Ghost Free & Singularity Free Theories of Gravity

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Phys. Rev. Lett. (2012), JCAP (2012, 2011), JCAP (2006)

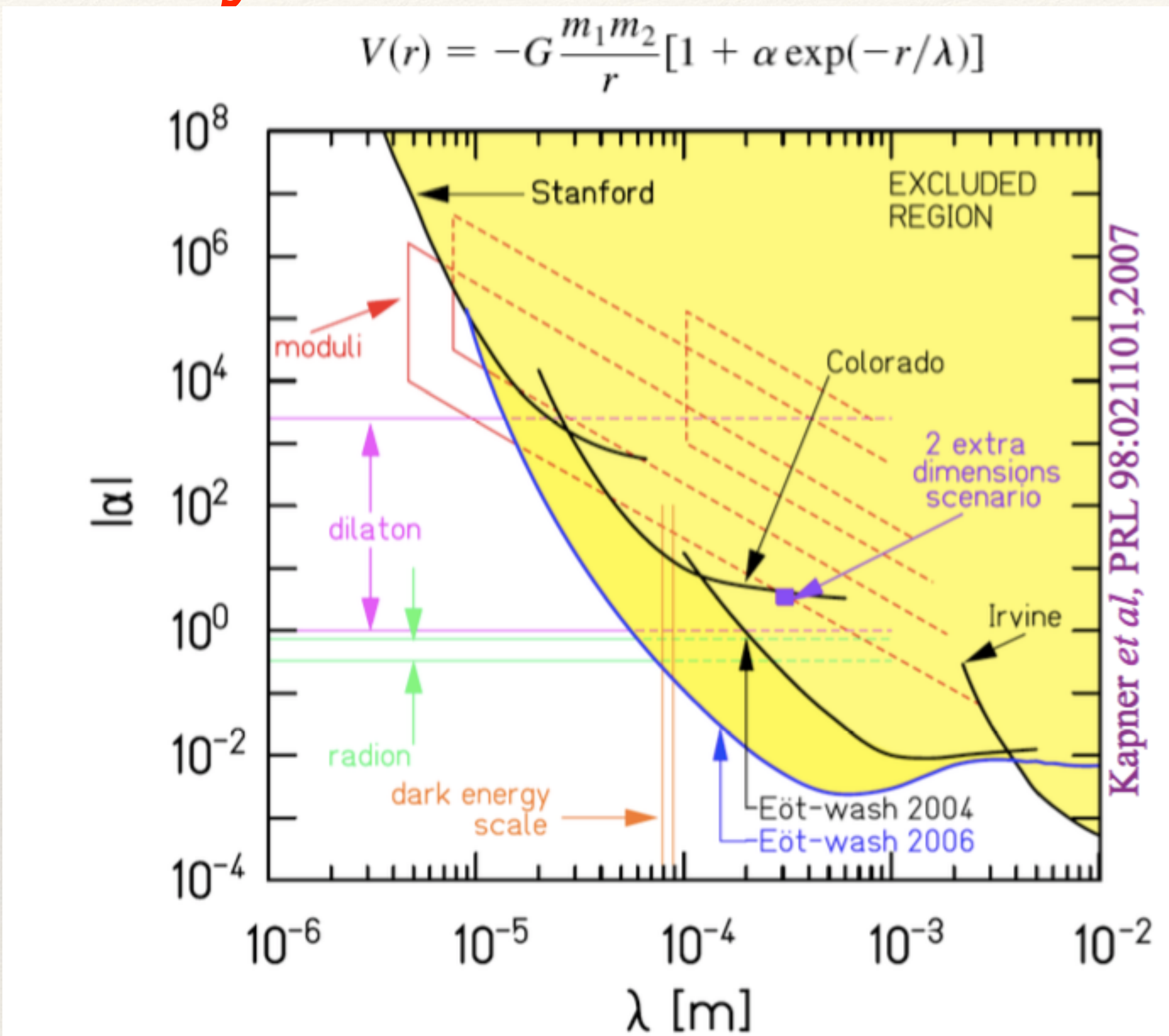
Class.Quant. Grav. (2013), Phys. Rev. D (2014), 1412.3467 (Class. Quant. Grav. 2014),

1503.05568 (Phys. Rev. Lett. 2015), 1509.01247 (Phys. Rev. D, 2015), 1602.08475,

1603.03440, 1604.01989

Einstein's GR is well behaved in IR, but UV is Pathetic;
Aim is to address the UV aspects of Gravity

Energy Ladder :
Very Little do we know about Gravity



$(10^{27}\ \mathrm{eV})^4$

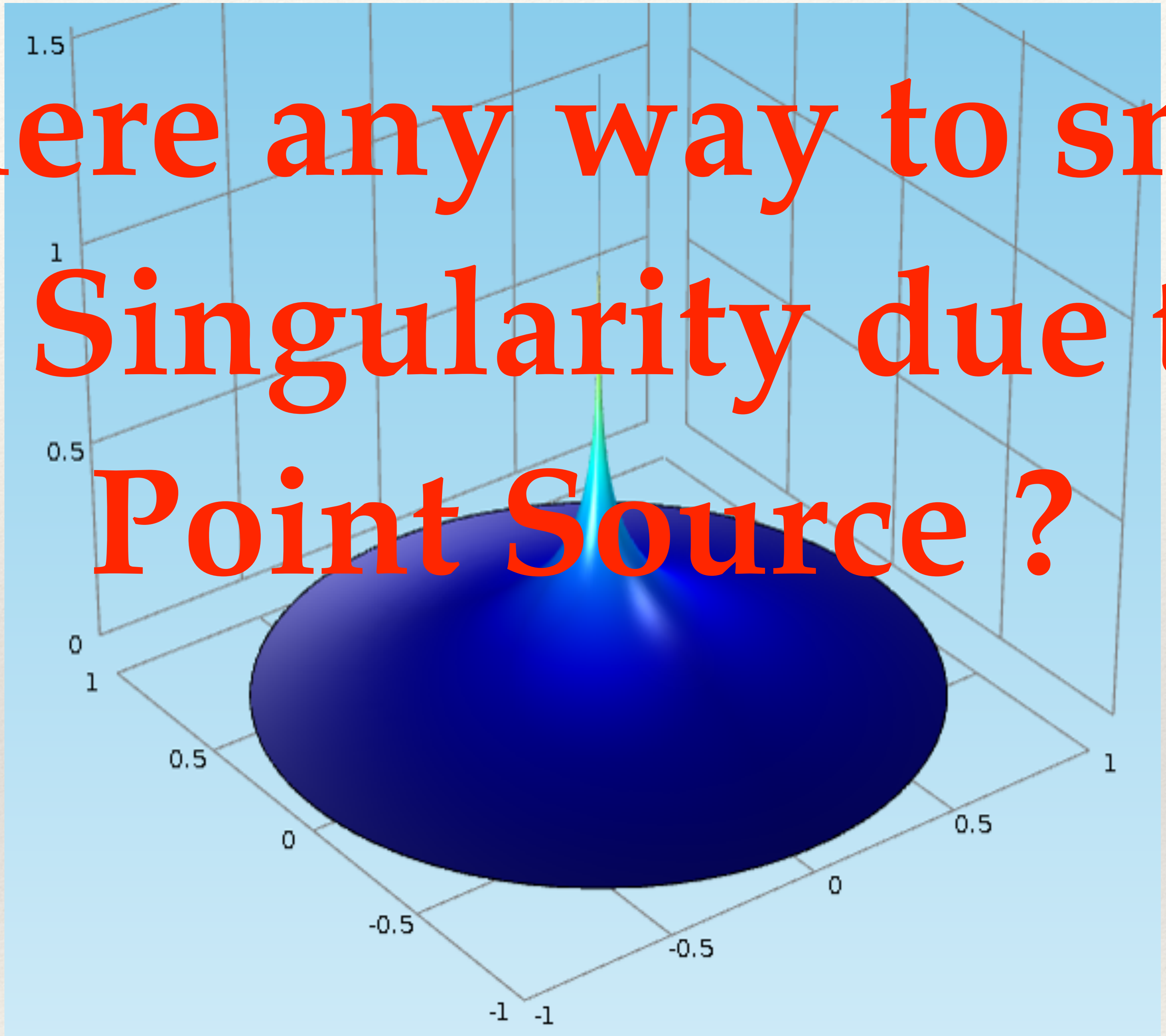
$(10^{-2}\ \mathrm{eV})^4$

$(10^{-3}\ \mathrm{eV})^4$

No departure from Newtonian Gravity up to

$10^{-5}\ \mathrm{m} \sim 100\ (\mathrm{eV})^{-1}$ or, $M \sim 10^{-2}\ \mathrm{eV}$

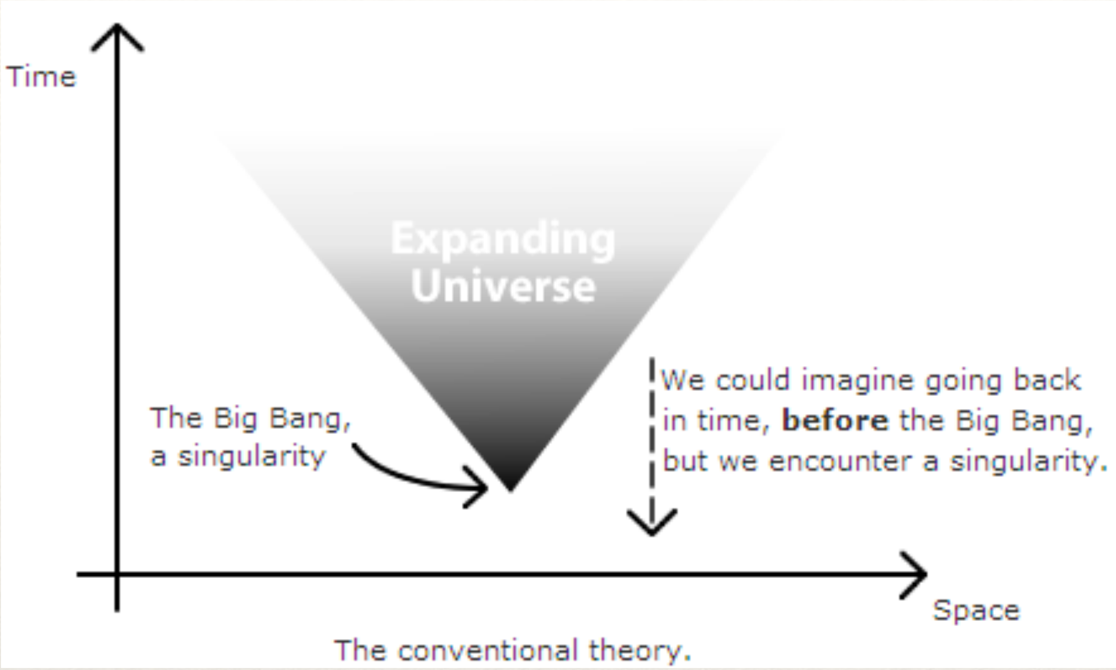
Einstein Gravity



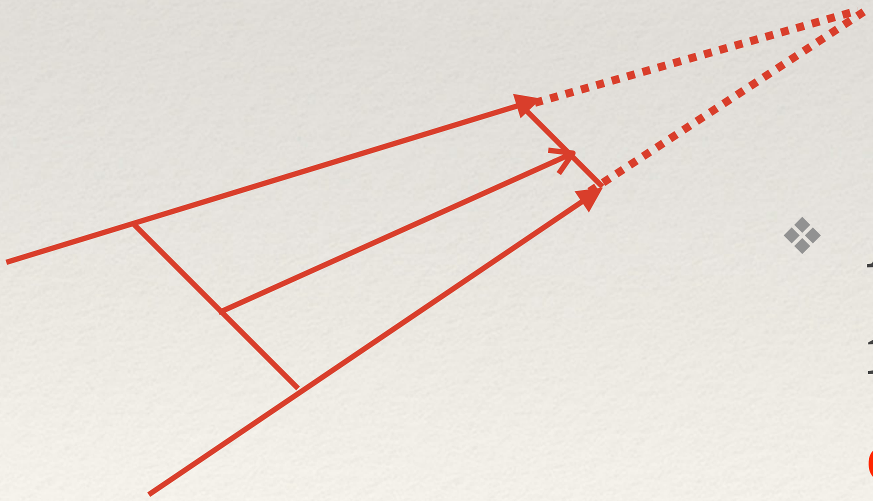
Is there any way to smear the Singularity due to a Point Source ?

