University of Southampton
School of Physics and Astronomy

invisibles
neutrinos, dark matter & dark energy physics

Unified Models of Neutrinos, Flavour and CP violation

Steve King
Corfu Holiday Palace,
2nd September 2015

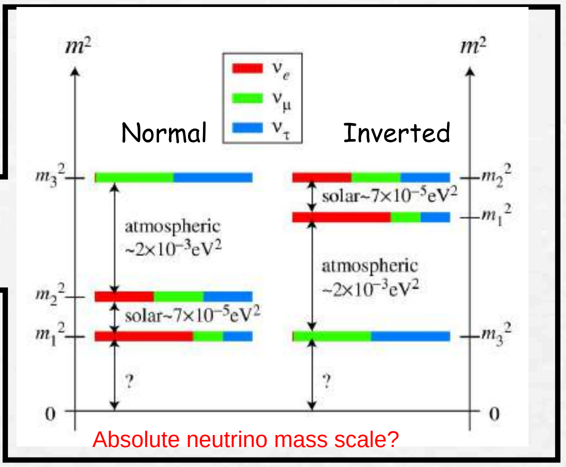
Lepton Mixing Matrix

Standard Model states

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

PMNS matrix

Neutrino mass states



Pontecorvo
Maki
Nakagawa
Sakata

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c^l_{23} & s^l_{23} \\ 0 & -s^l_{23} & c^l_{23} \end{pmatrix} \begin{pmatrix} c^l_{13} & 0 & s^l_{13}e^{-i\delta^l} \\ 0 & 1 & 0 \\ -s^l_{13}e^{i\delta^l} & 0 & c^l_{13} \end{pmatrix} \begin{pmatrix} c^l_{12} & s^l_{12} & 0 \\ -s^l_{12} & c^l_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\alpha_{21}}{2} & 0 \\ 0 & 0 & \frac{\alpha_{31}}{2} \end{pmatrix}$$

$s^l_{ij} = \sin(\theta^l_{ij})$

$c^l_{ij} = \cos(\theta^l_{ij})$

Atmospheric | Reactor | Solar | Majorana

Oscillation phase δ^l

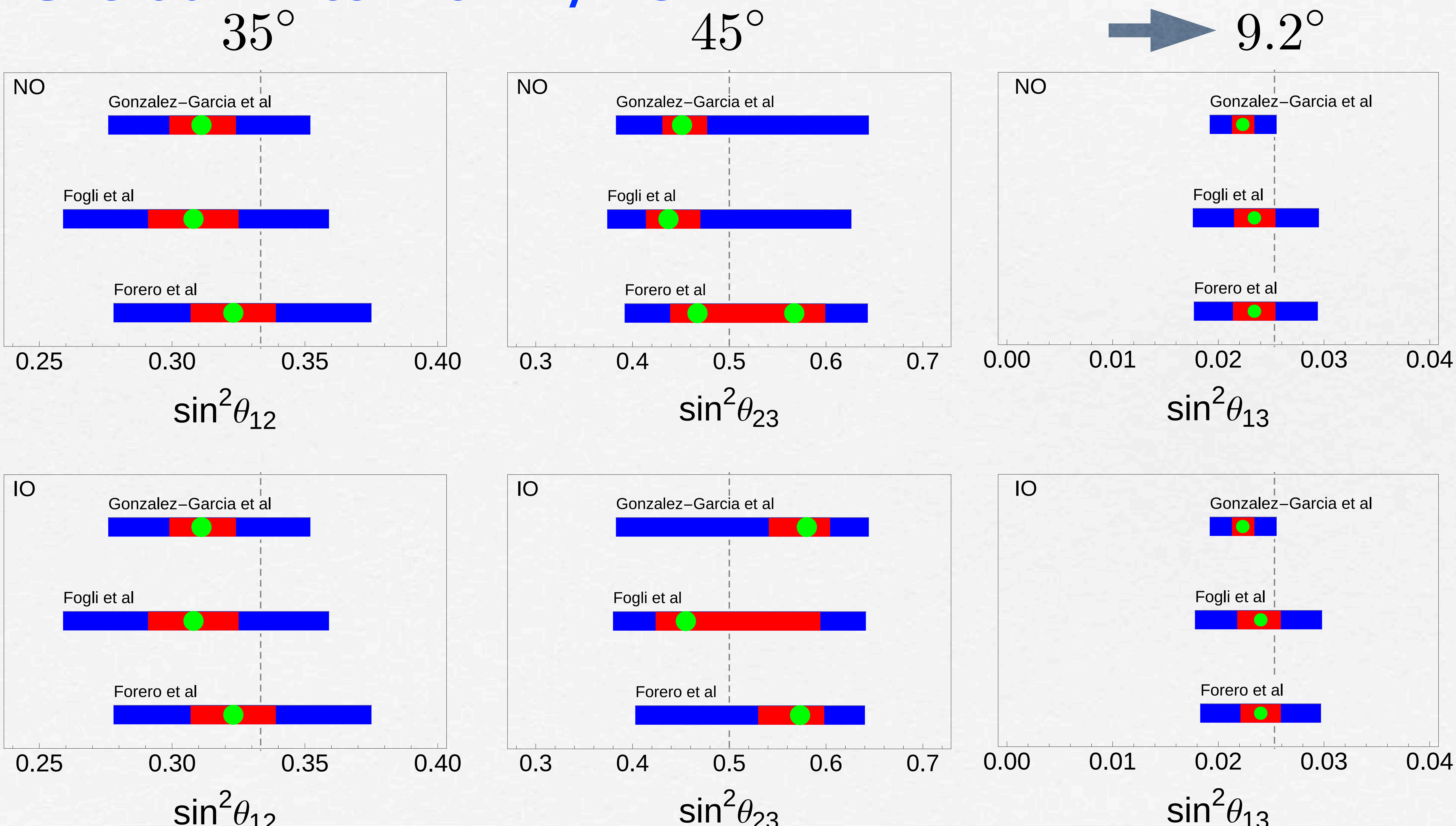
Majorana phases α_{21}, α_{31}

3 masses + 3 angles + 3 phases =
9 new parameters for SM

Gonzalez-Garcia et al = Gonzalez-Garcia, Maltoni, Salvado, Schwetz
Fogli et al = Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo
Forero et al = Forero, Tortola, Valle

Global Fits 2014/15

$$\theta_{13} = \frac{\theta_C}{\sqrt{2}}$$

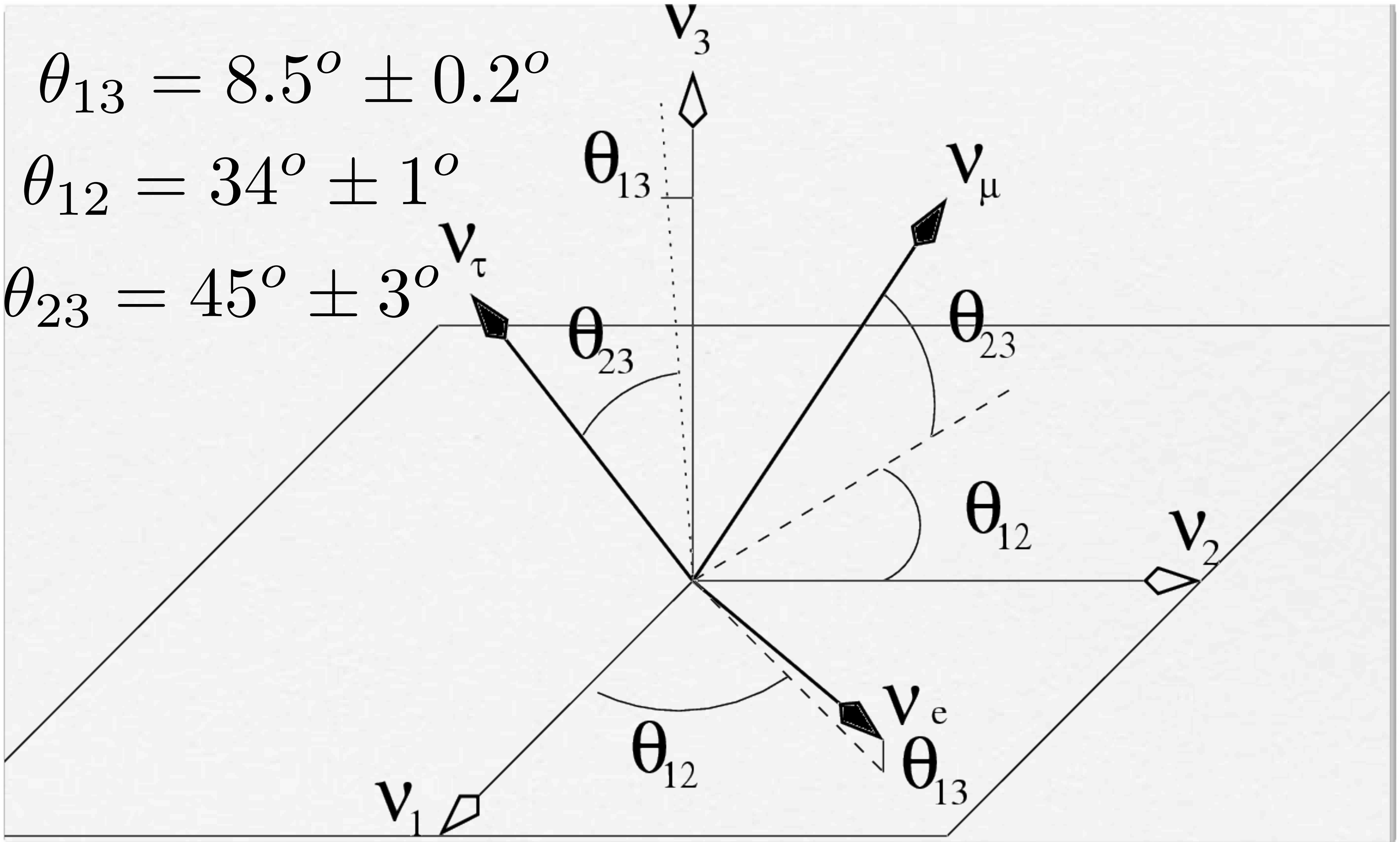


Lepton Mixing Angles (approx.)

$$\theta_{13} = 8.5^o \pm 0.2^o$$

$$\theta_{12} = 34^o \pm 1^o$$

$$\theta_{23} = 45^o \pm 3^o$$

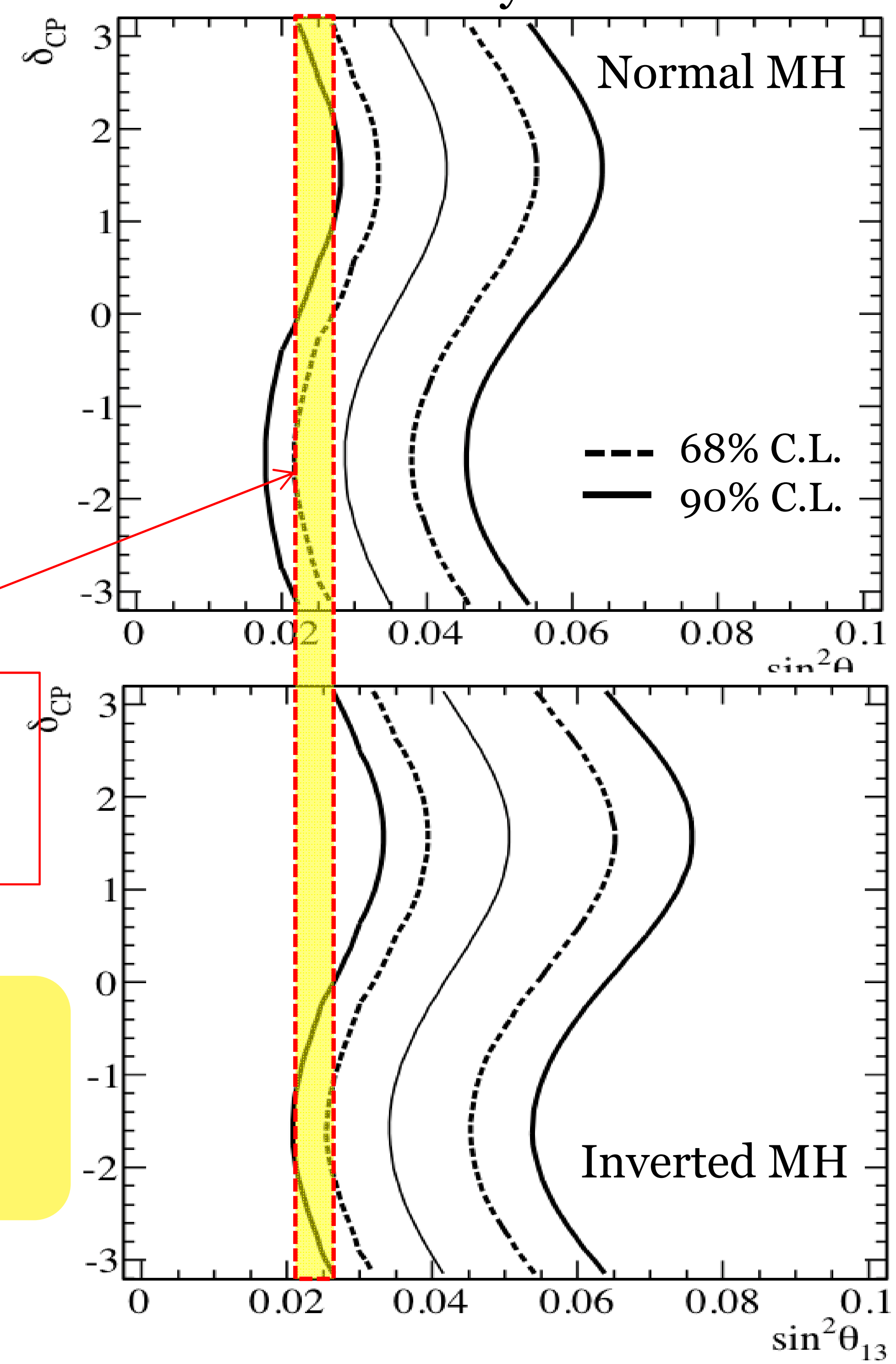


Hint for CP phase

T2K Run1-4 data
(data till 2013)
Preliminary

Fit ν_μ sample and ν_e sample simultaneously by $\sin^2\theta_{23}$,$\sin^2\theta_{13}$,δ and Δm^2

Now uncertainty of $\sin2\theta_{23}$ is included.



