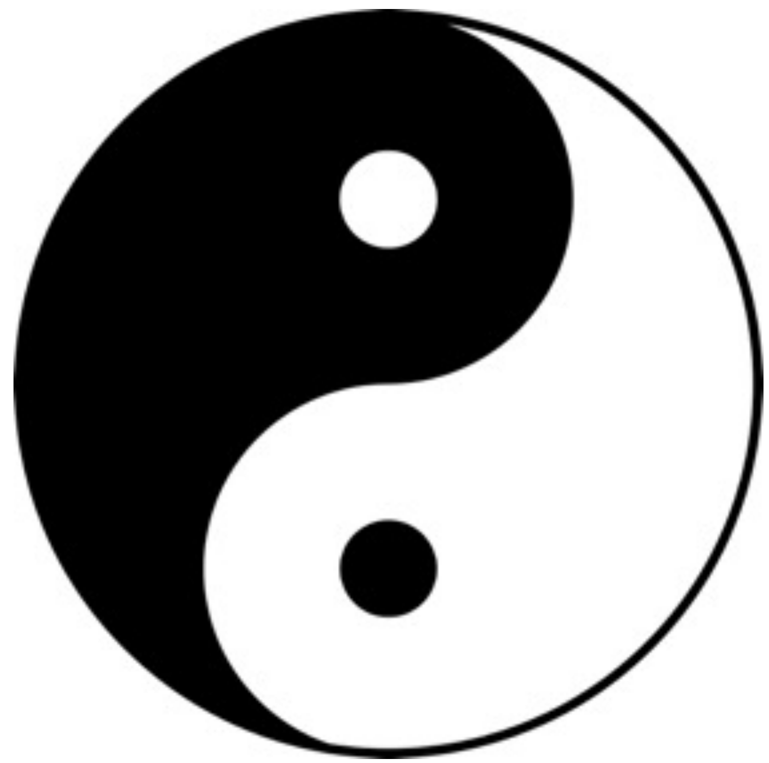
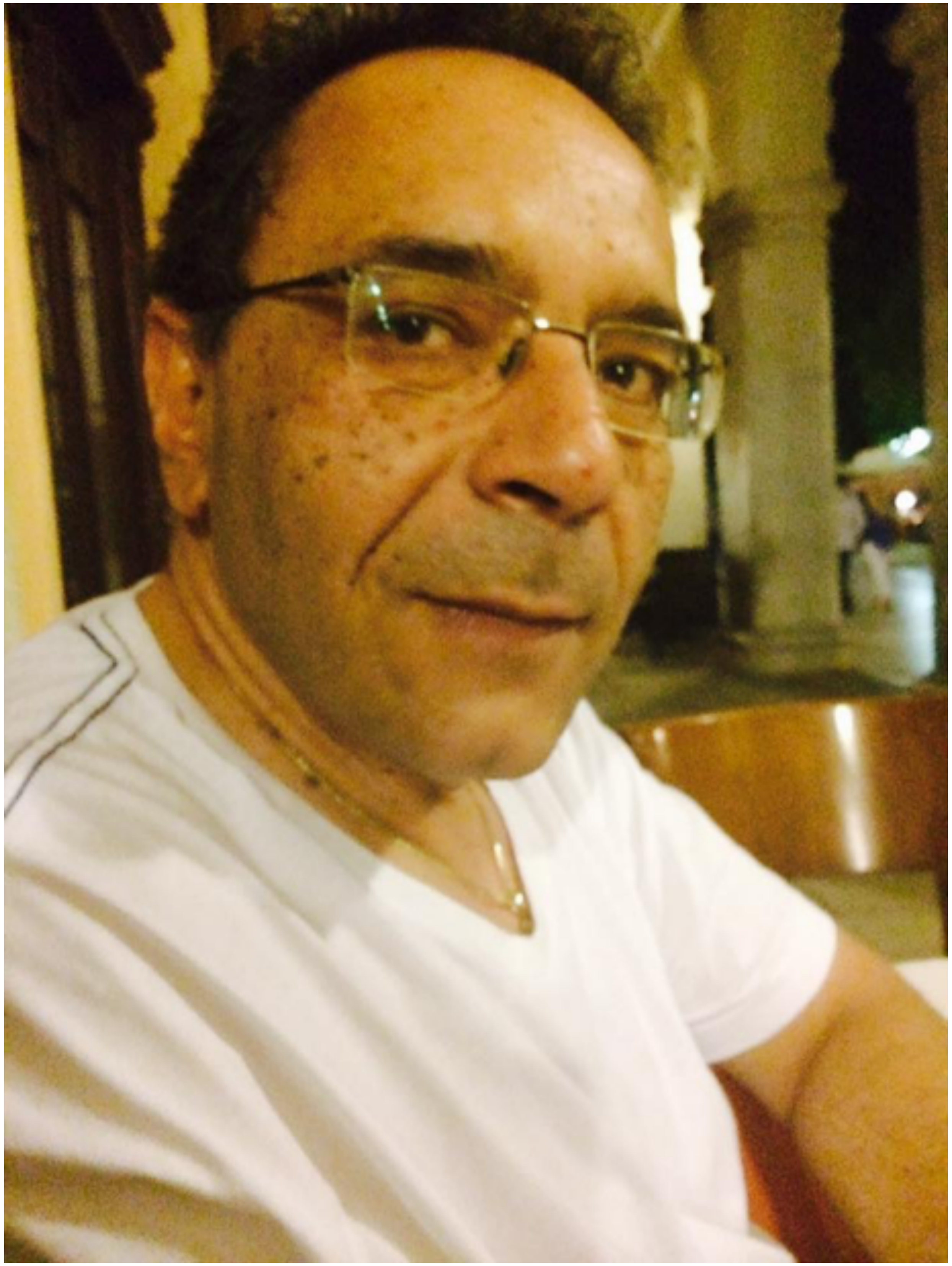
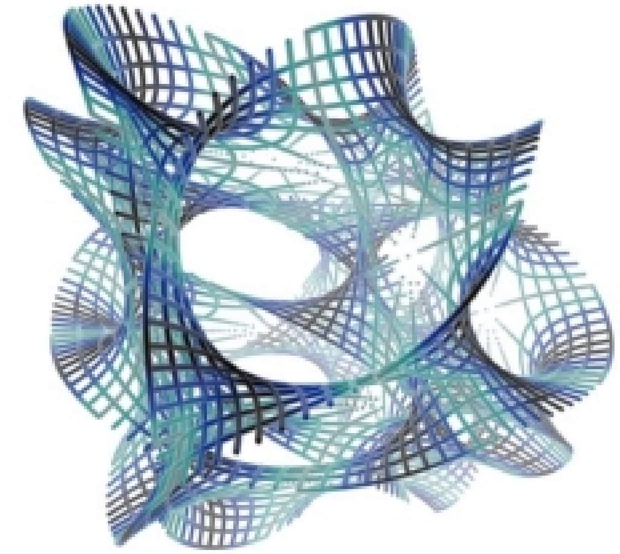
Double Field Theory





