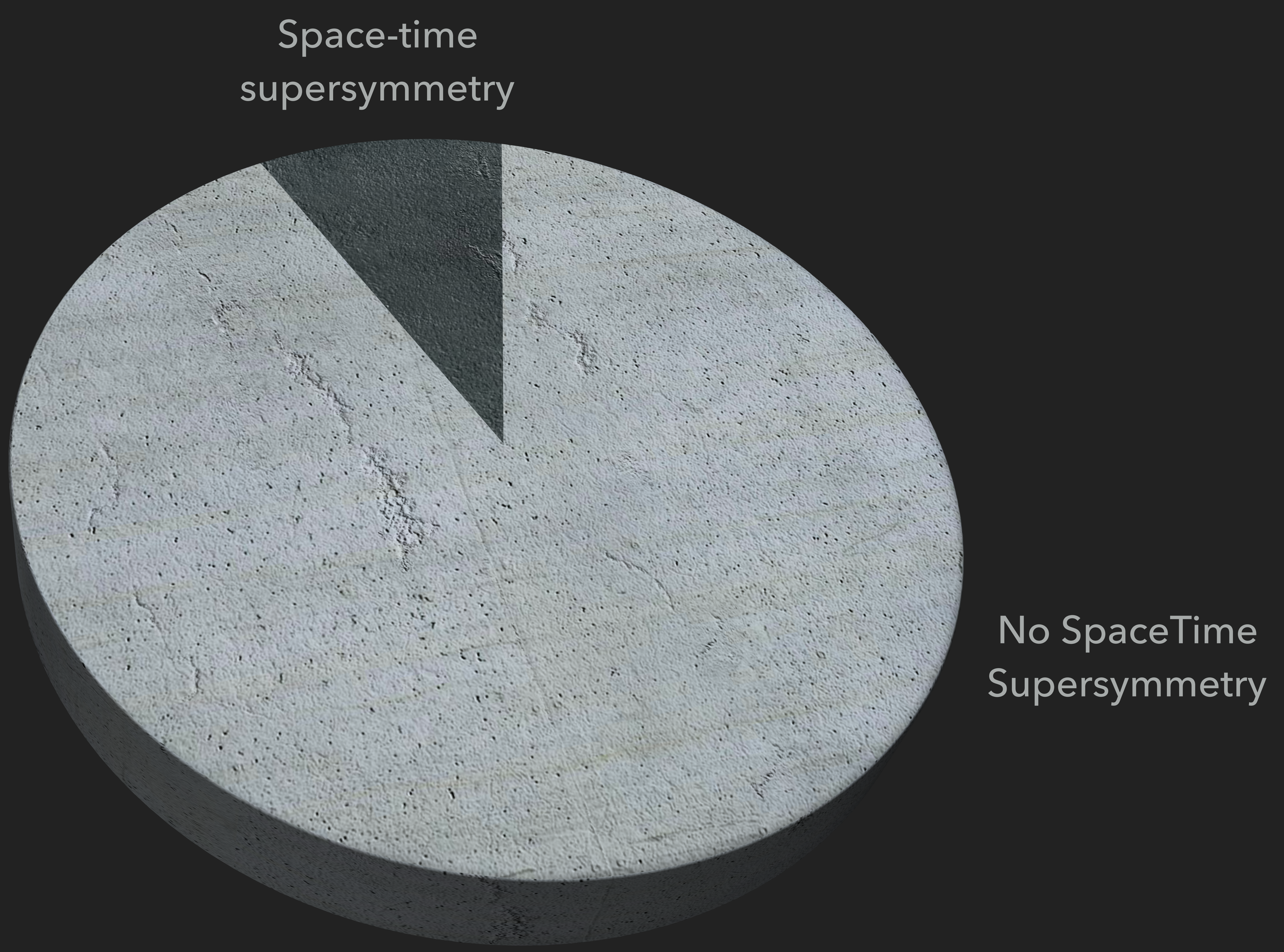
Quantum Fields & Strings

Corfu, September 15, 2016

PROPERTIES OF NON-SUPERSYMMETRIC STRINGS

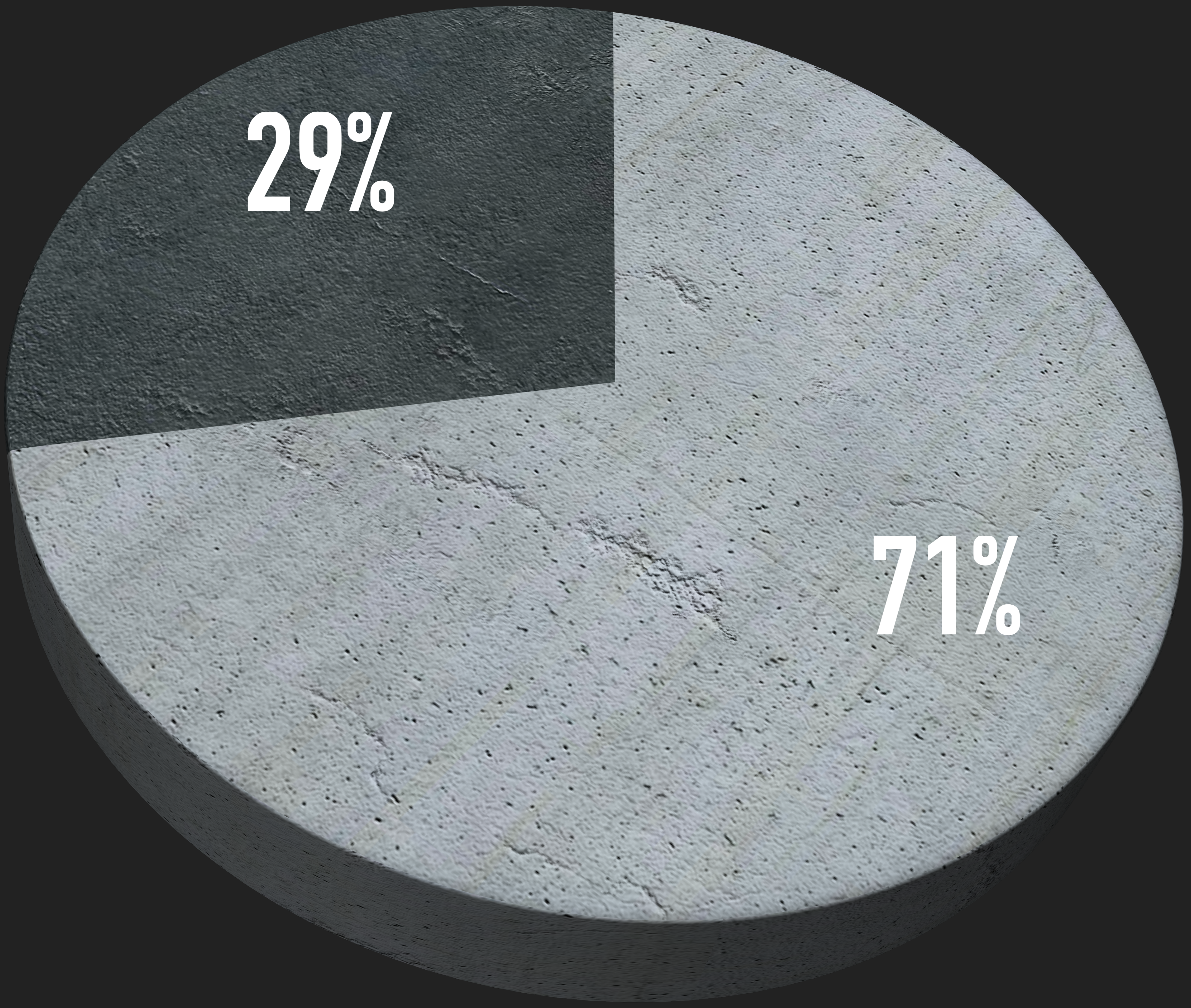
Carlo Angelantonj

The Landscape of (Perturbative) String Vacua



Already in D=10 …

Type IIA
Type IIB
Heterotic SO(32)
Heterotic $E_8 x E_8$
Type I



Type 0A
Type 0B
USp(32) BSB
0A Orientifold
0B Orientifold
0B′ Orientifold
0B′′ Orientifold
Heterotic SO(16) x SO(16)
Heterotic SO(16) x E_8
Heterotic E_7^2 x $SU(2)^2$
Heterotic SO(32)
Heterotic SO(24) x SO(8)

