aspects of the bosonic spectral action: successes & challenges

mairi sakellariadou

king's college london

QSPACE COST MP1405

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connes (1996, 2008)

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4dim ST with an internal kaluza-klein space attached to each point; the 5th dim is a discrete, 0dim space

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spectral triple

$$M \times F := (C^\infty(M, \mathcal{A}_F), L^2(M, S) \otimes \mathcal{H}_F, \not{D} \otimes \mathbb{I} + \gamma_5 \otimes D_F)$$

chamseddine, connes, marcolli (2007)

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$D_{\mathcal{F}}$ 96 x 96 matrix

in terms of 3x3 Yukawa mixing matrices and a real constant responsible for neutrino mass terms

$$D_F = \begin{pmatrix} S & T^* \\ T & \bar{S} \end{pmatrix}$$

$$S = \begin{pmatrix} & \begin{bmatrix} \Upsilon_v & \\ & \Upsilon_e \end{bmatrix} & & \\ \begin{bmatrix} \Upsilon_v^* & \\ & \Upsilon_e^* \end{bmatrix} & & & \\ & & & \begin{bmatrix} \Upsilon_u \otimes 1_3 & \\ & \Upsilon_d \otimes 1_3 \end{bmatrix} \\ & & \begin{bmatrix} \Upsilon_u^* \otimes 1_3 & \\ & \Upsilon_d^* \otimes 1_3 \end{bmatrix} & \end{pmatrix}$$

$$T = \begin{pmatrix} \begin{bmatrix} 0 & & & \\ & 0 & \Upsilon_R & \\ & & & 0 \end{bmatrix} & \\ & 0_4 \end{pmatrix}$$

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$$\mathcal{A}_{\mathcal{F}} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C})$$

$$k = 2a$$

$$k = 4$$

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chamseddine, connes (2007)

chamseddine, connes, marcolli (2007)

cutoff spectral action principle:

$$A = \sum_j a_j[\mathcal{D}_{\mathcal{F}}, b_j] \;\; , \;\; a_j, b_j \in \mathcal{A}_{\mathcal{F}}$$

$$\mathcal{D}_A = \mathcal{D}_{\mathcal{F}} + A + \epsilon' J A J^{-1}$$

the action functional depends only on the <u>spectrum</u> of the (generalised) Dirac operator and is of the form:

$$Tr\left(f\left(D_A^2 / \Lambda^2\right)\right)$$

bosonic part

$$f(x) = 1; x \leq \Lambda$$

$$f(x) = e^{-x}$$

chamseddine, connes (1996, 1997)

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evaluate trace with heat kernel techniques

$$\mathrm{Tr}\left(f\left(\frac{D_A^2}{\Lambda^2}\right)\right) \sim 2f_4\Lambda^4 a_0(D_A{}^2) + 2f_2\Lambda^2 a_2(D_A{}^2) + f(0)a_4(D_A{}^2) + O(\Lambda^{-2})$$

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chamseddine, connes (1996, 1997)

the rest follows after a long calculation ...

$$S_\Lambda = \int d^4x\sqrt{g}\,\Big(A_1\Lambda^4 \;+A_2\Lambda^2\left(\frac{5}{4}R - 2y_t^2H^2 - M^2\right)$$
$$+A_3\left(g_2^2 W^\alpha_{\mu\nu}W^{\alpha\;\mu\nu} + g_3^2 G^a_{\mu\nu}G^{a\;\mu\nu} + \frac{5}{3}g_1^2 B_{\mu\nu}B^{\mu\nu}\right)$$
$$+\text{other } \mathcal{O}(\Lambda^0) \;+\mathcal{O}\left(\Lambda^{-2}\right)\Big)$$

$$\frac{5}{3}g_1^2(\Lambda) = g_2^2(\Lambda) = g_3^2(\Lambda) \Longrightarrow \boxed{\Lambda \sim (10^{14} - 10^{17})\ \text{GeV}}$$

chamseddine, connes, marcolli (2007)

