Asymptotically Safe gravitational collapse: Kuroda-Papapetrou RG-improved model

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Gravitational collapse and singularities

- The gravitational collapse of a sufficiently massive star give rise to a gravitational singularity
- Is the singularity always covered by an event horizon?
- Can naked singularities arise from gravitational collapse?

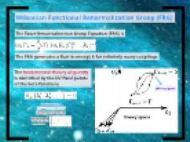
Cosmic Censorship Conjecture (CCC) - Penrose (1969)
The generic singularities arising in the gravitational collapse are always covered by an event horizon.

Asymptotic Safety Theory of Quantum Gravity

- Standard Model is an extremely well tested (renormalizable) theory, based on Quantum Field Theory;
- General Relativity describes the classical gravitational field

Problems in General Relativity:
- Singularity problem;
- Horizon and flatness problems;
- Small and positive cosmological constant;
- Einstein Gravity is a perturbatively non-renormalizable theory

Approaches to Quantum Gravity:
- New Physics (string theory, loop quantum gravity, etc...)
- Asymptotic Safety: by using non-perturbative methods, gravity makes sense as a Quantum Field Theory;
- ...



Gravitational collapse in the generalized Vaidya space-time: singularity strength

Mkenyeleye, Goswami, Maharaj. Phys. Rev. D 90, 064034 (2014)

- *What is the mathematical condition to have a naked singularity?*
- *What is its strength?*

Let us consider the generalized Vaidya space-time:

$$ds^2 = -f(r,v)\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$$

$$f(r,v) = 1 - \frac{2\,M(r,v)}{r}$$

Solving the geodesic equation: information on the astrophysical object (black hole or naked singularity)

$$\frac{dv}{dr} = 2\left(1 - \frac{2\,M(r,v)}{r}\right)^{-1}$$

AS Gravitational collapse: Kuroda-Papapetrou RG-improved model

Classical Vaidya space-time

$$ds^2 = -f_c(r,v)\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$$

$$f_c(r,v) = 1 - \frac{2\,m(v)\,G_0}{r}$$

By using the Exact Renormalization Group Equation (ERG)

$$G(k) = \frac{G_0}{1 + (G_0/g_*)\,k^2}$$

→ Running Newton's constant

M. Reuter, Phys. Rev. D 57, 971 (1998)

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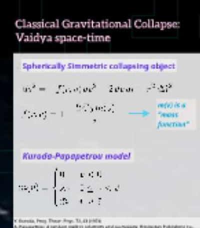
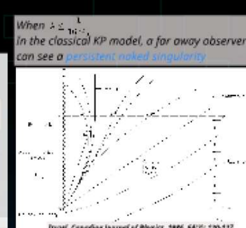




