Holographic Wilson loops in Lifshitz-like backgrounds

Anastasia Golubtsova[1]

based on works with
Dima Ageev, Irina Ia. Aref'eva and Eric Gourgoulhon
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[1]BLTP JINR

Recent Developments in Strings and Gravity,
Corfu 2016

Outline

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The quark-gluon plasma (2005)

- QGP is a strongly coupled fluid (LHC & RHIC experiments)
- QGP is created in time $\tau \sim 0.1$fm/c after the collision (short time of thermalization)
- QGP is anisotropic for a short time $0 < \tau_{therm} < \tau < \tau_{iso}$.

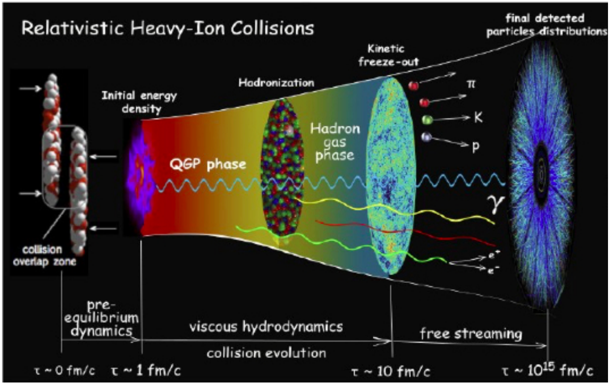


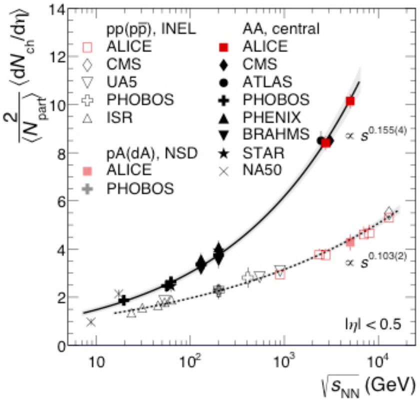
Figure: Picture from: P.Sorensen, C.Shen

- $D = 4$ Multiplicity is proportional to entropy of $D = 5$ BH Gubser'08

Experiment:
$S_{data} = \mathbf{s}_{NN}^{0.155}$
ALICE collaboration'15

Modified AdS:
$S_{data} = \mathbf{s}_{NN}^{0.12}$
Kiritis & Taliotis'11

Modified AdS+ ghosts:
$S_{data} = \mathbf{s}_{NN}^{0.16}$
Aref'eva et al.'14



ALICE collaboration'15

Broken scaling
$S_{data} = \mathbf{s}_{NN}^{0.16}$
Aref'eva & A.G.'14

Wilson loops

- The Wilson loop is a physical gauge invariant object
- It measures the interaction potential between the external quarks
- Transport coefficients of QGP (jet quenching parameter, drag force etc.)

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Holographic Wilson loop

The expectation value of WL in the fundamental representation calculated on the gravity side:

$$W[C] = \langle \mathrm{Tr}_F\, e^{i \oint_C dx_\mu A_\mu} \rangle = e^{-S_{string}[C]},$$

where C in a contour on the boundary, F – the fundamental representation, S is the minimal action of the string hanging from the contour C in the bulk.

J.M. Maldacena' 98

Holographic models

Holographic models

- Top-down approach: low-energy approximation of string theory (supergravity model) in asymptotically AdS backgrounds trying to reproduce features similar to QCD
  Examples:Sakai-Sugimoto model ($D4 - D8 - \bar{D}8$-branes), Mateos-Trancanelli model ($D3 - D7$-branes).

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  - asymptotically AdS spacetimes
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Breaking scale invariance

The AdS/CFT correspondence:

The Field Theory

- the conformal group $SO(D,2)$

of a D-dimensional CFT

$(t, x_i) \rightarrow (\lambda t, \lambda x_i)$, $i = 1, .., d-1$

The Gravitational Background

- the group of isometries

of AdS_{D+1}

$ds^2 = r^2 \left(-dt^2 + d\vec{x}_{d-1}^2\right) + \frac{dr^2}{r^2}$

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Lifshitz scaling: $t \to \lambda^\nu t, \qquad \vec{x} \to \lambda \vec{x}, \quad r \to \frac{1}{\lambda} r,$

where ν is the Lifshitz dynamical exponent

Lifsthiz fields theories

Lifshitz metric: $ds^2 = -r^{2\nu} dt^2 + \dfrac{dr^2}{r^2} + r^2 d\vec{x}^2_{d-1}$

Kachru, Liu, Milligan '08

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$\nu = 1$: AdS-metric.

