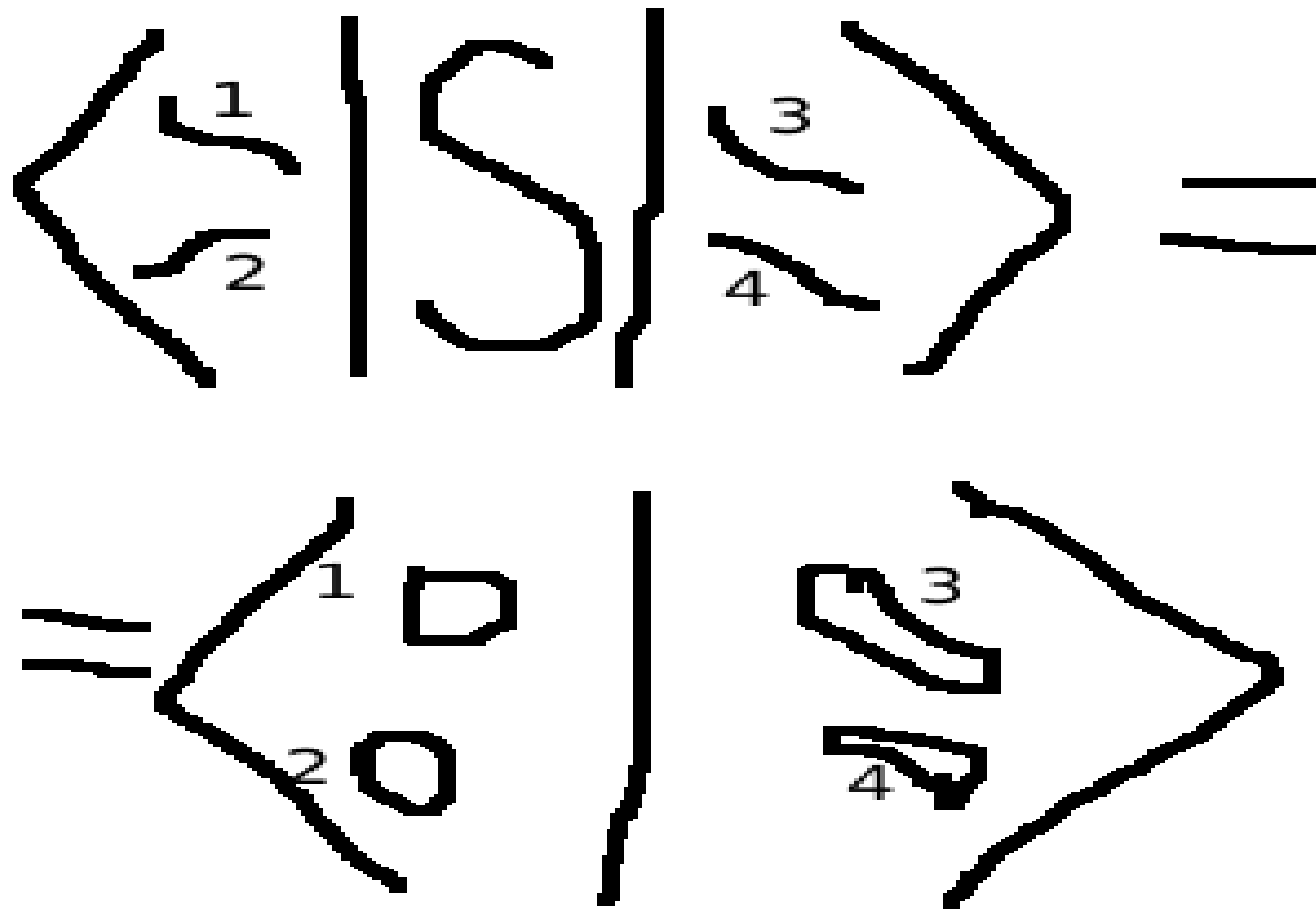


$$1 + 2 \dashrightarrow 3 + 4$$



Our S-matrix is just overlap of states  
in our ``object'' describing (Fock)  
space