Solving the ge
(black hole or

$$\frac{\mathrm{d}v}{\mathrm{d}r} = 2\left(1 - \frac{2\right.$$

Classical Gravitational Collapse: Vaidya space-time

Spherically Simmetric collapsing object

$$ds^2 = -f(r,v)\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$$

$$f(r,v) = 1 - \frac{2\,G_0\,m(v)}{r}$$

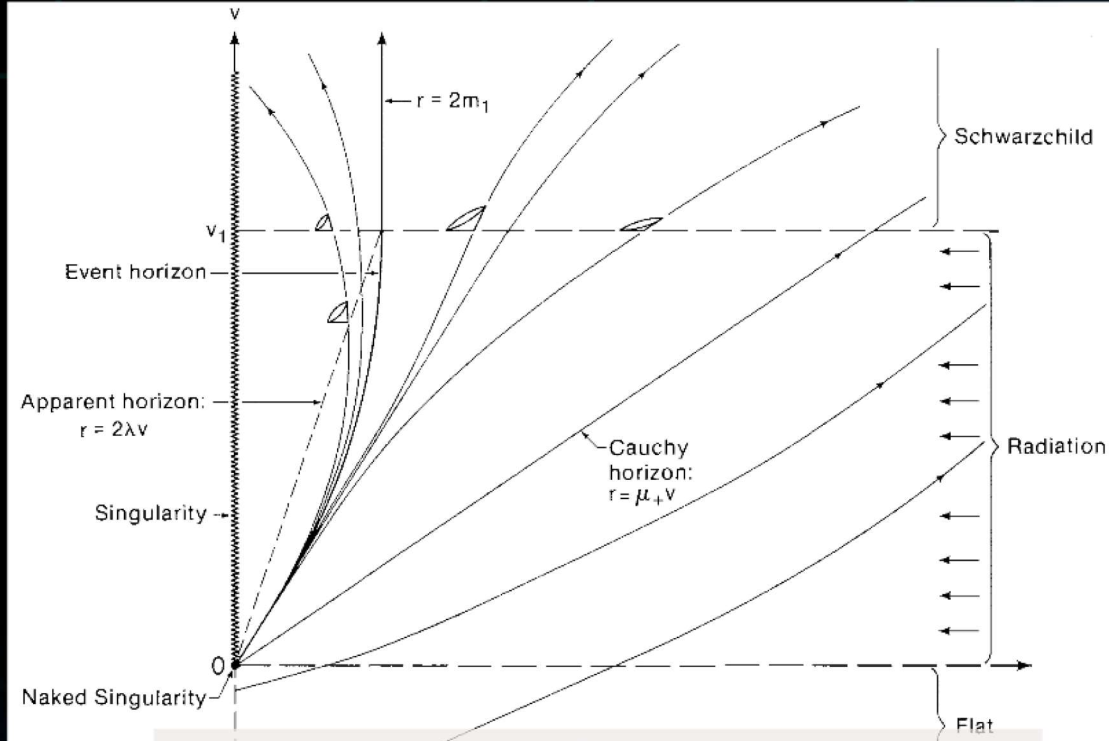
→ ***m(v) is a "mass function"***

Kuroda-Papapetrou model

$$m(v) = \begin{cases} 0 & v < 0 \\ \lambda v & 0 \le v < \bar{v} \\ \bar{m} & v \ge \bar{v} \end{cases}$$

Y. Kuroda, Prog. Theor. Phys. 72, 63 (1974)
A. Papapetrou, A random walk in relativity and cosmology. Hindustan Publishing Co., New Delhi, India (1985)

When $\lambda \le \frac{1}{16\,G_0}$
In the classical KP model, a far away observer can see a persistent naked singularity



Israel, Canadian Journal of Physics, 1986, 64(2): 120-127

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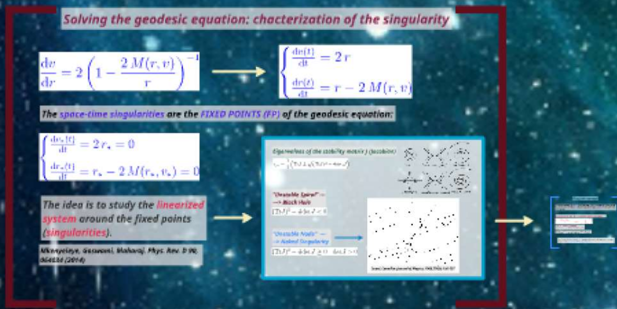


Solving the geodesic equation: chacterization of the singularity

$$\frac{\mathrm{d}v}{\mathrm{d}r} = 2\left(1 - \frac{2\,M(r,v)}{r}\right)^{-1} \longrightarrow \begin{cases} \dfrac{\mathrm{d}v(t)}{\mathrm{d}t} = 2\,r \\[2ex] \dfrac{\mathrm{d}r(t)}{\mathrm{d}t} = r - 2\,M(r,v) \end{cases}$$

The space-time singularities are the FIXED POINTS (FP) of the geodesic equation:

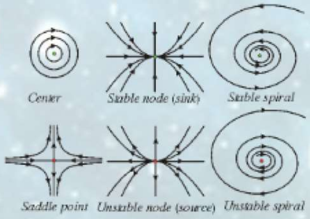
$$\begin{cases} \dfrac{\mathrm{d}v_*(t)}{\mathrm{d}t} = 2\,r_* = 0 \\[2ex] \dfrac{\mathrm{d}r_*(t)}{\mathrm{d}t} = r_* - 2\,M(r_*,v_*) = 0 \end{cases}$$

The idea is to study the linearized system around the fixed points (singularities).

