Coupling Topological Theories to the Standard Model

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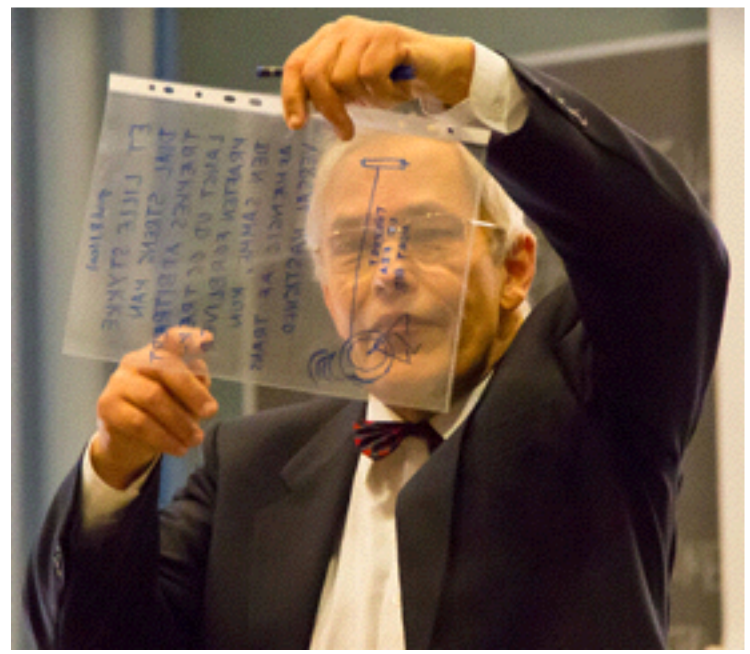
Corfu Workshop:

Recent Developments in Strings & Gravity

12-17 September 2016

Scientist Holger Bech Nielsen turns 75



Happy birthday Holger !

1. A (not so recent but) timely question: **B+L violation at LHC**

2. A new twist: **resonant tunneling** in QFT ? (Tye + Wong)

3. QM of the Chern-Simons number: **Energy bands?**

4. **BF theory** as decoupling limit of **axions**

5. More on **Volume operators**

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Well-known `problem' of perturbation theory:

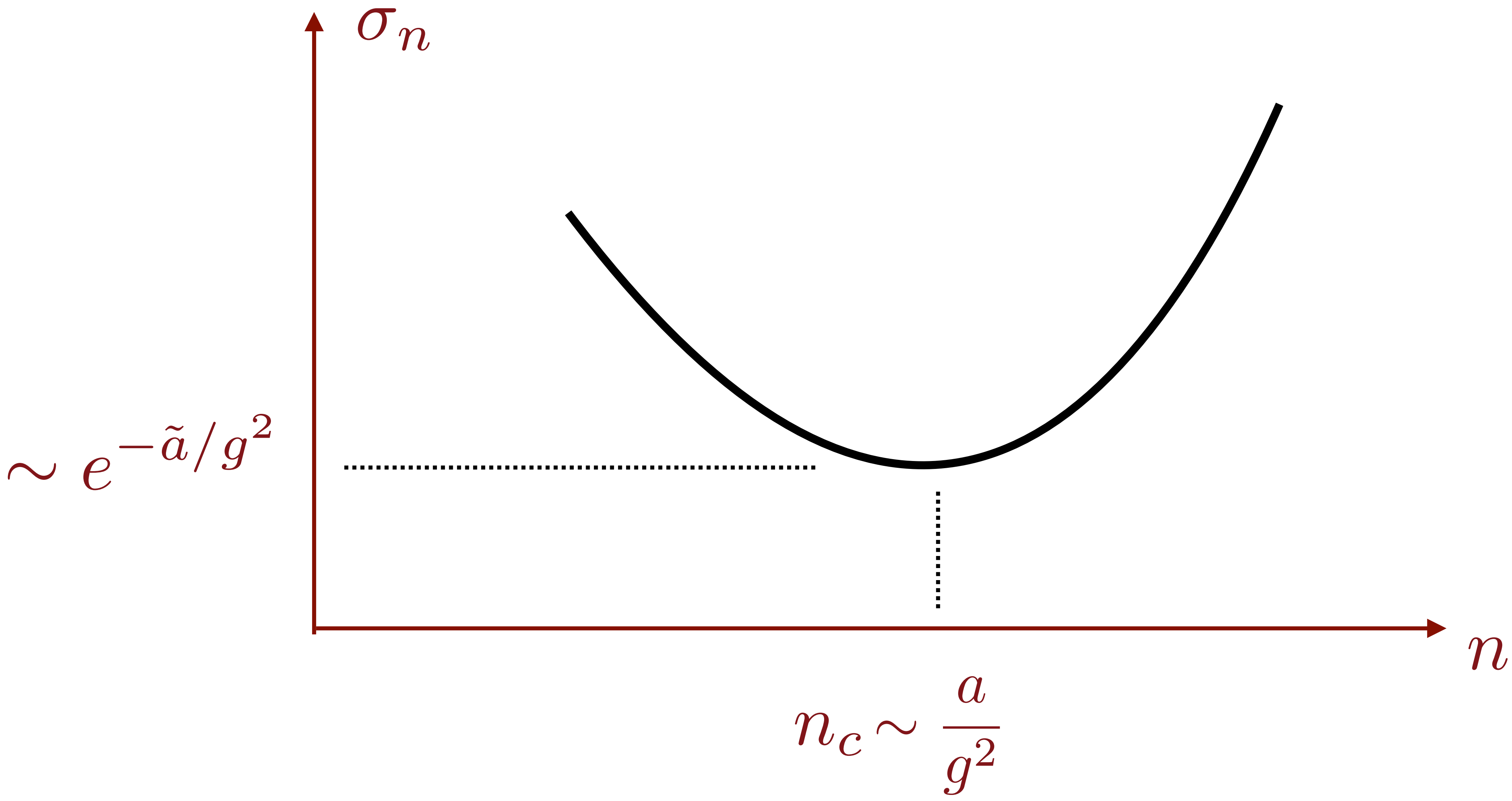
Hard to compute reliably processes with $2 \to n \sim \dfrac{1}{g^2}$ particles

. . . . even at very very weak coupling.

For ex. the tree-level cross-section behaves as

$$\sigma_n \sim g^{2n} n!\, f_n(E/M) \sim e^{n \log(g^2 n/a)}$$

reflecting the asymptotic behavior of large-order pert. theory.



Borel summability $\Longrightarrow$ σ_n stays exponentially small

Froissard bound $\Longrightarrow$ $\sigma_n \; < \; (\log E)^2$ much weaker

Related issue:

since **solitons** are made out of $\sim 1/g^2$ quanta, their production in High-E collisions should be exponentially small.

The only (unstable) soliton of the Standard Model is the **sphaleron** whose production leads to (B+L) violation.

Question: is instanton-induced B+L violation unsuppressed at LHC ?