Until now, no experimental evidence of space-time supersymmetry

It is interesting to explore string vacua with no space-time supersymmetry

Indeed, recently revamped interested in non-supersymmetric strings

[C.A., Florakis, Tsulaia; Faraggi, Kounnas, Partouche; Abel, Dienes; Groot Nibbelink et al.; …]

Outline

The problem of classical stability

Universality in gauge thresholds

based on

C.A., Florakis, Tsulaia, PLB 2014

C.A., Florakis, Tsulaia, NPB 2015

C.A., Bonansea, *in progress*

The problem of classical stability

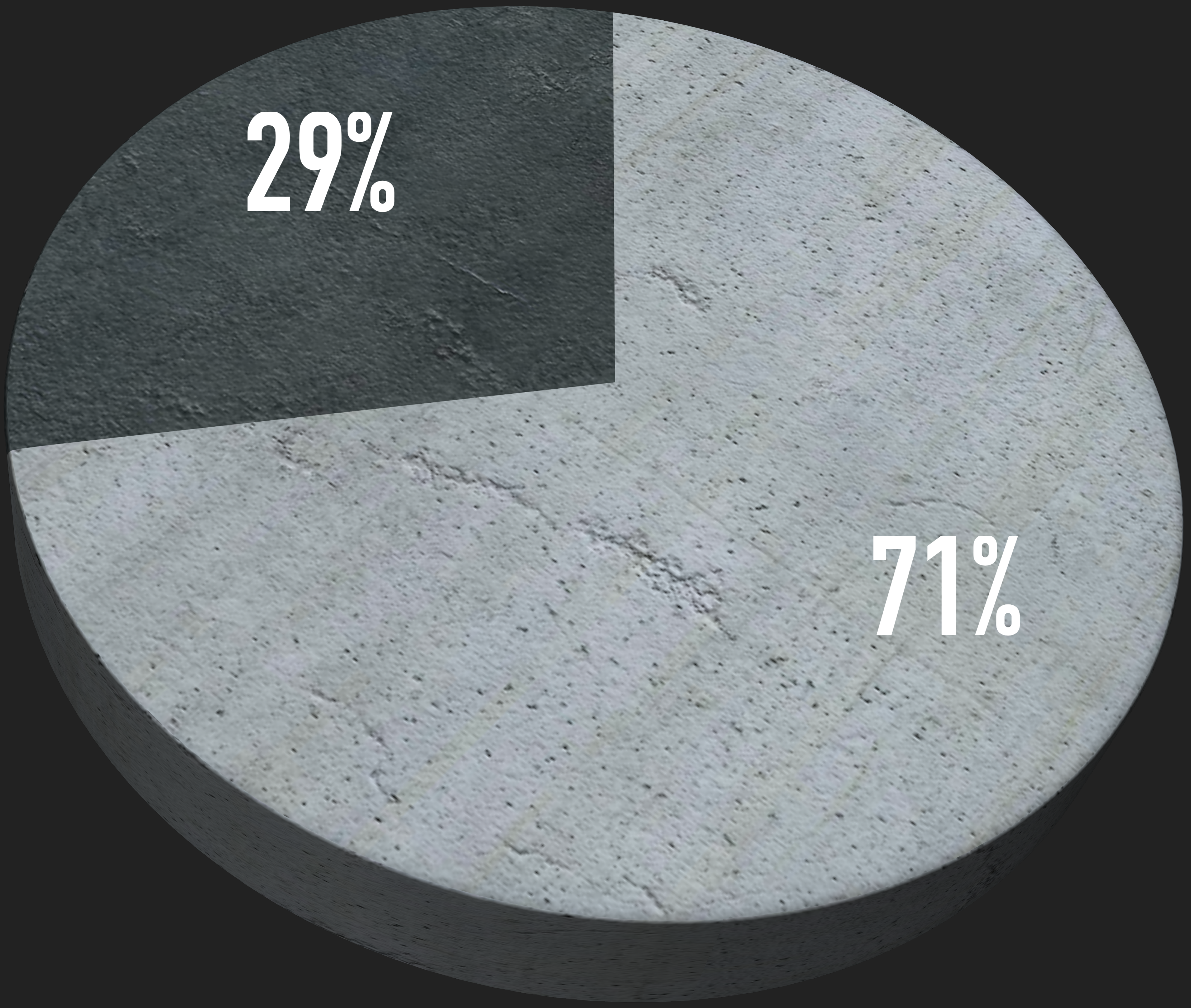
Type IIA

Type IIB

Heterotic SO(32)

Heterotic $E_8 x E_8$

Type I



Type 0A

Type 0B

USp(32) BSB

0A Orientifold

0B Orientifold

0B′ Orientifold

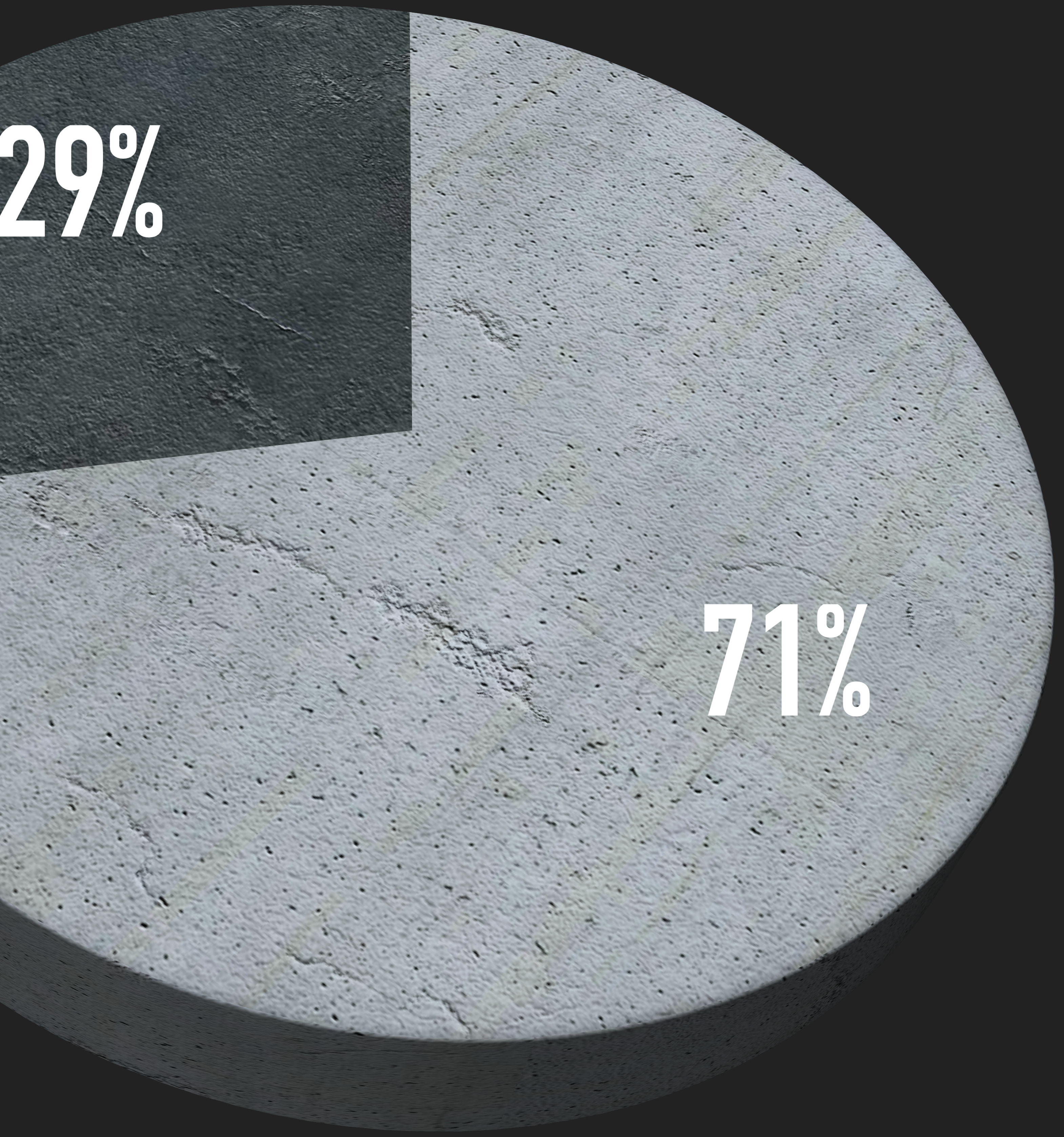
0B″ Orientifold

Heterotic SO(16) x SO(16)

Heterotic SO(16) x E_8

…

The problem of classical stability



Type 0A

Type 0B

USp(32) BSB

0A Orientifold

0B Orientifold

0B′ Orientifold

0B′′ Orientifold

Heterotic SO(16) x SO(16)

Heterotic SO(16) x E_8

…

have tachyons in the classical spectrum

Tachyon free!