68% allowed region from reactor measurement (PDG2013)

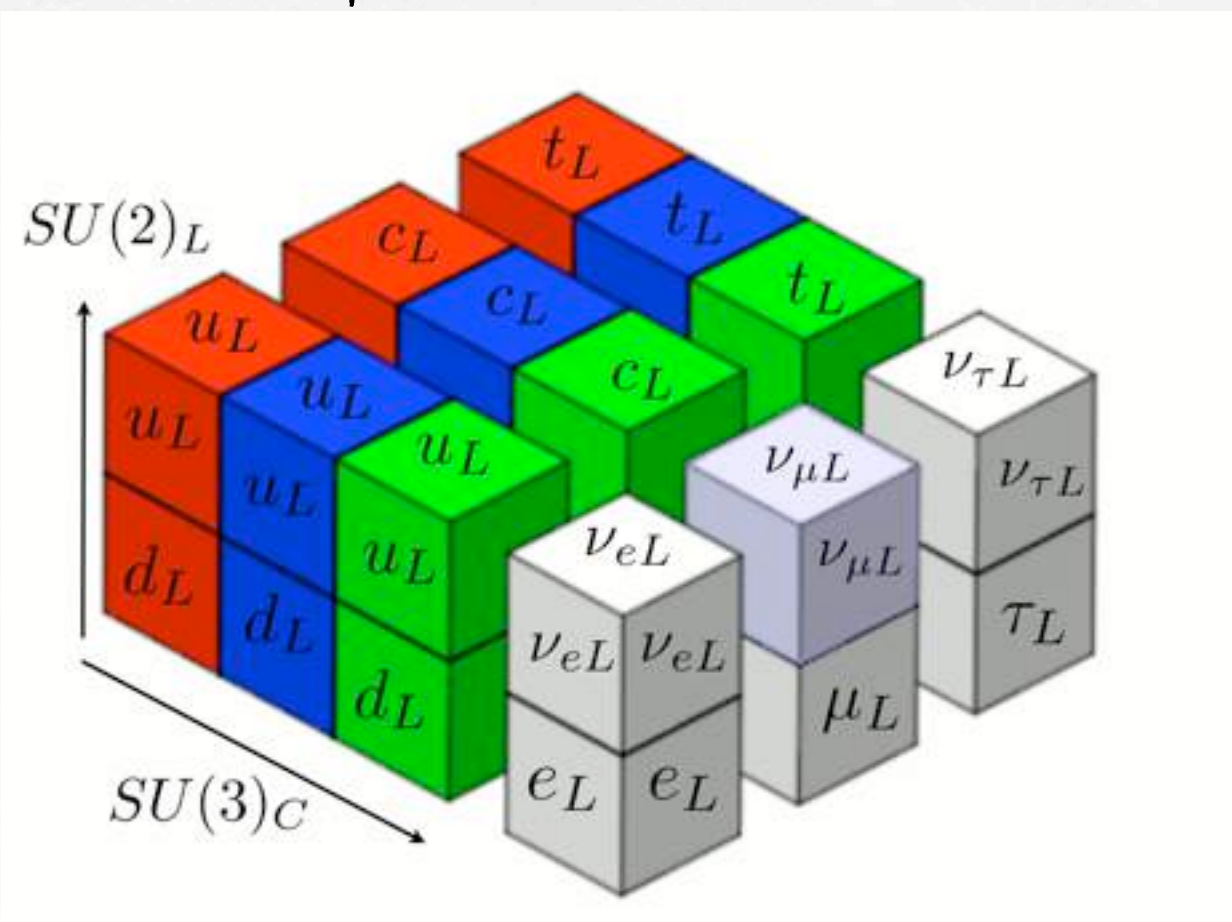
Hint for $\delta_{CP} \approx -90^\circ$

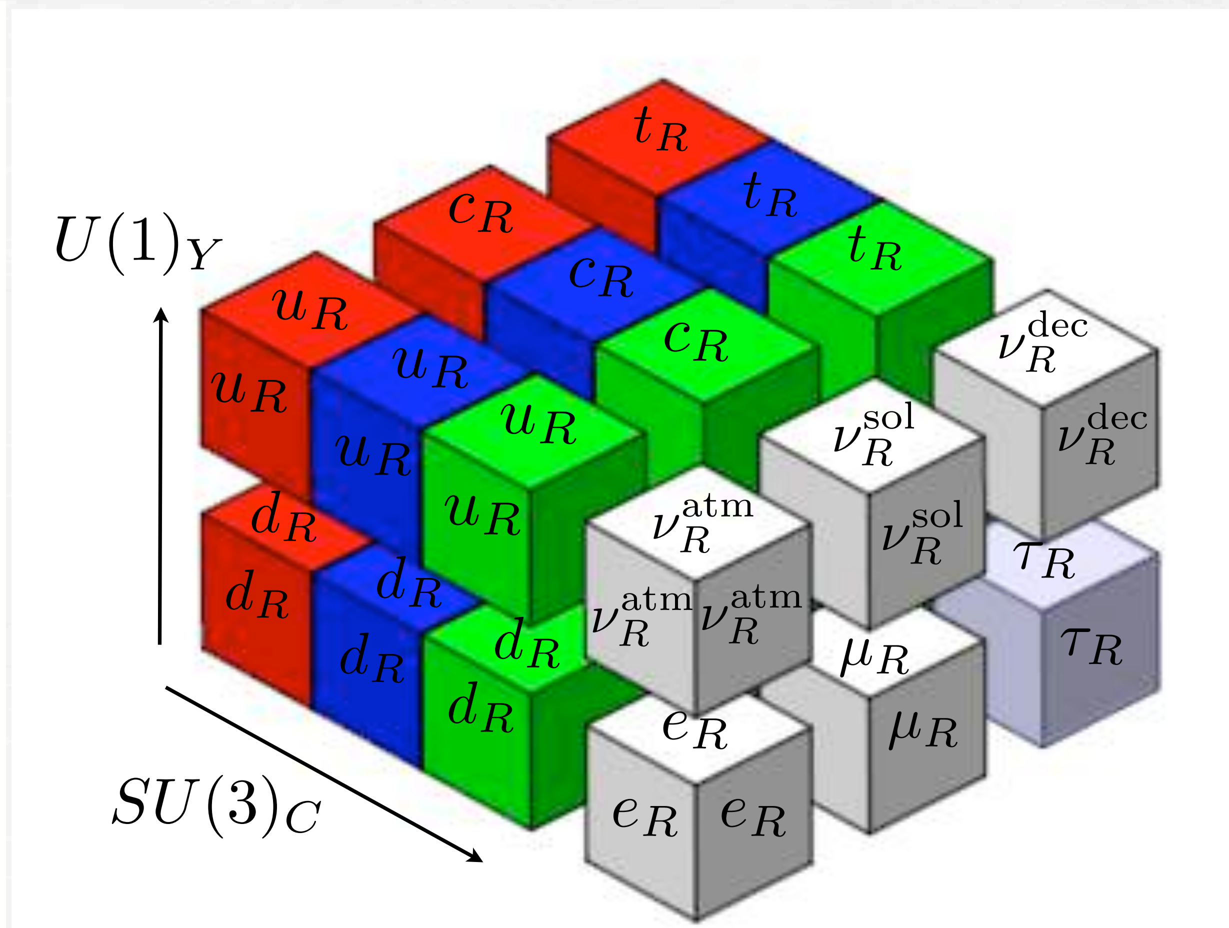
Seesaw motivates Standard Model with right-handed neutrinos

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Left-handed quarks and leptons (active neutrinos)

Right-handed quarks and leptons (sterile neutrinos)





The RH neutrino Spectrum

M_{GUT} $\uparrow M_R$

Classic See-Saw (Leptogenesis)

TeV

Low Scale See-Saw (LHC)

GeV

Nu-MSM

BAU (resonant lepto)

MeV

keV

WDM

eV

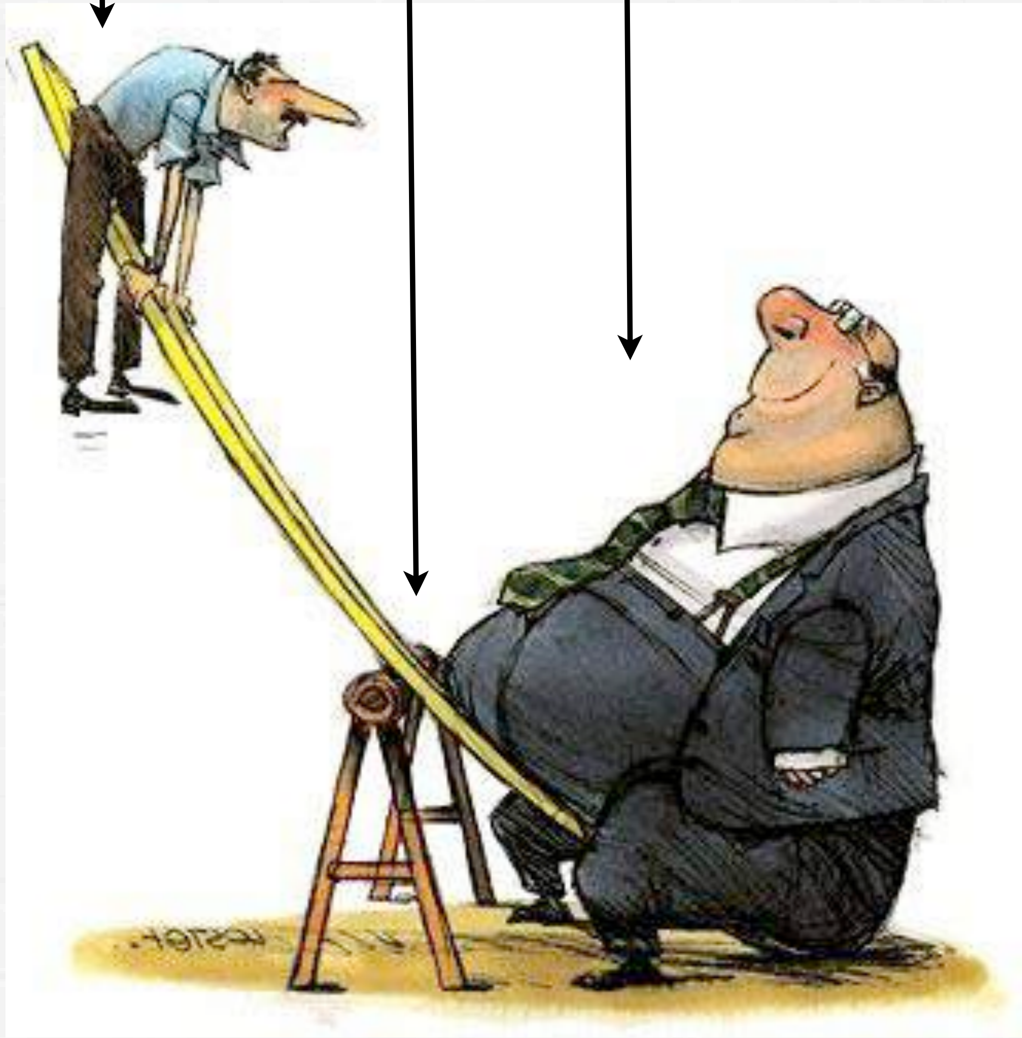
LSND, Reactor Anomaly,...

meV

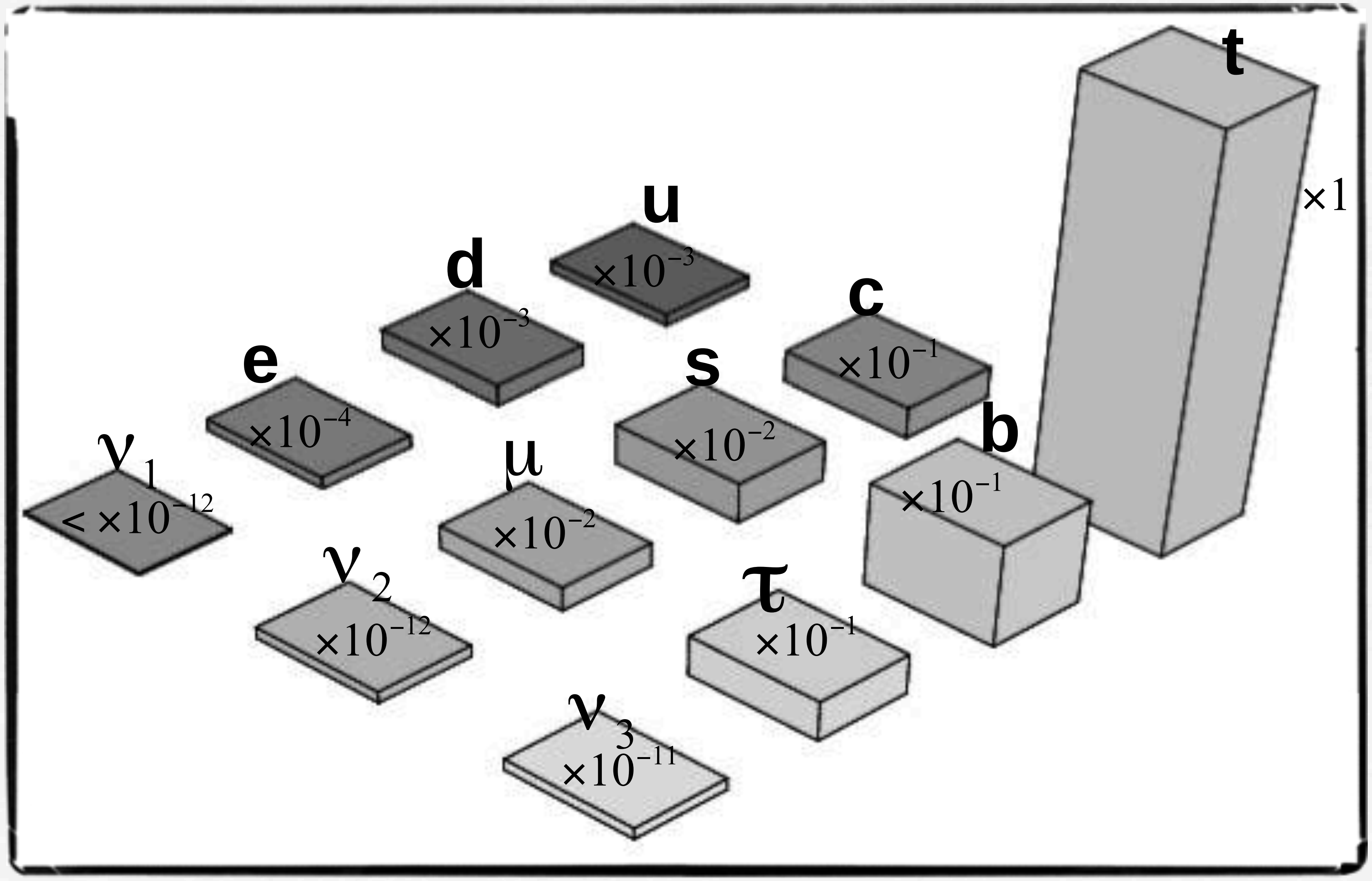
Extra radiation (Planck)

