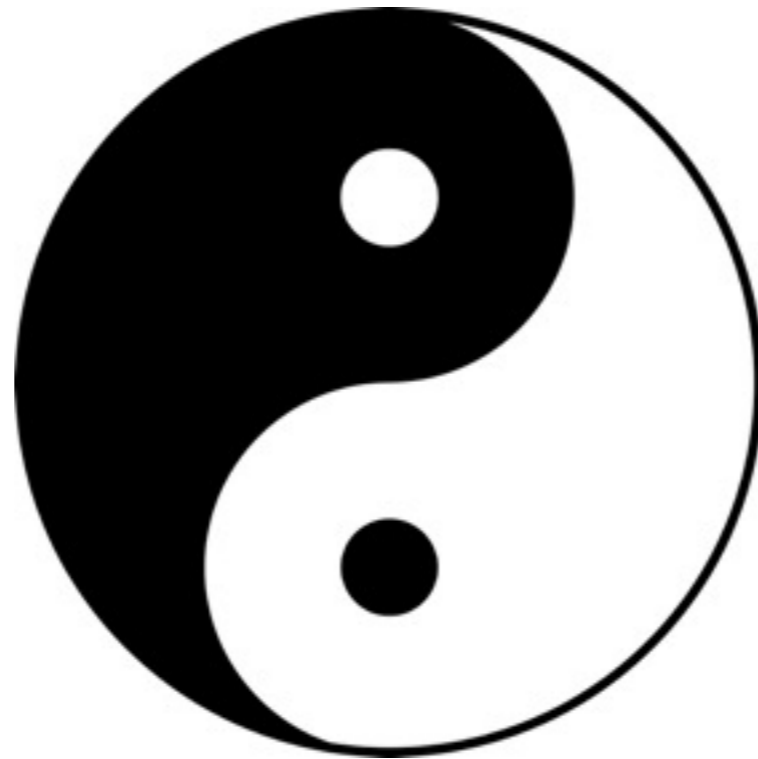


# Double Field Theory




# Double Field Theory

Hull & Zwiebach

- From sector of String Field Theory. Features some stringy physics, including T-duality, in simpler setting
- Strings see a doubled space-time
- Necessary consequence of string theory
- Needed for non-geometric backgrounds
- What is geometry and physics of doubled space?

# Strings on a Torus

- 
- States: momentum  $p$ , winding  $w$
  - String: Infinite set of fields  $\psi(p, w)$
  - Fourier transform to doubled space:  $\psi(x, \tilde{x})$
  - “Double Field Theory” from closed string field theory. Some non-locality in doubled space
  - Subsector? e.g.  $g_{ij}(x, \tilde{x})$ ,  $b_{ij}(x, \tilde{x})$ ,  $\phi(x, \tilde{x})$
  - T-duality is a manifest symmetry

# Double Field Theory

- Double field theory on doubled torus
- General solution of string theory: involves doubled fields  $\psi(x, \tilde{x})$
- *Real* dependence on *full* doubled geometry, dual dimensions not auxiliary or gauge artifact. Double geom. *physical* and *dynamical*
- *Strong constraint* restricts to subsector in which extra coordinates auxiliary: get conventional field theory locally. Recover **Siegel's** duality covariant formulation of (super)gravity

# Extra Dimensions

- Torus compactified theory has charges arising in SUSY algebra, carried by BPS states
- $P_M$ : Momentum in extra dimensions
- $Z_A$ : wrapped brane & wound string charges
- But  $P_M, Z_A$  related by dualities. Can  $Z_A$  be thought of as momenta for extra dimensions?
- Space with coordinates  $X^M, Y^A$  ?

# Extended Spacetime

- Supergravity can be rewritten in extended space with coordinates  $X^M, Y^A$ . Duality symmetry manifest.
- But fields depend only on  $X^M$  (or coords related to these by duality).
- Gives a geometry for non-geometry: T-folds
- Actual **stringy** symmetry of theory can be quite different from this sugra duality:  
**Background dependence?**
- In string theory, can do better... DOUBLE FIELD THEORY, fields depending on  $X^M, Y_M$ .

# M-Theory

- 11-d sugra can be written in extended space.
- Extension to full M-theory?
- If M-theory were a perturbative theory of membranes, would have extended fields depending on  $X^M$  and 2-brane coordinates  $Y_{MN}$
- But it doesn't seem to be such a theory
- Don't have e.g. formulation as infinite no. of fields. Only implicit construction as a limit.
- Extended field theory gives a duality-symmetric reformulation of supergravity

# Type IIA Supergravity Compactified on $T^4$

Duality symmetry  $SO(5,5)$

BPS charges in 16-dim rep

# Type IIA Supergravity on $M_4 \times M_6$

Can be written as “Extended Field Theory”

in space with 6 + 16 coordinates

with  $SO(5,5)$  Symmetry

Berman, Godzagar, Perry;  
Hohm, Samtleben

Is  $SO(5,5)$  a “real” symmetry for generic solutions  $M_4 \times M_6$ ?

