EXPONENTIAL AND POWER-LAW EXPANSION OF THE UNIVERSE FROM THE TYPE IIB MATRIX MODEL

Asato Tsuchiya

Shizuoka Univ.

Workshop on Noncommutative Field Theory and Gravity

@Corfu September 21-27, 2015

References

- T. Azuma, Y. Ito, S.-W. Kim, J. Nishimura and A.T., work in progress
- Y. Ito, J. Nishimura and A.T., arXiv:1506.04795
- H. Aoki, J. Nishimura and A.T., JHEP 1405, 131 (2014), arXiv:1401.7848
- Y. Ito, S.-W. Kim, Y. Koizuka, J. Nishimura and A.T.,

  PTEP 2014, 083B01 (2014), arXiv:1312.5415
- J. Nishimura and A.T., JHEP 1312, 002 (2013), arXiv:1305.5547
- S.-W. Kim, J. Nishimura and A.T., JHEP 1210, 147 (2012), arXiv:1208.0711
- J. Nishimura and A.T., PTEP 2013, 043B03 (2013), arXiv:1208.4910
- S.-W. Kim, J. Nishimura and A.T., Phys. Rev. D86, 027901 (2012), arXiv:1110.4803
- S.-W. Kim, J. Nishimura and A.T., Phys. Rev. Lett. 108, 011601 (2012), arXiv:1108.1540

Introduction

String theory as unified theory

- String theory is a promising candidate for unified theory including gravity
- One expects string theory to determine

  space-time on which strings propagate

  (in particular space-time dimensions)

  gauge group

  matter contents (number of generations)

  Inflaton potential (and initial condition)

  etc.

Issues in string theory

perturbation theory ＋ D-brane

⬇

Numerous vacua (Landscape)

There are numerous vacua that are theoretically allowed

various
- space-time dimensions
- gauge groups
- matter contents (number of generations)
- cosmological constants

Predictive power of string theory is restricted

Cosmic (initial) singularity Liu-Moore-Seiberg (’02), ……

In general, perturbation theory cannot resolve the cosmic singularity

Non-perturbative effects are important at the beginning of the universe

Nonperturbative formulation of string theory

Nonperturbative formulation of string theory

determine the true vacuum uniquely

resolve the cosmic singularity Cf.) lattice QCD for QCD

Here we study the type IIB matrix model which has bee proposed as a nonperturbative formulation of superstring theory

In particular, we consider a Lorentzian version of the model

Outline

1. Introduction
2. Review of the type IIB matrix model
3. Expanding (3+1)-dim. Universe from the type IIB matrix model
4. Large-scale numerical simulation of simplified models
5. Cutoff independence
6. Conclusion and outlook

Review of type IIB matrix model

Type IIB matrix model

Ishibashi-Kawai-Kitazawa-A.T. ('96)

$$S = -\frac{1}{g^2}\mathrm{tr}\left(\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{1}{2}\bar{\Psi}\Gamma^\mu[A_\mu, \Psi]\right)$$

Dimensional reduction of 10D U(N) N=1 SYM to zero dimension

$N \times N$ Hermitian matrices

A_μ : 10D Lorentz vector $(\mu = 0, 1, \cdots, 9)$

Ψ : 10D Majorana-Weyl spinor

Large-N limit is taken

Space-time does not exist a priori, but is emergent

Cf.) Steinacker's talk on Monday
Szabo, Chatzistavrakidis, Yang,…

Manifest SO(9,1) symmetry and manifest 10D N=2 SUSY

Correspondence with world-sheet action

Green-Schwarz action of Schild-type for type IIB superstring with κ symmetry fixed

$$S=\int d^2\sigma\left(\frac{1}{4}\{X_\mu,X_\nu\}^2-\frac{i}{2}\bar{\psi}\Gamma^\mu\{X_\mu,\psi\}\right) \qquad \{A,B\}=\frac{\partial A}{\partial\sigma_1}\frac{\partial B}{\partial\sigma_2}-\frac{\partial A}{\partial\sigma_2}\frac{\partial B}{\partial\sigma_1}$$

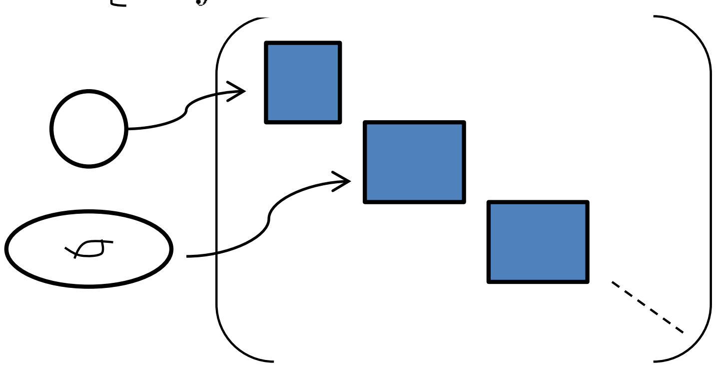
SO(9,1) symmetry＋10D N=2 SUSY

matrix regularization Cf.)O'Connor' talk

$$\begin{cases} X_\mu(\sigma_1,\sigma_2)\to A_\mu \\ \psi(\sigma_1,\sigma_2)\to\Psi \\ \{\ ,\ \}=-i[\ ,\ ] \\ \int d^2\sigma\to \mathrm{tr} \end{cases}$$

Type IIB matrix model

$$S=-\frac{1}{g^2}\mathrm{tr}\left(\frac{1}{4}[A_\mu,A_\nu]^2+\frac{1}{2}\bar{\Psi}\Gamma^\mu[A_\mu,\Psi]\right)$$



1) This regularization preserves all symmetries of the GS action
2) The matrix model can represent multi strings
   2nd quantized

10D N=2 SUSY

Corresponding to 10DN=2SUSY possessed by Green-Schwarz action

$$\begin{cases} \bar{\epsilon}Q^{(1)}A_\mu = i\bar{\epsilon}\Gamma_\mu\Psi \\ \bar{\epsilon}Q^{(1)}\Psi = \frac{i}{2}[A_\mu, A_\nu]\Gamma^{\mu\nu}\epsilon \end{cases}$$

$$\begin{cases} \bar{\xi}Q^{(2)}A_\mu = 0 \\ \bar{\xi}Q^{(2)}\Psi = \xi\,1 \end{cases}$$

$$\begin{cases} c_\nu P^\nu A_\mu = c_\mu\,1 \\ c_\nu P^\nu \Psi = 0 \end{cases}$$

dimensional reduction of
10D N=1 SUSY

translation of
eigenvalues

10D N=2 SUSY

$$[\bar{\epsilon}_1\tilde{Q}^{(i)}, \bar{\epsilon}_2\tilde{Q}^{(j)}] = 2i\delta^{ij}\bar{\epsilon}_2\Gamma_\mu\epsilon_1 P^\mu$$

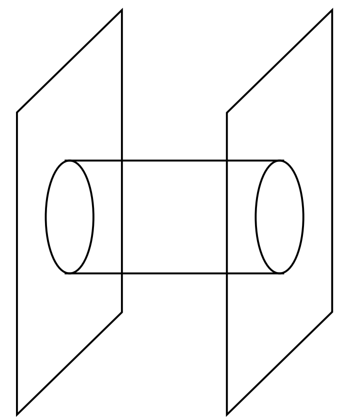
$$\tilde{Q}^{(1)} = Q^{(1)} + Q^{(2)}$$
$$\tilde{Q}^{(2)} = i(Q^{(1)} - Q^{(2)})$$

eigenvalues of A_μ are coordinates

strongly suggests that the model includes gravity

Interaction between D-branes

$$A_\mu = \begin{pmatrix} x_\mu^{(1)}1 + a_\mu^{(1)} & \\ & x_\mu^{(2)}1 + a_\mu^{(2)} \end{pmatrix}$$



$$W = \frac{1}{(x^{(1)} - x^{(2)})^8}\left(-12\mathrm{tr}(f_{\mu\lambda}^{(1)} f_{\lambda\nu}^{(1)})\mathrm{tr}(f_{\mu\rho}^{(2)} f_{\rho\nu}^{(2)}) + \frac{3}{2}\mathrm{tr}(f_{\mu\nu}^{(1)} f_{\mu\nu}^{(1)})\mathrm{tr}(f_{\lambda\rho}^{(2)} f_{\lambda\rho}^{(2)})\right)$$