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$$\frac{5}{3}g_1^2(\Lambda)=g_2^2(\Lambda)=g_3^2(\Lambda) \;\Rightarrow\; \Lambda\sim\left(10^{14}-10^{17}\right)\text{ GeV}$$

use RGE to get predictions for SM

chamseddine, connes, marcolli (2007)

compatible with 126 GeV higgs mass

stephan (2009)

connes, chamseddine (2012)

chamseddine, connes, van suijlekom (2013)

devastato, lizzi, martinetti (2013)

gravitational terms coupled to matter:

$$\mathscr{S}^{\mathrm{E}}_{\text{bosonic}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x \,,$$

$$\begin{aligned}
\kappa_0^2 &= \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}} \,, \\
\alpha_0 &= -\frac{3 f_0}{10\pi^2} \,, \\
\gamma_0 &= \frac{1}{\pi^2}\left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4}\mathfrak{d}\right) , \\
\tau_0 &= \frac{11 f_0}{60\pi^2} \,, \\
\mu_0^2 &= 2\Lambda^2 \frac{f_2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \,, \\
\boxed{\xi_0 = \frac{1}{12}} \\
\lambda_0 &= \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2} \,;
\end{aligned}$$

- EH action with a cosmological term
- topological term
- conformal gravity term with the weyl curvature tensor
- conformal coupling of higgs to gravity

the coefficients of the gravitational terms depend upon the yukawa parameters of the particle physics content

chamseddine, connes, marcolli (2007)

successes of the cutoff bosonic spectral action

$$Tr\left(f\left(D_A^2/\Lambda^2\right)\right)$$

- description of geometry in terms of spectral properties of operators

⇨ model of particle interactions very close to real phenomenology

- SM does and the Pati-Salam gauge groups fit into NCG model, but SU(5) or SO(10) do not

⇨ absence of large groups prevents proton decay

- - infer quantities related to (higgs) boson based only on input from fermionic parameters in fluctuated Dirac operator
  - the Dirac operator defines also the fermionic part of the action

$$S_{\mathrm{F}} = \langle J\psi | D_A | \psi \rangle$$

- gravitational terms coupled to matter:

$$\mathcal{S}^{\mathrm{E}}_{\text{bosonic}} = \int \left(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \right) \sqrt{g}\, d^4x \,,$$

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framework to address early universe cosmology

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framework to address early universe cosmology

assume a wick rotation to imaginary time
⟹ lorentzian signature

gravitational & coupling between Higgs field and Ricci curvature

equations of motion

neglect nonminimal coupling between geometry and higgs

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \boxed{\frac{1}{\beta^2}\delta_{cc}\left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa} R_{\lambda\kappa}\right]} = \kappa_0^2 \boxed{\delta_{cc}} T^{\mu\nu}_{\text{matter}}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2 \alpha_0}$$

$$\delta_{cc} \equiv [1 \cancel{- 2\kappa_0^2 \xi_0 \mathbf{H}^2}]^{-1}$$

$$\alpha_0 = \frac{-3 f_0}{10\pi^2}$$

nelson, sakellariadou, PRD 81 (2010) 085038

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FLRW $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\Sigma$

weyl tensor vanishes ⟹

NCSG corrections to einstein equations vanish

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gravitational & coupling between Higgs field and Ricci curvature

⇒ equations of motion

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NCSG corrections to einstein's eqs. are present only in inhomogeneous and anisotropic spacetimes

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at energies approaching higgs scale, the nonminimal coupling of higgs field to curvature cannot be neglected

$$\boxed{R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \kappa_0^2\left[\frac{1}{1-\kappa_0^2|\mathbf{H}|^2/6}\right]T^{\mu\nu}_{\text{matter}}}$$

