Tensor Field Theory, II

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Tensor track results 2010-2013

- Single scaling limit of tensor models at any D and double scaling limit at $d \leq 6$ have been solved and lead to branched polymers (Dartois, Gurau, R. Schaeffer...).
- Tensor field theories extend non-commutative field theory just as random tensors extend random matrices. They can be renormalized in many cases (up to rank/dimension 6) (Ben Geloun, R.). More surprisingly, TGFT's can also be renormalized in many cases (Carrozza, Lahoche, Oriti, R....). Their amplitudes are then the spin-foams of LQG.
- Some surprises: tensor $1/N$ expansion not topological, $d = 6$ critical dimension for tensorial double scaling, asymptotic freedom of quartic TFTs (Ben Geloun & co)
- Why are tensor field theories asymptotically free? see arXiv:1507.04190

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Tensor field theories = promising quantum field theories of space time with many nice features

- background independence
- sum over all topologies
- renormalizability
- asymptotic freedom

Random geometry program: look at critical point of leading graphs in $1/N$

Main open problem in 2013: leading graphs = melonic graphs

=> critical point = branched polymers. What lies beyond should incorporate sub-leading graphs

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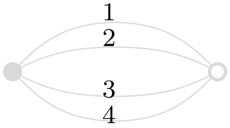

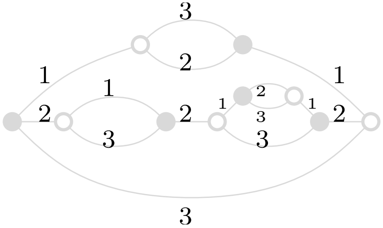

simple ”russian dolls” structure, governed by an underlying colored tree

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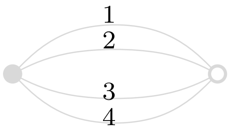

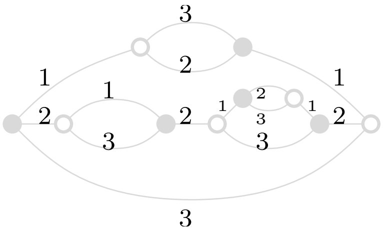

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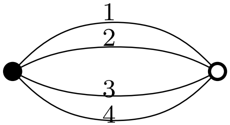

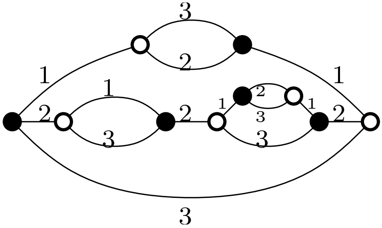

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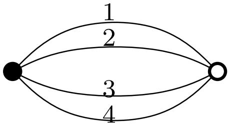

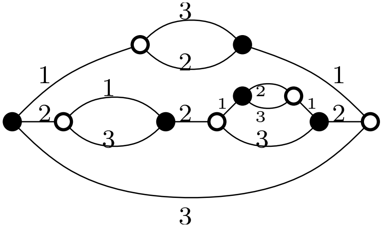

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- Phase transition and symmetry breaking analyzed at critical point (Benedetti, Gurau, Krajewski)
- Extended models in dimension 3 including non orientable geometries analyzed (Dartois, Fusy, Gurau, R. Tanasa, R. Youmans...)
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Intermediate field representation of scalar ϕ^4 theory

Intermediate field representation (IFR)

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\begin{aligned}
F &= \int_{-\infty}^{+\infty} e^{-\lambda x^4 - x^2/2} \frac{dx}{\sqrt{2\pi}} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-i\sqrt{2\lambda}\sigma x^2 - x^2/2 - \sigma^2/2} \frac{dx}{\sqrt{2\pi}} \frac{d\sigma}{\sqrt{2\pi}} \\
&= \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\log[1+i2\sqrt{2\lambda}\sigma] - \sigma^2/2} \frac{d\sigma}{\sqrt{2\pi}} \\
&= \int_{-\infty}^{+\infty} \sum_{n=0}^{\infty} \frac{V^n}{n!} d\mu(\sigma)
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$$V(\sigma) = -\frac{1}{2} \log[1 + i2\sqrt{2\lambda}\sigma]$$

