Standard Model Effective Field Theories at one loop: Some simple examples

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There are many excellent reviews on Effective Field Theories:

- S.Weinberg, Physica A96 (1979) 327; QTF 1995
- H. Georgi, Ann. Rev. Nucl. Part. Sci. 43 (1993) 209; WI&MPT 1984
- D.B. Kaplan, nucl-th/9506035
- A.V. Manohar, hep-ph/9508245
- A. Pich, hep-ph/9806303
- I.Z. Rothstein, hep-ph/0308266
- C.P. Burgess, hep-th/0701053
- W. Skiba, arXiv:1006.2142 [hep-ph]

 S. Dittmaier's (EW), A. Mitov's (QCD), A. Pich's (Flavor) and J.Valle's (V) lectures on the Standard Model, and H. Hollik's (SUSY), S. Pokorski's (SIH) and G. Ross' (GUT) lectures in Corfu 2016

Outline

- Standard Introduction
 - Operator basis: accidental symmetries and equations of motion
 - Decoupling and matching
- Recent developments on the Standard Model Effective Field Theory mainly deal with its phenomenological applications to confront the LHC data on top and especially Higgs physics, and with the calculation of the one-loop contributions of arbitrary additions of heavy particles with spin 0, 1/2 and 1:
 - Diagrammatic approach
 - Functional approach

- An effective field theory (EFT) is a model which is assumed to describe a physical system in a given range of energy (implying the existance of a heavy scale). It is more convenient because being precise enough, it is in general easier to deal and to calculate with.
- It is specified once the light fields and the symmetries of the interactions are fixed (assumed to be local).
- There is a renewed interest nowadays motivated by **experiment**: LHC is confirming a **gap** with increasing **precision**.
- We will assume that the Standard Model (SM) is the light sector of a weakly coupled more fundamental theory at a high scale Λ, above the TeV.
- In the following we review some of the ingredients to describe the **SM EFT up to one loop**.

Effective Lagrangian

accidental symmetries are not fulfiled by higher order operators -lepton number, ...-

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_{\dim \mathcal{O}_i > 4} \frac{c_i}{\Lambda^{\dim \mathcal{O}_i - 4}} \mathcal{O}_i$$

basis of local operators (no redundant)
-systematic use of equations of motion-

$$\left(\frac{E}{\Lambda}\right)^{\dim\mathcal{O}_i-4} \sim \epsilon \quad \Rightarrow$$

engineering dimension upper limit

$$\dim \mathcal{O}_i \sim 4 + \frac{\log \epsilon}{\log(E/\Lambda)} \quad \left(2 = \frac{\log 0.01}{\log \frac{100 \text{ GeV}}{1 \text{ TeV}}}\right)$$

finite number of independent operators

Accidental symmetries

Lepton number is an accidental symmetry of the (minimal) Standard Model because all renormalizable couplings among the electroweak quark and lepton doublets and singlets and invariant under the gauge symmetry group SU(3)XSU(2)XU(1) do also preserve **baryon** and **lepton number**.

However, already at next order there exists one dimension 5 operator with nonvanishing lepton number equal to 2:

S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566



P. Minkowski '77; M. Gell-Mann, P. Ramond and R. Slansky '79; T. Yanagida '79; R.N. Mohapatra and G. Senjanovic '80; J. Schechter and J.W.F. Valle '80

$$\frac{\delta \mathcal{L}}{\delta \overline{N_R}} = \mathrm{i} \not \!\!\!D N_R - MC(\overline{N_R})^T - \lambda \tilde{\phi}^{\dagger} \ell_L = 0$$

Writing the analogous equation for $\underset{\bigstar}{\mathsf{N}_{\mathsf{L}}}$ and combining them

$$[(i \not D)^2 - MM^{\dagger}]N_R - i \not D(\lambda \tilde{\phi}^{\dagger} \ell_L) - M\lambda^* C(\tilde{\phi}^{\dagger} \ell_L)^* = 0$$

$$N_R = -\left(\frac{1}{MM^{\dagger}} \left(1 - \frac{D^2}{MM^{\dagger}} + \dots\right) [i \not D(\lambda^{\dagger} \tilde{\phi}^{\dagger} \ell_L) + M\lambda^* C(\tilde{\phi}^{\dagger} \ell_L)^*\right)$$

$$\star N_L = (N_R)^c = C(\overline{N_R})^T, \quad N_R = (N_L)^c = C(\overline{N_L})^T$$

$$\mathcal{L}_{6} = \begin{bmatrix} \left(\alpha_{cd}^{(1)}\right)_{ij} \left[\left(\phi^{\dagger}\phi\right)\left(\overline{l_{L}^{t}}\phi^{\mu}l_{L}^{j}\right) + \left(\alpha_{cd}^{(3)}\right)_{ij}\right] \left(\phi^{\dagger}\phi\right)\left(\left(D_{\mu}\phi\right)^{\dagger}D^{\mu}\phi\right) + \alpha_{\phi}^{(3)}\left(\phi^{\dagger}D_{\mu}\phi\right) \left(\left(D^{\mu}\phi\right)^{\dagger}\phi\right) + \alpha_{\phi}^{(3)}\left(\phi^{\dagger}\phi\right)\right) \left(\left(D^{\mu}\phi\right)^{\dagger}\phi\right) + \alpha_{\phi}^{(3)}\left(\left(D^{\mu}\phi\right)^{\dagger}\phi\right) + \alpha_{\phi}^{(3)}\left(\left(D^{\mu}\phi\right)^{$$

F. del Aguila, J. de Blas, M. Perez-Victoria [arXiv:0803.4008 [hep-ph]]

The effective field theory does incorporate an expansion parameter v/ Λ which allows in principle to classify the size of the different effects.