Lifshitz-like spacetimes for holography

- A spatial extension of the Lifshitz scaling

$$(t, x, y, r) \to (\lambda^{\nu} t, \lambda^{\nu} x, \lambda y_1, \lambda y_2, \frac{r}{\lambda})$$

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$$ds^2 = r^{2\nu}\left(-dt^2 + dx^2\right) + r^2 dy_1^2 + r^2 dy_2^2 + \frac{dr^2}{r^2},$$

M. Taylor'08, Pal'09.

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The 5d Lifshitz-like metrics

$$\text{Type} - (\mathbf{1}, \mathbf{2}) \quad ds^2 = r^{2\nu}\left(-dt^2 + dx^2\right) + r^2\left(dy_1^2 + dy_2^2\right) + \frac{dr^2}{r^2}.$$

$$\text{Type} - (\mathbf{2}, \mathbf{1}) \quad ds^2 = r^{2\nu}\left(-dt^2 + dx_1^2 + dx_2^2\right) + r^2 dy^2 + \frac{dr^2}{r^2}.$$

Type IIB SUGRA, $D3-D7$-branes

$\mathcal{M} = M_5 \times X_5$: M_5 is a $5d$ Lifshitz-like metric, X_5 is an Einstein manifold.

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D3-D7 system, Azeyanagi, Lin, Takayanagi'09

$$ds^2 = \tilde{R}^2 \left[\rho^2 \left(-dt^2 + dx^2 + dy^2 \right) + \rho^{4/3} dw^2 + \frac{d\rho^2}{\rho^2} \right] + R^2 ds^2_{X_5}.$$

$$\nu = 3/2, \quad r \equiv \rho^{2/3}, \quad (t, x, y, w, \rho) \to \left(\lambda t, \lambda x, \lambda y, \lambda^{2/3} w, \frac{\rho}{\lambda} \right)$$

$\mathcal{M}_5 \times X_5$	t	x	y	r	w	s_1	s_2	s_3	s_4	s_5
D3	$\times$	$\times$	$\times$		$\times$					
D7	$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$

Type IIB SUGRA, $D3-D7$-branes: Anisotropic QGP

Mateos&Trancanelli'11

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H}dx_3^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}d\Omega_{S^5}^2.$$

The functions $\mathcal{F}, \mathcal{B}, \mathcal{H}$ depend on the radial direction u and the anisotropy α. At high temperatures $\alpha \ll T$:

$\mathcal{F}(u) =$

$$1 - \frac{u^4}{u_h^4} + \frac{\alpha^2}{24u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4)\log(1 + \frac{u^2}{u_h^2})\right]$$

$$\mathcal{B}(u) = 1 - \frac{\alpha^2}{24u_h^2}[\frac{10u^2}{u_h^2+u^2} + \log(1 + \frac{u^2}{u_h^2})], \ \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}.$$

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$D7$-probes in $D3$-background $Lif_{IR}/AdS_{5,UV} \times X_5 \Rightarrow$ deformed SYM.
$\alpha = 0 \Rightarrow$ isotropic $D3$-brane AdS/CFT: $AdS_5 \times S^5 \Rightarrow \mathcal{N} = 4$ SYM.
Jet quenching, drag force, potentials... see Giataganas et al.'12

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- difficult to study
- not cover experimental data

Outline

Black branes in Lifshiz-like spacetimes

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{|g|}\left(R[g] + \Lambda - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{4}e^{\lambda\phi}F^2_{(2)}\right),$$

Λ is negative cosmological constant.

The Einstein equations

$$R_{mn} = -\frac{\Lambda}{3}g_{mn} + \frac{1}{2}(\partial_m\phi)(\partial_n\phi) + \frac{1}{4}e^{\lambda\phi}\left(2F_{mp}F^p_n\right) - \frac{1}{12}e^{\lambda\phi}F^2 g_{mn}.$$

The scalar field equation

$$\Box\phi = \frac{1}{4}\lambda e^{\lambda\phi}F^2, \quad \text{with} \quad \Box\phi = \frac{1}{\sqrt{|g|}}\partial_m(g^{mn}\sqrt{|g|}\partial_n\phi).$$

