Calibratable standard candles

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17th September, 2015

Based on arxiv/1506.01354, 1508.07850
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Corfu Summer Institute 2015

Outline

The perfect standard candle

$\min \chi^2 \equiv \text{MLE}$

The standard candle by MLE

Calibrated standard candles by MLE

The perfect standard candle

Imagine placing lightbulbs of the exact same luminosity $L_{SN} \approx 10^{9.5} L_{\odot}$ in the universe.

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Given some experimental noise σ_{exp} on the measurement of the distance modulus $\mu = m - M(\sim \log \text{flux} - \log \text{luminosity})$ of the SNe, we can now construct a χ^2, which we minimise to find the cosmological parameters:

$$\chi^2 = \sum_{SNe} \left(\frac{m_{SN} - M_0 - \mu_{cosmo}(z_{SN}, \Omega_m, \Omega_\Lambda)}{\sigma_{exp}} \right)^2$$

$\min \chi^2 \equiv$ MLE

"why do we construct and minimise χ^2s?"

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$$\mathcal{L} = \text{probability density(data } | \text{ model)}$$

$\min \chi^2 \equiv$ MLE

Simple case: gaussian noise on the measurement of μ,

$$\mathcal{L} = \prod_{SNe} (2\pi\sigma_{exp}^2)^{-1/2} \exp\left[-\frac{1}{2}\left(\frac{m_{SN} - M_0 - \mu_{cosmo}(z_{SN}, \Omega_m, \Omega_\Lambda)}{\sigma_{exp}}\right)^2\right]$$

$$\Rightarrow \chi^2 = const - 2\log\mathcal{L}$$

The standard candle by MLE

What happens if the the candle is not perfect, but has an unknown intrinsic dispersion σ_{int}?
The immediate answer is of course "add the errors in quadrature!"
(and fit σ_{int} somehow...)

I suggest: *formulate the problem properly*, and see if we can write down the likelihood

The standard candle by MLE

We have an unmeasured distance modulus, μ of the SN, drawn from a gaussian with mean $\mu_0(z)$ and unknown variance σ_{int}^2. The observed data $\hat{\mu}$ (which has a hat) is contaminated by noise with variance σ_{exp}^2:

$$\underbrace{p(\mu)}_{\text{gaussian model}} \quad \rightarrow \quad \underbrace{\mu \rightarrow \hat{\mu}}_{\text{gaussian noise}}$$

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By using the *identity* $p(A) = \int p(A|B)p(B)dB$,

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By using the *identity* $p(A) = \int p(A|B)p(B)dB$, incorporate the unknown variable μ into our construction of the likelihood:

$$\mathcal{L} = \prod_{SNe} p(\hat{\mu}) = \prod_{SNe} \int \underbrace{p(\hat{\mu}|\mu)}_{noise} \underbrace{p(\mu)}_{intrinsic} \; d\mu$$

$$= \prod_{SNe} \int (2\pi\sigma_{int}\sigma_{exp})^{-1} \exp\left[-\frac{1}{2}\left(\frac{\hat{\mu}-\mu}{\sigma_{exp}}\right)^2 - \frac{1}{2}\left(\frac{\mu-\mu_0}{\sigma_{int}}\right)^2\right] d\mu$$

$$= \prod_{SNe} (2\pi[\sigma_{int}^2 + \sigma_{exp}^2])^{-1/2} \exp\left[-\frac{1}{2}\frac{(\hat{\mu}-\mu_0)^2}{\sigma_{int}^2 + \sigma_{exp}^2}\right]$$

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$$-2\log\mathcal{L} = const + \sum_{SNe}\left\{\underbrace{\frac{(\hat{\mu}-\mu_0(\Omega_m,\Omega_\Lambda))^2}{\sigma_{exp}^2+\sigma_{int}^2}}_{\text{à la eg. Astier et al., 2006}} + \log(\sigma_{int}^2+\sigma_{exp}^2)\right\}$$

Minimisation over cosmological parameters is as before — and we have an extra parameter, σ_{int}, for which we also find the minimum (enforcing "χ^2" $\approx$ # SNe − # parameters)

Bottomline is: *what we guessed was $\mathcal{O}$(right answer)!*
Now, we know exactly why!