low energy nu mass matrix

$$m^{\nu} = -m_D \frac{1}{M_R} m_D^T$$

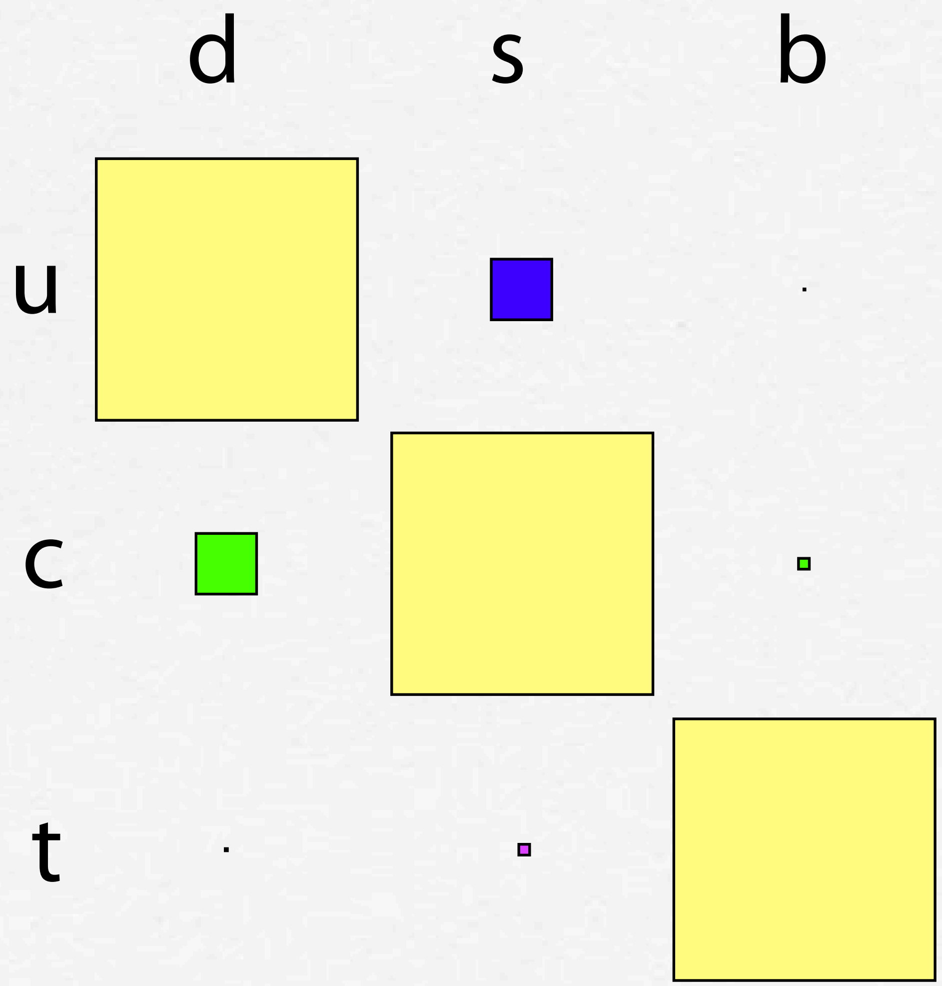


What is the origin of Quark and Lepton Masses?

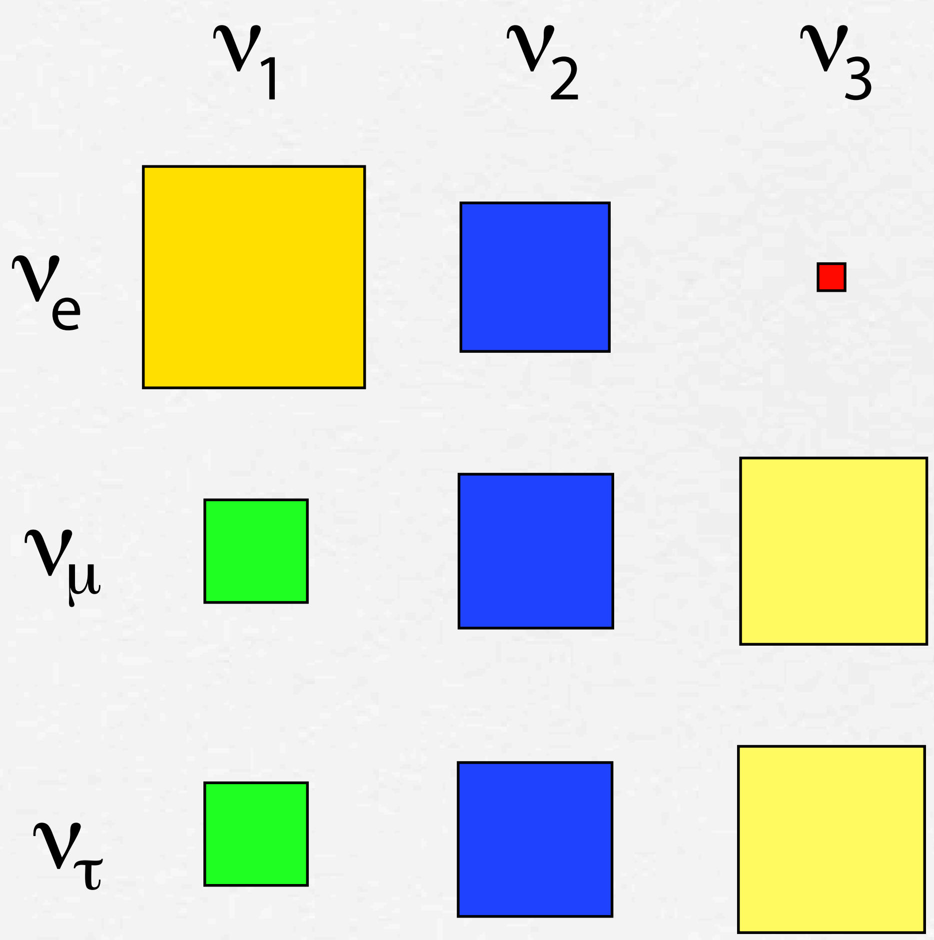


What is the origin of Quark and Lepton Mixing?