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- first step (extension): decompose each Feynman ϕ^4 graph with n vertices into 3^n combinatorial maps, with new "dashed edges" and loop vertices whose "corners" or "arcs" are the former graph edges;
- second step (collapse): contract every loop vertex to a fat black vertex $=>$ result expressed in terms of maps with dashed edges only and new black vertices

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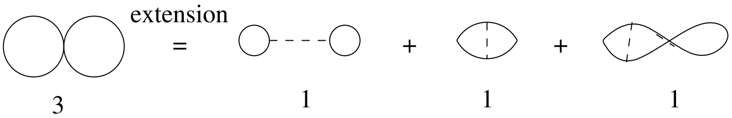
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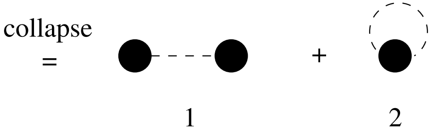
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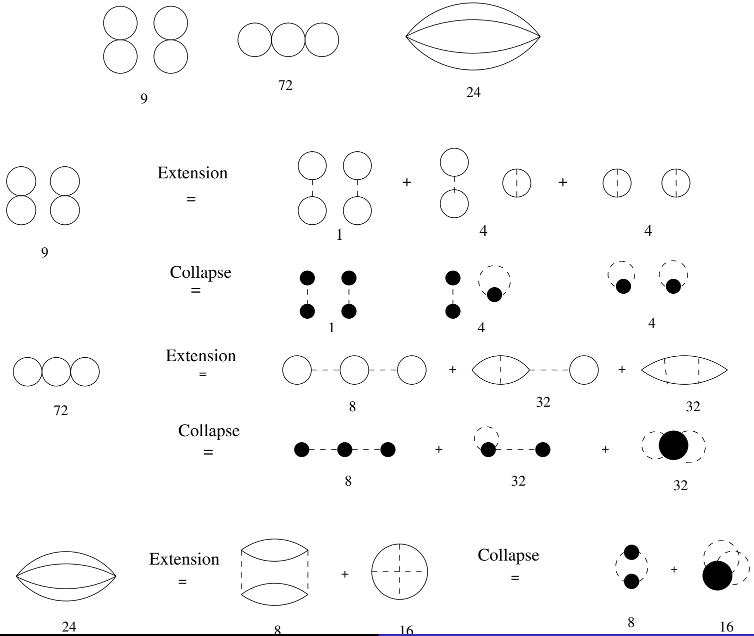
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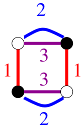
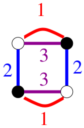
Quartic Melonic Case

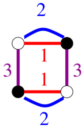
Partition function for rank d quartic melonic model

$$Z = \int_{(\mathbb{C}^N)^{\otimes d}} dT d\bar{T} e^{-N^{d-1}\left(\frac{1}{2}(\bar{T}\cdot T)+\frac{\lambda}{4}\sum_{c=1}^{d}(\bar{T}\cdot_{\hat{c}} T)\cdot_c(\bar{T}\cdot_{\hat{c}} T)\right)}.$$

Intermediate Hermitian matrix field M_c splits the melonic interaction term $(\bar{T}\cdot_{\hat{c}} T)\cdot_c(\bar{T}\cdot_{\hat{c}} T)$

Example: $d=3$, $c=1$: 

, $c=2$: 

, $c=3$: 

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$$e^{-N^{d-1}\frac{\lambda}{4}(\bar{T}\cdot_{\hat{c}} T)\cdot_c(\bar{T}\cdot_{\hat{c}} T)} = \int dM_c e^{-\frac{N^{d-1}}{2}\mathrm{Tr}M_c^2 - i\sqrt{\lambda/2}N^{d-1}\mathrm{Tr}(\bar{T}\cdot_{\hat{c}} T)M_c}.$$

Quartic Melonic Case

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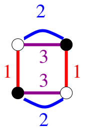
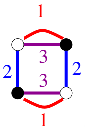
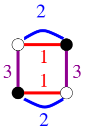
Example: $d = 3$, $c = 1$: 

, $c = 2$: 

, $c = 3$: 