$$ds^2 = \left(1 - \frac{2Gm}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2Gm}{r}\right)}$$

Cosmological Singularity



Big Bang Singularity, Space Time have an edge

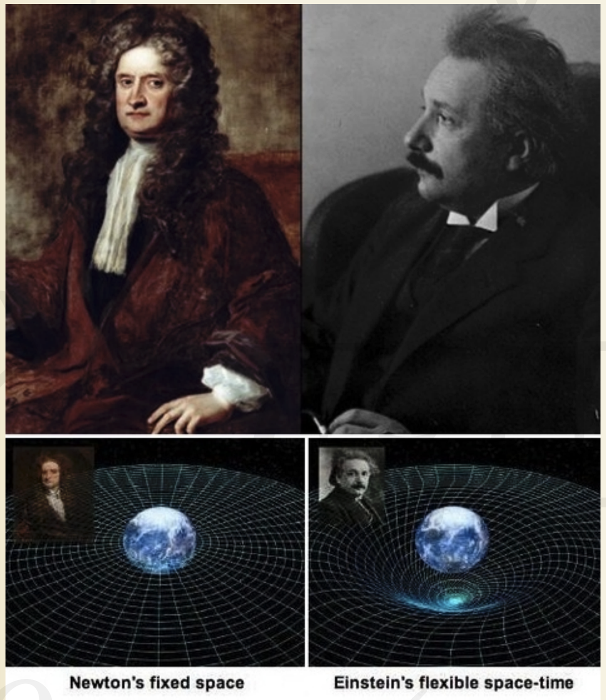


$$\rho + p \geq 0$$

- A singularity would always imply focusing of geodesics, but focusing alone cannot imply a singularity

"Inflation does not solve the singularity problem"

UV Modification of Gravity



UV is Pathological,

IR Part is Safe

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} + \cdots \right)$$

Gravity requires modification at small distances and at early times

While keeping the General Covariance

Motivated from String Field Theory, analogous to Born-Infeld theory of E & M

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

Three New Results

- Consistent theory of Gravity around Constant Curvature Backgrounds
- Criteria for resolving Cosmological Singularity
- Divergence structures in 1 and 2-loops in a scalar Toy model

Without SUSY and SUGRA : SUSY is broken for a generic time dependent scenarios

Consistent Covariant Quadratic Theories of Gravity with Stable Constant Curvature Backgrounds

“Perturbative Unitarity”

“Ghost Free”

“Tachyon Free”

“Correct degrees of freedom in Graviton Propagator”

Spin-2

&

Spin-0

components of a Graviton Propagator

4th Derivative Gravity & Power Counting renormalizability

$$I = \int d^4x\, \sqrt{g}\, \left[\lambda_0 + k\, R + a\, R_{\mu\nu} R^{\mu\nu} - \tfrac{1}{3}\,(b+a) R^2\right]$$

$$D \propto \frac{1}{k^4 + Ak^2} = \frac{1}{A}\left(\frac{1}{k^2} - \frac{1}{k^2 + A}\right)$$

Massive Spin-0 & Massive Spin-2 (Ghost) Stelle (1977)

Utiyama, De Witt (1961), Stelle (1977)

Modification of Einstein's GR

Modification of Graviton Propagator

Extra propagating degree of freedom

Challenge: to get rid of the extra dof

Ghosts

Higher Order Derivative Theory Generically Carry Ghosts (-ve Risidue) with real "m"(No-Tachyon)

$$S = \int d^4x\ \phi\Box(\Box + m^2)\phi \Rightarrow \Box(\Box + m^2)\phi = 0$$

$$\Delta(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2+m^2)}$$

Propagator with first order poles

Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!

$$\Box e^{-\Box}\phi = 0$$

No extra states other than the original dof.

Moffat (1991), Tomboulis (1997), Tseytlin (1997), Siegel (2003), Biswas, Grisaru, Siegel (2004), Biswas, Mazumdar, Siegel (2006)

Higher order construction of Gravity

$$S = S_E + S_q$$

$$S_q = \int d^4x\sqrt{-g}\left[R_{....}\mathcal{O}^{....}_{....}R^{....} + R_{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....} + R_{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....} + \cdots\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad R \sim \mathcal{O}(h)$$

$$S_q = \int d^4x\sqrt{-g}R_{\mu_1\nu_1\lambda_1\sigma_1}\mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2}R^{\mu_2\nu_2\lambda_2\sigma_2}$$

Covariant derivatives

Unknown Infinite Functions of Derivatives

Redundancies & Form Factors

$$
\begin{aligned}
S_q \;=\; \int d^4x\sqrt{-g}[& RF_1(\Box)R + RF_2(\Box)\nabla_\mu\nabla_\nu R^{\mu\nu} + R_{\mu\nu}F_3(\Box)R^{\mu\nu} + R^\nu_\mu F_4(\Box)\nabla_\nu\nabla_\lambda R^{\mu\lambda} \\
+ \; & R^{\lambda\sigma}F_5(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\lambda R^{\mu\nu} + RF_6(\Box)\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda}F_7(\Box)\nabla_\nu\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
+ \; & R^\rho_\lambda F_8(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1}F_9(\Box)\nabla_{\mu_1}\nabla_{\nu_1}\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
+ \; & R_{\mu\nu\lambda\sigma}F_{10}(\Box)R^{\mu\nu\lambda\sigma} + R^\rho_{\mu\nu\lambda}F_{11}(\Box)\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1}F_{12}(\Box)\nabla^{\rho_1}\nabla^{\sigma_1}\nabla_\rho\nabla_\sigma R^{\mu\rho\nu\sigma} \\
+ \; & R^{\nu_1\rho_1\sigma_1}_\mu F_{13}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1}F_{14}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma}
\end{aligned}
$$

$$
= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]
$$

UV completion of Starobinsky Inflation up to quadratic in curvature

(1) GR
(2) Weyl Gravity
(3) F(R) Gravity
(4) Gauss-Bonnet Gravity
(5) Ghost free Gravity

Biswas, Mazumdar, Siegel, 2006,

Chialva, Mazumdar, 2013,

Koshelev, Modesto, Rachwal, Starobinsky, 2016

Linearised Equations of Motion around Minkowski

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_q = -\int d^4x\Big[\frac{1}{2}h_{\mu\nu}a(\Box)\Box h^{\mu\nu} + h_\mu^\sigma b(\Box)\partial_\sigma\partial_\nu h^{\mu\nu} \qquad (3)$$
$$+ hc(\Box)\partial_\mu\partial_\nu h^{\mu\nu} + \frac{1}{2}hd(\Box)\Box h + h^{\lambda\sigma}\frac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu}\Big]$$

$$\begin{aligned}
a(\Box) &= 1 - \frac{1}{2}\mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box \\
b(\Box) &= -1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box + 2\mathcal{F}_3(\Box)\Box \\
c(\Box) &= 1 + 2\mathcal{F}_1(\Box)\Box + \frac{1}{2}\mathcal{F}_2(\Box)\Box \\
d(\Box) &= -1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box \\
f(\Box) &= -2\mathcal{F}_1(\Box)\Box - \mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box.
\end{aligned}$$

$$\begin{aligned}
R_{\mu\nu\lambda\sigma} &= \frac{1}{2}(\partial_{[\lambda}\partial_\nu h_{\mu\sigma]} - \partial_{[\lambda}\partial_\mu h_{\nu\sigma]}) \\
R_{\mu\nu} &= \frac{1}{2}(\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} - \partial_\nu\partial_\mu h - \Box h_{\mu\nu}) \\
R &= \partial_\nu\partial_\mu h^{\mu\nu} - \Box h
\end{aligned}$$

$$a + b = 0$$
$$c + d = 0$$
$$b + c + f = 0$$

Similar analysis has been derived for dS an AdS

Graviton Propagator around Minkowski

$$a(\Box)\Box h_{\mu\nu} \quad + \quad b(\Box)\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} + c(\Box)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h)$$
$$+\eta_{\mu\nu}d(\Box)\Box h \quad + \quad \frac{1}{4}f(\Box)\Box^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = -\kappa\tau_{\mu\nu}$$

$$-\kappa\tau\nabla_\mu\tau^\mu_\nu = 0 = (c \overset{=0}{+} d)\Box\partial_\nu h + (a \overset{=0}{+} b)\Box h^\mu_{\nu,\mu} + (b + c \overset{=0}{+} f)h^{\alpha\beta}_{,\alpha\beta\nu}$$

Bianchi Identity

$$a + b = 0$$
$$c + d = 0$$
$$b + c + f = 0$$

$$\Pi^{-1\lambda\sigma}_{\mu\nu}h_{\lambda\sigma} = \kappa\tau_{\mu\nu} \qquad h = h^{TT} + h^L + h^T$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P^0_s}{(a-3c)k^2}$$

Spin projection operators

Ph.D. Thesis by K. J. Barnes, 1963

R. J. Rivers (1963)

P. Van Nieuwenhuizen,

Nucl.Phys. B60 (1973), 478.

Let us introduce

$$\begin{aligned}
\mathcal{P}^2 &= \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma}+\theta_{\mu\sigma}\theta_{\nu\rho})-\frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}\,,\\
\mathcal{P}^1 &= \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma}+\theta_{\mu\sigma}\omega_{\nu\rho}+\theta_{\nu\rho}\omega_{\mu\sigma}+\theta_{\nu\sigma}\omega_{\mu\rho})\,,\\
\mathcal{P}^0_s &= \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}\,,\quad \mathcal{P}^0_w=\omega_{\mu\nu}\omega_{\rho\sigma}\,,\\
\mathcal{P}^0_{sw} &= \frac{1}{\sqrt{3}}\theta_{\mu\nu}\omega_{\rho\sigma}\,,\quad \mathcal{P}^0_{ws}=\frac{1}{\sqrt{3}}\omega_{\mu\nu}\theta_{\rho\sigma}\,,
\end{aligned} \tag{16}$$

where the transversal and longitudinal projectors in the momentum space are respectively

$$\theta_{\mu\nu}=\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}, \qquad \omega_{\mu\nu}=\frac{k_\mu k_\nu}{k^2}.$$

Note that the operators $\mathcal{P}^i$ are in fact 4-rank tensors, $\mathcal{P}^i_{\mu\nu\rho\sigma}$, but we have suppressed the index notation here.