⇒ effective gravitational constant

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^\alpha \mathbf{H}||D^\beta \mathbf{H}|g_{\alpha\beta} - \mu_0|\mathbf{H}|^2 + \lambda_0|\mathbf{H}|^4$$

$$\Rightarrow \quad -\mu_0|\mathbf{H}|^2 \to -\left(\mu_0 + \frac{R}{12}\right)|\mathbf{H}|^2$$

⇒ increases the higgs VEV

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$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4}$$

$$g_3^2 = g_2^2 = \frac{5}{3} g_1^2$$

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linear perturbations around minkowski

$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2 \left[-1, \left(\delta_{ij} + h_{ij}(x)\right)\right]\right)$$

$$a(t) = 1$$

$$\nabla_i h^{ij} = 0$$

$$\Downarrow$$

$$\boxed{\left(\Box - \beta^2\right) \Box h^{\mu\nu} = \beta^2 \frac{16\pi G}{c^4} T^{\mu\nu}_{\text{matter}}}$$

with conservation eqs: $\frac{\partial}{\partial x^\mu} T^\mu_{\ \nu} = 0$

$$\beta^2 = -\frac{1}{32\pi G \alpha_0}$$

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energy lost to gravitational radiation by orbiting binaries:

$$\boxed{-\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} \approx \frac{c^2}{20G}|\mathbf{r}|^2 \dot{h}_{ij}\dot{h}^{ij}}$$

strong deviations from GR at frequency scale

$$2\omega_c \equiv \beta c \sim (f_0 G)^{-1/2} c$$

set by the moments of the test function f

scale at which NCSG effects become dominant

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magnitude of deviations from GR < accuracy to which the rate of change of orbital period agrees with GR predictions $\Longrightarrow$ $\beta > 7.55 \times 10^{-13}\mathrm{m}^{-1}$

Binary	Distance (pc)	Orbital Period (hr)	Eccentricity	GR (%)
PSR J0737-3039	~ 500	2.454	0.088	0.2
PSR J1012-5307	~ 840	14.5	$< 10^{-6}$	10
PSR J1141-6545	> 3700	4.74	0.17	6
[illegible]	~ 6400	7.752	0.617	0.1
[illegible]	~ 1100	10.1	?	1
PSR B2127+11C	~ 9980	8.045	0.68	3

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GPB/LARES

$$\Omega_{\text{geodesic}} = \Omega_{\text{geodesic(GR)}} + \Omega_{\text{geodesic(NCG)}}$$

fixed to the GR predicted value

$$\Omega_{\text{geodesic(GR)}} = 6606 \ \text{mas/y}$$

$$|\Omega_{\text{geodesic(NCG)}}| \leq \delta\Omega_{\text{geodesic}}$$

$$\delta\Omega_{\text{geodesic}} = 18 \ \text{mas/y}$$

$$\Rightarrow \boxed{\beta > 7.1 \times 10^{-5}\text{m}^{-1}}$$

Effect	Measured	Predicted
Geodesic precession	6602 ± 18	6606
Lense-Thirring precession	37.2 ± 7.2	39.2

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

GPB/LARES

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constraints from torsion balance:

NCSG modifications to newtonian potentials are similar to those by a fifth force via

$$V(r) = -\frac{GMm}{r}(1 + \alpha e^{-\beta r})$$

eöt-wash and irvine experiments $\Rightarrow$ $\beta \geq 10^4 \text{ m}^{-1}$

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

inflation through the nonminimal coupling between the geometry and the higgs field

$$S^{\mathrm{L}}_{\mathrm{GH}} = \int \left[\frac{1-2\kappa_0^2\xi_0 H^2}{2\kappa_0^2} R - \frac{1}{2}(\nabla H)^2 - V(H)\right]\sqrt{-g}\, d^4x$$