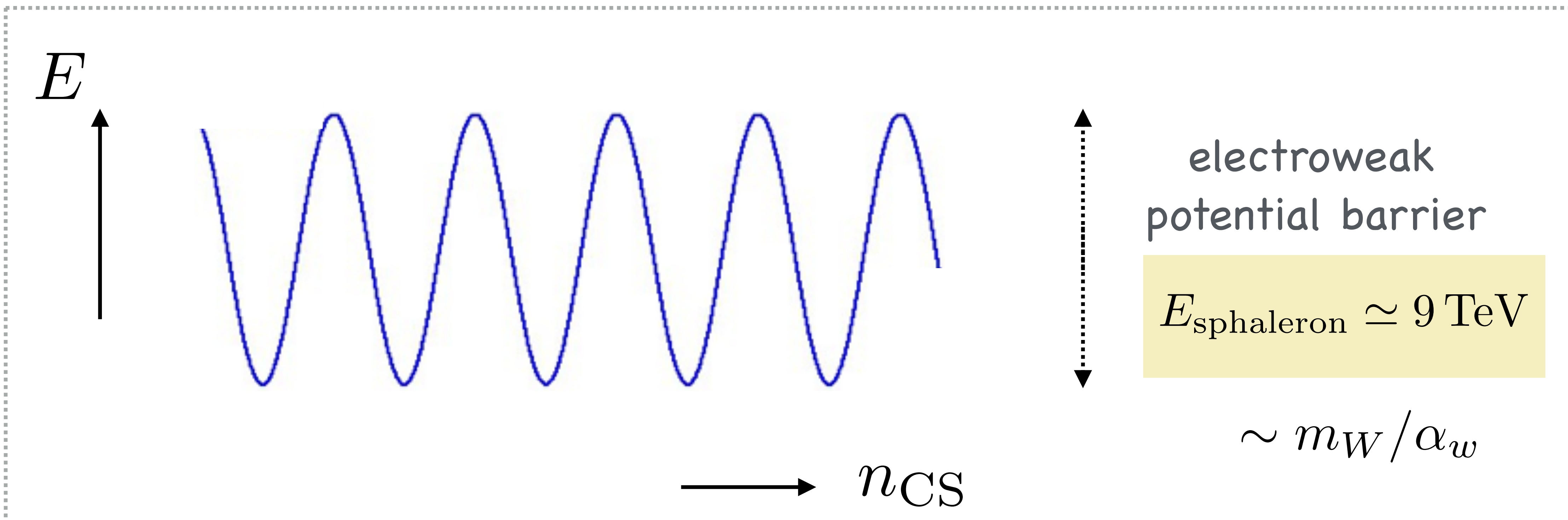
$$\partial_\mu\, j^\mu_{B_i} = \partial_\mu\, j^\mu_{L_i} = {}^*(F \wedge F)$$

$$\Longrightarrow \boxed{\Delta B = \Delta L = 3\,\Delta n_{\mathrm{CS}}}$$

't Hooft '76

instanton number

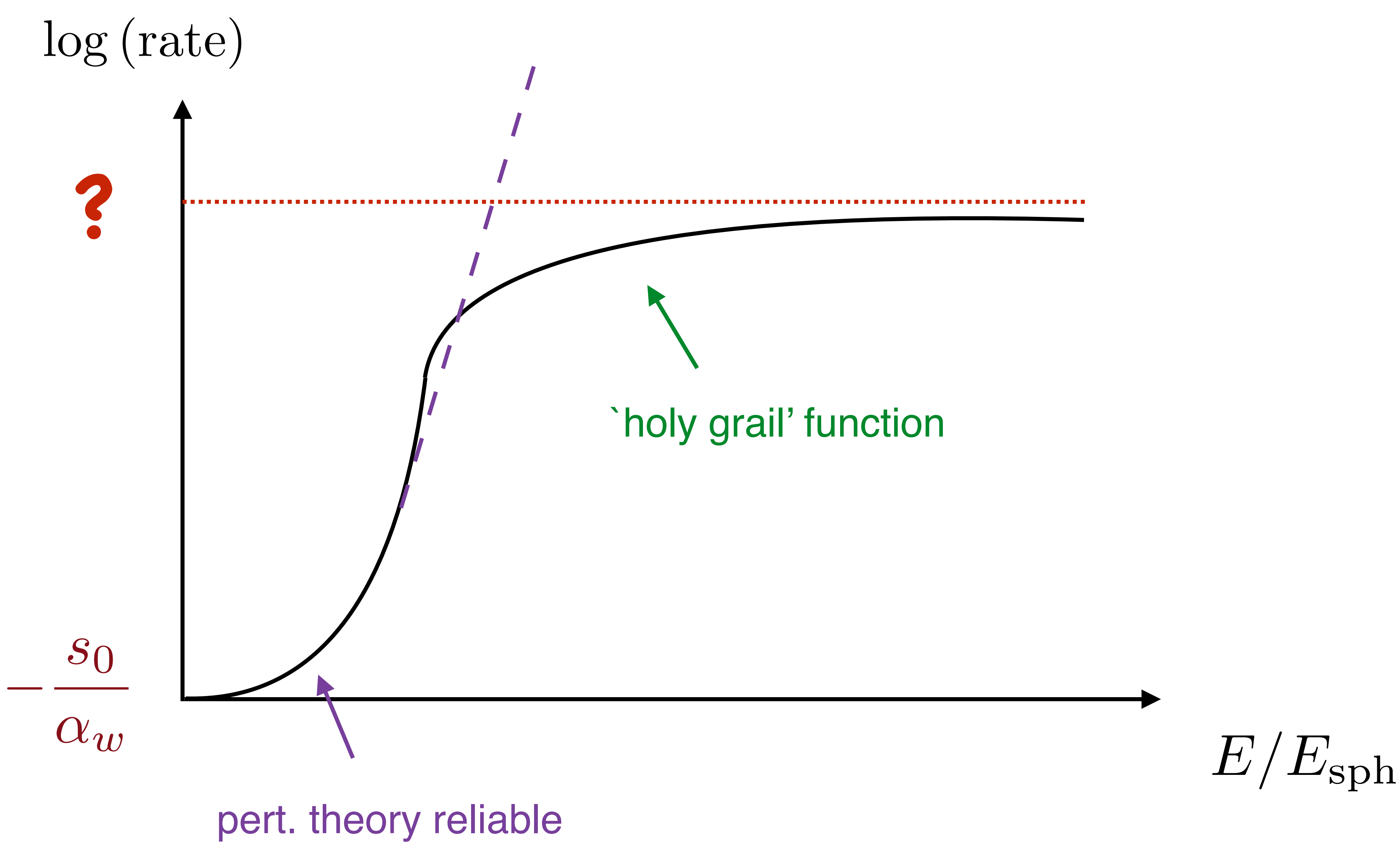
There is a characteristic scale in the problem because m_W puts an IR cutoff on the instanton size:



The C.M. energy at LHC is sufficient, but does tunneling occur ?

The question was debated in the early 90's, starting with a semiclassical calculation showing fast initial growth of the rate with collision energy.