In this case, what if, for instance, **no new physics is found up to 10**¹² **TeV** -already near the Planck scale- ?

- The new physics originating Majorana neutrino masses at so high scale is strongly interacting.
- Otherwise, neutrino masses are mainly **Dirac** -requiring the existance of 3 light right-handed neutrinos-.

$$\begin{pmatrix} 0 & m \\ m & M \sim \Lambda \end{pmatrix} \rightarrow \begin{pmatrix} 0 & m \\ m & m' \sim 0 \end{pmatrix}$$

Equations of motion and field redefinitions

Relevant (dimension < 4), marginal (dimension = 4) and irrelevant (dimension > 4) operators, which it is convenient to bring to a canonical form without redundancies:

- Necessary to make a meaningful comparison between different (phenomenological) analyses.
- Although there are subsets more suitable for given data subsets.

As in the case of lepton number violation, just discussed, new physics may only contribute at tree level to relatively large orders (in I/Λ) but in general its effects will also manifest at lower orders after quantum corrections are taken into account.

$$\mathcal{L}_{SM} = \overline{q_L} \mathbf{i} \mathcal{D} q_L + \overline{t_R} \mathbf{i} \mathcal{D} t_R - \lambda_t (\overline{q_L} \tilde{\phi} t_R + \overline{t_R} \tilde{\phi}^{\dagger} q_L) + \dots$$

$$\mathcal{O}_{\phi q}^{(1)} = \phi^{\dagger} \phi \ \overline{q_L} \text{ i } \not D \ q_L \xrightarrow{q_L \text{ EOM:}} \lambda_t \ (\mathcal{O}_{u\phi} = \phi^{\dagger} \phi \ \overline{q_L} \ \tilde{\phi} \ t_R)$$
$$i \not D q_L - \lambda_t \tilde{\phi} t_R = 0$$

$$\begin{split} \mathcal{Z}[J] &= \int D\varphi \, \exp\left\{\mathrm{i} \int dx \, \left[\mathcal{L}(\varphi(x), \partial_{\mu}\varphi(x)) + J(x)\varphi(x)\right]\right\} \text{ c. Arzt, hep-ph/9304230} \\ \mathcal{S}_{\mathrm{eff}} &= \int d^{4}x \, \mathcal{L}_{\mathrm{eff}} \\ \mathcal{L}_{\mathrm{eff}} &= \mathcal{L}_{\mathrm{SM}} + \sum_{\mathrm{dim}\mathcal{O}_{i} > 4} \frac{c_{i}}{\Lambda^{\mathrm{dim}\mathcal{O}_{i} - 4}} \mathcal{O}_{i} = \sum_{n=0}^{\infty} \mathcal{L}_{n} \end{split}$$

Any combination of terms which allows for the factorization of an equation of motion of a light field ϕ can be removed because it has no contribution to the S-matrix

$$\mathcal{L}_{m} = \frac{1}{\Lambda^{m}} \Big(\dots + f(\varphi, \partial_{\mu}\varphi) \Big(\frac{\delta \mathcal{L}_{\mathrm{SM}}}{\delta \varphi^{\dagger}} - \partial_{\mu} \frac{\delta \mathcal{L}_{\mathrm{SM}}}{\delta \partial_{\mu} \varphi^{\dagger}} \Big) \Big)$$
$$\frac{\delta \mathcal{L}_{\mathrm{SM}}}{\delta \varphi^{\dagger}} - \partial_{\mu} \frac{\delta \mathcal{L}_{\mathrm{SM}}}{\delta \partial_{\mu} \varphi^{\dagger}} = 0$$

This follows from the observation that the field redefinition

cancels such a combination without modifying

$$\varphi^{\dagger} \to \varphi^{\dagger} - \frac{1}{\Lambda^m} f(\varphi, \partial_{\mu} \varphi)$$
$$\mathcal{L} = \sum_{n=0}^m \mathcal{L}_n$$

$$\alpha_{\phi q}^{(1)} \left(\left(\mathcal{O}_{\phi q}^{(1)} = \phi^{\dagger} \phi \ \overline{q_L} \ i \not D \ q_L \right) \longrightarrow \lambda_t \left(\mathcal{O}_{u\phi} = \phi^{\dagger} \phi \ \overline{q_L} \ \tilde{\phi} \ t_R \right) \right)$$