But all have dilaton tadpole

Upon compactification all* may develop tachyonic instabilities

*with the exception of the USp(32) BSB

The problem of classical stability

The requirement of stability (plus modular invariance) actually imposes non-trivial constraints on the mass distribution of the string states

$$\mathscr{Z} = \tau_2^{1-D/2} \sum_i \sum_{m,n} a_{mn}^{(i)} q^m \bar{q}^n$$

$$a_{nn}^{(i)} \to \Phi^{(i)}(n)\,, \qquad n \in \mathbb{R}$$

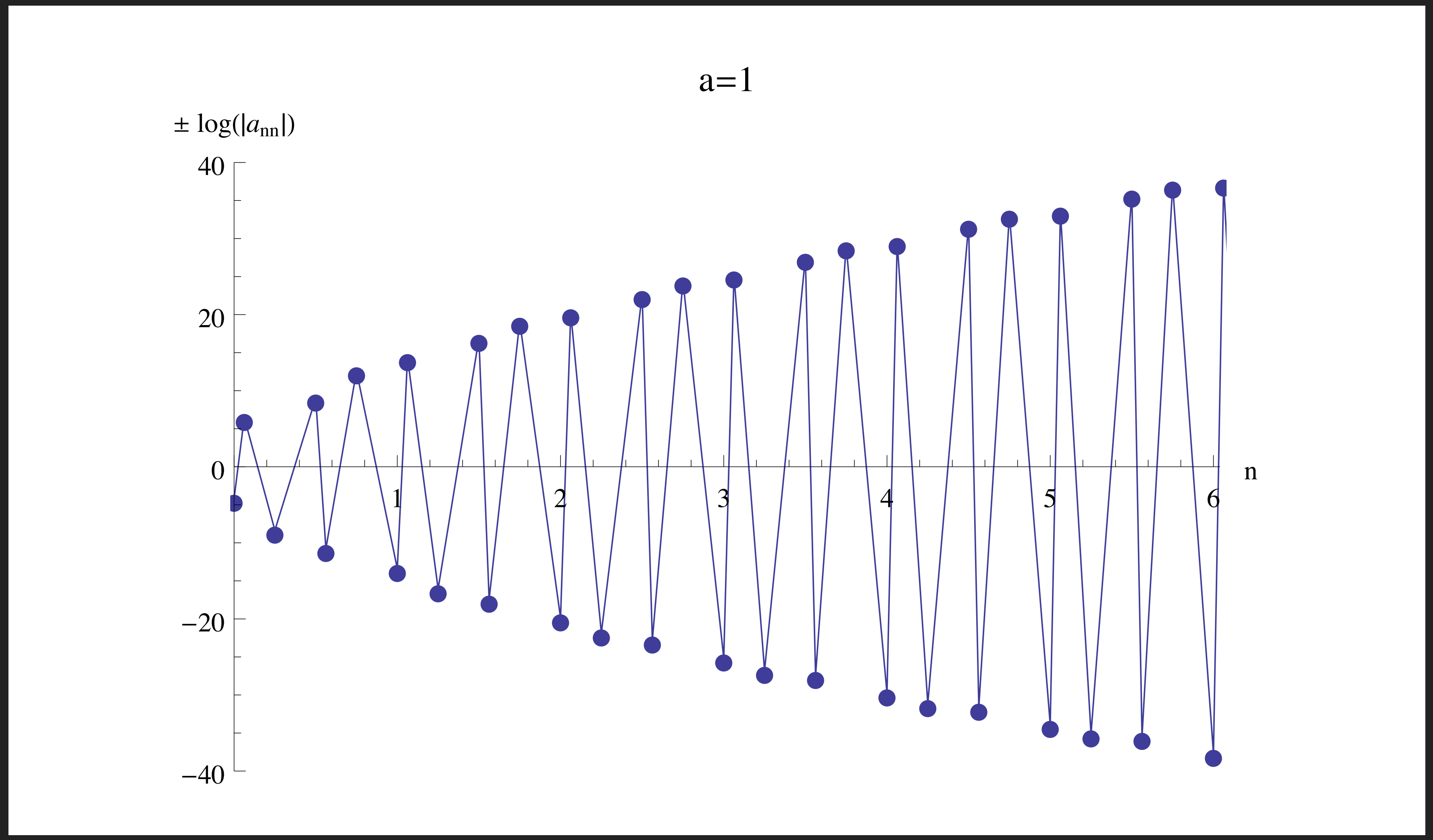
$$\langle a_{nn} \rangle \to \sum_i \Phi^{(i)}(n) = 0$$

Misaligned supersymmetry

[Dienes]

The problem of classical stability

The requirement of stability (plus modular invariance) actually imposes non-trivial constraints on the mass distribution of the string states



Misaligned supersymmetry

[Dienes]

The problem of classical stability

The requirement of stability (plus modular invariance) actually imposes non-trivial constraints on the mass distribution of the string states

The connection between misaligned supersymmetry and classical stability was then proven using properties of the Rankin-Selberg transform

[C.A., Cardella, Elitzur, Rabinovici]

$$\sum_{\{m^2\}} d(m^2) e^{-4\pi\tau_2 m^2/\Lambda^2} \sim \frac{3}{\pi}\Lambda^{2-d}\int_{\mathscr{F}} d\mu\, \mathscr{Z}(\tau,\bar{\tau}) + \sum_{\zeta^*(\rho)=0} C_\rho \Lambda^{\rho-d}$$