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The measurement is now not just the distance modulus, it is also an (empirically motivated) correction parameter, x_1.
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The data: μ *and* x_1 — a priori one is not priviledged!
Calibration of our prediction (linear!):

$$\mu_{corrected} = \mu_{cosmo} + \alpha x_1$$

α is some unspecified constant (to be fitted)

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As before; what we have are 'true' values contaminated by noise:

$$p(x_1) \to x_1 \to \hat{x}_1$$
$$\underbrace{p(\mu)}_{\text{corrected by } x_1} \to \mu \to \hat{\mu}$$

Calibrated standard candles by MLE

as before, we can write that explicitly in the likelihood:

$$\mathcal{L} = \prod_{SNe} p(\hat{\mu}, \hat{x}) = \prod_{SNe} \int d\mu \; dx \; \underbrace{p(\hat{\mu}, \hat{x}|\mu, x)}_{\text{noise}} \; \underbrace{p(\mu) \; p(x)}_{\text{modelling}}$$

noise, $p(\hat{\mu}, \hat{x}|\mu, x)$ is taken as gaussian as before, $p(\mu)$ gaussian as before.

— what about $p(x)$???

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As a prototype, I propose another gaussian:

(as used in eg. Kowalski et al 2008, although see eg. Howell et al 2007)

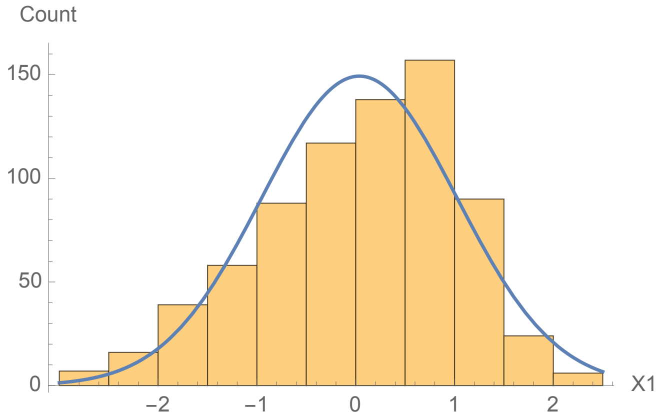

figure: X_1 data from Betoule et al. 2014

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We continue as before: construct likelihood!

$$\mathcal{L} = \prod_{SNe} p(\hat{\mu}, \hat{x}) = \prod_{SNe} \int d\mu\ dx\ \underbrace{p(\hat{\mu}, \hat{x}|\mu, x)}_{\text{noise}}\ \underbrace{p(\mu)\ p(x)}_{\text{modelling}}$$

noise, $p(\hat{\mu}, \hat{x}|\mu, x)$ is taken as gaussian as before, $p(\mu)$ gaussian as before.

— what about $p(x)$? $\rightarrow$ gaussian! $\rightarrow$ analytic integral!

(we fit the parameters of the new gaussian too!)

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adding a colour term in the same fashion we get to the analysis as is today:

$$\mathcal{L} = \left|2\pi(\Sigma_{\mathsf{d}} + A^{\mathsf{T}}\Sigma_l A)\right|^{-1/2} \\ \times \exp\left[-\frac{1}{2}(\hat{Z} - Y_0 A)(\Sigma_{\mathsf{d}} + A^{\mathsf{T}}\Sigma_l A)^{-1}(\hat{Z} - Y_0 A)^{\mathsf{T}}\right]$$

(where $\hat{Z} = \{\hat{m}_B - \mu, \hat{x}_1, \hat{c} \ldots\}$ etc.)

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(where $\hat{Z} = \{\hat{m}_B - \mu, \hat{x}_1, \hat{c} \ldots\}$ etc.)
fits both μ *and* x_1 and c distributions!

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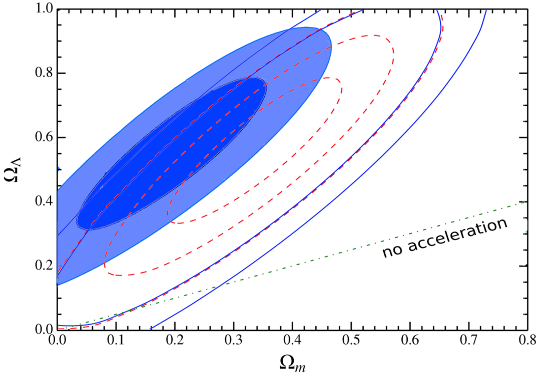

figure: Dashed red line: our MLE fit $(1, 2, 3\sigma)$, solid blue: JLA result $(1, 2\sigma)$. [full blue line: 10d projection $(1, 2\sigma)$]

In conclusion

- We are still learning about the shape/colour/etc.- corrections,
- our proposed method allows us explicitly to study the nature of these
- the cosmological parameters are *very* sensitive to our treatment and modelling of the corrections