Out of the six operators four of them, $\{\mathcal{P}^2,\mathcal{P}^1,\mathcal{P}^0_s,\mathcal{P}^0_w\}$, form a complete set of projection operators:

$$\mathcal{P}^i_a\mathcal{P}^j_b=\delta^{ij}\delta_{ab}\mathcal{P}^i_a \quad \text{and} \quad \mathcal{P}^2+\mathcal{P}^1+\mathcal{P}^0_s+\mathcal{P}^0_w=1\,, \tag{17}$$

$$\mathcal{P}^0_{ij}\mathcal{P}^0_k=\delta_{jk}\mathcal{P}^0_{ij}\,,\quad \mathcal{P}^0_{ij}\mathcal{P}^0_{kl}=\delta_{il}\delta_{jk}\mathcal{P}^0_k\,,\quad \mathcal{P}^0_k\mathcal{P}^0_{ij}=\delta_{ik}\mathcal{P}^0_{ij}\,,$$

For the above action, see:

Biswas, Koivisto, Mazumdar 1302.0532

Tree level Graviton Propagator

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}$$

No new propagating degree of freedom other than the massless Graviton

$$a(\Box) = c(\Box) \Rightarrow 2\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0$$

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + R\mathcal{F}_1(\Box)R - \frac{1}{2}R^{\mu\nu}\mathcal{F}_2(\Box)R_{\mu\nu}\right]$$

Without loss of generality either $\mathcal{F}_1,\ or\ \mathcal{F}_2,\ or\ \mathcal{F}_3 = 0$

Well known Higher Derivative limits

$$a(\Box) = 1 - \frac{1}{2}\mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box$$
$$b(\Box) = -1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box + 2\mathcal{F}_3(\Box)\Box$$
$$c(\Box) = 1 + 2\mathcal{F}_1(\Box)\Box + \frac{1}{2}\mathcal{F}_2(\Box)\Box$$
$$d(\Box) = -1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box$$
$$f(\Box) = -2\mathcal{F}_1(\Box)\Box - \mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box.$$

(1) GR:

$$a(0) = c(0) = -b(0) = -d(0) = 1$$

$$\lim_{k^2\to 0} \Pi = (\mathcal{P}^2/k^2) - (\mathcal{P}_s^0/2k^2) \equiv \Pi_{GR}$$

(2) F(R) Gravity:

$$\mathcal{L}(R) = \mathcal{L}(0) + \mathcal{L}'(0)R + \frac{1}{2}\mathcal{L}''(0)R^2 + \cdots$$

$$a = -b = 1, \qquad c = -d = 1 - \mathcal{L}''(0)\Box$$

$$\Pi = \frac{\mathcal{P}^2}{k^2} - \frac{\mathcal{P}_s^0}{2k^2(1+3\mathcal{L}''(0)k^2)} \qquad \Pi = \Pi_{GR} + \frac{1}{2}\frac{\mathcal{P}_s^0}{k^2+m^2}, \qquad m^2 = \frac{1}{3\mathcal{L}''(0)}$$

(3) GB Gravity:

$$\mathcal{L} = R + \alpha(\Box)G,$$

$$a = c = -b = -d = 1$$

$$\Pi = \Pi_{GR}$$

(4) Weyl Gravity:

$$\mathcal{L} = R - \frac{1}{m^2}C^2 \qquad C^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

$$a = -b = 1 - (k/m)^2$$

$$c = -d = 1 - (k/m)^2/3 \text{ and } f = -2(k/m)^2/3$$

$$\Pi = \frac{\mathcal{P}^2}{k^2(1-(k/m)^2)} - \frac{\mathcal{P}_s^0}{2k^2} = \Pi_{GR} - \frac{\mathcal{P}^2}{k^2+m^2}$$

Biswas, Koivisto, Mazumdar
1302.0532

Complete Field Equations

Ghost-free gravity

2.3. The Complete Field Equations

$$S=\int d^4x\sqrt{-g}\left(\frac{R}{2}+R\mathcal{F}_1(\Box)R+R^{\mu\nu}\mathcal{F}_2(\Box)R_{\mu\nu}+C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C_{\mu\nu\lambda\sigma}\right)$$

Following from this we find the equation of motion for the full action S in (1) to be a combination of S_0, S_1, S_2 and S_3 above

$$\begin{aligned}P^{\alpha\beta}=&\,G^{\alpha\beta}+4G^{\alpha\beta}\mathcal{F}_1(\Box)R+g^{\alpha\beta}R\mathcal{F}_1(\Box)R-4\left(\nabla^\alpha\nabla^\beta-g^{\alpha\beta}\Box\right)\mathcal{F}_1(\Box)R\\&-2\Omega_1^{\alpha\beta}+g^{\alpha\beta}(\Omega_{1\sigma}^{\sigma}+\bar{\Omega}_1)+4R_\mu^\alpha\mathcal{F}_2(\Box)R^{\mu\beta}\\&-g^{\alpha\beta}R_\nu^\mu\mathcal{F}_2(\Box)R_\mu^\nu-4\nabla_\mu\nabla^\beta(\mathcal{F}_2(\Box)R^{\mu\alpha})+2\Box(\mathcal{F}_2(\Box)R^{\alpha\beta})\\&+2g^{\alpha\beta}\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box)R^{\mu\nu})-2\Omega_2^{\alpha\beta}+g^{\alpha\beta}(\Omega_{2\sigma}^{\sigma}+\bar{\Omega}_2)-4\Delta_2^{\alpha\beta}\\&-g^{\alpha\beta}C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C_{\mu\nu\lambda\sigma}+4C^{\alpha}{}_{\mu\nu\sigma}\mathcal{F}_3(\Box)C^{\beta\mu\nu\sigma}\\&-4(R_{\mu\nu}+2\nabla_\mu\nabla_\nu)(\mathcal{F}_3(\Box)C^{\beta\mu\nu\alpha})-2\Omega_3^{\alpha\beta}+g^{\alpha\beta}(\Omega_{3\gamma}^{\gamma}+\bar{\Omega}_3)-8\Delta_3^{\alpha\beta}\\&=T^{\alpha\beta}\,,\end{aligned}\tag{52}$$

where $T^{\alpha\beta}$ is the stress energy tensor for the matter components in the universe and we have defined the following symmetric tensors:

$$\Omega_1^{\alpha\beta}=\sum_{n=1}^{\infty}f_{1_n}\sum_{l=0}^{n-1}\nabla^\alpha R^{(l)}\nabla^\beta R^{(n-l-1)},\quad \bar{\Omega}_1=\sum_{n=1}^{\infty}f_{1_n}\sum_{l=0}^{n-1}R^{(l)}R^{(n-l)},\tag{53}$$

$$\Omega_2^{\alpha\beta}=\sum_{n=1}^{\infty}f_{2_n}\sum_{l=0}^{n-1}R_\nu^{\mu;\alpha(l)}R_\mu^{\nu;\beta(n-l-1)},\quad \bar{\Omega}_2=\sum_{n=1}^{\infty}f_{2_n}\sum_{l=0}^{n-1}R_\nu^{\mu(l)}R_\mu^{\nu(n-l)}\,,\tag{54}$$

$$\Delta_2^{\alpha\beta}=\frac{1}{2}\sum_{n=1}^{\infty}f_{2_n}\sum_{l=0}^{n-1}[R_\sigma^{\nu(l)}R^{(\beta|\sigma|;\alpha)(n-l-1)}-R_\sigma^{\nu;(\alpha(l)}R^{\beta)\sigma(n-l-1)}]_{;\nu}\,,\tag{55}$$

$$\Omega_3^{\alpha\beta}=\sum_{n=1}^{\infty}f_{3_n}\sum_{l=0}^{n-1}C^{\mu;\alpha(l)}_{\nu\lambda\sigma}C_\mu{}^{\nu\lambda\sigma;\beta(n-l-1)},\ \bar{\Omega}_3=\sum_{n=1}^{\infty}f_{3_n}\sum_{l=0}^{n-1}C^{\mu(l)}_{\nu\lambda\sigma}C_\mu{}^{\nu\lambda\sigma(n-l)}\,,\tag{56}$$

$$\Delta_3^{\alpha\beta}=\frac{1}{2}\sum_{n=1}^{\infty}f_{3_n}\sum_{l=0}^{n-1}[C^{\lambda\nu(l)}_{\sigma\mu}C_\lambda{}^{(\beta|\sigma\mu|;\alpha)(n-l-1)}-C^{\lambda\nu}{}_{\sigma\mu}{}^{;(\alpha(l)}C_\lambda{}^{\beta)\sigma\mu(n-l-1)}]_{;\nu}\,.\tag{57}$$

The trace equation is often particularly useful and below we provide it for the general action (1):

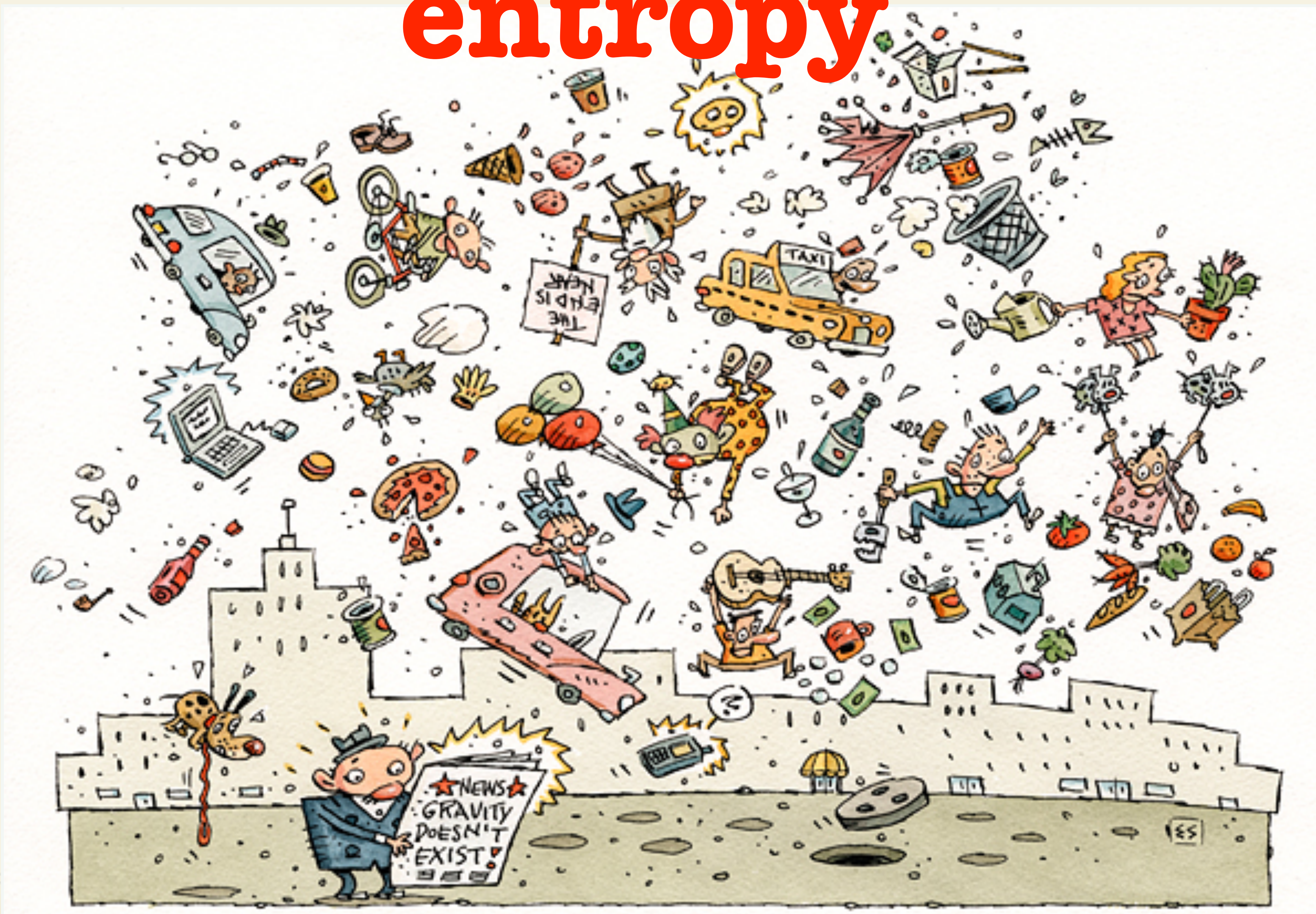
$$\begin{aligned}P=&-R+12\Box\mathcal{F}_1(\Box)R+2\Box(\mathcal{F}_2(\Box)R)+4\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box)R^{\mu\nu})\\&+2(\Omega_{1\sigma}^{\sigma}+2\bar{\Omega}_1)+2(\Omega_{2\sigma}^{\sigma}+2\bar{\Omega}_2)+2(\Omega_{3\sigma}^{\sigma}+2\bar{\Omega}_3)-4\Delta_{2\sigma}^{\sigma}-8\Delta_{3\sigma}^{\sigma}\\&=T\equiv g_{\alpha\beta}T^{\alpha\beta}\,.\end{aligned}\tag{58}$$