Ringwald '89
Espinosa '89

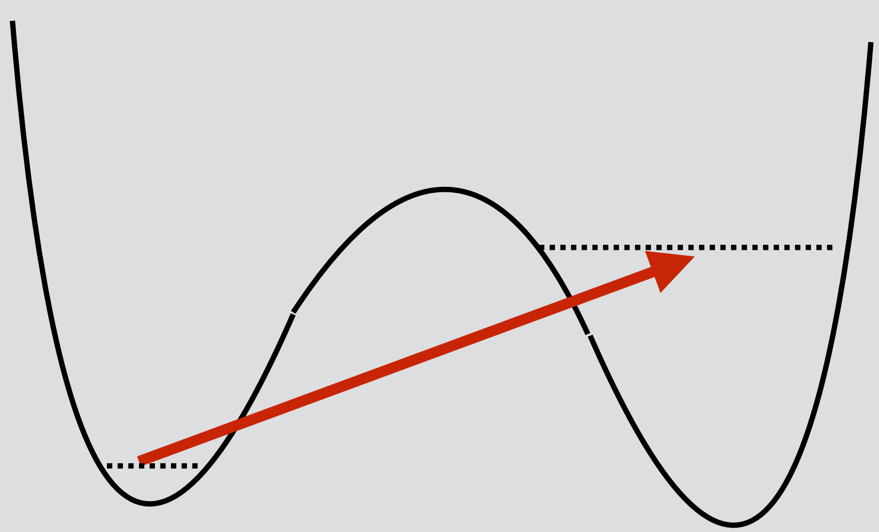


Consensus : rate stays exponentially small

reviews: Mattis '92; Guida, Konishi, Magnoli '94; Ringwald '02
Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov '03; …

A nice (analog) exercise for your QM class:

Excite ground state of unharmonic oscillator with very energetic linear source **(one `hard' quantum):**



$$\Delta V = x\, e^{i\Delta E t}$$

Relevant matrix element

'hard quantum'

$\sim \nu$

$$\langle 0|x|E\rangle \ = \exp(\frac{1}{g^2} f_0(\nu) + f_1(\nu) + \cdots)$$

$E = \nu/g^2$

Stays negative, approaches $\frac{1}{2} S_{\text{inst}}$ as $\nu \to \infty$

CB, Lazaridis, Shaffi, Tiktopoulos '91
CB '92

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So, shall one forget (B+L) violation at LHC?

most probably **YES,** but:

- Amazing that such a basic SM question has not been unambiguously settled.

amplitudes ?

- Analogy with Black-Hole creation in HE collision ?

launch a bet ?

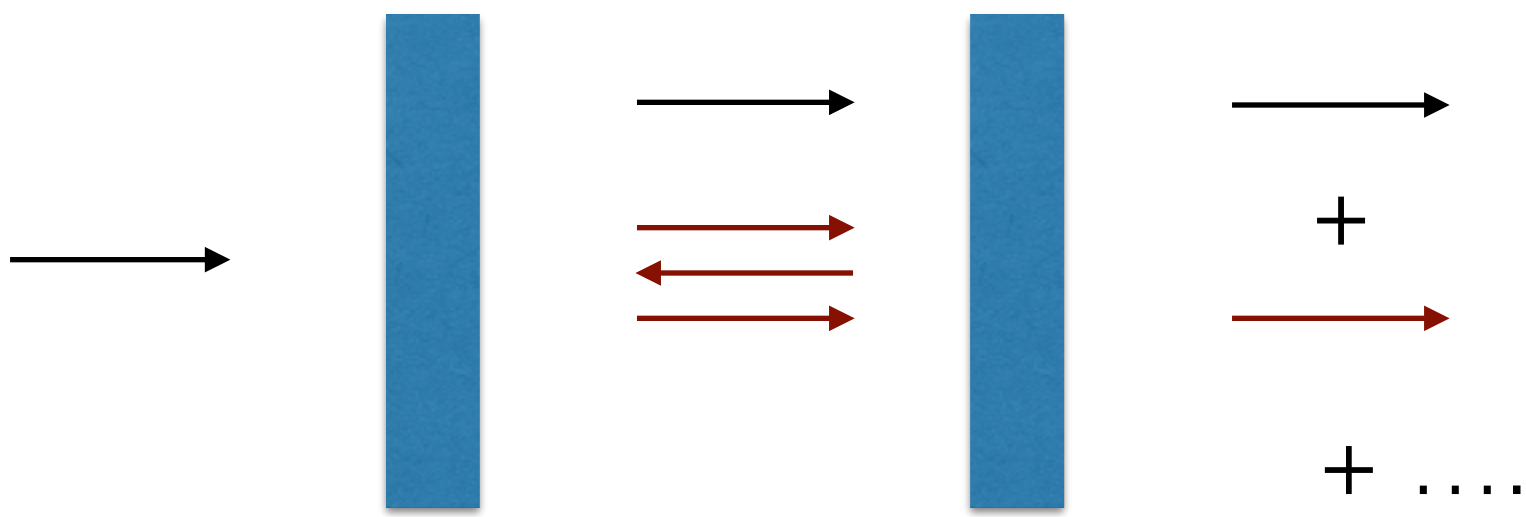
- New recent challenge to consensus by **Tye** and **Wong**

Bloch Wave Function for the Periodic Sphaleron Potential and Unsuppressed Baryon and Lepton Number Violating Processes, arXiv: 1505.03690 [hep-th] ; & arXiv: 1601.00418 [hep-th]

Tye + Wong :

– problem can be reduced to an effective QM in **periodic potential**

– fine tuning the energy can lead to **resonant tunneling**



$$\mathcal{A} \sim e^{-2S_0}(1 + e^{2i\phi(E)} + e^{4i\phi(E)} + \cdots)$$

In QM: double barrier totally transparent at selected energies

Diode

Could such a mechanism work in **Quantum Field Theory** ?