Warsaw basis

W. Buchmüller and D. Wyler, Nucl. Phys. B268 (1986) 621

Gauge-invariant dimension 6 operators constructed with the Standard Model fields, up to redundancies which are taking care using equations of motion for fermions and gauge and Higgs bosons, integration by parts, Pauli (SU(2)) and Gell-Mann (SU(3)) matrix properties and Fierz transformations

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B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, [arXiv:1008.4884 [hep-ph]]
J.A. Aguilar-Saavedra, [arXiv:1008.3562 [hep-ph]]
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Ignoring flavor indices and assuming that there are not light right-handed neutrinos (and that baryon number is conserved)

<u>Operator basis</u>	<u>1986</u>	<u>2010</u>
No fermions	16	15
2 fermions	35	19
4 fermions	29	25
Dimension 6	80	59

	X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 \varphi^3$
Q_G	$f^{ABC}G^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$	Q_{φ}	$(arphi^\dagger arphi)^3$	$Q_{e\varphi}$	$(\varphi^{\dagger}\varphi)(\bar{l}_{p}e_{r}\varphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}$	$Q_{\varphi \Box}$	$(\varphi^{\dagger}\varphi)\Box(\varphi^{\dagger}\varphi)$	$Q_{u\varphi}$	$(\varphi^{\dagger}\varphi)(\bar{q}_{p}u_{r}\widetilde{\varphi})$
Q_W	$\varepsilon^{IJK}W^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$	$Q_{\varphi D}$	$\left(\varphi^{\dagger}D^{\mu}\varphi\right)^{\star}\left(\varphi^{\dagger}D_{\mu}\varphi\right)$	$Q_{d\varphi}$	$(\varphi^{\dagger}\varphi)(\bar{q}_{p}d_{r}\varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$		******		
	$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$
$Q_{\varphi G}$	$\varphi^{\dagger}\varphiG^{A}_{\mu\nu}G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{l}_{p}\gamma^{\mu}l_{r})$
$Q_{\varphi \widetilde{G}}$	$\varphi^{\dagger}\varphi\widetilde{G}^{A}_{\mu\nu}G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^{\dagger}i\overleftrightarrow{D}^{I}_{\mu}\varphi)(\bar{l}_{p}\tau^{I}\gamma^{\mu}l_{r})$
$Q_{\varphi W}$	$\varphi^{\dagger}\varphiW^{I}_{\mu\nu}W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{\varphi} G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{e}_{p}\gamma^{\mu}e_{r})$
$Q_{\varphi \widetilde{W}}$	$\varphi^{\dagger}\varphi\widetilde{W}^{I}_{\mu\nu}W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{\varphi} W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{q}_{p}\gamma^{\mu}q_{r})$
$Q_{\varphi B}$	$\varphi^{\dagger}\varphi B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^{\dagger}i\overleftrightarrow{D}^{I}_{\mu}\varphi)(\bar{q}_{p}\tau^{I}\gamma^{\mu}q_{r})$
$Q_{\varphi \widetilde{B}}$	$\varphi^{\dagger}\varphi\widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{u}_{p}\gamma^{\mu}u_{r})$
$Q_{\varphi WB}$	$\varphi^{\dagger}\tau^{I}\varphiW^{I}_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\bar{d}_{p}\gamma^{\mu}d_{r})$
$Q_{\varphi \widetilde{W}B}$	$\varphi^{\dagger}\tau^{I}\varphi\widetilde{W}^{I}_{\mu\nu}B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^{\dagger}D_{\mu}\varphi)(\bar{u}_{p}\gamma^{\mu}d_{r})$

	$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r) (\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r) (\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r) (\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r) (\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r) (\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r) (\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r) (\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r) (\bar{d}_s \gamma^\mu T^A d_t)$
$(\bar{L}R)$	$(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		<i>B</i> -violating		
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^{\alpha})^T C u_r^{\beta}\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$		
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^{\gamma})^T C e_t\right]$		
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$		
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^{I}\varepsilon)_{jk}(\tau^{I}\varepsilon)_{mn}\left[(q_{p}^{\alpha j})^{T}Cq_{r}^{\beta k}\right]\left[(q_{s}^{\gamma m})^{T}Cl_{t}^{n}\right]$		
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma} \left[(d_p^{\alpha})^T C u_r^{\beta} \right] \left[(u_s^{\gamma})^T C e_t \right]$		

Flavor changing top couplings

J.A. Aguilar-Saavedra, [arXiv:0904.2387 [hep-ph]]

$$\mathcal{L}_{Ztc} = -\frac{g}{2c_W} \bar{c} \gamma^{\mu} \left(X_{ct}^L P_L + X_{ct}^R P_R \right) t Z_{\mu} - \frac{g}{2c_W} \bar{c} \frac{i\sigma^{\mu\nu} q_{\nu}}{M_Z} \left(\kappa_{ct}^L P_L + \kappa_{ct}^R P_R \right) t Z_{\mu} + \text{H.c.}$$

$$\begin{split} \delta X_{ct}^{L} &= \frac{1}{2} \left[C_{\phi q}^{(3,2+3)} - C_{\phi q}^{(1,2+3)} \right] \frac{v^{2}}{\Lambda^{2}} \,, \\ \delta X_{ct}^{R} &= -\frac{1}{2} C_{\phi u}^{2+3} \frac{v^{2}}{\Lambda^{2}} \,, \\ \delta \kappa_{ct}^{L} &= \sqrt{2} \left[c_{W} C_{uW}^{32*} - s_{W} C_{uB\phi}^{32*} \right] \frac{v^{2}}{\Lambda^{2}} \,, \end{split}$$