It is worth noting that we have checked special cases of our result against previous work in sixth order gravity given in [24] and found them to be equivalent at least to the cubic order (see Appendix C for details).

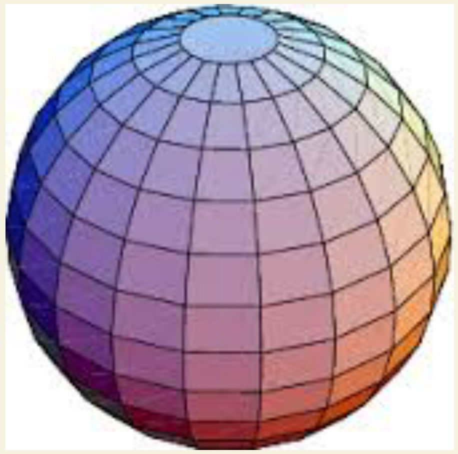
$$R^{(m)}\equiv\Box^m R$$

Biswas, Conroy, Koshelev, Mazumdar
1308.2319 Class.Quant. Grav. (2014)

Gravitational entropy



Gravitational Entropy



$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$S_W = -8\pi \oint_{r=r_H,\ t=\text{const}} \left(\frac{\partial \mathcal{L}}{\partial R_{rtrt}}\right) q(r) d\Omega^2$$

Wald (1990, 1993), Iyer, Wald (1993)

$$S_W = \frac{Area}{4G}\left[1 + \alpha\left(2\mathcal{F}_1 + \mathcal{F}_2 + \underbrace{2\mathcal{F}_3}_{=0}\right) R\right]$$

Holography is an IR effect

Higher order corrections yield zero entropy
"Ground State of Gravity"

Conroy, Mazumdar, Teimouri, 1503.05568, hep-th (Phys. Rev. Lett. 2015)

Consistent theories of Gravity around dS and Ads backgrounds

$$S = \int d^4x\sqrt{-g}\left[\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\widehat{\mathcal{O}}_{iI}\mathcal{Q}_{iI})\right]$$

Most generic action - "Parity Invariant" and "Torsion Free"

$$R = \bar{R} = \text{const}, \qquad R_{\mu\nu} = \frac{\bar{R}}{4}\bar{g}_{\mu\nu}, \qquad R^{\rho}_{\mu\sigma\nu} = \frac{\bar{R}}{12}(\delta^{\rho}_{\sigma}\bar{g}_{\mu\nu} - \delta^{\rho}_{\nu}\bar{g}_{\mu\sigma})$$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \Lambda + \frac{\lambda}{2}\left(R\mathcal{F}_1(\Box)R + S_{\mu\nu}\mathcal{F}_2(\Box)S^{\mu\nu} + C_{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C^{\mu\nu\lambda\sigma}\right)\right]$$

$$h_{\mu\nu} = h^{\perp}_{\mu\nu} + \bar{\nabla}_\mu A^{\perp}_\nu + \bar{\nabla}_\nu A^{\perp}_\mu + (\bar{\nabla}_\mu\bar{\nabla}_\nu - \frac{1}{4}\bar{g}_{\mu\nu}\bar{\Box})B + \frac{1}{4}\bar{g}_{\mu\nu}h$$

For pure EH action, see D'Hoker, Freedman, Mathur, Matusis, Rastelli (hep-th/9902042)

Full generalisation, see: Biswas, Koshelev, Mazumdar, 1602.08475

Quadratic order Action for spin-2 and spin-0 components

$$S_2 \equiv \frac{1}{2}\int dx^4\sqrt{-\bar{g}}\,\widetilde{h^{\perp}}^{\mu\nu}\left(\bar{\Box}-\frac{\bar{R}}{6}\right)$$
$$\left\{1+\frac{2}{M_p^2}\lambda c_{1,0}\bar{R}+\frac{\lambda}{M_p^2}\left[\left(\bar{\Box}-\frac{\bar{R}}{6}\right)\mathcal{F}_2(\bar{\Box})+2\left(\bar{\Box}-\frac{\bar{R}}{3}\right)\mathcal{F}_3\left(\bar{\Box}+\frac{\bar{R}}{3}\right)\right]\right\}\widetilde{h^{\perp}}_{\mu\nu}$$

$$S_0 \equiv -\frac{1}{2}\int dx^4\sqrt{-\bar{g}}\,\tilde{\phi}\left(\bar{\Box}+\frac{\bar{R}}{3}\right)$$
$$\left\{1+\frac{2}{M_p^2}\lambda c_{1,0}\bar{R}-\frac{\lambda}{M_p^2}\left[2(3\bar{\Box}+\bar{R})\mathcal{F}_1(\bar{\Box})+\frac{1}{2}\bar{\Box}\mathcal{F}_2\left(\bar{\Box}+\frac{2}{3}\bar{R}\right)\right]\right\}\tilde{\phi}$$

Minkowski limit matches with our earlier propagator

$$\Pi_2 = \frac{i}{p^2\left\{1-\frac{2p^2}{M_p^2}\left[\mathcal{F}_2(-p^2)+2\mathcal{F}_3(-p^2)\right]\right\}},$$

$$\Pi_0 = \frac{-i}{p^2\left\{1+\frac{2p^2}{M_p^2}\left[6\mathcal{F}_1(-p^2)+\frac{1}{2}\mathcal{F}_2(-p^2)\right]\right\}}$$

$$\widetilde{h^{\perp}}_{\mu\nu} = \frac{1}{2}M_p h^{\perp}_{\mu\nu}\,, \qquad \tilde{\phi} = \sqrt{\frac{3}{32}}M_p\phi$$

Biswas, Koshelev, Mazumdar

1602.08475

80th B'Day Celeb. of Carl Brans

Most generic Ghost FreeGraviton Propagator in dS/AdS

$$\mathcal{T}(\bar{R},\bar{\Box}) \equiv 1 + \frac{4\bar{R}}{M_p^2}c_{1,0} + \frac{2}{M_p^2}\left[\left(\bar{\Box} - \frac{\bar{R}}{6}\right)\mathcal{F}_2(\bar{\Box}) + 2\left(\bar{\Box} - \frac{\bar{R}}{3}\right)\mathcal{F}_3\left(\bar{\Box} + \frac{\bar{R}}{3}\right)\right]$$

$$\mathcal{S}(\bar{R},\bar{\Box}) \equiv 1 + \frac{4\bar{R}}{M_p^2}c_{1,0} - \frac{2}{M_p^2}\left[2(3\bar{\Box} + \bar{R})\mathcal{F}_1(\bar{\Box}) + \frac{1}{2}\bar{\Box}\mathcal{F}_2\left(\bar{\Box} + \frac{2}{3}\bar{R}\right)\right]$$

$$\mathcal{T}(\bar{R},\bar{\Box}) \equiv e^{\tau(\bar{\Box})},$$

$$\mathcal{S}(\bar{R},\bar{\Box}) \equiv \left(1 - \frac{\bar{\Box}}{m^2}\right)^{\epsilon} e^{\sigma(\bar{\Box})}$$

$\epsilon = 0,$ No scalar propagating d.o.f.

Biswas, Koshelev & AM, 1602.08475

Background Independent Action of Quadratic Action of Gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + \alpha_0(R, R_{\mu\nu}) + \alpha_1(R, R_{\mu\nu}) R \mathcal{F}_1(\Box) R \right.$$
$$\left. + \alpha_2(R, R_{\mu\nu}) R_{\mu\nu} \mathcal{F}_2(\Box) R^{\mu\nu} + \alpha_3(R, R_{\mu\nu}) C_{\mu\nu\lambda\sigma} \mathcal{F}_3 C^{\mu\nu\lambda\sigma} \right]$$

Proof is due

Newtonian Limit

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2} \qquad a(\Box) = c(\Box) = e^{-\Box/M^2}$$