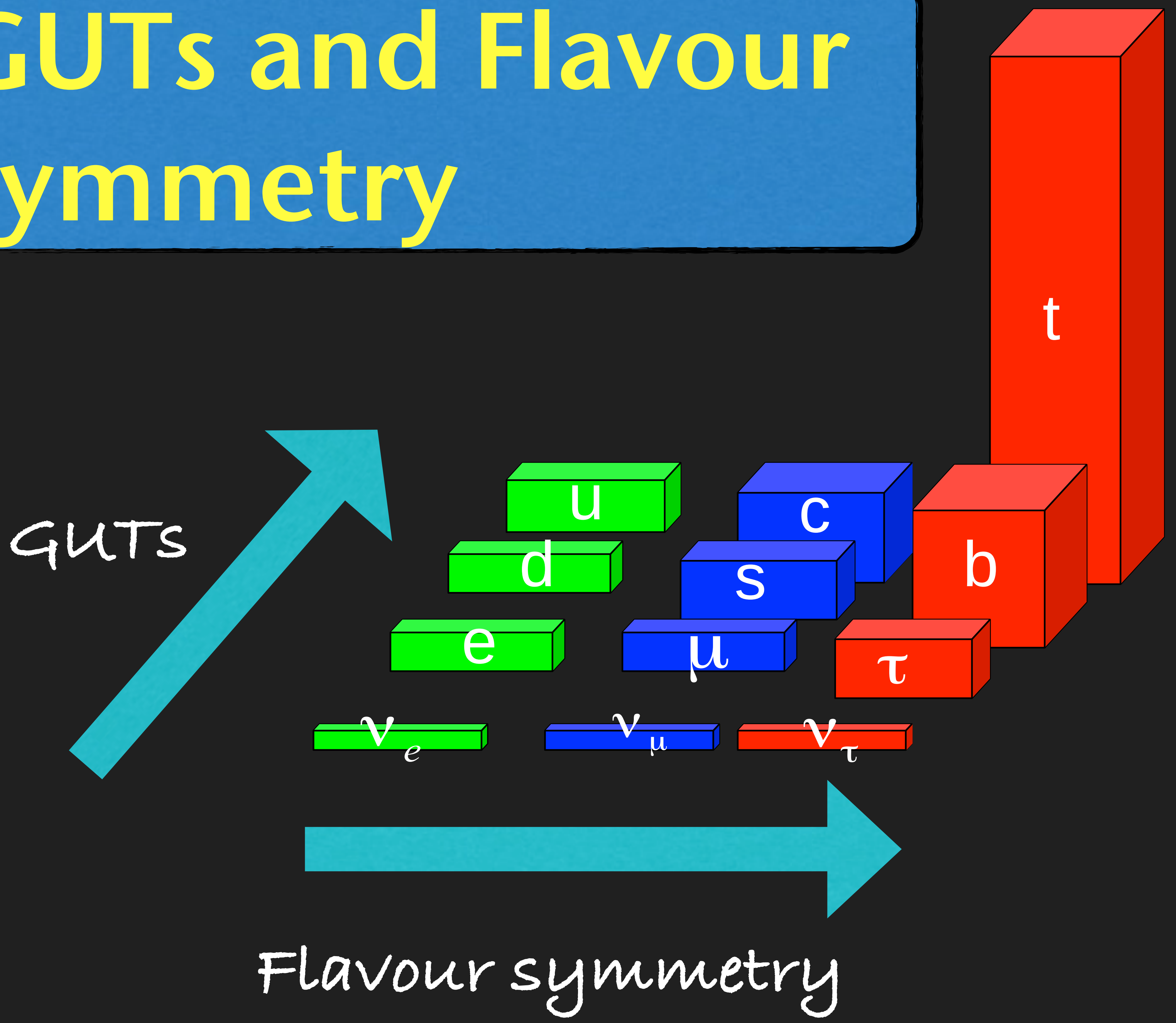
CKM



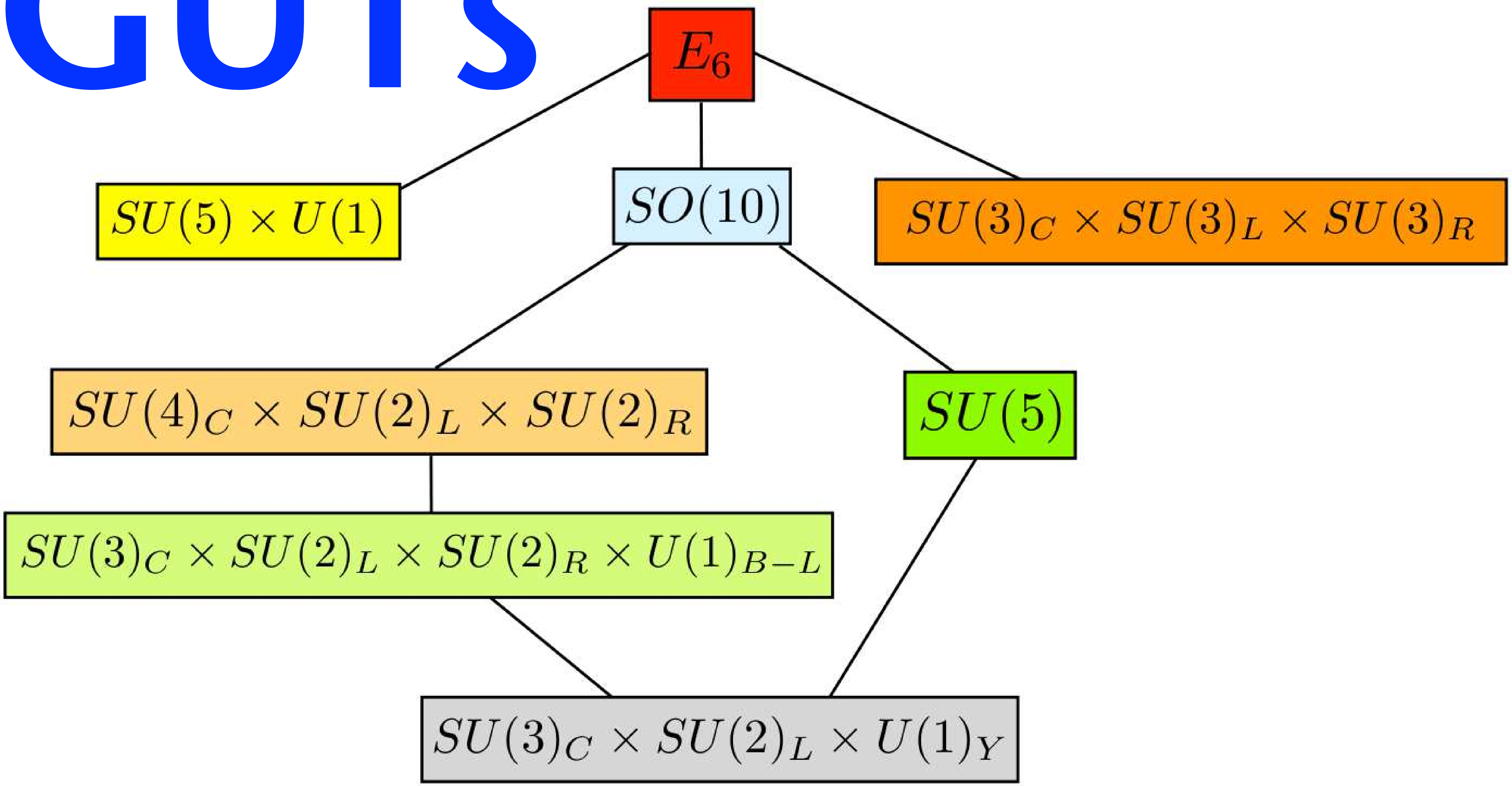
PMNS



GUTs and Flavour symmetry



GUTs

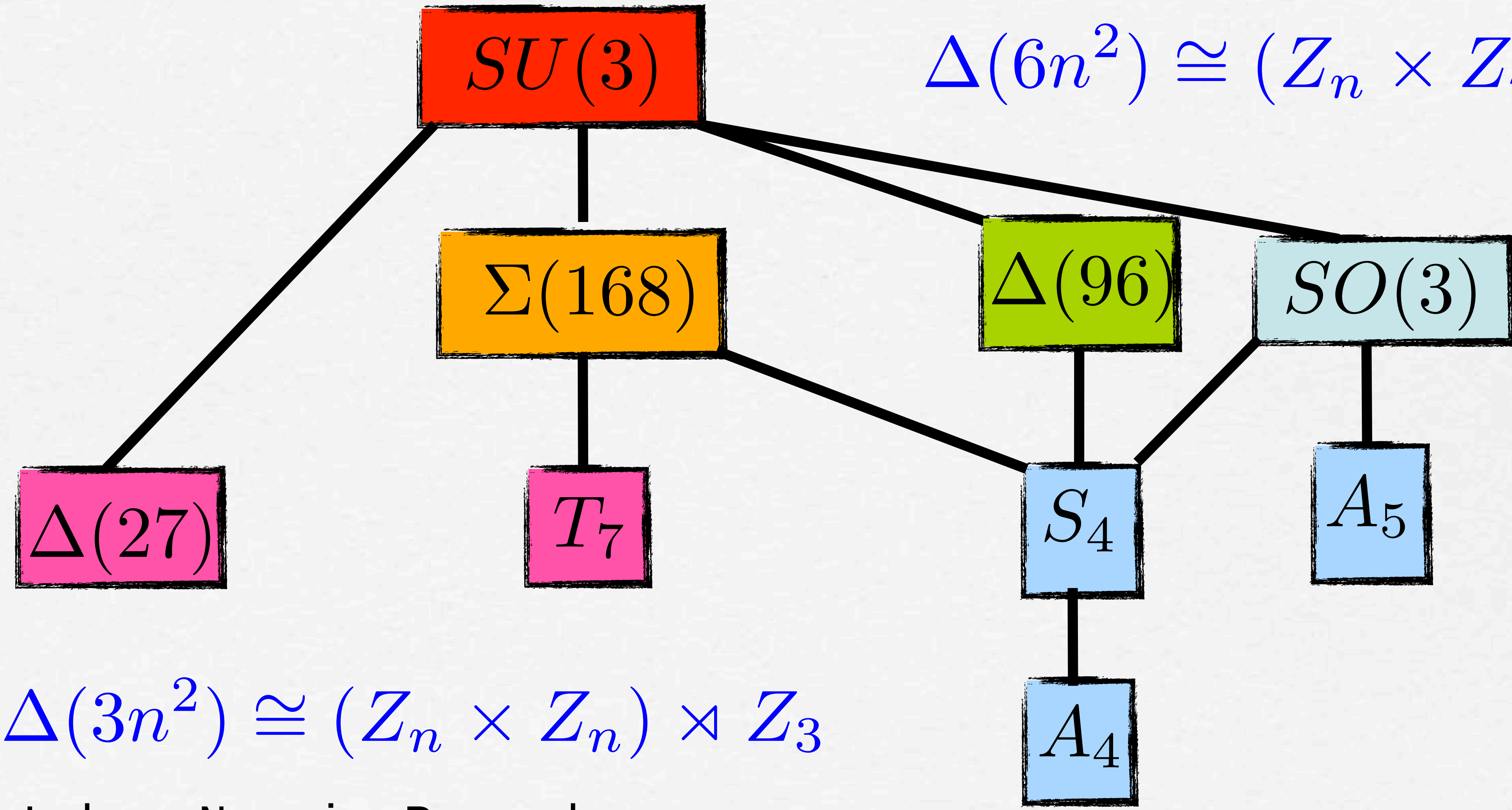


E_6
$SU(5) \times U(1)$
$SO(10)$
$SU(3)_C \times SU(3)_L \times SU(3)_R$
$SU(4)_C \times SU(2)_L \times SU(2)_R$
$SU(5)$
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$SU(3)_C \times SU(2)_L \times U(1)_Y$

Flavour Symmetry (FLASY)

Escobar, Luhn

$$\Delta(6n^2) \cong (Z_n \times Z_n) \rtimes S_3$$



$$\Delta(3n^2) \cong (Z_n \times Z_n) \rtimes Z_3$$

Luhn, Nasri, Ramond

The Klein Symmetry

Felix Klein

Phase symmetry of diagonal charged lepton mass matrix

$$T^\dagger (M_e^\dagger M_e) T = M_e^\dagger M_e \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{2i\pi/n}$$

Symmetry of Majorana matrix depends on PMNS

$$m_\nu = S^T m_\nu S \qquad m_\nu = U^T m_\nu U$$

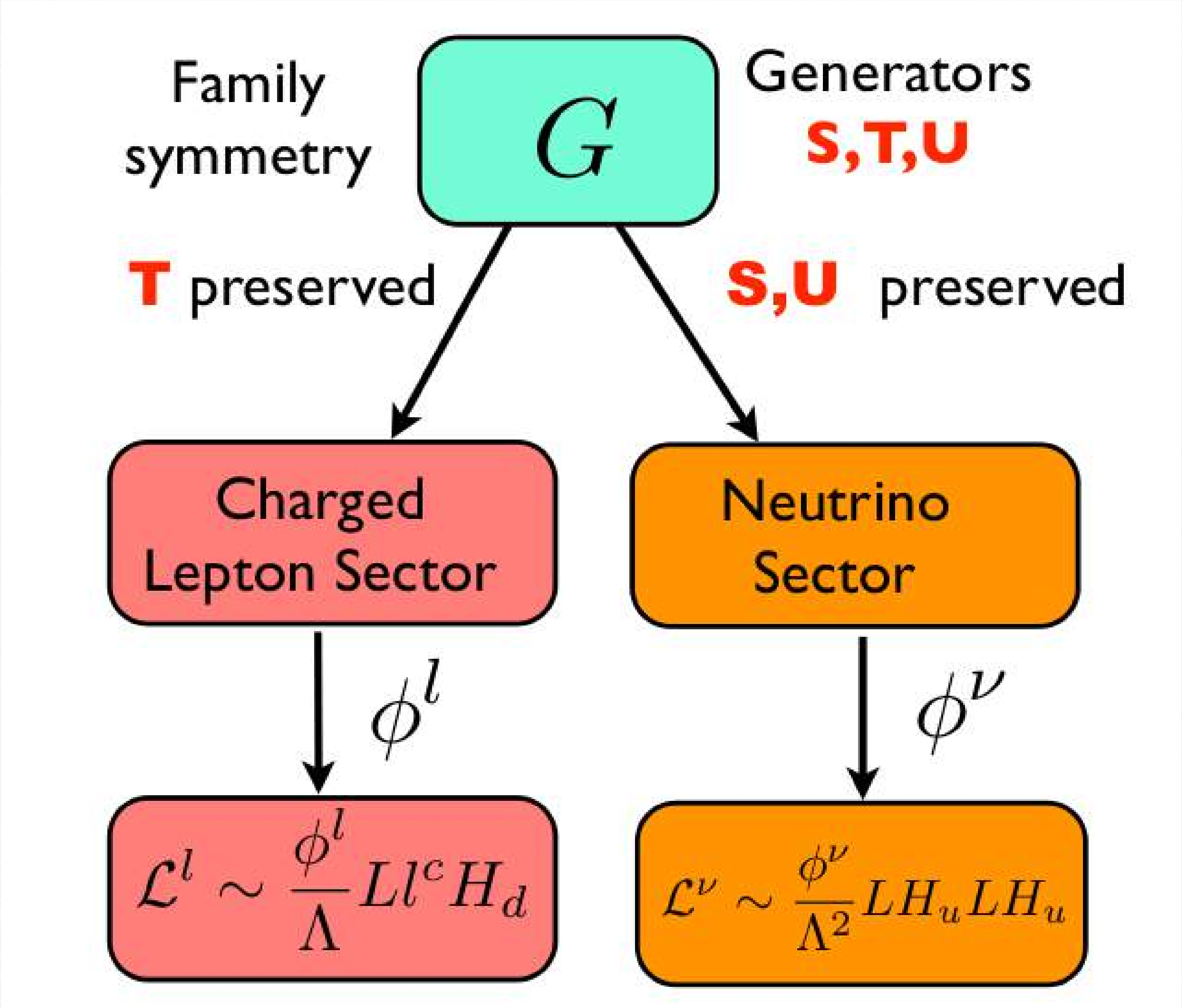
$$\left.\begin{aligned} S &= U^*_{\mathrm{PMNS}}\ \mathrm{diag}(+1,-1,-1)\ U^T_{\mathrm{PMNS}} \\ U &= U^*_{\mathrm{PMNS}}\ \mathrm{diag}(-1,+1,-1)\ U^T_{\mathrm{PMNS}} \\ SU &= U^*_{\mathrm{PMNS}}\ \mathrm{diag}(-1,-1,+1)\ U^T_{\mathrm{PMNS}} \end{aligned}\right\}$$

Klein Symmetry

$$\mathcal{K} = \{1, S, U, SU\}$$

$$Z_2 \times Z_2$$

Direct Models



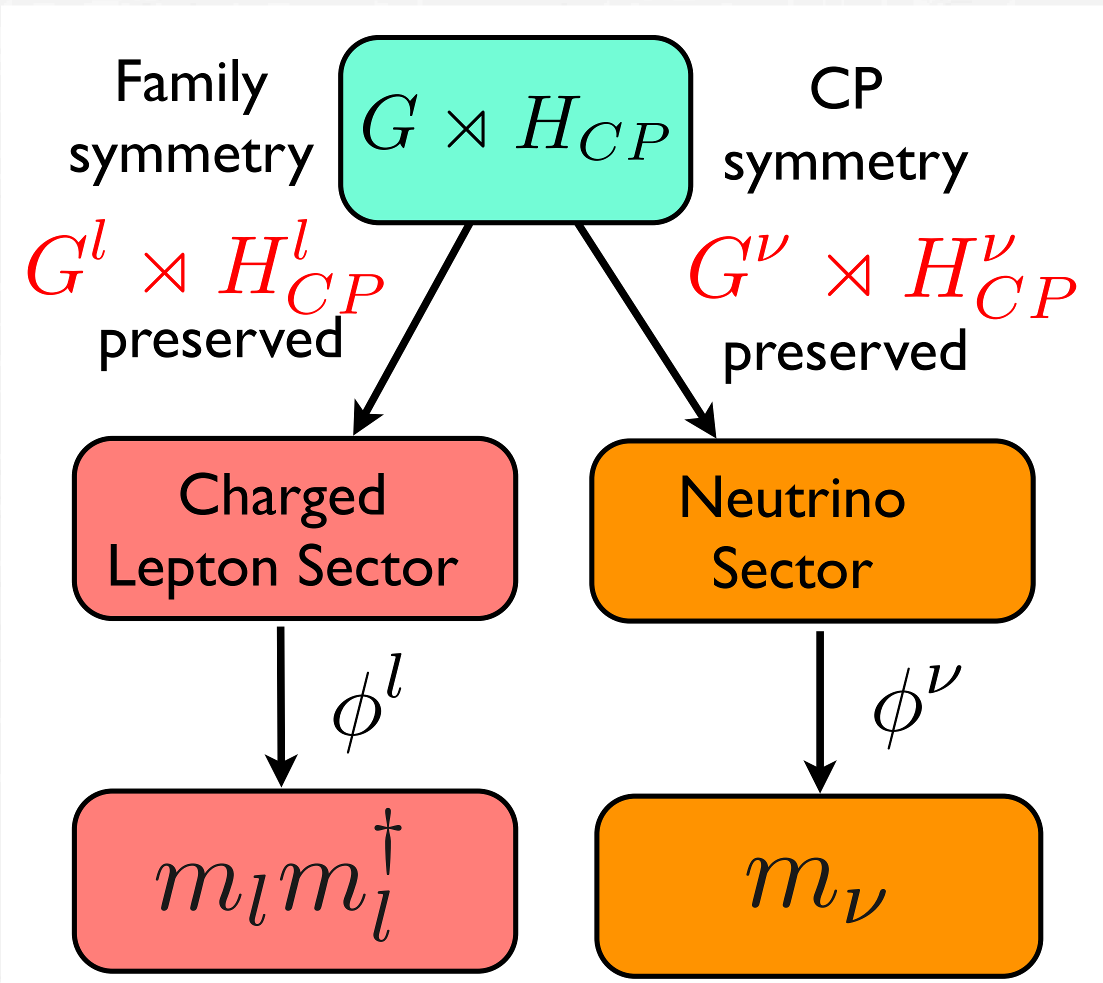
Klein symmetry S,U and T are each identified as subgroups of some family symmetry