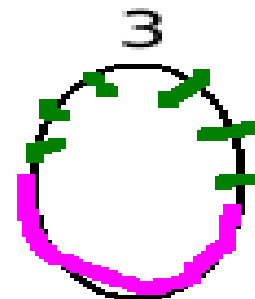
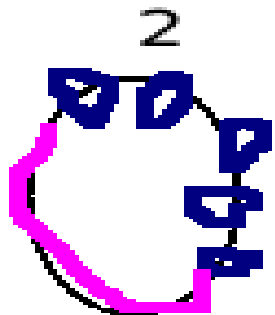
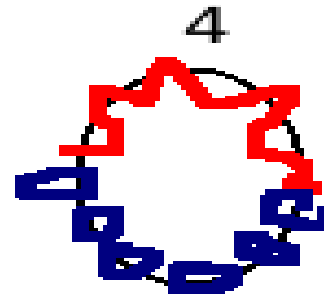
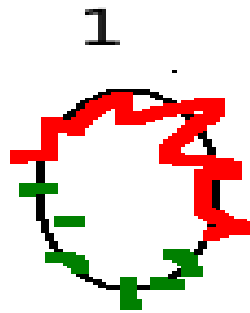
Calculate as if S-matrix were 1:



# ``Objects'' from an Initial String has Possibility of Going to Whatever String in Final State

- In our model all the EVEN ``objects'' are just like bosons identical particles.
- So any even ``object'' in the initial state can go to become identified with any one in the final state.
- Now we assume the approximation that such an identification scheme of initial with final even ``objects'' dominates more the more the neighboring ``objects'' follow each other. So dominantly long chain-pieces should go collectively over from one string to another one.

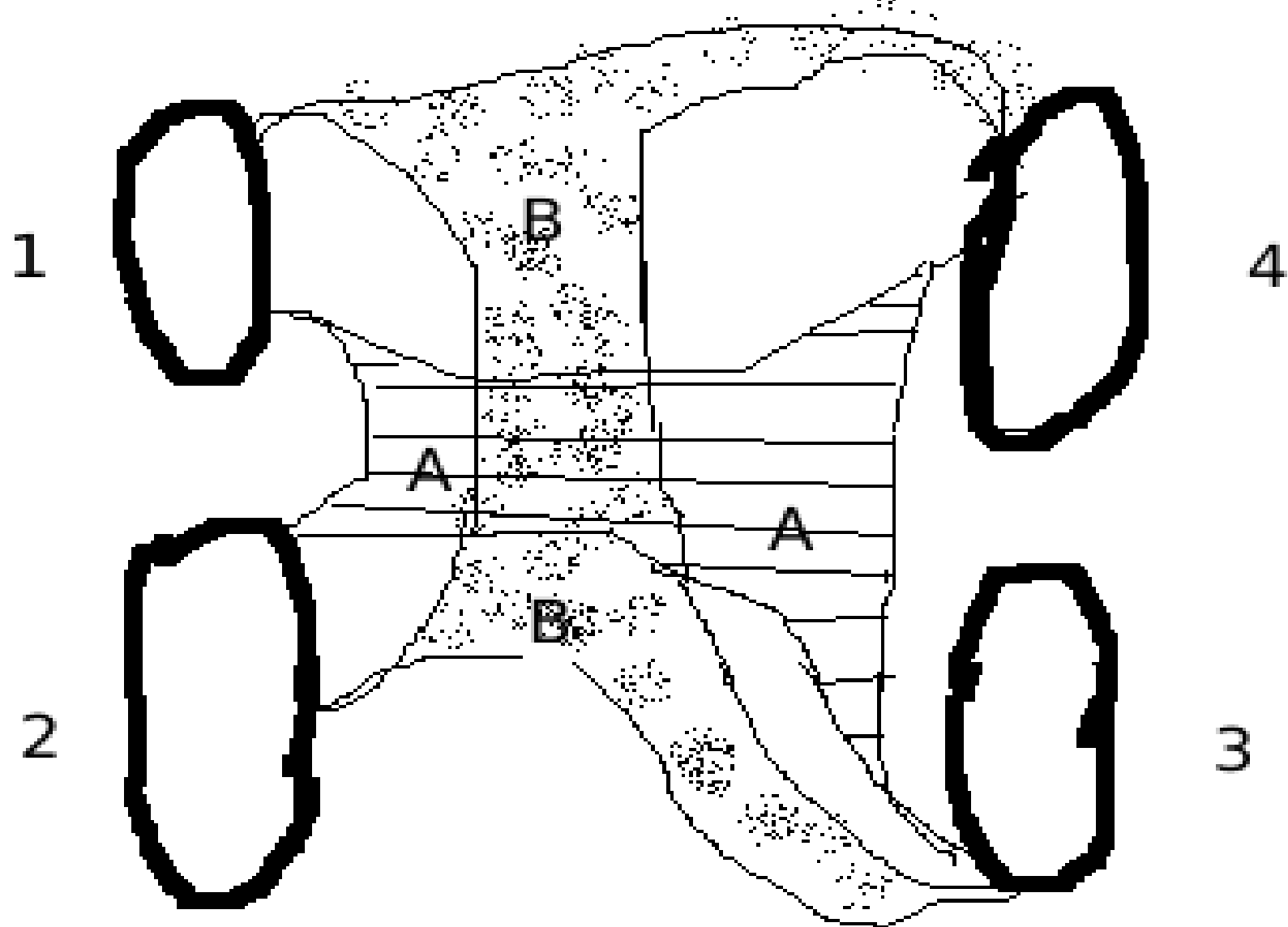
# Identifying ``Objects'' in Initial and Final Possibility



