Introduction to SUSY and MSSM

CORFU SUMMER INSTITUTE

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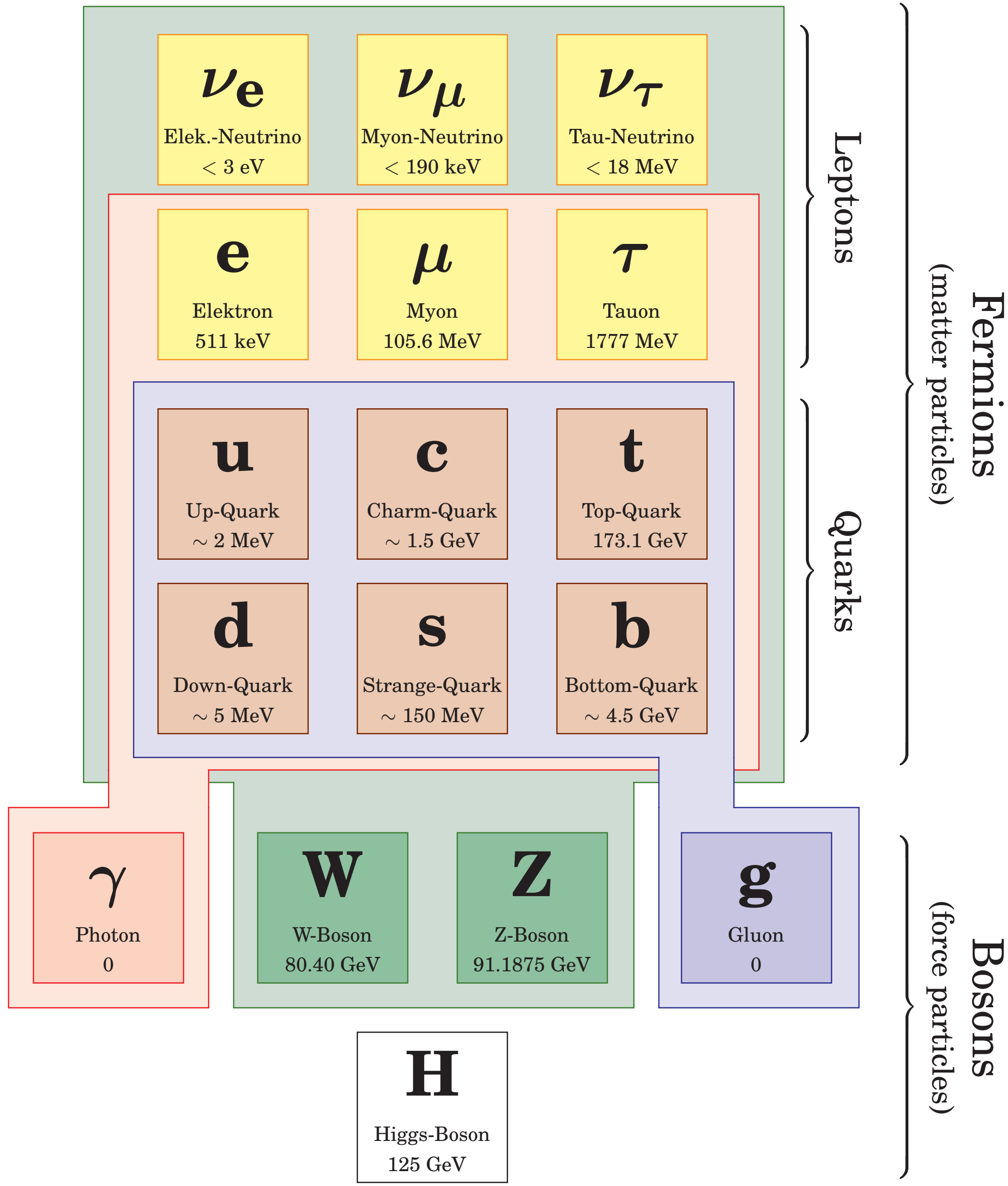
Standard Model input is now completely determined



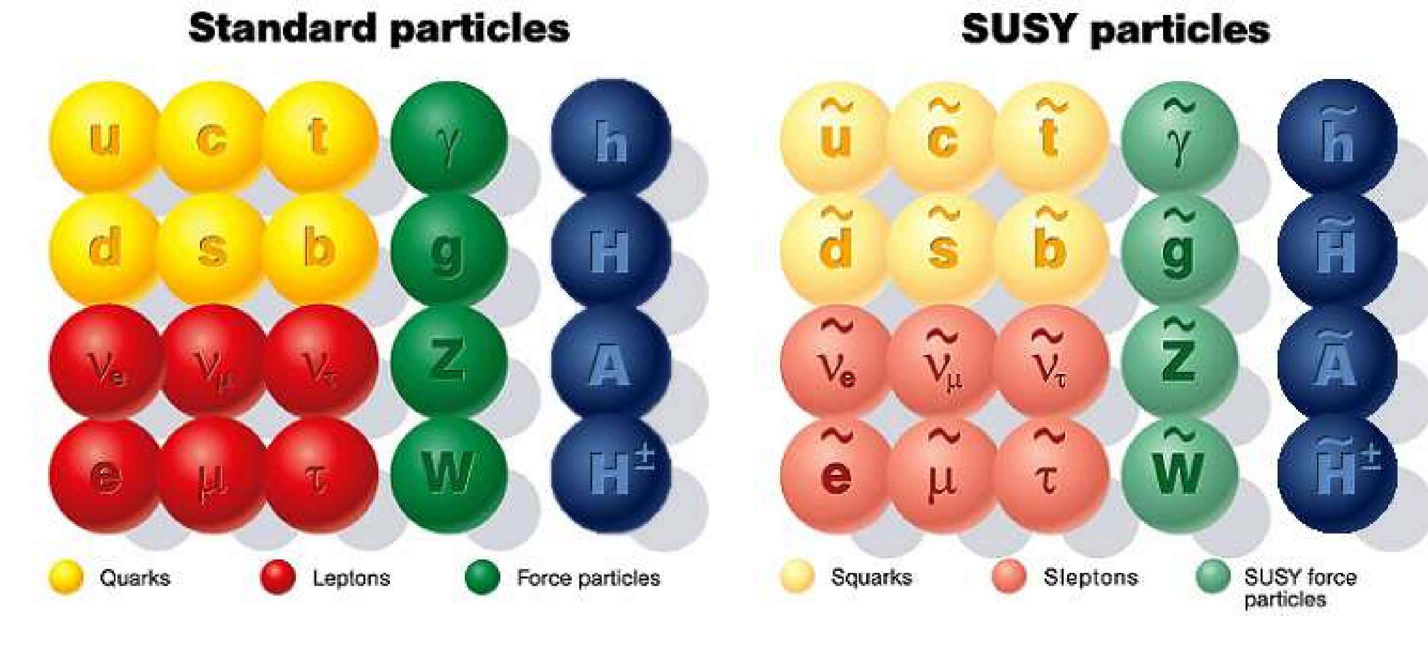
Why physics beyond the Standard Model?

many open issues

- hierarchy problem $v \ll M_{\text{Pl}}, \quad M_H \ll M_{\text{Pl}}$
- large number of free parameters $g_1,\, g_2,\, v,\, m_f,\, V_{\text{CKM}}$
- no further unification of forces
- missing link to gravity

- nature of dark matter?
- baryon asymmetry of the universe?

Supersymmetry



- gauge coupling unification
- dark matter candidate (lightest SUSY particle, LSP)
- physical Higgs bosons: $h^0, H^0, A^0, H^\pm$
  lightest Higgs boson h^0 < 130 GeV

Outline

1. SUSY algebra and representations

2. SUSY fields and Lagrangians

3. MSSM, formulation and content

4. Tests of the MSSM

1. SUSY algebra and representations

1.1. Space-time symmetry

Poincaré transformations

- translations a^μ, P^μ
- rotations $\alpha\,\vec{n}$, $\vec{J}$
- boosts $\phi\,\vec{n}$ $\vec{K}$
- reflexions

$\underbrace{\phantom{P^\mu\ \vec{J}\ \vec{K}}}_{\text{generators}}$

Generators: P^μ, $J^{\mu\nu}$

$J^{12} = J^3$, $J^{23} = J^1$, $J^{31} = J^2$

$J^{0k} = K^k$

Poincaré Algebra

$$\boxed{\begin{aligned}
[J^{\mu\nu}, J^{\rho\sigma}] &= i\left(g^{\mu\sigma} J^{\nu\rho} - g^{\mu\rho} J^{\nu\sigma} + g^{\nu\rho} J^{\mu\sigma} - g^{\nu\sigma} J^{\mu\rho}\right) \\
[P^\mu, J^{\nu\rho}] &= i\left(g^{\mu\rho} P^\sigma - g^{\mu\sigma} P^\rho\right) \\
[P^\mu, P^\nu] &= 0
\end{aligned}}$$

invariant operators

(Casimir operators)

commute with $\mathcal{J}^{\mu\nu}$, P^μ

(i) $P^2 = P_\mu P^\mu$ $\rightarrow$ mass

(ii) $W^2 = W_\mu W^\mu$, $W^\mu = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \mathcal{J}_{\nu\rho} P_\sigma$

$\rightarrow$ spin

Eigenspaces of Casimir operators are the irreducible representations of the Lie algebra

[Schur's Lemma]

$\rightarrow$ 1-particle spaces according to m, s

Internal symmetries

e.g. gauge symmetries

additional symmetry group G

with Lie-Algebra

$$[T_a, T_b] = i f_{abc} T_c$$

commute with P.A.

$$[P^\mu, T_a] = [J^{\mu\nu}, T_a] = 0$$

direct product $\mathcal{P} \times G$
of Poincare Group and
internal symmetry group G

only possibility to merge $\mathcal{P}$
with other symmetries
containing $[\,,\,]$

[Coleman-Mandula Theorem]

can be circumvented if the extra symmetry contains

$\{\cdot,\cdot\}$ instead of $[\cdot,\cdot]$

anti-commutators

$$\{F_1, F_2\} = F_1 F_2 + F_2 F_1$$

$\rightarrow$ graded Lie Algebra:

$[B, B'] = B''$

Lie-Algebra

lin. space L_0

$[\ ,\ ] \in L_0$

$\{F, F'\} = B$

$F \in L_1$

linear space

$\{\ ,\ \} \in L_0$

$$[B, F] = F' \in L_1$$

1.2. Spinors

Generators of Lorentz Group

$$J^l,\ K^l \longrightarrow$$

$$A^l = \frac{1}{2}\left(J^l + iK^l\right)$$

$$B^l = \frac{1}{2}\left(J^l - iK^l\right)$$

Lie Algebra:

$$[A^k, A^l] = i\,\varepsilon^{klm} A^m$$

$$[B^k, B^l] = i\,\varepsilon^{klm} B^m$$

$$[A^l, B^k] = 0$$

$$\boxed{SU(2) \times SU(2)}$$

irreducible representiations:

$$(j_1, j_2)\,, \qquad j = 0, \tfrac{1}{2}, \ldots$$

<u>spinor representations</u>

$j_1 = \frac{1}{2}, \quad j_2 = 0:$ $\quad \vec{A} = \frac{1}{2}\vec{\sigma}, \quad \vec{B} = 0$

$$\vec{J} = \frac{1}{2}\vec{\sigma}, \qquad \vec{K} = -\frac{i}{2}\vec{\sigma}$$

2×2- matrices,
act on 2-comp. spinor

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$ Weyl spinor

Lorentz transformation Λ :

$$\psi \xrightarrow{\Lambda} \psi' = D(\Lambda)\psi$$

$$D(\Lambda) = \begin{cases} e^{-\frac{i}{2}\alpha \vec{n}\cdot\vec{\sigma}} & \text{rotation} \\ e^{-\frac{1}{2}\phi \vec{n}\cdot\vec{\sigma}} & \text{boost} \end{cases}$$