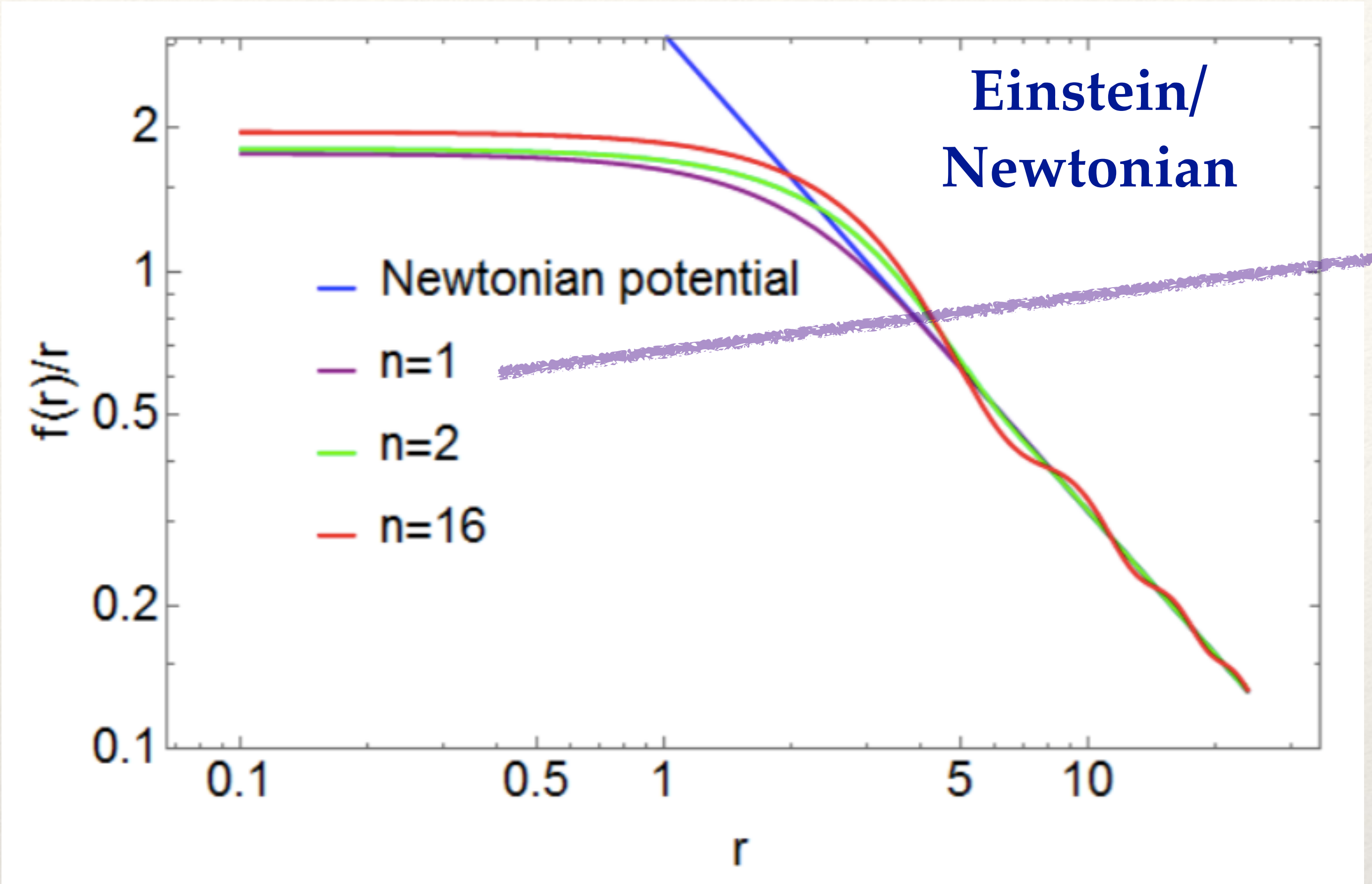
$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box}\right]R - 2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box}\right]R^{\mu\nu}\right]$$

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Psi)dr^2$$

$$\Phi = \Psi = \frac{Gm}{r}\mathbf{erf}\left(\frac{rM}{2}\right)$$

Biswas, Gerwick, Koivisto, Mazumdar, Phys. Rev. Lett. (2012)
(gr-qc/1110.5249)

Resolution of Singularity at short distances

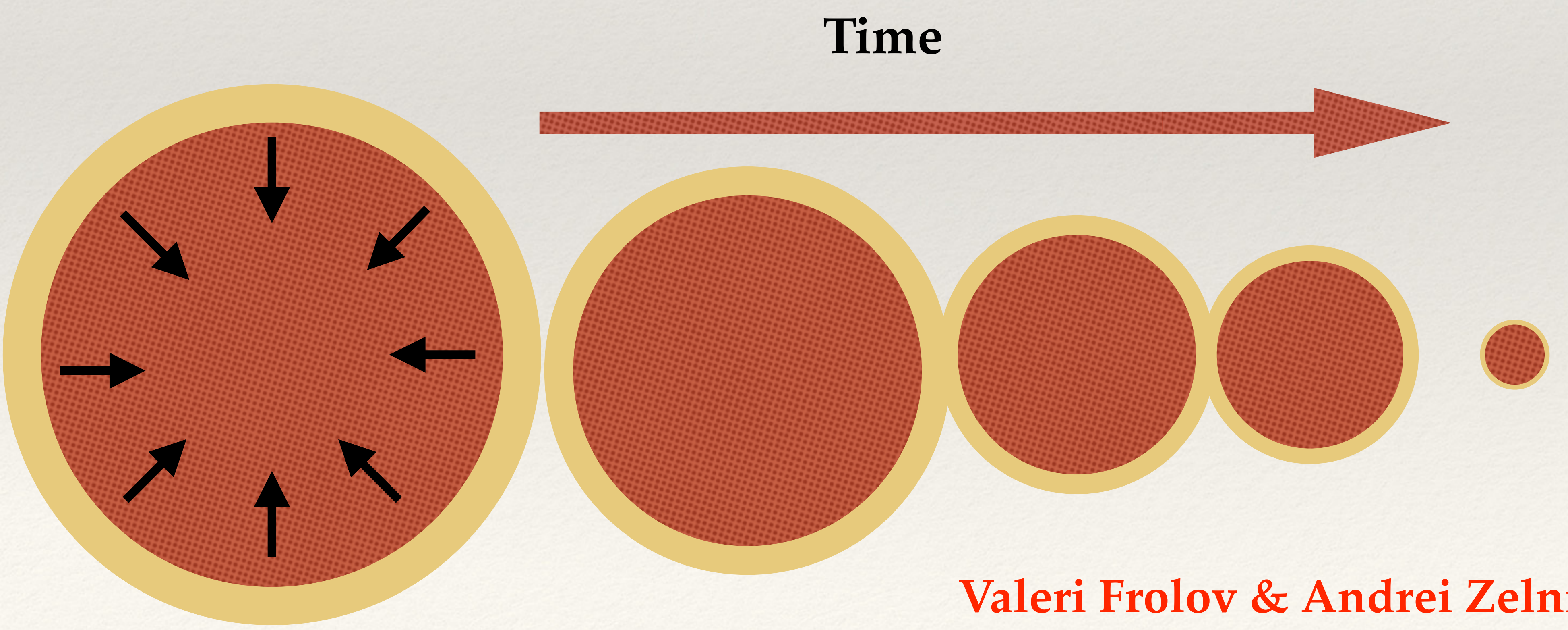


$$\Phi(r) = \Psi(r) = \frac{Gm}{r}\mathbf{erf}\left(\frac{rM}{2}\right)$$

$$mM \ll M_p^2 \implies m \ll M_p$$

Current Bound : M > 0.01 eV

Edholm, Koshelev, Mazumdar (2016)

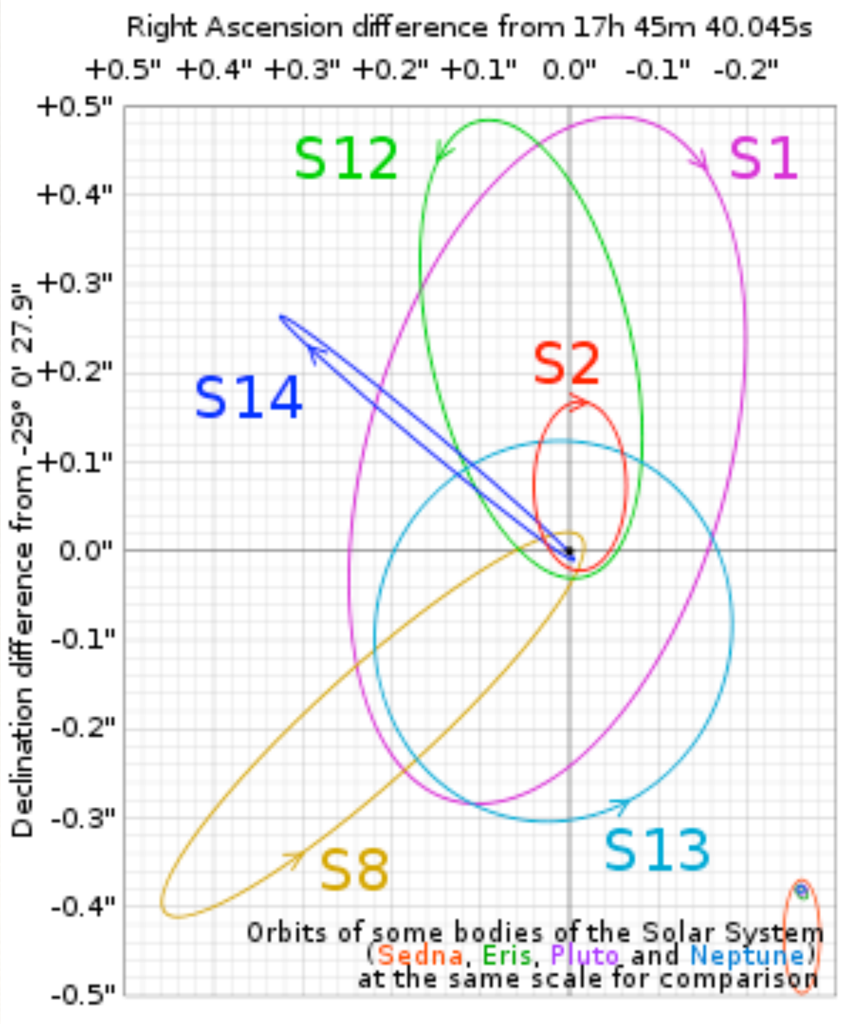


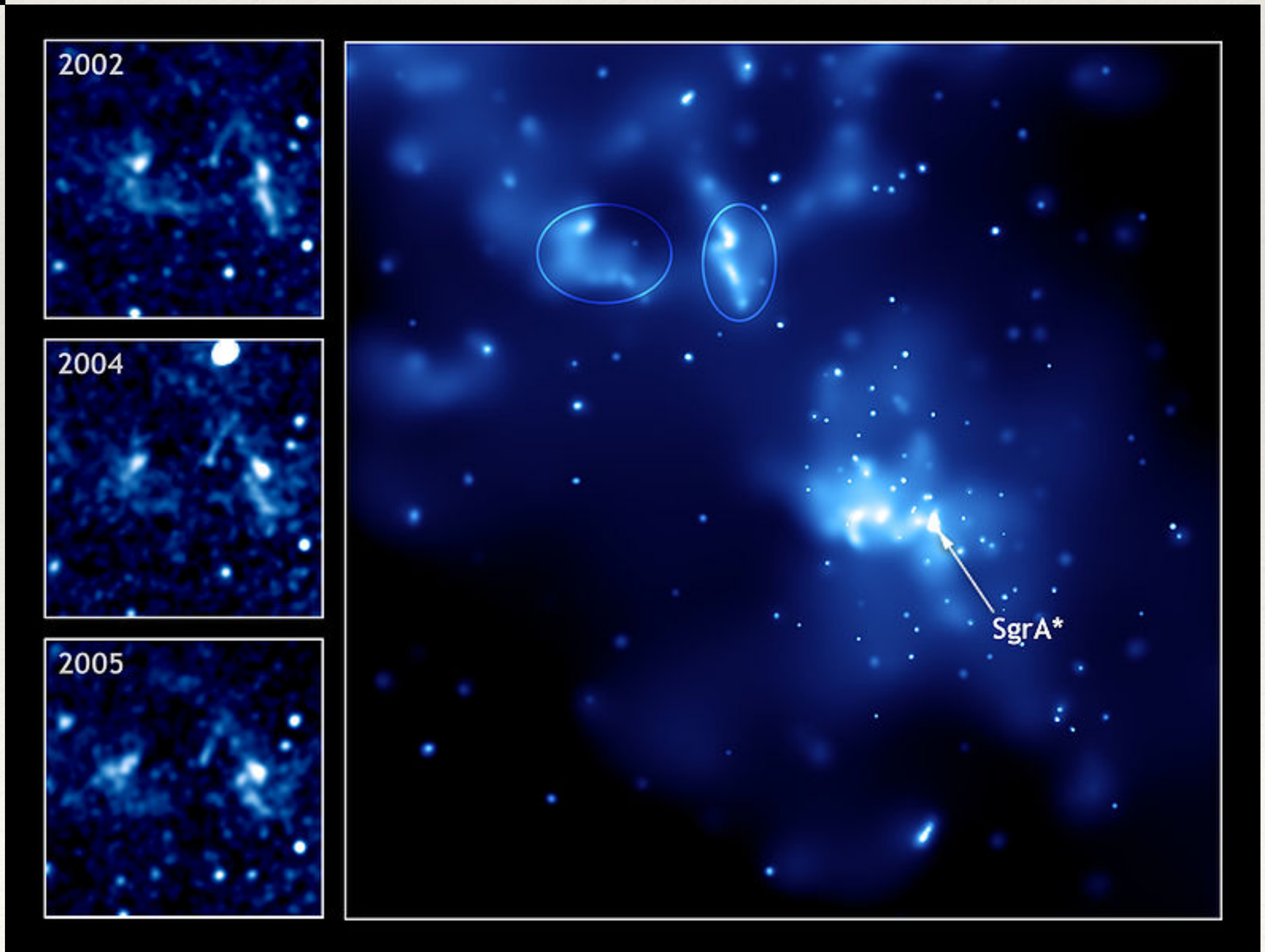
A lump of matter without horizon and without singularity