subject to radiative corrections as a function of energy

$$\xi_0 = \frac{1}{12}$$

$$V(H) = \lambda_0 H^4 - \mu_0^2 H^2$$

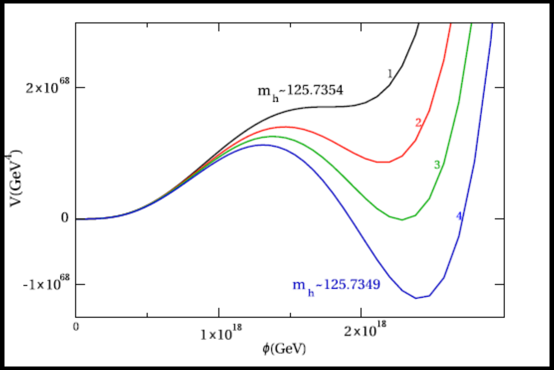
nelson, sakellariadou, PLB 680 (2009) 263

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

effective potential at high energies: $V(H) = \lambda(H) H^4$

running of the higgs self-coupling at two-loops:

$\Rightarrow$ slow-roll conditions satisfied



buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

effective potential at high energies: $V(H) = \lambda(H) H^4$

running of the higgs self-coupling at two-loops:

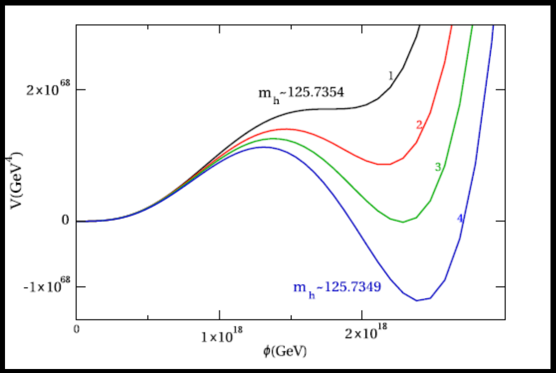
$\Rightarrow$ slow-roll conditions satisfied but...

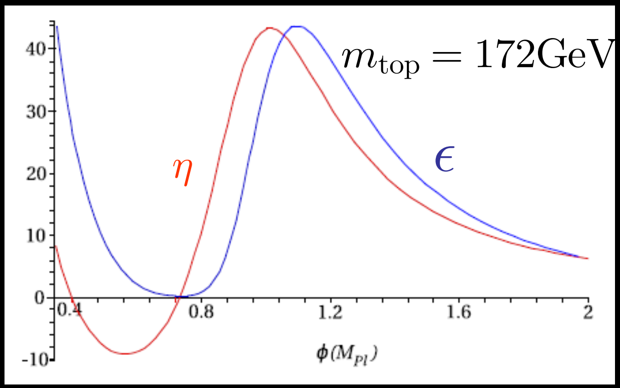
CMB constraints cannot be satisfied

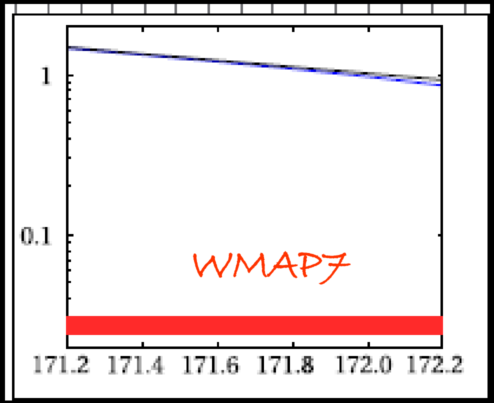
(incompatibility with top quark mass)

ϵ needs to be too small to allow for sufficient e-folds $N \sim \epsilon^{-1/2} \mathrm{d}\phi$

$\Rightarrow$ $(V_\star/\epsilon_\star)^{1/4}$ becomes too large to fit the CMB constraint







buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

some reasons to go beyond cutoff spectral action

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- expansion is valid when fields & their derivatives are small wrt Λ

weak-field approximation

what does it happen in the ultraviolet regime (high momenta)?

