Vacua of an S_3-symmetric scalar potential

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Consider 3 SU(2) doublets

- 3 fermion families, 3 scalar doublets?
- Perhaps “natural” dark matter?
- Spontaneous CP violation?
- Impose S_3 discrete symmetry
- Rich phenomenology

Arguments for S_3 symmetry

- General potential has 46 parameters
- Most general S_3 symmetric potential has 10
- More predictive!
- Symmetries help to control FCNC
- Symmetry may help stabilise Dark Matter

Early history

- Pakvasa and Sugavara, 1978
- Derman, 1979
- Kubo, Okada, Sakamaki, 2004
- Das and Dey, 2014
- + many others

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S_3 decomposition

S_3 can be decomposed

→ a singlet and a doublet (with respect to S_3)

→ a pseudosinglet and a doublet (with respect to S_3)

These two choices are very similar

Two "Frameworks"

May work with the

- reducible representation (Derman) or the
- irreducible representations (Pakvasa & Sugawara, Das & Dey)

There is a linear map from one framework to the other

Reducible representation

$$\phi_1, \quad \phi_2, \quad \phi_3$$

$$\phi_i = \begin{pmatrix} \varphi_i^+ \\ (\rho_i + \eta_i + i\chi_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, 3$$

$$V = V_2 + V_4$$

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \tfrac{1}{2}\gamma \sum_{i<j} [\phi_i^\dagger \phi_j + \mathrm{h.c.}],$$

$$\begin{aligned} V_4 = {} & A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i<j} \{C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \overline{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \tfrac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \mathrm{h.c.}]\} \\ & + \tfrac{1}{2} E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \mathrm{h.c.}] + \sum_{i \neq j \neq k \neq i, j<k} \{\tfrac{1}{2} E_2 [(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \mathrm{h.c.}] \\ & + \tfrac{1}{2} E_3 [(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \mathrm{h.c.}] + \tfrac{1}{2} E_4 [(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \mathrm{h.c.}]\} \end{aligned}$$

10 parameters

Irreducible representations

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 - \phi_2) \\ \frac{1}{\sqrt{6}}(\phi_1 + \phi_2 - 2\phi_3) \end{pmatrix} \qquad h_S = \frac{1}{\sqrt{3}}(\phi_1 + \phi_2 + \phi_3)$$

$$h_i = \begin{pmatrix} h_i^+ \\ (w_i + \tilde{\eta}_i + i\tilde{\chi}_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, \quad h_S = \begin{pmatrix} h_S^+ \\ (w_S + \tilde{\eta}_S + i\tilde{\chi}_S)/\sqrt{2} \end{pmatrix}$$

$$V_2 = \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2)$$

$$\begin{aligned} V_4 = {} & \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ & + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) \\ & + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\ & + \lambda_8 (h_S^\dagger h_S)^2 \end{aligned}$$

10 parameters

Note that irreducible representation chooses a particular “direction” among

$$\phi_1, \quad \phi_2, \quad \phi_3$$

Not unique — convention

This potential exhibits

$h_1 \to -h_1$ symmetry

but not $h_2 \to -h_2$

Equivalent doublet representation

$$\begin{pmatrix} \tilde{\chi}_1 \\ \tilde{\chi}_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

the above symmetry becomes

$$\tilde{\chi}_1 \leftrightarrow \tilde{\chi}_2$$

In the irreducible-rep framework

the case $\lambda_4 = 0$ SPECIAL

or, in the reducible-rep framework

$$4A - 2(C + \overline{C} + D) - E_1 + E_2 + E_3 + E_4 = 0$$

leads to a continuous SO(2) symmetry

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Massless states!

At this stage, the two frameworks are equivalent

However, introducing Yukawa couplings, for example, in terms of

$$\phi_1, \quad \phi_2, \quad \phi_3$$

or

$$h_1, \quad h_2, \quad h_S$$

they would naturally be different

The vevs are related

$$w_1 = \frac{1}{\sqrt{2}}(\rho_1 - \rho_2)$$

$$w_2 = \frac{1}{\sqrt{6}}(\rho_1 + \rho_2 - 2\rho_3)$$

$$w_S = \frac{1}{\sqrt{3}}(\rho_1 + \rho_2 + \rho_3)$$

$$\rho_1 = \frac{1}{\sqrt{3}}w_S + \frac{1}{\sqrt{2}}w_1 + \frac{1}{\sqrt{6}}w_2$$

$$\rho_2 = \frac{1}{\sqrt{3}}w_S - \frac{1}{\sqrt{2}}w_1 + \frac{1}{\sqrt{6}}w_2$$

$$\rho_3 = \frac{1}{\sqrt{3}}w_S - \frac{\sqrt{2}}{\sqrt{3}}w_2$$

Summary of representations

2 "frameworks"