Valeri Frolov & Andrei Zelnikov (2015)

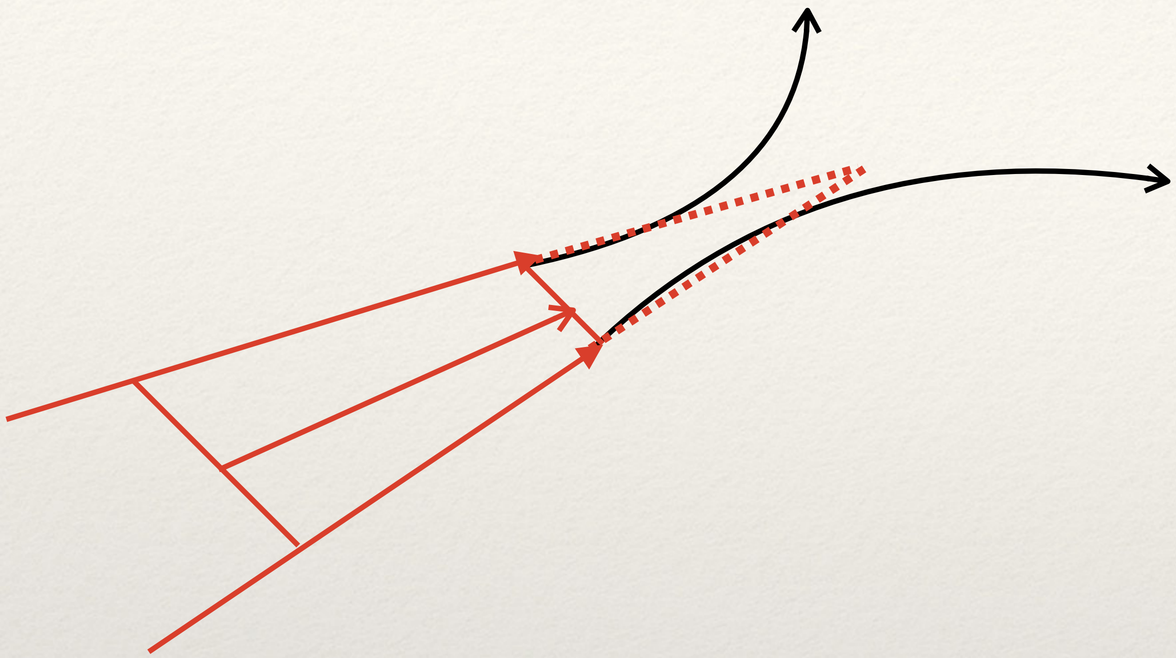
Puffy Horizon

Event Horizon Telescope

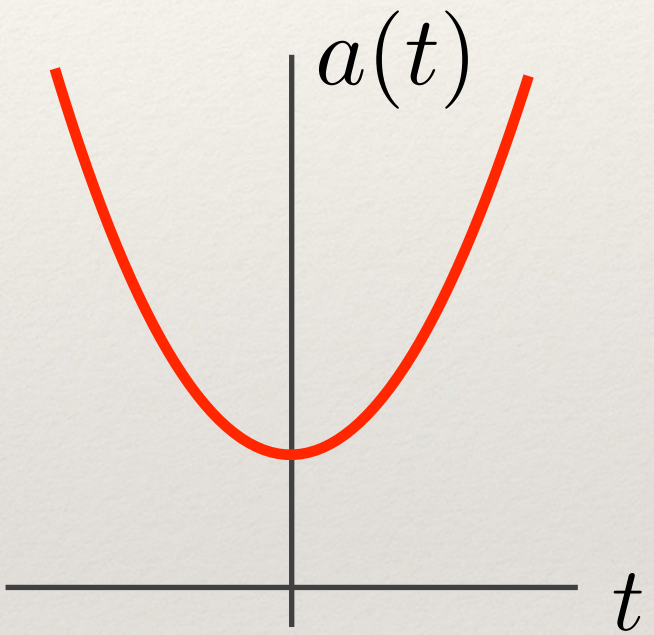




Non-Singular Bouncing Solutions: UV completion of Starobinsky Inflation



$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box}\right]R + \Lambda\right]$$



Linear Solution

$$h \sim \mathtt{diag}(0, A\sin\lambda t, A\sin\lambda t, A\sin\lambda t) \text{ with } A \ll 1$$

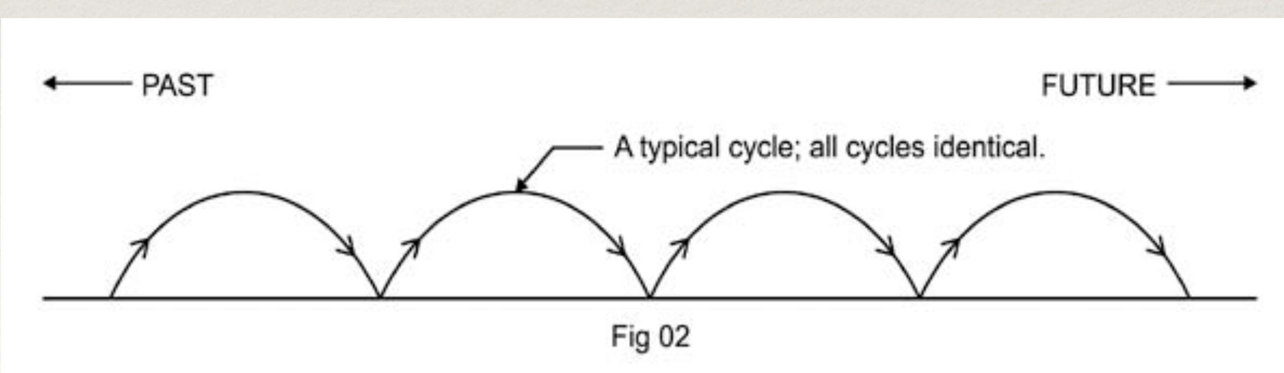


Fig 02

Biswas, Gerwick, Koivisto, Mazumdar,
Phys. Rev. Lett. (gr-qc/1110.5249)

Non-Linear Solution

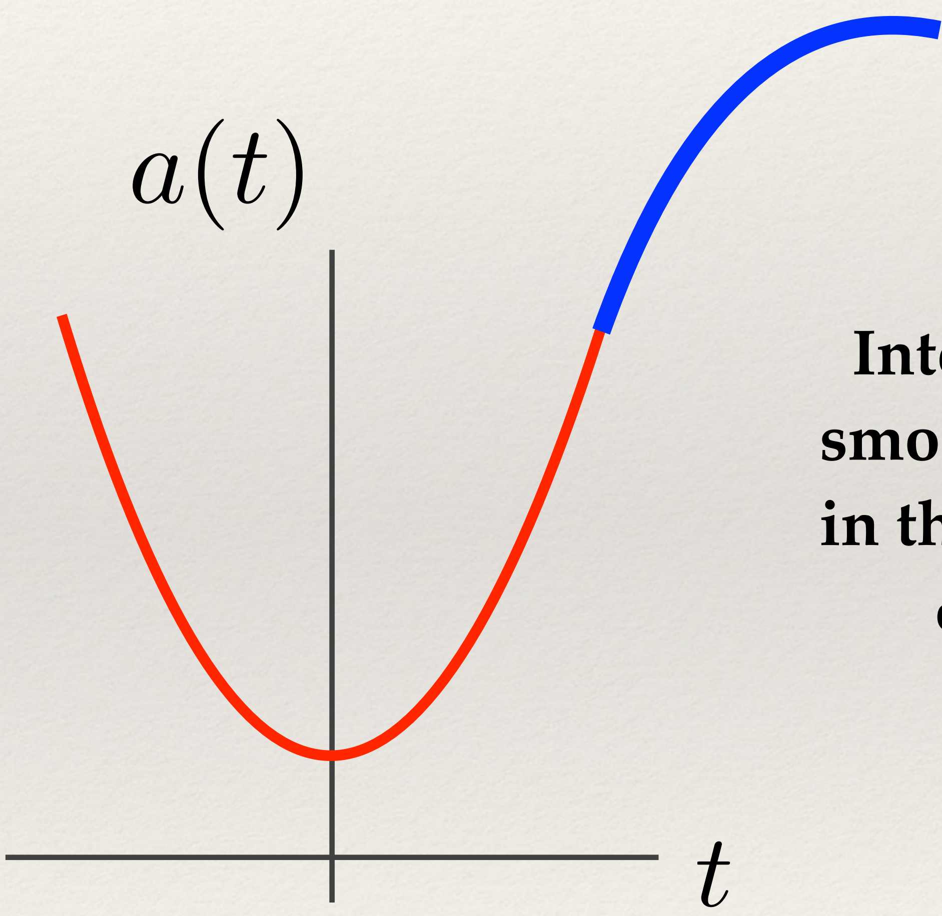
$$a(t) = \cosh\left(\sqrt{\frac{r_1}{2}}\,t\right)$$

Biswas, Mazumdar, Siegel, JCAP (2006)

Nonlocal Gravity & Cosmological Singularity

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}}-1}{\Box}\right]R + \Lambda\right]$$

$$a(t) = a_0 \cosh\left(\sqrt{\frac{\Lambda}{6M_{pl}^2}}t\right)$$



Interesting smoking gun in the Planck data !

Cosmological Constant at Bounce

$$M \sim \Lambda^{1/4}$$

Biswas, AM, PRD (2014)

"Einstein Gravity Does Not Permit Such Solution"

Hawking-Penrose Singularity Theorems & RayChaudhuri Equation

$$\frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq -R_{\mu\nu}k^\mu k^\nu \qquad \theta = \nabla_\mu k^\mu$$

$$R_{\mu\nu}k^\mu k^\nu = \kappa T_{\mu\nu}k^\mu k^\nu$$

General Relativity

$$R_{\mu\nu}k^\mu k^\nu \geq 0, \qquad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq 0$$

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$R^{(L)}_{\mu\nu}k^\mu k^\nu = \frac{1}{a(\bar{\Box})}\left[\kappa T_{\mu\nu}k^\mu k^\nu - \frac{(k^0)^2}{2}f(\bar{\Box})\Box R^{(L)}\right]$$

Defocusing: $R^L_{\mu\nu}k^\mu k^\nu \leq 0$

3 Criteria for Defocusing Null Congruences without Ghosts & Tachyons

$$\frac{f(\bar{\Box})\Box}{a(\bar{\Box})}R^{(L)} > 0 \Rightarrow \frac{a(\bar{\Box}) - c(\bar{\Box})}{a(\bar{\Box})}R^{(L)} > 0$$

$$c(\bar{\Box}) = \frac{a(\bar{\Box})}{3}\left[1 + 2\left(1 - \alpha M_P^{-2}\Box\right)\tilde{a}(\bar{\Box})\right]$$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_P^2 R + R\mathcal{F}_1(\bar{\Box})R\right]$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}$$

Massless Graviton for : a=c

(1) **Infinite Derivatives**

Locality leads to Starobinsky Model, which requires Tachyonic massive Spin-0 states to resolve singularity, but it cannot give Inflation !