The problem of classical stability

$$\sum_{\{m^2\}} d(m^2)\, e^{-4\pi\tau_2 m^2/\Lambda^2} \sim$$

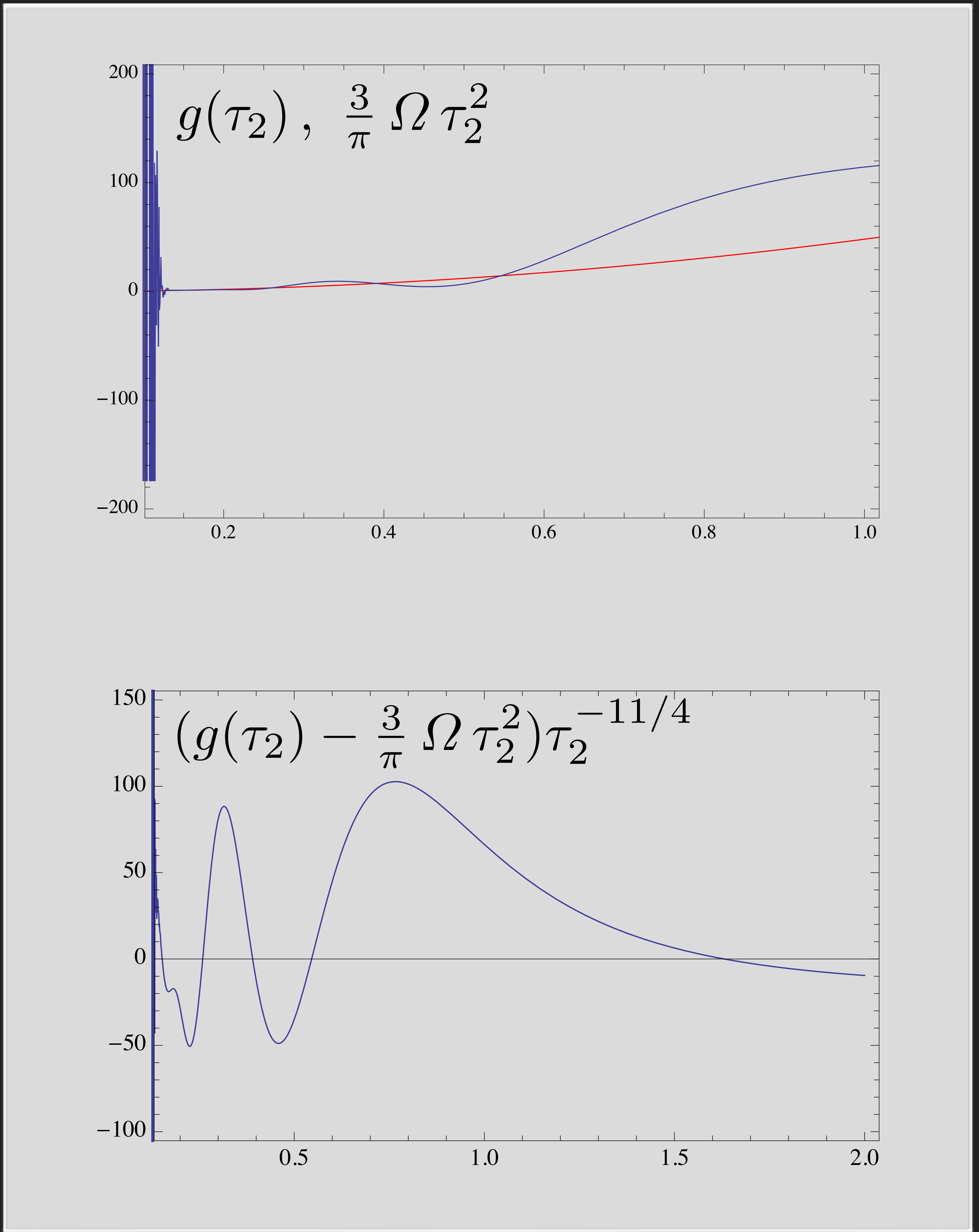
$$\frac{3}{\pi}\Lambda^{2-d}\int_{\mathscr{F}} d\mu\, \mathscr{Z}(\tau,\bar{\tau}) + \sum_{\zeta^*(\rho)=0} C_\rho \Lambda^{\rho-d}$$

In the limit $\Lambda \to \infty$

$$\sum_{\{m^2\}} d(m^2) \to 0$$

Asymptotic supersymmetry

[Kutasov, Seiberg]



The problem of classical stability

Asymptotic supersymmetry and Misaligned supersymmetry are *necessary conditions* for classical stability.

Are them also *sufficient conditions*? **NO!**

For instance, Scherk-Schwarz compactifications are “continuous” deformations of the spectrum.

Degrees of freedom are neither generated nor eliminated

The problem of classical stability

Degrees of freedom are neither generated nor eliminated, therefore asymptotic supersymmetry continues to hold

$$\sum_{\{m^2\}} d(m^2) \to 0$$

even when tachyons appear in the physical spectrum.

The problem of classical stability

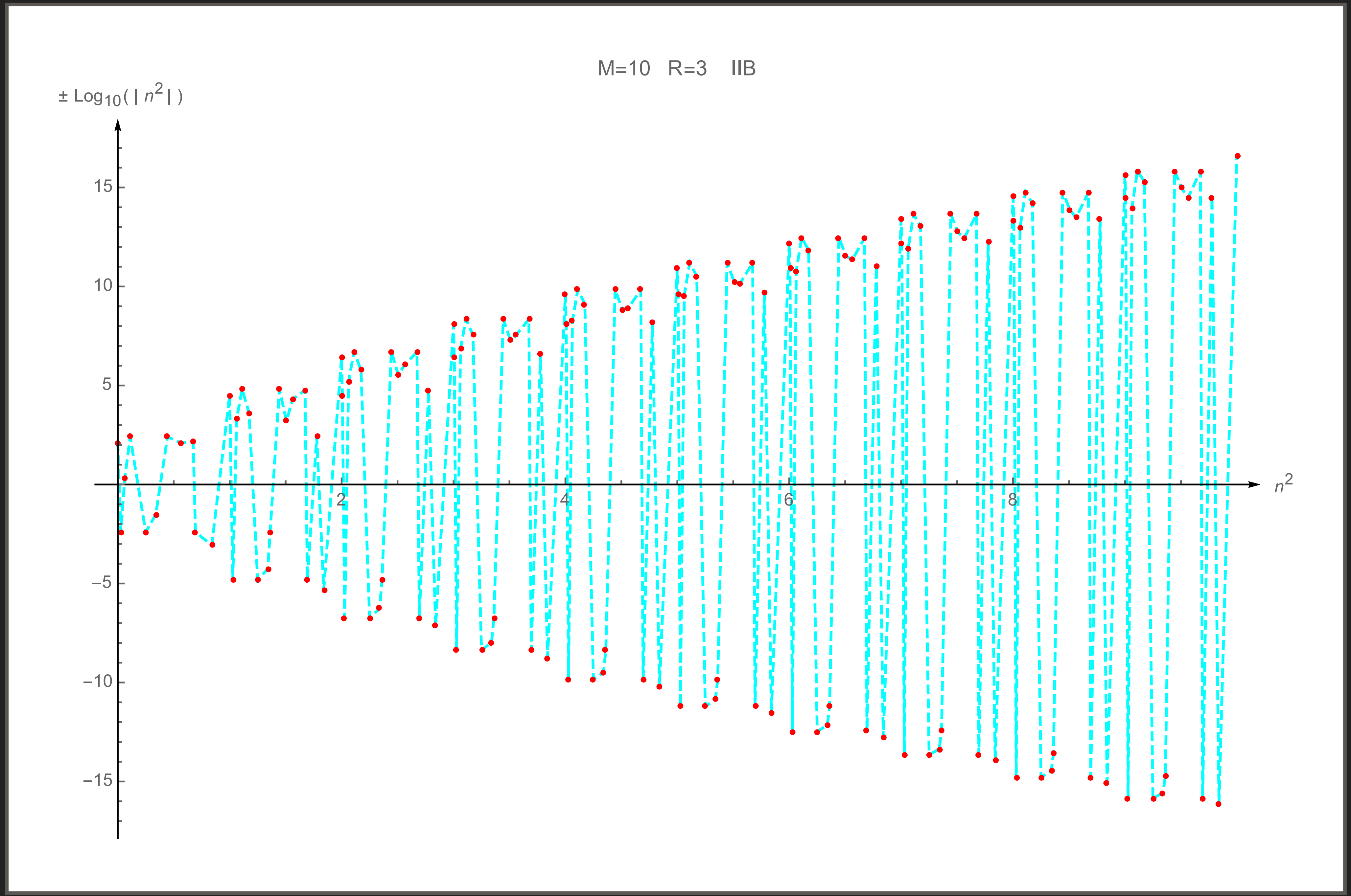
Similarly, since $\Phi^{(i)}(n)$ are continuous functions