Ioannis Bakas
1960-2016

LECTURE 1: World sheet approach



- Overview
- Symmetry, T-duality, Geometry
- T-folds, non-geometric backgrounds.
- Double sigma-models
- D-branes

LECTURE 2: Target space approach

- Double Field Theory
- Generalised T-duality, Gauge Symmetries
- Geometry

String/M Theory

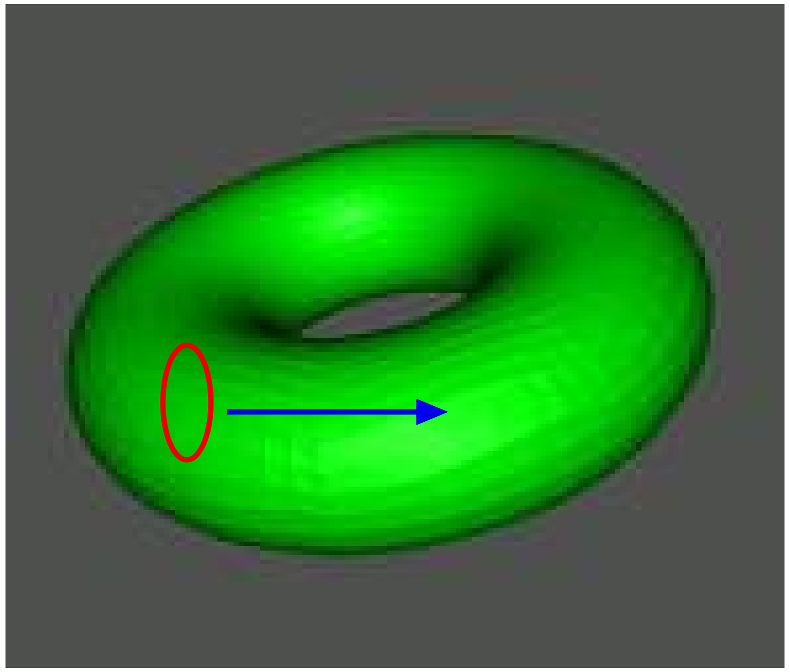
- Supergravity limit - misses stringy features
- Infinite set of fields: misses dualities,…
- World-sheet theory - only perturbative string
- String field theory: captures interactions, T-duality, algebraic structure
- Holography - only special spacetimes? AdS,…
- Matrix theory - non-perturbative, geometry hidden or emergent
- Fundamental formulation?

Strings in Geometric Background

Manifold, background tensor fields G_{ij}, H_{ijk}, Φ

Fluctuations: modes of string

Treat background and fluctuations the same?



Stringy geometry? Singularity resolution?

Dualities: mix geometric and stringy modes

Non-Geometric Background?

String theory: solutions that are not "geometric"

Moduli stabilisation. Richer landscape?

Symmetries

- (G, B, Φ) satisfying field eqns determine CFT
- Same CFT can be given by $(G, B, \Phi),\ (G', B', \Phi')$
- - if related by diffeos + B-field gauge transformations
- - if related by T-duality
- Same physics, so these are SYMMETRIES
- Stringy Equivalence Principle

T-duality

- Takes S^1 of radius R to S^1 of radius 1/R
- Exchanges momentum p and winding w
- Exchanges S^1 coordinate X and dual S^1 coordinate $\tilde{X}$
- Acts on "doubled circle" with coordinates $(X, \tilde{X})$

Duality Symmetries

- Supergravities: continuous classical symmetry, broken in quantum theory, and by gauging

- String theory: discrete quantum duality symmetries; not field theory symms

- T-duality: perturbative symmetry on torus, mixes momentum modes and winding states

- U-duality: non-perturbative symmetry of type II on torus, mixes momentum modes and wrapped brane states

Symmetry & Geometry

- Spacetime constructed from local patches
- All symmetries of physics used in patching
- Patching with diffeomorphisms, gives manifold
- Patching with gauge symmetries: bundles
- String theory has new symmetries, not present in field theory. New <u>non-geometric</u> string backgrounds
- Patching with T-duality: T-FOLDS
- Patching with U-duality: U-FOLDS

Extra Dimensions

- Kaluza-Klein theory: extra dimensions to spacetime, charges from KK momenta
- String theory in 10-d, M-theory in 11-d
- KK momenta dual to string winding modes and brane wrapping modes. Further dimensions? Strings on torus see doubled spacetime: double field theory
- Extended spacetime as arena for M-theory?

String Geometries

- D-dimensional **Manifold** N with tensor fields $g, H = db, \phi$
- **Generalised Geometry**: doubled tangent space E - a twisted form of $T \oplus T^*$ Action of O(D,D). Generalised metric $\mathcal{H}(g, b)$ and O(D,D) metric η on E.
- If N is D-**torus** T^D, T-duality symmetry O(D,D;Z) acts on D discrete momenta and D winding numbers. String theory "sees" double torus T^{2D}.
- If N is T^d **torus bundle**, string theory "sees" bundle with double torus fibres T^{2d}. T-duality O(d,d;Z) geometric.

- **T-folds**: torus bundles with T-duality transition functions. Good string backgrounds. g,H no longer tensors: “non-geometric”
- **Doubled torus bundle** geometric.
- More general “non-geometric” spaces with “R-flux”
- Non-toroidal backgrounds: **supergravity** can be rewritten in **duality covariant form** by introducing extra coordinates. What is geometry of enlarged spacetime if can’t be understood in terms of windings?

String Theory

- Standard: 2-d sigma model with target space (N,g,H,…)
- Gives infinite tower of fields on N
- **Doubled sigma model**, with target space double geometry

Tseytlin; CH

- Gives infinite tower of double fields on double space: **Double Field Theory**

Siegel; CH + Zwiebach

- Some physics can be studied using finite set of fields
- **Polarisations** and **Section Conditions**

- DFT written in terms of "generalised metric" $\mathcal{H}(g,b)$ and O(D,D) "metric" η

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix} . \qquad \eta_{MN} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

- Transform as "generalised tensors" via "generalised Lie derivative"

- Derived for torus, but gives duality covariantised supergravity for more general spaces. Meaning of extra coordinates?