Reducible representation (Derman):

$\phi_1, \ \phi_2, \ \phi_3$ $\rho_1, \ \rho_2, \ \rho_3$

Irreducible representation (Pakvasa & Sugawara, Das & Dey):

$h_1, \ h_2, \ h_S$ $w_1, \ w_2, \ w_S$

Vacua—a classification

Derivatives of potential wrt (complex) fields must vanish

Three complex derivatives = 0 or

Five real derivatives (3 moduli, 2 relative phases) = 0

The minimisation conditions must be consistent.
This is an important constraint on the potential.

May work in either framework

But a particular vacuum may look simpler in one framework than in the other.

Vacua—a classification

Derivatives of potential wrt (complex) fields must vanish

Three complex derivatives = 0 or

Five real derivatives (3 moduli, 2 relative phases) = 0

Note: Alternative classification given by Ivanov and Nishi, 1410.6139, JHEP

Symmetries of 3HDM vacua

Our approach

The 5 minimisation equations give 5 constraints on 10 potential parameters — for a given vacuum configuration

$$(w_1, w_2, w_S) \equiv (\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S)$$

↑ real (convention)

Irreducible framework.

Are the 5 equations independent?

Are they consistent?

5 equations

$$\left(\frac{\partial V}{\partial \hat{w}_1}\right) = \mu_1^2 \hat{w}_1 + \lambda_1 \hat{w}_1 (\hat{w}_1^2 + \hat{w}_2^2) + \lambda_2 \hat{w}_1 \hat{w}_2^2 [\cos(2\sigma_1 - 2\sigma_2) - 1] + \lambda_3 \hat{w}_1 [\hat{w}_1^2 + \hat{w}_2^2 \cos(2\sigma_1 - 2\sigma_2)]$$
$$+ \lambda_4 \hat{w}_1 \hat{w}_2 \hat{w}_S [\cos(2\sigma_1 - \sigma_2) + 2\cos\sigma_2] + \frac{1}{2}(\lambda_5 + \lambda_6)\hat{w}_1 \hat{w}_S^2 + \lambda_7 \hat{w}_1 \hat{w}_S^2 \cos 2\sigma_1,$$
$$\left(\frac{\partial V}{\partial \hat{w}_2}\right) = \mu_1^2 \hat{w}_2 + \lambda_1 \hat{w}_2 (\hat{w}_1^2 + \hat{w}_2^2) + \lambda_2 \hat{w}_1^2 \hat{w}_2 [\cos(2\sigma_1 - 2\sigma_2) - 1] + \lambda_3 \hat{w}_2 [\hat{w}_1^2 \cos(2\sigma_1 - 2\sigma_2) + \hat{w}_2^2]$$
$$+ \frac{\lambda_4}{2} \hat{w}_S [\hat{w}_1^2 \cos(2\sigma_1 - \sigma_2) + (2\hat{w}_1^2 - 3\hat{w}_2^2)\cos\sigma_2] + \frac{1}{2}(\lambda_5 + \lambda_6)\hat{w}_2 \hat{w}_S^2 + \lambda_7 \hat{w}_2 \hat{w}_S^2 \cos 2\sigma_2,$$
$$\left(\frac{\partial V}{\partial \hat{w}_S}\right) = \mu_0^2 \hat{w}_S + \frac{\lambda_4}{2} \hat{w}_2 [\hat{w}_1^2 \cos(2\sigma_1 - \sigma_2) + (2\hat{w}_1^2 - \hat{w}_2^2)\cos\sigma_2] + \frac{1}{2}(\lambda_5 + \lambda_6)(\hat{w}_1^2 + \hat{w}_2^2)\hat{w}_S$$
$$+ \lambda_7 \hat{w}_S [\hat{w}_1^2 \cos 2\sigma_1 + \hat{w}_2^2 \cos 2\sigma_2] + \lambda_8 \hat{w}_S^3,$$
$$\left(\frac{\partial V}{\partial \sigma_1}\right) = -(\lambda_2 + \lambda_3)\hat{w}_1^2 \hat{w}_2^2 \sin(2\sigma_1 - 2\sigma_2) - \lambda_4 \hat{w}_1^2 \hat{w}_2 \hat{w}_S \sin(2\sigma_1 - \sigma_2) - \lambda_7 \hat{w}_1^2 \hat{w}_S^2 \sin 2\sigma_1,$$
$$\left(\frac{\partial V}{\partial \sigma_2}\right) = (\lambda_2 + \lambda_3)\hat{w}_1^2 \hat{w}_2^2 \sin(2\sigma_1 - 2\sigma_2) + \frac{\lambda_4}{2} \hat{w}_2 \hat{w}_S [\hat{w}_1^2 \sin(2\sigma_1 - \sigma_2) - (2\hat{w}_1^2 - \hat{w}_2^2)\sin\sigma_2]$$
$$- \lambda_7 \hat{w}_2^2 \hat{w}_S^2 \sin 2\sigma_2.$$