How different pieces of cyclically ordered chains of objects may get exchanged.



# Diagrams to Describe Different Ways of Identification of ``objects'' in in and out



# A bit nontrivial: The Wave Function( $\alpha$ ) for the cyclically ordered object chains(describing strings)

- Depending on which say mass eigenstate of the (open) string we shall use as external particle, we need to put the cyclically ordered chain into that state, i.e. We need the appropriate wave function( $\alpha$ ).
- We shall here take the ground state – the tachyon of the bosonic string model - .
- Its wave functional can be represented by a functional integral (trick) (or by analogue model).
- Physically we could say: we propagate a string in our doubled formulation for an infinitely long purely imaginary time; then only the ground state would

# Functional Integral Represents the Wave Function

Our functional integrals to present wave functions are just like usual string theory functional integrals:

$$\int e^{-\int g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi} \mathcal{D}\varphi$$

where  $\mu$  and  $\nu$  runs over 1 and 2.

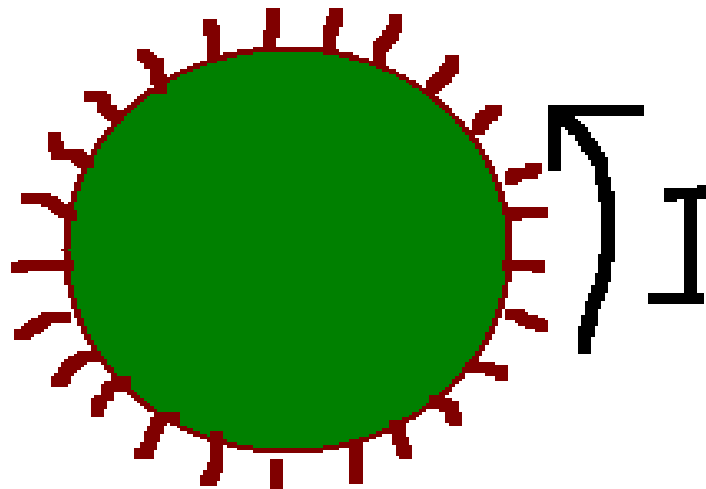
The functional integration is over  $(\sigma, \omega)$  while  $g^{\mu\nu}$  is a fixed metric in the  $(\sigma, \omega)$ -space.