graviton scalar

$$f_{\mu\nu}^{(i)} = i[a_\mu^{(i)}, a_\nu^{(i)}]$$

Light-cone string field theory

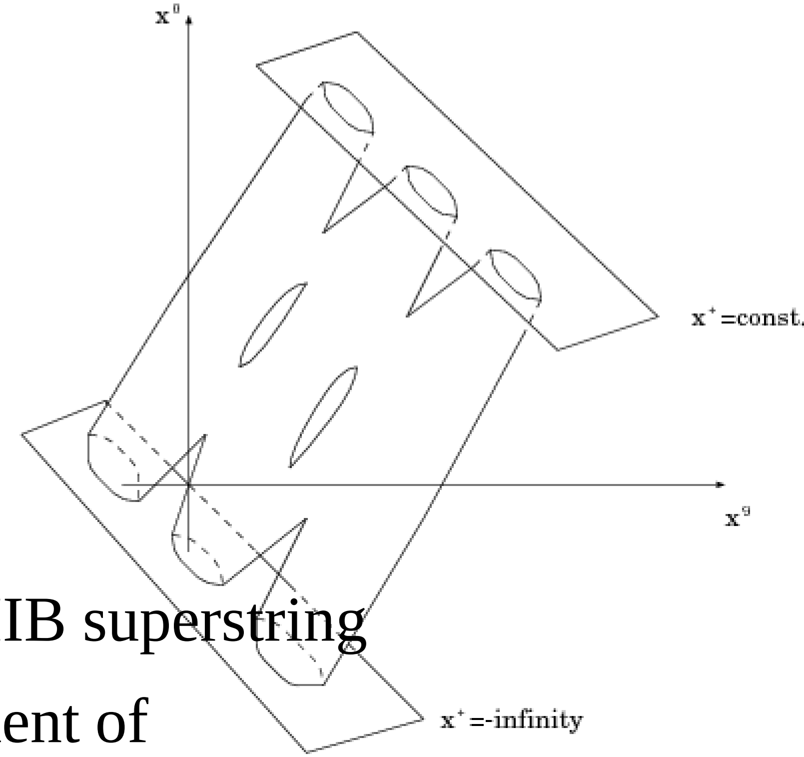
Fukuma-Kawai-Kitazawa-A.T. ('97)

$$W[k] = \mathrm{tr} P exp\left[i \int d\sigma (k^{\mu}(\sigma) A_{\mu} + (\mathrm{fermion}))\right] \sim \Psi[k(\cdot)]$$

Schwinger-Dyson equation for $W[k]$
on the light front

↓

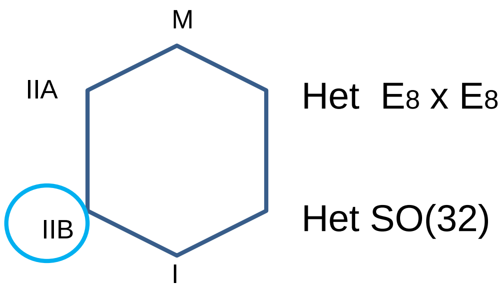
light-cone string field theory for
type IIB superstring



This implies that type IIB matrix model
reproduces perturbation theory of type IIB superstring

The definition of the model is independent of
perturbation theory, so the strong coupling regime is tractable

String duality



One can start from anywhere with nonperturbative formulation to tract strong coupling regime

Euclidean model

Lorentzian model

$$S_b \propto \mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) = -2\,\mathrm{tr}\,(F_{0i})^2 + \mathrm{tr}\,(F_{ij})^2$$

$$F_{\mu\nu} = -i[A_\mu, A_\nu]$$

opposite sign !

not bounded below

looks quite unstable

Euclideanization

Wick rotation

$$A_0 = iA_{10} \qquad \Gamma^0 = -i\Gamma_{10}$$

Euclidean model

manifest SO(10) symmetry

S_b : positive definite

Euclidean model is well-defined without cutoffs

Krauth-Nicolai-Staudacher ('98),
Austing-Wheater ('01)

People have been studying the Euclidean model

Expanding (3+1)-dim. Universe from the type IIB matrix model

Kim-Nishimura-A.T. ('11)

Why Lorentzian model?

- see time evolution of the Universe

  ~ need to study real time dynamics

- Wick rotation in gravitational theory is more subtle than field theory on flat space-time

ex.) causal dynamical triangulation（CDT) Ambjorn-Jurkiewicz-Loll ('05)

- Recent study of the Euclidean model using Gaussian expansion method suggests dynamical generation of 3-dimensional space-time

Nishimura-Okubo-Sugino ('11)

Here we study Lorentzian version of the type IIB matrix model

Regularization

- definition of path integral

Natural from the viewpoint of the Wick rotation on the worldsheet

$$Z=\int dA d\Psi e^{iS}=\int dA e^{iS_b}\mathrm{Pf}\mathcal{M}(A)$$

$$\sigma_2=i\sigma^0$$

Sign problem?

- $S_\mathrm{b}\propto \mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu})=-2\,\mathrm{tr}\,(F_{0i})^2+\mathrm{tr}\,(F_{ij})^2$

A_0 and A_i diverge ⟹ Introduce IR cutoffs

$$\frac{1}{N}\mathrm{tr}A_0^2\leq\kappa\frac{1}{N}\mathrm{tr}A_i^2 \qquad \frac{1}{N}\mathrm{tr}A_i^2\leq L^2$$

Discuss whether these cutoff are removed in the large-N limit

- Homogeneous action ⟹ integrate over scale factor of A_μ

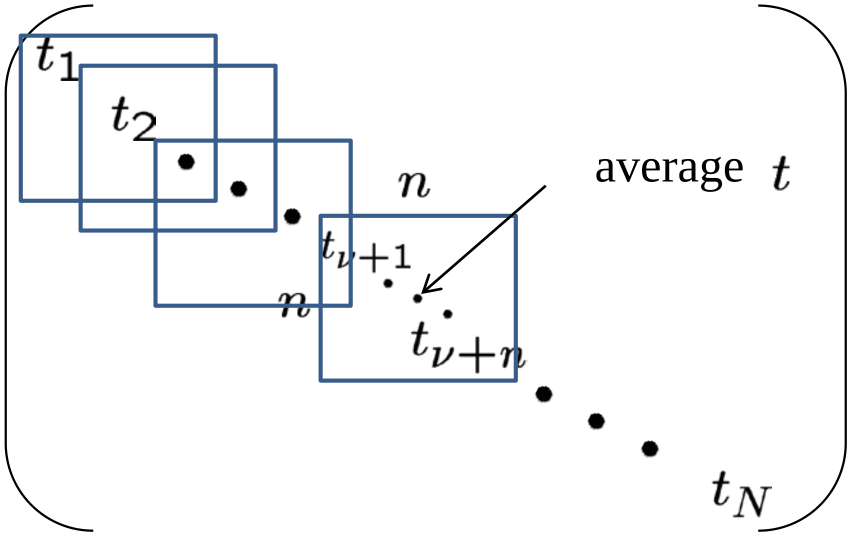
get $S_b=0$ constraint ⟹ no sign problem

$$Z=\int dA\ \mathrm{Pf}\mathcal{M}(A)\delta\left(\frac{1}{N}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})\right)\delta\left(\frac{1}{N}\mathrm{tr}(A_i^2)-1\right)\theta\left(\kappa-\frac{1}{N}\mathrm{tr}(A_0^2)\right)$$

Emergence of concept of `` … n”

Time does not exist a priori, so it is nontrivial whether concept of time evolution exists

