GAUGE THEORIES

and Non-Commutative geometry

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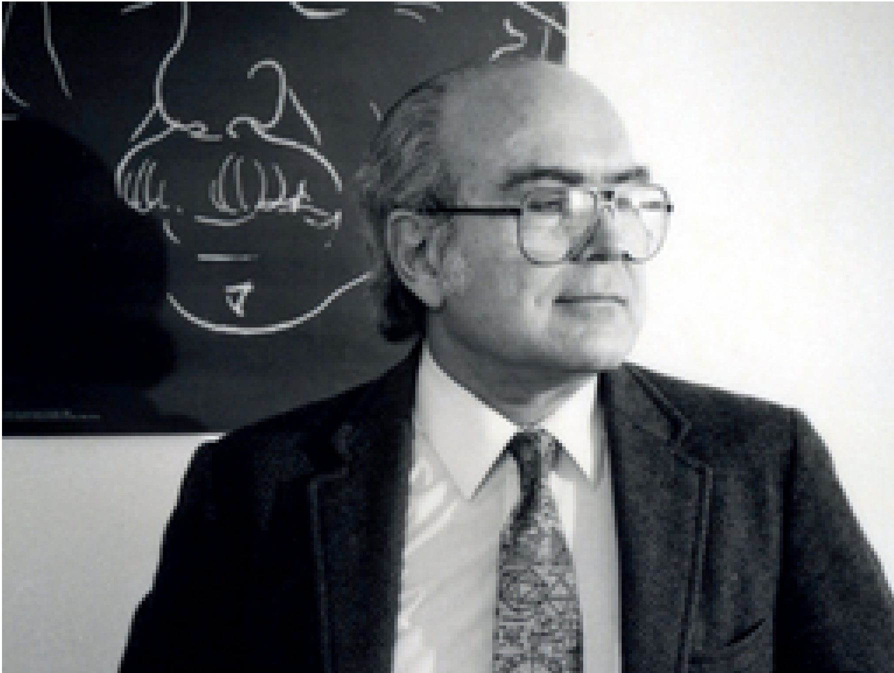

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- 3) Seiberg-Witten map.
- 4) Large N gauge theories and matrix models.
- 5) The construction of gauge theories using the techniques of non-commutative geometry.

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- $x_\mu x_\nu = q^{-1} R^{\rho\sigma}_{\mu\nu} x_\rho x_\sigma$ (quantum space case)
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  $\partial^\mu x_\nu = \delta^\mu_\nu \qquad [x_\mu, f(x)] = i\theta_{\mu\nu}\partial^\nu f(x)$

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- Define a * product
  $f * g = e^{\frac{i}{2}\frac{\partial}{x_\mu}\theta_{\mu\nu}\frac{\partial}{y_\nu}} f(x)g(y)|_{x=y}$

All computations can be viewed as expansions in θ
expansions in the external field

More efficient ways?

Quantum field theory in a space with non-commutative geometry?
BRS Symmetry?

Large N field theories

- $\phi^i(x)\ i = 1, ..., N\ ;\ N \to \infty$

  $\phi^i(x) \to \phi(\sigma, x)\ 0 \leq \sigma \leq 2\pi$

  $\sum_{i=1}^{\infty} \phi^i(x)\phi^i(x) \to \int_0^{2\pi} d\sigma(\phi(\sigma, x))^2$

  but

  $\phi^4 \to (\int)^2$

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- For a Yang-Mills theory, the resulting expression is local

Gauge theories on surfaces

E.G. Floratos and J.I.

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- Given an $SU(N)$ Yang-Mills theory in a $d-$dimensional space

$$A_\mu(x) \ = \ A^a_\mu(x) \ t_a$$

- there exists a reformulation in $d+2$ dimensions

$$A_\mu(x) \to \mathcal{A}_\mu(x, z_1, z_2) \qquad F_{\mu\nu}(x) \to \mathcal{F}_{\mu\nu}(x, z_1, z_2)$$

with $$[z_1, z_2] = \frac{2i}{N}$$

$$[A_\mu(x), A_\nu(x)] \to \{\mathcal{A}_\mu(x, z_1, z_2), \mathcal{A}_\nu(x, z_1, z_2)\}_{Moyal}$$

$$[A_\mu(x), \Omega(x)] \to \{\mathcal{A}_\mu(x, z_1, z_2), \Omega(x, z_1, z_2)\}_{Moyal}$$

$$\int d^4x \;\; Tr\,(F_{\mu\nu}(x)F^{\mu\nu}(x)) \quad \to \quad \int d^4x dz_1 dz_2 \;\; \mathcal{F}_{\mu\nu}(x, z_1, z_2) * \mathcal{F}^{\mu\nu}(x, z_1, z_2)$$

These expressions are defined for *all N*!

Not necessarily integer ???

I. Large N

-A simple algebraic result:

At large N

The $SU(N)$ algebra $\rightarrow$ The algebra of the area preserving diffeomorphisms of a closed surface. (sphere or torus).

-The structure constants of $[SDiff(S^2)]$ are the limits for large N of those of $SU(N)$.