# Functional Integral Represents wave function

- It shall be understood that we extract the wave function by **FIXING THE DERIVATIVE OF THE INTEGRATION VARIABLE ALONG A BOUNDARY.**
- To make precise sense a cutoff depending on the metric tensor is needed,
- although formally the functional integral looks invariant under a scaling of the metric tensor by a factor depending on the coordinates, this is not true due to the anomaly.
- But apart from this anomaly this type of functional integral is invariant under conformal transformations ( of the region over which the

# Wave Function for ``Objects'' from Functional Integral

We imagine the ``objects'' sitting along the edge of a disk, over which is defined a  $\varphi(\sigma^1, \sigma^2)$  to be functionally integrated over:



The  $J$ 's of the ``objects'' are related to the derivatives  $\partial_\mu \varphi$  at the edge.

# Our Oriented Continuity

- Because our orientation dependent “continuity condition” that the “objects” both even and odd only vary slowly along the chain we should strictly speaking have found a functional integral prescription ensuring that,
- But for easiness we take an unoriented functional integral prescription for the wave functional and argue the result should be the same if we keep to orientable two dimensional manifolds.
- Then we must only include identifying pieces of cyclically ordered chains of “objects” , when they have the SAME ORIENTATION.
- But that is anyway physically needed, since overlap of opposite orientations vanish.

# If Orientation of Chain Variable $l$ Inverted, ``Continuity'' is NOT Kept

The continuity condition - which is not even invariant under inversion of the enumeration  $l$  - means approximately (but Heisenberg uncertainty prevents too accurate continuity) slow variation of even and odd objects as going along the cyclically ordered chain:

$$J^i(l-1) \approx -\pi\alpha'(\Pi^i(l+1) - \Pi^i(l-1)) \approx J^i(l+1), \quad (1)$$

where  $l$  is odd.

The continuity condition - which is not even invariant under inversion of the enumeration  $l$  - means approximately (but Heisenberg uncertainty prevents too accurate continuity) slow variation of even and odd objects as going along the cyclically ordered chain:

$$J^i(l-1) \approx -\pi\alpha'(\Pi^i(l+1) - \Pi^i(l-1)) \approx J^i(l+1), \quad (1)$$

where  $l$  is odd.



# Conformal Transformations of the Two Dimensional Functional Integrals

- Have in mind that in our String Field Theory we have replaced the usual interval of sigma-coordinate describing the open string by one topologically circle shaped chain of “objects”, and thus already an open string looks by us more as a closed one in usual notation, while a closed string looks by us like TWO closed ones from usual notation.
- So to propagate an open string in imaginary time to make only its ground state survive becomes by us a half infinite cylinder ( while in usual a half infinite belt).
- By conformal invariance of the functional integrals (apart from the anomaly) an half infinite cylinder can be replaced by a unit disk say,

# Calculation Outline

- Notice already how the trick of introducing functional integrals for fields defined on two dimensional manifolds just because they can give the wave function for the string in our “object” description of the initial and final states is like getting the string in by a calculational trick ( There is in a way no true string in our formalism but we get it in because it is one smart trick to calculate presumably among many) !
- Our calculational stringlike formulation is though still doubled compared to say Mandelstams usual string time track functional integrals.

# Calculational Outline Continued

- The crux of the matter of our calculation of the Veneziano amplitude is to make the overlap of the in terms of “objects” described states of the several string states – the initial and the final states.
- Our “perturbation-like” approximation consists in not taking – as we should – all possible combinations of one even object in the initial state with one even object in the final state, but only include the identification schemes for objects having the longest pieces of chains following each other in the sense of going from e.g. string 1 to string 3
- For each system of correspondance between initial and final “objects” (keeping only the ones with long pieces going same way) we then calculate the overlap contribution from that correspondance by combining the

