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Beyond the Standard Model

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I. THE STANDARD MODEL

To date the LHC has not revealed any new states beyond the Standard Model (BSM) even though the searches are sensitive to new states of mass of $O(1TeV)$ with production cross sections comparable to that of the Standard Model states. A wide variety of possible new physics has been probed as may be seen from Figs 1 and 2 [1]

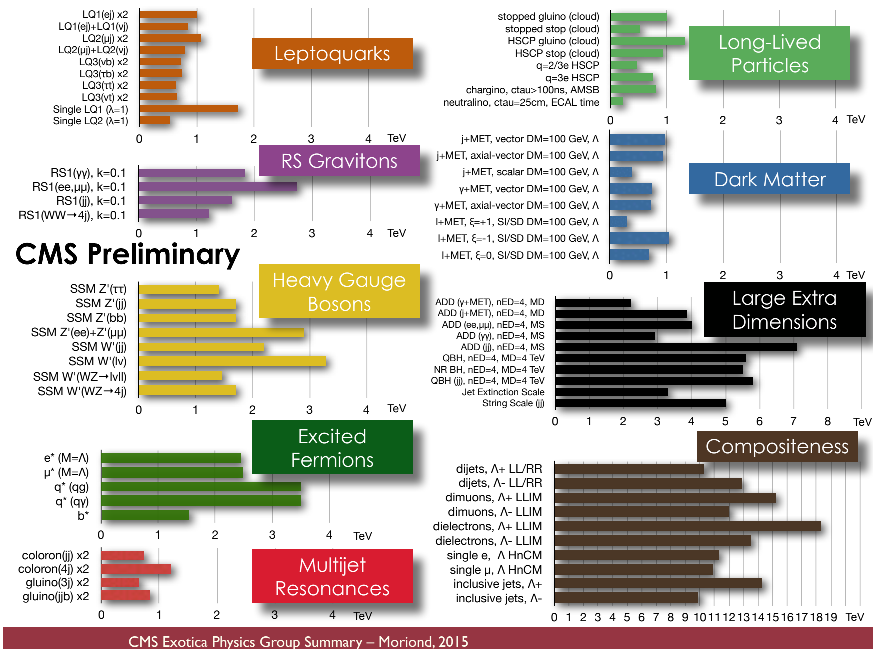

FIG. 1: *Summary of CMS limits on Exotica Beyond the Standard Model [23]*

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[1] Of course the CMS and ATLAS experiments both put limits on BSM states - here I have chosen to show only a subset of their results.

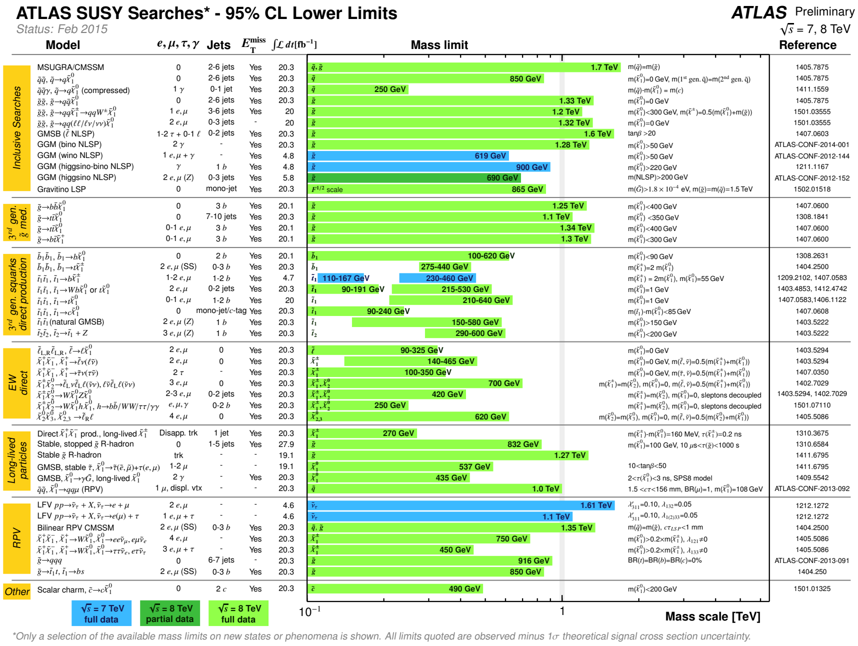

FIG. 2: *Mass reach of ATLAS searches for Supersymmetry [12].*

In addition the properties of the newly discovered Higgs fits very well[2] with those predicted by the Standard Model as it shown in the "pull-plot" of Figure 3a and the fit to the SM CKM matrix shown in Figure 3b. In particular the coupling of the Higgs to the states of the Standard Model is closely proportional to the mass as may be seen in Figure 3c.

The Higgs completes the Standard Model spectrum and its mass and now allows for a determination of the parameters of the Higgs potential

$$V(H) = -m^2|H|^2 + \lambda|H|^4 \tag{1}$$

giving $m \sim 89GeV$ and $\lambda \sim 0.13$ with the quartic coupling in the perturbative domain.

The absence of any sign of BSM physics suggests the possibility that the SM is complete up to high scales. Assuming this is the case it is instructive to determine the renormalisation group running of the quartic coupling to high energy scales [16] and this is shown in Figure 4a. It may be seen that the result is very sensitive to the top quark mass and, to a lesser extent, to the strong coupling constant. For their central values the coupling goes negative at a scale of $10^{10}GeV$ corresponding, in the absence of and BSM physics, to a potential unbounded from below or, if one cuts of at the Planck scale energy where gravity effects cannot be neglected, at least to the appearance of a second much deeper minimum at $O(10^{18}GeV)$ than the usual electroweak (EW) breaking minimum at $O(10^2GeV)$.

[2] There is a long-standing 3σ discrepancy with the measurement of $g-2$ of the muon [15]. This has an underlying theoretical uncertainty due to the difficulty in calculating light by light scattering and for the moment the discrepancy is not critical but could be a first indication of new physics.

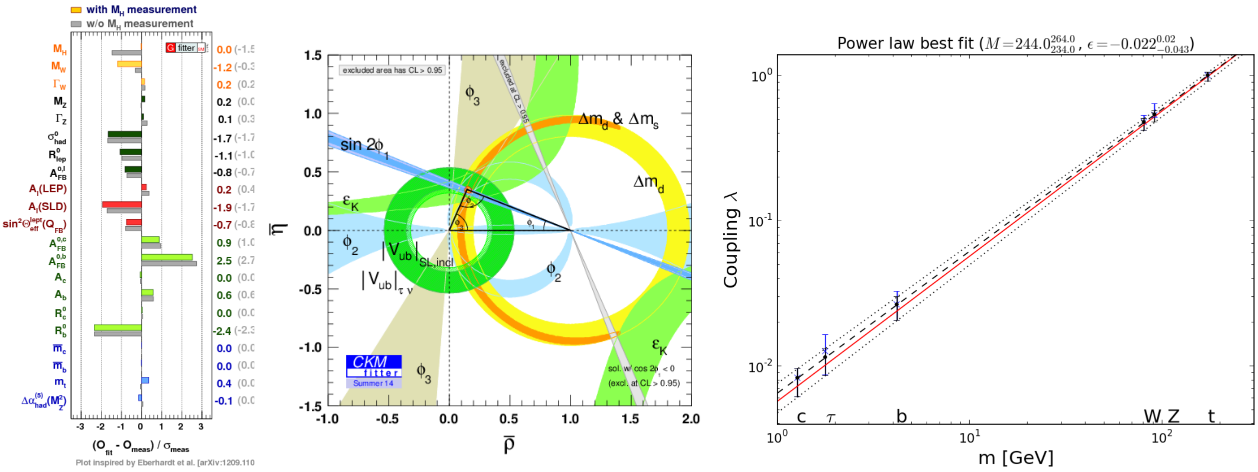

FIG. 3: *a)The pull plot showing the goodness of the SM fit to data [13] b)The fit to the SM CKM matrix [22] c)Mass dependence of the Higgs coupling [33].*

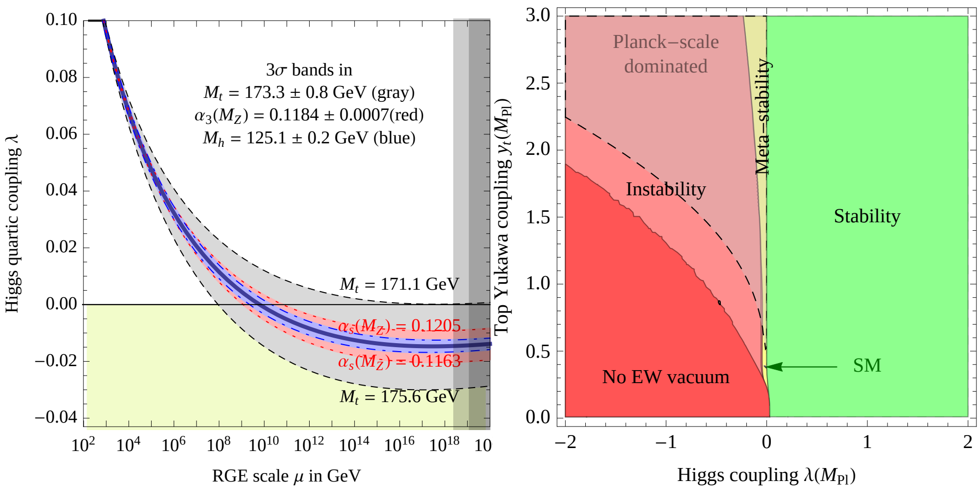

FIG. 4: *a)The RGE running of the quartic Higgs coupling in the SM varying m_t, $\alpha_s(M_Z)$ by $\pm 3\sigma$. b)SM phase diagram in terms of quartic Higgs coupling λ and top Yukawa coupling y_t renormalised at the Planck scale. The region where the instability scale Λ_I is larger than 10^{18} GeV is indicated as 'Planck-scale dominated'.*

However this does not necessarily imply BSM physics below $10^{10} GeV$ because it may be that the lifetime of the normal EW vacuum state is greater than the present lifetime of the universe so that if initially the universe is in this state it will not yet have tunnelled out [16]. This possibility is shown in Figure 4b. It may be seen that even with the quartic coupling negative at the Planck scale the SM is in the acceptable meta-stability region. On the other hand if the top quark mass is lower by 3σ than its central value the Higgs quartic coupling remains positive up to the Planck scale and the SM is in the stability region. Could there be a reason that physics at the Planck scale requires the quartic coupling to vanish at the Planck scale?

In summary all measurements to date support the validity of the Standard Model and no evidence for BSM physics has yet been found [3]. Given this is there any reason to expect new physics BSM?

II. WHY GO BEYOND THE STANDARD MODEL?

While the SM has been phenomenally successful as a theory of the strong, weak and electromagnetic interactions, it leaves many questions unanswered:

- Why the complicated multiplet structure of quarks and leptons?
- Why are quarks fractionally charged relative to the charged leptons?
- What is the origin of neutrino masses?
- Why are there so many parameters (28 including neutrino masses and mixings)?
- Why is there only partial unification of the forces?
- What solves the strong CP problem in the SM?
- Why is the electroweak breaking scale so much smaller than the Planck scale (the hierarchy problem)?
- What is the origin of dark matter, dark energy, baryogenesis, inflation, the cosmological constant?

These unanswered questions strongly suggest there must be physics BSM. The most immediate questions relate to the complicated multiplet structure and the associated profusion of parameters that suggests there may be further unification of the forces and matter.

The strong weak and electromagnetic forces are associated with the gauge bosons of the local $SU(3)\times SU(2)\times U(1)$ gauge symmetry yet only the neutral weak and electromagnetic interactions are directly related in the SM with the photon field given by the mixture of the SU(2) and U(1) gauge bosons:

$$A^{\gamma}_{\mu} = sin\theta_W W^3_{\mu} + cos\theta_W B_{\mu} \tag{2}$$

The matter multiplets, *c.f.* Figure 5, similarly show indications of an underlying unification with quarks and leptons both having their left-handed states in electroweak doublets while the right-handed partners are singlets corresponding to a violation of parity and charge conjugation. However there is no reason why C and P should be violated, why the quarks carry colour while the leptons do not or why there should be three families both in the quark and the lepton sectors. To address these questions we turn to the possibility of further unification called Grand Unification.

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[3] The observation of neutrino oscillations implies non-zero neutrino mass and a departure from the original SM. However, if one adds right-handed neutrinos to restore the symmetry in the spectrum between quarks and leptons, Dirac neutrino masses arise in the same way as for charged leptons and quarks. Here I refer to this natural extension of the original model as *the* Standard Model.

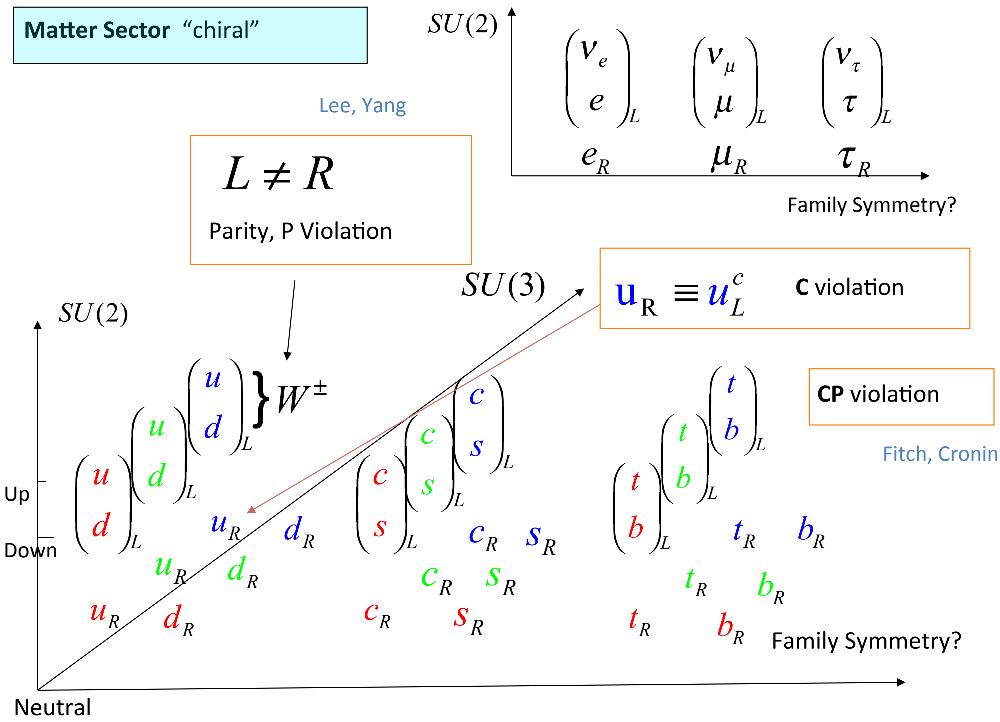

FIG. 5: *The multiplet structure of the SM matter and Higgs states.*

III. GRAND UNIFICATION

The first Grand Unified Theory (GUT) [4] to be studied was $SU(5)$ [41], the group of 5×5 complex unitary matrices with determinant 1. It has the minimum possible rank 4 with just 4 diagonal generators and contains the SM as a subgroup

$$SU(5) \supset SU(3) \times SU(2) \times U(1). \tag{3}$$

There are 24 independent matrices, U_i, making up the adjoint representation of $SU(5)$ and these can be expressed in terms of 24 hermitian matrices, the generators L_i

$$U = e^{-i\Sigma_{i=1}^{24}\beta_i L_i},\ U^\dagger U = 1 \Rightarrow L_i \ hermitian \tag{4}$$