Similar story for  $M_d \times M_{10-d}$  ;  $M_d \times M_{11-d}$



# Extension to String Theory?

Type IIA Superstring Compactified on  $T^4$

U-Duality symmetry  $SO(5,5;Z)$       6+16 dims?

But for other backgrounds, symmetry different

Type IIA Superstring Compactified on  $K3$

U-Duality symmetry  $SO(4,20;Z)$       6+24 dims?

Duality symmetry seems to be background dependent;  
makes background independent formulation problematic

# Double Field Theory

- String field theory gives complete formulation of perturbative closed string in CFT background Zwiebach
- Iterative construction of infinite number of interactions
- Non polynomial. Homotopy Lie algebra (violation of Jacobi's associativity etc)
- String field theory for torus gives infinite set of fields depending on doubled coordinates

# String Field Theory on Minkowski Space

Closed SFT:  
Zwiebach

String field  $\Phi[X(\sigma), c(\sigma)]$

$X^i(\sigma) \rightarrow x^i$ , oscillators

Expand to get infinite set of fields

$g_{ij}(x), b_{ij}(x), \phi(x), \dots, C_{ijk\dots l}(x), \dots$

Integrating out massive fields gives field theory for

$g_{ij}(x), b_{ij}(x), \phi(x)$

# String Field Theory on a torus

String field  $\Phi[X(\sigma), c(\sigma)]$

$X^i(\sigma) \rightarrow x^i, \tilde{x}_i$ , oscillators

Expand to get infinite set of double fields

$g_{ij}(x, \tilde{x}), b_{ij}(x, \tilde{x}), \phi(x, \tilde{x}), \dots, C_{ijk\dots l}(x, \tilde{x}), \dots$

Seek double field theory for

$g_{ij}(x, \tilde{x}), b_{ij}(x, \tilde{x}), \phi(x, \tilde{x})$

# Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

# Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

Treat as field equation, kinetic operator in doubled space

$$G^{ij} \frac{\partial^2}{\partial x^i \partial x^j} + G_{ij} \frac{\partial^2}{\partial \tilde{x}_i \partial \tilde{x}_j}$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Treat as constraint on double fields

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} \quad (\Delta - \mu)\psi = 0$$

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Laplacian for metric

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

$$ds^2 = G_{ij} dx^i dx^j + G^{ij} d\tilde{x}_i d\tilde{x}_j$$

Treat as constraint on double fields

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} \quad (\Delta - \mu)\psi = 0$$

Laplacian for metric

$$ds^2 = dx^i d\tilde{x}_i$$

$$g_{ij}(x, \tilde{x}), b_{ij}(x, \tilde{x}), \phi(x, \tilde{x})$$

$$N = \bar{N} = 1$$

$$p^2 + w^2 = 0$$

$$p \cdot w = 0$$

**“Double Massless”**



# DFT gives $O(D,D)$ covariant formulation

## $O(D,D)$ Covariant Notation

$$X^M \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix} \quad \partial_M \equiv \begin{pmatrix} \tilde{\partial}^i \\ \partial_i \end{pmatrix}$$

$$\eta_{MN} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad M = 1, \dots, 2D$$

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} = \frac{1}{2} \partial^M \partial_M$$

**Constraint**

$$\partial^M \partial_M A = 0$$

on all fields and parameters

Weak Constraint or  
weak section condition

Arises from string theory constraint

$$(L_0 - \bar{L}_0)\Psi = 0$$

- Weakly constrained DFT non-local.  
Constructed to cubic order **Hull & Zwiebach**
- ALL doubled geometry dynamical, evolution in all doubled dimensions
- Restrict to simpler theory: **STRONG CONSTRAINT**
- Fields then depend on only half the doubled coordinates
- Locally, just conventional SUGRA written in duality symmetric form

## Strong Constraint for DFT

Hohm, H & Z

$$\partial^M \partial_M (AB) = 0$$

$$(\partial^M A) (\partial_M B) = 0$$

on all fields and parameters

If impose this, then it implies weak form, but product of constrained fields satisfies constraint.

This gives **Restricted DFT**, a subtheory of DFT

Locally, it implies fields only depend on at most half of the coordinates, fields are restricted to null subspace  $N$ .