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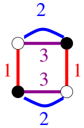
$$e^{-N^{d-1}\frac{\lambda}{4}(\bar{T}\cdot_{\hat{c}} T)\cdot_c(\bar{T}\cdot_{\hat{c}} T)} = \int dM_c e^{-\frac{N^{d-1}}{2}\mathrm{Tr}M_c^2 - i\sqrt{\lambda/2}N^{d-1}\mathrm{Tr}(\bar{T}\cdot_{\hat{c}} T)M_c}.$$

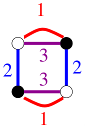
Quartic Melonic Case

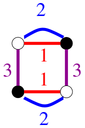
Partition function for rank d quartic melonic model

$$Z = \int_{(\mathbb{C}^N)^{\otimes d}} dT d\bar{T} e^{-N^{d-1}\left(\frac{1}{2}(\bar{T}\cdot T)+\frac{\lambda}{4}\sum_{c=1}^{d}(\bar{T}\cdot_{\hat{c}} T)\cdot_c(\bar{T}\cdot_{\hat{c}} T)\right)}.$$

Intermediate Hermitian matrix field M_c splits the melonic interaction term $(\bar{T} \cdot_{\hat{c}} T) \cdot_c (\bar{T} \cdot_{\hat{c}} T)$

Example: $d = 3$, $c = 1$: 

, $c = 2$: 

, $c = 3$: 

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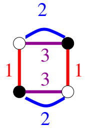
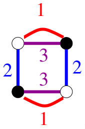
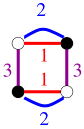
Example: $d = 3$, $c = 1$: 

, $c = 2$: 

, $c = 3$: 

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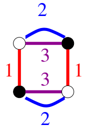
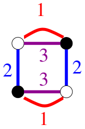
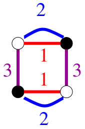
Example: $d = 3$, $c = 1$: 

, $c = 2$: 

, $c = 3$: 

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Quartic Melonic Case

Rewriting the tensor model using this representation of the interaction term

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where $\mathcal{M}_c = \mathbb{1}^{\otimes(c-1)} \otimes M_c \otimes \mathbb{1}^{\otimes(d-c)}$ for any $c \in [1, d]$.

Integrating out the T's:

$$Z = \int_{(H_N)^d} \prod_c dM_c e^{-\frac{1}{2}\sum_{c=1}^d \mathrm{Tr}\, \mathcal{M}_c^2} e^{-\mathrm{Tr}\log\left(\mathbb{1}^{\otimes D} + i\sqrt{\lambda/2}\sum_{c=1}^d \mathcal{M}_c\right)},$$

This is the intermediate field representation of the T^4 melonic tensor model.

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Results (Dartois, Eynard, Nguyen Viet-Anh)

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where $\Theta_c = (\bar{T} \cdot_{\hat{c}} T)$ and H_p is the Hermite polynomial of order p.

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The total resolvent $\mathcal{W}(x)$ of any M_c displays Catalan pole at leading order and Wigner's law at next-to-leading order

$$\mathcal{W}(x) = \frac{1}{x-\alpha} + \frac{1}{\sqrt{N^{d-2}}}(1-\alpha^2)\left(x \pm \sqrt{x^2 - \frac{1}{(1-\alpha^2)}}\right).$$

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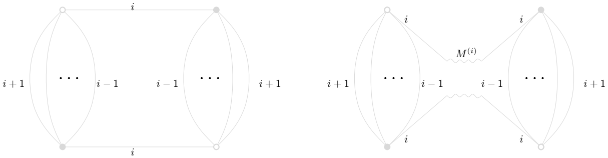

Left: a quartic melonic interaction term. Right: the same term split with the intermediate Hermitian matrix field M_i of color $i \in [1, d]$.