- If η were a metric on doubled space, it would define a flat geometry, so would be highly restrictive

Strings on a Torus

- States: momentum p, winding w
- String: Infinite set of fields $\psi(p,w)$
- Fourier transform to doubled space: $\psi(x,\tilde{x})$
- "Double Field Theory" from closed string field theory. Some non-locality in doubled space
- Infinite set of fields in doubled space

CH & Zwiebach

Double Field Theory

- Double field theory on doubled torus
- General solution of string theory: involves doubled fields $\psi(x, \tilde{x})$
- *Real* dependence on *full* doubled geometry, dual dimensions not auxiliary or gauge artifact. Double geom. *physical* and *dynamical* (with weak constraint)
- *Strong constraint* restricts to subsector in which extra coordinates auxiliary: get conventional field theory locally. Siegel's duality covariant formulation of (super)gravity, T-theory

Strings in Geometric Target

Sigma Model

$$S = \frac{1}{2}\int (g_{ij}dx^i \wedge *dx^j + b_{ij}dx^i \wedge dx^j)$$

Hamiltonian density

$$h = \frac{1}{2}\mathcal{H}_{MN}P^M P^N + \dots$$

$$\mathcal{H}_{MN} \quad = \quad \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix} \ . \qquad\qquad P^M = \begin{pmatrix} p_m \\ w^m \end{pmatrix}$$

Involves generalised metric

Hamiltonian density $\quad h = \frac{1}{2}\mathcal{H}_{MN}P^M P^N + \ldots$

Rewrite $\quad h = \frac{1}{2}\hat{\mathcal{H}}_{MN}\hat{P}^M \hat{P}^N + \ldots$

$$\hat{\mathcal{H}}_{MN} = \begin{pmatrix} g^{ij} & 0 \\ 0 & g_{ij} \end{pmatrix}$$

Natural metric on $T \oplus T^*$

$$\hat{P}^M = \begin{pmatrix} \hat{p}_m \\ w^m \end{pmatrix} = \begin{pmatrix} p_m - b_{mn}w^n \\ w^m \end{pmatrix}$$

Section of $T \oplus T^*$

P^M Section of E, with metric $\mathcal{H}_{MN}$

E is $T \oplus T^*$ twisted by b-field

Generalised Geometry. Gives transformations under Diff(N) and b-field gauge transformations, transition functions

Strings on Circle

$$M = S^1 \times X$$

Discrete momentum p=n/R
If it winds m times round S^1, winding energy w=mRT
Energy = p^2+w^2+....

<u>T-duality</u>: Symmetry of string theory

$$\begin{array}{ccc} p & \leftrightarrow & w \\ m & \leftrightarrow & n \\ R & \leftrightarrow & 1/RT \end{array}$$

- Fourier transf of discrete p,w gives periodic coordinates
  $X, \tilde{X}$ Circle + dual circle
- Stringy symmetry, not in field theory
- On d torus, T-duality group $O(d, d; \mathbb{Z})$

Strings on T^d

$$X = X_L(\sigma + \tau) + X_R(\sigma - \tau), \qquad \tilde{X} = X_L - X_R$$

X conjugate to momentum, $\tilde{X}$ to winding no.

$$dX = *d\tilde{X} \qquad\qquad \partial_a X = \epsilon_{ab}\partial^b \tilde{X}$$

Need "auxiliary" $\tilde{X}$ for interacting theory

Vertex operators $e^{ik_L \cdot X_L}, \; e^{ik_R \cdot X_R}$

Strings on T^d

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Strings on torus see **DOUBLED TORUS**

T-duality group $O(d,d;\mathbb{Z})$

Doubled Torus 2d coordinates

Transform linearly under $O(d,d;\mathbb{Z})$

$$X \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix}$$

DOUBLED GEOMETRY Duff;Tseytlin; Siegel;Hull;…

Strings on d-Torus

Target space $T^d \times \mathbb{R}^D$ T^d Coordinates $X^i, i = 1, ..., d$

Moduli on torus (constant) $G_{ij},\ B_{ij}$ $E_{ij} = G_{ij} + B_{ij}$

T-Duality Symmetry $O(d,d;\mathbb{Z})$

i) Large Diffeos $GL(d;\mathbb{Z})$

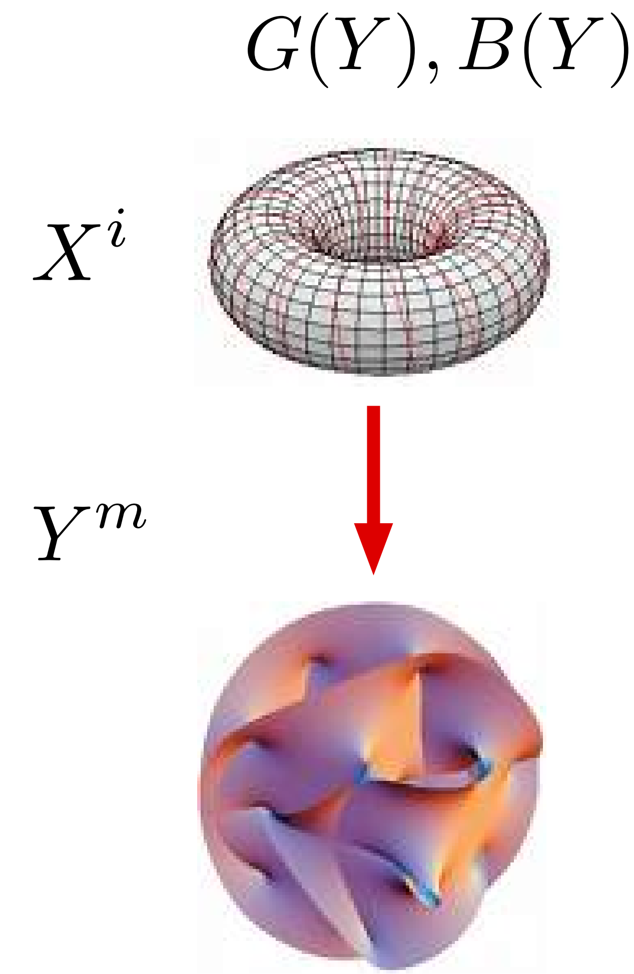
ii)B-shifts $B \rightarrow B + \Theta, \quad \Theta_{ij} \in \mathbb{Z}$

iii) Inversions $R_i \rightarrow 1/R_i$

$$E \rightarrow (aE+b)(cE+d)^{-1} \qquad h = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(d,d;Z)$$