- Superfluid Helium-3 A $\longrightarrow$ B phase decay time

WKB estimate :	$10^{20,000}$ $years$
laboratory :	$minutes$

Tye + Wohns '11

- Cosmological bubble formation in multiverse

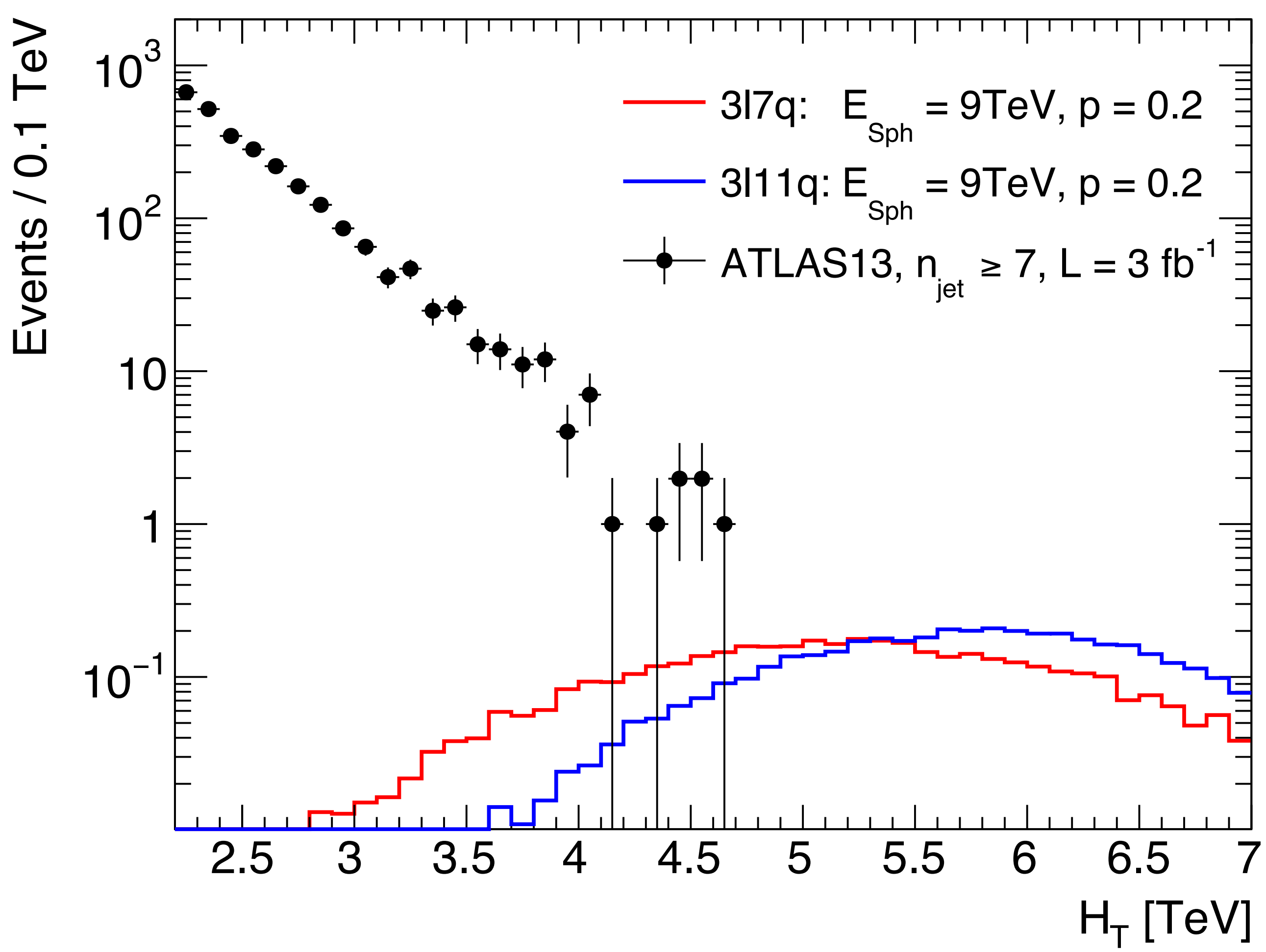
M. Sasaki et al '12

New chance for unsuppressed (B+L) violation ?

- scrutinize all possibilities, since theory not sealed

- **timely since LHC operates at sphaleron energies !**

Partial (negative) answer already from LHC2 :



$= \sum p_T^{\rm jets}$

Ellis+Sakurai arXiv:1601.03654

(search for Black Holes)

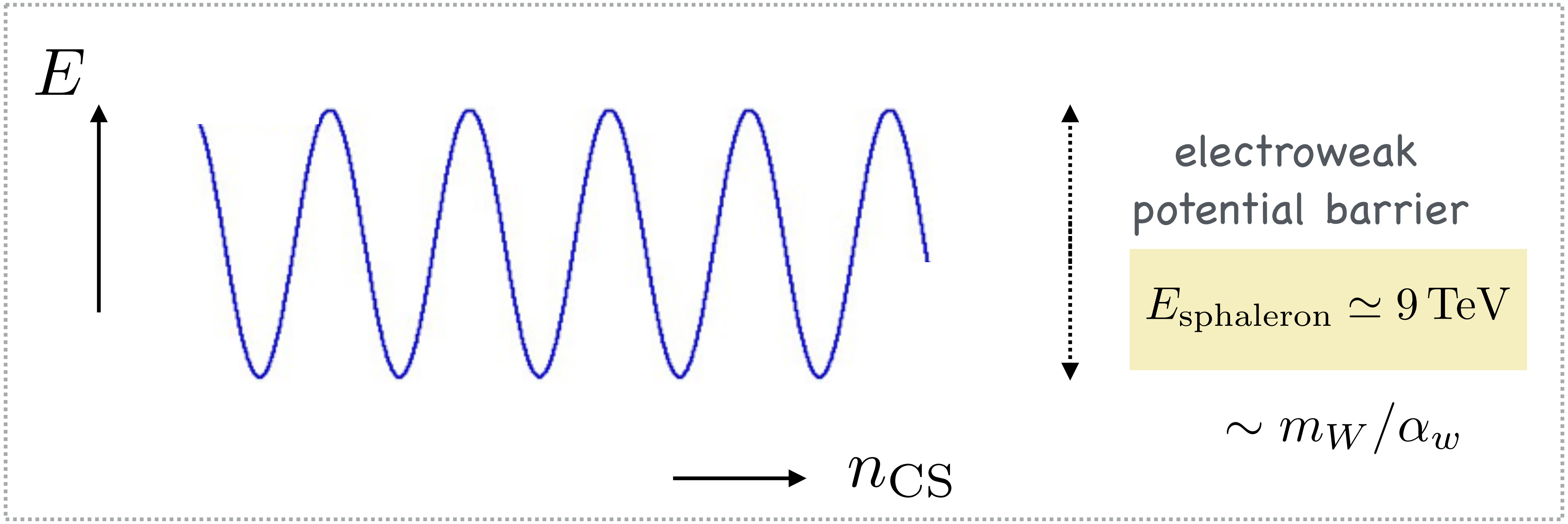
Rest of talk: few theoretical remarks

motivated by proposal of Tye and Wong

3 Tye + Wong proposed 2 steps:

- Reduction of dynamics in interaction region to single degree of freedom: **Chern-Simons number** of gauge field.

- QM in periodic potential: band structure & resonant tunneling



Does Yang-Mills theory have energy bands ?

or

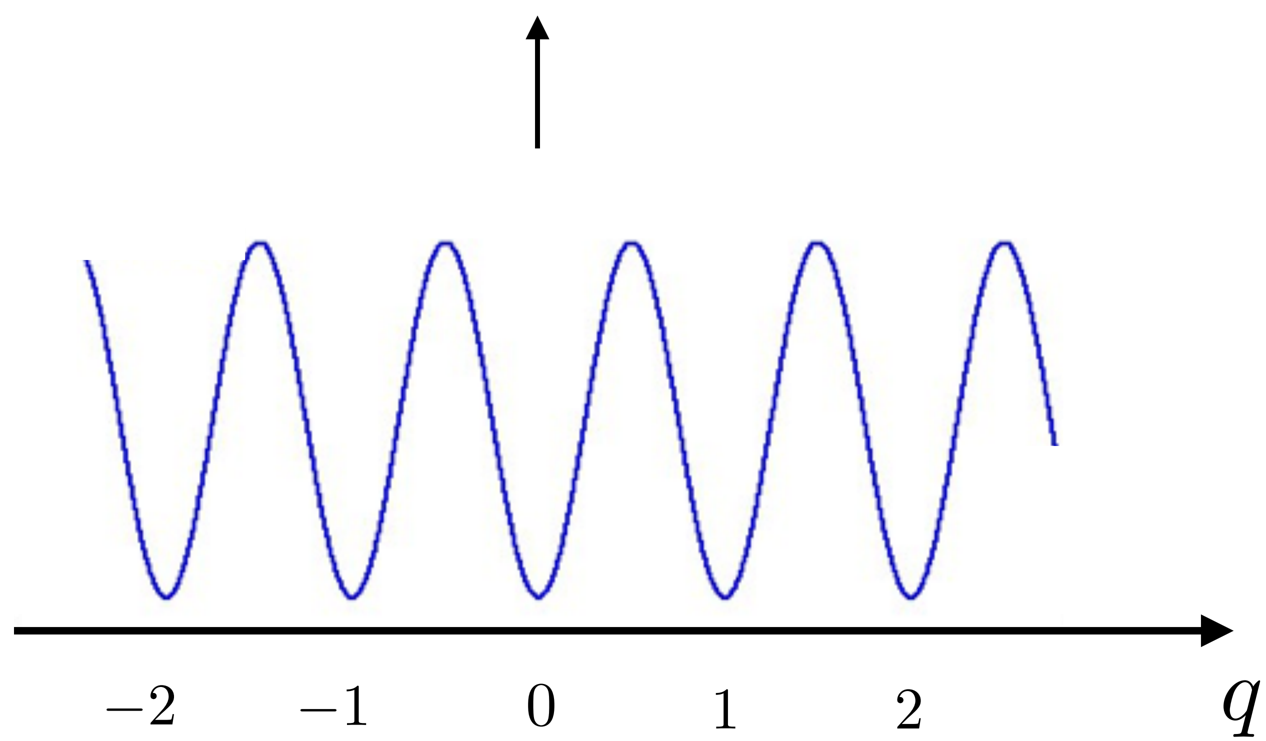
How to `ungauge' large gauge transformations ?