 $\delta \kappa_{ct}^R = \sqrt{2} \left[c_W C_{uW}^{23} - s_W C_{uB\phi}^{23} \right] \frac{v^2}{\Lambda^2}$

 $\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}^{I}\varphi \equiv i\varphi^{\dagger}\left(\tau^{I}D_{\mu}-\overleftarrow{D}_{\mu}\tau^{I}\right)\varphi$ $\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi \equiv i\varphi^{\dagger}\left(D_{\mu}-\overleftarrow{D}_{\mu}\right)\varphi$

3 (2) stands for t (c)

$$\mathcal{L}_{Htc} = -\frac{1}{\sqrt{2}} \,\bar{c} \left(\eta_{ct}^L P_L + \eta_{ct}^R P_R \right) t \, H + \text{H.c}$$

$$\delta\eta_{ct}^L = -\frac{3}{2} C_{u\phi}^{32*} \frac{v^2}{\Lambda^2}$$
$$\delta\eta_{ct}^R = -\frac{3}{2} C_{u\phi}^{23} \frac{v^2}{\Lambda^2}$$

Three-body top decays and single and pair top production

Pauli (T) and Gell-Mann (λ) matrices:

Fierz rearrangements:

$$\sum_{I=1}^{3} (\tau^{I})_{ij} (\tau^{I})_{kl} = 2 \left(\delta_{il} \delta_{kj} - \frac{1}{2} \delta_{ij} \delta_{kl} \right) \qquad (\bar{A}_{L} \gamma^{\mu} B_{L}) (\bar{C}_{L} \gamma_{\mu} D_{L}) = (\bar{A}_{L} \gamma^{\mu} D_{L}) (\bar{C}_{L} \gamma_{\mu} B_{L}) (\bar{A}_{R} \gamma^{\mu} B_{R}) (\bar{C}_{R} \gamma_{\mu} D_{R}) = (\bar{A}_{R} \gamma^{\mu} D_{R}) (\bar{C}_{R} \gamma_{\mu} B_{R}) \sum_{a=1}^{8} (\lambda^{a})_{ij} (\lambda^{a})_{kl} = 2 \left(\delta_{il} \delta_{kj} - \frac{1}{3} \delta_{ij} \delta_{kl} \right) (\bar{A}_{R} \gamma^{\mu} B_{R}) (\bar{C}_{L} \gamma_{\mu} D_{L}) = -2 (\bar{C}_{L} B_{R}) (\bar{A}_{R} D_{L}) two orderings$$

572 independent gauge-invariant four-fermion operators involving one or two top quarks (taking into account different fermion chiralities, colour contractions and flavour combinations) -out of the 25 different types of independent four-fermion operators-.

2

Loop expansion and small Wilson coefficients

The expansion of the effective lagrangian is on the small ratio E/ Λ between the available energy (E) in the process and the heavy scale (Λ) set by the new physics. However, the theory and then, the Wilson coefficients, are also an expansion on the interaction parameters $\alpha/4\pi$, i.e., on the number of loop corrections.

Hence, one must worry about the relative size of the different coefficients, too. As a matter of fact, only 39 of the 59 independent operators of dimension 6 considered are generated at tree level by exchange of particles of spin 0, 1/2 and 1 (interaction terms up to dimension 4).

C. Arzt, M.B. Einhorn and J. Wudka, hep-ph/9405214; M.B. Einhorn and J. Wudka, arXiv:1307.0478[hep-ph] The X^3 , $X^2\phi^2$ and $\psi^2X\phi$ sets are loop generated.

Analogously, other bases can be more convenient to account for particular new physics additions because they capture their contributions in a smaller set of independent operators. A prime example, very timely, is the case of universal heavy physics primarely manifesting in the Higgs sector.

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{c_{\varphi \Box}}{\Lambda^2} \mathcal{O}_{\varphi \Box} \rightarrow \frac{1}{2} (1 - 2c_{\varphi \Box} \frac{v^2}{\Lambda^2}) \partial_{\mu} h \partial^{\mu} h + \dots, \quad v \sim 246 \text{ GeV}$$
$$h \rightarrow (1 + c_{\varphi \Box} \frac{v^2}{\Lambda^2}) h$$
Similarly, the operators $\mathcal{O}_{e\varphi, u\varphi, d\varphi}$ (eventually flavor dependent) modify the Higgs couplings

G.F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi [hep-ph/0703164]
J. Elias-Miro, J.R. Espinosa, E. Masso, A. Pomarol [arXiv:1308.1879 [hep-ph]]
R. Contino, A. Falkowski, F. Goertz, C. Grojean and F. Riva [arXiv:1604.06444 [hep-ph]]

