Constrained Superfields and Applications

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Why constrained superfields?

$\rightarrow$ When SUSY is broken it is generically non-linearly realized.

$\rightarrow$ Constrained superfields give non-linear realizations.

$\rightarrow$ Consistency with superspace methods.

$\rightarrow$ Offer a "hint" to the UV theory.

The goldstino

SUSY breaking

- We can break SUSY with a chiral superfield

$$X = A + \sqrt{2}\,\theta^\alpha G_\alpha + \theta^\alpha \theta_\alpha F.$$

- We have a Lagrangian

$$\mathcal{L} = A\,\partial^2 \bar{A} + i\partial_m \bar{G}\bar{\sigma}^m G - V(A, \bar{A}) + \cdots$$

- SUSY broken:
  - Massless goldstino: $\delta G_\alpha = -f\,\epsilon_\alpha + \cdots$
  - $V(A_0, \bar{A}_0) = f^2 > 0.$
  - The scalar A is generically massive.

Nilpotent goldstino superfield

Rocek '78, Lindstrom, Rocek '79, Casalbuoni, De Curtis, Dominici, Feruglio, Gatto '89, Komargodski, Seiberg '09

- In IR the scalar can be removed by imposing the superspace constraint

$$X^2 = 0 \quad \to \quad A^2 + 2\sqrt{2}\theta(AG) + 2\,\theta^2(2AF - G^2) = 0$$
$$\to \quad X = \frac{G^2}{2F} + \sqrt{2}\,\theta^\alpha G_\alpha + \theta^\alpha\theta_\alpha F.$$

- Equivalent to sgoldstino decoupling.
- The simplest model for supersymmetry breaking is

$$\mathcal{L} = \int d^4\theta X\bar{X} + \left\{\int d^2\theta fX + c.c.\right\}$$
$$= -f^2 + i\partial_m\bar{G}\bar{\sigma}^m G + \frac{1}{4f^2}\bar{G}^2\partial^2 G^2 - \frac{1}{16f^6}G^2\bar{G}^2\partial^2 G^2\partial^2\bar{G}^2$$

and supersymmetry is non-linearly realized.

N=1 SUGRA and constrained superfields

- The same setup works in supergravity with

$$X^2 = 0,$$

and $K = X\overline{X}$ and $W = fX + W_0$.

- In the $G = 0$ gauge the component form is

$$e^{-1}\mathcal{L} = -\frac{1}{2}R + \frac{1}{2}\epsilon^{klmn}(\overline{\psi}_k\overline{\sigma}_l\mathcal{D}_m\psi_n - \psi_k\sigma_l\mathcal{D}_m\psi_n)$$

$$- W_0\,\overline{\psi}_a\overline{\sigma}^{ab}\overline{\psi}_b - \overline{W_0}\,\psi_a\sigma^{ab}\psi_b - |f|^2 + 3|W_0|^2.$$

Dudas, Ferrara, Kehagias, Sagnotti '15, Bergshoeff, Freedman, Kallosh, Van Proeyen '15, Bandos, Martucci, Sorokin, Tonin '15

- Bousso–Polchinski mechanism in SUGRA with three-form nilpotent chiral superfields. *FF, Kehagias, Racco, Riotto '16*

Constraints on matter superfields

Removing scalars

Brignole, Feruglio, Zwirner '97, Komargodski, Seiberg '09

- We have the matter chiral superfield

$$Y = y + \sqrt{2}\theta\chi^y + \theta^2 F^y .$$

- We impose

$$X\,Y = 0$$

which means

$$\left(\frac{G^2}{2F} + \sqrt{2}\,\theta G + \theta^2 F\right)\left(y + \sqrt{2}\theta\chi^y + \theta^2 F^y\right) = 0.$$

- This gives

$$y = \frac{G\,\chi^y}{F} - \frac{G^2}{2F^2} F^y .$$

A Zoo of constraints!

Komargodski, Seiberg '09

Keep only complex scalar

- For a chiral superfield $\mathcal{H}$ we impose

$$\overline{X} D_\alpha \mathcal{H} = 0 \;\rightarrow\; \mathcal{H}| = H + \text{fermions.}$$

Keep only real scalar

- For a chiral superfield $\mathcal{A}$

$$X\mathcal{A} = X\overline{\mathcal{A}} \;\rightarrow\; \mathcal{A}| = \phi + \text{fermions.}$$

Remove gaugino

- For a chiral superfield $W_\alpha = -\frac{1}{4}\overline{D}^2 D_\alpha V$

$$X\, W_\alpha = 0.$$

Can we organize?

Removing a single component

Dall'Agata, Dudas, FF '16

- For a generic superfield

$$Q = q + \theta\chi^q + \cdots$$

- We propose the constraint

$$X\overline{X}\,Q = 0.$$

- This removes only the lowest component

$$q = \frac{G\,\chi^q}{\sqrt{2}F} + \cdots$$

✓ Use to eliminate more components ($|X|^2 DQ = 0$ for χ^q).

