Heterotic Strings with Positive Cosmological Constant

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Based on work with J. Rizos

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Recent Developments in Strings & Gravity - Corfu

- Non-supersymmetric constructions have been extensively studied in the past
- Renewed interest in the context of String Phenomenology
- Attempts to construct non-supersymmetric heterotic vacua

with semi-realistic spectra

Two fundamental questions

- Tachyonic instabilities : either explicit, or spontaneous (Hagedorn, ...)
- Destabilisation of the classical vacuum : one loop tadpoles

I will return to these points later in the talk

Blaszczyk, Groot-Nibbelink Loukas, Ramos-Sanchez Abel, Dienes, Mavroudi Lukas, Lalak, Svanes Ashfaque, Athanasopoulos Faraggi, Sonmez, Ruehle Kounnas, Partouche Angelantonj, Tsulaia, Rizos A way to break supersymmetry

(stringy) Scherk-Schwarz mechanism

Scherk, Schwarz 1979 Rohm 1984 Kounnas, Porrati 1988 Kounnas, Rostand 1990

- Flat gauging of N=4 supergravity
- Spontaneous breaking of SUSY with exactly tractable worldsheet description
- Freely-acting orbifolds

Assume SUSY is (spontaneously) broken but the vacuum is classically stable

Study one-loop corrections to couplings in the low energy effective action

Scherk Schwarz mechanism

Deformation of vertex operators / fields by symmetry ${\boldsymbol{Q}}$

$$\Phi(X_5 + 2\pi R) = e^{iQ} \Phi(X_5)$$

$$\Phi(X_5) = e^{iQX_5/2\pi R} \sum_{m \in \mathbb{Z}} \Phi_m e^{imX_5/R}$$

$$\Phi(X_5)$$

Kaluza-Klein spectrum of charged states is shifted

 $M_{\rm KK} = \frac{|Q|}{2\pi R}$

Choose Q=F (spacetime fermion number)

Assigns different boundary conditions (& masses) to states within the same supermultiplet : spontaneous breaking of supersymmetry

Breaking scale $\sim I/R$, tied to the size of compact dimensions

Under certain well-defined conditions

Angelantonj, I.F., Tsulaia '14, '15

$$\Delta_{ab} = \sum_{i=1,2,3} \alpha_i \log \left[T_2^{(i)} U_2^{(i)} |\eta(T^{(i)})\eta(U^{(i)})|^4 \right] + \beta_i \log \left[T_2^{(i)} U_2^{(i)} |\vartheta_4(T^{(i)})\vartheta_2(U^{(i)})|^4 \right] + \gamma_i \log \left| j_2(T^{(i)}/2) - j_2(U^{(i)}) \right|^4$$

 α , β , γ $\,$: model dependent constants computable from the massless spectrum

Effective potential

- Scherk-Schwarz breaking exhibits no-scale structure
- The scale of SUSY breaking is not determined at tree level

$$m_{3/2} = rac{|U|}{\sqrt{T_2 U_2}}$$
 T, U are moduli at tree level

- Loop corrections to the effective potential may (de)stabilise the no-scale moduli
- Dynamical determination of SUSY breaking scale

What is the morphology of the one loop effective potential in such models ?

- Fixed points of the lattice (T-self dual) correspond to local extrema of the potential
- Natural scale in this problem : the string scale



- Standard Scherk-Schwarz breaking : fermions become massive $g = (-1)^{F_{
 m s.t.}} \delta$
- # bosons > # fermions at the massless level

$$V = -\int_{\mathcal{F}} d\mu \, Z(\tau, \bar{\tau}) \, \sim (n_F - n_B)/R^4$$

No decompactification problem : gauge couplings do not explode



SUSY is broken at the string scale $m_{3/2} \sim M_s$



Danger of encountering tachyonic modes ?

- Can we construct solutions with an abundance of massless fermions ?
- If so, we could expect a local maximum "spontaneous decompactification"



Opens the possibility for low scale SUSY breaking $m_{3/2} \sim 1/\sqrt{T_2}$



Favours large volume : no tachyons

- Can we construct solutions with an abundance of massless fermions ?
- If so, we could expect a local maximum "spontaneous decompactification"



For SUSY breaking at TeV range, the potential is still too large



Abel, Dienes, Mavroudi Kounnas, Partouche



I.F. and J. Rizos 2016

Question : Is it possible to construct such chiral models ?