CB, T. Tomaras 1603.08749

Simple QM with periodic potential:

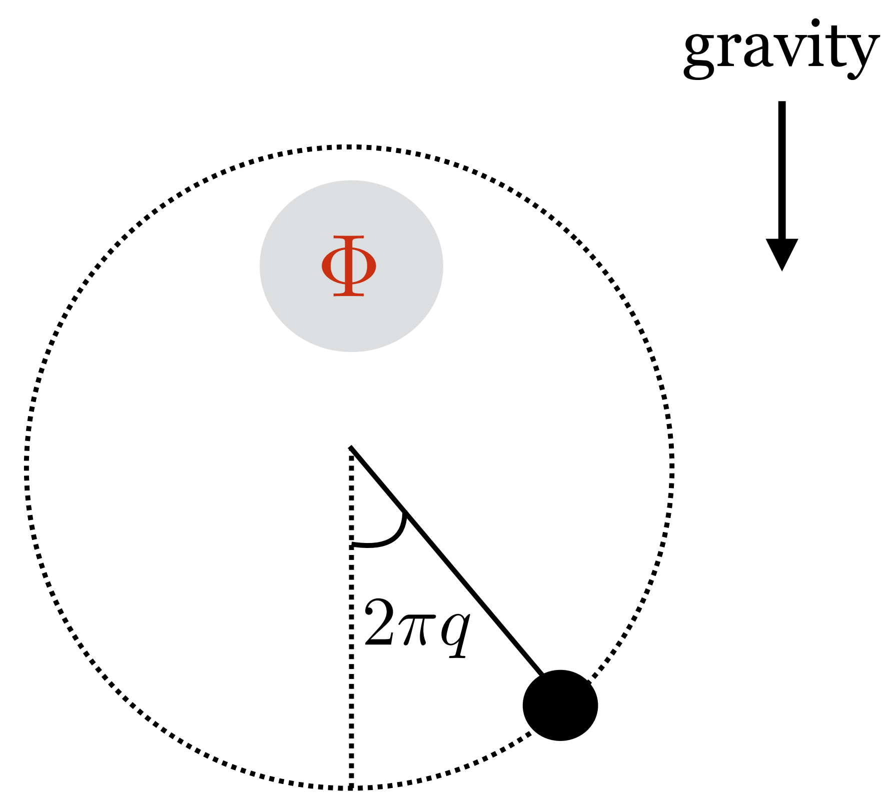
Bands (global $\mathbb{Z}$)

$$V = -A\cos(2\pi q)$$



Discrete spectrum

(gauged $\mathbb{Z}$)



Bands arise for **periodic** potential in **non-compact** dimension,

$$S = \int dt\, \mathcal{L} \;=\; \int dt\, [\frac{M}{2}\dot{q}^{\,2} - V(q) - \theta\, \dot{q}\,] \qquad\qquad V(q) = V(q+n),\; n \in \mathbb{Z}$$

$$p \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}} = M\dot{q} - \theta\ , \qquad \boxed{\mathcal{H}(\theta) \equiv p\dot{q} - \mathcal{L} = \frac{1}{2M}(p+\theta)^2 + V(q)}$$

Diagonalize generator of discrete translations organizes spectrum in **continuous bands:**

$$e^{ip}|k,\lambda\rangle = e^{ik}|k,\lambda\rangle$$

band index $\lambda = 0, 1, \cdots$

quasi-momentum $k \in [-\pi, \pi]$

Unitary equivalence:

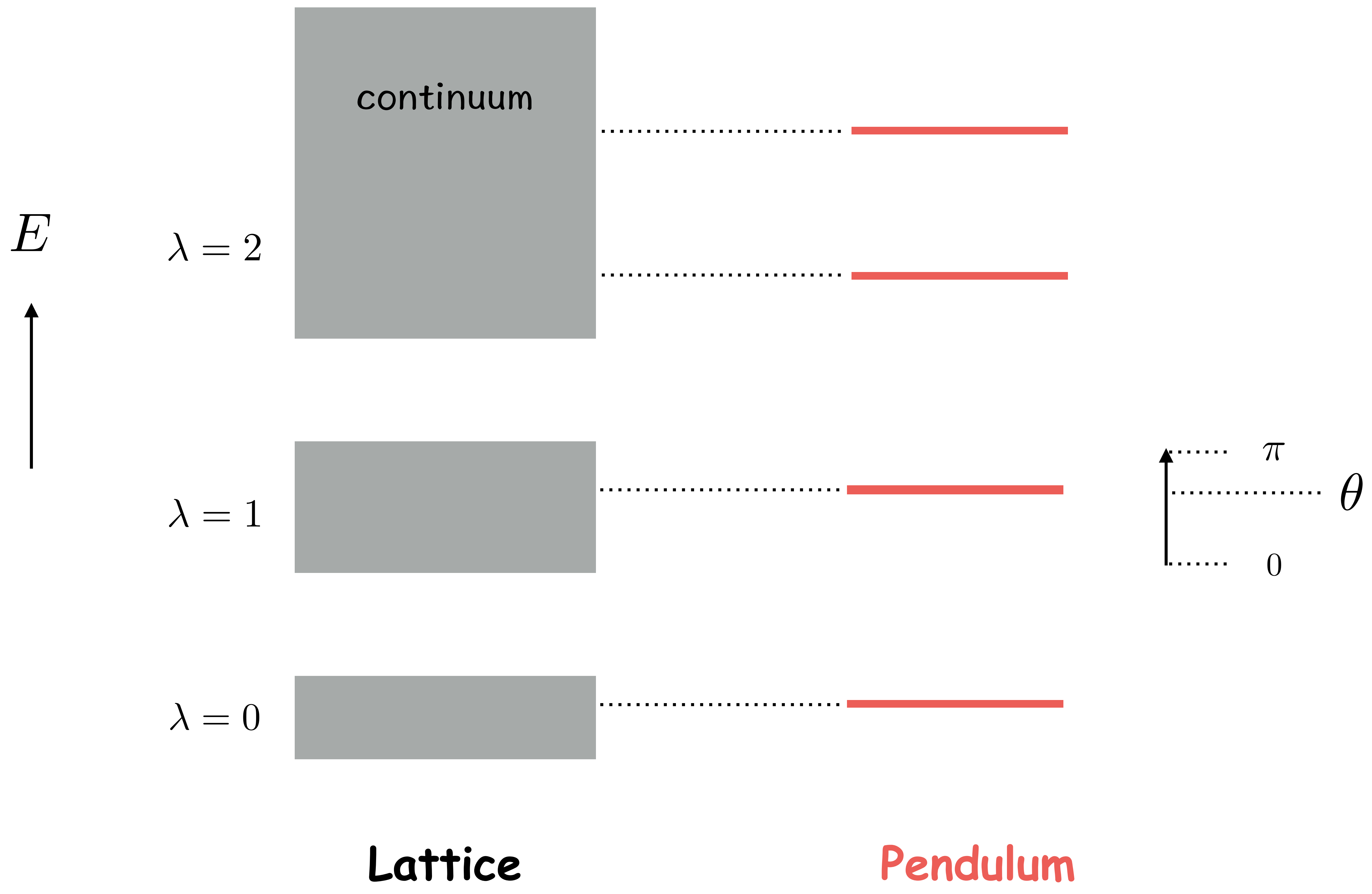
$$p + \theta = e^{-i\theta q}\, p\, e^{i\theta q} \qquad \Longrightarrow \qquad \mathcal{H}(\theta) = e^{-i\theta q}\, \mathcal{H}(0)\, e^{i\theta q}$$

$$\Longrightarrow \qquad \psi_{k,\lambda}(q,\theta) = e^{-i\theta q}\, \psi_{k+\theta,\lambda}(q,0) \qquad \text{and} \qquad E_{k,\lambda}(\theta) = E_{k+\theta,\lambda}(0)$$

So theta angle irrelevant, just reshuffles eigenstates by quasi-momentum shift.