The gauge field

$$D_m\left(e^{\lambda\phi}F^{mn}\right) = 0.$$

Gravity duals: black brane

The Lifshitz-like black brane

$$ds^2 = e^{2\nu r}\left(-f(r)dt^2 + dx^2\right) + e^{2r}\left(dy_1^2 + dy_2^2\right) + \frac{dr^2}{f(r)},$$

where $f(r) = 1 - me^{-(2\nu+2)r}$. Aref'eva,AG, Gourgoulhon'16

$$F_{(2)} = \frac{1}{2}q dy_1 \wedge dy_2, \quad \phi = \phi(r), \quad e^{\lambda\phi} = \mu e^{4r}.$$

The Hawking temperature of the black brane:

$$T = \frac{1}{\pi}\frac{(\nu+1)}{2\nu}\, m^{\frac{\nu}{2\nu+2}}.$$

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5d GAUGED $U(1)^3$ SUGRA

Donos, Gauntlett'12, Hoker, Kraus'12

Holographic Wilson Loops

- The expectation value of WL in the fundamental representation calculated on the gravity side:

$$W[C] = \langle \mathrm{Tr}_F\, e^{i \oint_C dx_\mu A_\mu} \rangle = e^{-S_{string}[C]}.$$

The Nambu-Goto action is

$$S_{string} = \frac{1}{2\pi\alpha'} \int d\sigma^1 d\sigma^2 \sqrt{-\det(h_{\alpha\beta})}, \tag{1}$$

with the induced metric of the world-sheet $h_{\alpha\beta}$ given by

$$h_{\alpha\beta} = g_{MN}\partial_\alpha X^M \partial_\beta X^N, \quad \alpha, \beta = 1, 2, \tag{2}$$

g_{MN} is the background metric, $X^M = X^M(\sigma^1, \sigma^2)$ specify the string, σ^1, σ^2 parametrize the worldsheet.

- The potential of the interquark interaction

$$W(T, X) = \langle \mathrm{Tr}\, e^{i \oint_{T\times X} dx_\mu A_\mu} \rangle \sim e^{-V(X)T}.$$

Holographic Wilson Loops

A similar operator to probe QCD is the spatial rectangular Wilson loop of size $X \times Y$ (for large Y)

$$W(X,Y) = \langle \mathrm{Tr} e^{i \oint_{X\times Y} dx_\mu A_\mu} \rangle = e^{-\mathcal{V}(X)Y}$$

defines the so called pseudopotential $\mathcal{V}$:

$$\mathcal{V}(X) = \frac{S_{string}}{Y}.$$

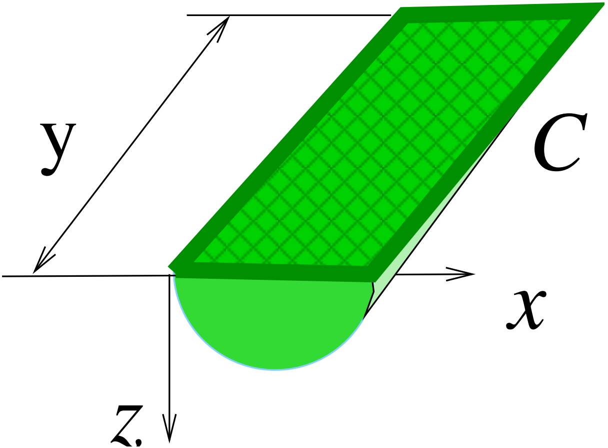
The spatial Wilson loops obey the area law at all temperature, i.e.

$$\mathcal{V}(X) \sim \sigma_s X,$$

where σ_s defines the spatial string tension

$$\sigma_s = \lim_{X\to\infty} \frac{\mathcal{V}(X)}{X}.$$

Holographic spatial Wilson loops



Spatial WL in Lifshitz-like backgrounds

Rectangular WL in the spatial planes xy_1(or xy_2) and y_1y_2.
Possible configurations: D.Ageev, I.Ya.Aref'eva,AG, E.Gourgoulhon'16

- a rectangular loop in the xy_1 (or xy_2) plane with a short side of the length ℓ in the longitudinal x direction and a long side of the length L_{y_1} along the transversal y_1 direction

$$x \in [0, \ell < L_x], \quad y_1 \in [0, L_{y_1}];$$

- a rectangular loop in the xy_1 plane with a short side of the length ℓ in the transversal y_1 direction and a long side of the length L_x along the longitudinal x direction:

$$x \in [0, L_x], \quad y_1 \in [0, \ell < L_{y_1}];$$

- a rectangular loop in the transversal y_1y_2 plane with a short side of the length ℓ in one of transversal directions (say y_1) and a long side of the length L_{y_2} along the other transversal direction y_2

$$y_1 \in [0, \ell < L_{y_1}], \quad y_1 \in [0, L_{y_2}].$$

Static WL. Case 1: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(x)$, $v = v(x)$.