$A_0 =$

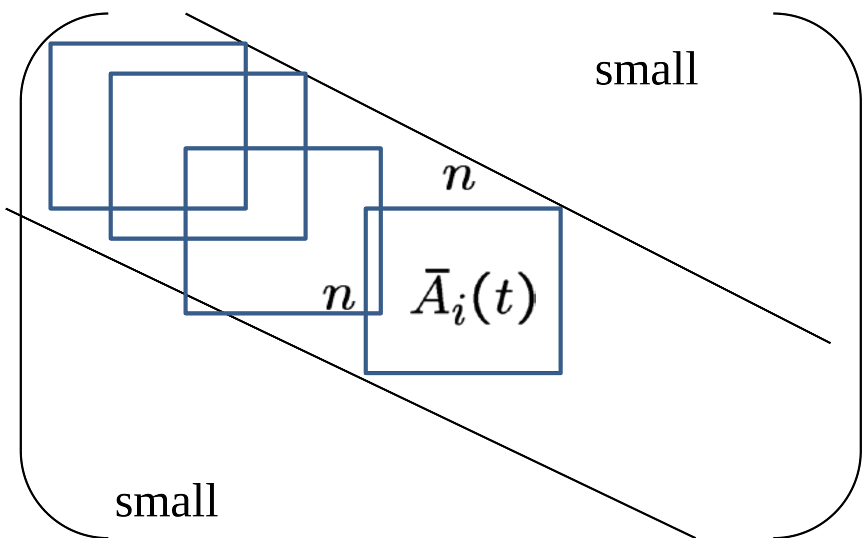


$t_1 < t_2 < \cdots < t_N$

These values are dynamically determined

$A_i =$

$(i = 1, \cdots, 9)$



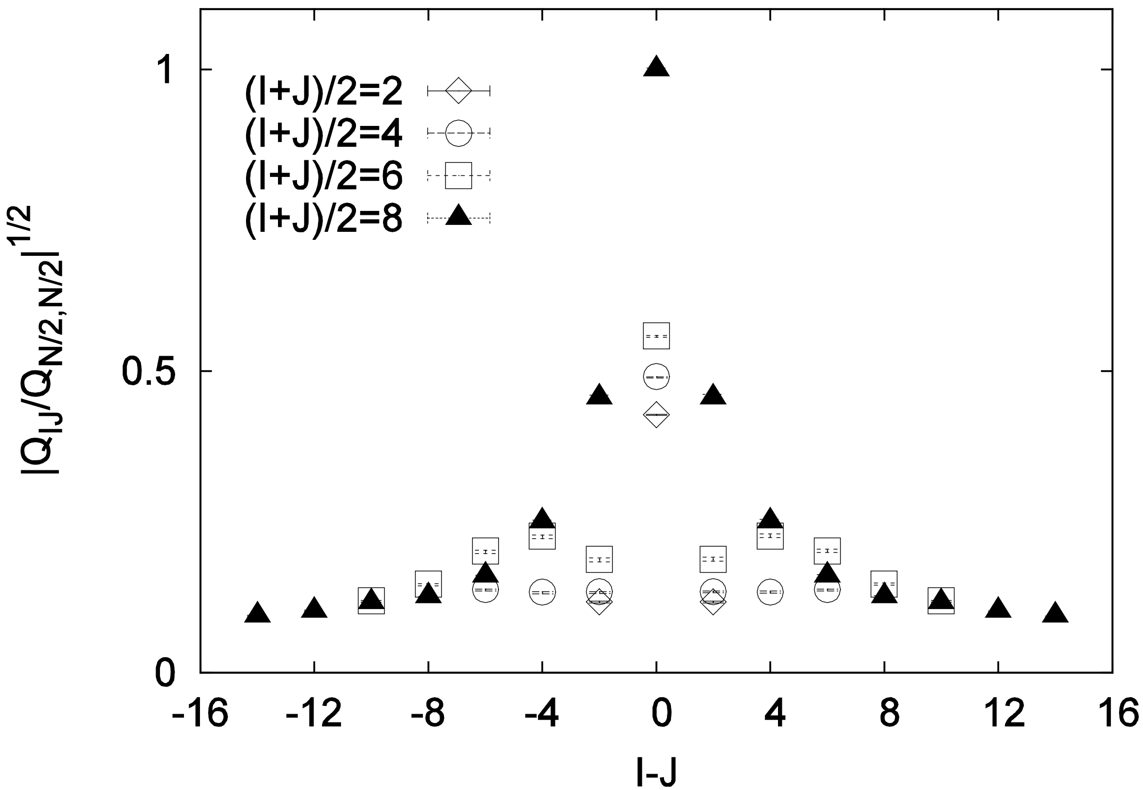
Band-diagonal structure is observed, which is nontrivial

$\bar{A}_i(t)$ represents space structure at time t

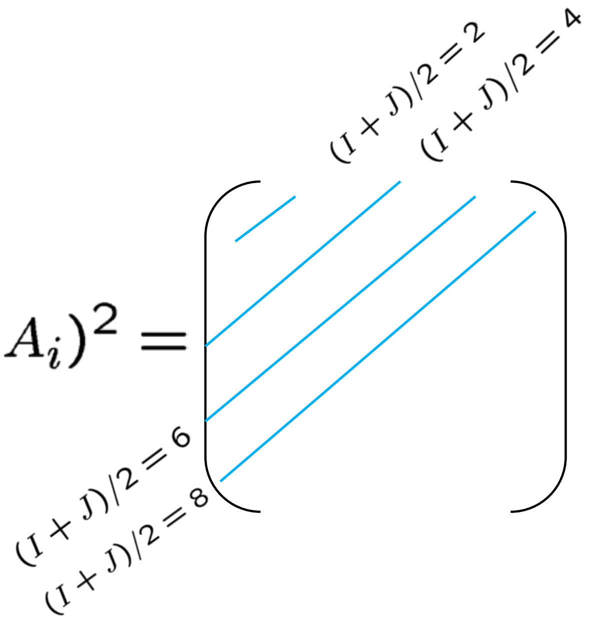
Locality of time is guaranteed

Determination of block size

$N = 16$



$(A_i)^2 =$



We take $n = 4$

SSB of SO(9) symmetry

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}(\bar{A}_i(t)\bar{A}_j(t))$$