Looks like conventional field theory on subspace  $N$

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- **Siegel's** duality covariant form of (super)gravity

- In string theory, T-duality acts on torus or fibres of torus fibration, relates local modes and winding
- Winding modes: doubling of torus or fibres
- Other topologies may not have windings, or have different numbers of momenta and windings. No T-duality. No doubling?
- DFT 'background independent' **HHZ**. Can write on doubling of any space. What is double if not derived from string theory?

# Generalised T-duality transformations: HHZ

$$X'^M \equiv \begin{pmatrix} \tilde{x}'_i \\ x'^i \end{pmatrix} = h X^M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix}$$

$h$  in  $O(d,d;\mathbf{Z})$  acts on toroidal coordinates only

$$\mathcal{E}_{ij} = g_{ij} + b_{ij}$$

$$\mathcal{E}'(X') = (a\mathcal{E}(X) + b)(c\mathcal{E}(X) + d)^{-1}$$

$$d'(X') = d(X)$$

Buscher if fields independent of toroidal coordinates  
Generalisation to case without isometries

$$X^M = \begin{pmatrix} \tilde{x}_m \\ x^m \end{pmatrix} \quad \xi^M = \begin{pmatrix} \tilde{\epsilon}_m \\ \epsilon^m \end{pmatrix}$$

## Linearised Gauge Transformations

$$\delta h_{ij} = \partial_i \epsilon_j + \partial_j \epsilon_i + \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i ,$$

$$\delta b_{ij} = -(\tilde{\partial}_i \epsilon_j - \tilde{\partial}_j \epsilon_i) - (\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i) ,$$

$$\delta d = -\partial \cdot \epsilon + \tilde{\partial} \cdot \tilde{\epsilon} . \quad \text{Invariance needs constraint}$$

Diffeos and B-field transformations mixed.

If fields indep of  $\tilde{x}_m$ , conventional theory  $g_{ij}(x), b_{ij}(x), d(x)$

$\epsilon^m$  parameter for diffeomorphisms

$\tilde{\epsilon}_m$  parameter for B-field gauge transformations

# Generalised Metric Formulation

Hohm, H & Z

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix}.$$

2 Metrics on double space  $\mathcal{H}_{MN}, \eta_{MN}$

$$\mathcal{H}^{MN} \equiv \eta^{MP}\mathcal{H}_{PQ}\eta^{QN}$$

Constrained metric  $\mathcal{H}^{MP}\mathcal{H}_{PN} = \delta^M_N$



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Constrained metric  $\mathcal{H}^{MP}\mathcal{H}_{PN} = \delta^M_N$

Covariant  $O(D,D)$  Transformation

$$h^P_M h^Q_N \mathcal{H}'_{PQ}(X') = \mathcal{H}_{MN}(X)$$

$$X' = hX \quad h \in O(D, D)$$

## O(D,D) covariant action

$$S = \int dx d\tilde{x} e^{-2d} L$$

$$L = \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_N \mathcal{H}^{KL} \partial_L \mathcal{H}_{MK} \\ - 2 \partial_M d \partial_N \mathcal{H}^{MN} + 4 \mathcal{H}^{MN} \partial_M d \partial_N d$$

- Lagrangian L CUBIC in fields!
- Indices raised and lowered with  $\eta_{MN}$
- O(D,D) covariant (in  $\mathbb{R}^{2D}$ )

## 2-derivative action

$$\mathcal{S} = \mathcal{S}^{(0)}(\partial, \partial) + \mathcal{S}^{(1)}(\partial, \tilde{\partial}) + \mathcal{S}^{(2)}(\tilde{\partial}, \tilde{\partial})$$

Write  $\mathcal{S}^{(0)}$  in terms of usual fields

Gives usual action (+ surface term)

$$\int dx \sqrt{-g} e^{-2\phi} \left[ R + 4(\partial\phi)^2 - \frac{1}{12} H^2 \right]$$

$$\mathcal{S}^{(0)} = \mathcal{S}(\mathcal{E}, d, \partial)$$

## 2-derivative action

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Gives usual action (+ surface term)

$$\int dx \sqrt{-g} e^{-2\phi} \left[ R + 4(\partial\phi)^2 - \frac{1}{12} H^2 \right]$$

$$\mathcal{S}^{(0)} = \mathcal{S}(\mathcal{E}, d, \partial)$$

$$\mathcal{S}^{(2)} = \mathcal{S}(\mathcal{E}^{-1}, d, \tilde{\partial}) \quad \text{T-dual!}$$

$$\mathcal{S}^{(1)} \quad \text{strange mixed terms}$$

## O(D,D) covariant action

$$S = \int dx d\tilde{x} e^{-2d} L$$

$$L = \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_N \mathcal{H}^{KL} \partial_L \mathcal{H}_{MK} \\ - 2 \partial_M d \partial_N \mathcal{H}^{MN} + 4 \mathcal{H}^{MN} \partial_M d \partial_N d$$