$$\Delta(6n^2)$$

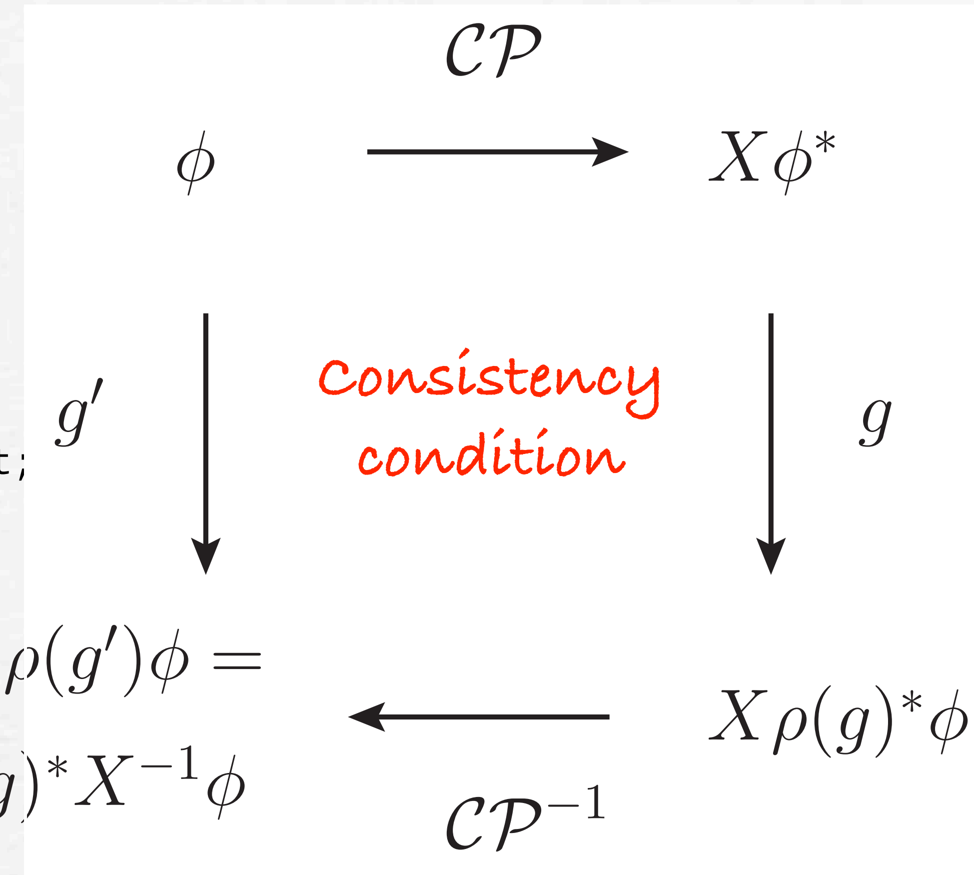
is the only viable symmetry class - predicts zero Dirac CPV but non-zero Majorana phases

Holthausen,Lim, Lindner;
SK,Neder,Stuart;
Lavoura,Ludl;
Fonseca,Grimus

CP violation



Feruglio,Hagedorn; Holthausen,Lindner Schmidt; Ding,SFK,Luhn,Stuart; Nishi,Xing; Hagedorn,Meroni, Molinaro; Ding,SFK,Neder; Chen et al...



Equivalently use CP invariants of the Lagrangian

$$I_1 \equiv \mathrm{Tr}\,[H_\nu, H_l]^3 = 0$$

$$H_\nu \equiv m_\nu m_\nu^\dagger \text{ and } H_l \equiv m_l m_l^\dagger$$

Branco, de Medeiros Varzielas and S.F.K., 1502.03105 1505.06165

- S_4 and A_4 models with CP symmetry are constructed,all the possible cases following from the model-independent analysis can be realized. **Dirac CP phase is predicted to be trivial or maximal**.

$\Delta(96), \Delta(6n^2), \Delta(3n^2)$ allows many possible predictions for Dirac CP phase in semi-direct models

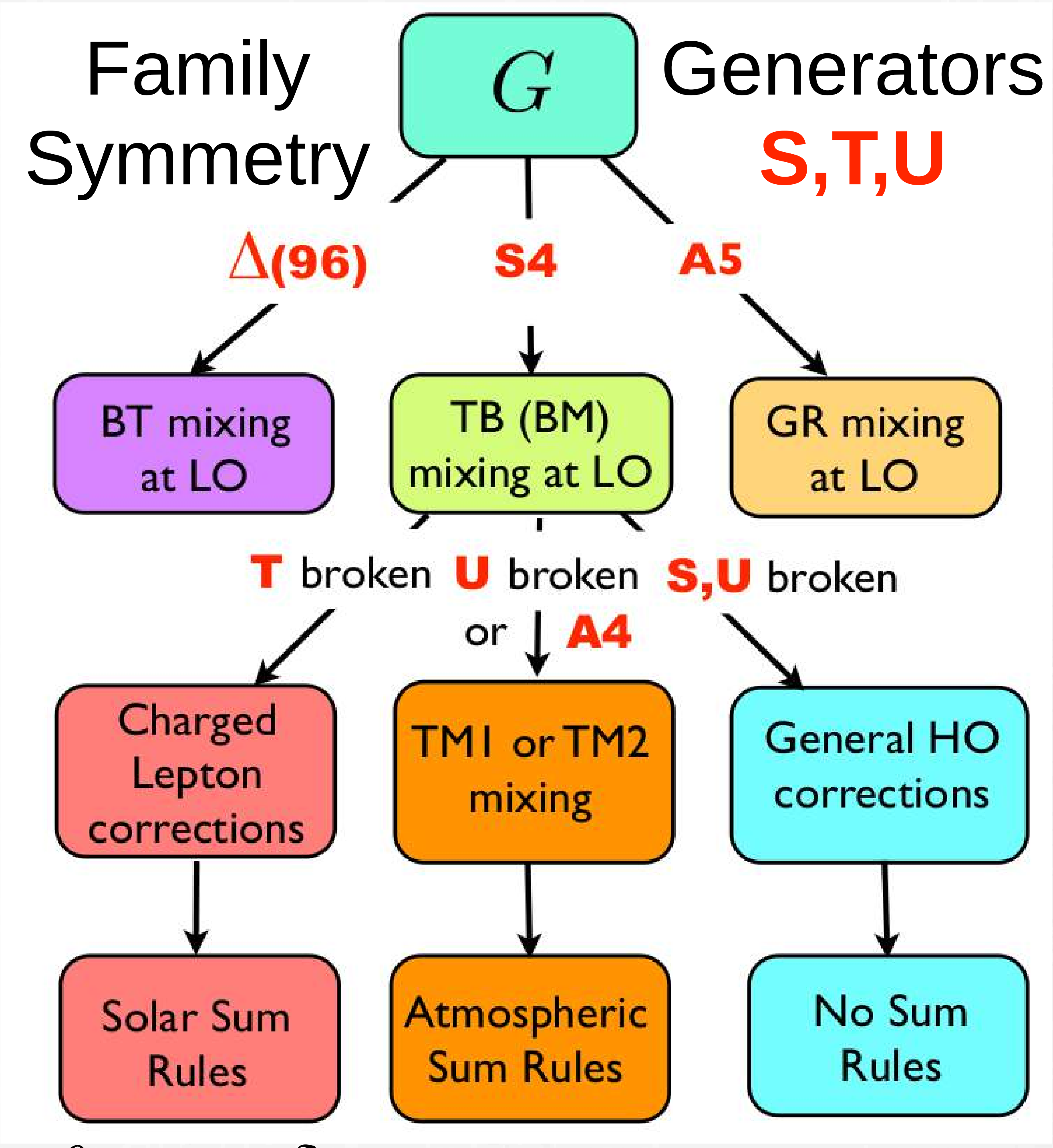
Semi-Direct Models

Klein symmetry and T are partly preserved as subgroups of some family symmetry

Reviews:

S.F.K.,Luhn 1301.1340;

S.F.K.,Merle, Morisi,Shimizu, Tanimoto, 1402.4271



Antusch,S.F.K. 0506297/0508044

$$\theta_{12} \approx \theta_{12}^{\nu} + \theta_{13} \cos\delta$$

$$\begin{array}{lcl} \mathrm{TB} & = & \text{tri-bimaximal} \\ \mathrm{BM} & = & \text{bimaximal} \\ \mathrm{GR} & = & \text{golden ratio} \\ \mathrm{BT} & = & \text{bi-trimaximal} \\ \mathrm{TM} & = & \text{trimaximal} \end{array}$$

	θ_{13}	θ_{23}	θ_{12}
TB	0°	45°	35.3°
BM	0°	45°	45°
GR	0°	45°	31.7°
BT	12.2°	36.2°	36.2°
TM	$\neq 0^\circ$	$\neq 45^\circ$	35.3°

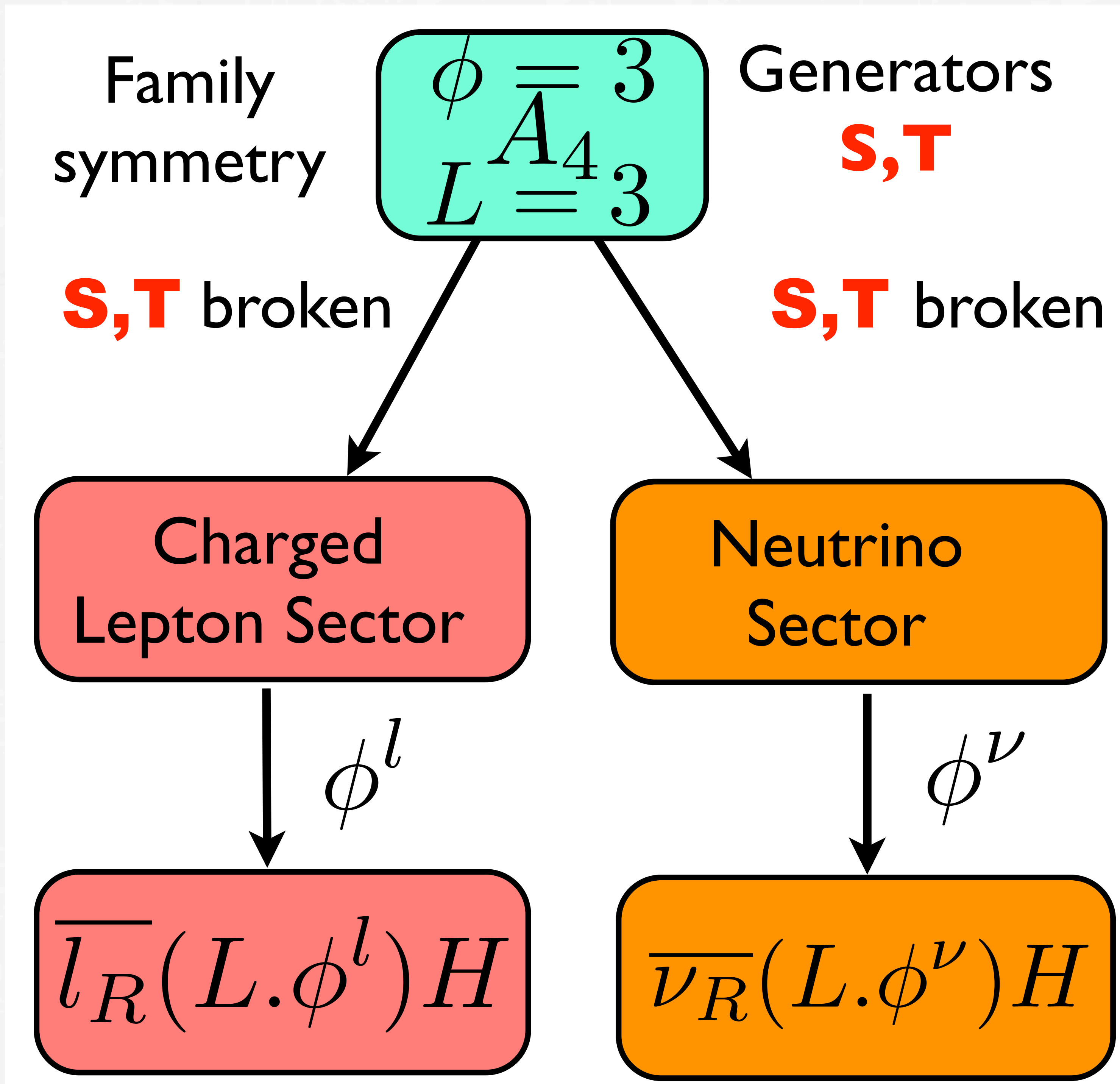
Indirect Models

Diagonal alignments

$$\phi_e = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}$$

$$\phi_\mu = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}$$

$$\phi_\tau = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$



NEW alignments Motivated by neutrino data

$$\phi_{\rm sol} = v \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix}$$

$$\phi_{\rm atm} = v \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$n = 3, 4$$

S.F.K. 1305.4846

A_4 Vacuum alignments

Symmetry preserving
eigenvectors of group elements

$$\langle\phi\rangle = \begin{pmatrix}0\\0\\v\end{pmatrix} \begin{pmatrix}0\\v\\0\end{pmatrix} \begin{pmatrix}v\\0\\0\end{pmatrix} \begin{pmatrix}\pm v\\\pm v\\\pm v\end{pmatrix}$$

Direct

Symmetry breaking
orthogonal alignments

$$\langle\phi\rangle = \begin{pmatrix}0\\v\\v\end{pmatrix} \perp \begin{pmatrix}v\\v\\-v\end{pmatrix}, \begin{pmatrix}v\\0\\0\end{pmatrix}$$