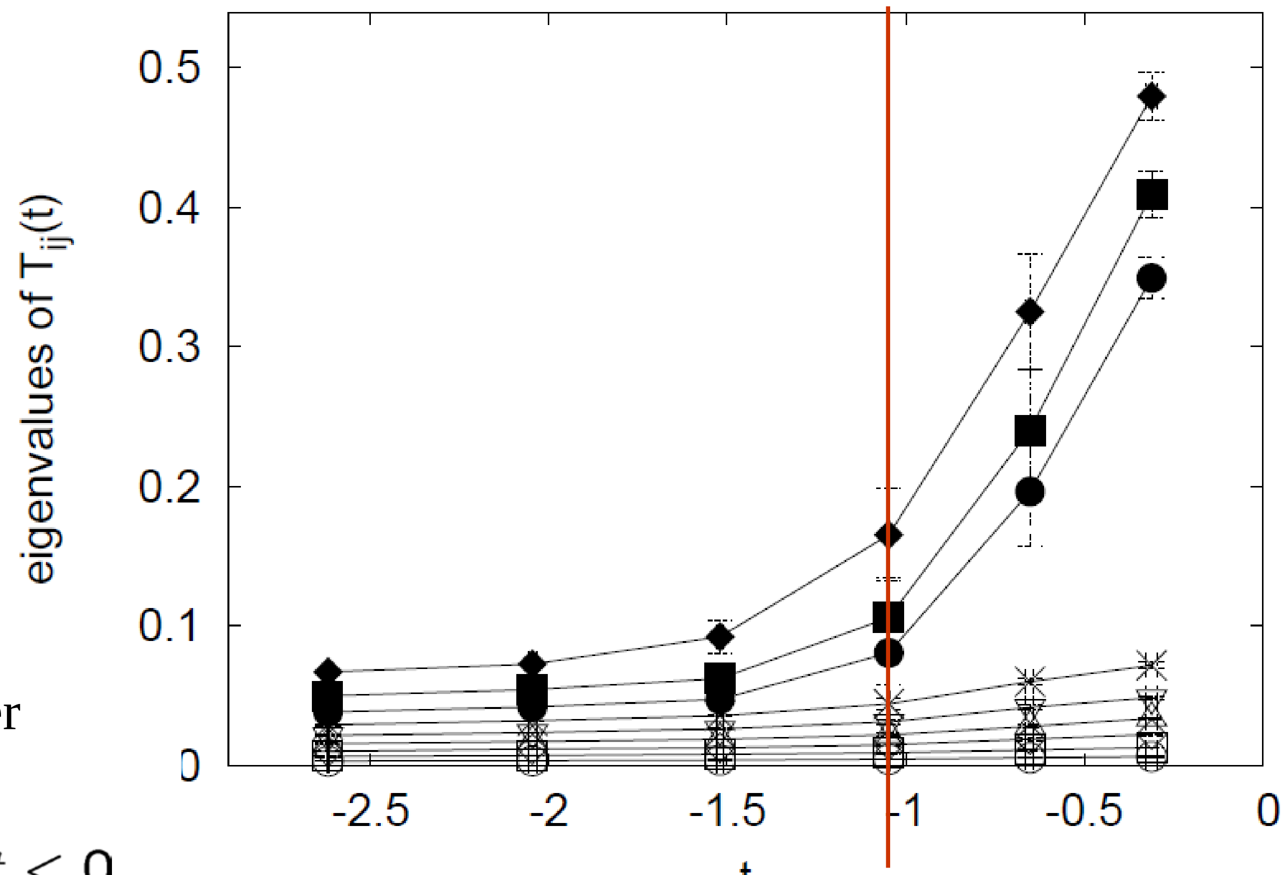
~Moment of inertia tensor

$$SO(9) \xrightarrow{\text{SSB}} SO(3)$$

i,j = 1~9

$N = 16$

$\kappa = 4.0$



"critical time" t_c

symmetric under

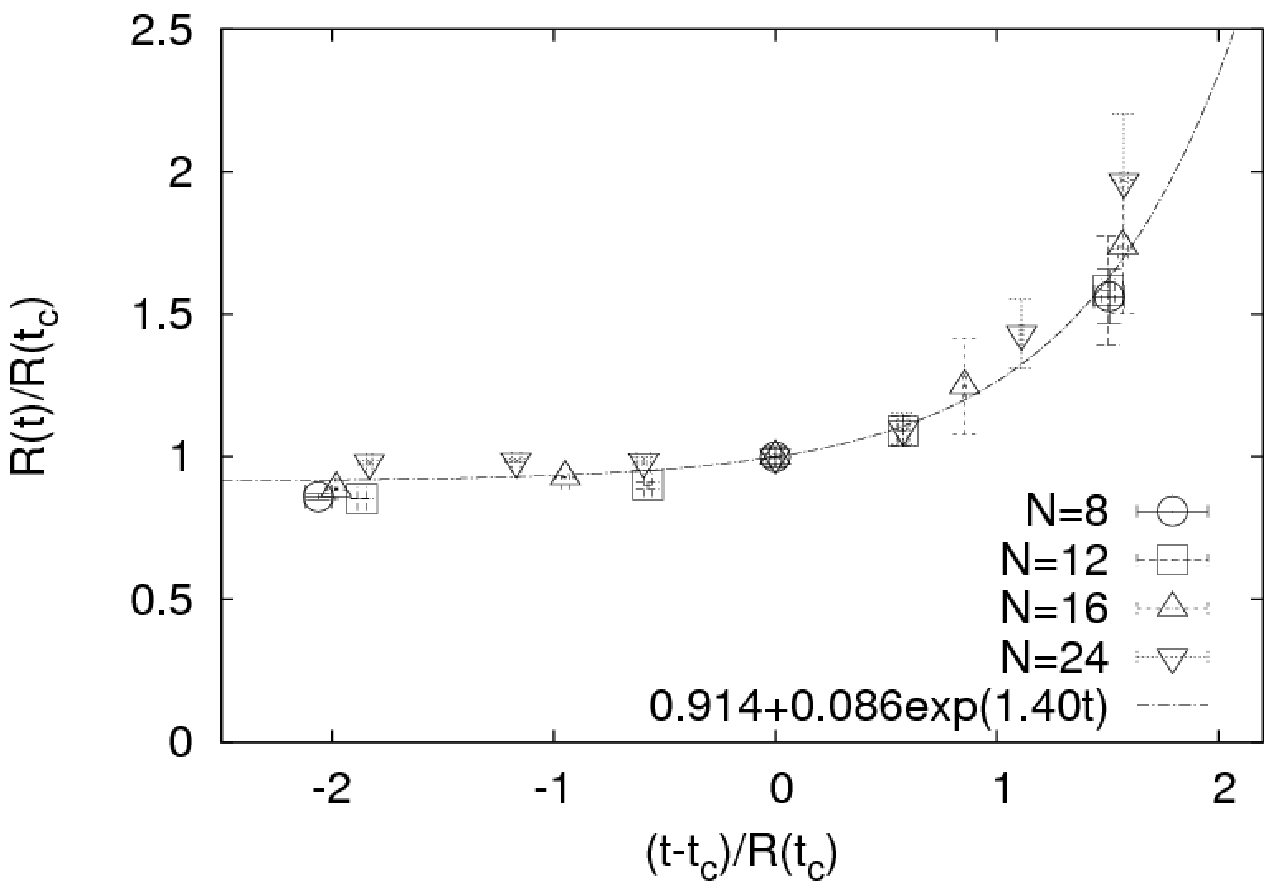
$t \to -t$

we only show $t < 0$

Exponential expansion

$$R(t)^2 \equiv \frac{1}{n}\mathrm{tr}\bar{A}_i(t)^2 = T_{ii}(t)$$

~extent of space at time t



$R(t_c)$

unique scale parameter ~size of the Universe at the beginning

Consistent with exponential expansion

Reminiscent of inflation

$$\kappa = 2.0 \times N^{1/4}$$

Large-scale numerical simulation of simplified models

Ito-Nishimura-A.T. ('15)

Simulating at large N

- Need to simulate at larger N to see later times
- Calculation time ～ N^5 ← Pfaffian

Here

- Make approximation for contribution of fermions

  → calculation time ～ N^3
- Large-scale parallel computation （Kei supercomputer)

  ⇨ N=512

Contribution of fermion

$$S_f = \mathrm{tr}(\bar{\Psi}\Gamma^{\mu}[A_{\mu}, \Psi])$$

$$= \underline{\mathrm{tr}(\bar{\Psi}\Gamma^{0}[A_{0}, \Psi])} + \underline{\mathrm{tr}(\bar{\Psi}\Gamma^{i}[A_{i}, \Psi])}$$

Dominant at early times

$$A_0 \gg A_i$$

Keep only 1st term

Simplified model for early times (VDM model)

$$\mathrm{Pf}\mathcal{M}(A) = \prod_{I<J}(t_I - t_J)^{2(d-1)}$$

Repulsive force between eigenvalues of A_0

Dominant at late times

$$A_0 \ll A_i$$

Simplified model for late times (Bosonic model)