• Answer: YES

BUT



although being necessary for suppressing the value of the cosmological constant, the condition for bose-fermi degeneracy is NOT sufficient

it turns out that non level-matched states around self-dual points crucially affect the shape of the potential, including its sign !

Using the fermionic construction with 9 basis vectors

$$\begin{split} v_1 &= \mathbf{1} = \{\psi^{\mu}, \, \chi^{1, \dots, 6}, \, y^{1, \dots, 6}, \, \omega^{1, \dots, 6} | \bar{y}^{1, \dots, 6}, \, \bar{\omega}^{1, \dots, 6}, \, \bar{\psi}^{1, \dots, 5}, \, \eta^{1, 2, 3}, \, \bar{\phi}^{1, \dots, 8} \} \\ v_2 &= S = \{\psi^{\mu}, \, \chi^{1, \dots, 6} \} \\ v_3 &= e_{12} = \{y^{1, 2}, \, \omega^{1, 2} | \bar{y}^{1, 2}, \, \bar{\omega}^{1, 2} \} \\ v_4 &= e_{34} = \{y^{3, 4}, \, \omega^{3, 4} | \bar{y}^{3, 4}, \, \bar{\omega}^{5, 6} \} \\ v_5 &= e_{56} = \{y^{5, 6}, \, \omega^{5, 6} | \bar{y}^{3, 4}, \, \bar{\omega}^{5, 6} \} \\ v_6 &= b_1 = \{\chi^{3, 4}, \, \chi^{5, 6}, \, y^{3, 4}, \, y^{5, 6} | \bar{y}^{3, 4}, \, \bar{y}^{5, 6}, \, \bar{\eta}^1, \, \bar{\psi}^{1, \dots, 5} \} \\ v_7 &= b_2 = \{\chi^{1, 2}, \, \chi^{5, 6}, \, y^{1, 2}, \, y^{5, 6} | \bar{y}^{1, 2}, \, \bar{y}^{5, 6}, \, \bar{\eta}^2, \, \bar{\psi}^{1, \dots, 5} \} \\ v_8 &= z_1 = \{\bar{\phi}^{1, \dots, 4} \} \\ v_9 &= z_2 = \{\bar{\phi}^{5, \dots, 8} \} \qquad + \text{choice of GGSO coefficients} \sim 10^8 \text{ models} \end{split}$$

• chirality

Conditions

- spontaneous breaking of N=1 SUSY
- no tachyons (at the fermionic point)

Model Classification

I.F. and J. Rizos 2016

Scan of random sample of 10⁶ models satisfying these conditions : 1135 models



Net # families

Example A

I.F. and J. Rizos 2016

Example A : net chirality 12 and $n_F > n_B$

$$Z = -312 - 12320q + \frac{2}{\bar{q}} + \frac{56q}{\bar{q}} + \frac{792q^2}{\bar{q}} - \frac{16q^{1/4}}{\bar{q}^{3/4}} - \frac{416q^{5/4}}{\bar{q}^{3/4}} - \frac{5520q^{9/4}}{\bar{q}^{3/4}} + \frac{32\sqrt{\bar{q}}}{\sqrt{\bar{q}}} - \frac{256q^{3/2}}{\sqrt{\bar{q}}} + \frac{512q^{3/4}}{\bar{q}^{1/4}} + \frac{11264q^{7/4}}{\bar{q}^{1/4}} + 4064q^{1/4}\bar{q}^{1/4} + 101568q^{5/4}\bar{q}^{1/4} + \dots$$

$$V = -\frac{1}{2(2\pi)^4} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} Z \approx +0.054$$