These derivatives do not depend on λ_5 and λ_6 separately, only on the sum, $\lambda_5 + \lambda_6$. Likewise, no dependence on λ_1, λ_2 and λ_3 separately, only on two combinations orthogonal to $\lambda_1 + \lambda_2 - 2\lambda_3 = 0$.

8 relevant parameters

5 equations

$$
\begin{aligned}
a_{11}P_1 + a_{12}P_2 + a_{13}P_3 + a_{14}P_4 + a_{15}P_5 &= b_1 \\
a_{21}P_1 + a_{22}P_2 + a_{23}P_3 + a_{24}P_4 + a_{25}P_5 &= b_2 \\
a_{31}P_1 + a_{32}P_2 + a_{33}P_3 + a_{34}P_4 + a_{35}P_5 &= b_3 \\
a_{41}P_1 + a_{42}P_2 + a_{43}P_3 + a_{44}P_4 + a_{45}P_5 &= b_4 \\
a_{51}P_1 + a_{52}P_2 + a_{53}P_3 + a_{54}P_4 + a_{55}P_5 &= b_5
\end{aligned}
$$

The P_i denote different parameters of the potential.

These five equations define five hyperplanes in the parameter space.

Are the 5 equations independent? Study determinant!

Not all of the possible $\binom{8}{5} = 56$ combinations will lead to five independent equations.

Are they consistent?

11 real vacua

constraints

reducible irreducible

Vacuum	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	Comment
R-0	$0, 0, 0$	$0, 0, 0$	Not interesting
R-I-1	x, x, x	$0, 0, w_S$	$\mu_0^2 = -\lambda_8 w_S^2$
R-I-2a	$x, -x, 0$	$w, 0, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_1^2$
R-I-2b	$x, 0, -x$	$w, \sqrt{3}w, 0$	$\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$
R-I-2c	$0, x, -x$	$w, -\sqrt{3}w, 0$	$\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$
R-II-1a	x, x, y	$0, w, w_S$	$\mu_0^2 = \frac{1}{2}\lambda_4 \frac{w_2^3}{w_S} - \frac{1}{2}\lambda_a w_2^2 - \lambda_8 w_S^2$, $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2 + \frac{3}{2}\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1b	x, y, x	$w, -w/\sqrt{3}, w_S$	$\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2$, $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1c	y, x, x	$w, w/\sqrt{3}, w_S$	$\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2$, $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-2	$x, x, -2x$	$0, w, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2,\ \lambda_4 = 0$
R-II-3	$x, y, -x-y$	$w_1, w_2, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right), \lambda_4 = 0$
R-III	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	$\mu_0^2 = -\frac{1}{2}\lambda_a(w_1^2 + w_2^2) - \lambda_8 w_S^2$, $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right) - \frac{1}{2}\lambda_a w_S^2$, $\lambda_4 = 0$

16 complex vacua

	IRF (Irreducible Rep.)	RRF (Reducible Rep.)
	w_1, w_2, w_S	ρ_1, ρ_2, ρ_3
C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm\frac{2\pi i}{3}}, xe^{\mp\frac{2\pi i}{3}}$
C-III-a	$0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$y, y, xe^{i\tau}$
C-III-b	$\pm i\hat{w}_1, 0, \hat{w}_S$	$x+iy, x-iy, x$
C-III-c	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$	$xe^{i\rho}-\frac{y}{2}, -xe^{i\rho}-\frac{y}{2}, y$
C-III-d,e	$\pm i\hat{w}_1, \epsilon\hat{w}_2, \hat{w}_S$	$xe^{i\tau}, xe^{-i\tau}, y$
C-III-f	$\pm i\hat{w}_1, i\hat{w}_2, \hat{w}_S$	$re^{i\rho}\pm ix, re^{i\rho}\mp ix, \frac{3}{2}re^{-i\rho}-\frac{1}{2}re^{i\rho}$
C-III-g	$\pm i\hat{w}_1, -i\hat{w}_2, \hat{w}_S$	$re^{-i\rho}\pm ix, re^{-i\rho}\mp ix, \frac{3}{2}re^{i\rho}-\frac{1}{2}re^{-i\rho}$
C-III-h	$\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau}, y, y$ $y, xe^{i\tau}, y$
C-III-i	$\sqrt{\frac{3(1+\tan^2\sigma_1)}{1+9\tan^2\sigma_1}}\hat{w}_2 e^{i\sigma_1},$ $\pm\hat{w}_2 e^{-i\arctan(3\tan\sigma_1)}, \hat{w}_S$	$x, ye^{i\tau}, ye^{-i\tau}$ $ye^{i\tau}, x, ye^{-i\tau}$