(2) **Massless Spin-2,**

(3) **Non-Tachyonic Massive Spin-0**

$$\Pi(-k^2) = \frac{1}{a(-k^2)}\left[\frac{\mathcal{P}^2}{k^2} - \frac{1}{2\tilde{a}(-k^2)}\left(\frac{\mathcal{P}_s^0}{k^2} - \frac{\mathcal{P}_s^0}{k^2+m^2}\right)\right]$$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_p^2 R + cR^2\right]$$

$$\Pi_{R^2} = \Pi_{GR} + \frac{1}{2}\frac{\mathcal{P}_s^0}{k^2+m^2}$$

Conroy, Koshelev, Mazumdar, 1604.01989

Quantum aspects

- **Superficial degree of divergence goes as**

$$E = V - I. \ \ \text{Use Topological relation}: \ L = 1 + I - V$$

$$E = 1 - L \qquad\qquad E < 0, \ \text{for } L > 1$$

- **At 1-loop, the theory requires counter term, the 1-loop, 2 point function yields Λ^4 divergence**
- **At 2-loops, the theory does not give rise to additional divergences, the UV behaviour becomes finite, at large external momentum, where dressed propagators gives rise to more suppression than the vertex factors**

Talaganis, Biswas, Mazumdar, (2014)

Toy model based on Symmetries

GR e.o.m : $g_{\mu\nu} \to \Omega\, g_{\mu\nu}$

Around Minkowski space the e.o.m are invariant under:

$$h_{\mu\nu} \to (1+\epsilon)h_{\mu\nu} + \epsilon\eta_{\mu\nu}$$

Construct a scalar field theory with infinite derivatives whose e.o.m are invariant under

$$\phi \to (1+\epsilon)\phi + \epsilon$$

$$S_{free} = \frac{1}{2}\int d^4x(\phi\Box a(\Box)\phi) \qquad a(\Box) = e^{-\Box/M^2}$$

$$S_{int} = \frac{1}{M_p}\int d^4x \left(\frac{1}{4}\phi\partial_\mu\phi\partial^\mu\phi + \frac{1}{4}\phi\Box\phi a(\Box)\phi - \frac{1}{4}\phi\partial_\mu\phi a(\Box)\partial^\mu\phi\right)$$

$$\Pi(k^2) = -\frac{i}{k^2 e^{\bar{k}^2}}$$

Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity

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Abstract

In this paper we will consider quantum aspects of a non-local, infinite derivative scalar field theory - a *toy model* depiction of a covariant infinite derivative, non-local extension of Einstein's general relativity which has previously been shown to be free from ghosts around the Minkowski background. The graviton propagator in this theory gets an exponential suppression making it *asymptotically free*, thus providing strong prospects of resolving various classical and quantum divergences. In particular, we will find that at 1-loop, the 2-point function is still divergent, but once this amplitude is renormalized by adding appropriate counter terms, the ultraviolet (UV) behavior of all other 1-loop diagrams as well as the 2-loop, 2-point function remains well under control. We will go on to discuss how one may be able to generalize our computations and arguments to arbitrary loops.

Ultra High Energy Scatterings of Gravitons

High-Energy Scatterings in Infinite-Derivative Field Theory and Ghost-Free Gravity

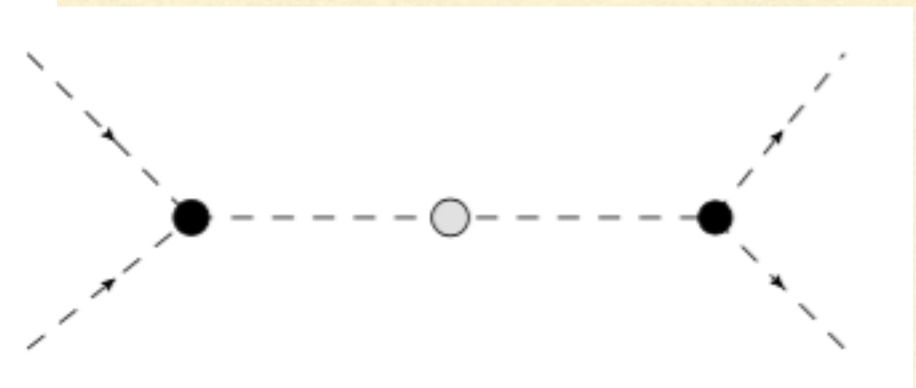
Spyridon Talaganis and Anupam Mazumdar

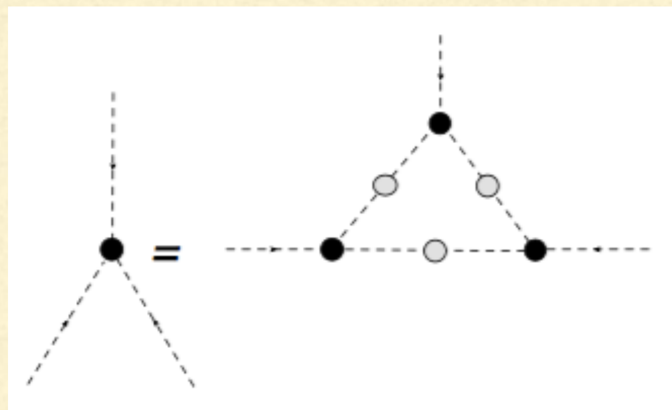
Consortium for Fundamental Physics, Lancaster University, LA1 4YB

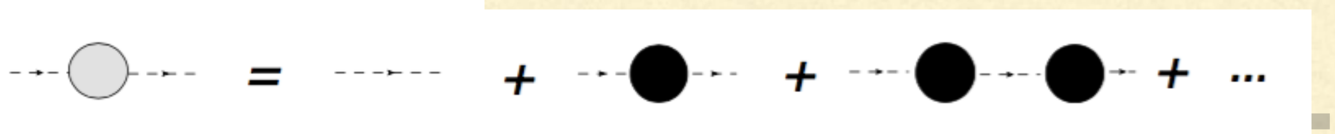
March 14, 2016

Abstract

In this paper, we will consider scattering diagrams in the context of infinite-derivative theories. First, we examine a finite-order higher-derivative scalar field theory and find that we cannot eliminate the external momentum divergences of scattering diagrams in the regime of large external momenta. Then, we employ an infinite-derivative scalar toy model and obtain that the external momentum dependence of scattering diagrams is convergent as the external momenta become very large. In order to eliminate the external momentum divergences, one has to dress the bare vertices of the scattering diagrams by considering renormalised propagator and vertex loop corrections to the bare vertices. Finally, we investigate scattering diagrams in the context of a scalar toy model which is inspired by a *ghost-free* and *singularity-free* infinite-derivative theory of gravity, where we conclude that infinite derivatives can eliminate the external momentum divergences of scattering diagrams and make the scattering diagrams convergent in the ultraviolet.







Conclusions

- **We have constructed a Ghost Free & Singularity Free Theory of Gravity around Constant Curvature Backgrounds.**
- **Studying singularity theorems, Hawking radiation, Non-Singular Bouncing Cosmology , ..., many interesting problems can be studied in this framework.**
- **Holography is not a property of UV, becomes part of an IR effect.**
- **Quantum computations also show that Infinite Derivative Gravity can ameliorate UV behaviour.**
- **Ultra-High energy graviton scatterings do not blow up.**

All these consequences have ramifications for Inflation & Quantum aspects of Gravity

Extra Slides

Does Higgs Play a Role During Inflation with Einstein Gravity ?

$$S \sim \int \sqrt{g} d^4x \left[R + \xi R H^2 + \cdots \right]$$

$$\xi \sim \mathcal{O}(10^3 - 10^4)$$

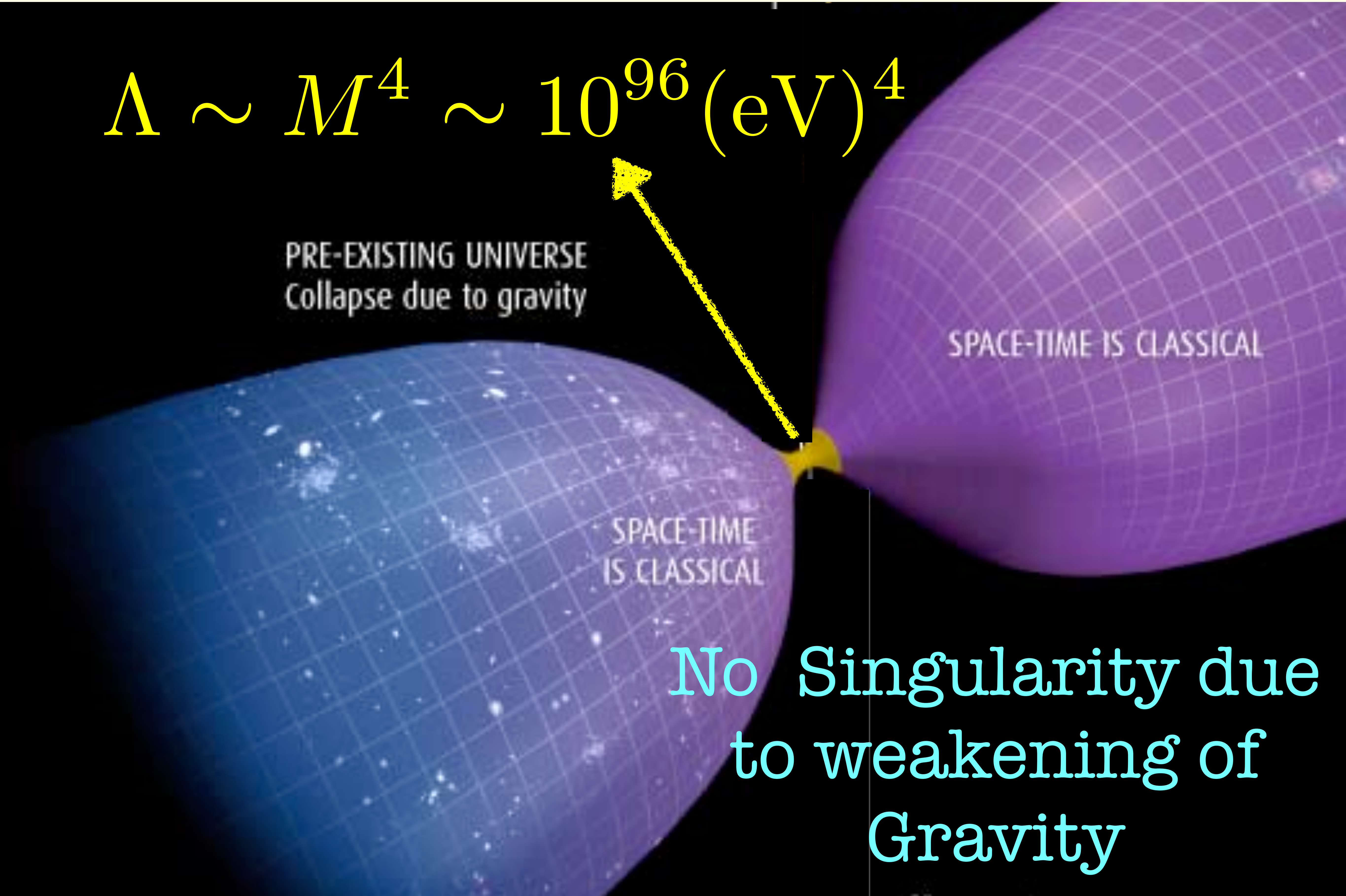
$$S \sim \int \sqrt{g} d^4x \left[R + \alpha_1 R^2 + \alpha_2 R^{\mu\nu} R_{\mu\nu} + \alpha_3 R^{\mu\nu\lambda\sigma} R_{\mu\nu\lambda\sigma} \right]$$

Higgs is Lost in the Myriad of Gravitational Terms !!!!