Mkenyeleye, Goswami, Maharaj. Phys. Rev. D 90, 064034 (2014)

Eigenvalues of the stability matrix J (Jacobian)

$$\chi_\pm = \frac{1}{2}\left(\mathrm{Tr}J \pm \sqrt{(\mathrm{Tr}J)^2 - 4\,\mathrm{det}J}\right)$$



"Unstable Spiral" ------> Black Hole

$$(\mathrm{Tr}J)^2 - 4\,\mathrm{det}J < 0$$

"Unstable Node" ------> Naked Singularity

$$(\mathrm{Tr}J)^2 - 4\,\mathrm{det}J \geq 0 \quad \mathrm{det}J > 0$$

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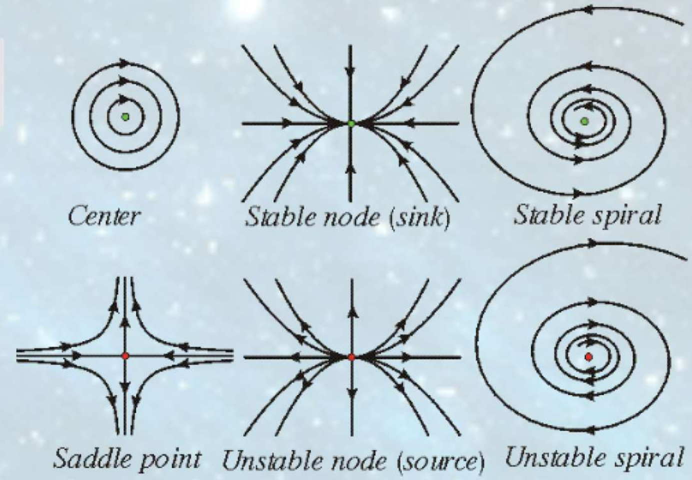

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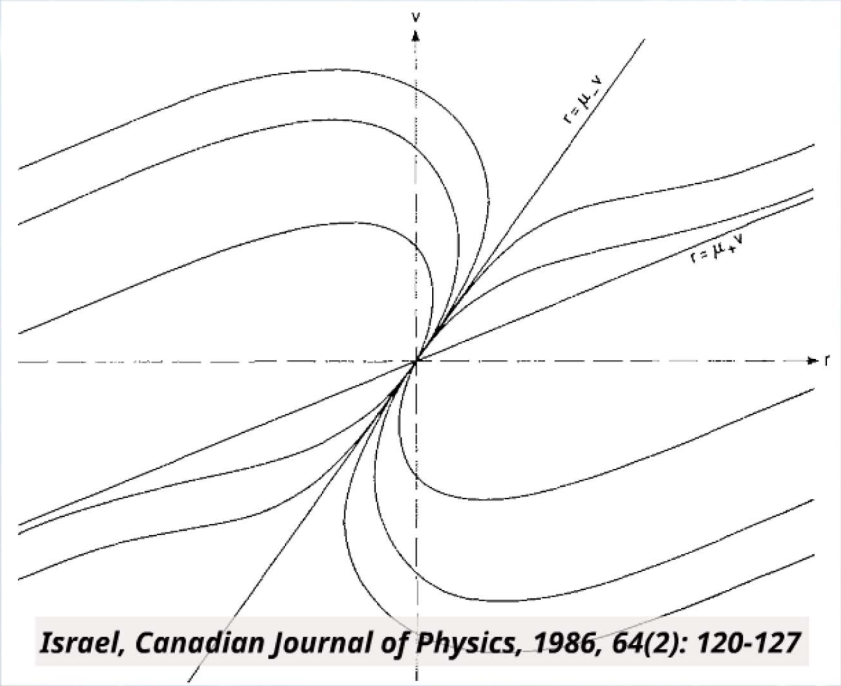

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Singularity strength

A singularity is said to be strong if an object falling into the singularity is destroyed by the gravitational tidal forces. Otherwise it is called weak or integrable (the space-time is extendable).