Indirect

New Orthogonal alignments

$$\langle\phi\rangle = \begin{pmatrix}2v\\-v\\v\end{pmatrix} \perp \begin{pmatrix}v\\v\\-v\end{pmatrix}, \begin{pmatrix}0\\v\\v\end{pmatrix}$$

$$\langle\phi\rangle = \begin{pmatrix}v\\nv\\(n-2)v\end{pmatrix} \perp \begin{pmatrix}2v\\-v\\v\end{pmatrix}$$

S.F.K. 1304.6264
Björkeroth and S.F.K. 1412.6996

Minimal Predictive Seesaw models

Two right-handed neutrinos ("minimal")

$$M_1 = M_{\rm atm} \text{ and } M_2 = M_{\rm sol}$$

3.3

$$H(L.\phi_{\rm atm})N^c_{\rm atm} + H(L.\phi_{\rm sol})N^c_{\rm sol} + M_{\rm atm}N^c_{\rm atm}N^c_{\rm atm} + M_{\rm sol}N^c_{\rm sol}N^c_{\rm sol}$$

$$\langle\phi_{\rm atm}\rangle = v_{\rm atm}\begin{pmatrix}0\\1\\1\end{pmatrix}, \quad \langle\phi_{\rm sol}\rangle = v_{\rm sol}\begin{pmatrix}1\\n\\n-2\end{pmatrix}$$

NEW indirect alignments ("predictive")

$$\lambda^\nu = \begin{pmatrix}0 & b\\ a & nb\\ a & (n-2)b\end{pmatrix}, \qquad M^c = \begin{pmatrix}M_1 & 0\\ 0 & M_2\end{pmatrix}$$

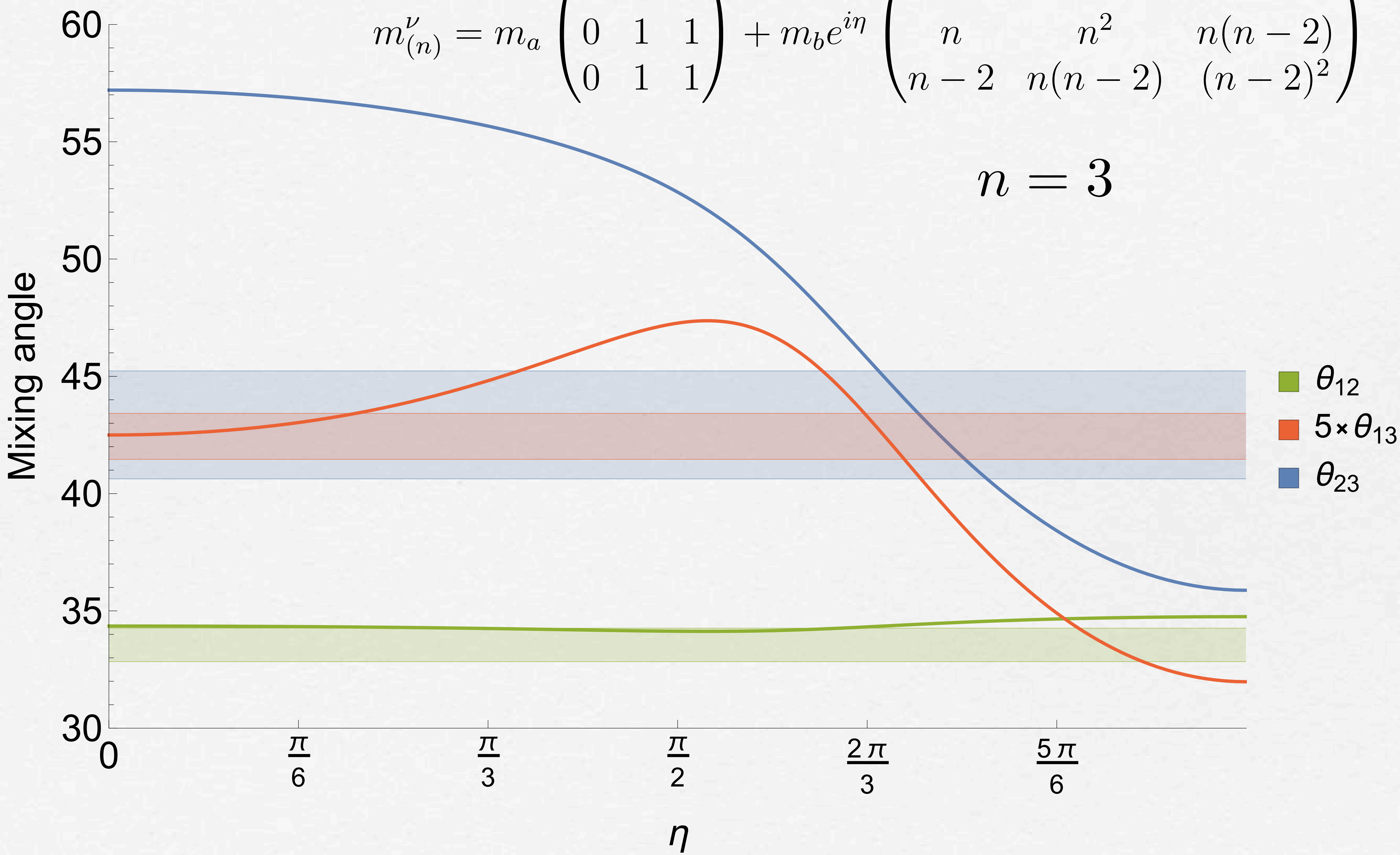
Seesaw matrices

Only one phase

$$m^\nu_{(n)} = m_a\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + m_b e^{i\eta}\begin{pmatrix}1 & n & n-2\\ n & n^2 & n(n-2)\\ n-2 & n(n-2) & (n-2)^2\end{pmatrix}$$

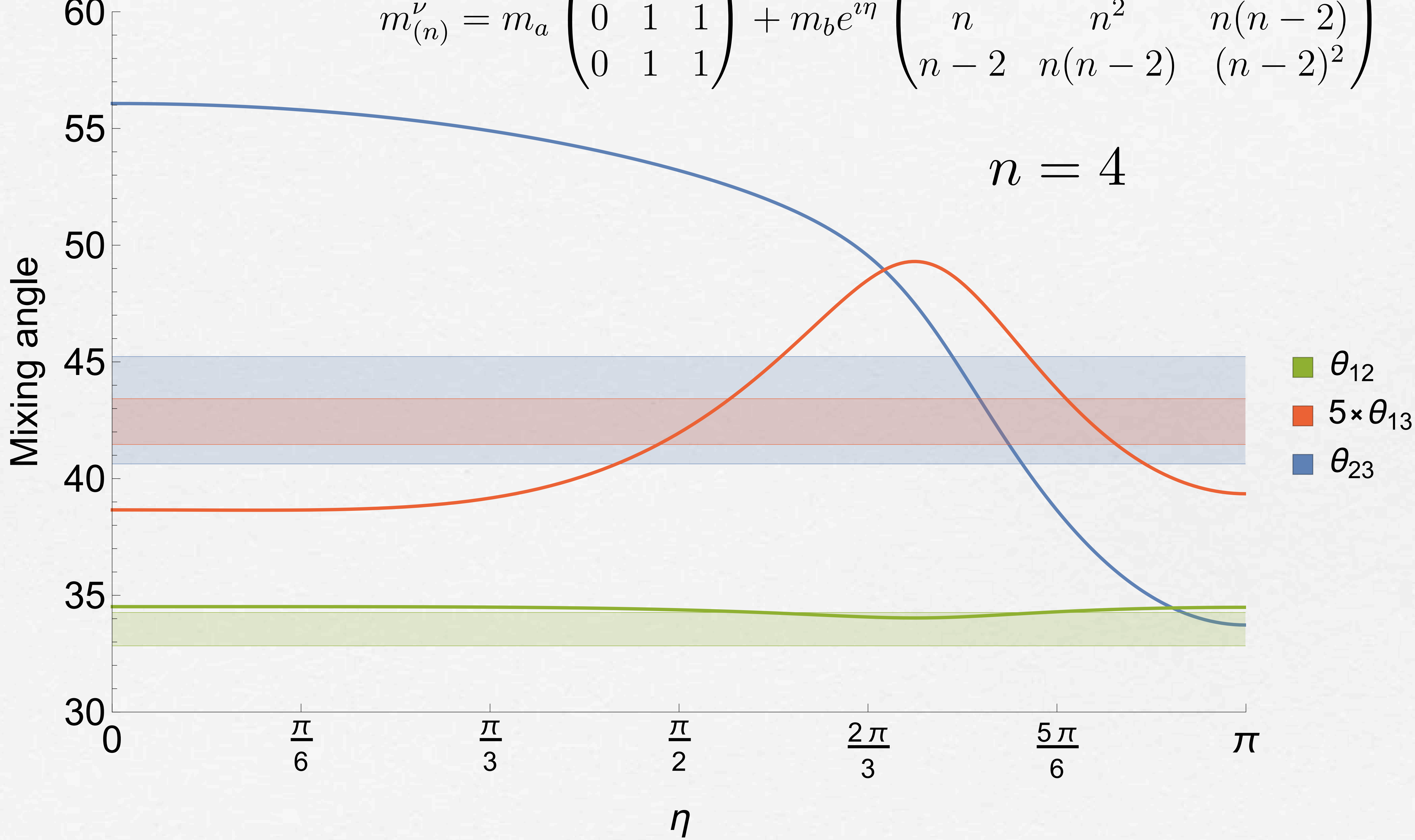
Neutrino mass matrix

$$m^{\nu}_{(n)} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & n & n-2 \\ n & n^2 & n(n-2) \\ n-2 & n(n-2) & (n-2)^2 \end{pmatrix}$$

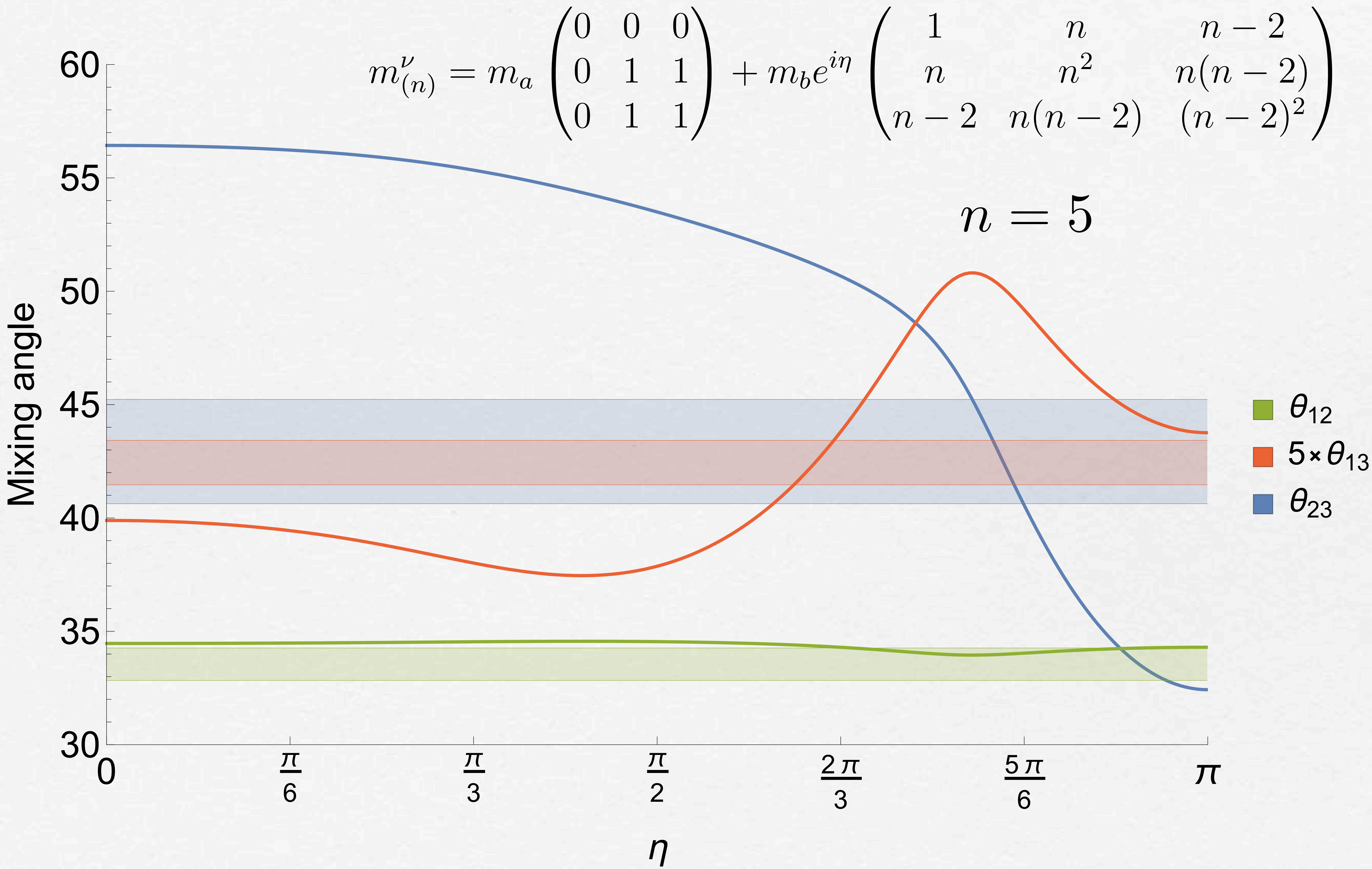


(a) CSD(3)

$$m^{\nu}_{(n)} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & n & n-2 \\ n & n^2 & n(n-2) \\ n-2 & n(n-2) & (n-2)^2 \end{pmatrix}$$



(b) CSD(4)



$$m^{\nu}_{(n)} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & n & n-2 \\ n & n^2 & n(n-2) \\ n-2 & n(n-2) & (n-2)^2 \end{pmatrix}$$
$n = 5$
Mixing angle
60
55
50
45
40
35
30
0
$\frac{\pi}{6}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{5\pi}{6}$
π
η
θ_{12}
$5\times\theta_{13}$
θ_{23}