-Alternatively: For the sphere

$$x_1 = cos\phi \ sin\theta, \quad x_2 = sin\phi \ sin\theta, \quad x_3 = cos\theta$$

$$Y_{l,m}(\theta,\phi) = \quad \sum_{\substack{i_k=1,2,3\\ k=1,\ldots,l}} \alpha^{(m)}_{i_1\ldots i_l} \ x_{i_1}\ldots x_{i_l}$$

where $\alpha^{(m)}_{i_1\ldots i_l}$ is a symmetric and traceless tensor.
For fixed l there are $2l+1$ linearly independent tensors $\alpha^{(m)}_{i_1\ldots i_l}$, $m = -l, ..., l$.

Choose, inside $SU(N)$, an $SU(2)$ subgroup.

$$[S_i, S_j] = i\epsilon_{ijk} S_k$$

A basis for $SU(N)$:

$$S^{(N)}_{l,m} = \quad \sum_{\substack{i_k=1,2,3 \\ k=1,\dots,l}} \alpha^{(m)}_{i_1 \dots i_l} \; S_{i_1} \dots S_{i_l}$$
$$[S^{(N)}_{l,m},\; S^{(N)}_{l',m'}] = i f^{(N) l'',m''}_{l,m;\; l',m'} \; S^{(N)}_{l'',m''}$$

The three $SU(2)$ generators S_i, rescaled by a factor proportional to $1/N$, will have well-defined limits as N goes to infinity.

$$S_i \to T_i = \frac{2}{N} S_i$$
$$[T_i, T_j] = \frac{2i}{N} \epsilon_{ijk} T_k$$
$$T^2 = T_1^2 + T_2^2 + T_3^2 = 1 - \frac{1}{N^2}$$

In other words: under the norm $\|x\|^2 = Trx^2$, the limits as N goes to infinity of the generators T_i are three objects x_i which commute and are constrained by

$$x_1^2 + x_2^2 + x_3^2 = 1$$

$$\frac{N}{2i}\,[f,g] \rightarrow \quad \epsilon_{ijk}\; x_i\; \frac{\partial f}{\partial x_j}\frac{\partial g}{\partial x_k}$$

$$\frac{N}{2i}\,[T^{(N)}_{l,m}, T^{(N)}_{l',m'}] \rightarrow \; \{Y_{l,m}, Y_{l',m'}\}$$

$$N[A_\mu, A_\nu] \rightarrow \quad \{A_\mu(x,\theta,\phi), A_\nu(x,\theta,\phi)\}$$

II. To all orders

We can parametrise the T_i's in terms of two operators, z_1 and z_2.

$$T_+ = T_1 + iT_2 = e^{\frac{iz_1}{2}}\ (1 - z_2^2)^{\frac{1}{2}}\ e^{\frac{iz_1}{2}}$$
$$T_- = T_1 - iT_2 = e^{-\frac{iz_1}{2}}\ (1 - z_2^2)^{\frac{1}{2}}\ e^{-\frac{iz_1}{2}}$$
$$T_3 = z_2$$

If we assume that z_1 and z_2 satisfy:

$[z_1, z_2] = \frac{2i}{N}$

The T_i's satisfy the $SU(2)$ algebra.

If we assume that the T_i's satisfy the $SU(2)$ algebra, the z_i's satisfy the Heisenberg algebra

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- Whether it will turn out to be convenient for us to use is still questionable.
- It will depend on our ability to simplify the mathematics sufficiently, or to master them deeply, in order to get new insights

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- The fermions are arbitrary, but their dynamics is not.
- Do we need a third world, The world of scalars?

  Many arbitrary parameters. Their masses are unstable Why??

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  Does not seem to work
- Supersymmetry. The scalars complete the massive vector supermultiplet.

  We do not know where and how it is broken.
- Could the scalars become also geometrical?

The techniques of non-com. geometry

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- Gauge transformations are:
- Diffeomorphisms *space-time*
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  Is Kaluza-Klein the answer?
- Question: Is there a space on which Internal symmetry transformations act as Diffeomorphisms?
- Answer: Yes, but it is a space with non-commutative geometry.

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- *New predictions for the B.E.H. mass?*

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- Related question: Is there a B.R.S. symmetry for this model?

The spectacular accuracy reached by experiments, as well as theoretical calculations, made particle physics a precision science

- *Quiz:* The experimental measurement of the W boson mass yields a value: $m_W = 80.385\text{GeV}$

  Suppose somebody comes and claims he has a theory which predicts $m_W = 81\text{GeV}$.
  Would you call it a successful prediction?