Mathematical characterization: singularity strength parameter

$$S = \frac{\dot{M}_{FP}\, X_{FP}^2}{2} \qquad X_{FP} \equiv \lim_{(r,v)\to \mathrm{FP}} \frac{v(r)}{r}$$

S>0 ----> Strong Singularity
Otherwise ----> Integrable Singularity

Mkenyeleye, Goswami, Maharaj. Phys. Rev. D 90, 064034 (2014)
Strokov, Lukash, Mikheeva. Int. J. Mod. Phys. A 31, 1641018 (2016)

In all the classical models, the gravitational collapse can always give rise to strong naked singularities.

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- **General Relativity** describes the classical gravitational field

Problems in General Relativity:

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Wilsonian Functional Renormalization Group (FRG)

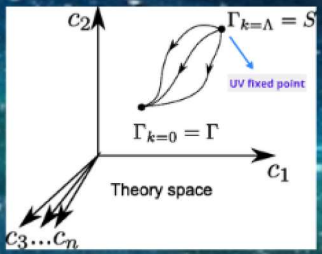
The Exact Renormalization Group Equation (ERG) is

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\ k\partial_k R_k \,(\Gamma_k^{(2)} + R_k)^{-1}$$

The FRG generates a flow in energy k for infinitely many couplings

The fundamental theory of gravity is identified by the UV fixed points of the beta functions

$$\beta_{\lambda_i}(\lambda_1^*, \lambda_2^*, \dots) = 0$$



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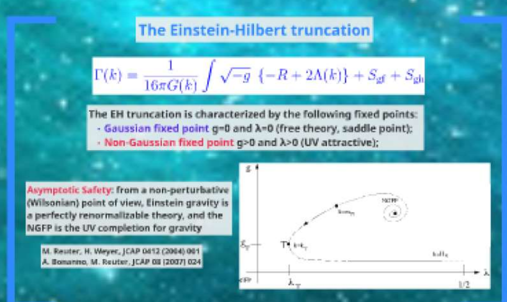
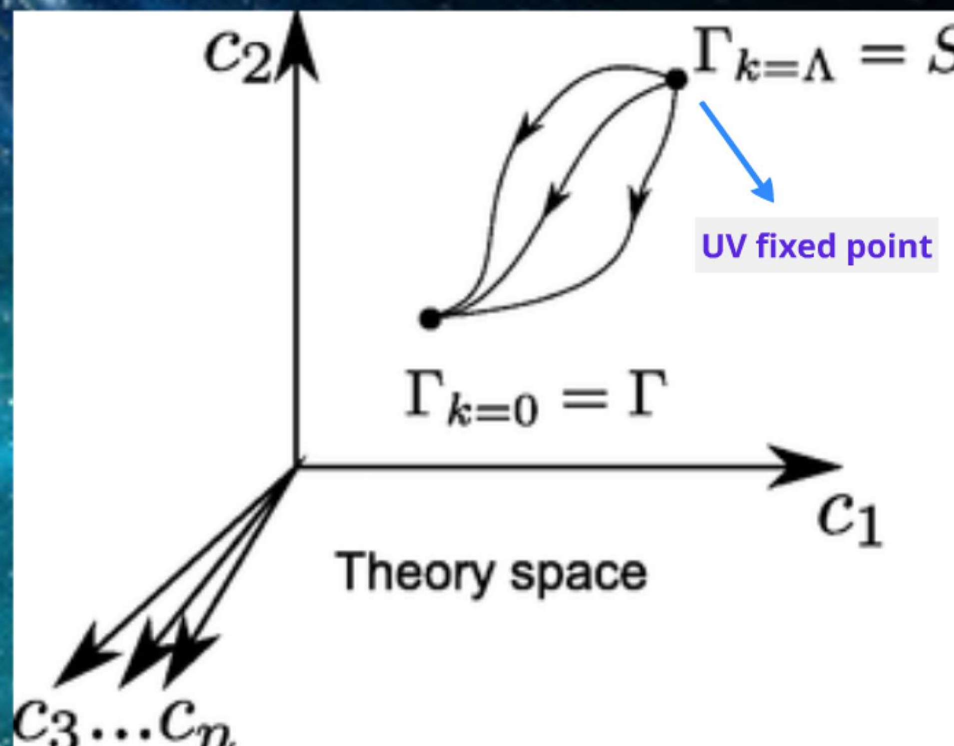