# Yet Computational Outline

- When we for  $1+2 \rightarrow 3+4$  identify pieces of the cyclically ordered chains of objects from initial and final states in the simplest (in the sense of having biggest unbroken pieces going between the various string-assigned cycles) and glue the corresponding disks for the functional integrals (giving the wave functions) together we obtain a functional integral for a two dimensional manifold, that turns out to be topologically a Riemann sphere.
- The inlets of the momenta for the four external strings(particles) sit of course in four points in this Riemann sphere.
- It turns out that they sit so that there is reflection symmetry – we talk about as a flattening symmetry – identifying pairs of points on the Riemann sphere, so that we for our calculational purpose can use a flattened two dimensional manifold of topology as a disk and with the four inlets from

# Yet Computational Outline

- Each way of distributing the initial state “objects” into the final state to be identified with the final state ones has to be summed up as separate contributions to the final “scattering” amplitude/ S-matrix / overlap.
- This becomes in the lowest order of our “perturbation like” approximation an integral, that ends up as the integration well known to represent Veneziano models. (A priori it would be the sum over permutations of the objects in initial state before being identified with final state ones)
- We strongly must use conformal transformations of the functional integrals used.
- But then there is an anomaly which actually is needed to get the right Veneziano model expression.

# How to perform the Conformal Transformations of the stringlike functional integrals?

- Remember that the ground state wave function representing functional integrals were on either a half infinite cylinder surface or conformally equivalently a disk or even we could use the complement of a disk, with infinity as its center.
- Part of the cyclically ordered chain of “objects” for string 1 ( one of the initial strings) go into 3 while another part goes to 4( the strings in the final state were called 3 and 4)
- Similarly for the other string 2 in the initial state part becomes a piece in 3 part in 4.

# Concretely Constructing the Conformal Composed Functional Integral Region

- We choose to use unit disks for the two initial state strings 1 and 2.
- We take complements of unit disks with center at infinity for the final state strings 3 and 4.
- We perform our calculation in a specially chosen frame or rather “gauge” so that all four external strings have equally many “objects” in the regularization and our each object a fixed “longitudinal momentum”  $P_+$ .

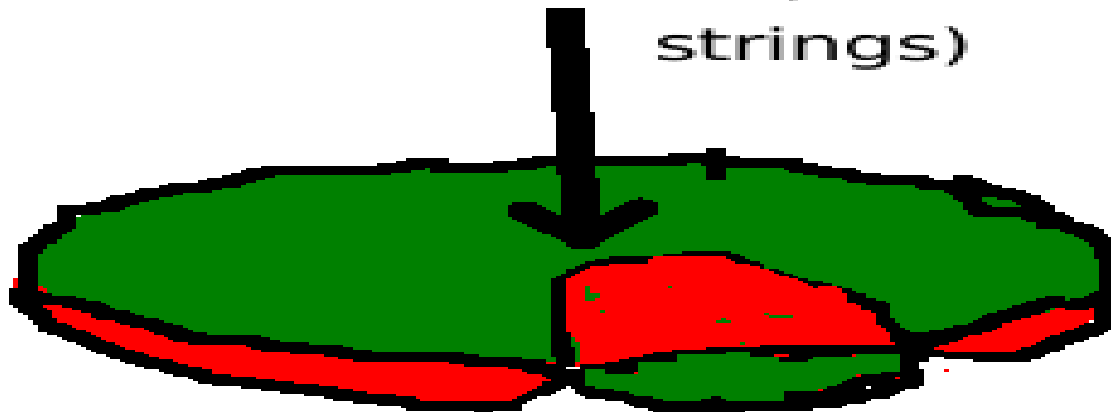
# Concretely to make Manifold for Functional Integral Composed from the Wave function producing ones

- Take the disks first of the initial state strings 1 and 2 and put them in two different layers in the complex plane.
- Next put the two complements of disks for the two final state strings in the remaining part of the complex plane including infinity (as the center) again into two different sheets/layers.
- Now the initial and the final state functional integral regions meet at the unit circle and lie on two layers. So we can very freely identify initial and final state "objects" by identifying these layers correspondingly.
- In the simplest case supposed to dominate we just have one piece of the unit circle where 1 goes 4 while



# Two Layers from Initial State Strings

Inlet of current/momentum  
for 1 and 2 (incoming  
strings)



Two unit disks lying two layers in the complex plane, seen in perspective, and prepared for being glued to complements of disks for outgoing 3 + 4.