If $SU(5)$ is a local gauge symmetry there are 24 gauge bosons, $V_\mu^{a=1..24}$ which couple to matter via the covariant derivative. Defining $V_\mu = \frac{1}{\sqrt{2}}\sum_{a=1}^{24} V_\mu^a L^a$ the action of the covariant derivative on a 5 dimensional representation of fermions is given by

$$(D_\mu \psi^5)_i = [\delta_i^j \partial_\mu - \frac{ig}{\sqrt{2}} V_{\mu,i}^j]\psi_5^j \tag{5}$$

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[4] For general reviews of GUTs see[59, 67]

where

$$V_\mu = \begin{bmatrix} G_1^1 - \frac{2B}{\sqrt{30}} & G_2^1 & G_2^1 & \overline{X}_1 & \overline{Y}_1 \\ G_1^2 & G_2^2 - \frac{2B}{\sqrt{30}} & G_3^2 & \overline{X}_2 & \overline{Y}_2 \\ G_1^3 & G_2^3 & G_3^3 - \frac{2B}{\sqrt{30}} & \overline{X}_3 & \overline{Y}_3 \\ X_1 & X_2 & X_3 & \frac{W_\mu^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{W_\mu^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} \end{bmatrix}_\mu , \tag{6}$$

Here $G^i_{\mu,j}, W^i_{\mu,j}, B_\mu$ are the $SU(3) \times SU(2) \times U(1)$ gauge boson fields; $SU(3)$ ($SU(2)$) has been embedded in the first 3 (last 2) rows and columns of V_μ. The 12 new gauge bosons in $SU(5)$ but not in the SM are $X_i, \bar{X}_i, Y_i, \bar{Y}_i$.

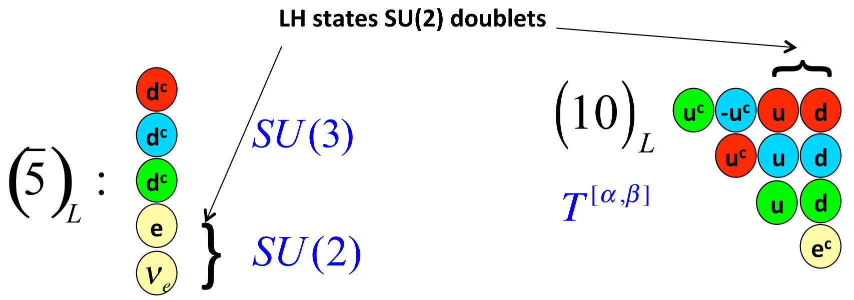

FIG. 6: *The SU(5) multiplet structure of the first SM family.*

The fermions of a single SM family fits nicely into a $\bar{5} + 10$ representation of $SU(5)$ as shown in Figure 6 where all states are left-handed (i.e. the right-handed (RH) quark and lepton states are rewritten as left-handed (LH) antiquark and anti lepton states). As one may see the $\bar{5}$ representation transforms as $(\bar{3}, 1) + (1, 2)$ under $SU(3) \times SU(2)$ and so necessarily requires that the right-handed quark states assigned to this representation must be singlets under $SU(2)$. The identification of these states follows from the observation that the charge operator corresponds to one of the SU(5) generators and must be traceless. This means that $3Q_{d^c} + Q_3 = 0$, i.e. $Q_d = \frac{1}{3}Q_e$, confirming that the coloured states in the $\bar{5}$ must be down antiquarks. Moreover the charge on the down quark is a third of that on the electron because there are 3 colours! The remaining 10 states of the SM fit into the 10 dimensional representation of $SU(5)$ as shown in Figure 6. We see that again the postdiction is that the RH states are $SU(2)$ singlets while the LH quark states are singlets.

The second GUT explored was $SO(10)$ [38, 40] which is a rank five group and contains $SU(5)$ as a subgroup, $SO(10) \supset SU(5) \times U(1)$. An alternative subgroup is the Pati-Salam group [62], $SO(10) \supset SU(4) \times SU(2)_L \times SU(2)_R$ in which lepton number is the fourth colour under the colour group $SU(4)$ and there is a L-R symmetry relating the two $SU(2)$ groups. In the context of the $SU(5)$ subgroup, it seems probable that the group should indeed be enlarged to $SO(10)$ because a single 16 dimensional representation of $SO(10)$ and explains the full structure of the 15 states in a SM family shown in Figure 5. The 16th state is an $SU(5)$ and hence SM singlet and neatly accommodates the right-handed neutrino filed that restores the symmetry between quarks and leptons both of which now have the same number of LH and RH states.

Thus we see that a GUT can significantly simplify the multiplet structure needed to accommodate the gauge bosons and matter fields of the SM. In doing so it answers some of the questions posed by the SM discussed above. There is still need to include three families of $SU(5)$ or $SO(10)$ representations and attempts were made to further increase the size of the GUT group in the hope that three families would be required to complete an irreducible representation of the GUT. However these attempts did not lead to convincing models and I will not pursue them here. Perhaps the most promising origin for the family origin is in the context of compactified string theories where the need to compactifiy leads to the need for family replication. Many three families models have been constructed but there is still no understanding of why these should be chosen over the many other models with different numbers of families.

A. The classic predictions of GUTs

Related to the simplification of multiplet structure is the prediction of relations between some of the parameters of the SM. The most immediate of these is the fact that $SU(5)$ has only a single gauge coupling constant so the three SM gauge couplings are predicted to be equal[5]. The evolution from this scale to the scale at which the couplings are measured is given by the RG equations and determined by the SM beta functions

$$\alpha_i^{-1}(\mu) = \alpha^{-1}(M_X) + \frac{1}{2\pi} b_i \ln\left(\frac{M_X}{\mu}\right) + .. \tag{7}$$

where

$$b_i^{SM} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_g \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + H \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix}$$

Here N_g is the number of generations and H the number of Higgs doublets. The result of solving the RG evolution is shown in Figure 7 One sees that, although the couplings do approach each other at high scales they miss by more than 11 standard deviations. While encoraging there is something missing - we will return to the question what this might be in Section VI.

GUTs also relate the fermion masses of states that belong to the same GUT representation. For example in the case of $SU(5)$ the group structure of the product of fermion representations that correspond to mass terms is

$$\begin{aligned} \overline{5} \times 10 &= 5 + \overline{45} \\ 10 \times 10 &= \overline{5} + 45 + 50 \end{aligned} \tag{8}$$

For the case that the Higgs are in a 5 representation the mass term in the Lagrangian is given by

$$L^5{}_{Yukawa} = \left(\psi^\dagger_{Ri\alpha}\right) m^D_{ij} \chi^{\alpha\beta}_{Lj} H^\dagger_\beta - \tfrac{1}{4}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\left(\chi^T\right)^{\alpha\beta}_{Li} \sigma^2 m^U_{ij} \chi^{\gamma\delta}_{Lj} H^\varepsilon + h.c. \tag{9}$$

[5] With the appropriate $SU(5)$ normalsation of the $U(1)$ factor

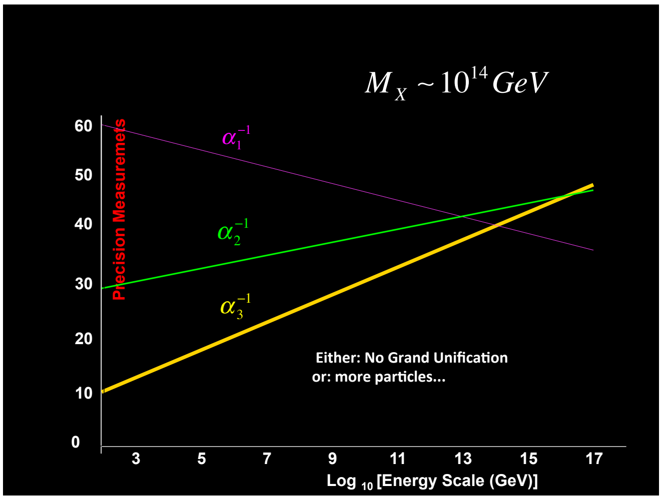

FIG. 7: *The evolution of the gauge couplings below the $SU(5)$ unification scale*

Since the down quarks and leptons belong to the same $\bar{5} + 10$ representations the first term leads to a relation between their masses given by

$$m_d = m_e, \; m_s = m_\mu, \; m_b = m_\tau. \tag{10}$$

This equation applies at the GUT scale so needs to be corrected by RG running. Due to the colour interactions the quarks get a relative enhancement by a factor of approximately 3 so the third relation is a reasonable approximation but the first two relations are wildly wrong. If, however, the Higgs belongs to the 45 representation of $SU(5)$ one sees from eq 8 that it will also give masses through the coupling

$$L_Y^{45} = \left(\psi^\dagger_{Ri\alpha}\right) m^d_{ij} \chi^{\beta\gamma}_{Lj} H^{\dagger\alpha}_{\beta\gamma} + \varepsilon_{\alpha\beta\gamma\rho\tau} \left(\chi^T\right)^{\alpha\beta}_{Li} \sigma^2 m^u_{ij} \psi^{\gamma\delta}_{Lj} H^{\rho\tau}_{\delta} + h.c. \tag{11}$$

giving the relation

$$-3m_d = m_e, \; -3m_s = m_\mu, \; -3m_b = m_\tau. \tag{12}$$

The sign can be absorbed in a redefinition go the fermion field so in this case, after including the RG running, the second relation is acceptable. Georgi and Jarlskog [42] proposed combining the two contributions with couplings restricted by a simple family symmetry, and obtained an interesting structure with the down quark and charged leptons having the mass matrix in family space given by

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} 0 & \varepsilon^3 & 0 \\ \varepsilon^3 & a\varepsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{13}$$

Here the "texture" zeros are due to Yukawa couplings being absent due to the imposeed family symmetry; such zeros are common in attempts to explain fermion masses by an underlying family symmetry. The parameter ϵ quantifes the spontaneous breaking of the family symmetry; in the unbroken limit only the Yukawa couplings of third generation of down quarks and leptons to is allowed but after spontaneous breaking other couplings appear at $O(\epsilon^n)$ where n depends on the charges assigned to a given family under the family symmetry. The (3,3), (12) and (2,1) entries come from the Yukawa coupling of eq(9) to the 5 component of the Higgs and so have equal coupling to down quarks and charged leptons. The (2,2) element comes from the Yukawa coupling of eq(11) to the 45 component of the Higgs and so the coefficient a is 1 for the strange quark and -3 for the τ lepton. The resulting mass matrix has eigenvalues given by

$$m_b \approx 3m_\tau,\ m_s \approx 3.\frac{1}{3}m_\mu,\ m_d \approx 3.3.m_e. \tag{14}$$

where I have included a factor of 3 to take approximate account of the QCD running to low scales. The last relation follows because of the texture zeros which mean that $DetM_d = DetM_l$ at the GUT scale because the determinant is given entirely by the 5 component of the Higgs. Solving the determinant equation for m_d then gives the relation shown. All three relations are in qualitative agreement with the measured quark and lepton masses although to make a quantitative comparison it is necessary to perform a detailed RG analysis including the other gauge interactions and threshold effects [66]. The Georgi Jarlskog model illustrates how acceptable relations between quark and lepton masses can result from a stage of Grand Unification. Their texture zero structure also illustrates how such zeros can lead to relations between masses and mixing angles. In addition to the determinant prediction the texture zero in the (1,1) position and the hermitian symmetry of the mass matrix leads to a prediction for the down quark contribution to the Cabbibo angle given by $\sin\theta_{Cabbibo} = \sqrt{\frac{m_d}{m_s}}$. This relation is remarkably good and if the top quark sector is included the relation for the Cabibbo angle has the form

$$\sin\theta_{Cabbibo} = \sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}} \tag{15}$$

where δ is the CKM matrix CP violating phase. This gives an excellent fit to the Cabbibo angle with near maximal CP violation, also in agreement with the experimental value.

Neutrino masses can also be well described via the see-saw mechanism[39, 58, 72] (see Section V A), the smallness of the neutrino mass following because the singlet neutrinos entering the see-saw mechanism are expected to have GUT scale masses.

One very important aspect of GUTs that is obvious from the form of the covariant derivative in eq(6) is that, when acting on the multiplets of figure 6, the X and Y gauge bosons couple quarks to leptons. In fact they have both baryon- and lepton-number violating interactions and mediate nucleon decay at a rate $\propto M_X^{-4}$. Before discussing this, however, I turn to an underlying problem that GUTs have that has a very significant effect on the expectation for the nucleon decay rate and decay channels.

IV. THE HIERARCHY PROBLEM

As we have seen GUTs provide a very plausible extension of the SM. However they suffer from a serious hierarchy problem, namely the difficulty in field theory of separating the electroweak scale from the GUT scale. As this problem applies to the SM on its own and as ideas for BSM physics are always strongly constrained by the hierarchy problem the subject deserves a separate discussion.

The SM is a renormalisable, spontaneously broken, local gauge quantum field theory. Much of its structure is explained if it is an effective field theory descending from a more complete theory, such as a GUT, relevant at a high scale, M. In this case at low scales the Lagrangian has the form

$$L_{eff}\left(\phi_{light}, \psi_{heavy}, M, E\right) \rightarrow L_{eff}\left(\phi_{light}, E\right) + O\left(\frac{1}{M}\right) \tag{16}$$

where one has integrated out the heavy degrees of freedom of mass M. Then it follows that the leading terms in L_{eff} have mass dimension ≤ 4 as in a renormalisable theory. The only states in L_{eff} are those that do not receive a mass of $O(M)$. It is significant that the only vector and fermonic states we observe have a symmetry that protects them from an $O(M)$ mass, namely local gauge symmetry for the vectors and chiral symmetry for the fermions (because LH and RH states have different $SU(2)$ quantum numbers). The looks very much a signal for an effective field theory. However the problem arises that the light Higgs is not protected by a symmetry because the SM symmetry does not forbid the Higgs mass term $M^2|H|^2$.