$\left| p_i, w^i \right\rangle$ Lie in 2d-lattice, action of $O(d,d;\mathbb{Z})$

T-Duality



- Space has d-torus fibration
- G,B on fibres
- T-Duality O(d,d;Z), mixes G,B
- Mixes Momentum and Winding
- Changes geometry and topology

$$E \to (aE+b)(cE+d)^{-1}$$

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T-Duality

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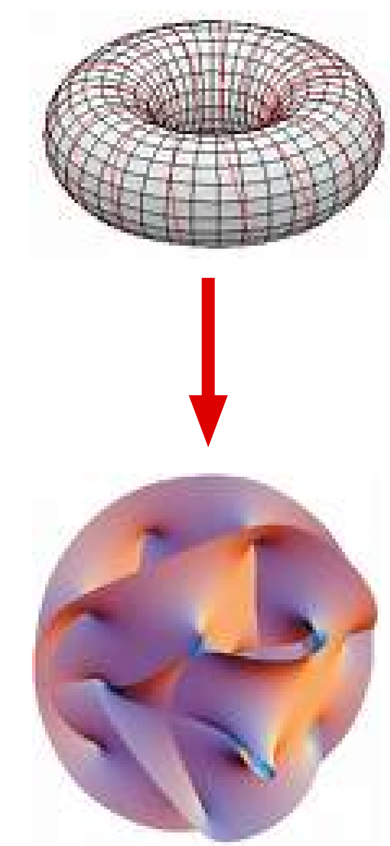
$G(Y), B(Y)$

X^i

Y^m



$$E \to (aE + b)(cE + d)^{-1}$$

$$h = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(d, d; Z) \qquad E_{ij} = G_{ij} + B_{ij}$$

$$\mathcal{H} \to h^t \mathcal{H} h$$

Dilaton

Careful 1-loop calculation gives shift in dilaton

$$\Phi \to \tilde{\Phi} = \Phi + \frac{1}{2} \log \frac{\det \tilde{G}}{\det G}$$

under T-duality

Conundrum

a) Dilaton shifts under T-duality
b) Dilaton expectation gives string coupling constant
c) T-duality is claimed to be a perturbative symmetry

$$g = \exp\langle\Phi\rangle$$

Conundrum

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Resolution

$$e^{-2d} = e^{-2\Phi}\sqrt{g}$$ is invariant under T-duality

Use $$g = \exp\langle d\rangle$$
as string coupling, invariant under T-duality

T-Duality & Cocycles

Suppose $R = \sqrt{\alpha'}$

$$p_L \equiv n - w, \quad p_R \equiv n + w$$ n,w integers

Naive T-duality T_0

$$X_L \to -X_L, \quad X_R \to X_R$$

$$n \leftrightarrow w \qquad (-1)^{\hat{N}_L}$$

Quantum T-duality T

$$|n, w, \tilde{N}_i, N_i\rangle \to \Omega_{n,w} \, (-1)^{N_L} \, |w, n, \tilde{N}_i, N_i\rangle$$

$$\Omega_{n,w}\Omega_{w,n} = 1$$ T_0 up to phase

Are Interactions Invariant Under T-Duality?

<u>Vertex Operators</u>

$$V^0_{(n,w)} = \exp\left(ip_L X_L + ip_R X_R\right)$$

Not mutually local

$$V^0_{(n,w)}(\sigma_1,\tau)\; V^0_{(n',w')}(\sigma_2,\tau)$$
$$= \exp \pi i(nw' + wn')V^0_{(n',w')}(\sigma_2,\tau)\; V^0_{(n,w)}(\sigma_1,\tau)$$

$V_{(n,w)} = \hat{C}_{(n,w)} \cdot V^{(0)}{}_{(n,w)}$ are mutually local

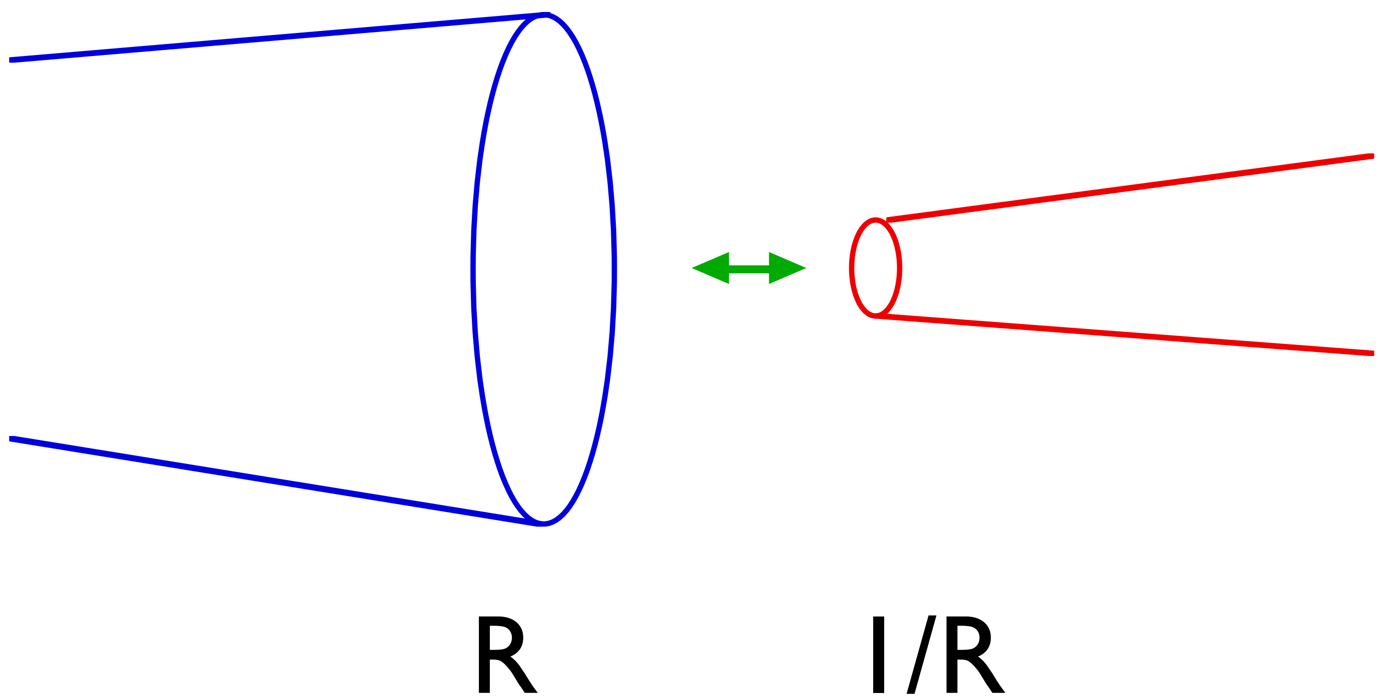
Cocyle $\qquad \hat{C}_{(n,w)} \equiv \exp\left(\pi i w\hat{n} - \frac{\pi i}{2} nw\right)$