The story is different when the **discrete symmetry** is **gauged**, i.e. the coordinate q is **compact**

In this case wavefunctions must be periodic, so we project onto the states $\psi_{0,\lambda}(q,\theta)$ with energy $E_{\theta,\lambda}(0)$


E
$\lambda = 2$
continuum
$\lambda = 1$
$\lambda = 0$
π
θ
0
Lattice
Pendulum

Simple trick to **ungauge** the symmetry:

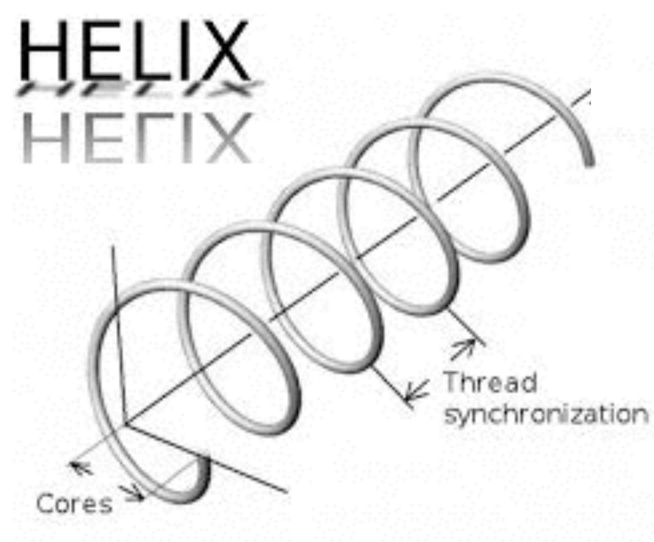
$$\Delta S = \int dt\, a(\dot{q} - \xi^{-1}\dot{b})$$

topological 'BF theory'

$$\Delta b = \xi\, \Delta q$$

$$a = \text{constant}$$

This converts the circle into a **helix** with thread ξ



b tag of 'winding vacua'

If $b \equiv b + 1$ and $\xi = 1/N$ the helix has finite length N

Apply to YM theory ?

Impose an IR cutoff by putting the theory on $S^3 \times \mathbb{R}$

radius $\sim m_W^{-1} \sim 1$

The role of q is played by the Chern-Simons number

$$n_{\rm CS} = -\frac{1}{8\pi^2}\int d^3x \, \epsilon^{ijk}\,{\rm tr}(A_i\partial_j A_k + \frac{2}{3}A_iA_jA_k) := \int_{S^3} \mathcal{C}(A)$$

$$F_{\mu\nu} = 0 \implies n_{\rm CS} = \text{winding}\ \# \in \mathbb{Z}$$

Large gauge transformations change $n_{\rm CS} \to n_{\rm CS} + 1$

so the variable $n_{\rm CS}$ is compact.

Can we `ungauge' large gauge transformations ?

First, derive an action for the QM degree of freedom $n_{\mathrm{CS}}(t)$

O(4)-invariant reduction on S^3 of SU(2) YM :

$$A_\tau = 0 \ , \qquad A_m = \frac{\bar{\sigma}_{\mu\nu}\,\hat{n}^\mu \partial_m \hat{n}^\nu}{1 + e^{-2y(\tau)}}$$

$$n_{\mathrm{CS}} = \frac{1}{2} + \frac{\sinh(3y) + 3\sinh y}{8\cosh^3 y}$$

magnetic

$$F_{lk} = \frac{\bar{\sigma}_{\mu\nu}\,\partial_l \hat{n}^\mu \partial_k \hat{n}^\nu}{2\cosh^2 y}$$

electric

$$F_{\tau m} = \frac{\bar{\sigma}_{\mu\nu}\,\hat{n}^\mu \partial_m \hat{n}^\nu}{2\cosh^2 y}\,\dot{y}$$

sphaleron

$$V(\tau) = -\frac{1}{2g^2}\int_{S^3} \mathrm{tr}(F_{lk}F^{lk}) = \frac{3\pi^2}{g^2}\,\frac{1}{\cosh^4 y} \ , \qquad \boxed{V_{\max} = \frac{3\pi^2}{g^2}}$$

also Gibbons & Steif '93

To compute the action use that $y = \tau - \tau_0$ is the Belavin et al instanton solution

so the Euclidean action is $\mathcal{L} = V(y)(\dot{y}^2 + 1)$

Canonical KE: $q = \frac{1}{2}(1 + \tanh y)$

so finally

$$\boxed{\begin{aligned} S(q,\dot{q}) \;&=\; \frac{12\pi^2}{g^2}\int dt\,\left[\,\dot{q}^{\,2} - 4q^2(q-1)^{\,2}\,\right] - \theta\int dt\,\dot{n}_{\mathrm{CS}}(q) \\ n_{\mathrm{CS}}(q) &= q^2(3-2q) \qquad\qquad n_{\mathrm{CS}}, q \in [0,1] \end{aligned}}$$

angle variables

glue two minima
twice differentiable

$S(q, \dot{q})$ is not an effective action, so its details do not matter.

Only two gross features are relevant:

- periodicity in $q \rightarrow q + 1$
- potential barrier $\sim 1/g^2$

Since $q \rightarrow q + 1$ is a large gauge transformation, q is a compact variable and there are **no bands**. Can we make this symmetry **global** by converting **circle into helix ?**

YES: Couple YM to a **Topological 3-form Theory:**

$$\Delta\mathcal{L}_{\mathrm{YM}} = \underset{\text{axion}}{a} \left[d\mathcal{C}(A) - \xi^{-1} d\underset{\text{3-form}}{B} \right]$$

\`Notes on gauging noneffective group actions\`

Pantev+Sharpe hep-th/0502027

\`Modifying the Sum Over Topological Sectors and Constraints on Supergravity\`'

Seiberg arXiv:1005.0002

In reduced QM this adds $$\Delta S = \int dt\; a\,[\,\dot{n}_{\rm CS} - \xi^{-1}\,\dot{b}\,]$$

$$b := \int_{S^3} B$$

so circle was converted to **helix**

Compact b and $\xi = 1/N$ $\Longleftrightarrow$ helix closes after N turns.

We succeeded to **`ungauge'** the non-connected component of the gauge group, with a **local renormalizable Lagrangian !**

Is this a different Standard Model ?

Yes, but dont get (overly) excited ....