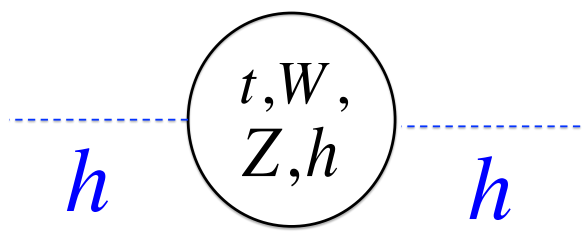

FIG. 8: *Radiative contributions to the Higgs mass.*

Even if one supposes the term is absent at tree level it will reappear in radiative order because in field theory virtual corrections contribute to the Higgs mass driving it to an unacceptably high scale. The mass correction coming from the graph of Figure 8 is given by

$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2\right)\Lambda^2 \sim m_h^2\left(\frac{\Lambda}{500GeV}\right)^2 \tag{17}$$

where m_h is of order the Higgs mass and Λ is a cut-off of the quadratically divergent mass; in an effective field theory it will be the scale, M, at which BSM physics alter the radiative corrections.

If the SM is *all* there is, the divergent contribution δm_h^2 is not measurable as only the renormalised mass $m^2 = m_0^2 + \delta m_h^2$ is physical where m_0^2 is the mass counter term. In this case one may argue that the case $m = 0$ is special, corresponding to "classical" scale invariance and in this case the measurable effects of radiative corrections to the mass are proportional to m_h^2 where m_h arises from spontaneous breaking of the classical scale invariance and can be chosen to be the required EW breaking scale.

I will return to a discussion of this "Just the SM" case later but it is reasonable to doubt the the SM can be all there is, *c.f.* our discussion of GUTs above. In the presence of heavy masses such as the X and Y gauge bosons of $SU(5)$ there is a measurable contribution[6], $\delta m^2_{h,X}$, to the Higgs mass coming from the loop of Figure 8 with the heavy states, X, in the loop, given by

$$\delta m^2_{h,X}(Q^2) \propto M_X^2 \ln\left(\frac{Q^2+M_X^2}{\Lambda^2}\right) \tag{18}$$

This term cannot be eliminated by renormalisation - the "real" hierarchy problem.

A. Solutions to the hierarchy problem

Various solutions to the hierarchy problem have been explored in recent years. These include

- "Just" the Standard Model (JSM). By "Just" I mean that one appeals to the idea of "classical" scale invariance mentioned above to eliminate the hierarchy problem. To answer some of the questions such as dark matter abundance it is necessary to add additional states but these should not be very heavy ro avoid the "real' hierarchy problem.'

- The Higgs and possibly other states are composite. In this case the Higgs coupling appearing in Figure 8 involves a form factor that is exponentially suppressed above the composite scale, $\Lambda_{composite}$ and $\Lambda \approx \Lambda_{composite}$ in eq(17). For this to be acceptable we need $\Lambda_{composite} \leq O(1TeV)$ and there will be new composite resonant states in this mass range providing a smoking gun for compositeness.

- The fundamental scale in the theory is low $M_* \sim 1TeV$ and high scales, such as the Planck scale setting the gravitational strength coupling, are derived from it. In this case $\Lambda \sim M_*$ in eq(17), avoiding a large hierarchy problem. For example, if space time is D dimensional with $d > 4$ and the extra dimensions are curled up on a length scale R, the gravitational potential has the form [10]

$$V(r) = \frac{1}{M_*^{2+d}R^d}\frac{m_1 m_2}{r}, \quad D = 4+d, \quad r \ll R \tag{19}$$

  implying

$$M^2_{Planck} = M_*^2(M_*R)^{\mathbf{d}}. \tag{20}$$

  For M_*R very large the induced Planck scale is large and the gravitational strength is weak because the flux lines spread into the additional dimensions and are diluted. An alternative version of extra dimensions involves a warp factor that suppresses the fundamental mass scale observed in our locality even though the theory has a fundamental large Planck scale

[6] By measuring the mass at two different scales, $Q^2_{1,2}$, one can establish the existence of this term.

[64, 65]. In both cases there will be Kaluza Klein modes at the TeV scale that will signal a departure from the SM.

- The Higgs is a (Pseudo) Goldstone boson [51]. The SM Higgs is associated with a spontaneously broken *global* symmetry factor of this group and, if it were an exact symmetry, it would be massless. However the global symmetry is broken in the SM by gauge interactions so the Higgs becomes a pseudo-Goldstone mode with a suppressed mass. Being only slightly suppressed compared to the underlying scale of new physics, this addresses the "little-hierarchy problem" explaining why we have not seen new physics below a TeV but not the large hierarchy problem explaining how Planck or GUT scale masses are avoided.

- The Higgs is protected by supersymmetry (SUSY). In this case the SM Higgs doublet belongs to a supermultiplet with a doublet fermion "Higgsino" partner. Supersymmetry requires the mass of the Higgs is the same as the mass of the Higgsino and, if there is a chiral symmetry forcing the fermion Higgsino to be massless, the Higgs will be massless too. This solution requires all the states of the SM to have SUSY partners and the fact they have not been seen requires that they acquire a mass of $O(1TeV)$ or greater. The graphs of Fig 8 with the heavy SUSY states in the loop serve to eliminate the quadratically divergent contribution to the Higgs mass[7]. However, being massive, these new states introduce a little "real" hierarchy problem with, for example, the residual Higgs mass contribution coming from the top, stop loop given by
$$\delta^{t+\tilde{t}} M_H^2 = \frac{h_t^2}{8\pi^2}\left(m_t^2 \ln(\frac{\Lambda^2}{m_t^2}) - m_{\tilde{t}}^2 \ln(\frac{\Lambda^2}{m_{\tilde{t}}^2})\right) \tag{21}$$
This is already uncomfortably large for $m_{\tilde{t}} = O(1TeV)$. In Section 10 I will give a quantitative estimate of the little hierarchy problem in SUSY.

V. "JUST" THE STANDARD MODEL

The discovery of the SM Higgs has immediate implications for the possibility the SM is complete, as was illustrated in Figure 4. One may see that if the top quark mass is at the lower end of its range the model could be complete to the Planck scale because the quartic coupling does not become negative over the range. As we discussed the quadratic divergence in the Higgs mass is not physical and the theory is defined by the renormalised mass. If the theory is classically scale invariant this mass vanishes but then the question is how the EW breaking scale is generated. An elegant answer was given by Coleman and Weinberg (CW) [25] who showed how radiative corrections can lead to spontaneous symmetry breakdown.

They presented the simple example of classically scale invariant scalar electrodynamics and determined the scalar potential arising from the graphs of the type shown in Figure 9:

[7] For example, due to SUSY, the top squark has the same couplings as the top quark in the couplings of Fig 17 but, being a scalar, has a relative minus sign in the loop contribution relative to the top fermion loop and thus cancels the quadratic divergence.

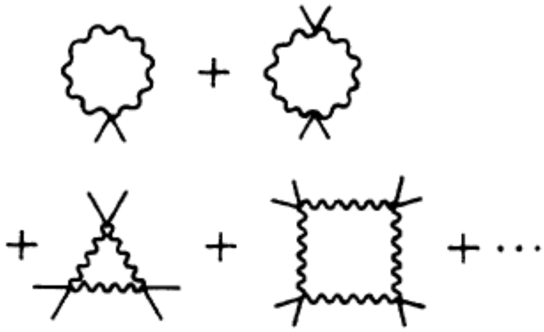

FIG. 9: *Radiative contributions to the Higgs potential in scalar electrodynamics.*

$$V = \left\{ \frac{\lambda}{4!}\phi^4 + \frac{3e^4}{64\pi^2}\phi^4 \ln\frac{\phi^2}{M^2} \right\} = \frac{3e^4}{64\pi^2}\phi^4 \left(\ln\frac{\phi^2}{\langle\phi\rangle^2} - \frac{1}{2} \right) \tag{22}$$

Here M is the scale at which the quartic coupling is defined. One may see that the radiative corrections drive the potential negative at some lower scale and it has a minimum at a point $\phi =< \phi >$ very close to this crossing point. This generates dynamical spontaneous symmetry breaking[8] of the classical scale invariance and generates a mass for both the "photon" and the scalar given by

$$m_\phi^2 = \frac{3e_\phi^2}{8\pi^2} m_\gamma^2 \ll m_\gamma^2 \tag{23}$$

Unfortunately this cannot be immediately applied to the SM as the scalar is much lighter than the gauge boson but many models have been constructed that do generate acceptable EW breaking via dynamical transmutation. I will give an example of this below.

In building "Just the SM" (JSM) it is essential that there should be no very heavy particles significantly coupled to the Higgs to avoid the "real" hierarchy problem. However there are several shortcomings of the SM that require new physics and the question is whether these can be avoided by extending the SM to include only light states. The most pressing questions to be answered are

- Neutrino masses?
- Baryogenesis?
- The strong CP problem?
- Dark matter?
- Gravity?

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[8] The generation of the symmetry breaking scale from the running of dimensionless couplings is often referred to as "dynamical transmutation".

A. Neutrino masses

If we add RH neutrinos to the SM, neutrino masses can be generated via the Lagrangian

$$L_{mass} = h_a \bar{l}_a \nu_{Ra} H + \frac{M_{ab}}{2} \nu_{Ra}^T C \, \nu_{Rb} \tag{24}$$

The first term is a Dirac mass, the analogue of the mass term for quarks and charged leptons. The second term is a Majorana mass term that violates lepton number but is allowed by the SM symmetry as the RH neutrinos are SM singlets. In GUTs the Dirac mass, m_D, can be comparable to the charged lepton masses but the smallness of neutrino mass is due to the choice of the Majorana mass, M, close to the GUT scale. In this case the couplings generate a Majorana mass for the LH neutrino states that is suppressed relative to the Dirac mass by the factor $\frac{m_D}{M}$ and is very small - the "see-saw" mechanism [39, 58, 72].

In the JSM one cannot allow such heavy Majorana masses but it is still possible to use the see-saw mechanism with light RH neutrinos if the Dirac couplings, h_a are ultra weak. For example for a $20GeV$ RH neutrino it is necessary to choose $h_a \sim 10^{-14}$ to get viable LH neutrino masses in the range $0.1 - 0.01eV$. Such small couplings are technically natural as the Dirac couplings are protected by a chiral symmetry and so radiative corrections will not drive the masses larger.

B. Baryogenesis

There have been several suggestions for producing the observed baryon excess via only low mass states. Here I give one possibility [5] that follows immediately from the terms in eq(24) generating neutrino masses. In the early universe, at temperatures above the RH neutrino masses, the Yukawa interactions of eq(24) produce a lepton number conserving thermal abundance of the right handed neutrinos.

$$L_A = L_B = L_C = 0 \tag{25}$$

where A, B, C label the three families. These neutrinos oscillate in the usual way with family changing interactions and, with CP violating terms in the RH neutrino sector, will cause asymmetric abundances to develop while, of course, preserving overall lepton number.

$$L_{A,B,C} \neq 0, \quad L_A + L_B + L_C = 0 \tag{26}$$

Consider the case that the interactions are such that, say, only $\nu_{R,C}$ drops out of thermal equilibrium at a temperature T_1 above the electroweak scale at which the baryon- and lepton-number violating sphaleron interactions [9] drop out of equilibrium. In this case in the period between T_1 and the electroweak scale the sphalerons will only convert the non-zero lepton number in the A, B channels to a non-zero baryon number

$$\Delta_{L\ AB} = L_A + L_B \rightarrow \quad \Delta B = \Delta_{L\ AB}/2. \tag{27}$$

[9] Sphalerons [52] in the SM non-perturbatively generate $B + L$ violating, $B - L$ conserving interactions [53] that are in thermal equilibrium above the EW breaking scale

For a range of choices of the couplings h_a giving acceptable neutrino masses the mechanism is able to generate the observed baryon asymmetry [5]. A bonus to the mechanism is the possibility that the lightest of the RH neutrinos is the dark matter of the universe. It turns out that this is possible but the parameter choice needed requires a resonant enhancement of baryogenesis through near degeneracy of two of the RH neutrino states [18]. However if another state, such as the axion, is the dark matter the original mechanism is perfectly viable.

C. The strong CP problem

One of the puzzles of the SM is the absence or near absence of a strong interaction term allowed by the symmetries of the SM. This is the CP violating θ term formed from the gluon field strength $G^a_{\mu\nu}$

$$L^\theta = \frac{\theta}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}, \quad \theta \leq 10^{-10}?? \tag{28}$$

This term contributes to the neutron dipole electric moment and the bound on its coefficient follows from the need to suppress its contribution below the observed experimental limit. The question is why should this term be so suppressed? This is all the more difficult to understand as there are unacceptably large contributions to θ coming from the fermion sector that arises via the triangle anomaly contribution to eq(28).

The most promising solution to the strong CP problem is to make θ a dynamical variable that is made small when minimising its potential [63]. This requires adding additional fields. The way relevant to the construction of the JSM [6] is to have two Higgs complex scalar doublets $H_{1,2}$ which give masses to the up and to the down quarks and leptons respectively together with a SM singlet complex scalar field S [30]. The interaction amongst these fields is given by

$$\begin{aligned} V(H_1, H_2) = \; & \frac{\lambda_1}{2}|H_1|^4 + \frac{\lambda_2}{2}|H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 \left| H_1^\dagger H_2 \right|^2 \\ & + \zeta_1 |S|^2 |H_1|^2 + \zeta_2 |S|^2 |H_2|^2 + \zeta_3 S^2 H_1 H_2 + \zeta_4 |S|^4 + h.c. \end{aligned} \tag{29}$$

Demanding that the theory be invariant under a PQ symmetry [63] that rotates the phase of the up quarks relative to the down quarks requires that $H_{1,2}$ have different phase rotations and this phase is in turn communicated to the S field when demanding the potential in eq(29) also be invariant under the PQ symmetry. As a result the singlet field $S = \left(\hat{S} + f_a\right) e^{i\frac{a}{f_a}}$ contains the axion, a, that replaces θ in eq(28)[10]. Up to small weak interaction corrections the vacuum expectation value (VEV) of a is set to zero when minimising the axion potential [71] thus solving the strong CP problem.

The axion interacts with matter and has been intensely searched for; it also contributes to the dark matter abundance which must not become too large. From the direct searches and cosmology

[10] This is due to the triangle anomaly involving the quarks which couple to a. The bare θ term can be absorbed into a by a field redefinition.

one establishes a bound on its coupling strength determined by f_a, the VEV of S, given by

$$10^{10}GeV \leq f_a \leq 10^{12}GeV. \tag{30}$$

Note that this large scale puts strong constraints on the couplings of the S field, requiring them to be ultra weak

$$\zeta_{1,2,3} \leq 10^{-20}\left(\frac{10^{12}GeV}{f_a}\right)^2. \tag{31}$$

However the ultra weak couplings are technically natural because the couplings can be forbidden by the shift symmetry $S \to S + \delta$.

In the case of the JSM the origin of the high scale is problematic as there are no explicit mass terms allowed by classical scale invariance. Luckily dimensional transmutation offers a scale invariant origin to this.