Naive T-duality T_0 does not preserve OPE's

Proper T-duality T does preserve OPE's

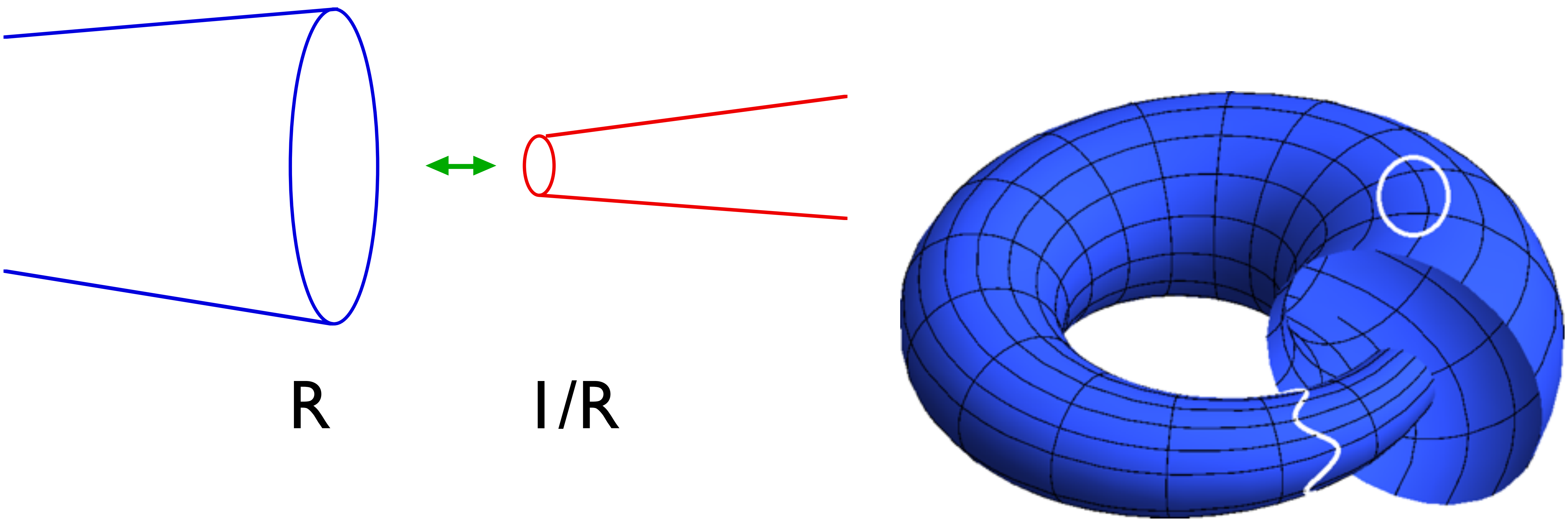
$$T \equiv T_0 \cdot (-1)^{\hat{n}\hat{w}}$$

T-fold patching



Glue big circle (R) to small (1/R)
Glue momentum modes to winding modes
(or linear combination of momentum and winding)
Not conventional smooth geometry

T-fold patching



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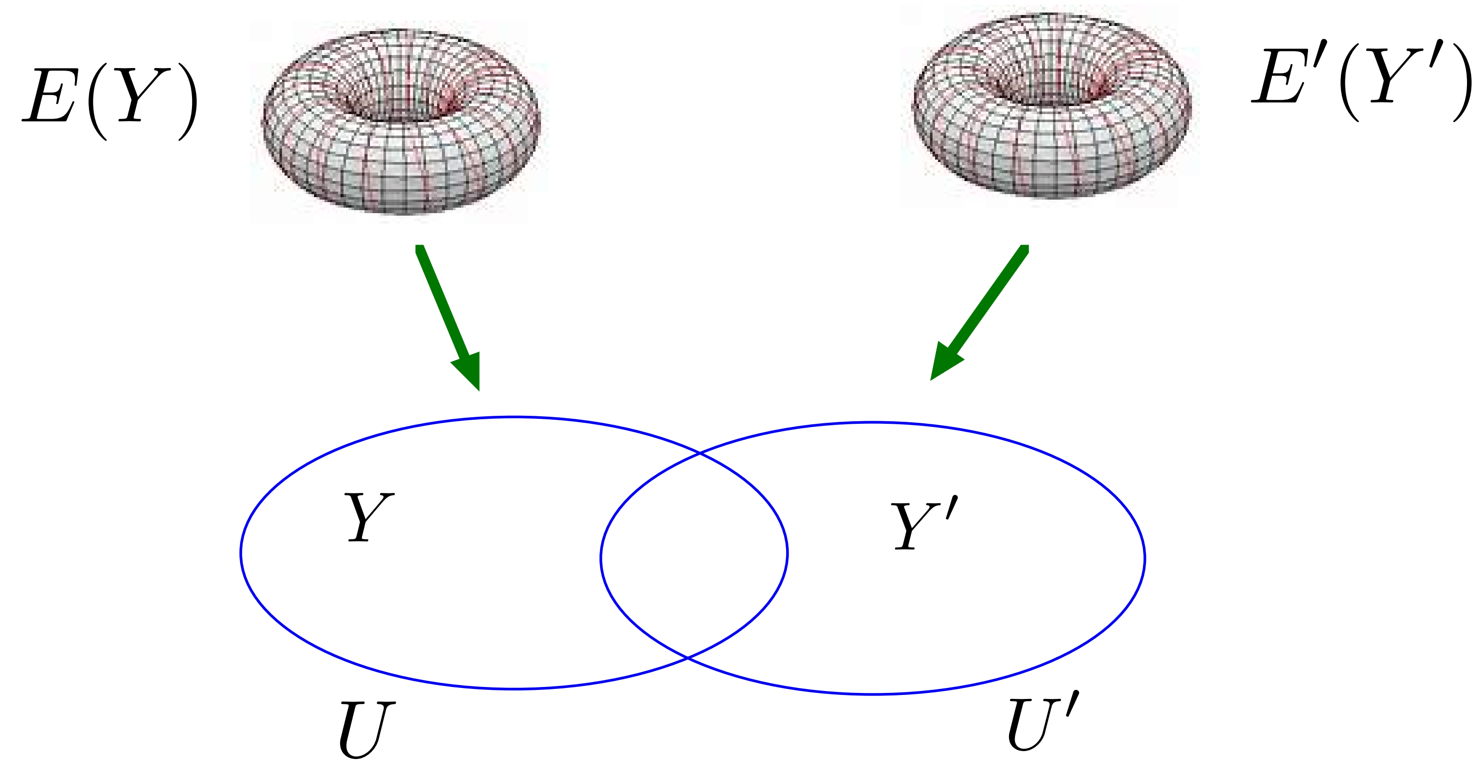