Example A

I.F. and J. Rizos 2016

Example A : net chirality I2 and $n_F > n_B$

$$T^2 \times T^2 \times T^2 / (\mathbb{Z}_2)^6$$

 $X^{1,2} \ X^{3,4} \ X^{5,6}$

$$\begin{split} \mathbb{Z}_{2}^{(1)} \times \mathbb{Z}_{2}^{(2)} + \text{standard embedding : explicitly breaking N=4 down to N=1} \\ \mathbb{Z}_{2}^{(3)} : (-1)^{F_{\text{s.t.}}+F_{2}} \delta_{1} \quad \text{Scherk-Schwarz breaking of N=1 to N=0} \\ \mathbb{Z}_{2}^{(4)} : (-1)^{F_{1}} \delta_{3} \quad \text{discrete Wilson line} \\ \mathbb{Z}_{2}^{(5)} : \delta_{5} \\ \mathbb{Z}_{2}^{(6)} : (-1)^{F_{1}} r = (0^{6}, 1^{2}; 0^{4}, \frac{1}{2}^{2}) \end{split}$$

+ a particular choice of discrete torsions

 $\epsilon(1,2), \epsilon(1,4), \epsilon(1,5), \epsilon(1,6), \epsilon(2,3), \epsilon(2,4), \epsilon(3,6), \epsilon(4,5), \epsilon(4,6)$

gauge group $SO(10) \times SO(8)^2 \times U(1)^3$



 $+\mathcal{O}(e^{-4\pi T_2})$

Counter Example B

Example B : net chirality 8 and $n_F = n_B$ at the generic point

$$\underbrace{Z = 8}_{= 8} + 1760q + \frac{2}{\bar{q}} + \frac{56q}{\bar{q}} - \frac{32q^{1/4}}{\bar{q}^{3/4}} + \frac{224\sqrt{q}}{\sqrt{\bar{q}}} - \frac{1024q^{3/4}}{\bar{q}^{1/4}} + 1984q^{1/4}\bar{q}^{1/4} + 30720\sqrt{q}\sqrt{\bar{q}} + \frac{2048\bar{q}^{3/4}}{q^{1/4}} + \dots$$

super no-scale model but with abundance of massless bosons at the fermionic point !

Example B : net chirality 8 and $n_F = n_B$ at the generic point

 $T^2 \times T^2 \times T^2 / (\mathbb{Z}_2)^6$ $X^{1,2} X^{3,4} X^{5,6}$

$$\begin{split} \mathbb{Z}_{2}^{(1)} \times \mathbb{Z}_{2}^{(2)} + \text{standard embedding : explicitly breaking N=4 down to N=1} \\ \mathbb{Z}_{2}^{(3)} : (-1)^{F_{\text{s.t.}} + F_1 + F_2} \delta_1 \quad \text{Scherk-Schwarz breaking of N=1 to N=0} \\ \mathbb{Z}_{2}^{(4)} : \delta_3 \\ \mathbb{Z}_{2}^{(5)} : \delta_5 \\ \mathbb{Z}_{2}^{(6)} : (0^8; 0^4, \frac{1}{2}^2) \end{split}$$

+ a particular choice of discrete torsions

 $\epsilon(1,4), \ \epsilon(1,5), \ \epsilon(2,3), \ \epsilon(2,4), \ \epsilon(3,4), \ \epsilon(5,6)$

gauge group $SO(10) \times SO(8)^2 \times U(1)^3$

Counter Example B

Example B : net chirality 8 and $n_F = n_B$ at the generic point



Counter Example B

Example B : net chirality 8 and $n_F = n_B$ at the generic point



what are the conditions for having the right shape for the potential ?

Small, positive cosmological constant?



exponentially suppressed vacuum energy for large volume $T_2 >> I$

how do we ensure that the potential has a positive maximum at the fermionic point?