Consider the S scalar potential including radiative corrections that follows from eq(29):

$$\begin{aligned} V_{DFSZ}(H_1, H_2, S) \simeq \; & \frac{\lambda_1}{2}\left(|H_1|^2 + \frac{\zeta_1}{\lambda_1}|S|^2\right)^2 + \frac{1}{64\pi^2}\left(\zeta_2|S|^2\right)^2\left(-\frac{1}{2} + \ln\frac{|S|^2}{f_a^2}\right) + \frac{\lambda_2}{2}|H_2|^4 \\ & + \zeta_3 S^2 H_1 H_2 + h.c. \end{aligned} \tag{32}$$

where $\zeta_2 > \zeta_1 > \zeta_3$ is assumed. The second term is of the form of eq(22) and has a minimum at $< S >= f_a$ where f_a is determined by the initial value of the ζ_2 coupling at the initial scale M. Since these are not determined we may treat f_a as a free parameter and choose it to satisfy eq(30). The model has 7 additional Higgs scalar states beyond the SM, one of which is the axion. The remaining fields have mass

$$m_H^2 = m_{H^\pm}^2 = m_A^2 = -\frac{\zeta_2}{2\zeta_1}m_h^2 \tag{33}$$

where m_h is the state identified with that found at the LHC, and an anomalously light state - the pseudo dilaton $|S|$ with mass

$$m_{|S|}^2 = -\left(\frac{\zeta_2^2}{32\pi^2\zeta_1}\right)^2 m_h^2 \simeq 13\left(\frac{10^{12}GeV}{\mathrm{v}_S}\right)^2\left(\frac{m_{H_2}}{m_h}\right)^4 eV^2 \tag{34}$$

Thus we see that the solution to the strong CP problem requires new light Higgs states. Because the couplings to the singlet S are ultra weak the collider phenomenology is independent of these couplings and is that of the Type II 2 Higgs doublet models but with a restricted mass spectrum as in eq(33). The light dilaton abundance turns out to be acceptable but it cannot be dark matter [6]. It may be that axion-like searches may be sensitive to it too but this has not yet been studied. The axion solves the strong CP problem and, depending on the value of f_a, may be the dark matter. There is a potential problem with axionic domain walls but it can be avoided through additional ultra weak PQ breaking interactions that are expected if the PQ symmetry is discrete rather than continuous [6].

D. Gravity

There remains the question whether it is possible to avoid the hierarchy problem in the JSM in the presence of gravity. Gravitational corrections to the Higgs mass are absent if one ignores the Higgs interactions because then the theory has a shift symmetry $H \to H + \delta$ that forbids the mass. Thus gravitational corrections must involve Higgs couplings that break the shift symmetry and thus appears first at two loop order $\delta m_h^2 \approx L G_N \Lambda_G^4$ where Λ_G is the cut-off of the quadratic divergence associated with the non-renormalisable gravitational interaction and L contains the loop factors and the SM Higgs couplings.

In the case of a classically scale invariant theory G_N is replaced by ϕ^2 where ϕ acquires a Planck mass VEV. The divergent radiative contributions to the Higgs mass proportional to powers of Λ_G are not measurable and combine with the counter term to make up the renormalised mass that is zero due to the scale invariance. Of course the scale invariant coupling $\epsilon\phi^2|H|^2$, that is invariant under the symmetries of the theory, must be ultra-weak to keep the Higgs light but this is technically natural if it is protected by the the shift symmetry $\phi \to \phi + \delta$. There are gravitational radiative corrections to the coupling but they do not come from the graviton coupling to the unsuppressed kinetic term of ϕ, which is invariant under the shift symmetry. However they are only generated by the graviton coupling to $\lambda\phi^4$ interaction that violates the shift symmetry. Thus λ will also have to be ultra-weak but this is also technically natural being protected by the shift symmetry. Thus if the couplings ϵ and λ are ultra-weak gravitational corrections to the Higgs mass will not generate an unacceptable contribution to the Higgs mass.

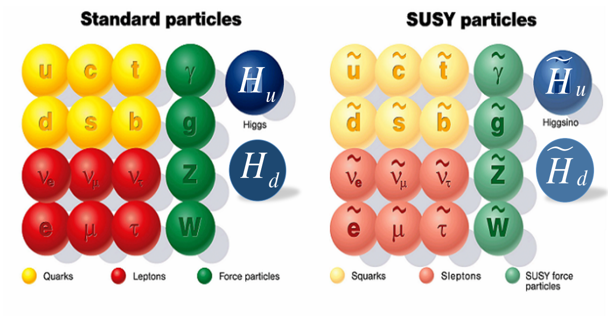
VI. SUPERSYMMETRY

By now, there are many excellent introductions to SUSY and the MSSM. One I particularly like is "A supersymmetry primer" by S. Martin [57] that is available on the web. For a recent impressive review of the theory and phenomenology of SUSY and extensive references see the book "Theory and Phenomenology of Sparticles" by M.Drees, R.Godbole and P. Roy [31].

The supersymmetric solution to the hierarchy problem is based on $N = 1$ SUSY with a single SUSY generator, Q, acting on either chiral supermultiplets that contain a complex scalar, ϕ, and a two component Weyl fermion, ψ or a vector supermultiplet containing a gauge field, A_μ and a Weyl fermion, χ:

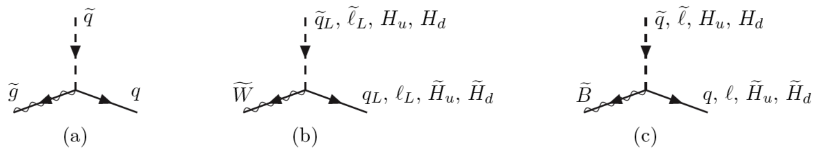
$$\begin{pmatrix} \psi \\ \phi \end{pmatrix}, \begin{pmatrix} A_\mu \\ \chi \end{pmatrix} \quad \updownarrow Q$$

The resulting spectrum for the Minimal Supersymmetric Standard Model (MSSM) is shown in Figure 10(I), the squarks and leptons partners of the quarks and leptons assigned to chiral supermultiplets, and the gluinos, Winos, Zino and photino partners of the W, Z and photon assigned to gauge supermultiplets. In order to generate fermion masses and to satisfy anomaly cancellation it is necessary to extend the SM to include two Higgs doublet representations, $H_{u,d}$, which generate the up and down masses respectively. These states are assigned to chiral supermultiplets with the Higgsinos their SUSY partners. Note that, as the group structure is a simple direct product,



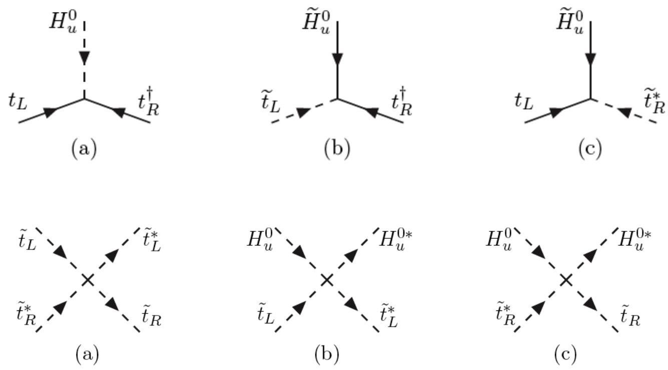
(I)

Gauge



Yukawa+F-terms

$$W_{\mathrm{MSSM}} = \overline{u}\mathbf{y_u}QH_u - \overline{d}\mathbf{y_d}QH_d - \overline{e}\mathbf{y_e}LH_d + \mu H_u H_d$$



(II)

FIG. 10: *(I) The MSSM spectrum (II)Interactions in the MSSM.*

$SU(3) \times SU(2) \times U(1) \times N = 1$ $SUSY$, the SM quantum numbers of the new SUSY states are the same as their SM partners. The gauge and Yukawa interactions of the new SUSY states are shown in Figure 10(II), with couplings the same as the related SM couplings.

A. SUSY GUTs

The GUTs discussed in Section III have very heavy states that couple strongly to the Higgs. As a result they suffer from the "real" hierarchy problem and cannot have the EW breaking scale far below the GUT scale. For this reason they must be modified to include low-scale SUSY to protect the Higgs mass from GUT scale corrections. This is readily achieved by assigning all the GUT states to N=1 vector or chiral supermultiplets with SUSY partners carrying the same GUT quantum numbers. These new SUSY states change the GUT phenomenology in significant ways. For example the prediction for gauge coupling unification is changes because, to solve the hierarchy problem, the SM partner states of Figure 10(I), must be light, $\leq O(1TeV)$, and so contribute to the running of the couplings up to the GUT scale. Since their gauge quantum numbers are determined there is no freedom in this apart from the precise choice of the masses of the new SUSY states. However the prediction is not very sensitive to this choice as the running is over some 16 orders of magnitude while the uncertainty in the SUSY spectrum is only of a single order. The result of including the SUSY states is shown in Figure 11 where it may be seen that

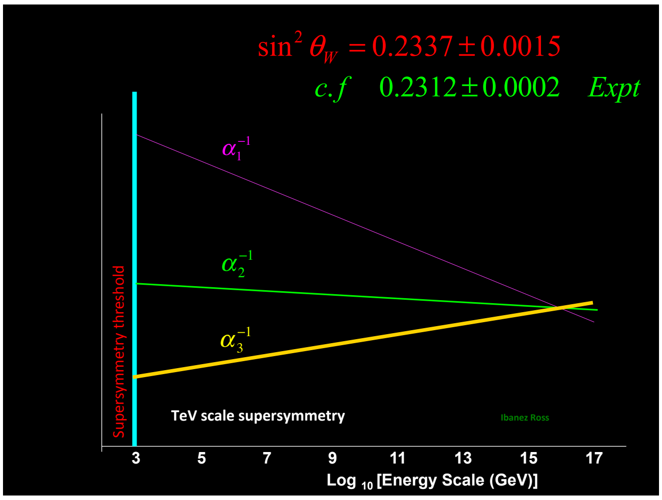
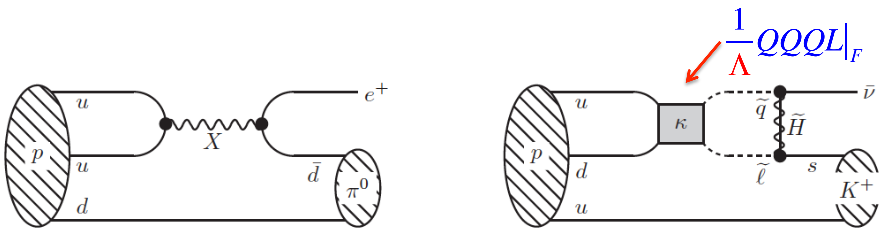


FIG. 11: *The SUSY GUT evolution of the gauge couplings below the $SU(5)$ unification scale*

the couplings unify very precisely, a dramatic improvement over the non-SUSY result shown in Figure 7.

The prediction for fermion masses and mixings follows similar lines to that discussed In Section III, modified by the changes in the radiative corrections due to the additional SUSY states, although in this case the change is not dramatic [66].

Having constructed a SUSY GUT that addresses the real hierarchy problem we are now in a position to discuss the new BSM processes that are due to the new X and Y bosons (and their SUSY partners) predicted by $SU(5)$. Figure 12a is given by X vector boson exchange and generates a dimension 6, four fermion, interaction. The amplitude is suppressed by M_X^{-2} and for $M_X > 10^{16}GeV$ the lifetime to the dominant decay mode is given by $\tau_{p\rightarrow\pi^0+e^+} > 10^{34}$years.



(a) Dimension 6. (b) Dimension 5.

FIG. 12: *Graphs generating nucleon decay in SUSY GUTs*

Figure 12b only occurs in the SUSY version as it involves squarks and sleptons[11]. It is generated by fermion exchange and generates a dimension 5, two fermion two scalar interaction that is only suppressed by one power of the exchanged particle (the coloured Higgsino partner of the coloured Higgs that is needed to complete the 5 dimensional representation of $SU(5)$). Writing Λ as the combination of the Higgsino mass with the inverse of the the Higgsino couplings involved in the diagram this operator is suppressed by a single power of Λ. The dominant Higgs cpupling is to heavy states and so the leading proton decay channel is $p \to K^+ + \bar{\nu}$ and to satisfy the present experimental lifetime bound $\tau_{p\to K^+ +\bar{\nu}} > 3 \times 10^{33}$years one needs $\Lambda > 10^{27} GeV$! Even though Λ is increased by small Yukawa couplings, it is difficult to reconcile this bound with a Higgsino mass of order the GUT scale. If the underlying theory is a string theory the problem is even more severe because one expects string scale mass (of $O(M_{Planck}) \sim 10^{18} GeV$) coloured Higgs triplets to couple unsuppressed to the SM states. However the amplitude requires coupling between the Higgsino states coupling to the up and down sectors and it is possible that the underlying theory has a symmetry that suppresses the amplitude. To study this we turn to a discussion of the additional symmetries that SUSY models may have.

B. SUSY extensions of the Standard Model

The SM symmetries allow the following terms of dimension ≤ 4 terms in the superpotential

$$\begin{aligned} W = \quad & h^E L H_d \overline{E} + h^D Q H_d \overline{D} + h^U Q H_u \overline{U} + \mu H_d H_u \\ & + \lambda L L \overline{E} + \lambda' L Q \overline{D} + \kappa L H_u + \lambda'' \overline{U} \overline{D} \overline{D} \\ & + \frac{1}{M}\left(QQQL + QQQH_d + Q\bar{U}\bar{E}H_d + LH_u LH_u\right) \end{aligned} \tag{35}$$

The terms of the first line are of dimension 4 in the Lagrangian and are needed to generate fermion masses and the Higgsino mass. However the latter should be of order the electroweak scale but

[11] This is the F-term of a QQQL operator, for details see [57].

the symmetries allow an arbitrarily high scale, the "μ' problem. The terms in the second line are also dimension 4 and are baryon- or lepton-number violating. If both types are present they generate nucleon decay suppressed in amplitude only by the square of a squark mass, completely unacceptable. The last line lists dimension 5 operators. The first violates both baryon- and lepton-number and generates proton decay and, as discussed in the last Section, required a suppression scale so high that it is preferably to suppress it. The second and third term violate baryon and lepton number respectively while the last term, which violates lepton number, is needed to generate Majorana mass for the LH neutrinos by the see-saw mechanism.

To obtain a phenomenologically viable theory it is necessary to suppress some of these operators via additional symmetries. The simplest possibility is via discrete symmetries and various possibilities have been suggested:

R-parity In the MSSM it is assumed there is a Z_2 discrete symmetry[12], "R" parity, under which only the Higgs supermultiplets are even [36]. This kills the terms in the second line but allows the "μ" term and all the dimension 5 terms. It also implies a very important restriction on SUSY phenomenology for it requires that the SUSY states can only be pair produced and that the lightest SUSY state (LSP) is stable and is a dark matter candidate.