Torus fibration

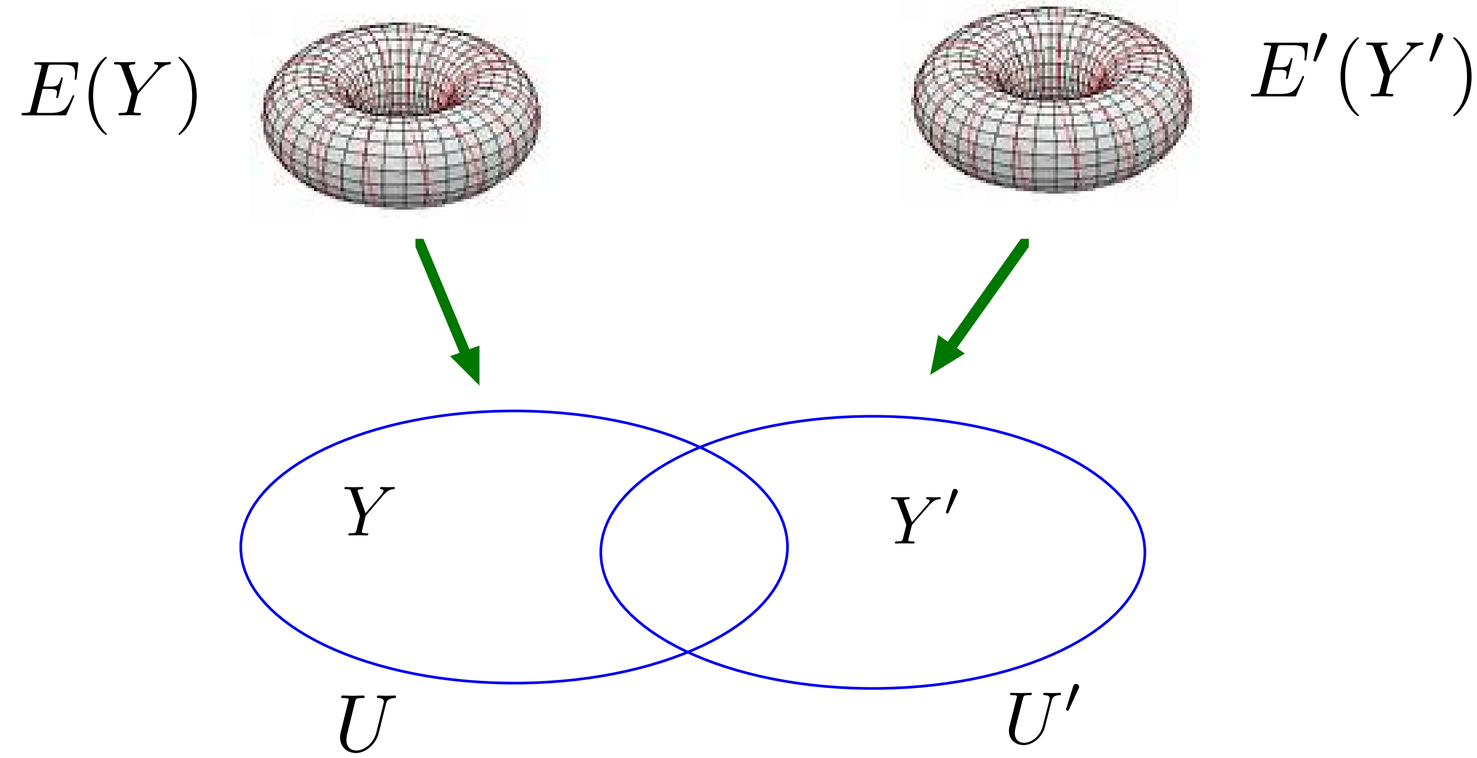
T-fold: Transition functions involve T-dualities

E=G+B Non-tensorial

$$O(d,d;\mathbb{Z}) \qquad E' = (aE+b)(cE+d)^{-1} \;\; \text{in} \;\; U \cap U'$$

Glue using T-dualities also → **T-fold**

Physics smooth, as T-duality a symmetry



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Glue using T-dualities also → **T-fold**

Physics smooth, as T-duality a symmetry

<u>Exotic Branes</u> can be T-folds — de Boer, Shigemori

E.g. T-dual of NS5-brane in transverse directions

Doubled Geometry for T-fold

CH

T^d torus fibres have doubled coords $\mathbb{X}^I = \begin{pmatrix} X^i \\ \widetilde{X}_i \end{pmatrix}$ $\quad I = 1, ..., 2d$

Transforms linearly under $O(d, d; \mathbb{Z})$
T-fold transition: mixes $X, \tilde{X}$
No global way of separating "real" space coordinate X from "auxiliary" $\widetilde{X}$

Duality covariant formulation in terms of $\mathbb{X}$
Transition functions $O(d, d; \mathbb{Z}) \subset GL(2d; \mathbb{Z})$
can be used to construct bundle with fibres T^{2d}

Doubled space can be a smooth manifold!

Sigma Model on doubled space. T-duality manifest.

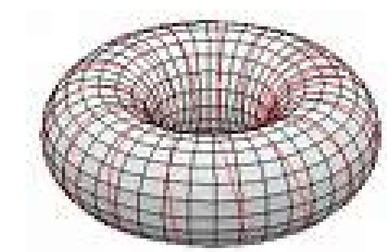
Doubled Bundle

T^{2d} bundle: doubled fibre

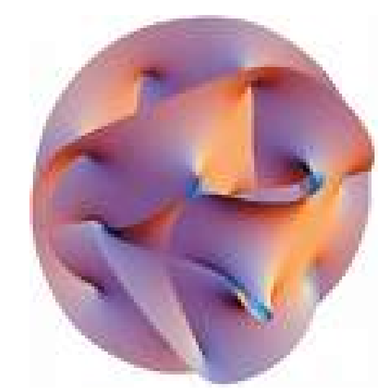
Construct duality-covariant sigma model on doubled space $(\mathbb{X}^I, Y^m)$

Constraint to halve degrees of freedom on fibre:

$\mathbb{X}^I$



Y^m



$$dX = *d\widetilde{X}$$ for free case

$$D\mathbb{X} = S(Y) * D\mathbb{X}$$ for general case

$$S(Y) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + O(Y) \qquad S^2 = 1$$

Double Sigma Model

Target space: doubled space
For torus bundle, double torus fibres
Formally, can "double everything" by introducing formal duals for non-toroidal dimensions.