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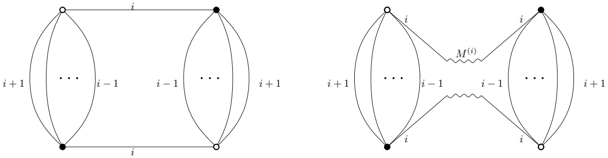

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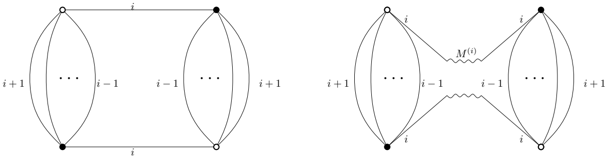

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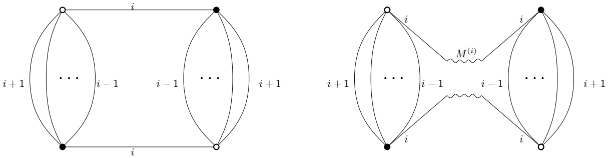

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Quartic Melonic Case, Graphical Representation

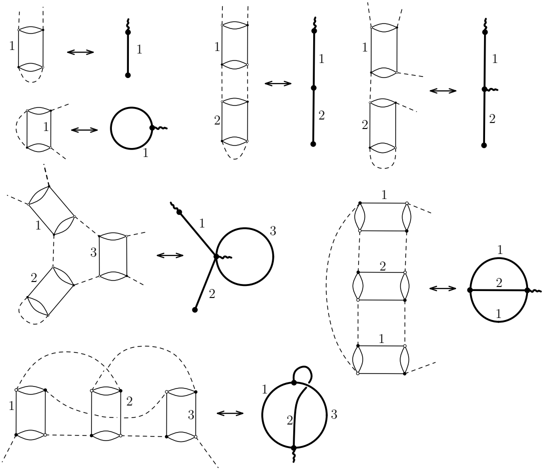

Figure: Intermediate Field Maps (courtesy L. Lionni)

Intermediate Field Representation, Summary

- Can be generalized beyond quartic invariants to any invariant (Stuffed Walsh Maps, Bonzom, Lionni, R.)
- Access to eigenvalues. Subleading order: Wigner-Dyson distribution
- Generalizations of Givental formula, topological recursion
- Good constructive formalism (Loop Vertex Expansion)
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Enhanced Rank Four Quartic Tensor Models

(joint work V. Bonzom and T. Delepouve, arXiv:1502.01365)

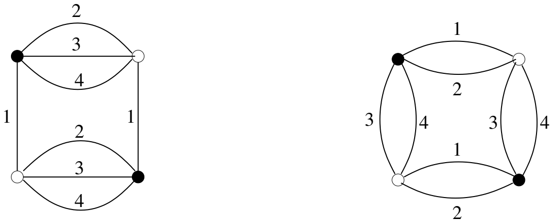

Figure: The quartic invariants at rank 4.

$$B_{\mathcal{C}_1}(\overline{\mathbf{T}},\mathbf{T}) = \sum_{n_1,\dots,n_4,n'_1,\dots,n'_4} \overline{T}_{n_1 n_2 n_3 n_4} T_{n_1 n'_2 n'_3 n'_4} \overline{T}_{n'_1 n'_2 n'_3 n'_4} T_{n'_1 n_2 n_3 n_4}$$

and three similar formulae for $B_{\mathcal{C}_2}$, $B_{\mathcal{C}_3}$ and $B_{\mathcal{C}_4}$. Also

$$B_{\mathcal{C}_{12}}(\overline{\mathbf{T}},\mathbf{T}) = \sum_{n_1,\dots,n_4,n'_1,\dots,n'_4} \overline{T}_{n_1 n_2 n_3 n_4} T_{n_1 n_2 n'_3 n'_4} \overline{T}_{n'_1 n'_2 n'_3 n'_4} T_{n'_1 n'_2 n_3 n_4}$$

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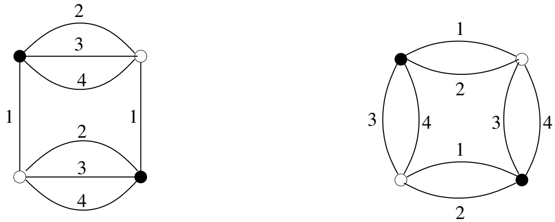

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Enhanced Rank Four Quartic Tensor Models

Standard general (color-symmetric) quartic tensor model at rank 4

$$d\mu_{standard} = d\mu_0 \; e^{-N^3 \lambda \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \; - \lambda' N^3 \sum_{i=2}^4 B_{\mathcal{C}_{1i}}(\overline{\mathbf{T}},\mathbf{T})} \; .$$

Borel summable uniformly in N for λ, λ' in cardioid domains (Delepouve, Gurau, R.).