$j_1 = 0, \ j_2 = \frac{1}{2}$: $\vec{A} = 0, \quad \vec{B} = \frac{1}{2}\vec{\sigma}$

$$\vec{J} = \frac{1}{2}\vec{\sigma}, \quad \vec{K} = \frac{i}{2}\vec{\sigma}$$

operate on 2-dim. space,

spinor $\quad \bar{\chi} = \begin{pmatrix} \bar{\chi}^1 \\ \bar{\chi}^2 \end{pmatrix}$

components: $\quad \bar{\chi}^a$ (often $\bar{\chi}^{\dot{a}}$)

Lorentz transf. Λ :

$$\bar{\chi} \xrightarrow{\Lambda} \bar{\chi}' = \bar{D}(\Lambda)\,\bar{\chi}$$

$$\bar{D}(\Lambda) = \begin{cases} e^{-\frac{i}{2}\alpha \vec{n}\vec{\sigma}} & \text{rotation} \\ e^{+\frac{1}{2}\phi\, \vec{n}\vec{\sigma}} & \text{boost} \end{cases}$$

D and $\bar{D}$ are inequivalent:

$$\bar{D} = T D^* T^{-1}, \quad T = i\sigma^2$$

$$\bar{D}^+ = D^{-1}, \quad \psi \to \bar{\psi} = -i\sigma^2 \psi^*$$

Pauli matrices: $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$

with $\sigma^0 := 1 \Rightarrow \sigma^\mu = (\sigma^0, \vec{\sigma})$

$$\bar{\sigma}^\mu = (\sigma^0, -\vec{\sigma})$$

4-vectors:

$$X^\mu = \bar{\chi}^+ \sigma^\mu \bar{\chi} = (\bar{\chi}^{1*}, \bar{\chi}^{2*}) \sigma^\mu \begin{pmatrix} \bar{\chi}^1 \\ \bar{\chi}^2 \end{pmatrix}$$

$$\bar{X}^\mu = \psi^+ \bar{\sigma}^\mu \psi = (\psi_1^*, \psi_2^*) \bar{\sigma}^\mu \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

scalars:

$$\bar{\chi}^+ \psi \xrightarrow{\Lambda} (\bar{D}\bar{\chi})^+ (D\psi)$$

$$= \bar{\chi}^+ \underbrace{\bar{D}^+ D}_{=1} \psi$$

$$\psi^+ \bar{\chi}$$

more spinor notations and conventions

definition: $$\psi^1 = -\psi_2, \quad \psi^2 = \psi_1$$

$$\bar\psi_1 = \bar\psi^2, \qquad \bar\psi_2 = -\bar\psi^1$$

$$\Rightarrow \quad \boxed{\bar\psi_a = \psi_a^*, \quad \bar\psi^a = \psi^{a\,*}}$$

$\Rightarrow$ *compact notations for Lorentz covariants*

$$\bar\chi^+\psi \;=\; \chi^a\psi_a \;\equiv\; \chi\psi$$

$$\psi^+\bar\chi \;=\; \bar\psi_a\bar\chi^a \;\equiv\; \bar\psi\bar\chi$$

$$\bar\psi^+\sigma^\mu\bar\psi \;=\; \psi\sigma^\mu\bar\psi$$

$$\psi^+\bar\sigma^\mu\psi \;=\; \bar\psi\bar\sigma^\mu\psi$$

4-component spinors

$$\Psi = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix} \xrightarrow{\Lambda} \left(\begin{array}{c:c} D(\Lambda) & 0 \\ \hdashline 0 & \bar{D}(\Lambda) \end{array}\right) \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix}$$

[Weyl representation]

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\bar{\psi} \neq \bar{\chi}$: Dirac spinor

$\bar{\psi} = \bar{\chi}$: Majorana spinor

$\mathbb{P}_L = \frac{1}{2}(1 - \gamma_5) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ projects on ψ

$\mathbb{P}_R = \frac{1}{2}(1 + \gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ projects on $\bar{\chi}$

$\begin{pmatrix} \psi \\ 0 \end{pmatrix}$: eigenstate of $\mathbb{P}_L$, left-chiral

$\begin{pmatrix} 0 \\ \bar{\chi} \end{pmatrix}$: eigenstate of $\mathbb{P}_R$, right-chiral

1.3. Poincaré Algebra → SUSY P.A.

introduce additional
spinor charges $Q, \bar{Q}$

$$Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}, \qquad \bar{Q} = \begin{pmatrix} \bar{Q}^1 \\ \bar{Q}^2 \end{pmatrix}$$

transform like Weyl spinors $\psi, \bar{\psi}$
under Lorentz transform.

$$Q \leftrightarrow D(\Lambda), \qquad \bar{Q} \leftrightarrow \bar{D}(\Lambda)$$

$$\bar{Q} = -i\sigma^2 Q^+$$

$$\begin{pmatrix} \bar{Q}^1 \\ \bar{Q}^2 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} Q_1^+ \\ Q_2^+ \end{pmatrix}$$

$$\bar{Q}_a = Q_a^+ \quad (a = 1, 2)$$

[generalization: Q^L, $L = 1, \dots N$]

SUSY Poincaré Algebra

$\boxed{\text{Poincaré Algebra}}$ (N = 1)

$$[Q, \mathcal{J}^{\mu\nu}] = \sigma^{\mu\nu} Q$$

$$[\bar{Q}, \mathcal{J}^{\mu\nu}] = \bar{\sigma}^{\mu\nu} \bar{Q}$$

$$[Q, P^{\mu}] = [\bar{Q}, P^{\mu}] = 0$$

$$\{Q_a, Q_b\} = \{\bar{Q}_a, \bar{Q}_b\} = 0$$

$$\{Q_a, \bar{Q}_b\} = 2(\sigma^{\mu})_{ab} P_{\mu}$$

$$\sigma^{\mu\nu} = \frac{i}{4}\left(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}\right)$$

$$\bar{\sigma}^{\mu\nu} = \frac{i}{4}\left(\bar{\sigma}^{\mu}\sigma^{\nu} - \bar{\sigma}^{\nu}\sigma^{\mu}\right)$$

1.4. Irreducible representations

Casimir operators $\rightarrow$ eigenspaces = irreducible reps.

- $P^2 = P_\mu P^\mu$ invariant under SUSY-P.A.

  $\Rightarrow$ mass m defines irr. rep.

- $W^2 = W_\mu W^\mu$ no Casimir operator of SUSY-P.A.

$$W^\mu = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} J_{\nu\rho} P_\sigma$$

- instead: $Y^\mu = W^\mu - \frac{1}{4} X^\mu$

$$X^\mu = Q^a \sigma^\mu_{ab} \bar{Q}^b \equiv Q\sigma^\mu\bar{Q}$$

$$= P^\mu - \bar{Q}\sigma^\mu Q$$

commutator:

$$[Y^\mu, Y^\nu] = i\,\varepsilon^{\mu\nu\rho\sigma} P_\rho Y_\sigma$$

rest system: $P_0 = m$, $\vec{P} = 0$

$$[Y^k, Y^l] = i\,m\,\varepsilon^{klj} Y^j$$

$\vec{Y}^2$ has eigenvalues

$m^2 y(y+1)$, $y = 0, \frac{1}{2}, 1, \ldots$

Casimir operator: $C^2 = C_{\mu\nu} C^{\mu\nu}$

$$C^{\mu\nu} = Y^\mu P^\nu - Y^\nu P^\mu$$

check: $[C^{\mu\nu}, P^\rho] = 0$

$[C^{\mu\nu}, Q] = 0$

$[C^2, J^{\mu\nu}] = 0$

C^2 invariant $\longrightarrow$ rest frame

$$C^2 = 2m^2 Y^2 - 2\,(Y_\mu P^\mu)$$

$$\rightarrow 2m^2 Y^2 - 2m^2 (Y_0)^2$$

$$= 2m^2 (Y^2 - Y_0^2) = 2m^2 (-\vec{Y}^2)$$

eigenvalues: $-2m^4 \cdot y(y+1)$

$[\, y = 0, \tfrac{1}{2}, 1, \dots]$

$\boxed{\text{irreducible representations are classified by } m, y}$

states: $|\, m, y; \vec{p}, y_3, \dots \rangle$

Spin ?

action of Q, $\bar{Q}$?

continue in rest frame, $P^\mu = (m, \vec{0})$

$$\{ Q_a , \bar{Q}_b \} = 2 (\sigma^0)_{ab} \, m = 2m \, \delta_{ab}$$

define $\quad f_a^- = \frac{1}{\sqrt{2m}} Q_a \quad (a = 1,2)$

$$f_a^+ = \frac{1}{\sqrt{2m}} \bar{Q}_a$$

$$\boxed{\begin{aligned} \{ f_a^- , f_b^+ \} &= \delta_{ab} \\ \{ f_a^- , f_b^- \} &= \{ f_a^+ , f_b^+ \} = 0 \end{aligned}}$$

anti-commutators for creation (f_a^+) and annihilation (f_a^-) operators of fermions

- vacuum (0-fermion state) $|\Omega\rangle$

$$f_a^- |\Omega\rangle = 0$$

- $f_a^+ |\Omega\rangle$ are 1-fermion states

- $f_1^+ f_2^+ |\Omega\rangle$ 2-fermion state

4 linearly independent states:

$$|\Omega\rangle, \quad f_a^+|\Omega\rangle, \quad f_1^+ f_2^+ |\Omega\rangle$$

for the same value y_3

[based on $[Q, Y^3] = [\bar{Q}, Y] = 0$]

$$Y^3 |\Omega\rangle = y_3 |\Omega\rangle$$

$$Y^3 f_a^+ |\Omega\rangle = f_a^+ Y^3 |\Omega\rangle = y_3 f_a^+ |\Omega\rangle$$

$$Y^3 f_1^+ f_2^+ |\Omega\rangle = f_1^+ f_2^+ Y^3 |\Omega\rangle$$

$$= y_3 f_1^+ f_2^+ |\Omega\rangle$$

$\Rightarrow$ each y_3 occurs as 4-fold degenerate

$$\boxed{\text{dimension of irred. rep.} = 4 \cdot (2y+1)}$$

Values of S^3:

rest frame: $W^3 = m S^3$

$X^k = -\bar{Q}\sigma^k Q \Rightarrow X^k |\Omega\rangle = 0$

1) state $|\Omega\rangle$:

$$Y^3 |\Omega\rangle = m y_3 |\Omega\rangle$$
$$= (W^3 - \tfrac{1}{4} X^3)|\Omega\rangle = m S^3 |\Omega\rangle$$

$$\boxed{S_3 = y_3}$$

2) states $f_a^+ |\Omega\rangle$:

$$W^3 f_1^+ |\Omega\rangle = m \left(y_3 + \tfrac{1}{2}\right) f_1^+ |\Omega\rangle$$

$$W^3 f_2^+ |\Omega\rangle = m \left(y_3 - \tfrac{1}{2}\right) f_2^+ |\Omega\rangle$$

$$\boxed{s_3 = y_3 \pm \tfrac{1}{2}}$$

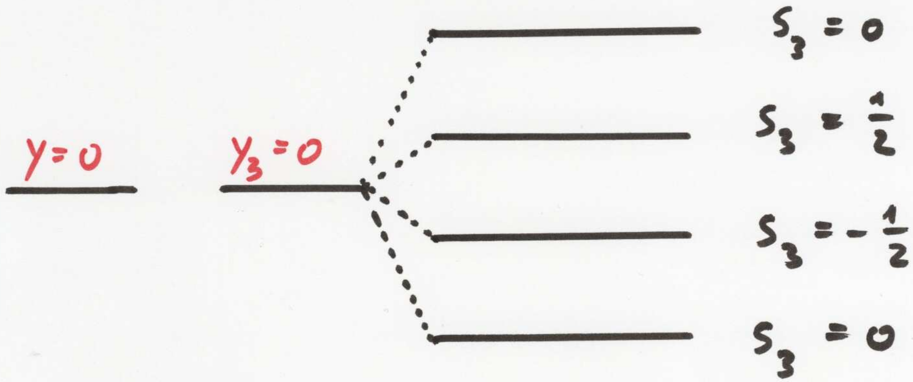
3) state $f_1^+ f_2^+ |\Omega\rangle$:

$$W^3 |\ldots\rangle = m\, y_3 |\ldots\rangle$$

$$\boxed{s_3 = y_3}$$

$\Rightarrow$ different spins in an irred. rep.