Green for 2; red for 1.

# Building up the Manifold for the Functional integral Composed

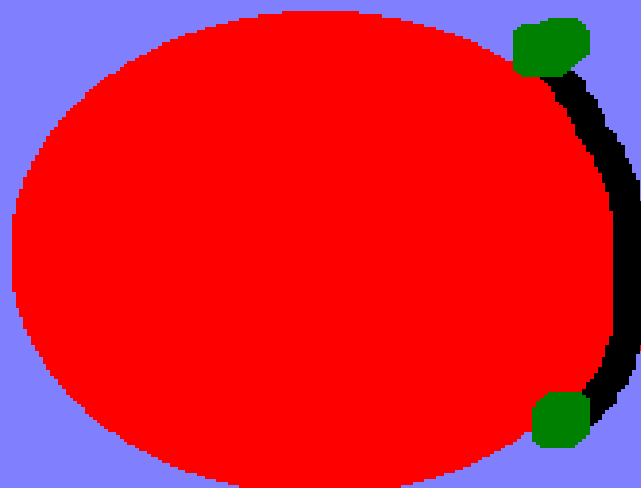
- Glue together the two layered disk with the edges permuted along part of the unit circle with the two layered complement of the unit disks representing the functional integral regions for the final state particles(strings), 3 and 4.
- Then we get the two dimensional region for the functional integral the result of which is the contribution from the identification correspondance related to the angle on the unit circle along which say 1 and 4 were identified.
- The functional integral is evaluated by using the essential conformal invariance and using analogy to Koba Nielsen disk and computing anharmonic ratios.
- But an anomaly correction has to be included. The conformal invariance is broken by the anomaly which

# Putting disks and complement disks into two layer complex plane

The complex plane with two sheets

Under the unit disk the disks of 1 & 2.

Under the exterior the 3 and 4 parts.



The cut  
where 1 & 4  
meet.

The cut where 1 &  
3 meet.

# Next use square root like transformation to get the two singular points straightened out

- The two singular points – green on the figure - are taken as singular points for a square root like function ( product of two square roots) and we get the two sheets mapped into just one Riemann sphere.
- Now we managed to get the unit circle mapped to the real axis.
- And the two singular points to the 0 and infinity.
- And we got the inlet points where the four external momenta are let in to lie on the (new) unit circle.
- Seeing the inversion on the circle as a symmetry we do not need the exterior and just got Koba Nielsen disk.

# The Surprise of our Calculation:

we found only one term in Veneziano model:

$$B(-\alpha(s), -\alpha(t))$$

But not those terms which have poles in the s-channel.

What went wrong?

Infinite momentum frame is bad!

One cannot have negative longitudinal P

# In First Calculation we Gauge fixed

We use the parametrization choice, which combined with our discretization, gives that each object carry a fixed amount of "longitudinal momentum"

$$P^+ = \frac{1}{2}(P^0 + P^2)$$

so that

$$J^+ = \frac{\alpha \cdot \alpha'}{2}$$

for all "objects".

# Is Usual IMF Formalism Wrong?

- Such gauge fixing that assigns the string or the cyclically ordered chain parameter to be proportional to the amount of longitudinal momentum is usual in Infinite Momentum Frame (IMF) formalism.
- But is it enough? We can never get the negative energy or longitudinal momentum frame states – such as the Dirac sea -.
- One has in IMF thrown vacuum away.
- That were presumably the goal but is it good enough also e.g. For our string field theory?
- We had chosen now to allow for “objects” with negative longitudinal momentum.

# We Change Our gauge Choice to Allow both Signs!

**Change** the parametrization choice, which combined with our discretization, gives that each object carry a fixed amount of "longitudinal momentum"

$$P^+ = \frac{1}{2} (P^0 + P^2 \xi)$$

so that we allow also negative long. m.

$$J^+ = \pm \frac{\alpha \cdot \alpha'}{2}$$

for "two different sorts of "objects".