Enhanced (maximally rescaled) general quartic model at rank 4

$$d\mu_{max} = d\mu_0 \; e^{-N^3 \lambda \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \; - N^4 \lambda' \sum_{i=2}^4 B_{\mathcal{C}_{1i}}(\overline{\mathbf{T}},\mathbf{T})} \; .$$

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Borel summable uniformly in N for λ, λ' in cardioid domains (Delepouve, Gurau, R.).

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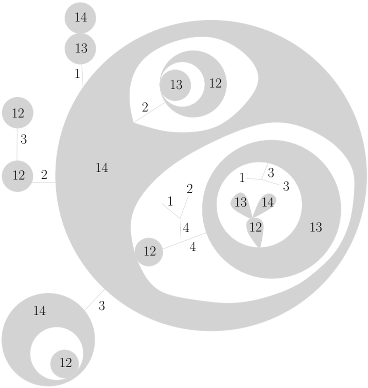
Leading Order Maps

Leading order maps in the IF representation: made of trees of unicolored edges which connect bicolored planar maps. The latter can touch one another at a single vertex at most, thus displaying a cactus structure.



Grey areas are connected components of given color types.

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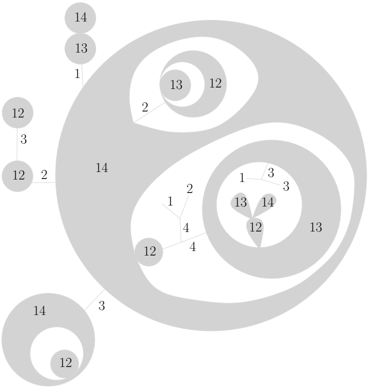

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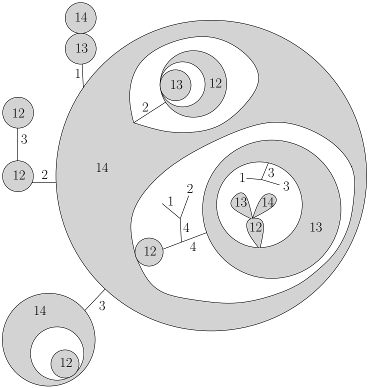

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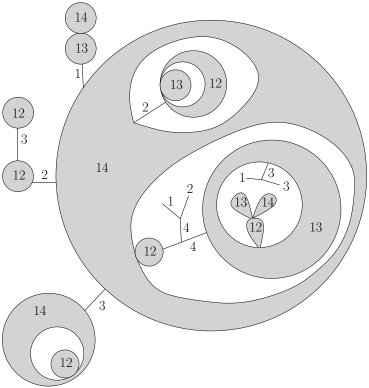

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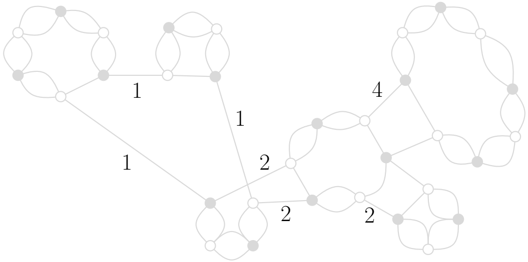
Universality

Inductive definition: A tree of necklaces of type $\{p_1, \ldots, p_n, p_{n+1}\}$ is obtained from a tree of necklaces of type $\{p_1, \ldots, p_n\}$ by removing any edge of color i and replacing it with the necklace of size p_{n+1} open on an edge of color i (preserving bipartite character).