The Einstein-Hilbert truncation

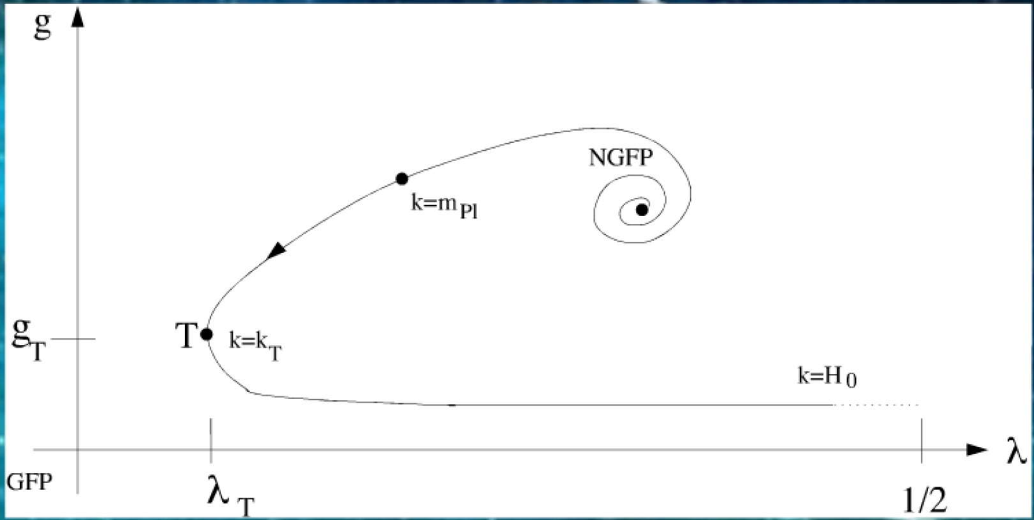
$$\Gamma(k) = \frac{1}{16\pi G(k)} \int \sqrt{-g}\ \{-R + 2\Lambda(k)\} + S_{\text{gf}} + S_{\text{gh}}$$

The EH truncation is characterized by the following fixed points:

- **Gaussian fixed point** g=0 and λ=0 (free theory, saddle point);
- **Non-Gaussian fixed point** g>0 and λ>0 (UV attractive);

Asymptotic Safety: from a non-perturbative (Wilsonian) point of view, Einstein gravity is a perfectly renormalizable theory, and the NGFP is the UV completion for gravity

M. Reuter, H. Weyer, JCAP 0412 (2004) 001
A. Bonanno, M. Reuter, JCAP 08 (2007) 024



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$$f_c(r,v) = 1 - \frac{2\,m(v)\,G_0}{r}$$

Metric improvement

$$f_c(r,v) \longrightarrow f_0(r,v) = 1 - \frac{2\,m(v)}{r}\,\frac{G_0}{1+\omega\,G_0\,[k(r)]^2}$$

What is, in this case, the correct cutoff identification?

The best choice is to relate k(r) with the energy density of a null free falling observer

$$k(r) = \xi\,\sqrt[4]{\rho(r,v)} = \xi\,\sqrt[4]{\frac{\dot{m}(v)}{4\pi r^2}}$$

By using this cutoff identification, and assuming $m(v)=\lambda v$ (KP model)

$$f_q(r,v) = 1 - \frac{2\,\lambda\,G_0\,v}{r+\alpha\sqrt{\lambda}} \qquad \alpha = \frac{\xi^2\,G_0}{\sqrt{4\pi\,g_*}}$$

$$f_c(r,v) = 1 - \frac{2\,\lambda\,G_0\,v}{r}$$

The effect of a running Newton's constant is to produce a shift in the radial coordinate r(v) → The critical value λ_c is greater than 1/16G

By using the **Exact Renormalization Group Equation (ERG)**

$$G(k) = \frac{G_0}{1 + (G_0/g_*)\,k^2}$$

→ **Running Newton's constant**

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Babic, Guberina, Horvat, Stefancic. Phys.Rev. D71 (2005) 124041
Bonanno, Esposito, Rubano, Scudellaro. Class. Quant. Grav. 23 (2006) 3103

By using this cutoff identification, and assuming m(v)=λv (KP model)

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The effect of a running Newton's constant is to produce a shift in the radial coordinate r(v) → **The critical value λc is greater than 1/16G**