# Extension of chains with Negative and Positive pieces

- Our introduction of negative  $P^+$  momenta, negative longitudinal momenta, allows us to add to any cyclically ordered chain of “objects” an equal amount of the negative  $P^+$  or  $J^+$  ones and of the old positive  $P^+$  or  $J^+$  ones.
- Such an extension opens the possibility for “annihilating” a negative chain piece in one incoming string, say 1 with a positive piece in another 2.
- It turns out that we must then also have an annihilation in the final state between positive and negative pieces.
- And remarkably by inclusion of such possibilities we

# Energy is Not Used Properly as Hamiltonian in Our model, Time Not Discussed

## Definition of Energy = $P^-$

In our formulation the even objects have each only 24 true physical degrees of freedom  $(J^i, \Pi^i)$  with  $i$  running through 1 to 24. The  $J^+ = \frac{a\alpha'}{2}$  is fixed as part of our gauge fixing, the "infinite momentum frame energy"  $P^-$  is constructed so as to ensure that

$$J^2 = g_{\mu\nu} J^\mu J^\nu = 2J^+ J^- - J^i J^i = 0 \quad (\text{sum over } i = 1, 2, \dots, 24 \text{ understood}). \quad (13)$$

So we obtain

$$\text{For "even objects": } J^- = \frac{4(J^i)^2}{a\alpha'} \quad (14)$$

$$\text{For an odd } l \text{ "object": } J^-(l) = \frac{4(-\pi\alpha')^2(\Pi^i(l+1) - \Pi^i(l-1))^2}{a\alpha'} \quad (15)$$

where the  $\Pi^i(l+1)$  e.g. is the  $i$ th component of the conjugate momenta to the "transverse" (i.e.  $i$  up to 24) for the "even object" just the next step after the odd one considered.

# Published Check of Mass Spectrum

- We published a paper – and best so far to read that paper to learn about our model – in which we used the energy as defined from lightlikeness and checked that we got the usual spectrum with number of states etc. As string theory for open strings.
- In our model wherein we need checks that our model is indeed as we attempted to construct it a rewriting of the string theory this result of checking the spectrum is not totally trivial.

# Some Warning Signals ?

- Although the detailed formula for the energy in our formalism as to be calculated from the “objects” is not so trivial as for true massless free scalar particles, it looks very hard how our formalism should be able to incorporate the Hagedorn temperature phenomenon of usual string theory or Veneziano models.
- It can only come in by THE INITIAL STATE IN OUR MODEL BEING STRONGLY RESTRICTED BY THE CONTINUITY CONDITION.
- So we need a so strong condition on the allowed STATES in our model, that it even modifies – and should be included in – the Boltzmann distribution.
- And then these INITIAL condition restrictions can produce seemingly a quite different phase.

# Initial Condition Must be Very Important in Our Model

Hagedorn Temperature Surprising in Novel String Field Theory of  
Ninomiya's and Mine

When our Novel SFT looks like free massless scalar bosons, how can it have a Hagedorn temperature so that  $T \leq T_{Hagedorn}$  ? Free massless would have thermodynamic properties like electrodynamics and show the Planck behavior:

$$E = \text{"energy"} \propto T^4 (\text{for fixed volume}) \quad (13)$$

This is not at all looking like the Hagedorn behavior at all. With Hagedorn temperature it should never be possible to exceed the Hagedorn temperature  $T_{Hagedorn}$  !

# Our SFT Model Gives Usual String Theory String Spectrum Very Different from Free Massless



## Hagedorn Temperature in String Theory

Wellknown that the spectrum of strings for high mass has the Hagedorn maximal temperature behavior:

$$\rho(m) \propto \exp(m/T_{\text{Hagedorn}}) \quad (14)$$

# Energy or P – Depends Even on Connection of the Cyclically Ordered Chains of “Objects”

- Although the P- for a whole (say open) string is given as a sum over all the “objects” in a chain corresponding to the string it strictly speaking depends on the ordering into such a chain of the “even objects”, because the ODD “objects” – which do not exist fundamentally, but are constructed from the conjugate momenta of the even ones – will depend on the ordering.
- The P- which is the infinite momentum frame energy is construct for each “object” - both even and odd ones- by the requirement that the J-twenty six vector shall be lightlike, i.e.  $J^2=0$  ( as required by the constraint conditions in string theory).