Baryon parity An alternative "Baryon parity" is based on a Z_3 discrete group under which the supermultilet charges are $Q = 1, \overline{D},\ H_u = \alpha,\ L, \overline{E}, \overline{U}, H_d = \alpha^2$ with α the cube root of -1 [49]. This forbids the baryon number violating operator in the second line and the first two dimension 5 operators. While baryon parity avoids nucleon decay it still requires that the lepton number violating coefficients be very small to be consistent with the bounds on lepton number violation. It also allows the μ term and an unstable LSP.

Proton hexality The Z_6 combination of these two symmetries is more promising as it forbids all lepton- and baryon-number terms apart from the necessary last term responsible for neutrino masses [32]. However it still allows the μ term leaving the question why it should be only of order the electroweak scale unanswered.

Discrete R-symmetry The simplest way of avoiding the μ problem together with the baryon- and lepton-number violating terms requires an "R" symmetry under which the upper and lower components of a supermultiplet have charge differing by unity; in this case the superpotential should have charge 2 under the symmetry.

There is a Z_4^R choice for an R-symmetry that commutes with the $SO(10)$ GUT and avoids the μ term[55, 56]. In this the quark and lepton supermultiplets carry R-charge 1 while the Higgs supermultiplets are neutral. One may readily check that the μ term together with baryon- and lepton-number violating operators, apart from the one generating neutrino masses, do not have R-charge 2 and are forbidden. This scheme also generates a μ term of the correct magnitude because, in the local SUSY case, the VEV of the superpotential is the order parameter for SUSY breaking with the graviton mass, $m_{3/2}$, given by $m_{3/2} =< W > /M_{Planck}$. Since the superpotential carries R-charge 2 this VEV breaks the $Z(4)_R$ symmetry to $Z(2)$ R-parity. Then the operator $H_u H_d W/M_{Planck}^2$ generates a μ term of $O(m_{3/2})$.

[12] All the discrete symmetries discussed here are discrete gauge symmetries in the sense that they are discrete anomaly free [49].

Since the dimension 4 terms are now forbidden only the dimension 6 operator of Figure 12a contributes to proton decay with the dominant channel. The current experimental bound is $\tau^{SuperK}_{p\rightarrow e^{+}\pi^{0}} > 1\times 10^{34}$yrs. This requires $M_X > 6\times 10^{15}GeV$ for the $SU(5)$ gauge boson mass to be compared with the unification scale $M_U = (2.5\pm 2).10^{16}GeV$. Thus the expectation of the $SU(5)$ SUSYGUT with the $Z(4)_R$ symmetry is that the proton decay lifetime should be very close to the present limit.

C. SUSY and flavour physics

In supersymmetric models flavour changing and CP violating effects can be significantly enhanced relative to the SM, driven by processes involving squarks and leptons. In particular such models introduce new sources of CP violation such as the phase of the μ term or of the diagonal A_0 parameters which, if unsuppressed, lead to unacceptable electric dipole moments (EDM) - the SUSY CP problem. They may also generate significant favour changing neutral currents (FCNC) leading to processes such as $K^0,\ \bar{K}^0$ mixing and $\mu\rightarrow e\gamma$. The latter may be suppressed by demanding that at least the first two families of squarks and leptons be degenerate [13] as in the CMSSM or in models with an underlying family symmetry. This implements a GIM cancellation in the scalar sector in a similar manner to that in the fermion sector. Even so significant FCNC can be generated through the running of the scalar masses from the initial scale to the EW scale.

The SUSY CP problem is more difficult to control. Initially it was argued that the CP violation could be made small through cancellations between different terms but this introduces further fine tuning and is not very satisfactory. A much more natural solution follows if one can build a SUSY model that approximates minimal flavour violation (MFV) in which the origin of CP violation is via Yukawa couplings in the flavour changing sector where it is observed to be large. In this case the CP violation in the flavour conserving sector that generates EDMs results from processes involving two flavour changing vertices and hence is suppressed by powers of small mixing angles.

Models achieving this can be built using a family symmetry to generate viable Yukawa couplings and their related masses and mixing angles. In this case one starts with a CP invariant theory (compactified 4D theories in string theory often are CP invariant - CP being a discrete relic of the higher dimensional Lorentz group). CP is then spontaneously broken by the familon vevs that spontaneously break the family symmetry [69]. Via the Froggatt Nielsen mechanism the familions generate the (CP violating) Yukawa couplings. The resulting models do not realise exact MFV as the soft A-terms do not have exactly the same structure as the Yukawa couplings and lead to additional FCNC and CP violating effects. However these corrections are also suppressed by powers of small mixing angles.

Detailed estimates for various SUSY models of this type have been made. The most challenging channels turns out to be the EDMs and $\mu\rightarrow e\gamma$. As an example of the expected rates we consider a supergravity model with an SU(3) family symmetry that, while unbroken, guarantees the

[13] Strictly the degeneracy need only apply to the left- and right- up- and down- squarks and leptons separately.

degeneracy of squarks and sleptons in a given representation of the gauge group [14]. CP violating and flavour changing couplings are generated when the symmetry is spontaneously broken. Then the rate for $\mu \to e\gamma$ characterised by the mass insertion parameter, $|(\delta^\ell_{LR})_{12}|$ [11] is given by

$$|(\delta^\ell_{LR})_{12}| \approx 1 \times 10^{-4} \frac{A_0}{100\,\mathrm{GeV}} \frac{(200\ \mathrm{GeV})^2}{\langle \tilde{m}_l \rangle^2_{LR}} \frac{10}{\tan\beta} \left(\frac{\bar{\varepsilon}}{0.13} \right)^3 |y_1|\, |x_{123} - x_{23} - x_\Sigma| \,. \tag{36}$$

where $\bar{\epsilon}$ is the expansion parameter determining the mixing in the down quark charged lepton sector [66] (of the order of the Cabibbo angle) and y_1 and $x_{123,23,\Sigma}$ are parameters that are typically of order 1. y_1 is the coefficient of the leading super potential term generating the lepton mixing and $x_{123,23,\Sigma}$ are the coefficients multiplying the natural magnitudes of the F-terms of the familon fields. For the EDM one finds for the relevant mass insertion parameters

$$\begin{aligned}
|\mathrm{Im}(\delta^u_{LR})_{11}| &\approx 2\times 10^{-7} \frac{A_0}{100\,\mathrm{GeV}} \left(\frac{500\,\mathrm{GeV}}{\langle \tilde{m}_u \rangle_{LR}} \right)^2 \left(\frac{\bar{\varepsilon}}{0.13} \right)^3 \left(\frac{\varepsilon}{0.05} \right)^2 |y^f_1 + y^f_2|\, |x_{123} - x_{23} - x_\Sigma| \sin\phi_1, \\
|\mathrm{Im}(\delta^d_{LR})_{11}| &\approx 5\times 10^{-7} \frac{A_0}{100\,\mathrm{GeV}} \left(\frac{500\,\mathrm{GeV}}{\langle \tilde{m}_d \rangle_{LR}} \right)^2 \left(\frac{\bar{\varepsilon}}{0.13} \right)^5 \frac{10}{\tan\beta} |y^f_1 + y^f_2|\, |x_{123} - x_{23} - x_\Sigma| \sin\phi_1, \qquad (37) \\
|\mathrm{Im}(\delta^\ell_{LR})_{11}| &\approx 2\times 10^{-7} \frac{A_0}{100\,\mathrm{GeV}} \left(\frac{200\,\mathrm{GeV}}{\langle \tilde{m}_e \rangle_{LR}} \right)^2 \left(\frac{\bar{\varepsilon}}{0.13} \right)^5 \frac{10}{\tan\beta} |y^f_1 + y^f_2|\, |x_{123} - x_{23} - x_\Sigma| \sin\phi_1,
\end{aligned}$$

where ϵ is the expansion parameter determining the mixing in the up quark sector, y^f_i are the analogues of y_1 in the quark sector and ϕ_1 is a CP phase associated to the VEV of the relevant familon field.

The present experimental bound from the non-observation of $\mu \to e\gamma$ is $|(\delta^\ell_{LR})_{12}| \leq 10^{-5}$ which is in some tension with this bound requiring, for example, $\tilde{m}_l = 600\,\mathrm{GeV}$ if the remaining factors in Eq. (36) are of $O(1)$. For the EDMs the most stringent bound comes from mercury and corresponds to $|\mathrm{Im}(\delta^d_{LR})_{11}| < 6.7 \times 10^{-8}$ and requires $\tilde{m}_d = 1500\,\mathrm{GeV}$ if the other factors are of $O(1)$.

It is interesting that the SUSY mass scales needed for consistency with experiment are close to the increased mass scales needed to accommodate the 125 *GeV* mass discussed above, suggesting that the experimental limits may be quite close to the actual rates! Of course this depends on the $O(1)$ assumption for the values of the parameters; while this is the most natural value for the parameters there is a mechanism capable of suppressing the rates a bit more, by an extra power of ε in $|\mathrm{Im}(\delta^u_{LR})_{11}|$ and of $\bar{\varepsilon}$ in $|\mathrm{Im}(\delta^d_{LR})_{11}|$ and $|\mathrm{Im}(\delta^l_{LR})_{11}|$ respectively (for further details c.f. [8])

D. The little hierarchy problem in SUSY

In SUSY the Higgs quartic coupling is not a free parameter but is determined by the gauge couplings. As a result the Higgs mass is determined and given by [45]

[14] For a more general analysis and comparison with MFV expectations, see [54]

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta + \frac{3}{(4\pi)^2}\left[\ln\frac{M_S^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{X_t^2}{12 m_{\tilde{t}}^2}\right)\right] \tag{38}$$

where $M_S^2 = m_{\tilde{t_L}} m_{\tilde{t_R}}$, $X_t = A_0 - \mu \cot\beta$, A_0 is the coefficient of the soft SUSY breaking trilinear terms in the superpotential, $\tilde{m}_i$ are the SUSY breaking masses and the parameters are evaluated at the EW scale. From this one may see that to get large radiative corrections requires a large stop mass and/or X_t, leading to very heavy coloured SUSY states in the TeV range. While SUSY GUTs allow for a Higgs to be much lighter than the unification scale there remains a problem in that there is a tension between the lower limits on SUSY particles masses coming from eq(38) and from experimental bounds and the requirement that the EW scale be acceptable. In particular there are corrections to $m_{H_u}^2$ that sets the EW scale given by

$$\delta m_{H_u}^2 \simeq -\tfrac{3y_t^2}{4\pi^2}\left(m_{stop}^2 + \tfrac{g_s^2}{3\pi^2} m_{gluino}^2 \log\left(\tfrac{\Lambda}{m_{gluino}}\right)\right)\log\left(\tfrac{\Lambda}{m_{stop}}\right) \tag{39}$$

where Λ is the cut-off scale that in a SUSY GUT is the unification scale. These corrections drive the EW breaking scale near the squark and gaugino mass limits, uncomfortably large - this is the little hierarchy problem.

1. *Fine tuning measures*

In order to quantify the fine tuning needed to keep the electroweak scale much lower than the SUSY masses several fine tuning measures have been suggested [14, 34]. Two frequently used are Δ_m and Δ_q where

$$\Delta_m = \max\left|\Delta_{\gamma_i}\right|, \quad \Delta_q = \left(\sum \Delta_{\gamma_i}^2\right)^{1/2}, \quad \Delta_{\gamma_i} = \frac{\partial \ln v^2}{\partial \ln \gamma_i^2}, \quad \gamma_i = m_0, m_{1/2}, \mu_0, A_0, ... \tag{40}$$

Here the basic measure Δ_{γ_i} roughly determines the relative magnitude of the terms contributing to the Higgs mass proportional to the parameter γ_i. A value of 100 means that the cancellation should be accurate to 1 part in 100. Typically one term dominates in which case Δ_m and Δ_q are nearly equal but in the case that there are several comparable terms Δ_q would seem the more reasonable measure.

Of course the difficult question to answer when using such measures to limit the SUSY spectrum is how large the fine tuning measure can reasonably be? However it has recently been shown how the measure arises when performing a likelihood fit to the data [17, 43, 44] and this allows us to give a quantitative estimate for acceptable fine tuning. In particular the integration over the nuisance variable corresponding to the constraint requiring the W mass be as measured has the likelihood inversely proportional to Δ_q times the unconstrained likelihood.

This shows that it is the constrained likelihood that should be maximised when fitting data, *i.e.* one should maximize the ratio of the unconstrained likelihood to the fine tuning Δ ($\Delta = \Delta_{\gamma_\kappa}$ or Δ_q). If the fine tuning is large it reduces the overall likelihood. In terms of the associated χ^2 ($\chi^2_{\rm new}$) and unconstrained ($\chi^2_{\rm old}$) likelihoods are related by

$$\chi^2_{\text{new}} = \chi^2_{\text{old}} + 2\ln\Delta. \tag{41}$$

This relation can be used to infer what can be regarded as the "acceptable" upper bound of the fine tuning requiring that $\Delta_{\gamma_\kappa} \ll \exp(N_d/2)$ where N_{df} is the number of degrees of freedom. If this is satisfied then χ^2 per degree of freedom will not be significantly worsened. For simple SUSY extensions of the SM such as the CMSSM $N_{df} = O(10)$ which requires $\Delta << 100$ [44].

E. Fine tuning of the CMSSM

The "Constrained" MSSM is the name given to the MSSM with a particular choice of SUSY breaking parameters corresponding to a common scalar mass, m_0, a common gaugino mass, $m_{1/2}$, plus trilinear and bilinear scalar couplings $A_0\tilde{W}^3$ and $B_0\mu H_u H_d$, where $\tilde{W}^3$ is the trilinear superpotial with the superfieds replaced by their scalar components.