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{v^2}{4} \text{tr} \left(D_{\mu} \Sigma^{\dagger} D^{\mu} \Sigma \right) \left(1 + 2\kappa_V \frac{H}{v} + \cdots \right) - m_i \bar{f}_L^i \left(1 + 2\kappa_f \frac{H}{v} + \cdots \right) f_R^i + \cdots \\ \Sigma &= \exp \left(i \tau^a \frac{\chi^a(x)}{v} \right) \qquad \text{In the SM } \kappa_{\text{V,f}} = \mathbf{I} \end{aligned}$$

J. de Blas, M. Ciuchini, E. Franco, S. Mishima, M. Pierini, L. Reina and L. Silvestrini [arXiv:1608.01509 [hep-ph]]



Figure 7. Left: constraints from individual channels at 95% probability. Right: two-dimensional probability distributions for κ_V and κ_f at 68%, 95%, and 99% (darker to lighter), obtained from the fit to the Higgs-boson signal strengths.



Figure 10. Two-dimensional probability distributions for κ_V and κ_ℓ , for κ_V and κ_u , and for κ_V and κ_d , at 68%, 95%, and 99% (darker to lighter), obtained from the fit to the Higgs-boson signal strengths only (top plots) or the combination of Higgs-boson signal strengths and EWPO (bottom plots).

Effective field theories can make calculations not only easier but also allow to parameterize any unknown heavy physics. Although, the increasingly larger number of higher order operators and hence, of possible cancellations, may reduce the significance of the experimental fits and contraints.

Decoupling

Decoupling is inherent to Effective Field Theories, the question being if the fundamental field theory, which we assume originates it, does really decouple.

Natural (small) parameter

$$\mathcal{L} = \overline{\psi} (\mathbf{i} \ \mathcal{D} - \mathbf{M}) \psi + \frac{1}{2} \partial_{\mu} \phi \ \partial^{\mu} \phi - \frac{\mathbf{m}^{2}}{2} \phi^{2} - \lambda \overline{\psi} \psi \phi \qquad \text{M.D. Schwartz, QFT&SM}$$

$$\underbrace{\mathbf{M} \cdot \mathbf{L} \cdot \mathbf{M} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{M} \cdot \mathbf{M} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{M} \cdot \mathbf{M} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{M} \cdot \mathbf{M} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{M} \cdot \mathbf{M} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{M} \cdot \mathbf{M} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{L} \cdot \mathbf{M} \cdot \mathbf{L} \cdot \mathbf{L$$

$$\cdots \underbrace{(\sum \lambda^2 (p^2) = -i\frac{\lambda^2}{4\pi^2} \left\{ \frac{-p^2 + 6M^2}{\epsilon} - \frac{p^2}{6} + \underbrace{M^2}_{0} + \int_0^1 dx 3[x(1-x)p^2 - \underbrace{M^2}_{0}] \log \underbrace{\underbrace{M^2 + x(1-x)p^2}_{\overline{\mu}^2}}_{\overline{\mu}^2} \right\}}$$

no protection: hierarchy must be enforced

Natural parameters according to P.A.M. Dirac or G.'t Hooft, technical naturalness (weakest).

The **decoupling theorem** states that low-energy effects of heavy particles are suppressed by inverse powers of their masses or can be absorbed into the renormalisation of the light fields and couplings.

This does not need to apply if a coupling becomes large, making the theory non-perturbative. It is also convenient to use a renormalisation scheme where the decoupling is manifest. In any case one must impose all the conditions needed to define the hierarchy.

$$\mathcal{L} = \overline{\psi}(i \ \partial - M)\psi + \frac{1}{2} (\sum_{j=1,2} \partial_{\mu}\phi_j \ \partial^{\mu}\phi_j - \frac{m_{jj}^2}{2}\phi_j^2) - (\lambda_1\phi_1 + \lambda_2\phi_2)\overline{\psi}\psi \\ \begin{pmatrix} m_{11}^2 & m_{12}^2 \\ m_{12}^2 & m_{22}^2 \end{pmatrix} + \begin{pmatrix} \sim \lambda_1^2 M^2 & \sim \lambda_1\lambda_2 M^2 \\ \sim \lambda_2\lambda_1 M^2 & \sim \lambda_2^2 M^2 \end{pmatrix} \\ \hline m_{11}'^2, \ m_{11}'^2 m_{22}' > m_{12}'^2 m_{21}'^2$$

F. del Aguila, M. Masip and M. Perez-Victoria, hep-ph/9507455 P.H. Chankowski, S. Pokorski and J. Wagner, hep-ph/0601097

The addition of an extra U(I) with the mass of the new Z' going to infinity in a spontaneously broken gauge symmetry implies that the vacuum expectation value of the corresponding new scalar σ is neutral under the SM. Otherwise, the Z mass will go to infinity, too. Hence, the Z-Z' mass matrix has the form: $\begin{pmatrix} & v_{\rm SM}^2 & & v_{\rm SM}^2 \\ & v_{\rm SM}^2 & & v_{\sigma}^2 \end{pmatrix}$, being then enough to constraint v_{SM}. A more detailed study is required for $\begin{pmatrix} m^2 & mM \\ mM & M^2 \end{pmatrix}$

Matching

As already emphasized by H. Georgi, the relevant question is: Which is the difference of integrating out heavy modes and using the resulting non-local theory from calculating with the effective field theory ?