The renormalized Nambu-Goto action

$$S_{x,y_{1(\infty)},ren} = \frac{L_{y_1}}{2\pi\alpha'}\frac{1}{z_*^{1/\nu}}\int_0^1 \frac{dw}{w^{1+1/\nu}}\left[\frac{1}{\sqrt{f(z_* w)\left(1-w^{2+2/\nu}\right)}}-1\right]-\frac{\nu}{z_*^{1/\nu}},$$

where $w = z/z_*$.The length scale is

$$\frac{\ell}{2} = 2z_* \int_{z_0/z_*}^{1} \frac{w^{1+1/\nu}\,dw}{f(z_* w)(1-w^{2+2/\nu})}.$$

Then pseudopotential $\mathcal{V}_{x,y_{1(\infty)}} = \dfrac{S_{x,y_{1(\infty)},ren}}{L_{y_1}}$.

For small ℓ – the deformed Coulomb part

$$\mathcal{V}_{x,y_{1(\infty)}}(\ell,\nu) \underset{\ell\sim 0}{\sim} -\frac{\mathcal{C}_1(\nu)}{\ell^{1/\nu}}.$$

For large ℓ

$$\mathcal{V}_{x,y_{1(\infty)}}(\ell,\nu) \underset{\ell\to\infty}{\sim} \sigma_{s,1}(\nu)\,\ell.$$

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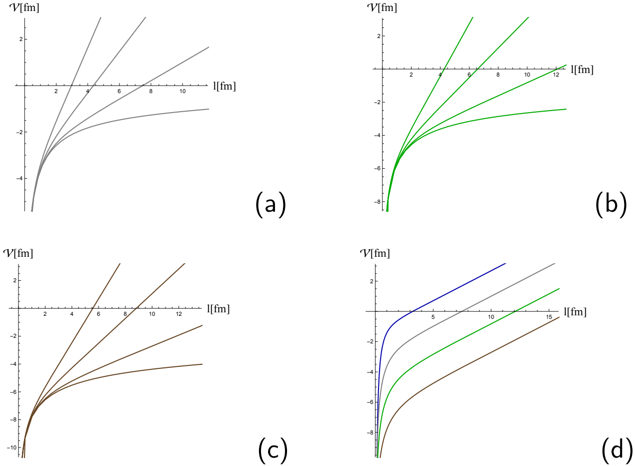

Figure: $\mathcal{V}_{x,y_{1(\infty)}}$ as a function of ℓ, $\nu = 2, 3, 4$((a),(b),(c)). The temperature $T = 30, 100, 150, 200$ MeV (from down to top) for all. In (d): $\mathcal{V}_{x,y_{1(\infty)}}$ for $\nu = 1, 2, 3, 4$ (from top to down) at $T = 100$ MeV.

Static WL. Case 2: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(y_1)$, $v = v(y_1)$

The renormalized Nambu-Goto action

$$S_{y_1\,,\, x_{(\infty)}\,,\, ren} = \frac{L_x}{2\pi\alpha'}\frac{1}{z_*}\int\limits_{z_0/z_*}^{1}\frac{dw}{w^2}\left[\frac{1}{\sqrt{f(z_* w)\left(1-w^{2+2/\nu}\right)}}-1\right]-\frac{1}{z_*}.$$

The length scale is

$$\ell = 2z_*^{1/\nu}\int\limits_0^1 \frac{w^{2/\nu}dw}{f(z_* w)\left(1-w^{2+2/\nu}\right)}$$

The pseudopotential $\mathcal{V}_{y_1\,,\, x_{(\infty)}} = \dfrac{S_{y_1\,,\, x_{(\infty)}\,,\, ren}}{L_x}$.

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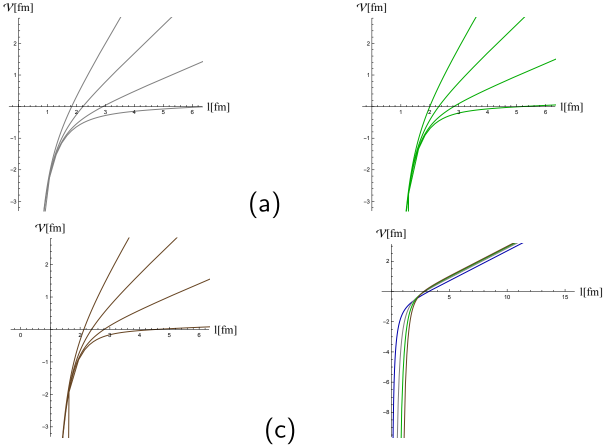