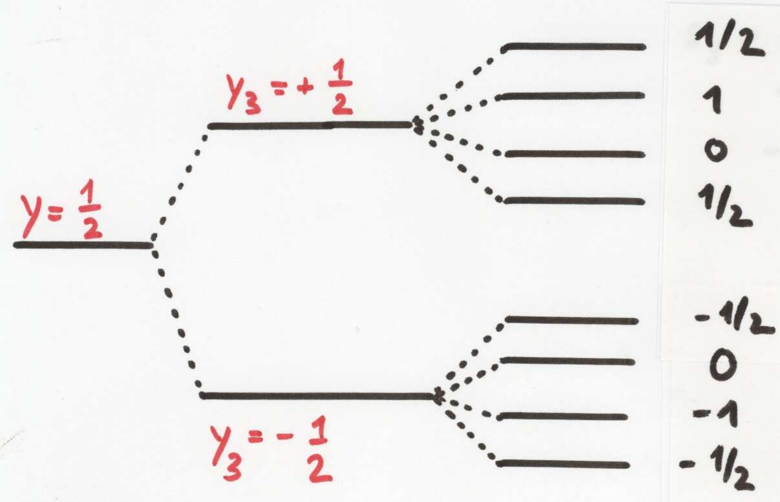
$\boxed{Y=0}$ chiral multiplet



2 bosons with spin 0

1 fermion with spin $\frac{1}{2}$

$\boxed{y = \frac{1}{2}}$ vector multiplet



2 fermions with spin $\frac{1}{2}$

1 boson with spin 1

1 boson with spin 0

Appendix to Section1

Useful formulae for spinors

Weyl-spinors with components ψ_a $(a = 1, 2)$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

form a 2-dimensional representation of the Lorentz Group. They transform under Lorentz transformations Λ according to

$$\Lambda : \qquad \psi \rightarrow D(\Lambda)\,\psi$$

with the matrix

$$D(\Lambda) = \begin{cases} e^{-\frac{i}{2}\alpha\,\vec{n}\vec{\sigma}} & \text{rotation} \\ e^{-\frac{1}{2}\phi\,\vec{n}\vec{\sigma}} & \text{boost.} \end{cases}$$

$\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ denote the Pauli matrices.

Weyl-spinors with components $\bar{\chi}^a$ $(a = 1, 2)$

$$\bar{\chi} = \begin{pmatrix} \bar{\chi}^1 \\ \bar{\chi}^2 \end{pmatrix}$$

belong to another, not equivalent, 2-dimensional representation of the Lorentz Group. Under the same Lorentz transformation Λ as above they transform according to

$$\Lambda : \qquad \bar{\chi} \rightarrow \bar{D}(\Lambda)\,\bar{\chi}$$

with the matrix

$$D(\Lambda) = \begin{cases} e^{-\frac{i}{2}\alpha\,\vec{n}\vec{\sigma}} & \text{rotation} \\ e^{+\frac{1}{2}\phi\,\vec{n}\vec{\sigma}} & \text{boost.} \end{cases}$$

The representation matrices are connected via

$$\bar{D} = T\,D^{\star}\,T^{-1}, \quad T = i\sigma^2$$

and fulfill the relation

$$D^{-1} = \bar{D}^{+}\,.$$

For each ψ tranforming with D, a $\bar{\psi}$ can be found transforming with $\bar{D}$, namely

$$\bar{\psi} = -i\,\sigma^2\,\psi^{\star}\,,$$

or explicitly,

$$\begin{pmatrix} \bar{\psi}^1 \\ \bar{\psi}^2 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_1^{\star} \\ \psi_2^{\star} \end{pmatrix}$$

The Pauli matrices, together with

$$\sigma^0 := \mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

can be summarized in terms of a 4-component quantity,

$$\sigma^\mu = (\sigma^0, \vec{\sigma})\,.$$

In addition, one defines

$$\bar{\sigma}^\mu = (\sigma^0, -\vec{\sigma})\,.$$

Lorentz covariants:

scalars:

$$\begin{array}{lll} \bar{\chi}^+ \psi & \text{mit} & \bar{\chi}^+ = (\bar{\chi}^{1\star}, \bar{\chi}^{2\star}) \\ \psi^+ \bar{\chi} & \text{mit} & \psi^+ = (\psi_1^\star, \psi_2^\star)\,. \end{array}$$

4-vectors:

$$\begin{array}{rcl} X^\mu & = & \bar{\chi}^+ \sigma^\mu \bar{\chi}\,, \\ \bar{X}^\mu & = & \psi^+ \bar{\sigma}^\mu \psi\,. \end{array}$$

Spinor notations:

In addition to the components ψ_a, $\bar{\psi}^a$ one defines:

$$\begin{array}{ll} \psi^1 = -\psi_2 & \psi^2 = \psi_1 \\ \bar{\psi}_1 = \bar{\psi}^2 & \bar{\psi}_2 = -\bar{\psi}^1\,. \end{array}$$

This yields

$$\bar{\psi}_a = \psi_a^\star, \quad \bar{\psi}^a = \psi^{a\star}$$

and a compact notations for the Lorentz covariants:

$$\begin{array}{rcl} \bar{\chi}^+\psi & = & \chi^1\psi_1 + \chi^2\psi_2 \equiv \chi^a\psi_a \equiv \chi\psi \\ \psi^+\bar{\chi} & = & \bar{\psi}_1\bar{\chi}^1 + \bar{\psi}_2\bar{\chi}^2 \equiv \bar{\psi}_a\bar{\chi}^a \equiv \bar{\psi}\bar{\chi} \end{array}$$

$$\begin{array}{rcl} \bar{\psi}^+\sigma^\mu\bar{\psi} & = & \psi^a\,(\sigma^\mu)_{ab}\,\bar{\psi}^b \equiv \psi\sigma^\mu\bar{\psi} \\ \psi^+\bar{\sigma}^\mu\psi & = & \bar{\psi}_a(\bar{\sigma}^\mu)^{ab}\psi_b \equiv \bar{\psi}\bar{\sigma}^\mu\psi \end{array}$$

The spinor products, expressed in terms of the original components, read

$$\begin{array}{rcl} \chi\psi & = & \chi_1\psi_2 - \chi_2\psi_1 \\ \bar{\psi}\bar{\chi} & = & \bar{\psi}^2\bar{\chi}^1 - \bar{\psi}^1\bar{\chi}^2\,. \end{array}$$

4-component spinors:

A Dirac spinor is composed of 2 Weyl spinors according to (Weyl representation)

$$\Psi = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix}$$

Under a Lorentz transformation Λ, it transforms as follows,

$$\Lambda : \qquad \Psi \rightarrow \begin{pmatrix} D(\Lambda) & \mathbf{0} \\ \mathbf{0} & \bar{D}(\Lambda) \end{pmatrix} \Psi$$

Dirac-Matrices in Weyl representation:

$$\gamma^\mu = \begin{pmatrix} \mathbf{0} & \sigma^\mu \\ \bar{\sigma}^\mu & \mathbf{0} \end{pmatrix}, \quad \gamma_5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

Chirale projektors:

$$\mathbf{P}_L = \frac{1-\gamma_5}{2} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$$
$$\mathbf{P}_R = \frac{1+\gamma_5}{2} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

The spinors

$$\begin{pmatrix} \psi \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \bar{\chi} \end{pmatrix}$$

are eigenspinors of $\mathbf{P}_L$ (left-chiral) and $\mathbf{P}_R$ (right-chiral). The representations D and $\bar{D}$ are thus left- and right-chiral representations

A Majorana spinor is a 4-component spinor with $\bar{\chi} = \bar{\psi}$:

$$\Psi_M = \begin{pmatrix} \psi \\ \bar{\psi} \end{pmatrix}$$

It obeys

$$\Psi_M = \mathbf{C}\,\overline{\Psi}_M^T$$

with

$$\overline{\Psi} = \Psi^+\gamma^0, \quad \mathbf{C} = \begin{pmatrix} \mathrm{i}\sigma^2 & \mathbf{0} \\ \mathbf{0} & -\mathrm{i}\sigma^2 \end{pmatrix}$$

2. SUSY fields and Lagrangians

2.1. Superspace

representation of Poincaré transf.

$$e^{-i\omega_{\mu\nu} J^{\mu\nu} - i a_\mu P^\mu}$$

translation:

$$\phi(x) \to \phi(x+a) = \phi(x) + a^\mu \frac{\partial \phi}{\partial x^\mu} + \cdots$$

$$= \left(1 - i a^\mu P_\mu\right) \phi(x)$$

$$P_\mu = i \frac{\partial}{\partial x^\mu} \equiv i \partial_\mu$$

Lie Algebra $\longrightarrow$ Group

$$[T_k, T_l] = \cdots \qquad e^{i a_k T_k}$$

$$e^A e^B =$$

$$\sum [\ldots [T_k, T_l] \ldots]$$

SUSY algebra contains $\{\ldots,\ldots\}$

$\rightarrow$ group? group elements?

strategy: multiply $Q, \bar{Q}$ by anti-commuting variables

$$\{\ ,\ \} \rightarrow [\ ,\ ]$$

New spinorial variables

$$\epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}, \quad \bar{\epsilon} = \begin{pmatrix} \bar{\epsilon}^1 \\ \bar{\epsilon}^2 \end{pmatrix}, \quad \theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \ldots$$

with $\{\epsilon_a, \epsilon_b\} = \{\bar{\epsilon}_a, \bar{\epsilon}_b\} = 0$

Grassmann variables

and $[\epsilon, P_\mu] = [\bar{\epsilon}, P_\mu] = 0$

$$\{Q_a, \epsilon_b\} = \{\bar{Q}_a, \bar{\epsilon}_b\} = \ldots = 0$$

then: $\{Q, \bar{Q}\} \rightarrow [\epsilon Q, \bar{\epsilon}\bar{Q}]$

group element:

$$e^{-i\xi^\mu P_\mu + i\epsilon Q + i\bar{\epsilon}\bar{Q}} \equiv g(\xi, \epsilon, \bar{\epsilon})$$

operates on a space with "coordinates" $\{x^\mu, \theta, \bar{\theta}\}$

<u>superspace</u>

- group multiplication:

$$g(0,\epsilon,\bar{\epsilon})\, g(x^\mu,\theta,\bar{\theta}) = \cancel{g(x^\mu+a^\mu, \theta+\epsilon,}$$

$$= g(x^\mu + a^\mu, \theta+\epsilon, \bar{\theta}+\bar{\epsilon})$$

with $a^\mu = i(\theta\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\theta})$

