STANDARD BIG BANG MODEL of COSMOLOGY

For "cosmic times" $t \gtrsim t_p \approx 10^{-44}$ sec $(\equiv M_p^{-1})$ after Big-Bang quantum fluctuations of gravity seize to exist

→ Gravity is described by "Classical General Relativity"
Strong, weak and electromagnetic interactions are, however described by Relativistic Quantum Field Theory (Gauge Theories).

Cosmological Principle: The universe is "homogeneous" and "isotropy" (strogest evidence from CBR)

→ "Robertson-Walker" metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right]$$

r, φ, θ are "comoving" polar coordinates (fixed for objects just following the general expansion).

k is the "scalar curvature" of 3-space:

$k = 0$ flat universe
$k > 0$ closed "
$k < 0$ open "

$a(t)$ is the "scale factor" of the universe (describes expansion)

normalized as $a(t_0) = 1$ (dimensionless)

Physical distance: (instantaneous) $R = a(t)\int_0^r \frac{dr}{(1-kr^2)^{1/2}}$ $\left(= a(t)\, r \text{ for } k=0\right)$.

"Hubble expansion" (flat case):

$$\bar{R} = a(t)\bar{r} \longrightarrow \bar{V} = \frac{d\bar{R}}{dt} = \frac{d}{dt}\left(a(t)\bar{r}\right) = \frac{\dot{a}}{a}\bar{R} + a\frac{d\bar{r}}{dt},$$

where $\bar{v} = a(t)\dfrac{d\bar{r}}{dt}$ = "peculiar velocity"

$$\bar{v} = \bar{0} \longrightarrow \bar{V} = \frac{\dot{a}(t)}{a(t)}\bar{R} \equiv H(t)\bar{R} \quad \text{(Hubble law)}$$

$H(t)$ = "Hubble parameter" $\quad H_0 = H(t_0) = 100h$ km/sec Mpc

$(0.45 \lesssim h \lesssim 1) \longrightarrow$ "1rst success of SBB".

"Friedmann equs."

Homogeneity + Isotropy of the universe $\longrightarrow$ "Energy Mom. tensor"
$T_\mu^{\ \nu} = \mathrm{diag}(-\rho, p, p, p)$ where ρ = energy density,
p = pressure.

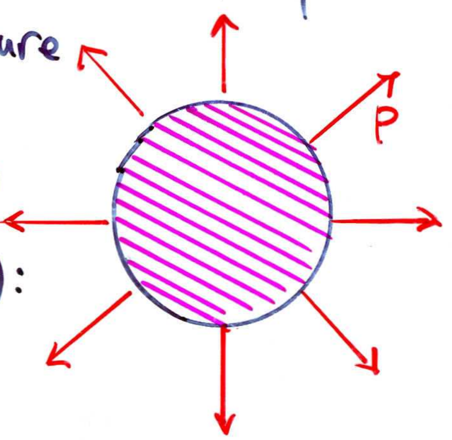
Energy Momentum Conservation $(T_{\mu\,;\nu}^{\ \nu} = 0)$

$$\longrightarrow \frac{d\rho}{dt} = -3\frac{\dot{a}}{a}(\rho + p) \quad \text{(Continuity equ)}$$

1rst term due to dilution of energy because of expansion
2nd " " " work done by pressure

$$\Longleftrightarrow d\left(\frac{4\pi}{3}a^3\rho\right) = -p\,4\pi a^2 da$$

"comoving volume"
of radius $\propto a(t)$:



Einstein equs $(R_\mu^{\ \nu} - \frac{1}{2}\delta_\mu^{\ \nu}R = 8\pi G T_\mu^{\ \nu})$

$$\Longrightarrow \left(\frac{\dot{a}(t)}{a(t)}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad \text{(Friedmann eq)}$$

"Expansion law":

$$\rho + p = \gamma\rho \longrightarrow \dot{\rho} = -3\frac{\dot{a}}{a}\gamma\rho \longrightarrow \frac{d\rho}{\rho} = -3\gamma\frac{da}{a}$$

$$\longrightarrow \ln\left(\frac{\rho}{\rho_0}\right) = -3\gamma\ln\left(\frac{a}{a_0}\right) \longrightarrow \rho \propto a^{-3\gamma}$$

"matter": $p = 0 \to \gamma = 1 \longrightarrow \rho \propto a^{-3}$ (dilution of a fixed # of particles in a 'comoving' volume)

"radiation": $p = \frac{1}{3}\rho \to \gamma = \frac{4}{3} \to \rho \propto a^{-4}$ (extra redshifting of wave-length due to expansion).

Friedmann equ with $\rho \propto a^{-3\gamma}$ $(k=0) \longrightarrow$

$$\frac{\dot{a}}{a} \propto a^{-3\gamma/2} \longrightarrow a \propto t^{2/3\gamma} \to a(t) = \left(\frac{t}{t_0}\right)^{2/3\gamma}$$

"matter": $a(t) = \left(\frac{t}{t_0}\right)^{2/3}$

"radiation": $a(t) = \left(\frac{t}{t_0}\right)^{1/2}$

Radiation dominated era (early universe):

$$\rho = \frac{\pi^2}{30}\left(N_b + \frac{7}{8}N_f\right)T^4 \equiv cT^4$$

$$s = \text{entropy density} = \frac{2\pi^2}{45}\left(N_b + \frac{7}{8}N_f\right)T^3$$