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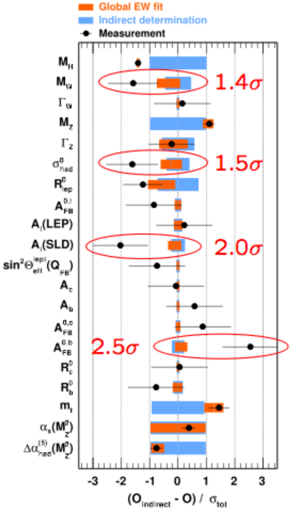
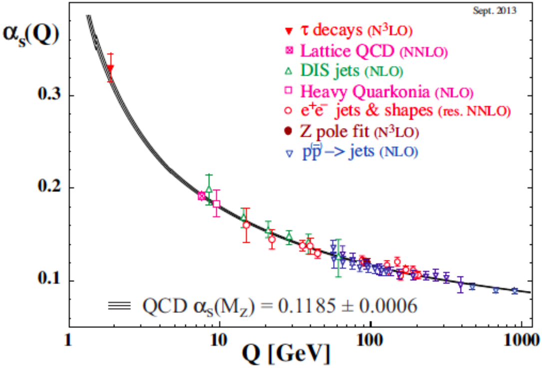
  Would you call it a successful prediction?

- *Answer:* NO! It is off by more than 40 Standard Deviations

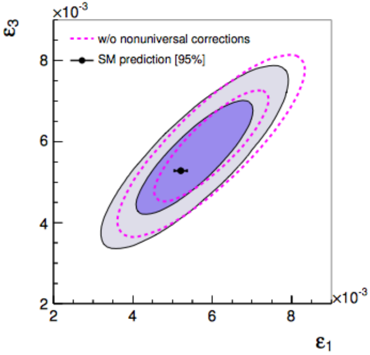
  The experimental result is $m_W = 80.385 \pm 0.015\text{GeV}$



Global EW fit
Indirect determination
Measurement
M_H
M_W
Γ_W
M_Z
Γ_Z
σ^0_{had}
R^0_{lep}
$A^{0,l}_{FB}$
$A_l(LEP)$
$A_l(SLD)$
$\sin^2\Theta^{lept}_{eff}(Q_{FB})$
A_c
A_b
$A^{0,c}_{FB}$
$A^{0,b}_{FB}$
R^0_c
R^0_b
m_t
$\alpha_s(M_Z^2)$
$\Delta\alpha^{(5)}_{had}(M_Z^2)$
1.4σ
1.5σ
2.0σ
2.5σ
-3 -2 -1 0 1 2 3
$(O_{Indirect} - O) / \sigma_{tot}$



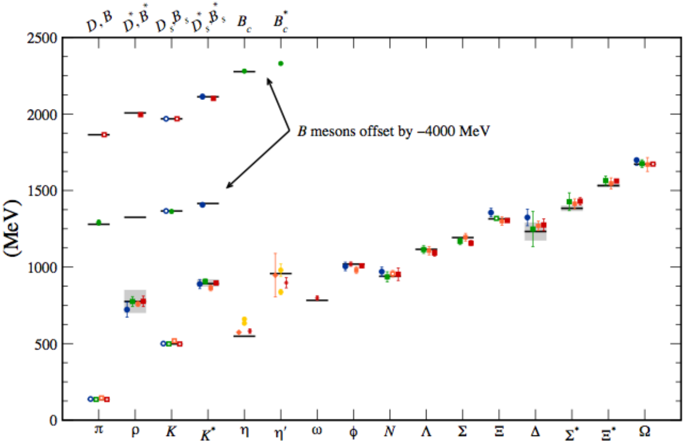
Sept. 2013
$\alpha_s(Q)$
τ decays (N³LO)
Lattice QCD (NNLO)
DIS jets (NLO)
Heavy Quarkonia (NLO)
e⁺e⁻ jets & shapes (res. NNLO)
Z pole fit (N³LO)
pp̄ –> jets (NLO)
0.3
0.2
0.1
QCD $\alpha_s(M_Z) = 0.1185 \pm 0.0006$
1
10
100
1000
Q [GeV]



ε_3
$\times 10^{-3}$
w/o nonuniversal corrections
SM prediction [95%]
8
6
4
2
2
4
6
8
$\times 10^{-3}$
ε_1

$$\epsilon_1 = \frac{3 G_F m_t^2}{8\sqrt{2}\pi^2} - \frac{3 G_F m_W^2}{4\sqrt{2}\pi^2} \tan^2\theta_W \ln\frac{m_H}{m_Z} + \ldots \qquad (1)$$

$$\epsilon_3 = \frac{G_F m_W^2}{12\sqrt{2}\pi^2} \ln\frac{m_H}{m_Z} - \frac{G_F m_W^2}{6\sqrt{2}\pi^2} \ln\frac{m_t}{m_Z} + \ldots \qquad (2)$$



D, B
D*, B*
Ds, Bs
Ds*, Bs*
B_c
B_c^*
2500
2000
1500
1000
500
0
(MeV)
B mesons offset by –4000 MeV
π
ρ
K
K*
η
η′
ω
ϕ
N
Λ
Σ
Ξ
Δ
Σ*
Ξ*
Ω

"Approximate" theories are no more sufficient!

A discrepancy by a few percent implies that we do not have the right theory!

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- **But, for the moment, we see no corner!**

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To end, a story

- **What is past, is prologue**
- **It means, you ain't heard nothing yet**