$\equiv$ shift in parameter space

function in superspace:

$$F(x^\mu, \theta, \bar\theta)$$

differentiation:

$$\frac{\partial}{\partial x^\mu} = \partial_\mu, \quad \frac{\partial}{\partial \theta^a} = \partial_a, \quad \frac{\partial}{\partial \bar\theta^a} = \bar\partial_a$$

basic rules:

$$\frac{\partial}{\partial \theta^a}\theta^b = \delta_a^{\ b}, \quad \frac{\partial}{\partial \bar\theta^a}\bar\theta^b = \delta_a^{\ b}$$

$$\frac{\partial}{\partial \theta^a}\bar\theta^b = 0, \quad \frac{\partial}{\partial \bar\theta^a}\theta^b = 0$$

note: $F(x, \theta, \bar\theta)$ is a polynomial in $\theta, \bar\theta$:

θ, $\bar\theta$, $\theta\theta$, $\bar\theta\bar\theta$, $(\theta\theta)\bar\theta$, $(\theta\bar\theta)\theta$, $(\theta\theta)(\bar\theta\bar\theta)$

pure SUSY transformation:

$$g(0, \epsilon, \bar{\epsilon}) = e^{i(\epsilon Q + \bar{\epsilon}\bar{Q})}$$

generates translations

$$F(x, \theta, \bar{\theta}) \longrightarrow g(0, \epsilon, \bar{\epsilon}) F(x, \theta, \bar{\theta})$$

$$= F(x + a, \theta + \epsilon, \bar{\theta} + \bar{\epsilon})$$

$$a^\mu = i(\theta \sigma^\mu \bar{\epsilon} - \epsilon \sigma^\mu \bar{\theta})$$

infinitesimal parameters:

$$F(x+a, \theta+\epsilon, \bar{\theta} + \bar{\epsilon})$$

$$= a^\mu \frac{\partial F}{\partial x^\mu} + \epsilon^a \frac{\partial F}{\partial \theta^a} + \bar{\epsilon}^a \frac{\partial F}{\partial \bar{\theta}^a}$$

$$= (1 + i\epsilon^a Q_a + i\bar{\epsilon}_a \bar{Q}^a) F$$

$\Rightarrow$ Q_a, $\bar{Q}_a$ as diff - operators

$$\boxed{\begin{aligned} iQ_a &= \frac{\partial}{\partial\theta^a} - i\sigma^\mu_{ab}\,\bar\theta^b\,\partial_\mu \\ i\bar Q_a &= -\frac{\partial}{\partial\bar\theta^a} + i\,\theta^b\,\sigma^\mu_{ba}\,\partial_\mu \end{aligned}}$$

define covariant derivative

$$\boxed{\begin{aligned} D_a &= \frac{\partial}{\partial\theta^a} + i\sigma^\mu_{ab}\,\bar\theta^b\,\partial_\mu \\ \bar D_a &= -\frac{\partial}{\partial\bar\theta^a} - i\,\theta^b\sigma^\mu_{ba}\,\partial_\mu \end{aligned}}$$

- D_a, $\bar D_a$ invariant under susy

$$[(\epsilon Q + \bar\epsilon\bar Q), D_a] = 0$$

$$[\epsilon Q + \bar\epsilon\bar Q, \bar D_a] = 0$$

2.2. Superfields

superfield = Lorentz scalar on superspace

$\Phi(x, \theta, \bar{\theta})$ polynomial in θ, $\bar{\theta}$

general form:

$$\Phi = \varphi(x) + \theta^a \psi_a(x) + \bar{\theta}_{\dot{a}} \bar{\chi}^{\dot{a}}(x)$$
$$+ (\theta\theta) F(x) + (\bar{\theta}\bar{\theta}) H(x)$$
$$+ (\theta \sigma^\mu \bar{\theta}) A_\mu(x)$$
$$+ (\theta\theta) \bar{\theta}_{\dot{a}} \bar{\lambda}^{\dot{a}}(x) + (\bar{\theta}\bar{\theta}) \theta^a \xi_a(x)$$
$$+ (\theta\theta)(\bar{\theta}\bar{\theta}) D(x)$$

$\left.\begin{array}{rl} \varphi, F, H, D: & \text{scalars} \\ A_\mu: & \text{vector} \\ \psi, \bar{\chi}, \bar{\lambda}, \xi: & \text{spinors} \end{array}\right\}$ components of Φ

general Φ is reducible

Irreducible superfields by suitable conditions (have to be invariant under susy transf.)

Conditions:

$\bar{D}_{\dot{a}} \Phi = 0$: (left-)chiral SF

$D_a \Phi^{+} = 0$: (right-) chiral SF

$\Phi = \Phi^{+}$: vector SF

$$\boxed{\bar{D}_{\dot a}\,\Phi = 0}$$ left-chiral SF

$$\Phi\Big|_{\bar{D}_{\dot a}\Phi=0} = \phi(z,\theta)$$

$$= \varphi(z) + \sqrt{2}\,\theta\,\psi(z) + \theta\theta\,F(z)$$

with $z^\mu = x^\mu + i\,\theta\sigma^\mu\bar\theta$

φ, F: scalars

ψ: Weyl spinor (left)

inf. SUSY transf. $\mathbb{1} + i\epsilon Q + i\bar\epsilon\bar Q$

$\Rightarrow$ transformation of components

$$\delta\varphi = \sqrt{2}\,\epsilon\,\psi$$

$$\delta\psi = i\sqrt{2}\,\sigma^\mu\bar\epsilon\,\partial_\mu\varphi + \sqrt{2}\,\epsilon\,F$$

$$\delta F = i\sqrt{2}\,\underbrace{\partial_\mu(\bar\epsilon\bar\sigma^\mu\psi)}_{\text{4-divergence}}$$

$$\boxed{D_a \Phi^+ = 0}$$ right-chiral SF

$$\Phi\Big|_{D_a \Phi^+ = 0} = \Phi^+(\bar{z}^\mu, \bar{\theta})$$

$$= \varphi^*(\bar{z}) + \sqrt{2}\, \bar{\theta}\, \bar{\psi}(\bar{z}) + \bar{\theta}\bar{\theta}\, F^*(\bar{z})$$

with $\bar{z} = x^\mu - i\, \theta \sigma^\mu \bar{\theta}$

$\bar{\psi}$: Weyl spinor (right-chiral)

$\boxed{V = V^\dagger}$ vector SF

$$\Phi = V(x^\mu, \theta, \bar{\theta}), \quad V = V^\dagger$$

has full expansion in $\theta, \bar{\theta}$
with constraints on components

φ, D : real scalar fields

$F = H^*$: scalar field

A_μ : vector field

$\psi = \chi, \; \xi = \lambda$: Weyl spinors

number of components can be
reduced by

susy gauge transformations

susy gauge transformation

$$V \to V' = V + (\phi + \phi^+)$$

$$\phi = a + \sqrt{2}\,\theta\,\xi + \theta\theta\, G$$

chiral SF

$$V = V^+ \quad \longrightarrow \quad V' = (V')^+$$

components of V':

$$\varphi' = \varphi + 2\,\mathrm{Re}\,a$$

$$\psi' = \psi + \sqrt{2}\,\xi$$

$$F' = F + G$$

$$A'_\mu = A_\mu - \partial_\mu (2\,\mathrm{Im}\,a)$$

$$\lambda' = \lambda + \frac{i}{\sqrt{2}}\,\sigma^\mu \partial_\mu \bar{\xi}$$

$$D' = D - \frac{1}{2}\,\Box (2\,\mathrm{Re}\,a)$$

eliminate $\varphi', \psi', F' \leftrightarrow \mathrm{Re}\,a, \xi, G$

in the gauge without the
φ, ψ, F components

$$V' = (\theta\sigma^\mu\bar\theta)\, A'_\mu(x)$$
$$+ (\theta\theta)\,\bar\theta\,\bar\lambda'(x)$$
$$+ (\bar\theta\bar\theta)\,\theta\,\lambda'(x)$$
$$+ (\theta\theta)(\bar\theta\bar\theta)\, D'(x)$$

$\boxed{\text{Wess-Zumino gauge}}$

in the following to be used

(drop ')

susy-transf. $\delta_\epsilon V$, $\delta_\epsilon = i\epsilon Q + i\bar\epsilon\bar Q$

$\Rightarrow$ $\delta A_\mu = \cdots$, $\delta\lambda = \cdots$,

$$\delta D = \frac{i}{2}\,\partial_\mu(\epsilon\sigma^\mu\bar\lambda - \lambda\sigma^\mu\bar\epsilon)$$

SUSY field strength

use covariant derivative $D, \bar{D}$

$$W_a := -\frac{1}{4}(\bar{D}\bar{D})\, D_a\, V(x, \theta, \bar{\theta})$$

- $\bar{D}_{\dot{a}} W_a = 0$
- invariant under SUSY-transf.
- components:

$$W_a = i\,\lambda_a(x) - 2\,\theta_a\, D(x) - (\sigma^{\mu\nu}\theta)_a\, F_{\mu\nu}(x) - (\theta\theta)(\sigma^\mu \partial_\mu \bar{\lambda})_a$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

2.3. SUSY Lagrangians

Lagrangian $\mathcal{L}$: scalar, hermitean

action: $S = \int d^4x \, \mathcal{L}$

SUSY: $\mathcal{L}$ resp. S invariant under SUSY-transf.

$$\mathcal{L} \xrightarrow{\text{susy}} \mathcal{L} + \underbrace{\partial_\mu K^\mu}_{\to 0 \text{ in } S}$$

products of SF $=$ SF

highest component in $\theta, \bar{\theta}$

$$\xrightarrow[\text{susy}]{} \quad + \partial_\mu (\dots)$$

under $\delta_\epsilon = i(\epsilon Q + \bar{\epsilon}\bar{Q})$:

$(\theta\theta) F \longrightarrow \delta F = \partial_\mu (\ldots)$

$(\theta\theta)(\bar{\theta}\bar{\theta}) D \longrightarrow \delta D = \partial_\mu (\ldots)$

<u>examples:</u>

$\phi = \varphi + \sqrt{2}\,\theta\psi + (\theta\theta) F$

$\phi^2 = \varphi^2 + 2\sqrt{2}\,\varphi\theta\psi + (\theta\theta)\underline{\underline{(2F\varphi - \psi\psi)}}$

$\phi^3 = \varphi^3 + \ldots + (\theta\theta)\cdot\underline{\underline{3(\varphi^2 F - \varphi\psi\psi)}}$

$\phi^\dagger\phi = \ldots + (\theta\theta)(\bar{\theta}\bar{\theta})\cdot[\ldots]$

$$[\ldots] = (\partial_\mu\varphi)^*(\partial^\mu\varphi)$$
$$- \frac{i}{2}\left(\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + \psi\sigma^\mu\partial_\mu\bar{\psi}\right)$$
$$+ F^* F$$