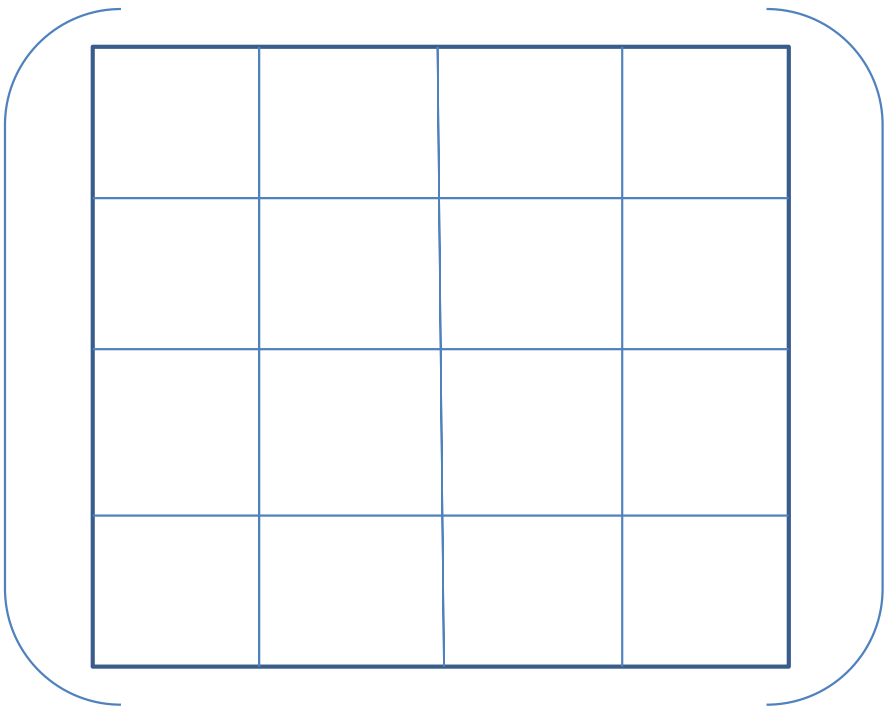
$$\mathrm{Pf}(\mathcal{M}) = 1$$

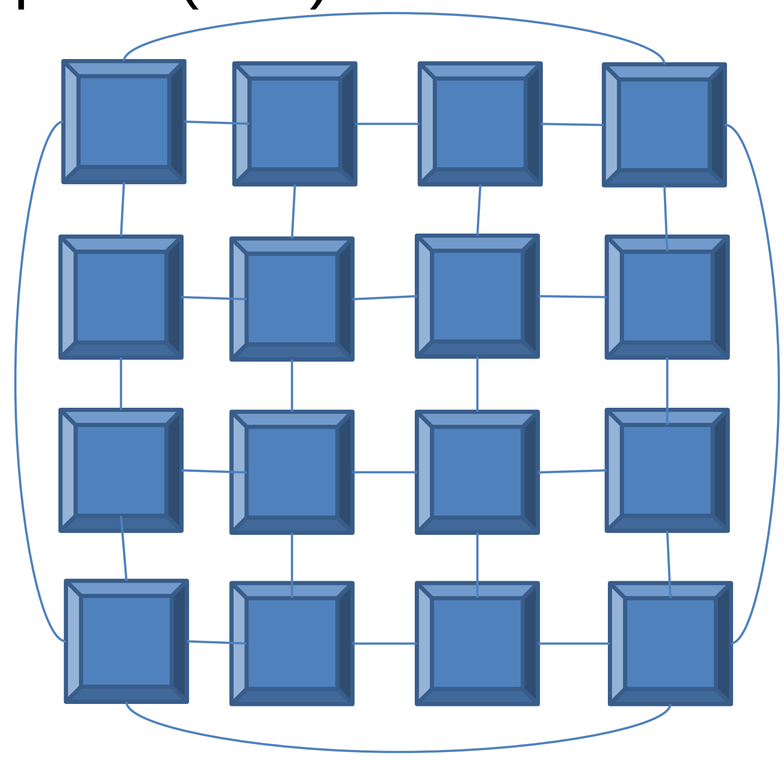
$\frac{1}{N}\mathrm{tr}A_0^2 \leq \kappa L^2$ not needed

Parallelization

Typically
32 × 32 = 1024nodes for N=1024
32 × 32 block matrix in each node

Computation at super parallel computer (Kei)