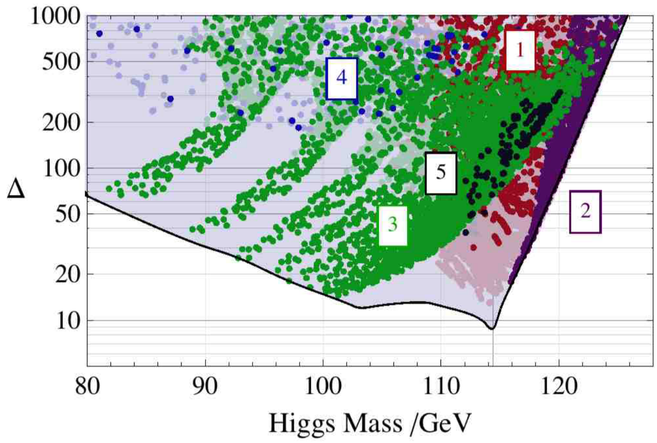


FIG. 13: *Two-loop fine-tuning versus Higgs mass for the scan over CMSSM parameters with no constraint on the Higgs mass. The solid line is the minimum fine-tuning with* $(\alpha_s, M_t) =$ *(0.1176, 173.1 GeV). The dark green, purple, crimson and black coloured regions have a dark matter density within* $\Omega h^2 = 0.1099 \pm 3 \times 0.0062$ *(i.e. 3σ saturation) while the lighter coloured versions of these regions lie below this bound. The colours and their associated numbers refer to different LSP structures as described in the text. Regions 1, 3, 4 and 5 have an LSP which is mostly bino-like. In region 2, the LSP has a significant higgsino component.*

In the CMSSM the Higgs mass is given by eq(38). The heavier the Higgs mass is the larger the radiative correction that is needed. Before the LHC start-up the bound on the Higgs mass was 114 GeV corresponding, for small X_t, to $M_S \approx 500\ GeV$. The measurement of the Higgs mass close to 126 GeV increases this to $M_s \approx 1TeV$. Thus the Higgs discovery has pushed the

SUSY threshold for the stops (or for X_t) up and this leads to the need for significantly greater fine tuning. Of course one must also allow for the X_t contribution and perform a fit to all the available data. The result of such a fit [20] that was performed before the LHC start-up is shown in Figure 13 where the fine tuning, $\Delta \equiv \Delta_m$, is shown as a function of the Higgs mass; note that the LEP bound on the Higgs mass was not included in the fit. The origin of the structure is due to two factors: the fall as the Higgs mass increases is due to the fact that the effective quartic interaction, λ_{eff}, increases, reducing the sensitivity of the EW breaking vev, $v^2 = m_{eff}^2/\lambda eff$ to changes in λ_{eff}. The sharp rise as the Higgs mass further increases is due to the fact that the sensitivity of m_{H_u} to m_0 increases rapidly as $Q^2 \sim m_{h^2}$ grows above 115 GeV^2.

It is instructive to see the origin of this sensitivity. The dominant terms in the RG equation for $m_{H_u}^2$, that sets the EW scale, involving m_0 are those proportional to the square of the top Yukawa coupling, y_t and can be integrated to give

$$\begin{aligned} m_{H_u}^2\left(Q^2\right) &= m_{H_u}^2\left(M_X^2\right) + \frac{1}{2}\left(m_{H_u}^2\left(M_X^2\right) + m_{Q_3}^2\left(M_X^2\right) + m_{\overline{u}_3}^2\left(M_X^2\right)\right)\left[\left(\frac{Q^2}{M_X^2}\right)^{\frac{3y_t^2}{4\pi^2}} - 1\right] \quad (42)\\ &= m_0^2\left(1 + \frac{3}{2}\left[\left(\frac{Q^2}{M_X^2}\right)^{\frac{3y_t^2}{4\pi^2}} - 1\right]\right) \quad (43)\end{aligned}$$

where we have used the fact that in the CMSSM all the scalar masses are equal unification scale. When the factor in square brackets is -2/3 the coefficient of m_0^2 vanishes - this is known as the focus point (FP) [37]. Remarkable the focus point is close to the electroweak scale! Clearly the appearance of the focus point affects the bounds on the SUSY spectrum coming from the hierarchy problem because the dependence of the scalars, the squarks and sleptons, on m_0^2 is suppressed and consequently, for models with the focus point, they can be much heavier than the Higgs. This nicely illustrates how correlations amongst the initial parameters can significantly reduce the fine tuning needed.

Also shown in Figure 13 is the dark matter abundance, colour coded [20] according to the dominant annihilation mechanism. The purple points with low fine tuning lie close to the focus point discussed above and one may see that, before the LHC startup, there were points in the parameter space scan with fine tuning less than 10, close to the LEP bound on the Higgs mass. As noted above, the points in the FP region have significant Higgsino component and the lowest fine tuned points are in conflict with the XENON100 bounds. However the most significant effect ruling out the low-fine-tuned points is the measurement of the Higgs mass giving

$$\Delta_{Min}^{CMSSM} > 350\,, \quad m_h = 125.6 \pm 3GeV, \quad (44)$$

unacceptably large given the constraint of eq(41).

F. Beyond the CMSSM

Of course the CMSSM is only one particular version of the MSSM, expressing the more than 100 SUSY parameters in terms of just 5. One may ask if there are other MSSM parameter choices with lower fine tuning that remain to be tested. However this is not so easy as the CMSSM has the scalar focus point that, *c.f.* eq(43), de-sensitizes the EW breaking scale to the common scalar mass m_0^2 and, in this sense, represents the class of models capable of minimising, at least part, of the fine tuning measure. In contrast gauge mediated supersymmetry breaking models do not have a common scalar mass and as a result the fine tuning in them is typically much larger [1] even though they may have a lower initial scale, M_X.

To do better than the CMSSM requires identifying a systematic way to reduce fine tuning. In the following subsection we discuss whether the fine tuning can be reduced by theoretically well-motivated modifications of the CMSSM boundary conditions for the SUSY breaking parameters. In the second subsection we consider the possibility that the fine tuning is reduced through an extension of the particle content of the MSSM.

1. Natural SUSY

In natural SUSY the universality of squark masses is relaxed with much lighter stop squarks than those associated with the first two generations [24, 29]. As we discussed in Section VI C the suppression of flavour changing neutral currents and CP violating effects place strong constraints on the first and second generation squarks favouring their mass to be in the TeV region. However the constraint on the stop squarks is very mild and this has led to the suggestion that they may be quite light, much less than a TeV. This is consistent with present LHC bounds due to the reduction in the E_T missing signals compared to that for the first two generation squarks. Since a large contribution to fine tuning comes from the sensitivity of the EW scale to the stop quark mass one may hope that fine tuning will be substantially reduced. However this turns out not to be the case because it is still necessary to have significant radiative corrections to the Higgs mass to drive it to 126 GeV and, for light stops, this must come from another sector of the theory, reintroducing large fine tuning. Recent studies [19, 46] find the fine tuning is at least 400 for the case the initial scale, M_X, at which the parameters are defined is close to the GUT scale, unacceptably large by the criterion in eq(41). Even in the case that the initial scale Λ in eq(39) is low there is no significant fine tuning advantage of a light stop if the gluino is in the TeV range.

2. Gaugino focus point

The second possibility that has been suggested is that there is a further focus point associated with the gauginos that reduces the sensitivity of the EW breaking scale to $m_{1/2}$. This can occur if the initial values of the gaugino masses have special, non-universal, ratios [48]. It has been shown that the required ratios are generated in specific GUT or string models and thus should not be considered as fine-tuned. The origin of the gaugino focus point may be seen from the RG equation

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\overline{u}_3}^2) + 2|a_t|^2\right) - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 \tag{45}$$

The first term on the RHS implicitly includes the effect of the gluino contribution to the squark masses. Although this is of higher order, since the QCD coupling is quite large, it gives a significant contribution that is comparable to that coming from the last two terms. Indeed if the gluino mass at the EW scale is similar to that of the Wino there is a cancellation between these terms that reduces the sensitivity of $m^2_{H_u}$ to the gaugino masses. Interestingly gauge coupling unification is also improved for lighter gluino mass due to the additional contribution at low scales of the gluino in the RG equations [70].

It has been shown [50] that there there are many models that naturally have the gaugino focus point at the EW scale and so it is interesting to ask how the overall fine tuning is affected. We consider the case of the MSSM with the CMSSM spectrum modified to allow for non-universal gaugino masses at the unification scale (the (C)MSSM) and requiring gauge coupling unification. The result of a fit to all the available data including the DM abundance gives

$$\Delta^{(C)MSSM}_{Min} = 60\ (500)\,, \quad m_h = 125.6 \pm 3GeV, \tag{46}$$

marginally acceptable by the criterion of eq(41).

Due to the additional flexibility in the gaugino sector, a large variety of LSP compositions is possible. For points satisfying the relic abundance upper bound the LSP is mainly composed of wino and higgsino, with typically only a very small bino component. Unlike the case for the CMSSM the direct detection cross section lies below the Xenon100 limit with the bulk of the points more than two orders of magnitude below. The correct relic abundance seems to be more easily achieved with a higgsino like LSP. A recent discussion of the phenomenology of the low-fine-tuned points can be found in [7].

3. Beyond the MSSM

The MSSM is the minimal extension of the SM, minimal in the sense that the fewest new states have been included when building a SUSY model. Could it be that non-minimal extensions reduce the fine-tuning constraints on SUSY and have not yet been experimentally tested?

Operator analysis

A useful way to look such extensions is to allow for a general modification of the MSSM by adding higher dimension operators that correspond to the effective field theory that results from integrating out additional heavy degrees of freedom and ask if such operators can reduce fine tuning. There is a unique leading dimension 5 operator with the form [21]

$$L = \frac{1}{M_*}\int d^2\theta f(X)(H_u H_d) \tag{47}$$

where $X = \theta\theta m_0$, θ the superspace parameter [31] and m_0 is the SUSY breaking scale.

This gives contributions to the scalar potential of the form

$$V = (|h_u|^2 + |h_d|^2)(\chi_1 h_u h_d + h.c.) + \frac{1}{2}(\chi_2 (h_u h_d)^2 + h.c.) \tag{48}$$

where $\chi_1 = 2f(0)\mu_{eff}/M_*$, $\chi_2 = -2f'(0)m_0/M_*$ and μ_{eff} is the effective μ term including the singlet contribution.

Note that the χ_1 term is supersymmetric so there are associated corrections involving Higgsinos that will generate Higgsino mass terms of the same order of magnitude as the correction to the Higgs mass terms (once the Higgs acquire their vevs). However in practice these corrections are going to be of O(10 Gev), important to get a Higgs mass of 125 GeV but small compared to the Higgsino mass coming from the μ_{eff} term. For this reason we concentrate on the effect in the scalar sector.

The fine tuning of this model has been analysed in [21] where it was shown that the fine tuning is significantly reduced by the first term of Eq.(48) while the second term only gives a modest reduction. The dominant effect comes from the contribution of Eq.(48) to the Higgs mass after electroweak breaking and, due to the fact that the first term involves an extra power of h_u, it gives the larger contribution.

The GNMSSM

The obvious question is what new physics can give rise to the first operator corresponding to this term. The answer is through the integration out of a new heavy gauge singlet or $SU(2)$ triplet superfield coupling to the Higgs sector. Interestingly the operator is *not* generated in the NMSSM, the simplest singlet extension of the MSSM, as it requires an explicit mass term for the singlet super field. We refer to this model as the generalised NMSSM (the GNMSSM).

The most general extension of the MSSM by a gauge singlet chiral superfield consistent with the SM gauge symmetry has a superpotential of the form

$$\mathcal{W} = \mathcal{W}_{\text{Yukawa}} + \frac{1}{3}\kappa S^3 + (\mu + \lambda S)H_u H_d + \xi S + \frac{1}{2}\mu_s S^2 \tag{49}$$

$$\equiv \mathcal{W}_{\text{NMSSM}} + \mu H_u H_d + \xi S + \frac{1}{2}\mu_s S^2 \tag{50}$$

where $\mathcal{W}_{\text{Yukawa}}$ is the MSSM superpotential generating the SM Yukawa couplings and $\mathcal{W}_{\text{NMSSM}}$ is the "normal" NMSSM with a Z_3 symmetry [35]. One of the dimensionful parameters can be eliminated by a shift in the vev v_s and can be used to set the linear term in S in the superpotential to zero, $\xi = 0$.

The form of eq(50) seems to make the hierarchy problem much worse as the SM symmetries do not prevent arbitrarily high scales for the dimensionfull mass terms. However these terms can be naturally of order the SUSY breaking scale if there is an underlying Z_4^R or Z_8^R symmetry [55, 56]. Implementing one of these R-symmetries forbids the last three terms of eq(50) so that, before SUSY breaking, the superpotential is of the NMSSM form. However, after supersymmetry breaking in a hidden sector with gravity mediation, soft superpotential terms are generated but with a scale of order the supersymmetry breaking scale in the visible sector characterised by the gravitino mass, $m_{3/2}$. With these the renormalisable terms of the superpotential take the form [56]

$$\mathcal{W}_{Z_4^R} \sim \mathcal{W}_{\text{NMSSM}} + m_{3/2}^2\, S + m_{3/2}\, S^2 + m_{3/2}\, H_u\, H_d\ , \tag{51}$$

$$\mathcal{W}_{Z_8^R} \sim \mathcal{W}_{\text{NMSSM}} + m_{3/2}^2\, S \tag{52}$$

where the $\sim$ denotes that the dimensional terms are specified up to $\mathcal{O}(1)$ coefficients. Clearly the Z_4^R case is equivalent to the GNMSSM. After eliminating the linear term in S the Z_8^R case gives a constrained version of the GNMSSM with $\mu_s/\mu = 2\kappa/\lambda$. Note that the SUSY breaking also breaks the discrete R symmetry but leaves the subgroup Z_2^R, corresponding to the usual matter parity, unbroken. As a result the lightest supersymmetric particle, the LSP, is stable and a candidate for dark matter.

Fine tuning in the GNMSSM

Fine tuning has been explored in detail for the simplified case of universal boundary conditions for the SUSY breaking parameters (CGNMSSM) [68]. Note that this goes beyond the operator analysis as we do not require that the singlet mass is large compared to the other parameters of the theory and thus cannot be integrated out. However, even allowing for the additional contribution to the Higgs mass coming from the singlet couplings, the regions of this model corresponding to low fine tuning have essentially been ruled out by a combination of LHC non-observation of SUSY and dark matter (DM) abundance. In particular the DM abundance has to be reduced below the "over-closure" limit and this is dominantly through stau co-annihilation that is only effective for relatively low m_0 and $m_{1/2}$ and hence sparticle masses in the reach of LHC8.

For the case of non-universal gaugino masses (the (C)GNMSSM) the situation changes because the LSP can now have significant Wino/higgsino components that ensures its efficient annihilation.