Lagrangian:

$$\mathcal{L} = \underbrace{\phi^\dagger \phi \Big|_{\theta\theta\bar\theta\bar\theta}}_{\text{kinetic}} \underbrace{- \frac{m}{2} \phi^2 \Big|_{\theta\theta}}_{\text{mass}} \underbrace{- \frac{g}{3} \phi^3 \Big|_{\theta\theta}}_{\text{interaction}}$$

$$= |\partial_\mu \varphi|^2 - \frac{i}{2}\left(\bar\psi \bar\sigma^\mu \partial_\mu \psi + \psi \sigma^\mu \partial_\mu \bar\psi\right) + F^* F$$

$$+ \frac{m}{2}\left(\psi\psi + \bar\psi\bar\psi\right) - m\left(\varphi F + \varphi^* F^*\right)$$

$$+ g\left(\varphi\psi\psi + \varphi^* \bar\psi\bar\psi - \varphi^2 F - \varphi^{*2} F^*\right)$$

F : no dynamical field ("auxiliary field")

eq. of motion for F:

$$\frac{\partial \mathcal{L}}{\partial F^*} = 0 : \qquad F - m\varphi^* - g\varphi^{*2} = 0$$

$$F = m\varphi^* + g\varphi^{*2}$$

insert in $\mathcal{L}$ $\longrightarrow$

$$\mathcal{L} = |\partial_\mu \varphi|^2 - m^2 \varphi^* \varphi$$

$$- \frac{i}{2}\left(\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi + \psi \sigma^\mu \partial_\mu \bar{\psi}\right) + \frac{m}{2}\left(\psi\psi + \bar{\psi}\bar{\psi}\right)$$

$$+ g\left(\varphi \psi \psi + \varphi^* \bar{\psi}\bar{\psi}\right)$$

$$- mg\,(\varphi^*\varphi)(\varphi + \varphi^*) - g\,(\varphi^*\varphi)^2$$

- scalar field φ } equal mass
- spinor field ψ }
- Yukawa interaction $\varphi - \psi - \psi$
- scalar self interaction
- universal coupling

("Wess Zumino model")

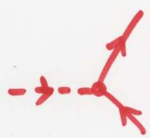
4-component notation:

$$\Psi = \begin{pmatrix} \psi \\ \bar\psi \end{pmatrix}, \qquad \psi\psi = \bar\Psi \, \mathbb{P}_L \Psi$$

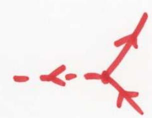
$$\bar\psi\bar\psi = \bar\Psi \, \mathbb{P}_R \Psi$$

$$\psi \sigma^\mu \partial_\mu \bar\psi + \bar\psi \bar\sigma^\mu \partial_\mu \psi = \bar\Psi \gamma^\mu \partial_\mu \Psi$$

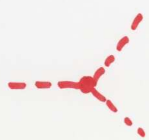
$$\begin{aligned} \mathcal{L} = {} & |\partial_\mu \varphi|^2 - m^2 |\varphi|^2 \\ & - \tfrac{1}{2} \bar\Psi \left(i\gamma^\mu \partial_\mu - m \right) \Psi \\ & + g \left(\varphi \bar\Psi \mathbb{P}_L \Psi + \varphi^* \bar\Psi \mathbb{P}_R \Psi \right) \\ & - mg \, (\varphi^* \varphi)(\varphi + \varphi^*) - g \, |\varphi|^4 \end{aligned}$$



$g \mathbb{P}_L$



$g \mathbb{P}_R$



example for vector field

components A_μ , λ, D

$$\mathcal{L} = \frac{1}{4} W^a W_a \Big|_{\theta\theta} + h.c.$$

$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$- \frac{i}{2} \left(\lambda \bar{\sigma}^\mu \partial_\mu \bar{\lambda} + \bar{\lambda} \sigma^\mu \partial_\mu \lambda \right)$$

$$+ 2 D^2 ,$$

D: auxiliary, eliminate ($D = 0$)

A_μ: photon $\qquad$ λ: photino

$$\Psi = \begin{pmatrix} \lambda \\ \bar{\lambda} \end{pmatrix} : \qquad \frac{1}{2} \bar{\Psi} i \gamma^\mu \partial_\mu \Psi$$

mass $= 0$

2.4. SUSY gauge theories

2.4.1. Abelian case

chiral SF ϕ, $\bar{D}_{\dot{a}} \phi = 0$

- SUSY gauge transf. $\phi \to \phi' = e^{-i\Lambda(x)} \phi$

$$\bar{D}_{\dot{a}} \phi' = 0 \iff \bar{D}_{\dot{a}} \Lambda = 0$$

Λ chiral SF

- with $\phi^{+\prime} = \phi^{+} e^{i\Lambda^{+}}$
the kinetic term of $\mathcal{L}$ changes:

$$\mathcal{L}_{kin} = \phi^{+}\phi\big|_{\theta\theta\bar{\theta}\bar{\theta}} \to \phi^{+} e^{i(\Lambda^{+}-\Lambda)} \phi\big|_{\theta\theta\bar{\theta}\bar{\theta}}$$

- get invariance by introducing vector field V in $\mathcal{L}_{kin}$:
$\to \mathcal{L}_{kin} = \phi^{+} e^{2gV} \phi\big|_{\theta\theta\bar{\theta}\bar{\theta}}$

with $V \to V' = V + \frac{i}{2g}(\Lambda - \Lambda^{+})$

local SUSY gauge transf.

$$\boxed{\begin{aligned}\phi &\to e^{-i\Lambda(x)}\phi \\ V &\to V + \frac{i}{2g}(\Lambda - \Lambda^\dagger)\end{aligned}}$$

$$e^{2gV} = 1 + 2gV + 2g^2 VV$$

$$\mathcal{L}_{kin} = \underbrace{\phi^\dagger\phi\Big|_{\theta\theta\bar\theta\bar\theta}}_{\substack{\text{free kin.}\\ \text{Term } \mathcal{L}_0}} + \underbrace{\phi^\dagger(2gV + 2g^2V^2)\phi\Big|_{\theta\theta\bar\theta\bar\theta}}_{\substack{\text{interaction}\\ \text{term } \mathcal{L}_1}}$$

$$\begin{aligned}\mathcal{L}_0 + \mathcal{L}_1 = {} & |D_\mu \varphi|^2 - \frac{i}{2}\left(\bar\psi\bar\sigma^\mu D_\mu\psi + \psi\sigma^\mu D_\mu\bar\psi\right) \\ & + ig\sqrt{2}\left(\varphi^*\psi\lambda - \varphi\bar\psi\bar\lambda\right) \\ & + FF^* + 2g(\varphi^*\varphi)D\end{aligned}$$

with $D_\mu = \partial_\mu - igA_\mu$

* add kinetic term for V

photon A_μ, photino λ

* 4-component notation

$$\Psi = \begin{pmatrix} \psi \\ \bar{\psi} \end{pmatrix}, \qquad \chi = \begin{pmatrix} \lambda \\ \bar{\lambda} \end{pmatrix}$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \bar{\chi} i \gamma^\mu \partial_\mu \chi$$

$$+ |D_\mu \varphi|^2 - \frac{1}{2} \bar{\Psi} i \gamma^\mu D_\mu \Psi$$

$$+ i g \sqrt{2} \left(\bar{\Psi} P_L \chi \varphi^* - \bar{\Psi} P_R \chi \varphi \right)$$

$$+ F F^* + 2g (\varphi^* \varphi) D + 2 D^2$$

* eliminate F, D

$$\frac{\partial \mathcal{L}}{\partial F^*} = 0 \Rightarrow F = 0$$

$$\frac{\partial \mathcal{L}}{\partial D} = 0 \Rightarrow D = -\frac{g}{2} (\varphi^* \varphi)$$

$|\varphi|^4$ term

SUSY-QED

needs 2 chiral SF: ϕ_+, ϕ_-

$$\phi_\pm \longrightarrow e^{\mp i\Lambda(x)} \phi_\pm$$

$$\left.\begin{array}{ll} \phi_+ : & \varphi_+, \psi_+, F_+ \\ \phi_- : & \varphi_-, \psi_-, F_- \end{array}\right\} \Psi = \begin{pmatrix} \psi_+ \\ \bar{\psi}_- \end{pmatrix}$$

$$\mathcal{L}_{SQED} = \frac{1}{4} W^a W_a \Big|_{\theta\theta} + \phi_+^+ e^{2eV} \phi_+ \Big|_{\theta\theta\bar\theta\bar\theta} + \phi_-^+ e^{-2eV} \phi_- \Big|_{\theta\theta\bar\theta\bar\theta} - \frac{m}{2} \left(\phi_+ \phi_- \Big|_{\theta\theta} + \phi_-^+ \phi_+^+ \Big|_{\bar\theta\bar\theta} \right)$$

eliminate F_+, F_-, D fields $\longrightarrow$

in 4-component notation:

$$
\begin{aligned}
\mathcal{L}_{SQED} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \bar{\chi} i \gamma^\mu \partial_\mu \chi \\
& + |D_\mu \varphi_+|^2 - m^2 |\varphi_+|^2 - \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi \\
& + |D_\mu \varphi_-|^2 - m^2 |\varphi_-|^2 \\
& - \frac{e^2}{2} \left(|\varphi_+|^2 - |\varphi_-|^2 \right)^2 \\
& - e\sqrt{2} \left(\bar{\chi} P_L \Psi \varphi_+^* + \bar{\Psi} P_R \chi \varphi_+ \right) \\
& + e\sqrt{2} \left(\bar{\Psi} P_L \chi \varphi_-^* + \bar{\chi} P_R \Psi \varphi_- \right)
\end{aligned}
$$

m: mass of e^-, e^+

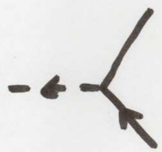
and of $\underbrace{\tilde{e}_L^-, \tilde{e}_L^+}_{\varphi_+}$, $\underbrace{\tilde{e}_R^-, \tilde{e}_R^+}_{\varphi_-^*}$

fermion-scalar interactions

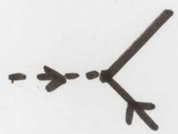
 $e^{\pm}$  χ

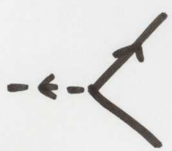
 φ_+ , φ_-

Vertices:

φ_+  $-ie\sqrt{2}\ \mathbb{P}_L$

φ_+  $-ie\sqrt{2}\ \mathbb{P}_R$

φ_-  $ie\sqrt{2}\ \mathbb{P}_R$

φ_-  $ie\sqrt{2}\ \mathbb{P}_L$

2.4.2. Non-Abelian case

gauge group G, generators $T_1, \dots T_N$

$$[T_a, T_b] = i f_{abc} T_c$$

matter: $\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix}$ n-dim. rep. of G

gauge fields: W_μ^α, $\alpha = 1, \dots N$

field strength: $W_{\mu\nu}^\alpha = \partial_\mu W_\nu^\alpha - \partial_\nu W_\mu^\alpha + g f_{abc} W_\mu^b W_\nu^c$

cov. derivative: $D_\mu = \partial_\mu - ig T_\alpha W^\alpha$

$$\mathcal{L} = -\frac{1}{4}(W_{\mu\nu}^\alpha)^2 + \bar{\Psi} i \gamma^\mu D_\mu \Psi + |D_\mu \varphi|^2 - m \bar{\Psi}\Psi - m^2 \varphi^\dagger \varphi$$

SUSY version

- matter fields

$$\Phi_\pm = \begin{pmatrix} \Phi_{1,\pm} \\ \vdots \\ \Phi_{n\pm} \end{pmatrix}$$ superfields (chiral)

$$\Phi_{k,\pm} = \varphi_{k,\pm} + \sqrt{2}\,\theta\,\psi_{k,\pm} + (\theta\theta)\,F_{k,\pm}$$

- gaugefields $V^1, \dots V^N$

vector SF

$$V^\alpha = (\theta\sigma^\mu\bar\theta)\,W_\mu^\alpha + (\bar\theta\bar\theta)(\theta\lambda^\alpha) + \cdots$$

$\alpha = 1, \dots N$ gauge index

[$a = 1, 2$: spinor ind.]