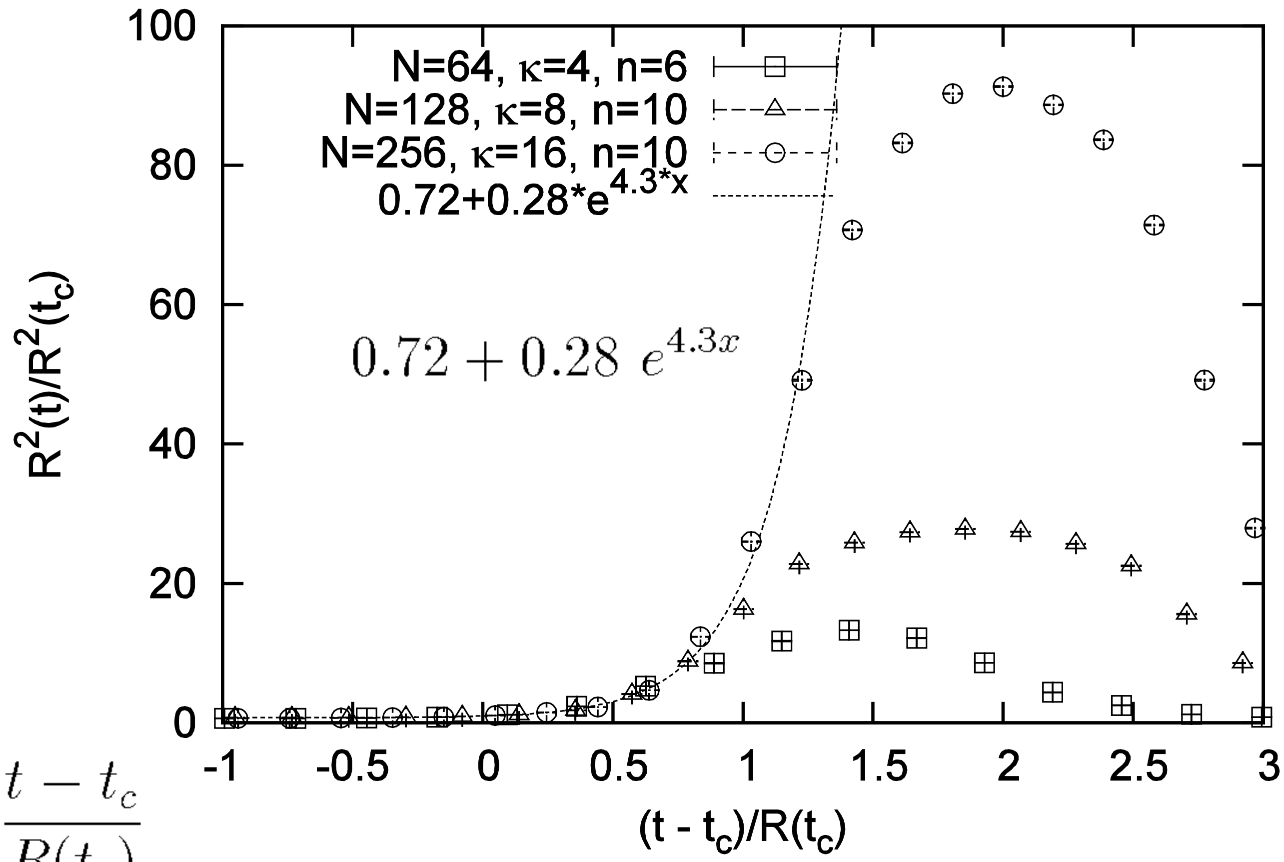
Each node takes care of each block

Two-dim. torus configuration of nodes

communication between nodes by MPI

Thread parallelization by OpenMP in each node

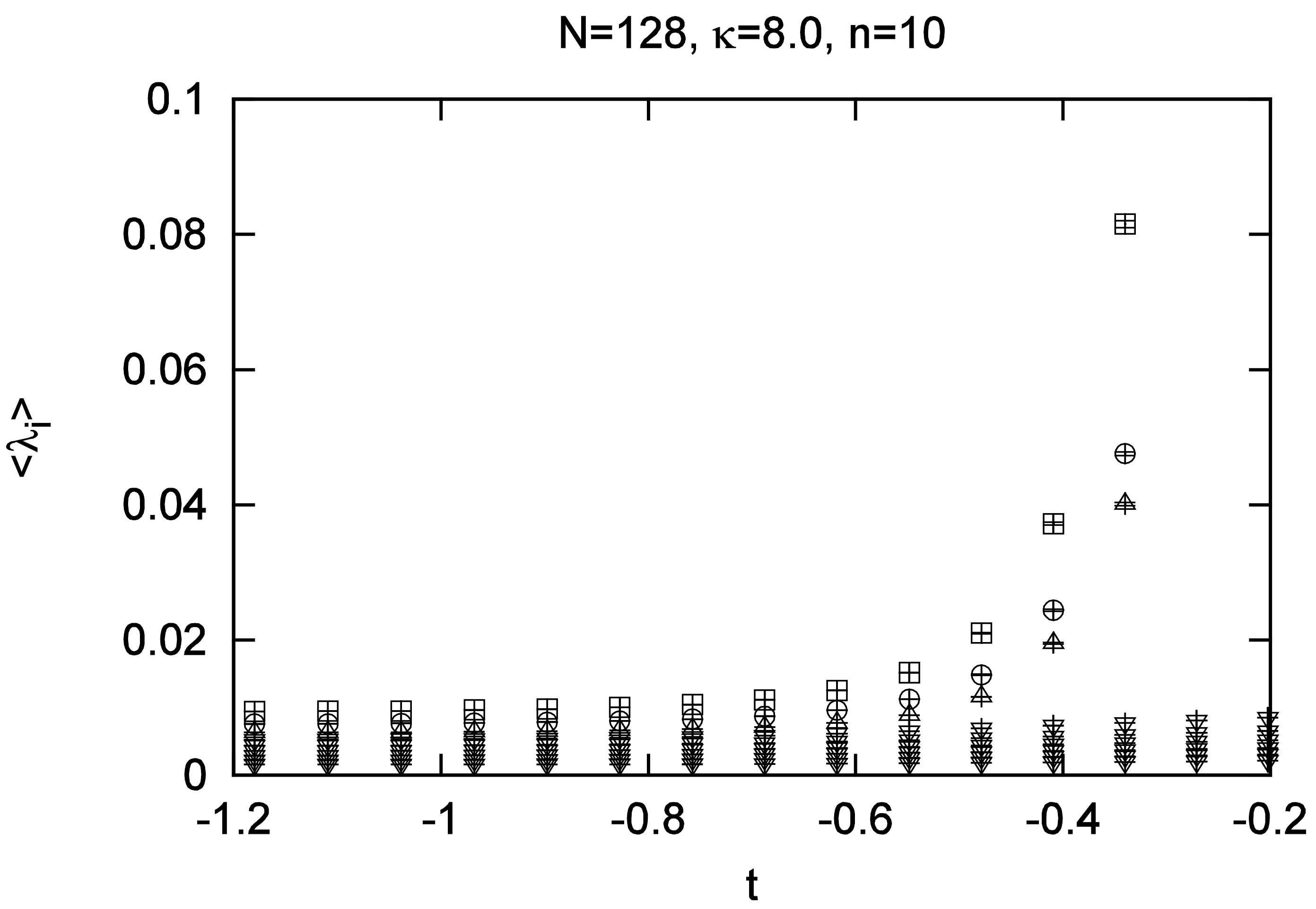
Exponential expansion in the VDM model



$$x = \frac{t - t_c}{R(t_c)}$$

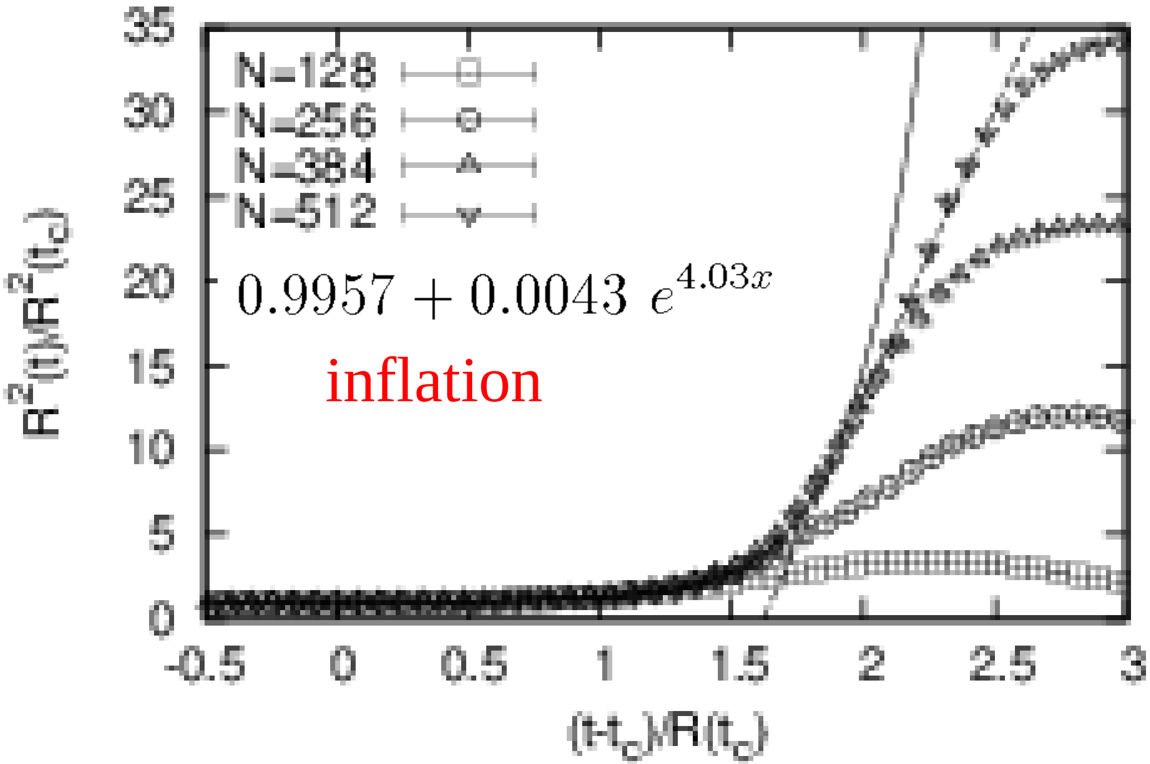
1st term contribute to exponential expansion

SSB of SO(9) to SO(3) in the VDM model



Power-law expansion in bosonic model

Ito-Nishimura-A.T ('16)



$R(t)^2 \sim t$

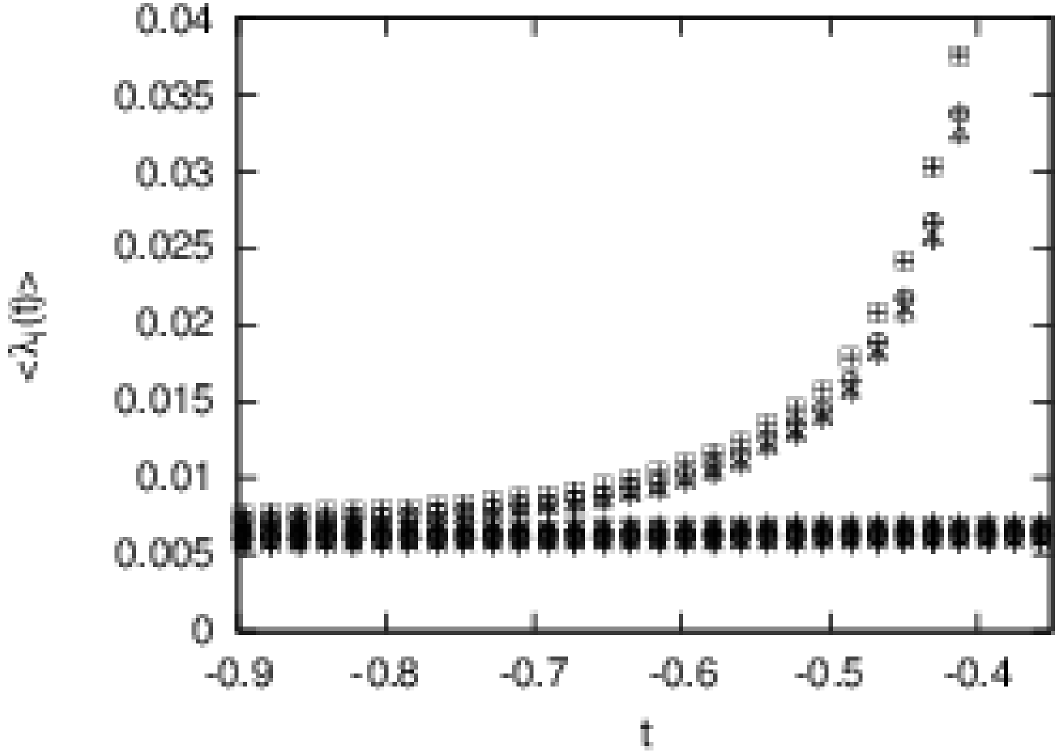
$\Rightarrow R(t) \sim t^{\frac{1}{2}}$

Reminiscent of radiation dominant universe

SSB of SO(9) to SO(3) in bosonic model

$N = 512$

$n = 32$



Other developments

- Developed a renormalization group like method

  (showed it works for the VDM model)