- In general easier calculations.
- Subtraction of infinities rather than renormalization (DR with \overline{MS}).
- Dimensional analysis applies

However, although the effective field theory in general allows for an easier description of the relevant physics, usually providing a better understanding of the important effects, the right physics must be put by hand requiring that the predictions of the effective field theory **match** those of the fundamental theory at the appropriate scale.

Let us review the different scenarios for lepton number violation beyond the Standard Model, already discussed at Corfu in 2012 by A. Santamaria:

F. del Aguila, A. Aparici, S.Bhattacharya, A. Santamaria and J. Wudka [arXiv:1305.4900 [hep-ph]]

LL:
$$\mathscr{O}^{(5)} = \left(\overline{\tilde{\ell}_{\alpha}}\phi\right)\left(\tilde{\phi}^{\dagger}\ell_{\beta}\right) = -v^{2}\overline{v_{\alpha L}^{c}}v_{\beta L} + \dots,$$

LR :
$$\mathscr{O}^{(7)} = \left(\phi^{\dagger} D_{\mu} \tilde{\phi}\right) \left(\phi^{\dagger} \overline{e_{\alpha R}} \gamma^{\mu} \tilde{\ell}_{\beta}\right) = i \frac{g v^{3}}{\sqrt{2}} W_{\mu}^{-} \overline{e_{\alpha R}} \gamma^{\mu} v_{\beta L}^{c} + \dots \text{ not involving quarks}$$

$$\operatorname{RR}: \qquad \mathscr{O}^{(9)} = \overline{e_{\alpha R}} e_{\beta R}^{c} \left(\phi^{\dagger} D \, \tilde{\phi} \right)^{2} = -\frac{g^{2} v^{4}}{2} W_{\mu}^{-} W^{-\mu} \overline{e_{\alpha R}} e_{\beta R}^{c} + \dots,$$



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F. del Aguila, A. Aparici, S. Bhattacharya, A. Santamaria and J. Wudka [arXiv:1111.6960 [hep-ph]]

$$\mathscr{L} = g_{\alpha\beta} \,\overline{e_{\alpha R}}^{c} e_{\beta R} \,\kappa - \mu_{\kappa} \,\kappa \mathrm{Tr} \left\{ \chi^{\dagger} \chi^{\dagger} \right\} - \lambda_{6} \,\sigma \,\phi^{\dagger} \chi \tilde{\phi} + \cdots$$







Matching

The matching conditions are fixed by requiring that the two theories describe the same physics. What is guaranteed requiring the equality of the ILPI Green functions of the fundamental and the effective field theories at the matching scale, fixed to minimize the logarithmic terms.



Heavy vector-like quark of charge 2/3 Large µ $\mathcal{L}_{SM} + \mathcal{L}_T$ $\mathcal{L}_{SM} = \overline{q_L} \mathbf{i} \mathcal{D} q_L + \overline{t_R} \mathbf{i} \mathcal{D} t_R - \lambda_t (\overline{q_L} \phi t_R + \overline{t_R} \phi^{\dagger} q_L) + \dots$ $\mathcal{L}_T = \overline{T}(i D - M)T - \lambda_T(\overline{q_L}\tilde{\phi}T_R + \overline{T_R}\tilde{\phi}^{\dagger}q_L)$ F. del Aguila, M. Perez-Victoria and J. Santiago [hep-ph/0007316] $\mathcal{L}_{SM} + \mathcal{L}_{\epsilon}^{(0l)}$ $\mathcal{L}_{6}^{(0l)} = \alpha_{\phi q}^{(1)} \mathcal{O}_{\phi q}^{(1)} + \alpha_{\phi q}^{(3)} \mathcal{O}_{\phi q}^{(3)} + \alpha_{u\phi} \mathcal{O}_{u\phi} + h.c.$ $\mathcal{O}_{\phi q}^{(1)} = \mathrm{i}\phi^{\dagger} D_{\mu}\phi \ \bar{q}\gamma^{\mu}q, \ \mathcal{O}_{\phi q}^{(3)} = \mathrm{i}\phi^{\dagger}\sigma^{a} D_{\mu}\phi \ \bar{q}\gamma^{\mu}\sigma^{a}q, \ \mathcal{O}_{u\phi} = \phi^{\dagger}\phi\bar{q}\tilde{\phi}t$ $\alpha_{\phi q}^{(1)} = -\alpha_{\phi q}^{(3)} = \frac{|\lambda_T|^2}{4M^2}, \ \alpha_{u\phi} = 2\lambda_t \alpha_{\phi q}^{(1)}$ p 1 Small µ