Notation: C-III-c

C → Complex

III → 3 independent constraints

16 complex vacua

C-IV-a*	$\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$	$re^{i\rho}+x, -re^{i\rho}+x, x$
C-IV-b	$\hat{w}_1, \pm i\hat{w}_2, \hat{w}_S$	$re^{i\rho}+x, -re^{-i\rho}+x, -re^{i\rho}+re^{-i\rho}+x$
C-IV-c	$\sqrt{1+2\cos^2\sigma_2}\hat{w}_2,$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho}+r\sqrt{3(1+2\cos^2\rho)}+x,$ $re^{i\rho}-r\sqrt{3(1+2\cos^2\rho)}+x, -2re^{i\rho}+x$
C-IV-d*	$\hat{w}_1 e^{i\sigma_1}, \pm\hat{w}_2 e^{i\sigma_1}, \hat{w}_S$	$r_1 e^{i\rho}+x, (r_2-r_1)e^{i\rho}+x, -r_2 e^{i\rho}+x$
C-IV-e	$\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}}\hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2}+re^{i\rho_1}\xi+x, re^{i\rho_2}-re^{i\rho_1}\xi+x,$ $-2re^{i\rho_2}+x$
C-IV-f	$\sqrt{2+\frac{\cos(\sigma_1-2\sigma_2)}{\cos\sigma_1}}\hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_1}+re^{i\rho_2}\psi+x,$ $re^{i\rho_1}-re^{i\rho_2}\psi+x, -2re^{i\rho_1}+x$
C-V*	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau_1}, ye^{i\tau_2}, z$

* C-IV-a, C-IV-d, C-V:
When constraints are imposed,
the vacuum turns out to be real!

16 complex vacua

Vacuum	Constraints
C-I-a	$\mu_1^2 = -2\left(\lambda_1 - \lambda_2\right)\hat{w}_1^2$
C-III-a	$\mu_0^2 = -\frac{1}{2}\lambda_b \hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2\lambda_7\right)\hat{w}_S^2,$ $\lambda_4 = \frac{4\cos\sigma_2 \hat{w}_S}{\hat{w}_2}\lambda_7$
C-III-b	$\mu_0^2 = -\frac{1}{2}\lambda_b \hat{w}_1^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\lambda_b \hat{w}_S^2,$ $\lambda_4 = 0$
C-III-c	$\mu_1^2 = -(\lambda_1 + \lambda_3)(\hat{w}_1^2 + \hat{w}_2^2),$ $\lambda_2 + \lambda_3 = 0, \lambda_4 = 0$
C-III-d,e	$\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{(\hat{w}_1^2 - \hat{w}_2^2)^2}{\hat{w}_S^2} - \epsilon\lambda_4 \frac{(\hat{w}_1^2 - \hat{w}_2^2)(\hat{w}_1^2 - 3\hat{w}_2^2)}{4\hat{w}_2\hat{w}_S}$ $-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \epsilon\lambda_4 \frac{\hat{w}_S(\hat{w}_1^2 - \hat{w}_2^2)}{4\hat{w}_2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_7 = \frac{\hat{w}_1^2 - \hat{w}_2^2}{\hat{w}_S^2}(\lambda_2 + \lambda_3) - \epsilon\frac{(\hat{w}_1^2 - 5\hat{w}_2^2)}{4\hat{w}_2\hat{w}_S}\lambda_4$
C-III-f,g	$\mu_0^2 = -\frac{1}{2}\lambda_b\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\lambda_b \hat{w}_S^2, \lambda_4 = 0$
C-III-h	$\mu_0^2 = -2\lambda_b \hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$

etc

Philosophy

Note that we do not consider the potential parameters “God given”, but rather specify the desired form of the vacuum (“designer vacuum”) and then ask:

Which choice of potential parameters can produce this vacuum?