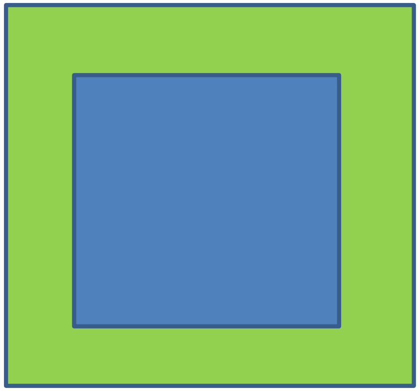
  Ito-Koizuka-Kim-Nishimura-A.T. ('13)
- Analyze classical solutions systematically and found some solutions which can resolve the cosmological constant problem

  (classical solutions are expected to be dominant at late times)

  Kim-Nishimura-A.T. ('12)   Cf.) Steinacker, Chatdistavrakidis, Stern
- Proposed a mechanism that the standard model

  particles appear   cf.) intersecting D-branes

  Chatzistavrakidis-Steinnacker-Zoupanos ('11)
  Nishimura-A.T. ('13)
  Aoki-Nishimura-A.T. ('14)
  Steinacker-Zahn (14)



Cutoff independence

Azuma-Ito-Nishimura-A.T. , work in progress

Generalizing IR cutoffs

$$\frac{1}{N}\mathrm{tr}(A_0^2)^p \leq \kappa^p \frac{1}{N}\mathrm{tr}(A_i^2)^p$$

$$\frac{1}{N}\mathrm{tr}(A_i^2)^p \leq L^{2p}$$

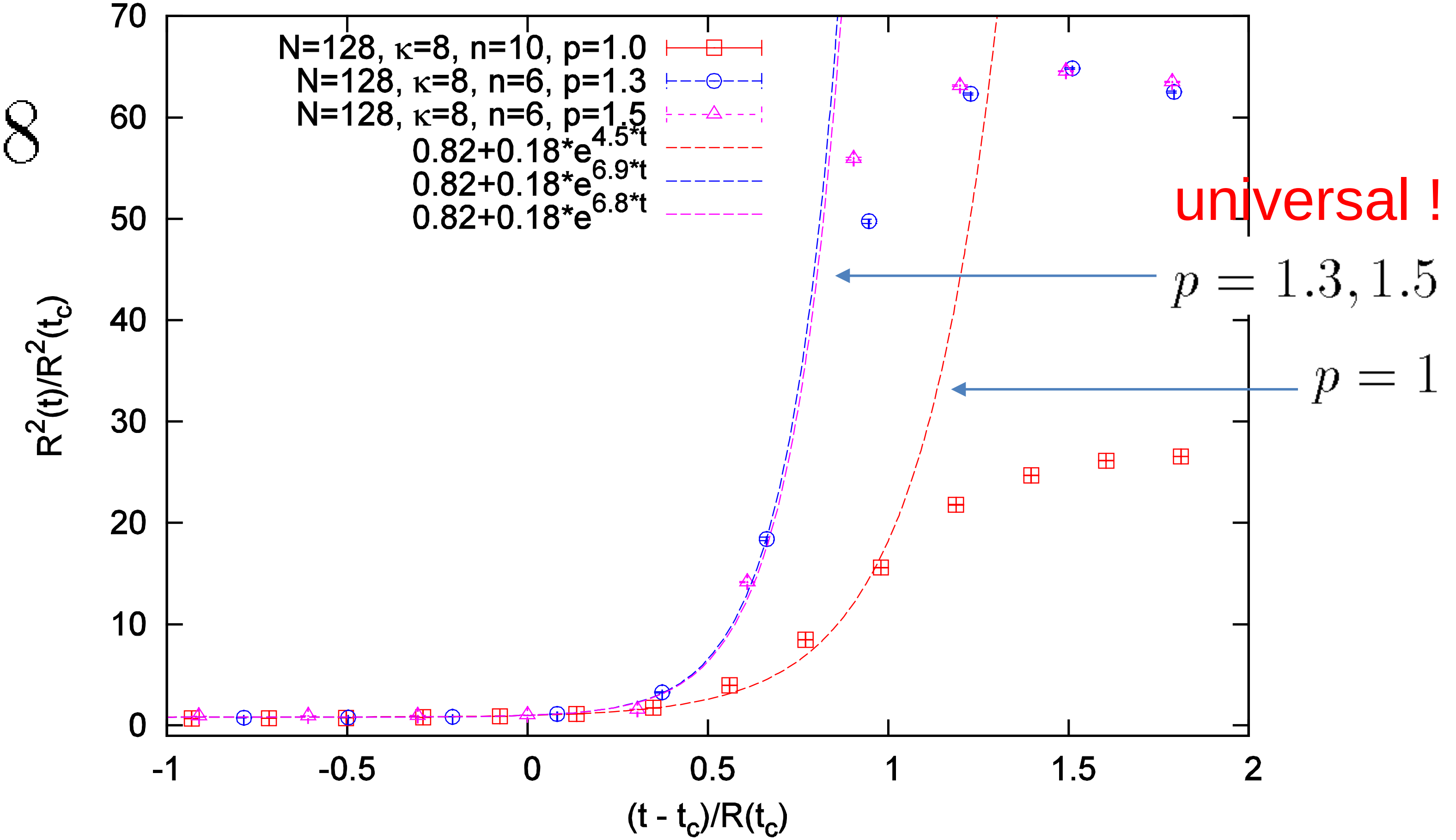
For larger p, larger eigenvalues are constrained more strongly

So far we have set $p = 1$

If results are independent of p $\rightarrow$ cutoff independence

Universal behavior of R in the VDM model

$$N = 128$$



Analysis of SD eqs. in the VDM model

SD eq for temporal direction

$$\int \prod_{k=1}^{N} d\alpha_k dA_i \frac{\delta}{\delta \alpha_M} \left(\alpha_m e^{-S[A]}\right) = 0$$

$$\left\langle \alpha_m \frac{\delta S_{\mathrm{tr}F}}{\delta \alpha_M} \right\rangle + \left\langle \alpha_m \frac{\delta S_{\mathrm{tr}A_0}}{\delta \alpha_M} \right\rangle + \left\langle \alpha_m \frac{\delta S_{\mathrm{vdm}}}{\delta \alpha_M} \right\rangle = \delta_{mM} - \frac{1}{N}$$

If these terms vanish in the large-N limit, the cutoff effect vanishes in the large-N limit