✓ Reproduces all known constraints.

Example

- Lets see something complicated

$$\overline{X}\mathcal{A} = \overline{X}\overline{\mathcal{A}}.$$

- This gives

$$X\overline{X}(\mathcal{A} - \overline{\mathcal{A}}) = 0 \;:\; \text{Removes real scalar}$$

and

$$X\overline{X}D_\alpha\mathcal{A} = 0 \;:\; \text{Removes fermion}$$

and

$$X\overline{X}D^2\mathcal{A} = 0 \;:\; \text{Removes aux. field.}$$

Inflationary models with X and $\mathcal{A}$

Kahn, Roberts, Thaler '15, Ferrara, Kallosh, Thaler '15, Carrasco, Kallosh, Linde '15, Dall'Agata, FF '15

- A simple example is

$$K = X\bar{X} - \frac{1}{4}(\mathcal{A} - \bar{\mathcal{A}})^2\,,\ W = g(\mathcal{A}) + X\, f(\mathcal{A})$$

  with $\overline{f(z)} = f(\overline{z})$ and $\overline{g(z)} = g(\overline{z})$.

- In the $G = 0$ gauge, the full Lagrangian is

$$e^{-1}\mathcal{L} = -\frac{1}{2}R + \epsilon^{klmn}\overline{\psi}_k\overline{\sigma}_l\mathcal{D}_m\psi_n - g(\phi)\left(\overline{\psi}_a\overline{\sigma}^{ab}\overline{\psi}_b + \psi_a\sigma^{ab}\psi_b\right)$$
$$-\frac{1}{2}\partial\phi\partial\phi - \left(f(\phi)^2 - 3g(\phi)^2\right).$$

- Model building becomes extremely easy, but there is almost no predictive power.

But SUSY is not only $N = 1$, what happens for $N > 1$?

Broken $N = 2$ in low energy

Cribiori, Dall'Agata, FF '16

- We need two goldstini

$$\delta G_\alpha = \xi_\alpha \mathcal{F} + \dots$$
$$\delta \tilde{G}_\alpha = \tilde{\xi}_\alpha \mathcal{F} + \dots$$

  and the aux. field $\mathcal{F}$ which gets the vev.
- In $N = 2$

$$\mathcal{X} = \frac{G^2 \tilde{G}^2}{\mathcal{F}^3} + \theta \frac{G \tilde{G}^2}{\mathcal{F}^2} + \tilde{\theta} \frac{\tilde{G} G^2}{\mathcal{F}^2} + \theta^2 \frac{\tilde{G}^2}{\mathcal{F}} + \tilde{\theta}^2 \frac{G^2}{\mathcal{F}} + \theta \tilde{\theta} \frac{G \tilde{G}}{\mathcal{F}} + \tilde{\theta}^2 \theta G + \theta^2 \tilde{\theta} \tilde{G} + \theta^2 \tilde{\theta}^2 \mathcal{F}.$$

- The theory is

$$\mathcal{L} = \int d^8\theta \mathcal{X} \bar{\mathcal{X}} + f \int d^4\theta \mathcal{X} + c.c.$$

  and in component form we find

$$\mathcal{L} = -f^2 + i \partial_m \bar{G} \bar{\sigma}^m G + i \partial_m \bar{\tilde{G}} \bar{\sigma}^m \tilde{G} + \cdots$$

Broken $N = 2$ as constrained $N = 1$

- We find (for $\bar{D}_{\dot{\alpha}} H_{\dot{\beta}} = 0$)

$$
\begin{aligned}
X^2 &= 0, \\
\bar{X} X \, D_\beta H_{\dot{\alpha}} &= 0, \\
\bar{X} X \, D^2 H_{\dot{\alpha}} &= 0.
\end{aligned}
$$

- From the constraints in $N = 1$ we easily understand what is the components.
- The theory is

$$
\begin{aligned}
\mathcal{L} = \int d^4\theta \left(X\bar{X} - \left| \partial_m \left(\frac{X\bar{H}^2}{2} \right) \right|^2 + i\partial_m (X\bar{H}^\alpha) \sigma^m_{\alpha\dot{\alpha}} (\bar{X} H^{\dot{\alpha}}) \right) \\
+ f \left(\int d^2\theta X + c.c. \right).
\end{aligned}
$$

- On top of the manifest $N = 1$ this theory has a second supersymmetry non-linearly realized!

We also have the structure for generic N. *Cribiori, Dall'Agata, FF '16*

$$\mathcal{X} = \frac{G^{2N}}{\mathcal{F}^{2N-1}} + \cdots + \theta_J^{2N-1} G^J + \theta^{2N} \mathcal{F}.$$

- We can now study low energy limits of broken N supersymmetric theories in superspace.
- Coupling to matter and gauge fields and to remove component fields.
- Coupling to supergravity?

Summary - Outlook

- We have discussed the constrained superfields formalism.
- We studied low energy limits for generic N.
- A lot of activity and applications in supergravity cosmology.
- Other directions?

Thank you!