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Static WL. Case 3: $\sigma^1 = y_1,\ \sigma^2 = y_2,\ z = z(y_1),\ v = v(y_1)$

The renormalized Nambu-Goto action

$$S_{y_1 y_{2(\infty)},ren} = \frac{L_{y_2}}{2\pi\alpha'} \frac{1}{z_*^{1/\nu}} \int\limits_{\frac{z_0}{z_*}}^{1} \frac{dw}{w^{1+1/\nu}} \left[\frac{1}{\sqrt{f(z_* w)\left(1 - w^{4/\nu}\right)}} - 1 \right] - \frac{\nu}{z_*^{1/\nu}}.$$

The length scale is

$$\ell = z_*^{1/\nu} \int \frac{dw}{w^{1-3/\nu}\sqrt{f(z_* w)\left(1 - w^{4/\nu}\right)}}.$$

The pseudopotential $\mathcal{V}_{y_1, y_{2(\infty)}} = \dfrac{S_{y_1, y_{2(\infty)}}}{L_{y_2}}$.

For small ℓ –

$$\mathcal{V}_{y_1, y_{2(\infty)}} \underset{\ell \to 0}{\sim} -\frac{\mathcal{C}_3(\nu)}{\ell}.$$

For large ℓ

$$\mathcal{V}_{y_1, y_{2(\infty)}}(\ell, \nu) \underset{\ell \to \infty}{\sim} \sigma_{s,3}(\nu)\, \ell.$$

Static WL.Case 3: $\sigma^1 = y_1$, $\sigma^2 = y_2$, $z = z(y_1)$, $v = v(y_1)$

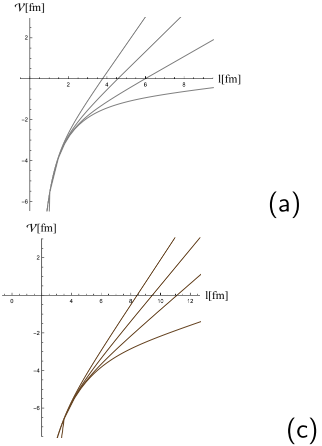

Figure: $\mathcal{V}_{y_1,\,y_{2(\infty)}}$ as a function of ℓ for $\nu = 2, 3, 4$ ((a),(b),(c), respectively). We take $T = 30, 100, 150, 200$ MeV from down to top, for (a),(b) and (c). In (d) $\mathcal{V}_{y_1,\,y_{2(\infty)}}$ for $\nu = 1, 2, 3, 4$ (from left to right) at $T = 100$ MeV.

Spatial string tension

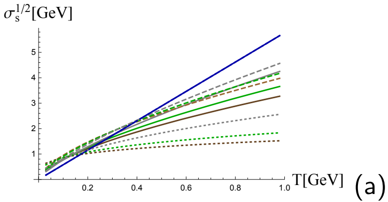

Figure: The dependence of the spatial string tension $\sqrt{\sigma_s}$ on orientation and temperature. The solid lines corresponds to the rectangular Wilson loop with a short extent in the x-direction, while the dashed lines correspond to a short extent in the y-direction. The dotted lines correspond to the rectangular Wilson loop in the transversal $y_1 y_2$ plane. (a) Blue line corresponds to $\nu = 1$, gray lines correspond to $\nu = 2$, green lines correspond to $\nu = 3$ and the brown ones correspond to $\nu = 4$. (b) The spatial string tension $\sqrt{\sigma_s}$ for different orientations for $\nu = 4$.

Alanen et al.'09,A. Dumitru et al.'13-14

Outline

The infalling shell background

The ingoing Eddington-Finkelstein coordinates

$$dv = dt + \frac{dz}{f(z)}.$$

The Vaidya solution in Lifshitz background

$$\begin{aligned} ds^2 &= -z^{-2}f(z)dv^2 - 2z^{-2}dvdz + z^{-2}dx^2 + z^{-2/\nu}(dy_1^2 + dy_2^2), \\ f &= 1 - m(v)z^{2+2/\nu}, v < 0 - \text{inside the shell}, v > 0 - \text{outside}, \\ f(v,z) &= 1 - \frac{M}{2}\left(1 + \tanh\frac{v}{\alpha}\right) z^{2+\frac{2}{\nu}} \end{aligned}$$

The infalling shell background

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- The Vaidya solution interpolates between the black hole (outside the shell) and the Lifshitz-like vacuum (inside the shell).