W_μ^α : vector bosons, λ^α: gauginos

$$V = T_\alpha V^\alpha$$ $n \times n$ matrix

- gauge transformation

$$\Phi_+' = e^{-i\Lambda}\Phi_+, \quad \Lambda = T_\alpha \Lambda^\alpha(x)$$

$$\Phi_-' = e^{i\Lambda}\Phi_-$$

- $\mathcal{L}_{\text{matter}} =$

$$\Phi_+^\dagger \, e^{2gV} \, \Phi_+ + \Phi_-^\dagger \, e^{-2gV} \Phi_-$$

$$(\phi_{+1}^\dagger, \ldots \phi_{+n}^\dagger) \, (n \times n) \begin{pmatrix} \phi_{+1} \\ \vdots \\ \phi_{+n} \end{pmatrix}$$

- $\mathcal{L}_{gauge}$:

$$W_a = -\frac{1}{4}(\bar{D}\bar{D})\, e^{-2gV} D_a\, e^{2gV}$$

field strength

$$\mathcal{L}_{gauge} = \frac{1}{4} W_a W^a \Big|_{\theta\theta} + \text{h.c.}$$

$$= -\frac{1}{4} W^{\alpha}_{\mu\nu} W^{\mu\nu,\alpha}$$

$$- \frac{i}{2}\left(\bar{\lambda}^{\alpha} \bar{\sigma}^{\mu} D_{\mu} \lambda^{\alpha} + \lambda^{\alpha} \sigma^{\mu} D_{\mu} \bar{\lambda}^{\alpha}\right)$$

$$+ 2\left(D^{\alpha}\right)^2$$

choose $G = SU(3)$:

SUSY-QCD

2.5. SUSY breaking

physics: $m_{bos} \neq m_{ferm}$

SUSY partners of standard particles heavy, need extra mass terms

here: formal description
[can be realized in models]

soft breaking:

- $M^2_{ij} \, \phi^+_i \phi_j$ mass for scalars
- $B_{ij} \, \phi_i \phi_j + A_{ijk} \, \phi_i \phi_j \phi_k + \text{h.c.}$ for scalars
- $\frac{1}{2} (M_\lambda \lambda\lambda + \text{h.c.})$ mass for gauginos

do not lead to quadratic divergences in loop contributions to masses, renormalizable

SUSY formulation

soft terms = part of SUSY interaction terms

introduce external chiral SF

"spurion field"

$$\eta(z,\theta) = a(z) + \sqrt{2}\,\theta\chi(z) + \theta\theta\,\hat{f}(z)$$

$$\hat{f}(z) = f_0 + f(z), \qquad f_0 \text{ const}$$

- $\tilde{M}_{ij}^2\, \eta^+\eta\, \Phi_i^+ e^{2gV} \Phi_j \Big|_{\theta\theta\bar{\theta}\bar{\theta}}$

  $\rightarrow \tilde{M}_{ij}^2 f_0^2\, \phi_i^+ \phi_j$ for $a = \chi = f = 0$

- $\underbrace{\tilde{B}_{ij}\,\eta}_{\tilde{B}_{ij} f_0}\, \Phi_i \Phi_j + \tilde{A}_{ijk}\,\eta\, \Phi_i \Phi_j \Phi_k \Big|_{\theta\theta} + \text{h.c.}$

  $\tilde{B}_{ij} f_0\, \phi_i \phi_j + \tilde{A}_{ijk} f_0\, \phi_i \phi_j \phi_k$

- $\frac{1}{2}\underbrace{\tilde{M}_\lambda\,\eta}_{\tilde{M}_\lambda f_0}\, W_a W^a \Big|_{\theta\theta} + \text{h.c.}$

  $\tilde{M}_\lambda f_0\, (\lambda_\alpha \lambda_\alpha + \bar{\lambda}_\alpha \bar{\lambda}_\alpha)$

$\dim[\eta] = 0$

interaction terms are supersymmetric and (power-counting) renormalizable

useful for quantization and proof of renormalizability

3. MSSM: formulation and content

$\boxed{\text{gauge boson content}}$

$SU(2)_I$: generators $T_{\mathrm{I}}^1,\ T_{\mathrm{I}}^2,\ T_{\mathrm{I}}^3, \quad T_{\mathrm{I}}^a = \frac{1}{2}\sigma_a$

gauge fields $W_\mu^1,\ W_\mu^2,\ W_\mu^3$

also: $W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i\, W_\mu^2\right),\ W_\mu^3$

$U(1)_Y$: generator Y

gauge field B_μ

$SU(3)_C$: generators $T^a = \frac{1}{2}\lambda_a \quad (a = 1, \ldots 8)$

gauge fields $G_\mu^a, \quad (a = 1, \cdots 8)$

matter fields and quantum numbers

$SU(2)_I$: weak isospin, generators $T_{\mathrm{I}}^a = \frac{1}{2}\,\sigma^a$ *for* L, $= 0$ *for* R

$U(1)_Y$: weak hypercharge, generator Y $\qquad \boxed{T_{\mathrm{I}}^3 + Y/2 = Q}$

fermion content (ignoring possibe right-handed neutrinos)

					T_{I}^3	Y
leptons:	$\Psi_L^{\mathrm{L}} =$	$\begin{pmatrix} \nu_e^{\mathrm{L}} \\ e^{\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} \nu_\mu^{\mathrm{L}} \\ \mu^{\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} \nu_\tau^{\mathrm{L}} \\ \tau^{\mathrm{L}} \end{pmatrix}$	$+\frac{1}{2}$, $-\frac{1}{2}$	-1, -1
	$\psi_l^{\mathrm{R}} =$	e^{R}	μ^{R}	τ^{R}	0	-2
quarks:	$\Psi_Q^{\mathrm{L}} =$	$\begin{pmatrix} u^{\mathrm{L}} \\ d^{\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} c^{\mathrm{L}} \\ s^{\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} t^{\mathrm{L}} \\ b^{\mathrm{L}} \end{pmatrix}$	$+\frac{1}{2}$, $-\frac{1}{2}$	$+\frac{1}{3}$, $+\frac{1}{3}$
	$\psi_u^{\mathrm{R}} =$	u^{R}	c^{R}	t^{R}	0	$+\frac{4}{3}$
	$\psi_d^{\mathrm{R}} =$	d^{R}	s^{R}	b^{R}	0	$-\frac{2}{3}$

Particle Content of the MSSM

Superfield	Bosons		Fermions		$SU_c(3)$	$SU_L(2)$	$U_Y(1)$
Gauge							
$\mathbf{G^a}$	gluon	g^a	gluino	$\tilde{g}^a$	8	1	0
$\mathbf{V^k}$	Weak	$W^k \quad (W^\pm, Z)$	wino, zino	$\tilde{w}^k \quad (\tilde{w}^\pm, \tilde{z})$	1	3	0
$\mathbf{V'}$	Hypercharge	$B \ (\gamma)$	bino	$\tilde{b}(\tilde{\gamma})$	1	1	0
Matter							
$\mathbf{L_i}$	sleptons	$\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$	leptons	$L_i = (\nu, e)_L$	1	2	-1
$\mathbf{E_i}$		$\tilde{E}_i = \tilde{e}_R$		$E_i = e_R$	1	1	2
$\mathbf{Q_i}$	squarks	$\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$	quarks	$Q_i = (u, d)_L$	3	2	1/3
$\mathbf{U_i}$		$\tilde{U}_i = \tilde{u}_R$		$U_i = u_R^c$	3^*	1	$-4/3$
$\mathbf{D_i}$		$\tilde{D}_i = \tilde{d}_R$		$D_i = d_R^c$	3^*	1	2/3
Higgs							
$\mathbf{H_1}$	Higgses	H_1	higgsinos	$\tilde{H}_1$	1	2	-1
$\mathbf{H_2}$		H_2		$\tilde{H}_2$	1	2	1

superfields for matter

$$\mathbf{Q} = \begin{pmatrix} \mathbf{Q}^1 \\ \mathbf{Q}^2 \end{pmatrix}, \mathbf{U}, \mathbf{D} \quad \text{(quarks)} \qquad \mathbf{L} = \begin{pmatrix} \mathbf{L}^1 \\ \mathbf{L}^2 \end{pmatrix}, \mathbf{E} \quad \text{(leptons)}$$

$$\mathbf{Q^i} = \tilde{Q}^i + \sqrt{2}\,(\theta q_L^i) + (\theta\theta) F_L^i$$

$$\mathbf{U} = \tilde{U} + \sqrt{2}\,(\theta u_R) + (\theta\theta) F_R^u$$

$$\mathbf{U}^\dagger = \tilde{U}^* + \sqrt{2}\,(\bar{\theta}\bar{u}_R) + (\bar{\theta}\bar{\theta}) F_R^{u\,*}$$

scalar *spinor* *auxiliary*

$\tilde{Q}^i = \tilde{q}_L^i$: *u- and d-squarks, “left-handed”*

$\tilde{U}^* = \tilde{u}_R$, *u-squark, “right-handed”*

4-component quark spinors: $\Psi_u = \begin{pmatrix} u_L \\ \bar{u}_R \end{pmatrix}, \quad \Psi_u^c = \begin{pmatrix} u_R \\ \bar{u}_L \end{pmatrix}$