## Gauge Transformation

$$\delta_\xi \mathcal{H}^{MN} = \xi^P \partial_P \mathcal{H}^{MN} \\ + (\partial^M \xi_P - \partial_P \xi^M) \mathcal{H}^{PN} + (\partial^N \xi_P - \partial_P \xi^N) \mathcal{H}^{MP}$$

Write as “Generalised Lie Derivative”

$$\delta_\xi \mathcal{H}^{MN} = \hat{\mathcal{L}}_\xi \mathcal{H}^{MN}$$

# Generalised Lie Derivative

$$A_{N_1 \dots}^{M_1 \dots}$$

$$\begin{aligned} \widehat{\mathcal{L}}_{\xi} A_M^N &\equiv \xi^P \partial_P A_M^N \\ &+ (\partial_M \xi^P - \partial^P \xi_M) A_P^N + (\partial^N \xi_P - \partial_P \xi^N) A_M^P \end{aligned}$$

Usual Lie derivative, plus terms involving  $\eta_{MN}$

$$\begin{aligned} \widehat{\mathcal{L}}_{\xi} A_M^N &= \mathcal{L}_{\xi} A_M^N \\ &- \eta^{PQ} \eta_{MR} \partial_Q \xi^R A_P^N \\ &+ \eta_{PQ} \eta^{NR} \partial_R \xi^Q A_M^P \end{aligned}$$

Strong Constraint: Gauge symm  $\sim$  diffeos and b-field trans

$$\underline{O(D,D)} \quad X' = hX$$

Symmetry for flat doubled space  $M = \mathbb{R}^{2D}$

B-shifts and  $GL(D, \mathbb{R})$  arise from local symmetries.

Isometries: if fields indep of some coords, more of  $O(D,D)$  can arise from local symmetries HHZ

Torus spacetime  $N = \mathbb{R}^{n-1,1} \times T^d$   $M = \mathbb{R}^{2n-2,2} \times T^{2d}$

$O(D,D)$  broken to subgroup containing B-shifts and

$$O(n, n) \times O(d, d; \mathbb{Z})$$

General Spacetime: No natural action of  $O(D,D)$

Strong Constraint: Gauge symm  $\sim$  diffeos and b-field trans

O(D,D)  $X' = hX$

Symmetry for doubled space  $M = \mathbb{R}^{2D}$

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O(D,D) broken to subgroup containing B-shifts and

$$O(n, n) \times O(d, d; \mathbb{Z})$$

B-shifts and  $GL(n, \mathbb{R}) \times GL(d, \mathbb{Z})$

arise from local symmetries.

General Spacetime: No natural action of O(D,D)



## Generalized scalar curvature

$$\begin{aligned}\mathcal{R} \equiv & 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} \\ & - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d \\ & + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL}\end{aligned}$$

$$S = \int dx d\tilde{x} e^{-2d} \mathcal{R}$$

Gauge Symmetry

$$\delta_\xi \mathcal{R} = \hat{\mathcal{L}}_\xi \mathcal{R} = \xi^M \partial_M \mathcal{R}$$

$$\delta_\xi e^{-2d} = \partial_M (\xi^M e^{-2d})$$

## Generalized scalar curvature

$$\begin{aligned}\mathcal{R} \equiv & 4 \mathcal{H}^{MN} \partial_M \partial_N d - \partial_M \partial_N \mathcal{H}^{MN} \\ & - 4 \mathcal{H}^{MN} \partial_M d \partial_N d + 4 \partial_M \mathcal{H}^{MN} \partial_N d \\ & + \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_K \mathcal{H}_{NL}\end{aligned}$$

$$S = \int dx d\tilde{x} e^{-2d} \mathcal{R}$$

Gauge Symmetry

$$\delta_\xi \mathcal{R} = \hat{\mathcal{L}}_\xi \mathcal{R} = \xi^M \partial_M \mathcal{R}$$

$$\delta_\xi e^{-2d} = \partial_M (\xi^M e^{-2d})$$

Field equations give gen. Ricci tensor

# Gauge Algebra

Parameters  $\Sigma^M$

Gauge Algebra  $[\delta_{\Sigma_1}, \delta_{\Sigma_2}] = \delta_{[\Sigma_1, \Sigma_2]_C}$

$$[\hat{\mathcal{L}}_{\xi_1}, \hat{\mathcal{L}}_{\xi_2}] = -\hat{\mathcal{L}}_{[\xi_1, \xi_2]_C}$$