# Make Our Novel SFT Supersymmetric?

Presumably extension of our model to to become a rewriting - with a nul set of information thrown away - of some of the superstrings will be very easy. One way that could be used would be to *bosonize* the fermionic degrees of freedom and then use our method as we did for the bosonic degrees of freedom.

Compactifying a single dimension in our formalism would be easy. We might at the end re-fermionize the bosonic degrees of freedom at the "object" -level to obtain presumably spin one half "objects".



# Conclusion

- We have constructed a new – since throws away information compared to older ones – string field theory, in the sense of a theory with potentially an arbitrary number of strings present.
- Our formalism is based on a “Fock space” of states for the universe of the strings, but formulated in terms of “objects”, in terms of which we have truly a Fock space: There can be any non-negative integer number of “objects” in any state for objects.
- The state of a single object is given its 24 transverse  $J^i$  components ( and then their 24 conjugate variables).

# Conclusion Continued

- The two last J-components for an “object” were reconstructed just by mathematical definition from the transverse – the 24 - components :  $J_+$  is just fixed as a “gauge choice” and the  $J_-$  is determined to make the total J 26-vector lightlike.
- It is only the EVEN “objects” that truly exist in our model-formulation; the ODD “objects” are only derived by a formula from the conjugate variables to the even ones.

# Conclusion Main Result

- We calculated the scattering amplitude – under a special assumption of all four strings having same longitudinal momentum – as just THE OVERLAP OF THE INITIAL AND THE FINAL STATE FORMULATED IN OUR MODEL “FOCK SPACE”!
- First we got only ONE of the three terms expected, but then introduction of “objects” with NEGATIVE longitudinal momentum  $P_+$  or say negative  $J_+$  led to also obtaining the two missing terms; and thereby a more crossing symmetric and boson symmetric scattering amplitude.

# Conclusion, Forward Looking

- It is not totally trivial, that our model gives string theory, but the derivation of the Veneziano model amplitude is a strong indication, that our model IS indeed string theory rewritten.
- Our model has the character of being a SOLUTION to string theory, even for many strings, string field theory, in the sense that it is in a Heisenberg-like formulation and nothing happens at all to our ``objects'' as time goes on. They are more like just the numbers in a certain solution to the string dynamical theory.

# Conclusion, Hopes

- Since our string field theory is like a solution, you could expect it to solve also say the AdS world string theory in the Maldacena conjecture and thus probably lead to revealing, that the Maldacena conjecture could be understood by such solution.
- I consider it an almost trivial thing to extend our model to include SUSY by giving our “objects” probably some spin.
- At the end we hopefully could also see through to an interpretation of branes in our formulation.
- But for the moment we concentrated too much on convincing ourselves and others, that our model works even just for the bosonic strings

# Conclusion, Phenomenological Question

- Could one believe that the true model for Nature were a SOLVABLE model? Well a priori random initial conditions could make the world random enough anyway.
- Could we possibly use solvability to argue, that the effective dimension never would go down to four instead of say 10 in the susy version (yet to be constructed)? Could complicated initial conditions arrange for the wanted effective dimension?

# Conclusion Question: Where is the String?

- Since our Fock space with fewer or more “objects” is essentially a free quantum field theory of massless scalar bosons, one may with good reason worry: where are the strings?
- Since the strings only appear as continuous one-dimensional structures because of the continuity in the cyclically ordered chains of “objects”, it is this continuity of chains of “objects”, that is the basis of the strings.
- Thus the strings are only there because of feature of the STATE of the “object” system.
- This means it is the INITIAL STATE assumed to have such continuity that makes up that there are strings. I.e. The strings are put in via an INITIAL STATE property!