(analogous for d-quarks and leptons)

superfields for Higgs

$$\mathbf{H}_1 = \begin{pmatrix} \mathbf{H}_1^1 \\ \mathbf{H}_1^2 \end{pmatrix} \text{ with } Y = -1, \qquad \mathbf{H}_2 = \begin{pmatrix} \mathbf{H}_2^1 \\ \mathbf{H}_2^2 \end{pmatrix} \text{ with } Y = +1$$

$$\mathbf{H}_i^k = H_i^k + \sqrt{2}\,(\theta \psi_{H_i^k}) + (\theta\theta) F_i^k$$

scalar *spinor* *auxiliary*

(Higgs) *(Higgsino)*

superfields for Higgs

$$\mathbf{H}_1 = \begin{pmatrix} \mathbf{H}_1^1 \\ \mathbf{H}_1^2 \end{pmatrix} \text{ with } Y = -1, \qquad \mathbf{H}_2 = \begin{pmatrix} \mathbf{H}_2^1 \\ \mathbf{H}_2^2 \end{pmatrix} \text{ with } Y = +1$$

$$\mathbf{H}_i^k = H_i^k + \sqrt{2}\,(\theta\psi_{H_i^k}) + (\theta\theta)F_i^k$$

scalar *spinor* *auxiliary*

(Higgs) *(Higgsino)*

electric charge: $\mathbf{Q}_{H_1} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \mathbf{Q}_{H_2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$$\Rightarrow \qquad \mathbf{H}_1 = \begin{pmatrix} \mathbf{H}_1^0 \\ \mathbf{H}_1^- \end{pmatrix}, \qquad \mathbf{H}_2 = \begin{pmatrix} \mathbf{H}_2^+ \\ \mathbf{H}_2^0 \end{pmatrix}$$

constructing the MSSM Lagrangian

[*notation:* $\mathbf{V}_i = T_\alpha \mathbf{V}_i^\alpha, \quad \mathbf{W}_a = T_\alpha \mathbf{W}_a^\alpha$]

$$\sum_{SU(3),SU(2),U(1)} \tfrac{1}{4}\,\mathrm{Tr}(\mathbf{W}_a \mathbf{W}^a) \,+\, h.c.$$

$$+\, \sum_{\mathrm{matter}} \, \mathbf{\Phi}_i^\dagger \, e^{2(g_3 \mathbf{V}_3 + g_2 \mathbf{V}_2 + g_1 \mathbf{V}_1)} \, \mathbf{\Phi}_i$$

$$+\, \sum_{\mathrm{Higgs}} \, \mathbf{H}_i^\dagger \, e^{2(g_2 \mathbf{V}_2 + g_1 \mathbf{V}_1)} \, \mathbf{H}_i$$

$$+\, \mathcal{W} \qquad \text{superpotential}$$

$$\mathcal{W} \,=\, \varepsilon_{ij}\, \mu\, \mathbf{H}_1^i \mathbf{H}_2^j$$

$$+\, \varepsilon_{ij} \left(Y_U\, \mathbf{Q}^j \mathbf{U}\, \mathbf{H}_2^i \,+\, Y_D\, \mathbf{Q}^j \mathbf{D}\, \mathbf{H}_1^i \,+\, Y_E\, \mathbf{L}^j \mathbf{E}\, \mathbf{H}_1^i \right)$$

- $\mathcal{W}$ conserves R-parity: $P_R = (-1)^{3(B-L)+2s}$

- P_R-violating interactions
  – induce baryon- or lepton-number violating processes
  – interactions must be suppressed
  – interactions are absent if P_R-conservation is postulated

- phenomenologically, P_R-violating terms can be present, with couplings (small) as free parameters

- minimal choice (MSSM) contains only R-parity conserving terms

- all SM particles have even, all SUSY particles have odd P_R $\Rightarrow$
  – SUSY-particles can only be produced in pairs
  – lightest SUSY particle ("LSP") is stable

soft breaking terms

$$\mathcal{L}_{\rm soft} = \sum_i m_i^2 \, |\varphi_i|^2$$

$$+ \sum_{SU(3),SU(2),U(1)} \tfrac{1}{2} M_\lambda \, \lambda_\alpha \lambda_\alpha$$

$$+ B \, \varepsilon_{ij} \, H_1^i H_2^j + h.c.$$

$$+ \varepsilon_{ij} \left(A_U \, \tilde{Q}^j \tilde{U} \, H_2^i + A_D \, \tilde{Q}^j \tilde{D} \, H_1^i + A_E \, \tilde{L}^j \tilde{E} \, H_1^i \right)$$

φ_i : all scalar fields

λ_α : all gaugino fields

$\tilde{U}, \tilde{D}, \tilde{E}$: scalar quark/lepton fields

$\tilde{Q}, \tilde{E}$: doublets of scalar quarks/leptons

general: coeffcients A are 3×3-matrices in generation space

- essentially all masses and mixings of superpartners are free parameters

- soft parameters can be treated as independent free parameters

- or: fixed by some (ad-hoc) assumptions

- or: derived from specific models of SUSY breaking

- essentially all masses and mixings of superpartners are free parameters
- soft parameters can be treated as independent free parameters
- or: fixed by some ad-hoc/ well motivated assumptions
- or: derived from specific models of SUSY breaking

- parameters M_λ, A_f can be complex
- new sources of CP-violation
- phenomenological constraints from electric dipole moments and from flavor physics

Higgs fields

two scalar doublets from $\mathbf{H}_1, \mathbf{H}_2$ superfields:

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} H_1^0 \\ \phi_1^- \end{pmatrix}, \qquad < H_1 >_0 = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ H_2^0 \end{pmatrix}, \qquad < H_2 >_0 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$$V_H^{\rm susy} = \mu^2 H_1^\dagger H_1 + \mu^2 H_2^\dagger H_2$$
$$+ \frac{g_1^2 + g_2^2}{8} \left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \frac{g_2^2}{2} |H_1^\dagger H_2|^2 \,,$$

$$V_H^{\rm soft} = m_1^2 H_1^\dagger H_1 + m_1^2 H_2^\dagger H_2 - m_3^2 \varepsilon_{ij} \left(H_1^i H_2^j + h.c.\right)$$

Higgs potential: $V_H = V_H^{\mathrm{susy}} + V_H^{\mathrm{soft}}$

$$= (\mu^2 + m_1^2)\, H_1^\dagger H_1 + (\mu^2 + m_2^2)\, H_2^\dagger H_2 - m_3^2\, \varepsilon_{ij}\left(H_1^i\, H_2^j + h.c.\right)$$

$$+ \frac{g_1^2+g_2^2}{8}\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \frac{g_2^2}{2}\,|H_1^\dagger H_2|^2$$

EW symmetry breaking: minimum of V_H at

$$H_1^0 = v_1 \neq 0, \quad H_2^0 = v_2 \neq 0, \quad \Phi_1^- = 0, \quad \Phi_2^+ = 0$$

necessary condition: $m_3^4 > (\mu^2 + m_1^2)\,(\mu^2 + m_2^2)$

requires $m_3^2 \neq 0$

- SUSY breaking required for EW symmetry breaking

Higgs potential: $V_H = V_H^{\rm susy} + V_H^{\rm soft}$

$$= (\mu^2 + m_1^2)\, H_1^\dagger H_1 + (\mu^2 + m_2^2)\, H_2^\dagger H_2 - m_3^2\, \varepsilon_{ij}\, (H_1^i\, H_2^j + h.c.)$$

$$+ \frac{g_1^2 + g_2^2}{8} \left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \frac{g_2^2}{2}\, |H_1^\dagger H_2|^2$$

EW symmetry breaking: minimum of V_H at

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necessary condition: $m_3^4 > (\mu^2 + m_1^2)\,(\mu^2 + m_2^2)$

requires $m_3^2 \neq 0$

- SUSY breaking required for EW symmetry breaking

SM particle masses:

$$M_{W,Z}^2 \sim v_1^2 + v_2^2, \quad m_d, m_e \sim v_1, \quad m_u \sim v_2$$

new parameter: $\tan\beta = \frac{v_2}{v_1}$

mass spectrum: 3 unphysical + 5 physical degrees of freedom

- 3 Goldstone bosons $G^0,\ G^\pm$
- 2 neutral CP-even Higgs bosons $h^0,\ H^0$
- 1 neutral CP-odd Higgs boson A^0 *“pseudoscalar”'*

$$M_A^2 = m_3^2\,(\cot\beta + \tan\beta)$$

conventional input parameters: $M_A,\ \tan\beta = \frac{v_2}{v_1}$

other masses $m_h, m_H, m_{H^\pm}$ predicted, not independent

mass eigenstates are linear combinations of the doublet components, with $\phi_1^+ = (\phi_1^-)^\dagger,\ \phi_2^- = (\phi_2^+)^\dagger$

$$H_1^0 = v_1 + \frac{1}{\sqrt{2}}\,(\phi_1 + i\chi_1)$$

$$H_2^0 = v_2 + \frac{1}{\sqrt{2}}\,(\phi_2 + i\chi_1)$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

$$\tan 2\alpha = \tan 2\beta \, \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \qquad -\frac{\pi}{2} < \alpha < 0$$

- predictions for dependent masses (tree-level):

$$m_{H^\pm}^2 = M_A^2 + M_W^2$$

$$m_{H,h}^2 = \frac{1}{2}\left(M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right)$$

$$m_h < M_Z \, |\cos(2\beta)| < M_Z \quad \textbf{(!)}$$

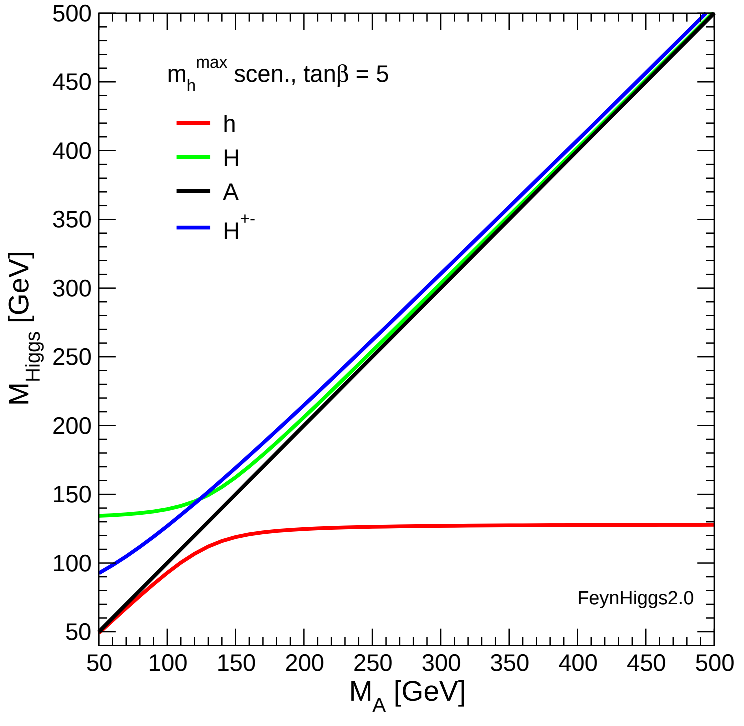
- substantial higher-order corrections:

dominant one-loop term $\quad \Delta m_h^2 \sim G_{\mathrm{F}}\, m_t^4 \, \log(m_{\tilde{t}}^2/m_t^2)$
from the Yukawa sector

all other sectors also contribute

$\boxed{\boldsymbol{m_h} = \text{observable sensitive to (still) unknown SUSY particles}}$

Higgs bosons in the MSSM: $h^0, H^0, A^0, H^\pm$



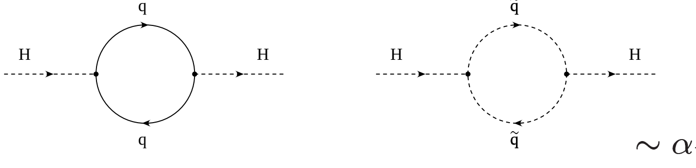
- *light Higgs boson* h^0

  $$m_h \le m_Z |\cos(2\beta)| + \Delta m_{h^0}$$

- *for heavy* $A^0, H^0, H^\pm$*:*

  h^0 *like Standard Model Higgs boson*

m_h^0 strongly influenced by quantum effects, e.g. t, $\tilde{t}$



$\sim \alpha_t$

gauginos and Higgsinos

mass terms = bilinear terms in gaugino and Higgsino fields

notation: *gluino* $\tilde{g}_a$ *, winos* $\tilde{W}^{\pm}, \tilde{W}^3$ *, bino* $\tilde{B}^0$ *, Higgsinos* $\tilde{H}_{1,2}^{\pm,0}$

$$\mathcal{L}_{\text{gaugino,Higgsino}} = \frac{1}{2} M_3 \tilde{g}_a \tilde{g}_a + \frac{1}{2}\, \chi^{\,\mathrm{T}}\, \mathbf{M^{(0)}} \chi + \psi_-^{\,\mathrm{T}}\, \mathbf{M^{(c)}} \psi_+ \, + \mathrm{h.c.}$$