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van sujielkom (2011) *iochum, levy, vassilevich (2012)*

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- magnitude of dimensionful parameters (hierarchy of scales):

cosmological constant << higgs VEV << planck scale

the ζ spectral action

$$S_\zeta \equiv \lim_{s\to 0} \operatorname{Tr} D^{-2s} \equiv \zeta(0, D^2) = a_4\left[D^2\right] = \int d^4x\,\sqrt{g}\,L$$

kurkov, lizzi, sakellariadou, watcharangkool, PRD91 (2015) 065013

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the lagrangian density obtained from the ζ spectral action is of the form:

$$\begin{aligned} L(x) &= a_4(D^2, x) \\ &= \alpha_1 M^4 + \alpha_2 M^2 R + \alpha_3 M^2 H^2 \\ &\quad + \alpha_4 B_{\mu\nu} B^{\mu\nu} + \alpha_5 W^{\alpha}_{\mu\nu} W^{\mu\nu\,\alpha} + \alpha_6 G^{a}_{\mu\nu} G^{\mu\nu\,a} \\ &\quad + \alpha_7\, H\left(-\nabla^2 - \frac{R}{6}\right) H + \alpha_8 H^4 + \alpha_9 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \alpha_{10} R^* R^* \end{aligned}$$

$\alpha_1, \cdots, \alpha_{10}$ dimensionless constants

kurkov, lizzi, sakellariadou, watcharangkool, PRD91 (2015) 065013

the ζ spectral action

M: dimensional constant in the Dirac operator $\Longrightarrow$ majorana mass of right handed neutrino

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kurkov, lizzi, sakellariadou, watcharangkool, PRD91 (2015) 065013

the ζ spectral action

- all dimensionless constants are normalised to spectral values
- 3 dimensionful parameters normalised from experiments

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$$
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\end{aligned}
$$

$$\boxed{\Lambda \sim (10^{14} - 10^{17})\ \mathrm{GeV}}$$

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kurkov, lizzi, sakellariadou, watcharangkool, PRD91 (2015) 065013

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- no higher dim (>4) operators $\Longrightarrow$ renormalisable and local
- no issues about asymptotic expansion and convergence
- S_ζ is purely spectral with no dependence on cutoff function

kurkov, lizzi, sakellariadou, watcharangkool, PRD91 (2015) 065013

spectral dimension

spectral dimension

- cutoff spectral action

taking the full momentum-dependence of the propagators

⟹ $D_S(T) = 0$ for all spins

⟹ one needs a UV completion (e.g., asymptotic safety)

alkofer, saueressig, zanusso (2014)

high energy bosons do not propagate

kurkov, lizzi, vassilevich (2013)

spectral dimension

- ζ spectral action

actions for higgs scalar, gauge fields have same bahaviour in UV & IR

⟹ spectral dim = topological dim of manifold (=4)

kurkov, lizzi, sakellariadou, watcharangkool, PRD91 (2015) 065013

spectral dimension

- ζ spectral action

actions for higgs scalar, gauge fields have same bahaviour in UV & IR

⟹ spectral dim = topological dim of manifold (=4)

gravitational spectral dim = 2

⟹ gravitational propagators decrease faster at infinity due to presence of fourth derivative (improvement on UV convergence)

there exists "low-energy" limit for which gravitational spectral dim=4

⟹ at very low energies dynamics does not feel weyl square terms

kurkov, lizzi, sakellariadou, watcharangkool, PRD91 (2015) 065013

<u>remark:</u> to get higgs quadratic term, we need a term in dirac operator corresponding to neutrino majorana mass

this term is also necessary to get correct higgs mass

$$a_i \psi^c \sigma(x) \psi$$

$$\psi^c (a_i \sigma(x) + M_i) \psi$$

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$\Downarrow$

these constant mass terms lead to the introduction of M^4, $M^2 H^2$ and $M^2 R$ terms in the action, and the corresponding counter terms in UV renormalisation

$$\begin{aligned} L(x) &= \boxed{\alpha_1 M^4 + \alpha_2 M^2 R + \alpha_3 M^2 H^2} \\ &+\alpha_4 B_{\mu\nu} B^{\mu\nu} + \alpha_5 W^{\alpha}_{\mu\nu} W^{\mu\nu\,\alpha} + \alpha_6 G^{a}_{\mu\nu} G^{\mu\nu\,a} \\ &+\alpha_7 H \left(-\nabla^2 - \frac{R}{6} \right) H + \alpha_8 H^4 + \alpha_9 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \alpha_{10} R^* R^* \end{aligned}$$