WL in time-dependent backgrounds.Case 1

$$v = v(x), \quad z = z(x), \quad f = f(v,z).$$

$$S_{x,y_{1(\infty)}} = \frac{L_y}{2\pi\alpha'}\int \frac{dx}{z^{1+1/\nu}}\sqrt{1 - f(z,v)v'^{\,2} - v'z'}, \quad ' \equiv \frac{d}{dx}.$$

The corresponding equations of motion are

$$\begin{aligned} v'' &= \frac{1}{2}\frac{\partial f}{\partial z}v'^2 + \frac{(\nu+1)}{\nu z}(1 - fv'^2 - 2v'z'), \\ z'' &= -\frac{\nu+1}{\nu}\frac{f}{z} + \frac{\nu+1}{\nu}\frac{f^2v'^2}{z} - \frac{1}{2}\frac{\partial f}{\partial v}v'^2 - \frac{1}{2}fv'^2\frac{\partial f}{\partial z} - v'z'\frac{\partial f}{\partial z}, \\ &+ 2\frac{(\nu+1)}{\nu z}fv'z'. \end{aligned}$$

The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0, v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{x,y_{1(\infty)}} = \frac{S_{x,y_{1(\infty)},ren}}{L_{y_1}}$$

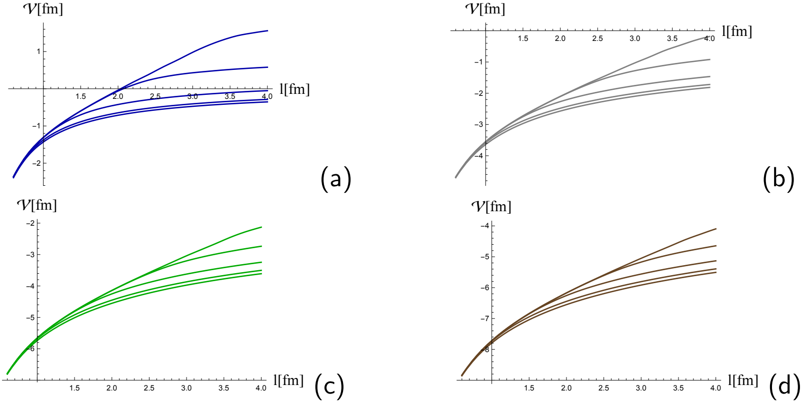

Figure: $\mathcal{V}_{x,y_{1(\infty)}}$ as a function of ℓ at fixed values of t for $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d), respectively). Different curves correspond to time $t = 0.1, 0.5, 0.9, 1.4, 2$ (from down to top, respectively).

$$\delta\mathcal{V}_1(x,t) = \mathcal{V}_{x,y_{1(\infty)}}(x,t) - \mathcal{V}_{x,y_{1(\infty)}}(x,t_f).$$

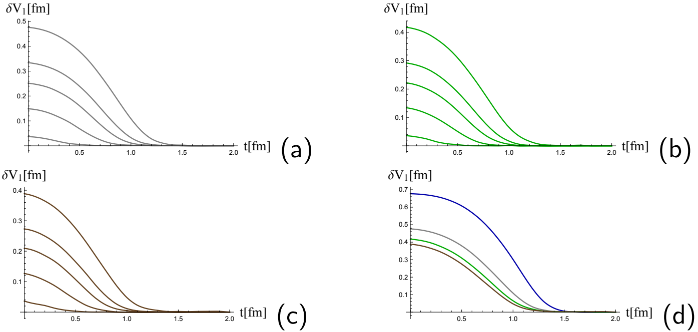

Figure: The time dependence of $-\delta\mathcal{V}_1(x,t)$, for different values of the length ℓ, $\nu = 2,3,4$ ((a),(b),(c), respectively). Different curves correspond to $\ell = 0.7, 1.2, 1.5, 1.7, 2$ (from down to top, respectively). In (d) we have shown $-\delta\mathcal{V}_1(x,t)$ as a function of t at $\ell = 2$ for $\nu = 1,2,3,4$ (from top to down).