EOM (no T_L interaction): $T_L = \frac{1}{M} (-\lambda_T \tilde{\phi}^{\dagger} q_L), \ T_R = \frac{\mathrm{i} \not D}{M^2} (-\lambda_T \tilde{\phi}^{\dagger} q_L)$

★ use t EOM to write the dimension 6 effective Lagrangian in the Warsaw basis SM EFT 28

Heavy vector-like quark of charge 2/3 Large µ $\mathcal{L}_{SM} + \mathcal{L}_T$ $\mathcal{L}_{SM} = \overline{q_L} \mathbf{i} \mathcal{D} q_L + \overline{t_R} \mathbf{i} \mathcal{D} t_R - \lambda_t (\overline{q_L} \phi t_R + \overline{t_R} \phi^{\dagger} q_L) + \dots$ $\mathcal{L}_T = \overline{T}(i \not D - M)T - \lambda_T (\overline{q_L} \tilde{\phi} T_R + \overline{T_R} \tilde{\phi}^{\dagger} q_L)$ F. del Aguila, Z. Kunszt and J. Santiago [arXiv:1602.00126 [hep-ph]] C. Anastasiou, A. Carmona, A. Lazopoulos and J. Santiago MatchMaker $\mathcal{L}_{SM} + \mathcal{L}_{6}^{(1l)}$ $\mathcal{L}_{6}^{(0l)} = \alpha_{\phi a}^{(1)} \mathcal{O}_{\phi a}^{(1)} + \alpha_{\phi a}^{(3)} \mathcal{O}_{\phi a}^{(3)} + \alpha_{u\phi} \mathcal{O}_{u\phi} + h.c.$ $\mathcal{O}_{\phi q}^{(1)} = \mathrm{i}\phi^{\dagger} D_{\mu}\phi \ \bar{q}\gamma^{\mu}q, \ \mathcal{O}_{\phi q}^{(3)} = \mathrm{i}\phi^{\dagger}\sigma^{a} D_{\mu}\phi \ \bar{q}\gamma^{\mu}\sigma^{a}q, \ \mathcal{O}_{u\phi} = \phi^{\dagger}\phi\bar{q}\tilde{\phi}t$

 $\mathcal{O}_{1} = |\phi^{\dagger} D_{\mu} \phi|^{2}, \quad \mathcal{O}_{2} = \phi^{\dagger} \phi \ \partial^{2} (\phi^{\dagger} \phi), \quad \mathcal{R} = \phi^{\ddagger} \phi - \phi^{\dagger} - D^{2} \phi \rightarrow \text{use } \Phi \text{ EOM}$ $\alpha_{\mathcal{R}}^{(1l)} = \frac{N_{C} |\lambda_{T}|^{2}}{16\pi^{2} M^{2}} \left(-\frac{1}{2} \lambda_{t}^{2} + \frac{1}{2} |\lambda_{T}|^{2} \right)$

$$\alpha_1^{(1l)} = \frac{N_C |\lambda_T|^2}{16\pi^2 M^2} \left(\frac{1}{2}\lambda_t^2 - \frac{1}{2}|\lambda_T|^2\right), \ \alpha_2^{(1l)} = \frac{N_C |\lambda_T|^2}{16\pi^2 M^2} \left(\frac{3}{2}\lambda_t^2 - \frac{1}{3}|\lambda_T|^2\right)$$



Small µ

Heavy vector-like quark of charge 2/3 Large µ $\mathcal{L}_{SM} + \mathcal{L}_T$ $\mathcal{L}_{SM} = \overline{q_L} \mathbf{i} D q_L + \overline{t_R} \mathbf{i} D t_R - \lambda_t (\overline{q_L} \phi t_R + \overline{t_R} \phi^{\dagger} q_L) + \dots$ $\mathcal{L}_T = \overline{T}(i \not D - M)T - \lambda_T (\overline{q_L} \tilde{\phi} T_R + \overline{T_R} \tilde{\phi}^{\dagger} q_L)$ F. del Aguila, Z. Kunszt and J. Santiago [arXiv:1602.00126 [hep-ph]] C. Anastasiou, A. Carmona, A. Lazopoulos and J. Santiago MatchMaker $\mathcal{L}_{SM} + \mathcal{L}_{6}^{(1l)}$ $\mathcal{L}_{6}^{(0l)} = \alpha_{\phi q}^{(1)} \mathcal{O}_{\phi q}^{(1)} + \alpha_{\phi q}^{(3)} \mathcal{O}_{\phi q}^{(3)} + \alpha_{u\phi} \mathcal{O}_{u\phi} + h.c.$ $\mathcal{O}_{\phi a}^{(1)} = \mathrm{i}\phi^{\dagger} D_{\mu}\phi \ \bar{q}\gamma^{\mu}q$ $\mathcal{O}_1 = |\phi^{\dagger} D_{\mu} \phi|^2$ $16\pi^2 \frac{\mathrm{d} \alpha_1}{\mathrm{d} \log \mu} = 8N_C \lambda_t^2 \alpha_{\phi q}^{(1)} + \dots$ mt $\alpha_1(m_t) = \alpha_1(M) - \frac{N_C \lambda_t^2 \alpha_{\phi q}^{(1)}(M)}{2\pi^2} \log\left(\frac{M}{m_t}\right) = \frac{N_C}{32\pi^2 M^2} \left[\lambda_t^2 |\lambda_T|^2 - |\lambda_T|^4 - 2\lambda_t^2 |\lambda_T|^2 \log\left(\frac{M^2}{m_t^2}\right) \right]$

To complete the calculation one has to integrate out the top, too

Automated one-loop matching

Recently, further effort has been dedicated to automate the one-loop calculations in the SM EFT, to account for arbitrary additions of heavy particles of spin 0, 1/2 and 1.