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$\psi^c (a_i \sigma(x) + M_i) \psi$

there are no dim 0 and dim 2 operators in the classical action

$$\begin{aligned} S_\zeta \;=\; & \int dx \sqrt{g} \left(\gamma_1 B_{\mu\nu} B^{\mu\nu} + \gamma_2 W^\alpha_{\mu\nu} W^{\mu\nu\,\alpha} + \gamma_3 G^a_{\mu\nu} G^{\mu\nu\,a} + \gamma_4 \, H \left(-\nabla^2 - \frac{R}{6} \right) H \right. \\ & \left. + \gamma_5 H^4 + \gamma_6 \, \sigma \left(-\nabla^2 - \frac{R}{6} \right) \sigma + \gamma_7 \sigma^4 + \gamma_8 H^2 \sigma^2 \gamma_9 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_{10} R^* R^* \right) \end{aligned}$$

scale invariant theory

- dynamical generation of the 3 scales upon quantisation

sakharov (1967) | coleman, weinberg (1973)

- remain without EH action and the cosmological constant (conformal gravity)

maldacena (2011)

$$\mathscr{S}^{\mathrm{E}}_{\text{bosonic}} = \int \left(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^{\star} R^{\star} + \frac{1}{4} G^{i}_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^{\alpha}_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \right) \sqrt{g}\, d^4x \,,$$

do the quadratic curvature terms imply negative energy massive graviton modes ?

ostrogradski (linear) instability

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do the quadratic curvature terms imply negative energy massive graviton modes ? *ostrogradski (linear) instability*

generalise a higher derivative theory into an SO(2,4) gauge theory and find conditions so that the e.o.m reduces to vacuum einstein's eq.

varying independently all connection fields, and not only the metric, Weyl gravity transforms from a 4th order theory into a theory of conformal equivalence classes of solutions to GR, provided the torsion vanishes

wheeler 2015)

$$\mathscr{S}^{\mathrm{E}}_{\text{bosonic}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^{\alpha}_{\mu\nu} F^{\mu\nu\alpha}$$
$$+ \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x \,,$$

depends on spin connection

$$S(\text{triad, spin connection}) = S_{\text{grav}}(\text{triad, spin connection}) + S(H, \psi)$$

Levi-Civita connection + term depending on torsion

hanisch, pfaffle, stephan (2010)

sakellariadou, watcharangkool (in progress)

remaining challenges

- almost commutative manifolds are based on riemannian ST

⇒ to do physics we need a generalisation to pseudo-riemannian ST (e.g., lorentzian, minkowskian)

hawkins (1997)
moretti (2003)
pashke, verch (2004)
van suijlekom (2004)
paschke, sitarz (2006)
strohmaier (2006)
franco (2010)
van den dungen, pashke,, rennie (2012)

- almost commutative geometry describes physics at classical level

combine ideas from (L)QG and quantum gauge theories

aastrup, grimstrup (2006, 2007)
marcolli, zainy, yasry (2008)
aastrup, rimstrup, nest (2009)

remaining challenges

- almost commutative manifolds are based on riemannian ST

⇒ to do physics we need a generalisation to pseudo-riemannian ST (e.g., lorentzian, minkowskian)

get within NCG setup a unified and geometrical formulation of QG with matter

- almost comm … classical level

combine ideas from (L)QG and quantum gauge theories thank you

ευχαριστώ