Of special interest:

- Complex vacuum (Spontaneous CP violation?)
- Vacuum with zero vevs (DM candidate)

Some complex vacua are related to a real vacuum, as a "generalization" (but note more constraints)

Complex	Real "origin"
C-I-a	none
C-III-a	R-II-1a
C-III-b	none
C-III-c	R-I-2a,2b,2c, R-II-3
C-III-d,e	none
C-III-f	none
C-III-g	none
C-III-h	R-II-1b,1c
C-III-i	R-II-1b,1c
C-IV-a*	R-III
C-IV-b	none
C-IV-c	R-II-1b,1c
C-IV-d*	R-III
C-IV-e	none
C-IV-f	R-II-1b,1c
C-V*	R-III

- Complex vevs are no guarantee for SCPV
- The symmetry of the Lagrangian could "hide" the complex conjugation

Example: C-I-a $(\rho_1, \rho_2, \rho_3) = x(1, e^{2i\pi/3}, e^{-2i\pi/3})$

Complex conjugation:

$$x(1, e^{2i\pi/3}, e^{-2i\pi/3}) \Rightarrow x(1, e^{-2i\pi/3}, e^{2i\pi/3})$$

But the Lagrangian has a symmetry:

$$\phi_2 \leftrightarrow \phi_3 \quad \text{and} \quad \rho_2 \leftrightarrow \rho_3$$

which will undo the complex conjugation

Two special complex vacua

Pakvasa & Sugawara (1978)

$$(w_1, w_2, w_S) \equiv (\hat{w}e^{i\sigma}, \hat{w}e^{-i\sigma}, \hat{w}_S)$$

Ivanov & Nishi (2014)

$$(w_1, w_2, w_S) \equiv (\hat{w}e^{i\sigma}, \hat{w}e^{i\sigma}, \hat{w}_S)$$

Neither violates CP

Both these vacua require $\lambda_4 = 0$

PS vacuum, for example

$$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{-i\sigma}, \hat{w}_S) \xrightarrow{\text{c. c.}} (\hat{w}e^{-i\sigma}, \hat{w}e^{i\sigma}, \hat{w}_S)$$

When $\lambda_4 = 0$ have symmetry

$$h_1 \leftrightarrow h_2$$

Several complex vacua represent spontaneous CP violation

Vacua with $\lambda_4 = 0$ conserve CP

(massless states, must break S_3)

Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV
C-I-a	X	no	C-III-f,g	0	no	C-IV-c	X	yes
C-III-a	X	yes	C-III-h	X	yes	C-IV-d	0	no
C-III-b	0	no	C-III-i	X	no	C-IV-e	0	no
C-III-c	0	no	C-IV-a	0	no	C-IV-f	X	yes
C-III-d,e	X	no	C-IV-b	0	no	C-V	0	no

Several complex vacua represent CP conservation

Some of these require $\lambda_4 = 0$

(massless states, must break S_3)

	Irred rep	Reducible rep
C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm\frac{2\pi i}{3}}, xe^{\mp\frac{2\pi i}{3}}$

$h_2 \leftrightarrow -h_2$ $\phi_2 \leftrightarrow \phi_3$

since $\lambda_4 = 0$

Several complex vacua represent CP conservation

	Irred rep	Reducible rep
C-III-b	$\pm i\hat{w}_1, 0, \hat{w}_S$	$x+iy, x-iy, x$

$$h_1 \leftrightarrow -h_1 \qquad \phi_2 \leftrightarrow \phi_3$$

Several complex vacua represent CP conservation

	Irred rep	Reducible rep
C-III-c	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$	$xe^{i\rho} - \frac{y}{2}, -xe^{i\rho} - \frac{y}{2}, y$

No CP violation

No obvious symmetry to explain it

Transform to the Higgs basis: $(w_1, w_2, w_S) = (v, 0, 0)$ real!

"Magic": potential can be made real by rotation of phases of fields that have no vev

Formal argument

CP is conserved if one can find a transformation U such that

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

which is also a symmetry of the Lagrangian

Branco, Gerard, Grimus, 1984

Several complex vacua represent CP conservation

	Irred rep	Reducible rep
C-IV-e	$\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2} + re^{i\rho_1}\xi + x, re^{i\rho_2} - re^{i\rho_1}\xi + x$, $-2re^{i\rho_2} + x$

More complicated to show CP conservation

One approach:
Transform to the Higgs basis, potential can be made real

Several complex vacua represent CP conservation

Less obvious explanation:

With $\lambda_4 = 0$ there is an SO(2) symmetry within $h_1, \; h_2$

Exploit this to transform such that vevs get same modulus

Invoke relation between moduli of vevs of doublet,
get equal and opposite phases:

$$(\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, w_S) \rightarrow (a e^{i\gamma}, a e^{-i\gamma}, w_S)$$

As a result $\quad U_{ij} \langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$

is satisfied. No CP violation!