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The 3-form can couple to external **membrane sources**

'Wilson-volume' operators: $\mathcal{W}_q = e^{2\pi i q \int_{S^3} B}$

If we denote lowest-band states $|\theta\rangle \sim \sum_n e^{in\theta} |n, b = \frac{n}{N}\rangle$

then $\mathcal{W}_m|\theta\rangle = |\theta + \frac{2\pi m}{N}\rangle$

and the partition function reads $Z_{\mathrm{YM+TFT}} = \sum_{k=1}^{N} Z_{\mathrm{YM}}(\theta + \frac{2\pi k}{N})$

$\mathcal{W}_m$ are **interface operators** that change the θ **angle**

Different theta sectors `coexist', but are only mixed by non-local operators — **usual superselection rule still holds**

... unless we make the **membranes dynamical.**

This is the case if the axion acquires dynamics. Indeed the topological theory is an **axion theory in the limit**

$$f_a \to \infty \qquad (\gg \text{all other masses})$$

Longish argument

TFT + QFT may look contrived, so let's look at a more realistic theory:

$$\Delta S = \int dt \left[\frac{f_a^2}{2}\, \dot{a}^2 - a\, \dot{n}_{\rm CS} \right]$$

Peccei-Quinn scale

Target space parametrized by **torus**

$$(a/2\pi, n_{\rm CS}) := (x, y) \equiv (x+1, y) \equiv (x, y+1)$$

and axion coupling is one (or more) units of background **magnetic flux**

cf Hofstadter's problem

$$A = -2\pi x\, dy \;\sim\; A = 2\pi y\, dx$$

Quantization does not depend on gauge

The new Hamiltonian reads

$$\mathcal{H} = \frac{1}{2f_a^2}\,[p_a - n_{\rm CS}]^2 + \mathcal{H}_0(\theta)$$

axion momentum

pure YM Hamiltonian

Diagonalize $p_a = n_a \in \mathbb{Z}$ so particle moving in a 1D potential

$$V(q, n_a) = \frac{1}{2f_a^2}\,[n_a - n_{\rm CS}(q)]^2 + V_0(q)$$

small perturbation for
$f_a \gg 1$

If we neglect the perturbation and the compactness of $q \sim n_{\rm CS}$
we find the same spectrum of continuous bands for each n_a

Perturbation puts particle in box of size $\sim f_a$
which discretizes the bands.

The perturbation lifts the discrete symmetry $n_{\text{CS}} \to n_{\text{CS}} + 1$

so to **restaure compactness** we must identify

$$(n_{\text{CS}}, n_a) \sim (n_{\text{CS}} + 1, n_a + 1)$$

Familiar from the study of Landau states on the torus:

$$\mathcal{H} = \frac{1}{2}(p_x - By)^2 + \frac{1}{2}p_y^2$$

commutes with torus translations:

$$U = e^{ip_x} \qquad \text{and} \qquad V = e^{ip_y}e^{-iBx}$$

gauge transfo that patches y & $y+1$

$$[\mathcal{H}, U] = [\mathcal{H}, V] = [U, V] = 0$$

if B quantized

So translation of y coupled with translation of p_x

conclude that $$(n_{\rm CS}+1, n_a) \sim (n_{\rm CS}, n_a - 1)$$

so the **axion momentum is the (almost) zero mode** that acts as flag and converts the circle in config. space into a helix.

When $f_a \to \infty$ the potential for p_a vanishes, and the zero mode is exact

qed

Short argument

Add a **mass term** for the topological 3-form

$$\mathcal{L}_{\rm mass} = (\xi f_a)^{-2} B_{\mu\nu\rho} B^{\mu\nu\rho}/12$$

Limit $f_a \to \infty$ gives back TFT + QFT of Seiberg.

Integrating out B gives standard axion theory:

$$B_{\mu\nu\rho} = \xi f_a^2\, \epsilon_{\mu\nu\rho\sigma} \partial^\sigma a$$

Interface operatoros read

$$\boxed{\mathcal{W}_\gamma = e^{i\gamma \int_{S^3} *da} \qquad \forall \gamma \in \mathbb{R}}$$

no quantization

5

Step back and think

- **Coupling YM to Topological Theory** forces <u>coexistence of theta sectors</u>

- Can be obtained as **decoupling limit of axion theory**

- **Interesting extended operators (observables)** - say few words if time in end

Can this make any difference in the (B+L) - violation problem ?

Hard to find bands & resonant tunneling at E $\ll$ Peccei-Quinn scale

in the QM problem for $n_{\rm CS}$

But if all collision energy could be streamlined into a single QM

variable, no need for resonant tunneling for E exceeding $E_{\rm sph}$

So my conclusion here is not optimistic ...

Hide problems of hierarchy under the rug ?

(no new light particles coupling observably to SM)

cf talk of Gia Dvali

= **Planck-era cosmology ?**

' 87

Precedent: **Brown + Teitelboim's** mechanism for relaxing the cosmo. constant

(non-dynamical 3-form, nucleation of membranes)

Any SM parameter can be made into decoupling field

$$S_0 + S_{\rm top} = \lambda_0 \int \mathcal{O} + \int a(\mathcal{O} - \partial_\mu B^\mu) + f^2 B_\mu B^\mu \quad + V(a)$$

$$\infty$$

so that it may relax to given value by membrane nucleation and choice of V(a). **The decoupling limit is a topological theory.**

- String theory embedding ? quantization of charge?

- `naturalness' without anthropic arguments ?

More on Volume Operators

$\mathcal{W}_\gamma$ are interesting observables that have not been studied in oridnary YM

Example of extended operators that play a role in checks of dualities

Close cousins of **Superconformal Interface** operators in

- N=4 d=4 SYM

DeWolfe, Freedman, Ooguri hep-th/0111135
Erdmenger, Guralnik, Kirsch hep-th/0203020
. . . .
Gaiotto+Witten arXiv:0804.2902, 2907; 0807.3720

$g_{\rm YM}$ and (in general) N_c also jump

Dual exact Supergravity Solutions (NS5-D5-D3)

(relevant for gravity non-localization)

Lunin; ; D'Hoker, Estes, Gutperle '07
. . . ; Assel, CB, Estes, Gomis '11, '12

- d=3 ABJM

Berman; Niarchos; . . . ?

Dual Supergravity Solutions (M2-M5-M5′)

$$(AdS_3 \times S^3 \times S^3) \times_w \Sigma$$

Estes, Feldman, Krym arXiv:1209.1845

CB, D′Hoker, Estes, Krym arXiv:1312.547

(First ?) solns with localized branes in compact space

- d=2 N=(2,2) CFTs

CB, de Boer, Dijkgraaf, Ooguri hep-th/0111210

Brunner, Roggenkamp arXiv: 0707.0922; 0806.4734

Exact field-theory computations

. . . .