Naively : one might think about additionally imposing $n_F > n_B$ at the fermionic point



this will not work ! (chirality & super no-scale structure)

expand the partition function as

 $Z = \sum_{\substack{n \in \mathbb{Z}/2 \\ n \ge -1/2}} \left[\sum_{\substack{m = -[n]-1 \\ model \text{ dependent} \\ \text{ degeneracies}}}^{\text{``asymmetry''}} \operatorname{``mass level''} \operatorname{``mass level''} \right]$

$$\tau = \tau_1 + i\tau_2$$

complex structure of the worldsheet torus

 $q_{\rm r} = e^{-2\pi\tau_2}$ $q_{\rm i} = e^{2\pi i\tau_1}$



intuition from field theory is based on S_1 but is S_2 negligible at self-dual points?

expand the integrals as

$$V = \sum_{n} Z_{n,0} I_{n,0}^{1} + \sum_{n,m} Z_{n,m} I_{n,m}^{2}$$

$$\begin{split} I_{n,m}^{1} &\equiv -\delta_{m,0} \int_{1}^{\infty} dy \, \frac{e^{-2\pi ny}}{y^{3}} = -\delta_{m,0} \, (2\pi n)^{2} \, \Gamma(-2, 2\pi n) \,, \\ I_{n,m}^{2} &\equiv -\int_{-1/2}^{1/2} dx \, e^{2\pi i |m|x} \, \int_{\sqrt{1-x^{2}}}^{1} dy \, \frac{e^{-2\pi ny}}{y^{3}} \end{split}$$

consider level-matched states first

 $I_{0,0}^1 = -1/2$

$$I_{n,0}^1 \sim -\frac{e^{-2\pi n}}{2\pi n}$$

negative contributions, as expected from field theory exponentially suppressed with increasing level n

$$I_{0,0}^2 = -(\log 3 - 1)/2$$

$$I_{n,0}^2 \sim -\frac{4}{3} \frac{e^{-\pi n\sqrt{3}}}{\pi n\sqrt{3}} \left[1 - \frac{3\sqrt{3}}{4\sqrt{2}} (\sqrt{3} - 1) \left(1 - \frac{2 + \sqrt{3}}{2\pi n} \right) \right] + \frac{e^{-2\pi n}}{2\pi n}$$

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expand the integrals as

$$V = \sum_{n} Z_{n,0} I_{n,0}^{1} + \sum_{n,m} Z_{n,m} I_{n,m}^{2}$$



 $I_{0,m}^2 \sim (-1)^{m+1} \left(\frac{2}{3\pi m}\right)^2$

exponentially suppressed with increasing level n

unlike field theory : alternating signs !

$$I_{n,m}^2 \sim \frac{(-1)^{m+1}}{2(\pi m)^2} \left(1 - \frac{1}{2m}\right) \frac{e^{-2\pi n \sqrt{1 - \frac{1}{4} \left(1 - \frac{1}{2m}\right)^2}}}{\left[1 - \frac{1}{4} \left(1 - \frac{1}{2m}\right)^2\right]^2}$$

unphysical states of low "mass" can reverse the sign of the potential !

an Anatomy of the Vacuum Energy

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expand the integrals as

 $V = \sum_{n} Z_{n,0} I_{n,0}^{1} + \sum_{n,m} Z_{n,m} I_{n,m}^{2}$

n	$\mid m$	0	±1	± 2	± 3	n	$\mid m$	0	±1	± 2	±3
-1	N/	Ά	0	N/A	N/A	-1		N/A	12.2	N/A	N/A
$-\frac{1}{2}$	N/	Ά	0	N/A	N/A	$-\frac{1}{2}$		N/A	0.617	N/A	N/A
0	-0.5	00	0	0	N/A	0		-0.0493	0.0315	-0.00989	N/A
$\frac{1}{2}$	-0.007	55	0	0	N/A	$\frac{1}{2}$		-0.00245	0.00163	-0.000587	N/A
1	-0.0002	08	0	0	0	1		-0.000123	0.0000846	-0.0000346	0.0000180
$\frac{3}{2}$	-6.61×10	-6	0	0	0	$\frac{3}{2}$		-6.24×10^{-6}	4.45×10^{-6}	-2.02×10^{-6}	1.11×10^{-6}

 $I_{n,m}^2$

 $I_{n,m}^1$

an Anatomy of the Vacuum Energy

I.F. and J. Rizos 2016

expand the integrals as

$$V = \sum_{n} Z_{n,0} I_{n,0}^{1} + \sum_{n,m} Z_{n,m} I_{n,m}^{2}$$

n	Model	А	В
-1		24.4	24.4
$-\frac{1}{2}$		-9.87	-19.7
0		172.	2.11
$\frac{1}{2}$		-29.6	-17.7
1		3.13	-2.73
$\frac{3}{2}$		9.71	8.18
Total		+170.	-5.47

unphysical tachyons have significant contribution !

the first few massive levels should be taken into account as well !