$$\langle a_{nn} \rangle \to \sum_i \Phi^{(i)}(n) = 0$$

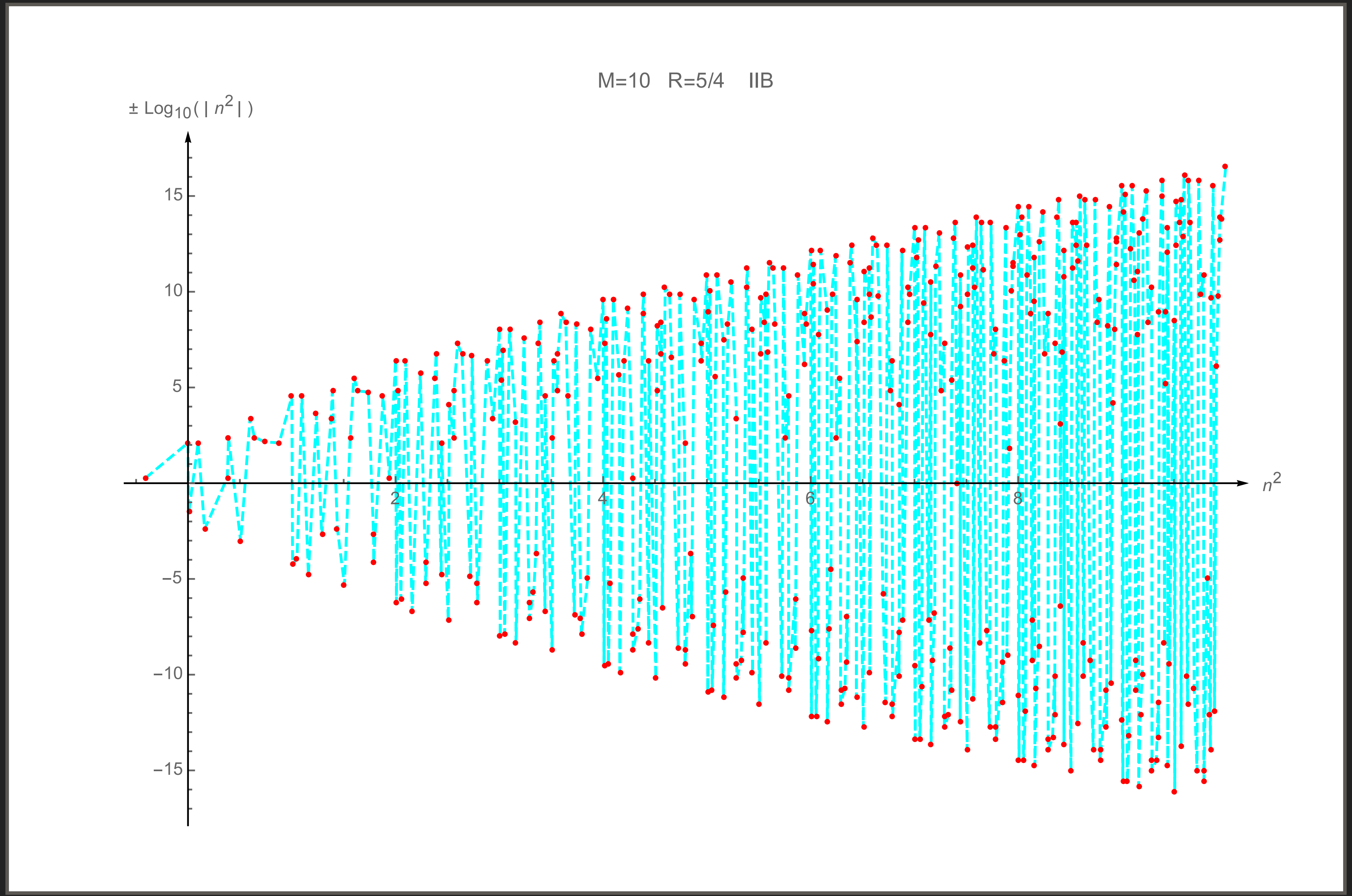
even when tachyons appear in the physical spectrum.

[CA, Bonansea]

The problem of classical stability



non tachyonic



tachyonic

The problem of classical stability

Therefore, asymptotic and misaligned supersymmetry are not a trademark of classical stability since also tachyonic vacua are expected to share these properties

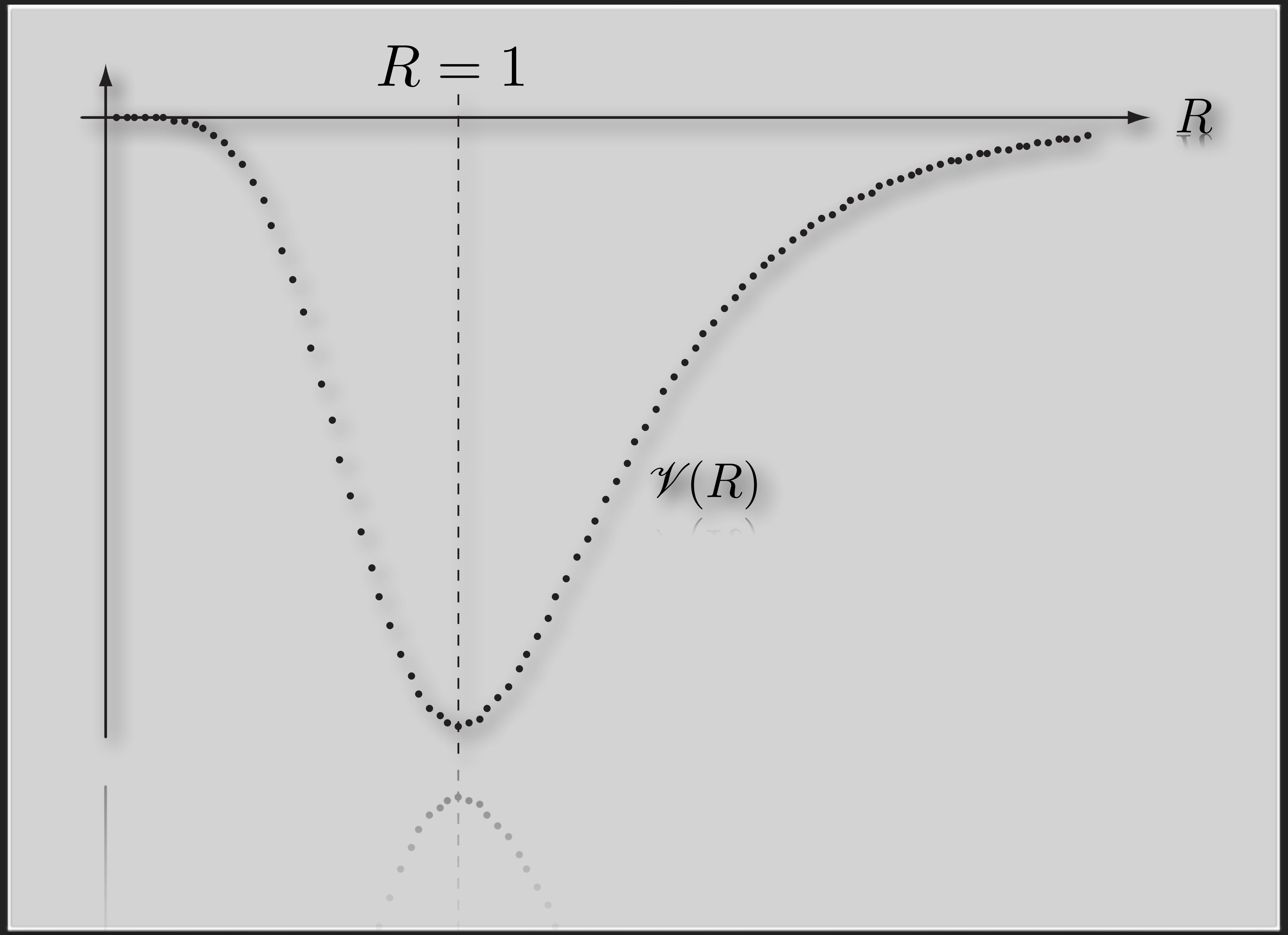
Moreover, they are necessary conditions only for closed string vacua, and are not shared by orientifold vacua, where one can have tachyon-free constructions with only bosonic excitations (see 0'B)

[Sagnotti; Israel, Niarchos]

The problem of classical stability

Difficult to achieve at any point in moduli space!

(Very) few known cases in lower dimensions



[C.A., Cardella, Irges]

Asymmetric Scherk-Schwarz in d=4,6

Tachyons actually arise is off-diagonal components of the metric are turned on!

Suitable orientifold action needed to eliminate the dangerous directions

The problem of classical stability

We shall always restrict our analysis at regions in moduli space where the classical vacua are stable.

Quantum destabilisation of the construction is an (important) open problem.