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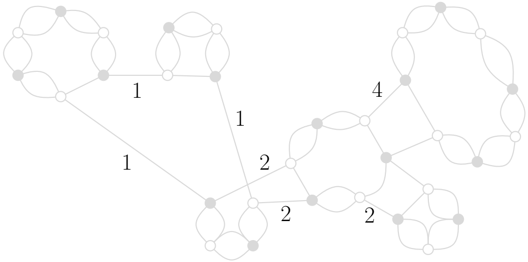

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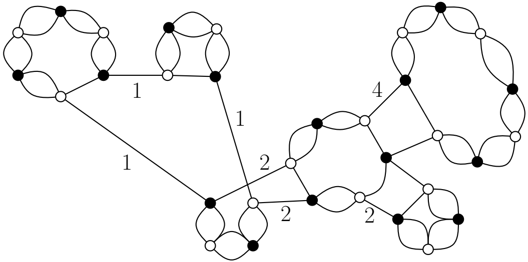

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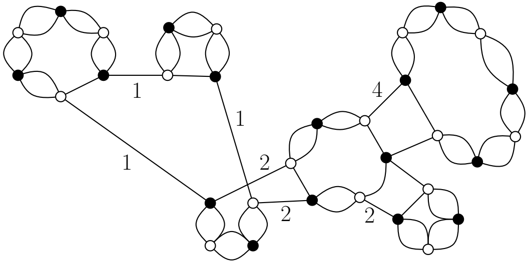

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Enhancement of trees of necklaces

Let us denote a generic tree of necklaces by $\mathcal{L}$. If it is of type $\{p_1, \ldots, p_n\}$, the enhancement it requires to contribute at large N is

$$\omega(\mathcal{L}) = \sum_{k=1}^{n} (2 + p_k) - 3(n-1) = 3 - n + \sum_{k=1}^{n} p_k .$$

Generalized model has measure

$$d\mu(\mathbf{T}, \overline{\mathbf{T}}) = \exp\Big(- \sum_{\mathcal{L}} N^{\omega(\mathcal{L})} \, t_{\mathcal{L}} \, B_{\mathcal{L}}(\mathbf{T}, \overline{\mathbf{T}}) \Big) \; d\mu_0(\mathbf{T}, \overline{\mathbf{T}}).$$

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Factorization

Theorem

Let us denote the expectation of the necklace of size p as

$$C_p = \frac{N^{2+p}}{N^4} \left\langle B_{12}^{(p)}(\mathbf{T}, \overline{\mathbf{T}}) \right\rangle = \frac{N^{2+p}}{N^4} \frac{\int d\mu(\mathbf{T}, \overline{\mathbf{T}}) B_{12}^{(p)}(\mathbf{T}, \overline{\mathbf{T}})}{\int d\mu(\mathbf{T}, \overline{\mathbf{T}})}.$$

Then the expectation of any tree of necklaces $\mathcal{L}_{\{p_1,\dots,p_n\}}$ factorizes in the large N limit

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Schwinger-Dyson equations at leading order

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$$C_p = \sum_{k=0}^{p-1} C_k \, C_{p-k-1} + \sum_{j\geq 1} j \, \partial_j V(C_1, C_2, C_3, \dots) \; C_{j+p-1}$$

where V is some polynomial, and C_p is the number of maps with root vertex of degree p. The quadratic term corresponds, as usual for equations à la Tutte, to the case where the root edge is a bridge.

The second term extends the length of the boundary face from p to $p+j-1$, which is also usual for planar maps. However, it is more complicated due to the branching process.

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Free energy behaves like $(g - g_c)^{2-\gamma}$, where γ is the entropy exponent.

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Conclusion, Open Problems

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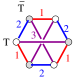
- Include terms beyond melonic LPA approximation in Wetterich equation $=>$ see how planar phase and baby universes appear in RG flow
- Organize next sub-leading levels in $1/N$ so as to see truly three dimensional random-geometric effects emerge $=>$ QG_3 

- Understand the pregeometric analog of Osterwalder-Schrader positivity, to select a subclass of tensor models which admit a real-time continuation
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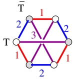
- Organize next sub-leading levels in $1/N$ so as to see truly three dimensional random-geometric effects emerge => QG_3 

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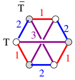

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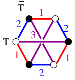

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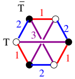

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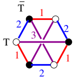
- Organize next sub-leading levels in $1/N$ so as to see truly three dimensional random-geometric effects emerge => QG_3 

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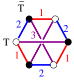
- Organize next sub-leading levels in $1/N$ so as to see truly three dimensional random-geometric effects emerge $=>$ QG_3 

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