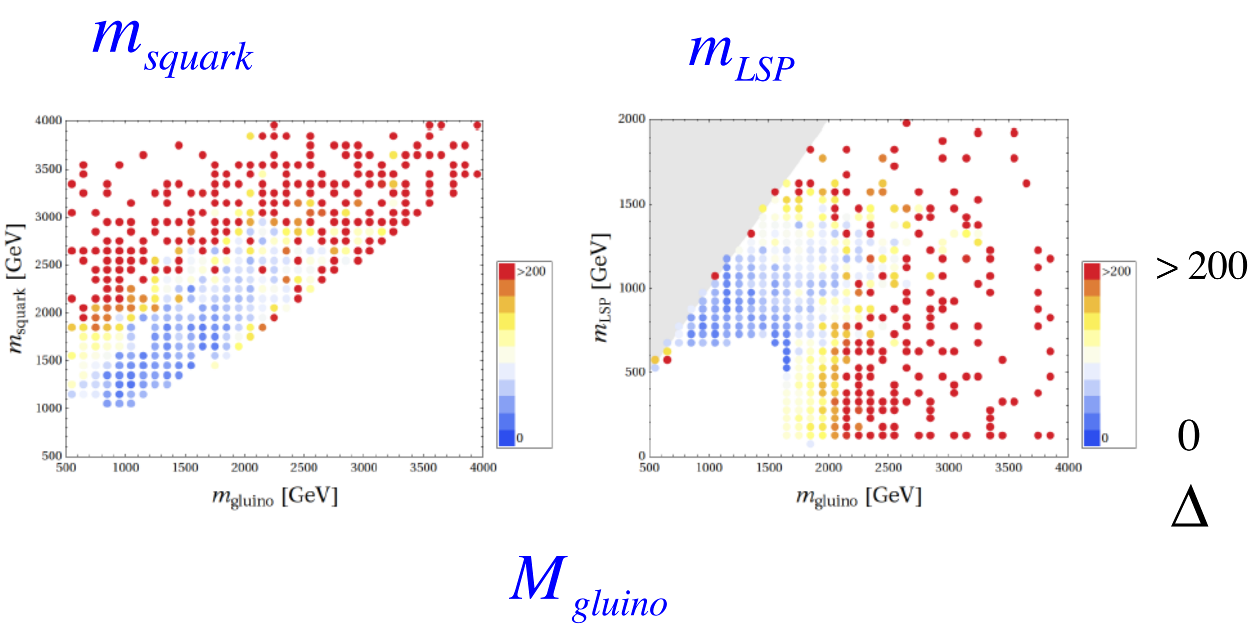


FIG. 14: *The fine tuning of acceptable points in the (C)GNMSSM plotted in a) the m_{squark}, m_{gluino} plane and b) the m_{LSP}, m_{gluino} plane.*

The minimal fine tuning after the cuts were imposed is given by

$$\Delta_{Min} = 20(25)\,, \quad m_h = 125.6 \pm 3 GeV, \tag{53}$$

perfectly acceptable by the criterion of eq(41), and there are significant areas of low fine tuning remaining to be explored by LHC14.

It is of interest to determine why the model is able to evade the current LHC bounds to see if there are lessons for future searches. In Fig. 14 we show typical masses of the superpartners in the low fine tuned region. It can be seen that points in blue with fine tuning below 100 can have gluino masses beyond 2 TeV and squark masses around 3 TeV. The most significant aspect is the fact that the points have large LSP mass ranging from $550GeV$ to $1250GeV$, corresponding to a compressed SUSY spectrum that is often the case for non-universal gaugino masses.

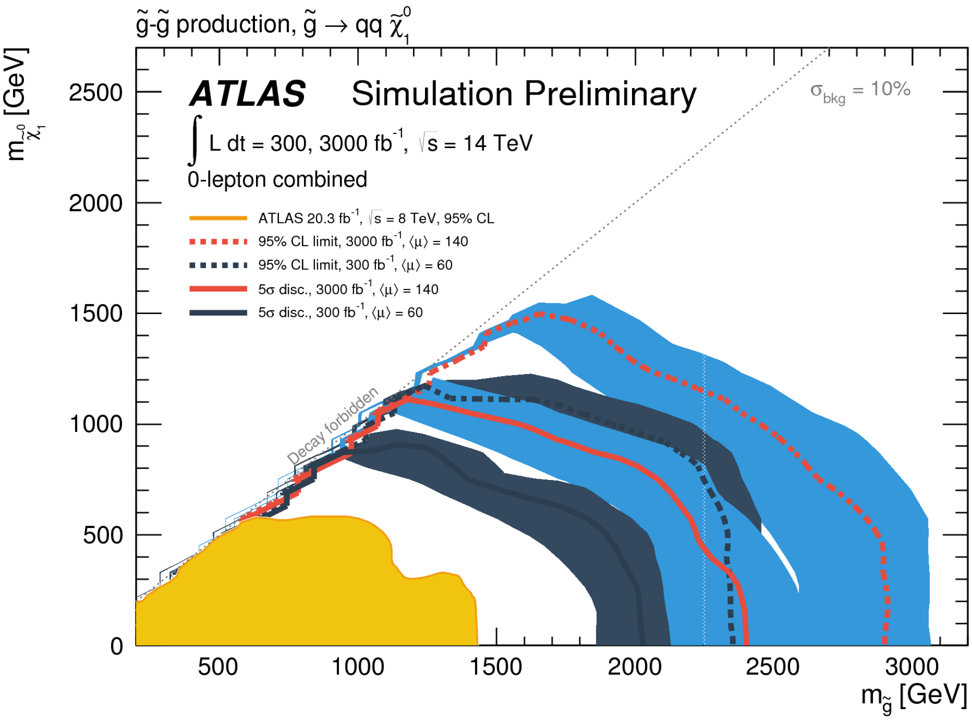


FIG. 15: *Present and future sensitivity of gluino searches to the LSP mass at the LHC.*

The reason these points escape the LHC search is obvious from Figure 15. The current LHC bound is shown in yellow and one may see that if the LSP is above $550GeV$ there is no LHC sensitivity to gluinos. In this example the squarks are taken to be heavier than the gluino but a similar conclusion applies if the opposite is true. The reason for this structure is a combination of effects: For gluino masses close to the LSP mass the missing energy is insufficient for the missing energy searches. For much heavier gluinos the missing energy is substantial but the production rate drops below the LHC sensitivity.

Will the future runs of the LHC be able to cover the full low-fine-tuned region of Figure 15? Figure 15 also shows the expected sensitivity at $14TeV$. One may see that with $300fb^{-1}$ the LSP sensitivity extends to $1200GeV$ and with the high luminosity $3000fb^{-1}$ it is $1500GeV$. Comparing with Figure 14, one sees that most, but not entirely all, of the the low fine tuned blue-region will be probed.

Dark Matter

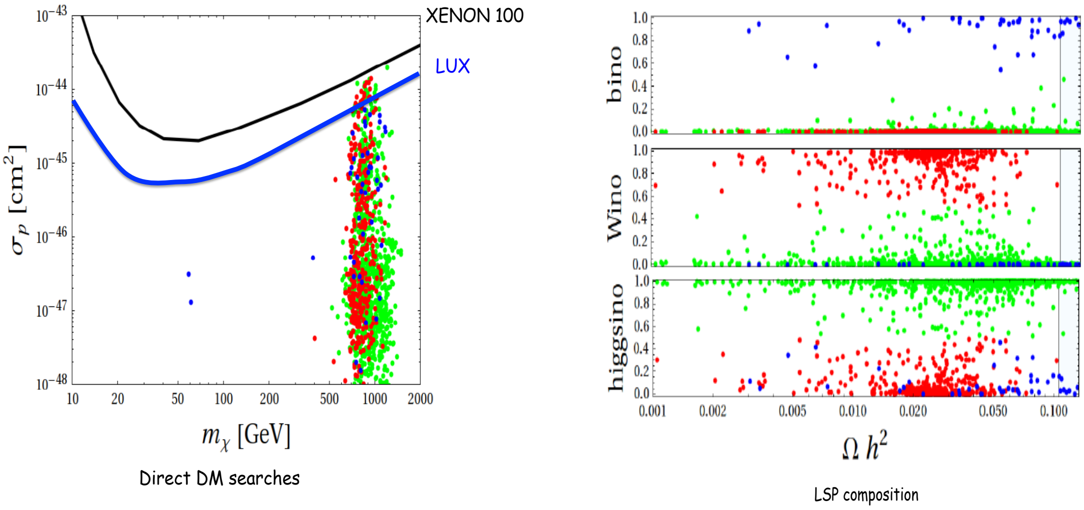


FIG. 16: *(i)The dark matter direct detection cross section as a function of the neutralino mass. It has been scaled (i.e. multiplied with* $(\Omega h^2)^{th}/0.1199$*) to account for cases with underabundant neutralinos. Also shown is the latest bound from XENON100 [9] and LUX [47]. (ii)The dark matter composition as a function of the relic density. Mostly bino-like LSPs are shown in blue, mostly Wino-like LSPs are shown in red and mostly higgsino-like LSPs are shown in green. For all points, in addition to the SUSY and Higgs cuts, a fine tuning* $\Delta < 100$ *was imposed.*

As mentioned above, the region of parameter space of the CGNMSSM that solves the little hierarchy problem has essentially been ruled out by a combination of LHC non-observation of SUSY and dark matter abundance. For the case of non-universal gaugino masses the situation changes because the LSP can now have significant non-bino component to allow for its efficient annihilation. In Figure 16 we show the direct detection cross section vs. the mass of the lightest neutralino together with the latest bound from XENON100 [9] as well as the dark matter composition as a function of the relic density. In this case direct dark matter searches are only able to probe a small part of the low-fine-tuned region of parameter space.

VII. THE COMPOSITE SOLUTION TO THE HIERARCHY PROBLEM

The composite solution to the hierarchy problem relies on having a form factor for the Higgs coupling appearing in Figure 8 that is exponentially suppressed above the composite scale which, to avoid fine tuning, should be $\leq O(1TeV)$. To achieve this there have to be be new composite resonant states in this mass range that provide characteristic experimental signatures for compositeness. The effective field theory describing composite theories below the mass scale of these states will have deviations from the SM structure that may show up in precision tests before the scale for production of the new states is reached.

As an introduction to composite theories and its description as an effective field theory[15] it is helpful to note that, while in the SM the fundamental interactions are symmetric under $SU(2) \times U(1)$, the observed mass spectrum is not. In the SM the masses come from the Higgs interactions through spontaneous symmetry breakdown that may be written in a chiral Lagrangian form (*without* the Higgs boson)

$$L_{mass} = \frac{\mathrm{v}^2}{4} Tr\left[(D_\mu\Sigma)^\dagger(D^\mu\Sigma)\right] - \frac{\mathrm{v}}{\sqrt{2}}\sum_{i,j}\left(\overline{u}_L^i d_L^i\right)\Sigma\begin{pmatrix}\lambda_{ij}^u u_R^j \\ \lambda_{ij}^d d_R^j\end{pmatrix} + h.c. \tag{54}$$

where Σ contains the would-be Goldstone modes χ^a that ultimately are eaten by the Higgs mechanism to provide the longitudinal degrees of freedom of the massive gauge bosons and

$$\Sigma(x) = \exp\left(i\sigma^a\chi^a(x)/\mathrm{v}\right), \quad D_\mu\Sigma = \partial_\mu\Sigma - ig\frac{\sigma^a}{2}W_\mu^a\Sigma + ig'\frac{\sigma^3}{2}B_\mu. \tag{55}$$

In this form the local $SU(2) \times U(1)$ invariance is now manifest:

$$\begin{aligned} &\Sigma \to U_L(x)\Sigma U_Y^\dagger(x) \\ &U_L(x) = \exp\left(i\alpha_L^a(x)\sigma^a/2\right), \quad U_Y(x) = \exp\left(i\alpha_Y(x)\sigma^3/2\right) \end{aligned} \tag{56}$$

In the unitary gauge $\langle\sum\rangle = 1$ and then it is clear that $\rho \equiv \frac{M_W^2}{M_Z^2\cos^2\theta_W} = 1$, a relation that has been verified to high precision. This relation follows because, *c.f.* eq(56), after SSB the Higgs sector has an enlarged custodial symmetry $SU(2)_L \times SU(2)_R \to SU(2)_{Custodial}$ under which χ^a form a triplet representation. In the chiral Lagrangian form this symmetry forbids the term $\mathrm{v}^2 Tr\left[\sum^\dagger D_\mu \sum \sigma^3\right]^2$ that would spoil the $\rho = 1$ prediction. In building composite models of the Higgs it is important to maintain the custodial symmetry.

Perturbative Unitarity

However the chiral Lagrangian of eq(54) violates perturbative unitarity as follows directly in the ξ gauge where one has quartic χ interactions from the gauge fixing terms

$$L_{GF} = -\tfrac{1}{2\xi}(\partial_\mu W_\mu^3 + \xi\tfrac{g\mathrm{v}}{2}\chi^3)^2 - \tfrac{1}{2\xi}(\partial_\mu B_\mu + \xi\tfrac{g'\mathrm{v}}{2}\chi^3)^2 - \tfrac{1}{2\xi}(\partial_\mu W_\mu^+ + \xi\tfrac{g\mathrm{v}}{2}\chi^+)^2 \tag{57}$$

that give the untarity violating amplitude $A(\chi^+\chi^- \to \chi^+\chi^-) = \frac{1}{\mathrm{v}^2}(s+t)$ where s, t are Mandelstam variables.

[15] In preparing this discussion of composite theories I have relied heavily on a superb review "The Higgs as a Composite Nambu-Goldstone boson" by R.Contino [26] and for a much more detailed discussion and extensive references to the original work I recommend studying this review.

The most economical way of restoring perturbative unitarity is to add a SM singlet scalar field $h(x)$ with the most general couplings of the form

$$\begin{aligned} L_H &= \frac{1}{2}(\partial_\mu h)^2 + V(h) + \frac{\mathrm{v}^2}{4} Tr\left[(D_\mu\Sigma)^\dagger(D^\mu\Sigma)\right]\left(1 + 2a\frac{h}{\mathrm{v}} + b\frac{h^2}{\mathrm{v}^2} + ...\right) \\ &- \frac{\mathrm{v}}{\sqrt{2}}\sum_{i,j}\left(\overline{u}_L^i d_L^i\right)\left(1 + c\frac{h}{\mathrm{v}}\right)\Sigma\begin{pmatrix}\lambda_{ij}^u u_R^j \\ \lambda_{ij}^d d_R^j\end{pmatrix} + h.c. \end{aligned} \tag{58}$$

These give additional contributions to $A(\chi^+\chi^- \to \chi^+\chi^-)$ as shown in Figure 17. In the SM

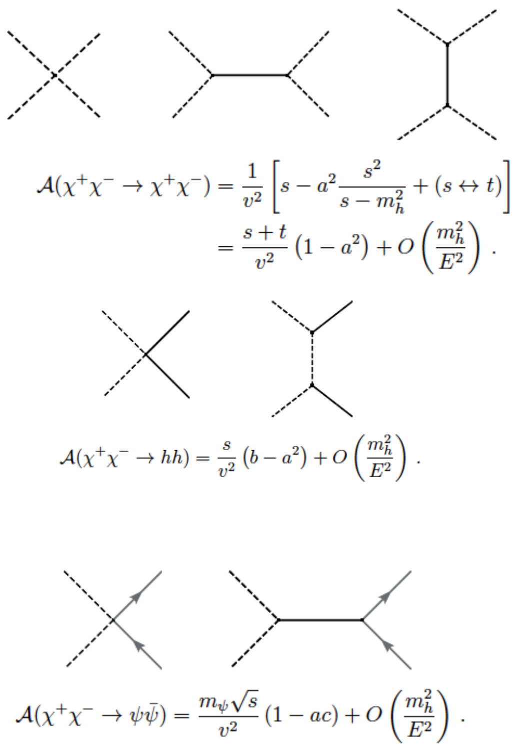


FIG. 17: *Contributions to* $A(\chi^+\chi^- \to \chi^+\chi^-)$.