[See Florakis, Partouche for interesting progress]

The non-supersymmetric vacua I'll be interested in are freely acting orbifolds of the heterotic string

$$\Omega_{\rm SB} = \gamma_{\rm SB}\,\delta$$

$$\gamma_{\rm SB} = (-1)^{F_{\rm st}}$$

$$\delta\ :\qquad X \to X + \pi R$$

These orbifolds implement the Scherk-Schwarz mechanism in String Theory (coordinate dependent compactifications)

These constructions actually interpolate among (most of) the 10d supersymmetric and non-supersymmetric vacua

$$(V_8\bar{V}_8 + S_8\bar{S}_8)\,\mathcal{E}_0 - (V_8\bar{S}_8 + S_8\bar{V}_8)\,\mathcal{O}_0$$
$$(O_8\bar{O}_8 + C_8\bar{C}_8)\,\mathcal{E}_{\frac{1}{2}} - (O_8\bar{C}_8 + C_8\bar{O}_8)\,\mathcal{O}_{\frac{1}{2}}$$

$$\mathcal{E}_0 = \sum_{m,n} \Gamma_{2m,n}$$
$$\mathcal{O}_0 = \sum_{m,n} \Gamma_{2m+1,n}$$
$$\mathcal{E}_{\frac{1}{2}} = \sum_{m,n} \Gamma_{2m,n+\frac{1}{2}}$$
$$\mathcal{O}_{\frac{1}{2}} = \sum_{m,n} \Gamma_{2m+1,n+\frac{1}{2}}$$

$R \to \infty$

$R \to 0$

$$|V_8 - S_8|^2$$

Type IIB

$$|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2$$

Type 0B

Universality in gauge thresholds

The aim is to reconstruct the low-energy effective action for the light modes including one-loop corrections

This subject has been successfully investigated in the 90’s and led to seminal results

The analysis was limited to vacua with space-time supersymmetry, where quantum corrections are high constrained

We shall focus here on the one-loop corrections to gauge couplings in heterotic vacua

$$\frac{16\pi^2}{g_\alpha^2(\mu)} = \frac{16\pi^2}{g_\mathrm{s}^2} + \beta_\alpha \log \frac{M_s^2}{\mu^2} + \Delta_\alpha$$

light states

heavy states

In particular, we shall be interested on the (moduli dependent) threshold corrections induced by the infinite tower of massive string states

$$\Delta_\alpha = \text{R.N.} \int_{\mathcal{F}} d\mu \frac{i\tau_2}{\pi\,\eta^2\bar\eta^2} \sum_{a,b} \partial_\tau \left(\frac{\theta\left[{a \atop b}\right]}{\eta} \right) \mathrm{Tr}_{\mathcal{H}\left[{a \atop b}\right]} \left[\left(Q_\alpha^2 - \frac{1}{4\pi\tau_2} \right) q^{L_0 - c/24}\, \bar q^{\bar L_0 - \bar c/24} \right]$$

A simple example …

The heterotic string compactified on $T^4 \times T^2/\mathbb{Z}_2 \times \mathbb{Z}_2$

singular limit of K3

responsible for SUSY breaking $(-1)^{F_{\rm st}+F_1+F_2}\,\delta$

$$
\begin{aligned}
\mathcal{Z} = & V_4 O_4 \left[(\bar O_{12}\bar O_4 \bar O_{16} + \bar C_{12}\bar C_4 \bar S_{16})\mathcal{E}_0 + (\bar O_{12}\bar O_4 \bar S_{16} + \bar C_{12}\bar C_4 \bar O_{16})\mathcal{O}_0\right] \\
& - C_4 C_4 \left[(\bar O_{12}\bar O_4 \bar S_{16} + \bar C_{12}\bar C_4 \bar O_{16})\mathcal{E}_0 + (\bar O_{12}\bar O_4 \bar O_{16} + \bar C_{12}\bar C_4 \bar S_{16})\mathcal{O}_0\right] \\
& + O_4 V_4 \left[(\bar V_{12}\bar V_4 \bar O_{16} + \bar S_{12}\bar S_4 \bar S_{16})\mathcal{E}_0 + (\bar V_{12}\bar V_4 \bar S_{16} + \bar S_{12}\bar S_4 \bar O_{16})\mathcal{O}_0\right] \\
& - S_4 S_4 \left[(\bar V_{12}\bar V_4 \bar S_{16} + \bar S_{12}\bar S_4 \bar O_{16})\mathcal{E}_0 + (\bar V_{12}\bar V_4 \bar O_{16} + \bar S_{12}\bar S_4 \bar S_{16})\mathcal{O}_0\right] + \ldots
\end{aligned}
$$

A simple example ...

The heterotic string compactified on $T^4 \times T^2/\mathbb{Z}_2 \times \mathbb{Z}_2$

singular limit of K3

responsible for SUSY breaking
$(-1)^{F_{\rm st}+F_1+F_2}\,\delta$

$$G = \mathrm{SO}(12) \times \mathrm{SO}(16)[\times \mathrm{SO}(4)]$$

at the massless level:

scalars: $(12, 1, 4)$

fermions: $(32, 1, 2) + (32', 1, 2') + (1, 128, 1)$

A simple example …

The heterotic string compactified on $T^4 \times T^2 / \mathbb{Z}_2 \times \mathbb{Z}_2$

singular limit of K3

responsible for SUSY breaking
$(-1)^{F_{\rm st}+F_1+F_2}\,\delta$

$$G = \mathrm{SO}(12) \times \mathrm{SO}(16) [\times \mathrm{SO}(4)]$$

at the massless level:

extra scalars: $(12, 16, 1)$ at the point $R = \sqrt{2}$ $(T = 2U)$

$$
\begin{aligned}
\Delta_{\mathrm{SO}(16)} = &- \tfrac{1}{24}\Gamma_{2,2}\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right] \frac{\hat{\bar{E}}_2\bar{E}_4\bar{E}_6 - \bar{E}_6^2}{\bar{\eta}^{24}} \\
&- \tfrac{1}{576}\Gamma_{2,2}\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right] \frac{\Lambda^{K3}\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]}{\eta^{12}\bar{\eta}^{24}}(\vartheta_3^8 - \vartheta_4^8)\bar{\vartheta}_3^4\bar{\vartheta}_4^4\left[(\hat{\bar{E}}_2 - \bar{\vartheta}_3^4)\bar{\vartheta}_3^4\bar{\vartheta}_4^4 + 8\bar{\eta}^{12}\right] \\
&- \tfrac{1}{48}\Gamma_{2,2}\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right] \frac{\bar{\vartheta}_3^4\bar{\vartheta}_4^4(\bar{\vartheta}_3^4 + \bar{\vartheta}_4^4)\left[(\hat{\bar{E}}_2 - \bar{\vartheta}_3^4)\bar{\vartheta}_3^4\bar{\vartheta}_4^4 + 8\bar{\eta}^{12}\right]}{\bar{\eta}^{24}} \\
&- \tfrac{1}{72}\Gamma_{2,2}\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right] \frac{\vartheta_2^4(\vartheta_3^8 - \vartheta_4^8)}{\eta^{12}} \frac{(\hat{\bar{E}}_2 - \bar{\vartheta}_3^4)\bar{\vartheta}_3^4\bar{\vartheta}_4^4 + 8\bar{\eta}^{12}}{\bar{\eta}^{12}} + \ldots
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{\mathrm{SO}(12)} = &- \tfrac{1}{24}\Gamma_{2,2}\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right] \frac{\hat{\bar{E}}_2\bar{E}_4\bar{E}_6 - \bar{E}_4^3}{\bar{\eta}^{24}} \\
&- \tfrac{1}{576}\Gamma_{2,2}\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right] \frac{\Lambda^{K3}\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]}{\eta^{12}\bar{\eta}^{24}} (\vartheta_3^8 - \vartheta_4^8)\bar{\vartheta}_3^4\bar{\vartheta}_4^4 \left[(\hat{\bar{E}}_2 - \bar{\vartheta}_3^4)\bar{\vartheta}_3^4\bar{\vartheta}_4^4 + 8\bar{\eta}^{12}\right] \\
&- \tfrac{1}{48}\Gamma_{2,2}\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right] \frac{\bar{\vartheta}_3^8\bar{\vartheta}_4^8\left[\hat{\bar{E}}_2(\bar{\vartheta}_3^4 + \bar{\vartheta}_4^4) + \bar{\vartheta}_2^8 - 2\bar{\vartheta}_3^4\bar{\vartheta}_4^4\right]}{\bar{\eta}^{24}} \\
&- \tfrac{1}{72}\Gamma_{2,2}\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right] \left(\frac{\vartheta_2^4(\vartheta_3^8 - \vartheta_4^8)}{\eta^{12}} \frac{\hat{\bar{E}}_2\bar{\vartheta}_3^4\bar{\vartheta}_4^4}{\bar{\eta}^{12}} \right. \\
&\qquad\qquad \left. + \frac{\vartheta_2^4\vartheta_4^4|\vartheta_2^4 - \vartheta_4^4|^2 - \vartheta_2^4\vartheta_3^4|\vartheta_2^4 + \vartheta_3^4|^2}{\eta^{12}\,\bar{\eta}^{12}}\,\bar{\vartheta}_3^4\bar{\vartheta}_4^4 \right) + \ldots
\end{aligned}
$$