On the nature of the singularity

Classical Kuroda-Papapetrou model:

$$\mathrm{Tr}J = 1 \quad \det J = 4\,\lambda\,G_0 \quad \Rightarrow \quad \chi_{\pm} = \frac{1}{2}\left(1 \pm \sqrt{1 - 16\,\lambda\,G_0}\right)$$

The origin $(0,0)$ is a **naked singularity** if $\lambda \leq \frac{1}{16\,G_0}$, and $S > 0$

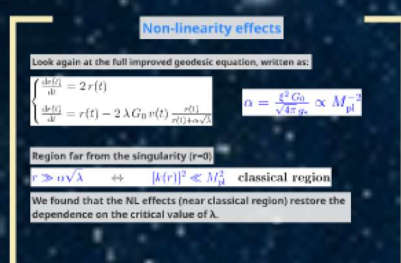
Improved Kuroda-Papapetrou model:

$$\mathrm{Tr}J = 1 - \frac{2\lambda v_0\,G_0}{\alpha\sqrt{\lambda}} \qquad \det J \propto G(r)]_{r\to 0} = 0$$ **Fixed Points line**

Strength: $S \propto G(0) = 0$ **Integrable!**

There is no dependence on the critical value λ_c

Why?


Non-linearity effects

Look again at the full improved geodesic equation, written as:

$$\alpha = \frac{\xi^2 G_0}{\sqrt{4\pi}\,g_*} \propto M_{\rm pl}^{-2}$$

Region far from the singularity (r=0)

$r \gg \alpha\sqrt{\lambda} \quad \Leftrightarrow \quad [k(r)]^2 \ll M_{\rm pl}^2$ classical region

We found that the NL effects (near classical region) restore the dependence on the critical value of λ.

Reminder

- **Eigenvalues of the stability matrix**

$$\chi_{\pm} = \frac{1}{2}\left(\mathrm{Tr}J \pm \sqrt{(\mathrm{Tr}J)^2 - 4\det J}\right)$$

- **Strength of the singularity**

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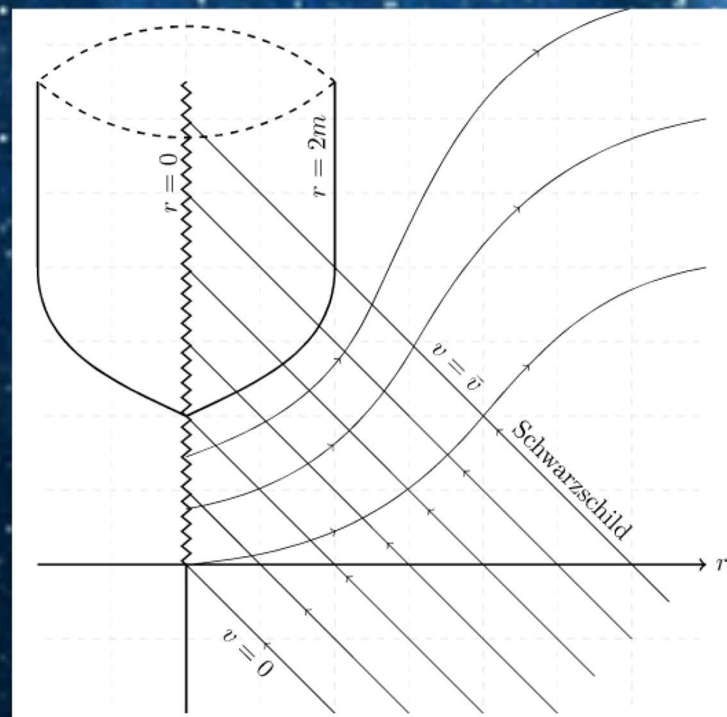

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We found that the NL effects (near classical region) restore the dependence on the critical value of λ.

Conclusions

- We studied the **RG-improved Kuroda-Papapetrou model**
- We found that the only effect of a running Newton constant is to ***turn a strong naked singularity into a line of integrable singularities***
- The space-time is then **extandable** beyond the singularity r = 0
- The presence of the limiting value λc is a purely **classical effect**: the formation of naked singularities in the KP model is due to the gravitational collapse dynamics in the classical region.