Example C : net chirality I2 and $n_F = n_B$ at the generic point

$$Z_{(C)} = \frac{2q_{i}}{q_{r}} - \frac{16q_{i}}{\sqrt{q_{r}}} + (40 + 64q_{i} + 56q_{i}^{2}) + \left(224 + \frac{6912}{q_{i}} + 768q_{i} - 672q_{i}^{2}\right)\sqrt{q_{r}}$$

$$+ \left(14336 + \frac{9216}{q_{i}^{2}} + \frac{118656}{q_{i}} - 10144q_{i} + 3072q_{i}^{2} + 792q_{i}^{3}\right)q_{r}$$

$$+ \left(-203776 + \frac{934400}{q_{i}^{2}} + \frac{498224}{q_{i}} - 39744q_{i} + 12800q_{i}^{2} - 10128q_{i}^{3}\right)q_{r}^{3/2} + \dots$$

super no-scale model with abundance of massless bosons at the fermionic point ! naively, worse than Model B !!!

Example C : net chirality I2 and $n_F = n_B$ at the generic point



the unphysical states manage to reverse the sign of the potential - checked to very high orders !

Example C : net chirality I2 and $n_F = n_B$ at the generic point

 $T^2 \times T^2 \times T^2 / (\mathbb{Z}_2)^6$ $X^{1,2} X^{3,4} X^{5,6}$

$$\begin{aligned} \mathbb{Z}_{2}^{(1)} &: X^{1,2,5,6} \to -X^{1,2,5,6} \\ \mathbb{Z}_{2}^{(2)} &: X^{3,4,5,6} \to -X^{3,4,5,6} \\ \mathbb{Z}_{2}^{(3)} &: (-1)^{F_{\text{s.t.}}+F_{2}} \delta_{1} , \delta_{1} : \{X_{1} \to X_{1} + \pi R_{1}\} \\ \mathbb{Z}_{2}^{(4)} &: (-1)^{F_{2}} \delta_{3} , \delta_{3} : \{X_{3} \to X_{3} + \pi R_{3}\} \\ \mathbb{Z}_{2}^{(5)} &: (-1)^{F_{1}+F_{2}} \delta_{5} , \delta_{5} : \{X^{5} \to X^{5} + \pi R_{5}\} \\ \mathbb{Z}_{2}^{(6)} &: (-1)^{F_{1}} r , r : (0^{8}; 0^{4}, \frac{1}{2}^{2}) \end{aligned}$$

+ a particular choice of discrete torsions

$$\epsilon(2,3), \ \epsilon(2,5), \ \epsilon(4,5), \ \epsilon(5,6)$$

Example C : net chirality I2 and $n_F = n_B$ at the generic point





what happens around $T_2=I$? is the "false vacuum" really stable?

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Example C : net chirality 12 and $n_F = n_B$ at the generic point



mass formula for lowest tachyonic states

$$M_{\rm BPS}^2 = \frac{1}{2} \left(T_2 + \frac{1}{T_2} \right) \left(U_2 + \frac{1}{4U_2} - \left| U_2 - \frac{1}{4U_2} \right| \right) - 1$$

tachyon free region in T,U parameter space

$$\left(T_2 + \frac{1}{T_2}\right)^{-1} \le U_2 \le \frac{1}{4}\left(T_2 + \frac{1}{T_2}\right)$$

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Example C : net chirality I2 and $n_F = n_B$ at the generic point



Example C : net chirality 12 and $n_F = n_B$ at the generic point



Some open questions

- Running of couplings : decompactification problem
- Linked to the presence of N=2 sectors and chirality
- Could it be that accidental cancellations occur in the beta functions for specific models ?
- Can this picture be coupled to a viable mechanism to stop the roll ?

Abel 2016



Thank you !