Something remarkable happens when taking the difference of thresholds

$$\Delta_{\mathrm{SO}(16)} - \Delta_{\mathrm{SO}(12)} = -24\, \Gamma_{2,2}\left[{0 \atop 0}\right] - \tfrac{1}{3}\, \Gamma_{2,2}\left[{0 \atop 1}\right] \left(\frac{\vartheta_2^{12}}{\eta^{12}} - 8\right)$$

(plus images under S and TS transformations)

Something remarkable happens when taking the difference of thresholds

$$\mathit{\Delta}_{\mathrm{SO}(16)} - \mathit{\Delta}_{\mathrm{SO}(12)} = -24\, \mathit{\Gamma}_{2,2} \left[{0 \atop 0} \right] - \tfrac{1}{3}\, \mathit{\Gamma}_{2,2} \left[{0 \atop 1} \right] \left(\frac{\vartheta_2^{12}}{\eta^{12}} - 8 \right)$$

Aside from the lattice contribution, the threshold difference only involves holomorphic functions of the Teichmüller parameter

Something remarkable happens when taking the difference of thresholds

$$\Delta_{\mathrm{SO}(16)} - \Delta_{\mathrm{SO}(12)} = -24\, \Gamma_{2,2} \left[{0 \atop 0} \right] - \frac{1}{3}\, \Gamma_{2,2} \left[{0 \atop 1} \right] \left(\frac{\vartheta_2^{12}}{\eta^{12}} - 8 \right)$$

Only a sub-sector of string states effectively contributes to the threshold difference!

Reminiscent of what happens in supersymmetric vacua

Upon evaluating the modular integrals

$$\begin{aligned}\Delta_{\mathrm{SO}(16)} - \Delta_{\mathrm{SO}(12)} =& 72 \log \left[T_2 U_2 |\eta(T)\eta(U)|^4\right] - \tfrac{8}{3} \log \left[T_2 U_2 |\vartheta_4(T)\vartheta_2(U)|^4\right] \\ &+ \tfrac{2}{3} \log |j_\infty(T/2) - j_\infty(U)|^4\end{aligned}$$

Upon evaluating the modular integrals

$$\Delta_{\mathrm{SO}(16)} - \Delta_{\mathrm{SO}(12)} = 72 \log\left[T_2 U_2 |\eta(T)\eta(U)|^4\right] - \frac{8}{3}\log\left[T_2 U_2 |\vartheta_4(T)\vartheta_2(U)|^4\right] + \frac{2}{3}\log|j_\infty(T/2) - j_\infty(U)|^4$$

$$j_\infty(q) \sim \frac{1}{q} + 276q - 2048q^2 + 11202q^3 + O(q^4)$$

is the equivalent of the Klein j-function for $\Gamma_0(2)$ attached to che cusp at infinity

Upon evaluating the modular integrals

$$\begin{aligned}\Delta_{\mathrm{SO}(16)} - \Delta_{\mathrm{SO}(12)} = & a \log \left[T_2 U_2 |\eta(T)\eta(U)|^4\right] + b \log \left[T_2 U_2 |\vartheta_4(T)\vartheta_2(U)|^4\right] \\ & + c \log |j_\infty(T/2) - j_\infty(U)|^4\end{aligned}$$

This result (modulo the overall coefficients) actually holds for a large class of non-supersymmetric vacua

Upon evaluating the modular integrals

$$\Delta_{\mathrm{SO}(16)} - \Delta_{\mathrm{SO}(12)} = a \log\left[T_2 U_2 |\eta(T)\eta(U)|^4\right] + b \log\left[T_2 U_2 |\vartheta_4(T)\vartheta_2(U)|^4\right] + c \log|j_\infty(T/2) - j_\infty(U)|^4$$

Gauge threshold differences are universal also when supersymmetry is absent!

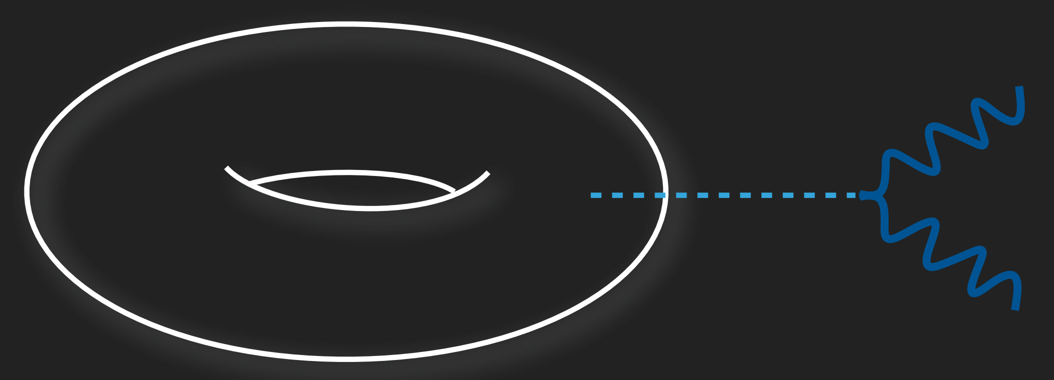
What is behind the non-susy universality?

When does it occur?

The anatomy of gauge thresholds

$$\Delta_\alpha = \text{R.N.} \int_{\mathcal{F}} d\mu \frac{i\tau_2}{\pi\,\eta^2\bar{\eta}^2} \sum_{a,b} \partial_\tau \left(\frac{\theta\left[\begin{smallmatrix} a \\ b \end{smallmatrix}\right]}{\eta} \right) \mathrm{Tr}_{\mathcal{H}\left[\begin{smallmatrix} a \\ b \end{smallmatrix}\right]} \left[\left(Q_\alpha^2 - \frac{1}{4\pi\tau_2} \right) q^{L_0 - c/24}\, \bar{q}^{\bar{L}_0 - \bar{c}/24} \right]$$

Contact term in the JJ correlation independent of the gauge group



universal dilaton exchange

As a result, the threshold differences read

$$\Delta_{\alpha\beta} = \int_{\mathcal{F}} d\mu \sum_{h,g} L\left[{}^{h}_{g}\right](\tau)\, \bar{\Phi}\left[{}^{h}_{g}\right](\bar{\tau})\, \Gamma\left[{}^{h}_{g}\right](G,B)$$

sum over orbifold sectors

helicity supertrace

group trace

shifted Narain lattice

Allow here for a spontaneous breaking of N=4, or compactification on non factorisable tori

(Generalised) Universality in N=2 vacua

In N=2 supersymmetric vacua the F^2 term is highly protected

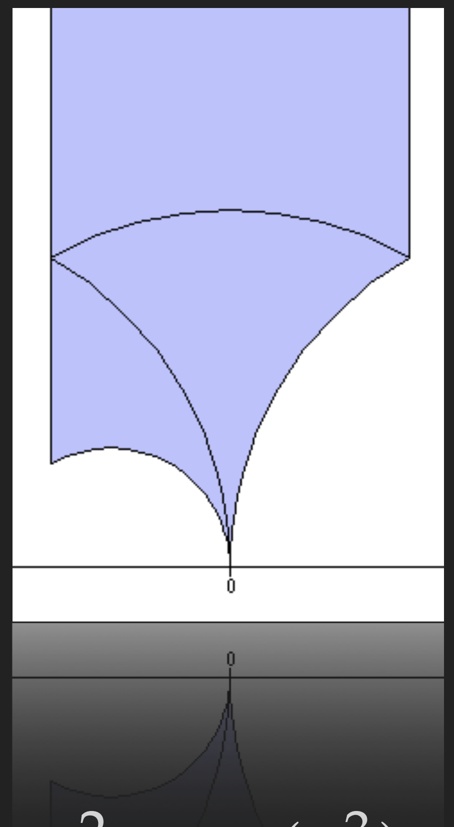
Only BPS states contribute to its radiative correction

$$\Delta_{\alpha\beta} = \int_{\mathcal{F}} d\mu \sum_{h,g} \bar{\Phi}\left[{h \atop g}\right](\bar{\tau})\, \Gamma\left[{h \atop g}\right](G,B)$$