(c) CSD(5)

Björkeroth, de Anda, de Medieoros Varzielas and S.F.K. 1503.03306

Towards a complete A_4xSU(5) SUSY GUT

- Renormalisable at GUT scale, SU(5) breaking potential, spontaneously broken CP.
- The MSSM is reproduced with R-parity emerging from a discrete Z_4^R .
- Doublet-triplet splitting is achieved through the Missing Partner mechanism.
- mu term is generated at the correct scale.
- Proton decay is sufficiently suppressed.
- It solves the strong CP problem through the Nelson-Barr mechanism .
- Explains quark mass hierarchies, mixing angles and the CP phase.
- Reproduces minimal predictive seesaw model with CSD(3) alignment.
- Two right-handed neutrinos, lighter one dominantly giving atmospheric neutrino mass.
- Z_9 flavour symmetry fixes the phase η to be one of ninth roots of unity, choose 2pi/3

Björkeroth, de Anda, de Medieoros Varzielas and S.F.K. 1503.03306

A_4xSU(5) SUSY GUT

Fermions Field	Representation A_4	SU(5)	$\mathbb{Z}_9$	$\mathbb{Z}_6$	$\mathbb{Z}_4^R$
F	3	$\bar{5}$	0	0	1
T_1	1	10	5	0	1
T_2	1	10	7	0	1
T_3	1	10	0	0	1
N_1^c	1	1	7	3	1
N_2^c	1	1	8	3	1
Γ	1	1	0	3	1

Higgs Field	Representation A_4	SU(5)	$\mathbb{Z}_9$	$\mathbb{Z}_6$	$\mathbb{Z}_4^R$
H_5	1	5	0	0	0
$H_{\bar{5}}$	1	$\bar{5}$	2	0	0
H_{24}	$1'$	24	3	0	0
Λ_{24}	$1'$	24	0	0	0
H_{45}	1	45	4	0	2
$H_{\overline{45}}$	1	$\overline{45}$	5	0	0

Flavons Field	Representation A_4	SU(5)	$\mathbb{Z}_9$	$\mathbb{Z}_6$	$\mathbb{Z}_4^R$
ξ	1	1	2	0	0
θ_1	1	1	1	3	0
θ_2	1	1	1	4	0
ϕ_e	3	1	0	0	0
ϕ_μ	3	1	3	0	0
ϕ_τ	3	1	7	0	0

$$W_{\rm down} = d_{33}T_3\frac{H_{\bar{5}}\phi_\tau}{M}F + d_{22}T_2\frac{H_{\overline{45}}H_{24}\phi_\mu}{M^2}F + d_{11}T_1\frac{H_{\bar{5}}\xi\phi_e}{\langle\Lambda_{24}\rangle^2}F + d_{12}T_1\frac{H_{\bar{5}}\xi\phi_\mu}{\langle\Lambda_{24}\rangle\langle H_{24}\rangle}F$$

$$W_{\rm up} = u_{ij}H_5T_iT_j\left(\frac{\xi}{M}\right)^{n_{ij}}$$

$$Y^u_{ij} = u_{ij}\left(\frac{\langle\xi\rangle}{M}\right)^{n_{ij}} \sim \begin{pmatrix} \tilde{\xi}^4 & \tilde{\xi}^3 & \tilde{\xi}^2 \\ & \tilde{\xi}^2 & \tilde{\xi} \\ & & 1 \end{pmatrix}$$

strong hierarchy of up quark masses

Asymmetric matrix
-small e mixing
- Nelson-Barr det Y=real

$$Y^d_{LR} \sim Y^e_{RL} \sim \begin{pmatrix} \frac{\langle\xi\rangle v_e}{v^2_{\Lambda_{24}}} & \frac{\langle\xi\rangle v_\mu}{v_{\Lambda_{24}}v_{H_{24}}} & 0 \\ 0 & \frac{v_{H_{24}}v_\mu}{M^2} & 0 \\ 0 & 0 & \frac{v_\tau}{M} \end{pmatrix}$$

Clebsch relations

$$\frac{Y^e_{33}}{Y^d_{33}} = 1, \qquad \frac{Y^e_{22}}{Y^d_{22}} = \frac{9}{2}, \qquad \frac{Y^e_{11}}{Y^d_{11}} = \frac{Y^e_{21}}{Y^d_{12}} = \frac{4}{9}.$$

A4xSU(5) SUSY GUT

Fermions

Field	Representation				
	A_4	SU(5)	$\mathbb{Z}_9$	$\mathbb{Z}_6$	$\mathbb{Z}_4^R$
F	3	$\bar{5}$	0	0	1
T_1	1	10	5	0	1
T_2	1	10	7	0	1
T_3	1	10	0	0	1
N^c_{atm}	1	1	7	3	1
N^c_{sol}	1	1	8	3	1
Γ	1	1	0	3	1

Higgs+flavons

Field	Representation				
	A_4	SU(5)	$\mathbb{Z}_9$	$\mathbb{Z}_6$	$\mathbb{Z}_4^R$
H_5	1	5	0	0	0
$H_{\bar{5}}$	1	$\bar{5}$	2	0	0
H_{45}	1	45	4	0	2
$H_{\overline{45}}$	1	$\overline{45}$	5	0	0
ξ	1	1	2	0	0
θ_2	1	1	1	4	0
ϕ_{atm}	3	1	3	1	0
ϕ_{sol}	3	1	2	1	0

$$W_\nu = y_1 H_5 F \frac{\phi_{\text{atm}}}{\langle\theta_2\rangle} N^c_{\text{atm}} + y_2 H_5 F \frac{\phi_{\text{sol}}}{\langle\theta_2\rangle} N^c_{\text{sol}} + y_3 \frac{\xi^2}{M_\Gamma} N^c_{\text{atm}} N^c_{\text{atm}} + y_4 \xi N^c_{\text{sol}} N^c_{\text{sol}}$$

$$\langle\phi_{\text{atm}}\rangle = v_{\text{atm}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \text{CSD(3)} \quad \langle\phi_{\text{sol}}\rangle = v_{\text{sol}} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$$

$$\lambda^\nu = \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix}, \quad M^c = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix},$$

$M_1 = M_{\text{atm}}$ and $M_2 = M_{\text{sol}}$

Reproduces minimal predictive seesaw with CSD(3)

Björkeroth, de Anda, de Medeiros Varzielas and S.F.K. 1505.05504

Leptogenesis in minimal predictive seesaw

n	m_a (meV)	m_b (meV)	η (rad)	θ_{12} (°)	θ_{13} (°)	θ_{23} (°)	δ_{CP} (°)	m_2 (meV)	m_3 (meV)
3	26.57	2.684	$\frac{2\pi}{3}$	34.3	8.67	45.8	-86.7	8.59	49.8

Excellent fit to oscillation data!

low energy nu mass matrix

$$m^{\nu} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

same phase

$$m_a = \frac{|a|^2 v^2}{M_1}, \quad m_b = \frac{|b|^2 v^2}{M_2}$$

$$\eta \equiv -\arg[a^{*2} b^2]$$

BAU

$$Y_B \approx 2.5 \times 10^{-11} \sin\eta \left[\frac{M_1}{10^{10}\ \text{GeV}}\right]$$

Leptogenesis - PMNS link

$$\eta = \frac{2\pi}{3} \leftrightarrow \delta_{CP} \sim -\frac{\pi}{2}$$

✶The phase eta is the only source of CP violation in neutrino oscillations and leptogenesis

✶Positive eta is associated with positive baryon asymmetry and negative oscillation phase

Testing SUSY flavour models

In the diagonal down quark basis (Super CKM basis) the down squark mass matrix is

$$\begin{pmatrix} m^2_{\tilde{d}_L} & m_d(A_d - \mu\tan\beta) & (\Delta^d_{12})_{LL} & (\Delta^d_{12})_{LR} & (\Delta^d_{13})_{LL} & (\Delta^d_{13})_{LR} \\ & m^2_{\tilde{d}_R} & (\Delta^d_{12})_{RL} & (\Delta^d_{12})_{RR} & (\Delta^d_{13})_{RL} & (\Delta^d_{13})_{RR} \\ & & m^2_{\tilde{s}_L} & m_s(A_s - \mu\tan\beta) & (\Delta^d_{23})_{LL} & (\Delta^d_{23})_{LR} \\ & & & m^2_{\tilde{s}_R} & (\Delta^d_{23})_{RL} & (\Delta^d_{23})_{RR} \\ & & & & m^2_{\tilde{b}_L} & m_b(A_b - \mu\tan\beta) \\ & & & & & m^2_{\tilde{b}_R} \end{pmatrix}$$

LHC direct measurement/limits

Matrix is diagonal corresponds to "minimal flavour violation"

we say that SUSY is "flavour blind"

Constrain off-diagonal elements from rare/FC processes

$$(\delta^d_{ij})_{LL} = \frac{(\Delta^d_{ij})_{LL}}{m_{\tilde{d}_{i\,L}} m_{\tilde{d}_{j\,L}}} \qquad (\delta^d_{ij})_{RR} = \frac{(\Delta^d_{ij})_{RR}}{m_{\tilde{d}_{i\,R}} m_{\tilde{d}_{j\,R}}} \qquad (\delta^d_{ij})_{LR} = \frac{(\Delta^d_{ij})_{LR}}{m_{\tilde{d}_{i\,L}} m_{\tilde{d}_{j\,R}}}$$

Dimou, Hagedorn, S.F.K., Luhn (to appear)

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semi direct model	Matter fields				Higgs fields			Flavon fields								
	T_3	T	F	ν^c	H_5	$H_{\overline{5}}$	$H_{\overline{45}}$	ϕ^u_2	$\widetilde{\phi}^u_2$	ϕ^d_3	$\widetilde{\phi}^d_3$	ϕ^d_2	$\phi^\nu_{3'}$	ϕ^ν_2	ϕ^ν_1	η
$SU(5)$	**10**	**10**	$\overline{\mathbf{5}}$	**1**	**5**	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
S_4	**1**	**2**	**3**	**3**	**1**	**1**	**1**	**2**	**2**	**3**	**3**	**2**	$\mathbf{3'}$	**2**	**1**	**1**
$U(1)$	0	5	4	-4	0	0	1	-10	0	-4	-11	1	8	8	8	7

$$\delta^u_{LL} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^u_{RR} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^u_{LR} \sim \begin{pmatrix} \lambda^8 & 0 & \lambda^7 \\ 0 & \lambda^4 & \lambda^6 \\ 0 & \lambda^7 & 1 \end{pmatrix},$$

Mimics Minimal Flavour Violation (MFV) due to high powers of $\lambda \approx 0.22$

$$\delta^d_{LL} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^4 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^d_{RR} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^d_{LR} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^6 & \lambda^2 \end{pmatrix},$$

$$\delta^e_{LL} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^e_{RR} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^4 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^e_{LR} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^2 \end{pmatrix}.$$

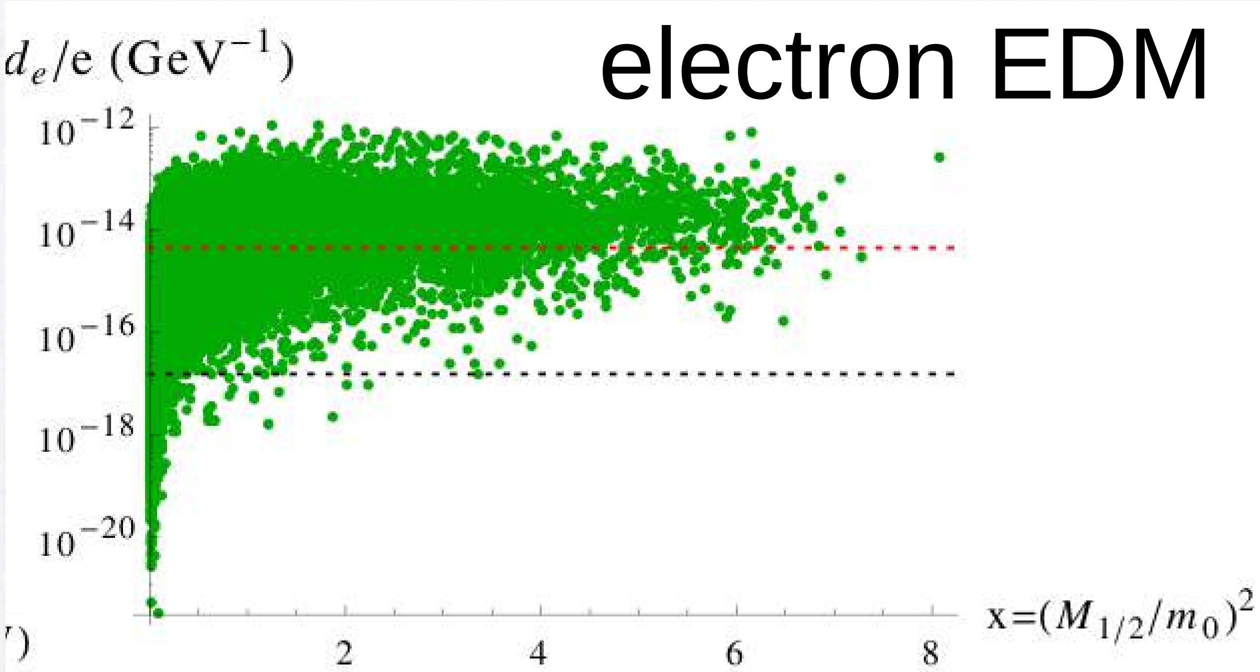
$$(\delta^f_{LL})_{ij} = \frac{(m^2_{\tilde{f}_{LL}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{LL}}$$