SM Higgs or 750 GeV Scalar at best plays a role of a Curvaton, but not as an Inflaton

Big Bounce & Cosmological Constant

$$\Lambda \sim M^4 \sim 10^{96}(\mathrm{eV})^4$$



No Singularity due to weakening of Gravity

Gravitational Entropy for (A)dS

$$S = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left[R - 2\Lambda + \alpha \left(R\mathcal{F}_1 R + R_{\mu\nu}\mathcal{F}_2 R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_3 R^{\mu\nu\lambda\sigma} \right) \right]$$

$$\Lambda = \pm \frac{(D-1)(D-2)}{2\ell^2}$$

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

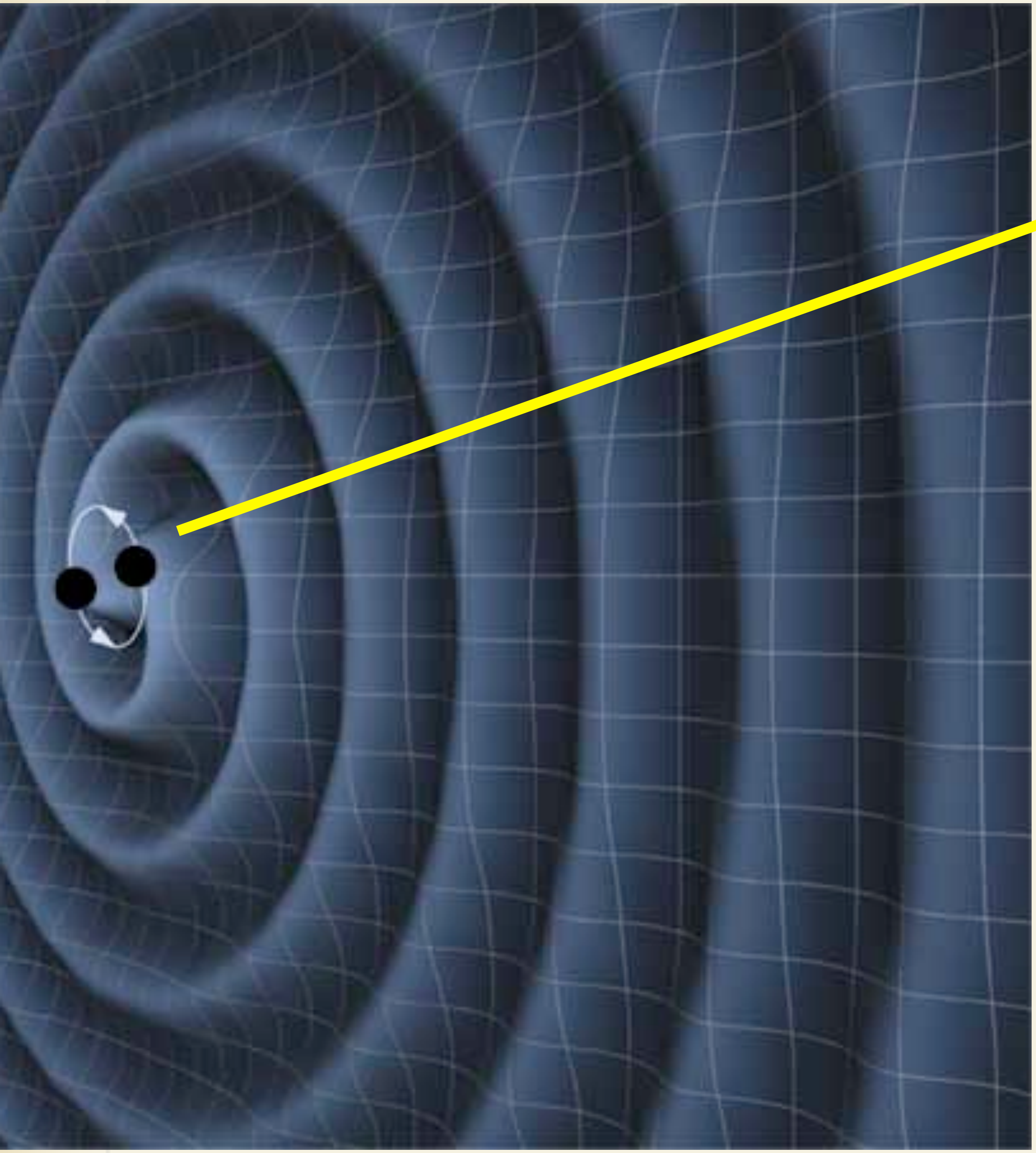
$$f(r) = \left(1 \mp \frac{r}{\ell^2} \right)$$

$$S_W^{(A)dS} = \frac{A_H^{(A)dS}}{4G_D} \left(1 \pm \frac{2\alpha}{\ell^2} \left(f_{1_0} D(D-1) + f_{2_0}(D-1) + 2 f_{3_0} \right) \right)$$

For $+$ α, dS entropy can be 0

This has important consequences for a non-singular cosmology

Gravitational Waves



$$\bar{h}_{jk} \approx G\frac{\omega^2(ML^2)}{r}$$

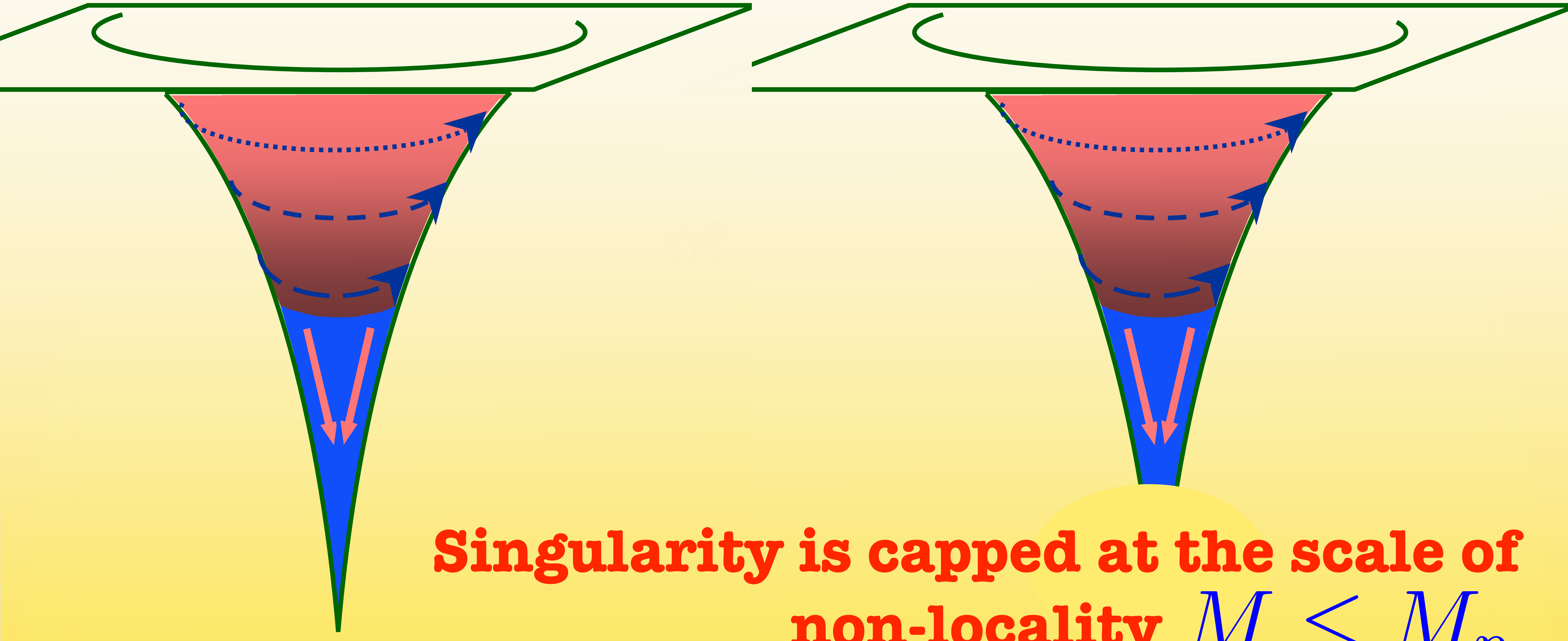
Large r
limit

$$\bar{h}_{jk} \approx G\frac{\omega^2(ML^2)}{r}\mathrm{erf}\left(\frac{rM_P}{2}\right)$$

$r \Longrightarrow 0,$ **No Singularity**

Biswas, Gerwick, Koivisto, AM,
Phys. Rev. Lett. (gr-qc/1110.5249)

Where would you expect the modifications?



Singularity is capped at the scale of non-locality $M \leq M_p$

Remnants of stringy Gravity

M_p

m_W

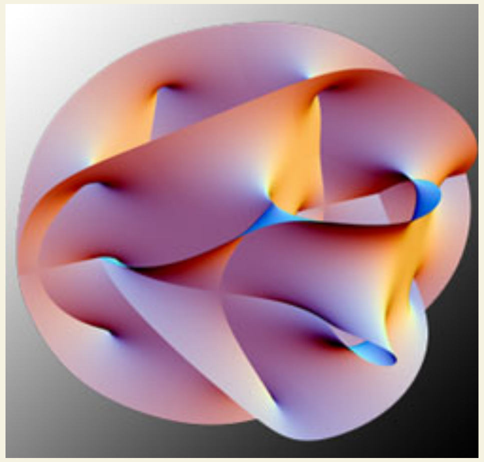
m_s

m_{KK}

$$\mathcal{L}^{10\text{d}} \sim R + R^4 + \cdots \qquad \kappa^2 = g_s^2(\alpha')^4$$

Perturbative string theory has α' & g_{s} corrections

For all orders : String field theory



$$\mathcal{L}^{4\text{d}} \sim R + \sum_i c_i R \left(\frac{\Box}{m_{kk}}\right)^i R + \cdots$$

$1-$ loop in g_{s} all orders in α'

Witten (1998) , Tseytlin (1995), Zwiebach (2000), Sigel (1998, 2003), ...

Loop quantum gravity

or

Dynamical Triangulation approach

↓

Wilson loops

↓

Non-local objects

It would be interesting to establish the connection