WL in time-dependent backgrounds.Case 2

$$v = v(y_1), \quad z = z(y_1), \quad f = f(v,z)$$

$$S_{y_1,\,x_{(\infty)}} = \frac{L_x}{2\pi\alpha'}\int dy_1 \frac{1}{z^2}\sqrt{\left(\frac{1}{z^{2/\nu-2}} - f(z,v)(v')^2 - 2v'z'\right)}, \quad {}' \equiv \frac{d}{dy_1}.$$

The corresponding equations of motion are

$$\begin{aligned} v'' &= \frac{1}{2}\frac{\partial f}{\partial z}v'^2 + \frac{\nu+1}{\nu z}\left(z^{2-2/\nu} - \frac{2\nu}{(1+\nu)}fv'^2 - 2v'z'\right), \\ z'' &= -\frac{\nu+1}{\nu}fz^{1-2/\nu} + \frac{2(\nu-1)z'^2}{\nu} + \frac{2}{\nu}\frac{f^2v'^2}{z} - \frac{1}{2\nu}\frac{\partial f}{\partial v}v'^2 - \frac{1}{2\nu}f\frac{\partial f}{\partial z}v'^2 \\ &- z'v'\frac{\partial f}{\partial z} + \frac{4}{z}fz'v'. \end{aligned}$$

The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0, v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{y_1,\,x_{(\infty)}} = \frac{S_{y_1,\,x_{(\infty)},ren}}{L_{y_1}}.$$

WL in time-dependent backgrounds. Case 2

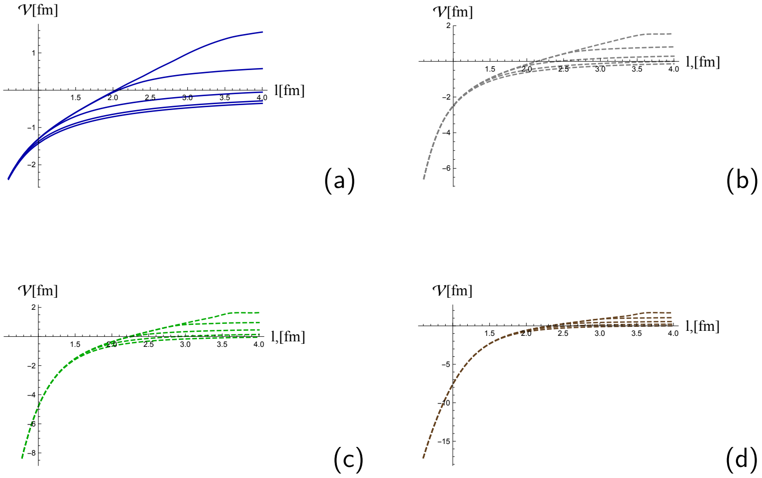

Figure: $\mathcal{V}_{y_1 x_\infty}$ as a function of ℓ at fixed values of t for $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d), respectively). Different curves correspond to $t = 0.1, 0.5, 0.9, 1.4, 2$ from down to top.

WL in time-dependent backgrounds. Case 2

$$\delta\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t)=\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t)-\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t_f).$$

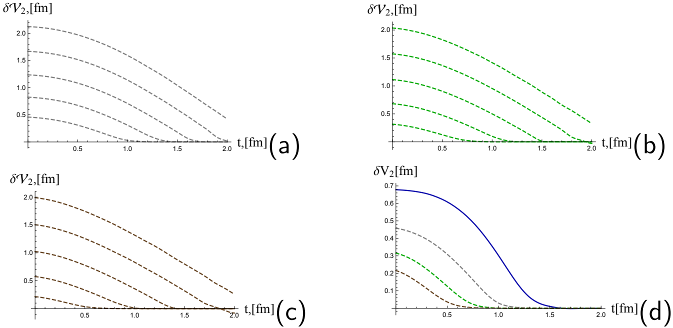

Figure: The time dependence of $-\delta\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t)$ for different values of the length ℓ, $\nu=2,3,4$ ((a),(b),(c), respectively). Different curves correspond to $\ell=2,2.5,3,3.5,4$ (from down to top, respectively). In (d) $-\delta\mathcal{V}_2(x,t)$ as a function of t at $\ell=2$ for $\nu=1,2,3,4$ (from top to down, respectively).