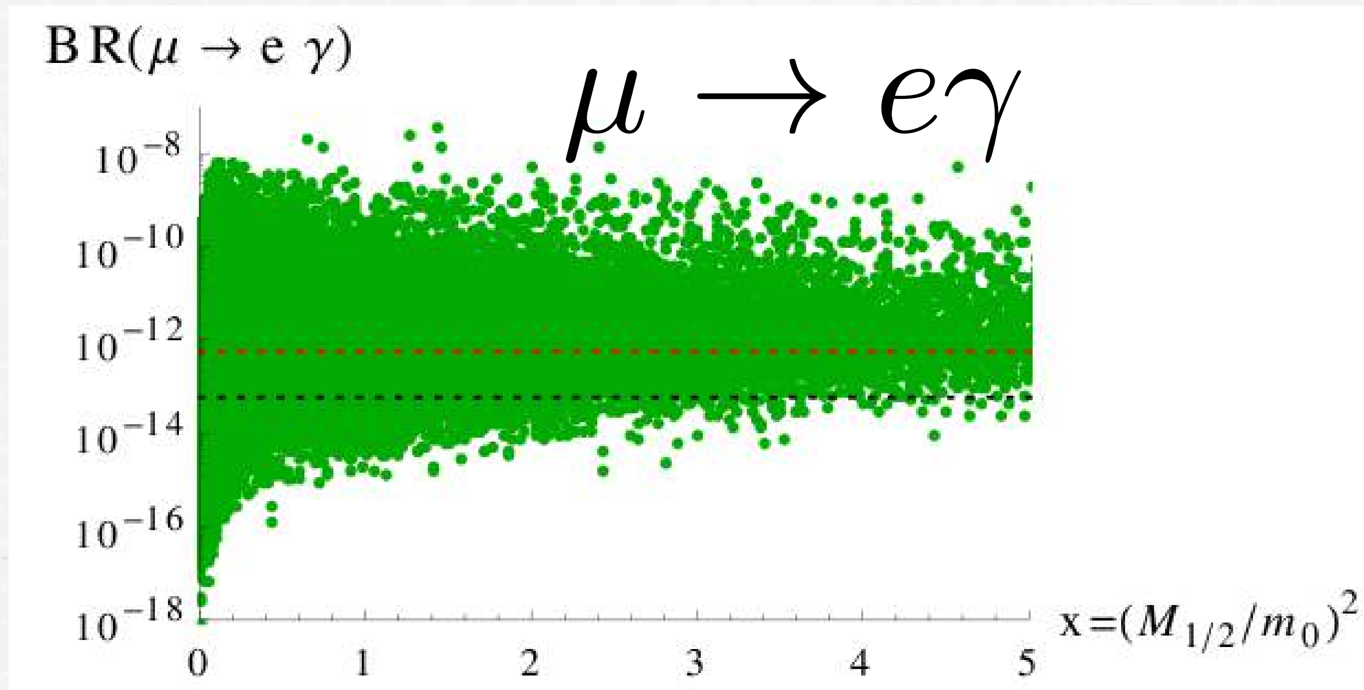
$$(\delta^f_{RR})_{ij} = \frac{(m^2_{\tilde{f}_{RR}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{RR}}$$

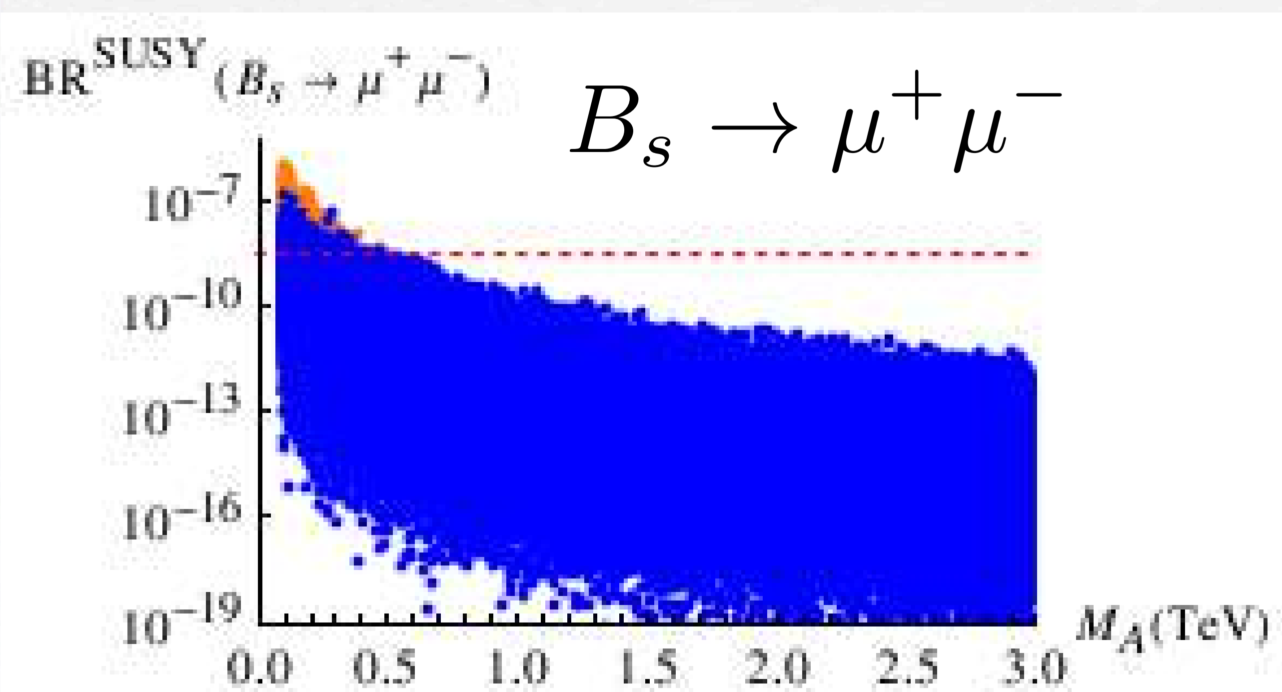
$$(\delta^f_{LR})_{ij} = \frac{(m^2_{\tilde{f}_{LR}})_{ij}}{\langle m_{\tilde{f}} \rangle^2_{LR}}$$

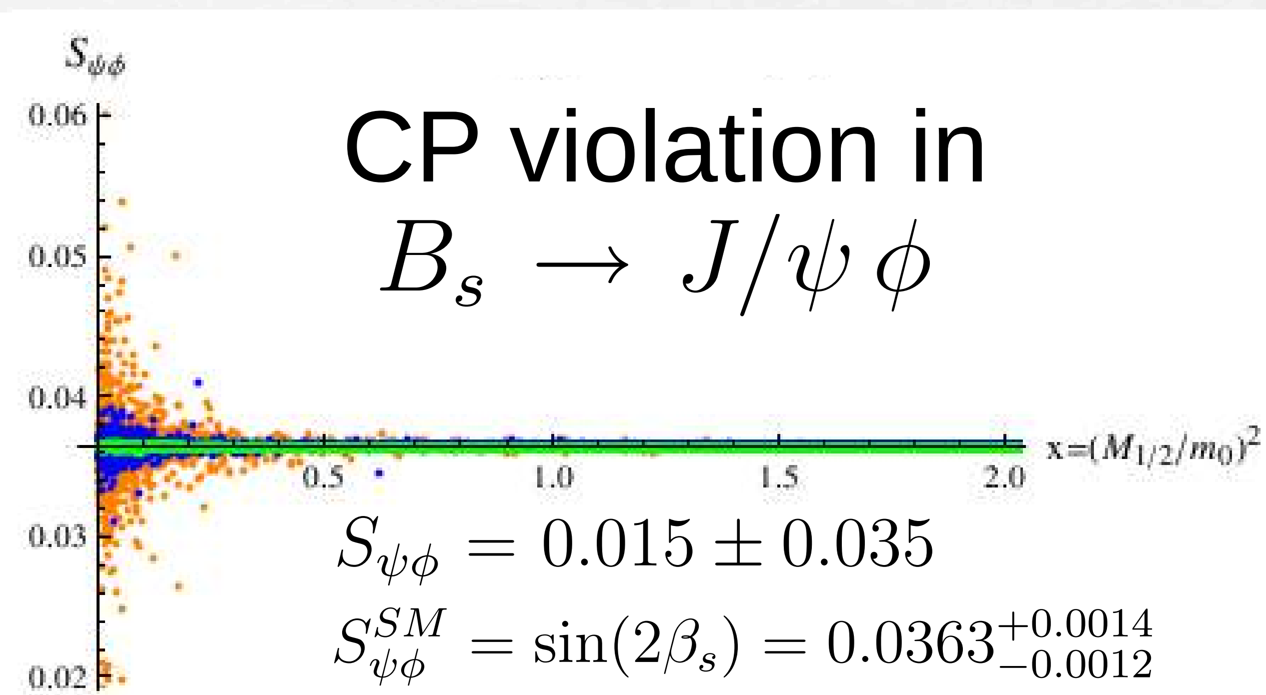
Dimou, Hagedorn, S.F.K., Luhn (to appear)

Testing SUSY flavour models





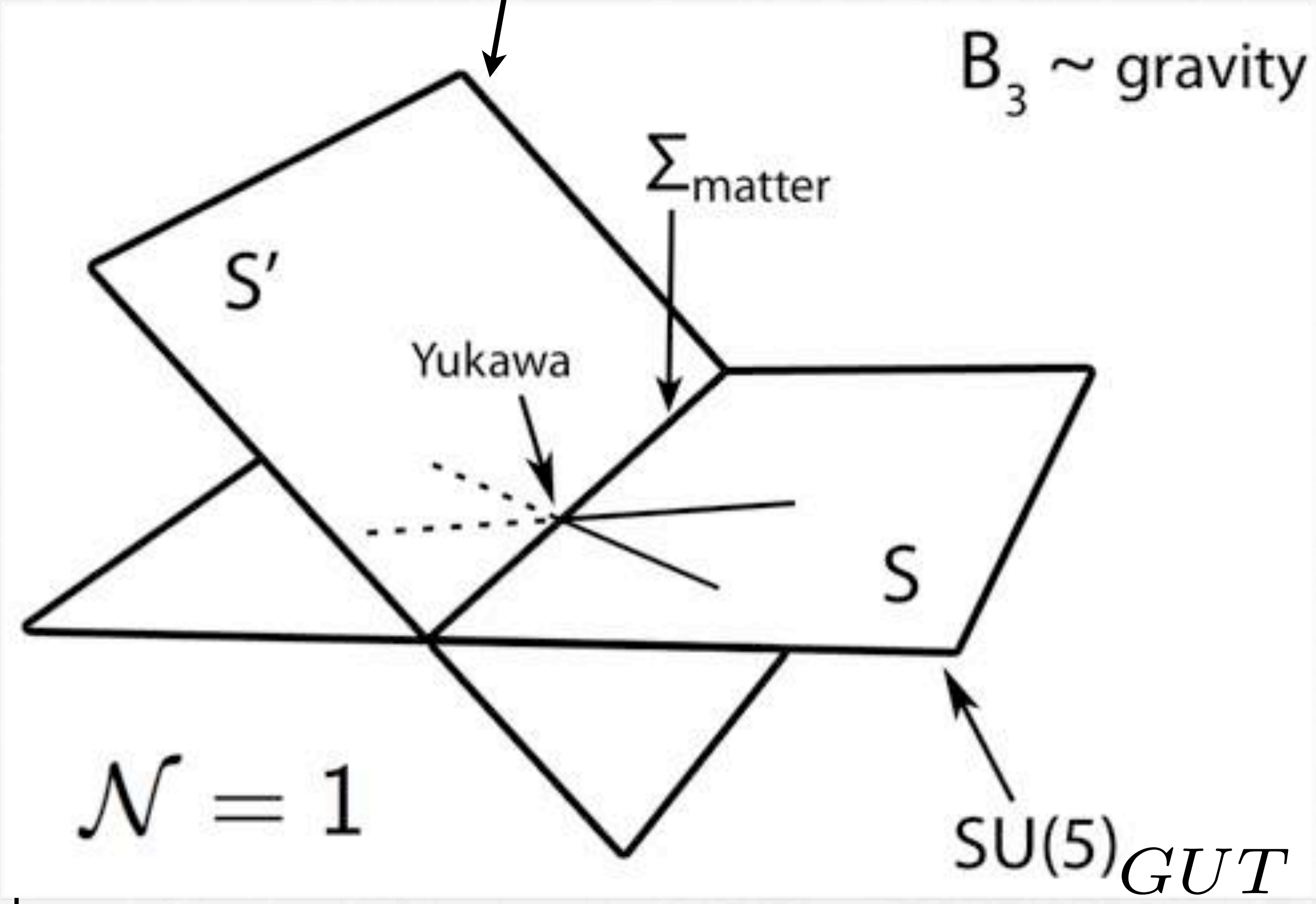




F-theory SU(5) with flavour symmetry

G.K.Leontaris, ``The origin of discrete symmetries in F-theory models,'' arXiv:1501.06499 [hep-th].

$SU(5)_\perp$



$$E_8 \to SU(5)_{\mathrm{GUT}} \times SU(5)_\perp$$

Conventionally

Heckman and Vafa

$$SU(5)_\perp \to U(1)_\perp^4$$

New possibilities

Antoniadis and Leontaris

$$SU(5)_\perp \to S_4 \times U(1)_\perp$$

$$SU(5)_\perp \to A_4 \times U(1)_\perp$$

$$SU(5)_\perp \to D_4 \times U(1)_\perp$$

S_4, A_4, D_4

Karozas, S.F.K.,Leontaris and Meadowcroft, 1505.00937, 1406.6290

Identified as discrete family symmetries

Conclusions

- GUT x Discrete Family Symmetry very predictive framework
- Direct models: Klein and T from Delta$(6n^2)$, zero Dirac phase
- Semi-direct models: partial symmetry S or SU, allows smaller groups, lepton mixing sum rules, CP predictions
- Indirect models: allows A_4 broken by orthogonal CSD(n) alignments, gives minimal predictive seesaw
- A4xSU(5) SUSY GUT based on CSD(3), predictive, complete
- SUSY flavour models mimic MFV with testable deviations
- SU(5) with discrete flavour symmetry from F-theory