(Generalised) Universality in *N*=2 vacua

$$\Delta_{\alpha\beta} = \int_{\mathcal{F}} d\mu \sum_{h,g} \bar{\Phi}\begin{bmatrix} h \\ g \end{bmatrix}(\bar{\tau})\, \Gamma\begin{bmatrix} h \\ g \end{bmatrix}(G,B)$$

The functions $\bar{\Phi}\begin{bmatrix} h \\ g \end{bmatrix}(\bar{\tau})$ must be regular in the deep infra-red
This, together with modular invariance highly constrain them



$$\bar{\Phi}\begin{bmatrix} 0 \\ 1 \end{bmatrix}(\bar{\tau}) = a + \sum_{\mathfrak{a}\neq\infty} b_{\mathfrak{a}}\, \bar{j}_{\mathfrak{a}}(\bar{\tau})$$

$$j_0(q) = 24 + 4096\, q + 98304\, q^2 + O(q^3)$$

(Generalised) Universality in N=2 vacua

Upon performing the modular integral

$$\Delta_{\alpha\beta} = -(a+24b_0)\,\log\left[T_2U_2|\vartheta_4(T)\vartheta_2(U)|^4\right] - 2b_0\,\log|j_\infty(T/2)-j_\infty(U)|^4$$

Signals the presence of extra massless states at the point T=2U (plus images)

What does this imply for
non supersymmetric vacua?

(Generalised) Universality in non-susy vacua

A generic 4d non-supersymmetric vacuum can be built as

$$T^6/\Omega_{\rm S} \times \Omega_{\rm SB} \qquad \text{with} \qquad \Omega_{\rm SB} \ni g_{\rm SB} = \gamma^{\rm L}_{\rm SB} \otimes \gamma^{\rm R}_{\rm gauge}$$

The F^2 term is not any longer protected and all states contribute to the thresholds

$$
\begin{aligned}
\Delta_{\mathrm{SO}(16)} = &- \tfrac{1}{24}\Gamma_{2,2}\big[\begin{smallmatrix}0\\0\end{smallmatrix}\big]\, \frac{\hat{\bar{E}}_2\bar{E}_4\bar{E}_6 - \bar{E}_6^2}{\bar{\eta}^{24}} \\
&- \tfrac{1}{576}\Gamma_{2,2}\big[\begin{smallmatrix}0\\1\end{smallmatrix}\big]\, \frac{\Lambda^{K3}\big[\begin{smallmatrix}0\\0\end{smallmatrix}\big]}{\eta^{12}\bar{\eta}^{24}}(\vartheta_3^8 - \vartheta_4^8)\bar{\vartheta}_3^4\bar{\vartheta}_4^4\left[(\hat{\bar{E}}_2 - \bar{\vartheta}_3^4)\bar{\vartheta}_3^4\bar{\vartheta}_4^4 + 8\bar{\eta}^{12}\right] \\
&- \tfrac{1}{48}\Gamma_{2,2}\big[\begin{smallmatrix}0\\1\end{smallmatrix}\big]\, \frac{\bar{\vartheta}_3^4\bar{\vartheta}_4^4(\bar{\vartheta}_3^4 + \bar{\vartheta}_4^4)\left[(\hat{\bar{E}}_2 - \bar{\vartheta}_3^4)\bar{\vartheta}_3^4\bar{\vartheta}_4^4 + 8\bar{\eta}^{12}\right]}{\bar{\eta}^{24}} \\
&- \tfrac{1}{72}\Gamma_{2,2}\big[\begin{smallmatrix}0\\1\end{smallmatrix}\big]\, \frac{\vartheta_2^4(\vartheta_3^8 - \vartheta_4^8)}{\eta^{12}}\, \frac{(\hat{\bar{E}}_2 - \bar{\vartheta}_3^4)\bar{\vartheta}_3^4\bar{\vartheta}_4^4 + 8\bar{\eta}^{12}}{\bar{\eta}^{12}} + \ldots
\end{aligned}
$$

(Generalised) Universality in non-susy vacua

$$\Delta_{\alpha\beta} = \int_{\mathcal{F}} d\mu \sum_{h,g} L\left[{}^h_g\right](\tau)\, \bar{\Phi}\left[{}^h_g\right](\bar{\tau})\, \Gamma\left[{}^h_g\right](G,B)$$

The functions $L\left[{}^h_g\right](\tau)$ are no longer constants and universality is lost … unless the $\bar{\Phi}\left[{}^h_g\right](\bar{\tau})$ are!

The functions $\bar{\Phi}\left[{h \atop g}\right](\bar{\tau})$ are determined by the action of the orbifold group on the gauge degrees of freedom

They are indeed constant in supersymmetric vacua
(as long as no symmetry enhancement occurs)

How does this constrain the way one breaks supersymmetry?

$$T^6/\Omega_{\rm S} \times \Omega_{\rm SB}$$

When are $\bar{\Phi}\left[\begin{smallmatrix} g_{\mathrm{S}} \\ g_{\mathrm{SB}} \end{smallmatrix}\right](\bar{\tau})$ and $\bar{\Phi}\left[\begin{smallmatrix} g'_{\mathrm{SB}} \\ g_{\mathrm{SB}} \end{smallmatrix}\right](\bar{\tau})$ constant?

$$g_{\mathrm{SB}} = \gamma^{\mathrm{L}}_{\mathrm{SB}} \otimes \gamma^{\mathrm{R}}_{\mathrm{gauge}} \qquad \rightarrow \qquad \tilde{g}_{\mathrm{S}} = \tilde{\gamma}^{\mathrm{L}}_{\mathrm{S}} \otimes \gamma^{\mathrm{R}}_{\mathrm{gauge}}$$

In this case, the orbifold $T^6/\Omega_{\mathrm{S}} \times \tilde{\Omega}_{\mathrm{S}}$ preserves supersymmetry and universality is guaranteed!
The $\bar{\Phi}\left[\begin{smallmatrix} h \\ g \end{smallmatrix}\right](\bar{\tau})$ are ***all*** constant as long as there is no gauge symmetry enhancement

Universality Theorem:

Any non-supersymmetric heterotic orbifold $T^6/\Omega_{\rm S} \times \Omega_{\rm SB}$ *yields a universal behaviour in the difference of gauge thresholds* $\Delta_{\alpha\beta}$ *for gauge groups* G_α *and* G_β*, of rank larger than one, if* $\Omega_{\rm SB}$ *can be consistently replaced by a supersymmetric orbifold* $\tilde{\Omega}_{\rm S}$ *with the very same action on the right-moving degrees of freedom, and provided no extra massless states charged with respect to* $G_\alpha \times G_\beta$ *emerge in the bulk of the moduli space of the supersymmetric orbifold* $T^6/\Omega_{\rm S} \times \tilde{\Omega}_{\rm S}$

The space of solutions is actually not very large:

$$g_{\mathrm{SB}} = (-1)^{F_{\mathrm{st}}} \begin{cases} (0^8;0^8) & \mathrm{E}_8 \times \mathrm{E}_8 \\ (1,0^7;1,0^7) & \mathrm{SO}(16) \times \mathrm{SO}(16) \\ (\frac{1}{2}^2,0^6;\frac{1}{2}^2,0^6) & [\mathrm{E}_7 \times \mathrm{SU}(2)]^2 \end{cases}$$

Conclusions

When supersymmetry is (spontaneously) broken a residual misaligned supersymmetry survives. However, it is not a signature of classical stability

We have studied radiative corrections to classically stable non-supersymmetric heterotic vacua

Remarkably, also in the absence of supersymmetry some quantities are “protected” and display a universal structure

Outlook

What about radiative corrections to other low-energy couplings?

Does universality survive in phenomenologically viable constructions?

What about higher-order (quantum) stability?

Is stability and calculability compatible with interesting phenomenology?

Thank you!