C-Bracket:

$$[\Sigma_1, \Sigma_2]_C \equiv [\Sigma_1, \Sigma_2] - \frac{1}{2} \eta^{MN} \eta_{PQ} \Sigma_{[1}^P \partial_N \Sigma_{2]}^Q$$

Lie bracket + metric term

Parameters  $\Sigma^M(X)$  restricted to N

Decompose into vector + 1-form on N

C-bracket reduces to **Courant bracket** on N

Same covariant form of gauge algebra found in similar context by **Siegel**

## Jacobi Identities not satisfied!

$$J(\Sigma_1, \Sigma_2, \Sigma_3) \equiv [ [\Sigma_1, \Sigma_2], \Sigma_3 ] + \text{cyclic} \neq 0$$

for both C-bracket and Courant-bracket

How can bracket be realised as a symmetry algebra?

$$[ [\delta_{\Sigma_1}, \delta_{\Sigma_2}], \delta_{\Sigma_3} ] + \text{cyclic} = \delta_{J(\Sigma_1, \Sigma_2, \Sigma_3)}$$

# Symmetry is Reducible

Parameters of the form  $\Sigma^M = \eta^{MN} \partial_N \chi$   
do not act

Gauge algebra determined up to such transformations

cf 2-form gauge field  $\delta B = d\alpha$

Parameters of the form  $\alpha = d\beta$   
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cf 2-form gauge field  $\delta B = d\alpha$

Parameters of the form  $\alpha = d\beta$   
do not act

Resolution:

$$J(\Sigma_1, \Sigma_2, \Sigma_3)^M = \eta^{MN} \partial_N \chi$$

$\delta_{J(\Sigma_1, \Sigma_2, \Sigma_3)}$  does not act on fields

**D-Bracket**

$$[A, B]_{\text{D}} \equiv \hat{\mathcal{L}}_A B$$

$$[A, B]_{\text{D}}^M = [A, B]_{\text{C}}^M + \frac{1}{2} \partial^M (B^N A_N)$$

Not skew, but satisfies Jacobi-like identity

$$[A, [B, C]_{\text{D}}]_{\text{D}} = [[A, B]_{\text{D}}, C]_{\text{D}} + [B, [A, C]_{\text{D}}]_{\text{D}}$$

On restricting to null subspace N

C-bracket  $\rightarrow$  Courant bracket

D-bracket  $\rightarrow$  Dorfman bracket

Gen Lie Derivative  $\rightarrow$  GLD of Grana, Minasian, Petrini  
and Waldram

# DFT geometry

[arXiv:1406.7794](https://arxiv.org/abs/1406.7794)

- Simple explicit form of finite gauge transformations. Associative and commutative.
- Doubled space is a manifold, not flat, despite constant 'metric'  $\eta$  in DFT.
- Gives geometric understanding of 'generalised tensors' & relation to generalised geometry
- Transition functions give global picture
- T-folds: non-geometric backgrounds included



# Further Developments

- Action for F-theory **Berman, Blair, Malek, Rudolph**  
Theory in 12 dimensions with  $SL(2)$  symmetry;  
section condition gives 12-D theory
- Superstring **Hohm & Zwiebach,...**
- DFT (super)geometry, curved metric  $\eta$  **Cederwall**
- Generalised parallelizability, susy flux  
backgrounds, exceptional CY,... **Waldram et al**
- Susy AdS, exceptional Sasaki-Einstein,...  
**Ashmore, Petrini, Waldram**
- Extension to WZW models **Blumenhagen, Hassler & Lust**  
Scherk-Schwarz,... **Grana et al**

# Conclusions

- Duality symmetries lead to extension of geometry to allow “non-geometric” solutions
- String theory on torus: T-duality symmetry. Winding modes: doubled geometry, infinite number of doubled fields
- DFT: with strong constraint, get conventional sugra in duality symmetric formulation
- More generally, this applies locally in patches. Use DFT gauge and  $O(D,D)$  symmetries in transition functions. Get T-folds etc.

- Full theory with weak constraint: non-local, stringy
- How much of this is special to tori?
- Other topologies may not have windings, or have different numbers of momenta and windings. No T-duality? No doubling?
- Duality symmetry gives deep insight into stringy geometry. But seems to be very different on different backgrounds, e.g.  $T^4$ , K3
- Much remains to be understood about string/M theory