WL in time-dependent backgrounds.Case 3

$$v = v(y_1), \quad z = z(y_1), \quad f = f(v,z).$$

$$S_{y_1,\,y_{2,(\infty)}} = \frac{L_{y_2}}{2\pi\alpha'}\int dy_1 \frac{1}{z^{1+1/\nu}}\sqrt{\left(\frac{1}{z^{2/\nu-2}} - f(v')^2 - 2v'z'\right)}.$$

The corresponding equations of motion are

$$\begin{aligned}
v'' &= \frac{1}{2}\frac{\partial f}{\partial z}v'^2 + \frac{2}{z\nu}\left(z^{2-2/\nu} - \frac{\nu+1}{2}fv'^2 - 2v'z'\right),\\
z'' &= -\frac{2}{\nu}fz^{1-2/\nu} + 2\frac{\nu-1}{\nu}\frac{z'^2}{z} + \frac{\nu+1}{\nu z}f^2v'^2 - \frac{1}{2}\frac{\partial f}{\partial v}v'^2 - \frac{1}{2}f\frac{\partial f}{\partial z}v'^2\\
&- z'v'\frac{\partial f}{\partial z} + \frac{2(\nu+1)}{\nu z}fv'z'. \qquad (3)
\end{aligned}$$

The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0, v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{y_1,\,y_{2,(\infty)}}(t,\ell) = \frac{S_{y_1,\,y_{2,(\infty)},ren}}{L_{y_2}}.$$

WL in time-dependent backgrounds. Case 3

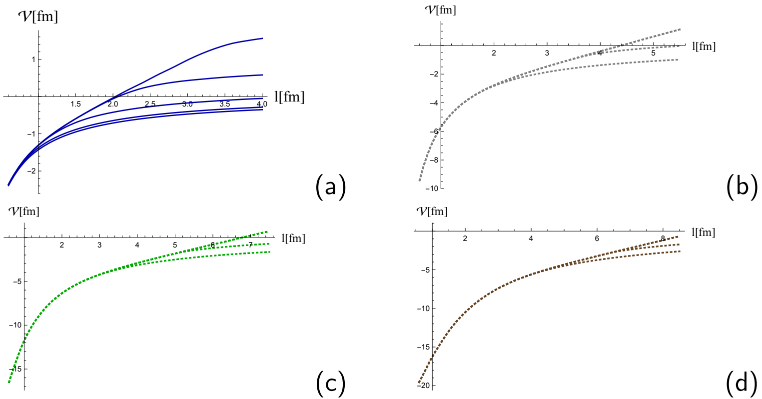

Figure: $\mathcal{V}_{y_1, y_{2,(\infty)}}(l, t)$ as a function of the length ℓ at fixed values of t, $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d)). (a): we take $t = 0.1, 0.5, 0.9, 1.4, 2$ from down to top, respectively; for plots (b),(c),(d): $t = 0.4, 1.5, 2.5, 3.34, 4$ from down to top, respectively.

WL in time-dependent backgrounds.Case 3

$$\delta\mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t) = \mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t) - \mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t_f).$$

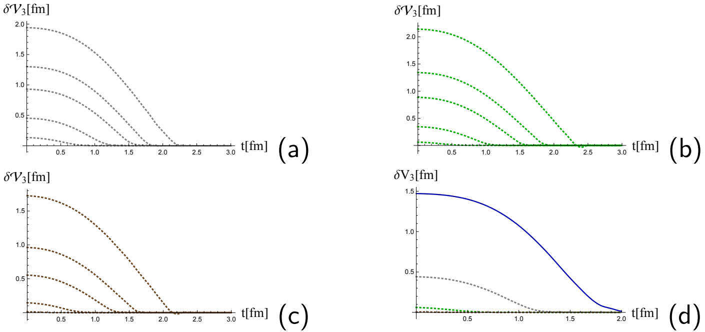

Figure: $-\delta\mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t)$ on t for different ℓ, $\nu = 2,3,4$ ((a),(b),(c)). (a): $l = 2.2, 3, 3.85, 4.4, 5.2$ from top to down; (b): $l = 3, 4.1, 5.2, 6, 7.1$ from top to down; (c): $l = 3.4, 4.6, 5.9, 6.8, 8$ from top to down. In (d): $-\delta\mathcal{V}_3(x,t)$ as a function of t at $\ell = 3$ for $\nu = 1,2,3,4$ (from top to down, respectively).

Summary and Outloook

Done

1. Black brane and shell solutions with Lifshitz-like asymptotics
2. Wilson loops in the Lifshitz-like backgrounds
3. Pseudopotentials and spatial string tensions

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Open questions

1. Time-like Willson loops, potentials, quarkonium spectrum that CAN FIT experemental data for multiplicity
2. Jet quenching parameter for anizotropic background that CAN FIT experemental data for multiplicity
3. Generalization for non-zero chemical potential (non-zero barionic density)
4. Isotropization, holographic RG-flow from $AdS_3 \times R^2$ to Lif_5?
5. Any supergravity embeddings?

Thank you for your attention!