($N_{b(f)}$ = # of b(f) degrees of freedom)

Adiabatic evolution ($S = sa^3 = \text{const.}$)

$$\longrightarrow aT = \text{const.}$$

$\rightarrow$ temperature – time relation during "radiation" dominance ($k=0$)

$$T^2 = \frac{M_P}{2(8\pi c/3)^{1/2}t} \longrightarrow$$ Expansion starts at $t=0$ with $T=\infty$, $a=0$ $(t \gtrsim t_P)$

"Important parameters":

(i) H_0 = present Hubble parameter

(ii) Friedmann equ with $k=0$ (flat universe) $\rightarrow$

$\rho = 3H^2/8\pi G \equiv \rho_c$ = critical density

Define $$\Omega = \frac{\rho}{\rho_c} = 1 + \frac{k}{a^2 H^2}$$

$\Omega = 1 \longrightarrow$ flat universe ($k=0$)

$\Omega > 1 \longrightarrow$ closed " ($k>0$)

$\Omega < 1 \longrightarrow$ open " ($k<0$)

Present value of Ω : $\Omega_0 = 1$ (inflation)

But $\Omega_b \approx 0.05 - 0.1$

Most matter is "dark" (!!)

(iii) Deceleration parameter $q = -\left(\frac{\ddot{a}}{\dot{a}}\right) / \left(\frac{\dot{a}}{a}\right) = \frac{\rho + 3p}{2\rho_c}$

$\rightarrow$ For matter ($p=0$) $\rightarrow q = \frac{1}{2}\Omega$

So Inflation $\rightarrow q_0 = 1/2$

"Particle Horizon":

The light travels only a finite distance $d_H(t)$ from the big-bang ($t=0$) till some cosmic time t.

From Robertson-Walker metric along the radial direction

$\longrightarrow a(t)\,dr = dt$ (propagation of a light ray)

$$\longrightarrow \quad d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')} \quad = \text{particle horizon at } t$$

= size of the universe which we have already seen at t

= distance at which causal contact has been established at t.

Now $a(t) = \left(\frac{t}{t_0}\right)^{2/3\gamma} \longrightarrow d_H(t) = \frac{3\gamma}{3\gamma-2}\, t \quad (\gamma \neq 2/3)$

" $\longrightarrow H(t) = \frac{2}{3\gamma}\, t^{-1}$

$$\Longrightarrow \quad d_H(t) = \frac{2}{3\gamma-2}\, H^{-1}(t)$$

"matter" $(\gamma = 1)$: $d_H(t) = 3t = 2H^{-1}(t)$

"radiation" $(\gamma = 4/3)$: $d_H(t) = 2t = H^{-1}(t)$

'present horizon' : $d_H(t_0) = 2H_0^{-1} = 6{,}000\, h^{-1}$ Mpc

' " time ' : $t_0 = 2H_0^{-1}/3 = 6.7 \times 10^9\, h^{-1}$ years

' " critical density': $\rho_c = 3H_0^2/8\pi G = 1.9 \times 10^{-29}\, h^{-2}$ gm/cm^3

A brief history of the universe according to GUTS

Assume a GUT based on the gauge group G $(= SU(5), SO(10), SU(3)^3, \ldots)$ with or without SUSY. G breaks as follows:

$$G \xrightarrow[M_X]{\langle\Phi\rangle} G_S = SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow[M_W]{} SU(3)_c \times U(1)_{em}$$

GUTs + Standard Big-Bang Cosmological model (based on classical gravitation) can describe the early history of the universe for cosmic times $t \gtrsim 10^{-44}$ sec.

They predict that the universe as it cools down after big-bang undergoes a series of phase transitions during which the initial gauge symmetry is gradually reduced and "important phenomena" take place :

$t=0$ — G — $t \sim 10^{-37}$ s — G_S — $t \sim 10^{-10}$ s

$*$ BIG BANG

$T = \infty$

A hot soup of massless q, ℓ, W, Z, gluons, $X, Y, \ldots \equiv$ "light"

$T \sim 10^{16}$ GeV

($\sim 10^{28}$ K)

$\Uparrow$

(i) BAU (success)

(ii) Magnetic Monopoles (+ other extended objects)

X, Y are massive

$T \sim 100$ GeV

$t \sim 10^{-4}$ s — $t \sim 180$ s

$SU(3)_c \times U(1)_{em}$

$T \sim 1$ GeV

($\sim 10^{12}$ K)

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Confinement

(quarks $\longrightarrow$ hadrons)

$T \sim 1$ MeV

'Nucleosynthesis'

(Abundance of light elements)

"2^{nd} success"

$\downarrow$

Depends on (i) n_B/n_γ, (ii) # of light ($m \lesssim 1$ MeV) particle species, i.e., N_ν = # of neutrinos.

Agreement with D, ^{3}He, ^{4}He, ^{7}Li if $n_B/n_\gamma \sim (6\text{-}7) \times 10^{-10}$, $N_\nu \lesssim 3.4$

$t_{eq.} \simeq 3{,}000$ years — $t \approx 200{,}000\, h^{-1}$ yrs — $t \sim 2 \times 10^8$ yrs

'Matter domination'

$T \approx 3{,}000$ K

Decoupling of matter and radiation (= recombination of atoms) $\Rightarrow$ last scattering surface of CBR ($T_0 \approx 2.73$ K)

"3^{rd} success"

"Galaxy formation"