This requires the corresponding matching between the fundamental theory and the effective one. Work has been done using both the diagrammatic and the functional approaches. No public codes are still available in either case:

•	Diagrammatic	C. Anastasiou, A. Carmona, A. Lazopoulos and J. Santiago, MatchMaker	
	8	F. del Aguila, Z. Kunszt and J. Santiago [arXiv:1602.00126 [hep-ph]]	
		(M. Boggia, R. Gomez-Ambrosio and G. Passarino [arXiv:1603.03660 [hep-ph]])	

• Functional B. Henning, X. Lu and H. Murayama [arXiv:1412.1837 [hep-ph]], [arXiv:1604.01019 [hep-ph]] A. Drozd, J. Ellis, J. Quevillon and T. You [arXiv:1512.03003 [hep-ph]], S.A.R. Ellis, J. Quevillon, T. You and Z. Zhang [arXiv:1604.02445 [hep-ph]] J. Fuentes-Martin, J. Portoles and P. Ruiz-Femenia [arXiv:1607.02142 [hep-ph]]

Diagrammatic method

MatchMaker will allow to perform tree-level and one-loop matching once the fundamental theory and the effective field theory operator basis are fixed, the SM EFT being only a particular case.

It uses standard tools and the matching is performed off-shell assuming (to start with) massless light particles and hence, the unbroken phase of the SM.



Functional method

J. Fuentes-Martin, J. Portoles and P. Ruiz-Femenia [arXiv:1607.02142 [hep-ph]]

Integrating out the heavy particles and heavy modes of the light ones: M, p_{hard} in light propagator >> m, ∂

$$\det \begin{pmatrix} \Delta_{h} & X_{lh}^{\dagger} \\ X_{lh} & \Delta_{l} \end{pmatrix} = \begin{pmatrix} I & X_{lh}^{\dagger} \Delta_{l}^{-1} \\ 0 & I \end{pmatrix} \begin{pmatrix} \Delta_{h} - X_{lh}^{\dagger} \Delta_{l}^{-1} X_{lh} & 0 \\ 0 & \Delta_{l} \end{pmatrix} \begin{pmatrix} I & 0 \\ \Delta_{l}^{-1} X_{lh} & I \end{pmatrix}$$
$$= \begin{pmatrix} I & 0 \\ X_{lh} \Delta_{h}^{-1} & I \end{pmatrix} \begin{pmatrix} \Delta_{h} & 0 \\ 0 & \Delta_{l} - X_{lh} \Delta_{h}^{-1} X_{lh}^{\dagger} \end{pmatrix} \begin{pmatrix} I & \Delta_{h}^{-1} X_{lh}^{\dagger} \\ 0 & I \end{pmatrix}$$
$$= U \begin{pmatrix} \Delta_{+} & 0 \\ 0 & \Delta_{-} \end{pmatrix} U^{\dagger} \xrightarrow{\text{B. Henning, X. Lu and H. Murayama}}_{\text{[arXiv:1604.01019 [hep-ph]]}}$$
Dropping the local counterpart

I.Z. Rothstein, hep-ph/0308266 <u>C.P. Burgess, hep-th/0701053</u>

$$V(l,h) = \frac{1}{2}m^{2}l^{2} + \frac{g_{l}}{4!}l^{4} + \frac{1}{2}M^{2}h^{2} + \frac{\tilde{g}_{h}M}{3!}h^{3} + \frac{g_{h}}{4!}h^{4} + \frac{\tilde{g}_{lh}M}{2}l^{2}h + \frac{g_{lh}}{4!}l^{2}h^{2}$$

The one-loop ILPI generator is independent of the couplings of h³ and l²h at leading order on I/M, when expressed as a function of the renormalised couplings.

Summary

- The SM EFT is the most general operator product expansion involving the SM fields and preserving the SM symmetries. It allows to encode in a model independent way possible deviations from the SM, eventually observable at the LHC. Although it should be modified if new fermions, scalars or interactions are eventually observed, enlarging the set of light fields.
- Present experimental limits and precision in general require to consider up to dimension 6 operators only. Without accounting for different flavours, there are 59 independent operators of this dimension. Their implications on top and Higgs physics have been extensively analysed.
- There are currently efforts to provide automated tools for calculating this effective theory up to one-loop for arbitrary additions of heavy particles of spin 0, 1/2 and 1.

Thanks for your attention