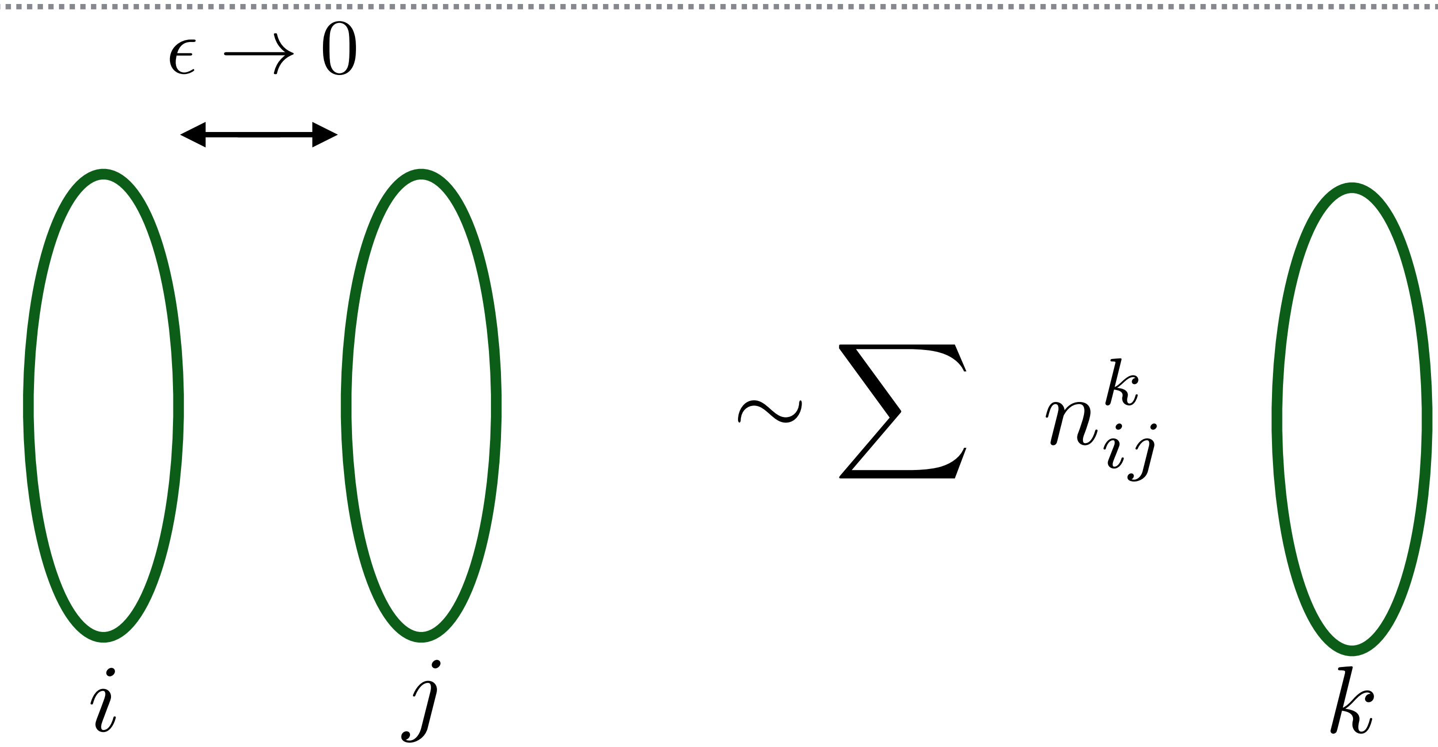
(formalism of boundary states)

CB, Brunner, Roggenkamp arXiv: 1205.4647

Questions of interest

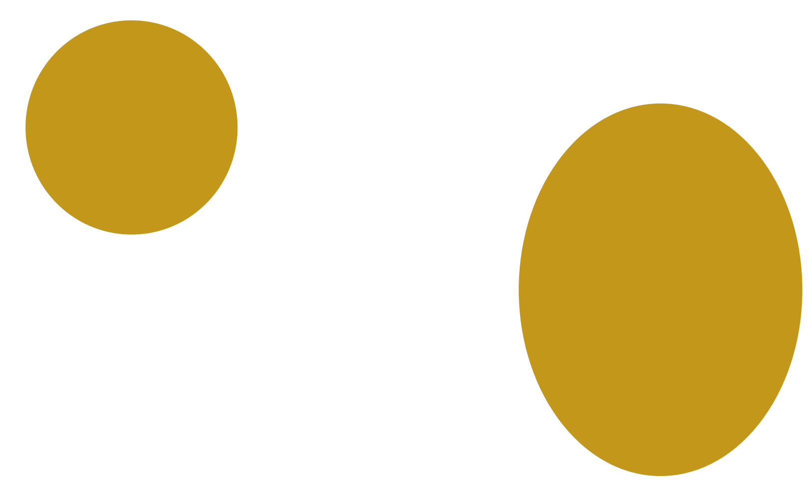
Fusion:

CB, Brunner '07

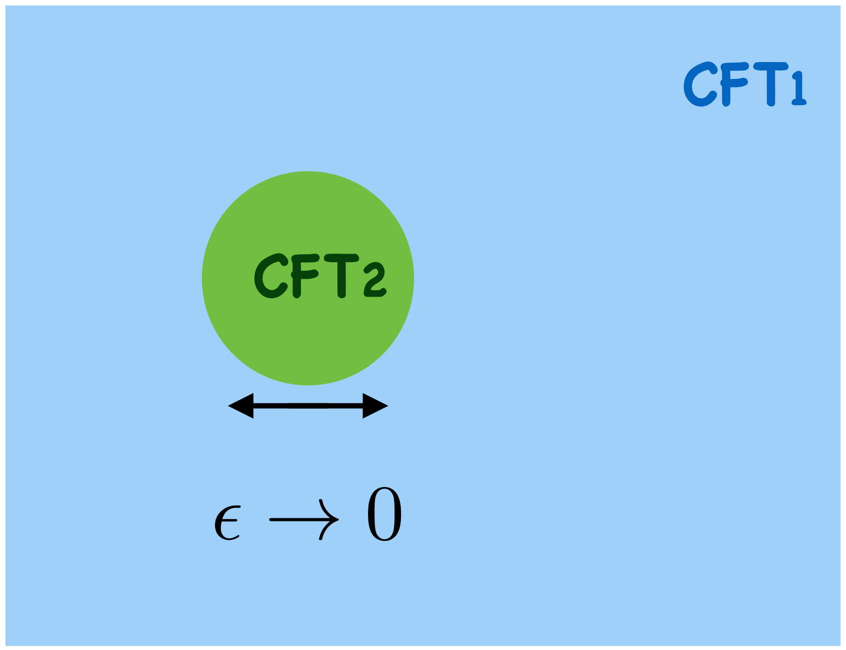


Casimir forces:

e.g. Machta, Veatch, Sethna (Nature '12)



Local Operator expansion:



$$\simeq \; g\mathbf{1} + \epsilon^{n_1}\mathcal{O}_1 + \cdots$$

g-factor

ops of CFT1

Nice formula: When two theories differ only in complex structure or only in Kähler structure, one finds

$$-2\log\; g = \boxed{K(\lambda_1,\bar\lambda_1) + K(\lambda_2,\bar\lambda_2) - K(\lambda_1,\bar\lambda_2) - K(\lambda_2,\bar\lambda_1)}$$

CB, Brunner, Douglas, Rastelli arXiv: 1311.2202

Holographic proof: D'Hoker, Gutperle arXiv:1406.5124

Quantity known in complex geometry as **Calabi's Diastasis**

$$\mathcal{D}(\lambda_1,\bar\lambda_1,\lambda_2,\bar\lambda_2) = K(\lambda_1,\bar\lambda_1) + K(\lambda_2,\bar\lambda_2) - K(\lambda_1,\bar\lambda_2) - K(\lambda_2,\bar\lambda_1)$$

E. Calabi, "Isometric Imbedding of Complex Manifolds," Ann. Math. 58, 1 (1953).

(Had to show that analytic extension makes sense)

$\mathcal{D}$ has very nice properties:

- reduces to square distance $\partial_I \bar\partial_{\bar J} K\ \delta\lambda^I \delta\bar\lambda^J$ when $\lambda_1 \to \lambda_2$
- it is invariant under complex embeddings
- it is invariant under Kahler-Weyl transformations

$$K(\lambda,\bar\lambda) \to K(\lambda,\bar\lambda) + F(\lambda) + \bar F(\bar\lambda)$$

Thanks for your attention