$a = b = c = 1$ and (*c.f.* Figure 17) perturbative unitarity is restored. This is due to the custodial symmetry because χ^a, h form a linear representation under $SU(2)_L \times SU(2)_R$. The Higgs potential is a function of the $SO(4)$ invariant $H^\dagger H = \sum_i h_i^2$ and so when one component of h acquires a VEV we have the breaking pattern

$$SO(4) \sim SU(2)_L \times SU(2)_R \to SO(3) \sim SU(2)_{Custodial} \tag{59}$$

In this case the three would-be Goldstone modes of the SM are those of the quotient group $SO(4)/SO(3)$. We will see later how this generalised for the case the Higgs is itself a pseudo Goldstone mode.

In composite models there will be corrections generating deviations from the SM values of a, b, c because there is no reason to have perturbative unitarity in the presence of new strong interactions.

These corrections are measurable in Higgs interactions and, as discussed below, there has been considerable effort to determine their expected magnitude in composite models.

A. Technicolour

Technicolour (TC) proposes that there is a new strong interaction in which the Higgs is a bound state of new technifermions. It is modelled on QCD which, even in the absence of the Higgs boson, does spontaneously breaks the SM and give masses to the W and Z. This occurs because QCD drives a quark condensate $< \bar{q}q >$ at the scale at which it becomes strong which causes the spontaneous breakdown of the global symmetry of the strong interactions (in the absence of quark masses)

$$SU(2)_L \times SU(2)_R \times U(1)_{Baryon\ \#B} \overset{< \bar{q}q >}{\longrightarrow} SU(2)_{V=L+R} \times U(1)_B \tag{60}$$

with the pions, π^a, the (pseudo-)Goldstone modes, $\Sigma(x) = \exp(i\sigma^a \pi^a(x)/f_\pi)$, $\quad f_\pi = 92MeV$. Since the quarks carry SM charges this breaks the SM gauge symmetry $SU(2) \times U(1)$ giving W and Z mass, $M_W = gf_\pi/2 = 29MeV$, with electromagnetism ($Q = T_{3L} + T_{3R} + B/2$) unbroken. The residual symmetry $SU(2)_V$ acts as the custodial symmetry[16].

Copying this structure Technicolour starts with a new TC group, $SU(N_{TC})$, with technigluons coupling to new techniquarks that carry SM charges as well as TC charges. Thus when the TC drives a techniquark condensate it will spontaneously break the SM group at the scale, v, at which the coupling becomes large. The RG equation for the TC coupling, starting with the value at the GUT scale determines the scale, v,

$$\mu \frac{d}{d\mu} \frac{1}{g_{TC}^2}(\mu) = -\frac{\beta_0}{8\pi^2} \Rightarrow \mathrm{v} = \mathrm{M}_{Planck} \exp\left(\frac{8\pi^2}{g_{TC}^2\left(M_{Planck}\right)(-\beta_0)}\right) \tag{61}$$

and it is assumed in Technicolour that the condensate scale lies at a higher scale than in QCD with $f_\pi \to F_\pi = 246\ GeV$ thus generating acceptable W and Z masses. As for QCD, $SU(2)_V$ acts as a custodial symmetry.

However TC has several problems in generating a phenomenologically acceptable model. Apart from the difficulty in explaining why, unlike QCD, there is a light scalar state to play the role of the Higgs, it tends to give too large corrections to the variables that are used to provide precision tests of the SM. For example the S-variable, defined in terms of the vacuum polarisation, gets a contribtion

$$S \equiv -16\pi \frac{\partial}{\partial q^2} \Pi_{3B}\left(q^2\right)_{q^2=0} \sim \frac{N_{TC} T_{\mathrm{TechniDoublets}}}{\pi} \geq O(1) \tag{62}$$

to be compared to the experimental limit $S \leq 0.3$ @ 99% CL.

Another severe problem for TC is the need to generate fermion masses. In TC this requires an extension of the gauge group under which the SM quarks belong to the same multiplet as

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[16] However we know that in QCD there is no light scalar resonance to play the role of the Higgs and unitarity is enforced by a tower of heavy resonances, ρ....

the techniquarks. This extended technicolour (ETC) group is broken at an new ETC scale $SU(N_{ETC}) \to SU(3)_C \times SU(N_{TC})$. The ETC interactions couple the two sectors together giving $L_{\rm int} = \frac{g_{ETC}^2}{\Lambda_{ETC}^2}(\overline{q}q)(\overline{\psi}_{TC}\psi_{TC})$ so that the TC condensate drives fermion masses

$$m_q = \frac{g_{ETC}^2}{\Lambda_{ETC}^2} < \overline{\psi}_{TC}\psi_{TC} > \sim F_\pi \left(\frac{F_\pi}{\Lambda_{ETC}}\right)^2 \tag{63}$$

The problem arises because the new ETC interactions also generate new flavour changing neutral current (FCNC) interactions amongst the SM quarks via the 4-quark operators, $\frac{(\overline{q}q)^2}{\Lambda_{ETC}^2}$. The bounds on FCNC require $\Lambda_{ETC} \geq 10^5 TeV$ which is too large to generate acceptable quark masses via eq(63). While there are suggestions for avoiding this conflict by assuming TC behaviour is *not* like QCD, these attempts are not entirely convincing.

B. The Higgs as a pseudo-Goldstone boson

There have been significant recent developments in the study of composite models that addresses the shortcomings of Technicolour. To try to explain why there should be a light scalar state with mass below the strong binding threshold it is postulated that the Higgs should be a pseudo-Goldstone boson with mass zero up to relatively small corrections from interactions not respecting the underlying global symmetry [51]. To generate fermion masses it is assumed that fermions are (partially) composite [4, 28]. The composite components of the LH and RH fermions couple strongly to the composite Higgs, generating the fermion mass. The mass depends on the fermion composite proportion allowing for an hierarchical fermion mass structure both for the quarks, charged leptons and neutrinos.

1. *Pseudo Goldstone structure*

The idea is to generalise the structure below eq(59) to obtain additional Goldstone modes via the quotient structure G/H. For $G = SO(4),\ H = SO(3)$ one has the 3 Goldstone modes of the SM which generate the longitudinal modes $W_L^\pm,\ Z_L$. The choice $G = SO(5),\ H = SO(3)$ has 4 Goldstone modes which may be identified with $W_L^\pm,\ Z_L,\ h$ and clearly one may obtain additional scalar states by choosing larger groups.

For the case of $SO(5)/SO(3)$ the nonlinear representation has the form

$$\Sigma(x) = \Sigma_0 \exp(\Pi(x)/f), \quad \Pi(x) = -iT^a h^a(x)\sqrt{2} \tag{64}$$

In this model the coefficients of eq(58) have been computed to give [2, 27]

$$a = \sqrt{1-\xi},\ b = 1 - 2\xi,\ \xi \equiv \frac{v^2}{f^2} \tag{65}$$

The corrections to the SM are already known to be small, $\leq 20\%$, so it is necessary to choose f much larger than the Higgs VEV. What about the precision tests? The $T \equiv \Delta\rho$ parameter is under control due to the custodial symmetry. The S parameter is given by

$$S \sim 4\pi(1.36)\left(\frac{\text{v}}{m_\rho}\right)^2 \tag{66}$$

which requires $m_\rho > 2.5TeV$ for the new composite ρ resonance.

2. Fermion masses

Fermion masses are particularly interesting in this model. The expectation is that the pseudo Goldstone Higgs is a composite state, a bound state of technifermions as in the TC model. However it is now understood that it is not necessary to have an ETC group to generate fermion masses. Instead it is argued that elementary SM fermions will mix with composite techni-fermion resonances via mixing terms of the form

$$L_{mix} = f\, q_L^\alpha (\lambda_L)_I^\alpha O_L^{qI} + f\, t_R^\alpha (\lambda_R)_I^\alpha O_L^{tI} + h.c. \tag{67}$$

where $O_{L,R}$ are techni-fermion composite operators. As a result of this mixing the fermion mass eigenstates are "partially composite" [4, 28]:

$$t_{L,R}^{mass\ eigenstate} \approx t_{L,R} + \varepsilon_{L,R} O_{L,R}^t, \quad \epsilon_{L,R} = \frac{\lambda_{L,R} f}{m_\psi} \tag{68}$$

where m_ψ is the techni-fermion mass. The Yukawa couplings, h_{q_i}, of the SM fermions proceeds through the strong coupling between the composite components of the fermions and the Higgs and so has the form $h_{q_i} \propto \epsilon_L \epsilon_R$. Thus heavy quark states are dominantly composite in nature while light quark states are dominantly elementary.

5D Analogue

There is an interesting correspondence between such composite models and higher dimensional theories that is helpful in understanding composite phenomenology [3]. Consider a field ϕ in a flat extra dimension ($0 \le y \le a$) with 5D mass parameter, M. The states of the SM are zero modes in 4D and can be written as $\phi = \phi^0(y)e^{ip.x}$ with the 4D momentum satisfying $p^2 = 0$. ϕ^0 then satisfies

$$\left(\partial^2|_{5D} + M^2\right)\phi^0 e^{ip.x} = e^{ip.x}\left(-\partial_y^2 + M^2\right)\phi^0 = 0 \tag{69}$$

Solving this equation with appropriate boundary conditions gives two types of solution, $\phi^0\,(y) \propto e^{-My}$ or e^{+My}, corresponding to the wave function being peaked towards the LH or RH boundary. The interpretation in terms of the 4D composite model is that the LH boundary is where the elementary fields live while the states on the RH boundary are composite. The Kaluza Klein (KK) modes that live in the bulk are identified with the techni-resonances in the 4D composite analogue.

The 4D Yukawa couplings are then given by the overlap of the LH and RH fermions $\psi_{L,R}^0$ with the Higgs wave function H^0 (*c.f.* Figure 18).

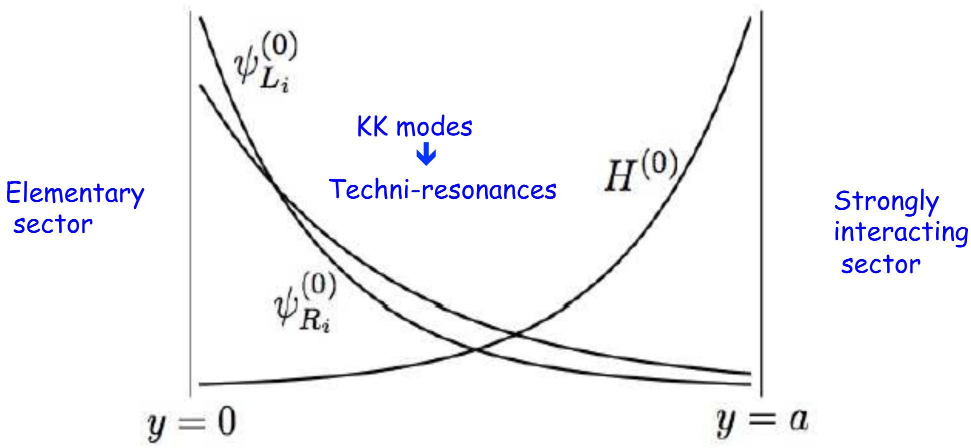

FIG. 18: *The 5D structure of fermion and Higgs states.*

$$Y_{4D,ij} \sim \int_0^a dy Y_{5D,ij}(y) e^{-(M_{L_i}+M_{R_j})y + M_H(y-a)} \tag{70}$$

Then there are two classes of coupling depending on the 5D mass parameters (which fix the 4D compositeness fractions, $\epsilon_{L,R}$):

$$\begin{aligned} Y^L_{4D,ij} &\sim Y(0)_{ij} e^{-M_H a}, \quad M_{L_i} + M_{R_j} > M_H \\ Y^R_{4D,ij} &\sim Y(a)_{ij} e^{-\left(M_{L_i}+M_{R_j}\right)a}, \quad M_{L_i} + M_{R_j} < M_H \end{aligned} \tag{71}$$

The first, with the integral dominated by $y \sim 0$, is the case the fermions are more elementary than in the second with $y \sim a$. Note that there is a strong ordering of the Yukawas, $Y^L << Y^R$.

The second case with more composite fermions is capable of describing charged fermion masses very well with large flavour hierarchies between the masses and small mixing angles. The case with more elementary fermions is capable of explaining neutrino masses and mixing because the Yukawas are much smaller and have small flavour sensitivity leading to neutrinos close in mass with large mixing angles. In this picture there is significant mixing of the Higgs with the techini-resonance bulk states.

3. The hierarchy problem and light top quark partners

As is clear from eq(17), radiative corrections to the Higgs require a cut-off scale $\leq O(1TeV)$ to avoid the need for significant fine tuning. In composite models the cut-off is due to the Higgs form factor and for this to produce a low cut-off scale there must be composite states at the same scale. However, from eq(66), there are strong lower bounds on the techni-resonances in the Higgs channel (e.g.$m_\rho > 2.5TeV$) and so one needs another source for the low-scale form factor. For the case of the top quark loop which gives the dominant contribution to the radiative Higgs correction, this must come from the composite top structure and requires light techni-fermion top quark partners of mass $\leq O(1TeV)$. Detailed studies of this [60, 61] show that the fine tuning needed is sensitive to the representation content of the top quark resonances. The minimum fine tuning occurs for the case they make up an $SO(4)$ 9-plet in which case the fine tuning is given by

$$\Delta_{composite} \sim \xi^{-1} \geq 10 \tag{72}$$

Top techni-resonance phenomenology [60, 61] Since the top partners carry colour their production cross section is large

$$QCD\ pair\ production: \quad \sigma_{m_t=500GeV} = 570fb, \quad \sigma_{m_t=1TeV} = 1.3fb \quad (8TeV\ C.M.) \tag{73}$$

Moreover the top partners carry large charge

$$9(SO(4) \sim 3_{5/3} \oplus 3_{2/3} \oplus 3_{-1/3} \quad (SU(2)_L \times U(1)_Y) \quad \supset 2 \times Q_{5/3} + Q_{8/3} \tag{74}$$

The large charge can lead to characteristic signals; for example $t_{8/3} \to 3W^+ + b + ...$ leading to significant multi-lepton branching ratios:

$$BR\left(Q_{5/3(8/3)} \to l^+l^+..\right) = 5(6)\%, \quad BR\left(Q_{5/3(8/3)} \to lll..\right) = 3(6.5)\% \tag{75}$$

The present bound coming from the first LHC run is $m_t > 770GeV$ (95%) but the run at $13TeV$ will be able to extend this above $1TeV$.

VIII. SUMMARY

The discovery of the Higgs in good agreement with the SM Higgs properties has put significant constraints on possible physics beyond the Standard Model:

- It is necessary to invoke a symmetry to keep the Higgs light: scale symmetry, supersymmetry or Nambu-Goldstone symmetry.
- All these cases require new states: more Higgs and/or Higgs interactions, SUSY partners or top quark partners.
- The fine tuning limits imply all these states should be discoverable at the LHC running at $14TeV$.
- Grand Unification is still viable but requires that we find SUSY.

The next few years will be exciting and I hope will point the way "Beyond the Standard Model"!

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