Geometry: Use generalised metric, gives O(d,d) covariant formalism.

Constraint/gauging: halves the doubled degrees of freedom

Rocek-Verlinde sigma model: doubled space, different geometry, only some of O(d,d) manifest.

$$\mathcal{L}_k = \frac{1}{4}\mathcal{H}_{IJ}\,(d\mathbb{X}^I + \mathcal{A}^I) \wedge *(d\mathbb{X}^J + \mathcal{A}^J) \quad +\mathcal{L}(Y)$$

CH

$$\mathcal{L}_{WZ} = -\frac{1}{2}L_{IJ}d\mathbb{X}^I \wedge \mathcal{A}^J \qquad \mathcal{L}_{top} = \frac{1}{2}\Omega_{IJ}d\mathbb{X}^I \wedge d\mathbb{X}^J$$

Generalised metric

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

O(d,d) metric

$$L = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

2d connections

$$\mathcal{A}^I = \begin{pmatrix} A^i \\ \widetilde{A}_i \end{pmatrix} \qquad \begin{array}{l} A^i \sim G_{mi}dY^m \\ \tilde{A}^i \sim B_{mi}dY^m \end{array}$$

O(d,d) Covariant

$$\mathcal{H} \to h^t\mathcal{H}h \quad \mathbb{X} \to h^{-1}\mathbb{X} \quad \mathcal{A} \to h^{-1}\mathcal{A}$$

Product structure

$$S^I{}_J = L^{IK}\mathcal{H}_{KJ} \qquad S^2 = 1$$

Doubled Everything

Torus Bundle: Fibre doubled, base not. $(y, x, \tilde{x})$

Formally, can “double” base. Introduce “dual” coordinate $\tilde{y}$ for base $X^M = (y, \tilde{y}, x, \tilde{x})$

All fields independent of $\tilde{y}$, no freedom of polarisation

Rewrite in terms of full generalised metric

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix} .$$

$$S = \frac{1}{4} \int \mathcal{H}_{MN}\, dX^M \wedge * dX^N$$

$$\mathcal{S} dX = * dX, \qquad \mathcal{S} = \eta^{-1}\mathcal{H}$$

Quantisation

How should we impose constraint?

$$d\mathbb{X} + \mathcal{A} = S(Y) * (d\mathbb{X} + \mathcal{A})$$

X: d left-movers and d right-movers: chiral bosons

1) Floreanini-Jackiw action for chiral bosons gives Tseytlin doubled sigma model.
2) Absence of conformal and Lorentz anomalies gives field equations Berman, Copland, Thompson,…
3) Gives DFT field equations at one-loop Copland
4) PST approach gives this on gauge fixing
5) Issues with SUSY, higher genus?
6) Canonical Hackett-Jones & Moutsopolos

More Quantisation

How should we impose constraint?

$$d\mathbb{X} + \mathcal{A} = S(Y) * (d\mathbb{X} + \mathcal{A})$$

CH

1) Chose polarisation locally $\mathbb{X} \to \{X^i, \tilde{X}_i\}$
2) Constraint generates shifts in $\widetilde{X}$
   Gauge these shifts: sigma-model $\mathcal{L}(Y, X)$
3)

$$\exp(i \int \mathcal{L}_{top}) = \exp(\pi i n \widetilde{n}) = \pm 1$$

Gives equivalence on arbitrary Riemann surface
4) Extends proof of T-duality to fibrewise case, with Killing vectors only locally defined
5) SUSY straightforward

Circle and Dual Circle

$$g = R^2 dX^2$$ Circle

$$\tilde{g} = \frac{1}{R^2} d\tilde{X}^2$$ Dual Circle

$$\mathcal{H} = R^2 dX^2 + \frac{1}{R^2} d\tilde{X}^2$$ Doubled Geometry

$$X \to \tilde{X},\ \tilde{X} \to X,\ R \to 1/R$$ Invariance

Polarisation: Choose which coordinate is “spacetime”

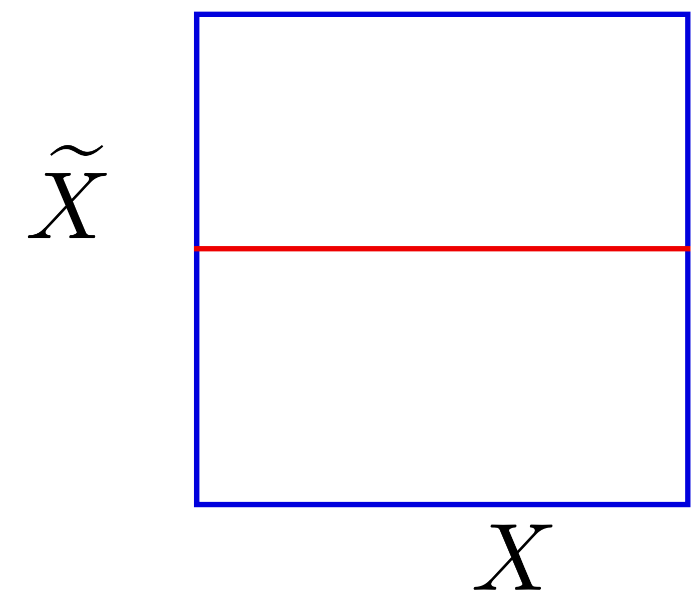
T-duality: keep doubled space fixed and change polarisation

OR: keep polarisation fixed and change $X \leftrightarrow \tilde{X}$

Polarisation

To recover conventional formulation, split into "fundamental" and "auxiliary":