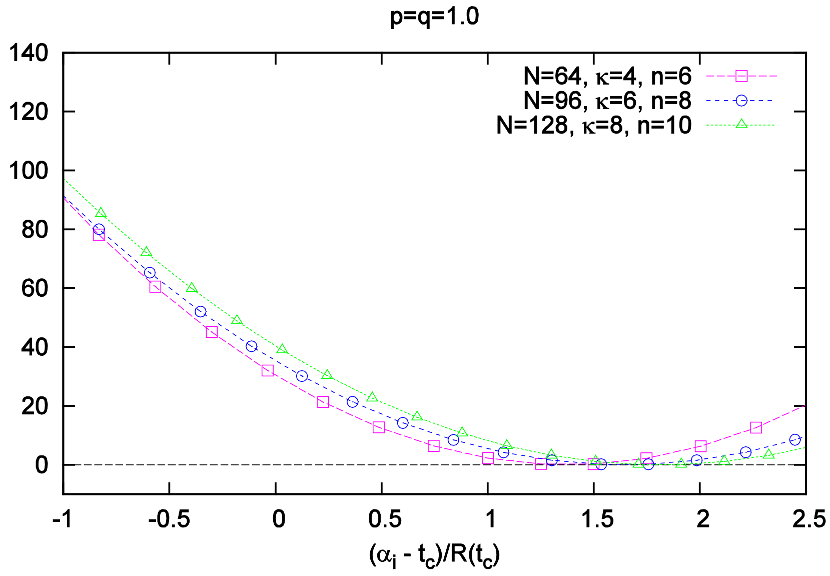
SD eq for spatial directions

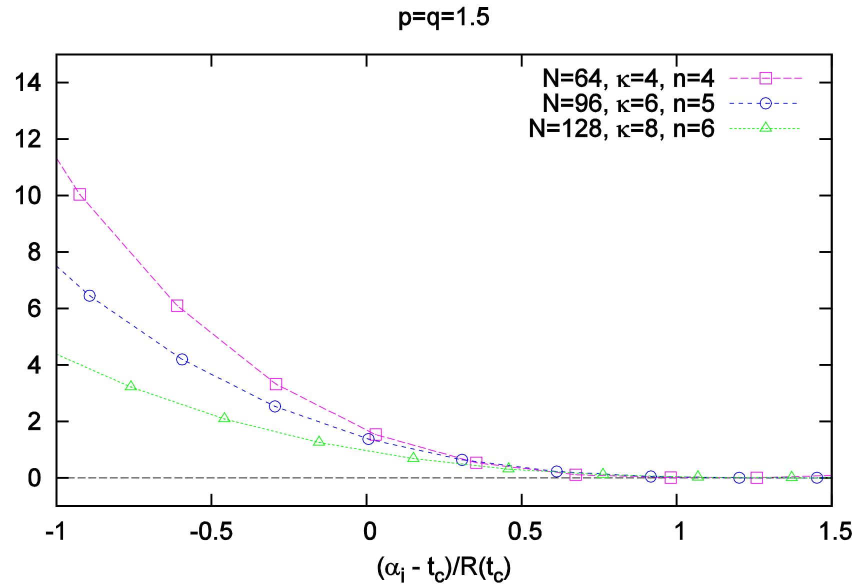
$$\int \prod_{m=1}^{N} d\alpha_m dA_i \frac{\delta}{\delta A^I_{MN}} \left(A^i_{mn} e^{-S[A]}\right) = 0$$

$$\sum_{i=I} \sum_{n=N} \left(\left\langle A^i_{mn} \frac{\delta S_{\mathrm{tr}F}}{\delta A^I_{MN}} \right\rangle + \left\langle A^i_{mn} \frac{\delta S_{\mathrm{tr}A_i}}{\delta A^I_{MN}} \right\rangle + \left\langle A^i_{mn} \frac{\delta S_{\mathrm{sym}}}{\delta A^I_{MN}} \right\rangle \right) = dN \left(1 - \frac{1}{N^2}\right) \delta_{mM}$$

Contribution of cutoff term of time direction

diagonal part



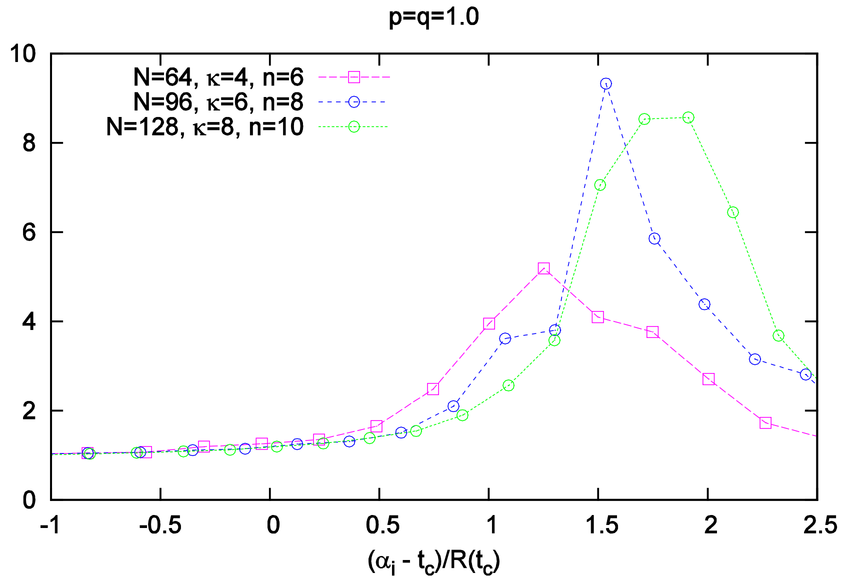


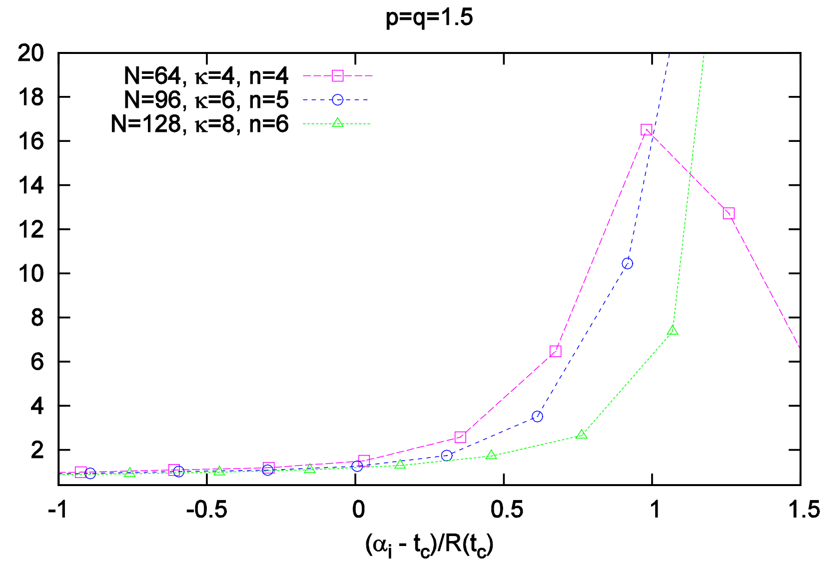
p=1

p=1.5

Contribution of cutoff term for spatial direction

diagonal part





p=1

p=1.5

Cutoff independence

Observation in the VDM model

- SSB of SO(9) to SO(3) occurs for p=1~1.5

  SSB of SO(9) to SO(5) for p=2

  ～critical value p_2~1.8
- R(t) agree at N=128 for p=1.3 and1.5 (exponential expansion)
- Analysis of SD equation

  cutoff effects almost scale with N for p=1

  cutoff effects decrease with N for p=1.5

  → critical value p_1~1

conjecture

cutoff effects vanish in the large-N limit for p_1<p<p_2

analogous things are true for the original model

results obtained for p=1 so far are qualitatively correct

Conclusion and outlook

Conclusion

- We studied Lorentzian version of the type IIB matrix model
- The Lorentzian model is well-defined and has no parameters except one scale parameter

  This property is expected in nonperturbative string theory
- the concept of ``time evolution” emerges

  when A_0 is made diagonal, $A_i \ (i = 1, \cdots, 9)$ have band-diagonal structure
- After a critical time, SO(9) symmetry of 9 dimensions is spontaneously broken down to SO(3) and 3 out of 9 dimensions start to expand rapidly
- The result is unique, no initial condition is imposed unique vacuum?
- The expansion is consistent with an exponential one
- No singular structures are observed cosmic singularity resolved?

Conclusion (cont'd)

- We performed large-scale parallel computation for simplified models.

  VDM model exponential expansion ~ inflation

  bosonic model exponential expansion → $\sqrt{t}$ expansion

  ~matter dominant universe

- We obtained evidences that the cutoff effects disappear in the large-N limit for a finite range of p in the VDM model

  We expect that analogous things are true for the original model

Outlook

- Numerical study

large-scale parallel computation of the original model
renormalization group method

→ find a classical solution smoothly connected to numerical simulation

establish cutoff independence by examining SD eq.

- Analytic study of the theory around the classical solution

how the Standard Model appears?

metric? Cf.) Ishiki's talk, Steinacker, Szabo, Chazistavrakidis, Yang, Hanada-Kawai-Kimura, …..

e-foldings? dark energy? dark matter? ………