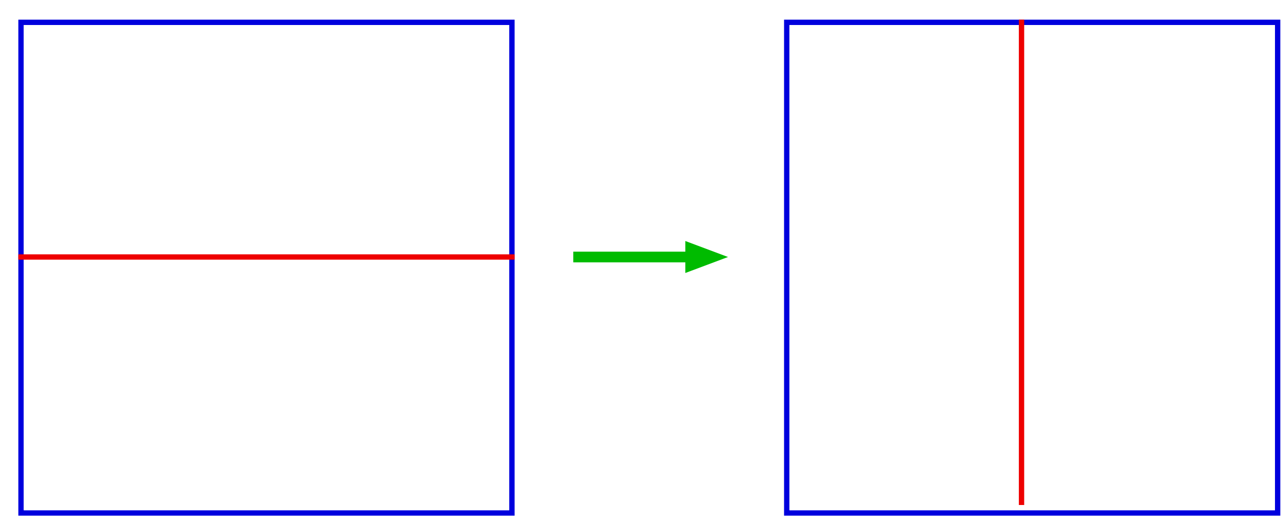
$$\mathbb{X} \to \{X^i, \tilde{X}_i\}$$



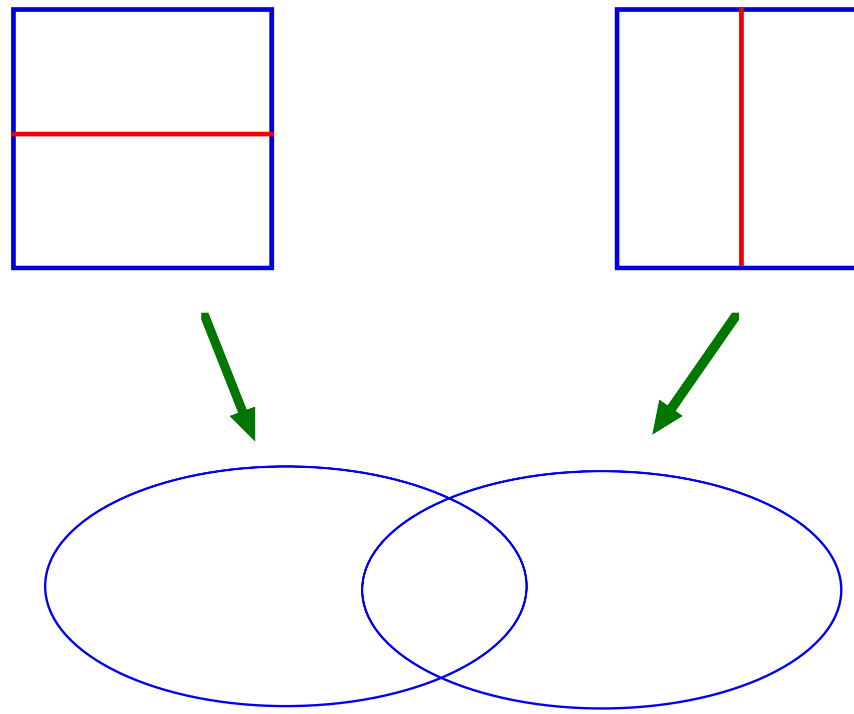
Pick "real spacetime", $T^d \subset T^{2d}$

T-duality rotates polarisation.

T-duality symmetry: physics independent of polarisation.



T-fold



Pick polarisation over each patch in base.

T-duality transitions: polarisation changes from patch to patch.

Geometric: there is global spacetime submanifold

Non-geometric if there is no global polarisation.

D-Branes & Open Strings

If X Neumann, T-dual $\tilde{X}$ is Dirichlet
If $\tilde{X}$ Dirichlet, T-dual X is Neumann

e.g. d=9, $R_{time} \times T^9$

$$\mathbb{X}^I = (X^i{}_D , X^i{}_N)$$

9 D coordinates, 9 N ones.
Universal 9-brane, lagrangian cycle

Polarisation chooses some number p of the Neumann directions as physical.
Interpret as p-brane

$X^1{}_N , X^2{}_N , X^3{}_N \ldots. X^9{}_N$, $X^1{}_D , X^2{}_D , X^3{}_D \ldots X^9{}_D$

Polarisation chooses 9 of 18 coords as “physical”

$X^1{}_N, X^2{}_N, X^3{}_N \ldots X^9{}_N$, $X^1{}_D, X^2{}_D, X^3{}_D \ldots X^9{}_D$

Polarisation chooses 9 of 18 coords as "physical"

$X^1{}_D, X^2{}_D, X^3{}_D \ldots X^9{}_D$ All 9 coords Dirichlet, 0-brane

$X^1{}_N, X^2{}_D, X^3{}_D \ldots X^9{}_D$ 1-brane

$X^1{}_N, X^2{}_N, X^3{}_D \ldots X^9{}_D$ 2-brane

$X^1{}_N, X^2{}_N, X^3{}_N \ldots X^9{}_N$ 9-brane

$X^1{}_N, X^2{}_N, X^3{}_N \ldots X^9{}_N,\ X^1{}_D, X^2{}_D, X^3{}_D \ldots X^9{}_D$

Polarisation chooses 9 of 18 coords as "physical"

$X^1{}_D, X^2{}_D, X^3{}_D \ldots X^9{}_D$	All 9 coords Dirichlet, 0-brane
$X^1{}_N, X^2{}_D, X^3{}_D \ldots X^9{}_D$	1-brane
$X^1{}_N, X^2{}_N, X^3{}_D \ldots X^9{}_D$	2-brane
$X^1{}_N, X^2{}_N, X^3{}_N \ldots X^9{}_N$	9-brane

T-fold transition: Glue Dp-brane to Dq-brane
Doubled picture: glue universal 9-branes together smoothly, but polarisation jumps