$\mathbf{M^{(c)}}$, $\mathbf{M^{(0)}}$ non diagonal in the components

$$\psi_+ = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_2{}^+ \end{pmatrix}, \quad \psi_- = \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_1{}^- \end{pmatrix}, \quad \chi = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix}$$

diagonalization $\rightarrow$ mass eigenstates:

- charginos $\chi_{1,2}^{\pm}$, neutralinos $\chi_{1,2,3,4}^0$

- chargino masses: $m_{\tilde{\chi}^{\pm}_{1,2}}$ from M_2, μ

$$\begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}$$

- neutralino masses: $m_{\tilde{\chi}^0_{1,2,3,4}}$ from M_1, M_2, μ

$$\begin{pmatrix} M_1 & 0 & -M_Z s_W \cos\beta & M_Z s_W \sin\beta \\ 0 & M_2 & M_Z c_W \cos\beta & -M_Z c_W \sin\beta \\ -M_Z s_W \cos\beta & M_Z c_W \cos\beta & 0 & -\mu \\ M_Z s_W \sin\beta & -M_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}$$

- sfermion masses: $m_{\tilde{f}_{1,2}}$ from $M_L, M_{\tilde{f}_R}, A_f$

$$\begin{pmatrix} m_f^2 + M_L^2 + M_Z^2 c_{2\beta}(I_f^3 - Q_f s_W^2) & m_f(A_f - \mu\kappa) \\ m_f(A_f - \mu\kappa) & m_f^2 + M_{\tilde{f}_R}^2 + M_Z^2 c_{2\beta} Q_f s_W^2 \end{pmatrix}$$

with

$$\kappa = \{\cot\beta; \tan\beta\} \quad \text{for} \quad f = \{u, d\}$$

note: M_L *equal for both* $\tilde{u}$ *and* $\tilde{d}$ *of a doublet*

$$M_L, M_{\tilde{u}_R}, A_u \quad \rightarrow m_{\tilde{u}_{1,2}},\ \theta_u$$

$$M_L, M_{\tilde{d}_R}, A_d \quad \rightarrow m_{\tilde{d}_{1,2}},\ \theta_d$$

$\Rightarrow \quad m_{\tilde{u}_{1,2}}, m_{\tilde{d}_{1,2}}$ *not independent*

Quantization and renormalization

conventional gauge theories

gauge group G, generators T_a, structure constants f_{abc}

for quantization: $\mathcal{L} = \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{ghost}}$

$$\mathcal{L}_{\text{fix}} = \tfrac{1}{2}\, F_a^2, \qquad F_a = \partial_\mu W^{a,\mu}$$

requires ghost fields c_a and anti-ghosts $\bar{c}_a$

$$\mathcal{L}_{\text{ghost}} = (\partial^\mu \bar{c}_a)\, (D_\mu^{\text{adj}})_{ab}\, c_b, \qquad D_\mu^{\text{adj}} = \partial_\mu - ig\, W_\mu^r\, T_r^{\text{adj}}$$

- $\mathcal{L}$ is symmetric under BRS transformations

$$sW_\mu^a = (D_\mu^{\text{adj}})_{ab}\, c_b \qquad\qquad [sW_\mu^a \equiv \delta W_\mu^a \quad \texttt{etc.}]$$

$$sc_a = -\partial^\nu W_\nu^a, \qquad s\bar{c}_a = -\tfrac{1}{2} g\, f_{abc}\, c_b c_c$$

BRS *[Becchi, Rouet, Stora]* symmetry guarantees

– renormalizability

– gauge invariant and unitary S matrix

important: ST identities = symmetry relations between Green functions, valid to all orders

basic quantity: effective action $\Gamma(\mathcal{L})$

generating functional of vertex functions

$$\frac{\delta\Gamma}{\delta\varphi_i\delta\varphi_j\ldots} = \Gamma_{\varphi_i\varphi_j\ldots}$$

classical action: $\Gamma_{\mathrm{cl}}(\mathcal{L}) = \int \mathrm{d}^4x\, \mathcal{L}$

$\Rightarrow$ tree level vertices

general: vertex functions with loop contributions, building blocks for renormalization

BRS symmetry: invariance of Γ under BRS transformations,

$$S(\Gamma) = \int \mathrm{d}^4x \left[\frac{\delta\Gamma}{\delta\varphi_i}\, s\varphi_i + \cdots\right] = 0$$

S: ST-operator

$$\Rightarrow \quad \frac{\delta S(\Gamma)}{\delta\varphi_j \ldots} = 0$$

relations between vertex functions

ST identities

$\Rightarrow$ all UV divergences in vertex functions can be removed by (multiplicative) renormalization of parameters and fields in the classical Lagrangian/action

SUSY gauge theories

SUSY transformation modify BRS transformations,

$\mathcal{L}_{\rm fix} + \mathcal{L}_{\rm ghost}$
not invariant under SUSY transformations

- BRS transformations $\rightarrow$ SUSY-BRS transformations

combine BRS and SUSY transformations

SUSY gauge theories

SUSY transformation modify BRS transformations,

$\mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{ghost}}$
not invariant under SUSY transformations

- BRS transformations $\rightarrow$ SUSY-BRS transformations

combine BRS and SUSY transformations

SUSY BRS symmetry $\Rightarrow$ ST identities

ST id must be fulfilled at any order, including counterterms
$\Rightarrow$ structure of counterterms

result:

$\Rightarrow$ all UV divergences in vertex functions can be removed by (multiplicative) renormalization of parameters and fields in the classical Lagrangian/action.

Parameters to be renormalized:
supersymmetric and soft-breaking parameters.

counterms fulfill the ST id $\Leftrightarrow$ the regularization scheme for loop calculations is symmetric

otherwise: symmetry-restoring counterterms needed, determined by the ST id

- important for practical calculations

practical calculations are done in

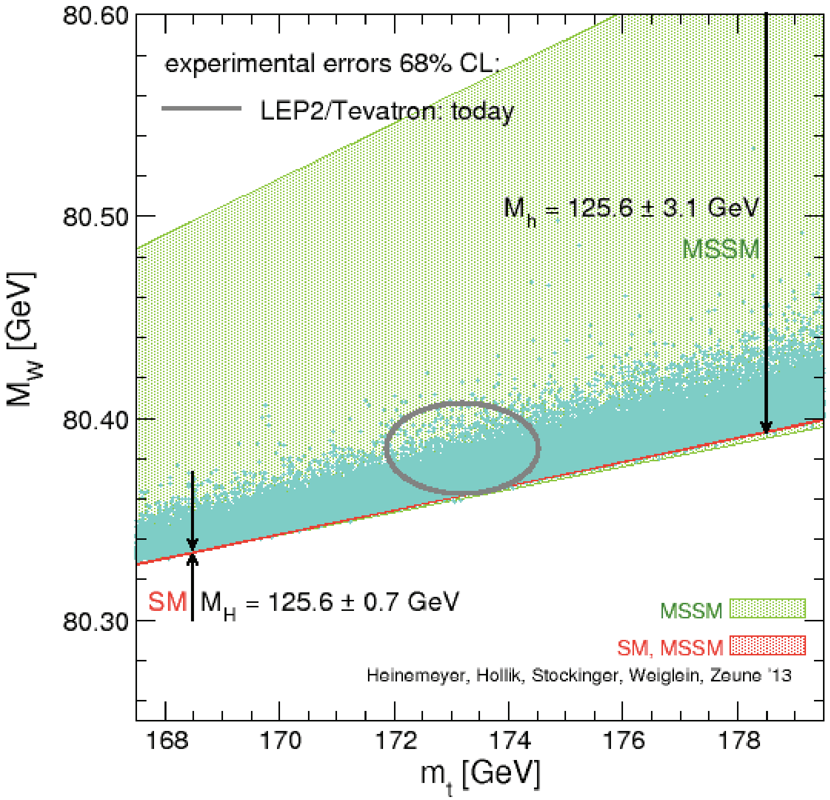
- dimensional regularization D_{reg}:
  $p^\mu, A^\mu, \gamma^\mu, g_{\mu\nu}$ in D dimensions
  not supersymmetric,
  needs symmetry-restoring counterterms
- dimensional reduction D_{red}:
  only momenta in D dimensions,
  no symmetry-restoring counterterms needed (at one-loop), beyond one-loop no general proof yet

4. Tests of the MSSM

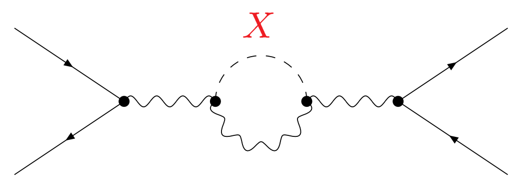
- SUSY parameters $\rightarrow$ mass spectrum + mixing matrices
- interaction terms $\rightarrow$ Feynman rules for the MSSM
- calculate processes with SUSY particles
  – production cross sections for colliders
  – decay widths/ branching ratios

  in terms of the model parameters
- confront predictions with experimental results: direct searches
- calculate electroweak precision observables (PO) with virtual SUSY particles *M_W, Z observables, muon $g-2$, and M_h (!)*
- compare predictions with experimental results for PO: indirect searches

indirect: precision observables with SUSY quantum loops



muon decay $\mu^- \to e^- \, \nu_\mu \, \bar{\nu}_e$



X = Higgs bosons, SUSY particles

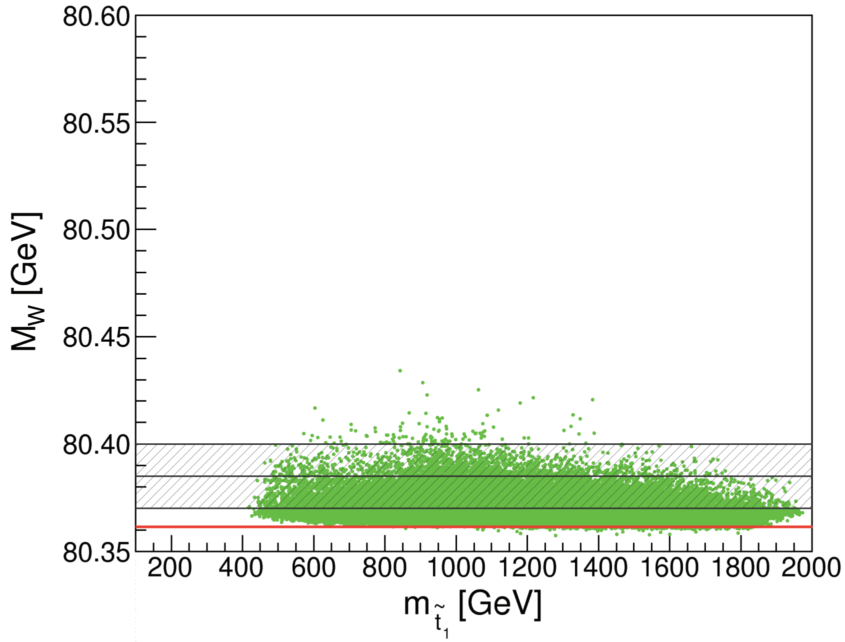
$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2 \left(1 - M_W^2/M_Z^2\right)} \cdot \left[1 + \Delta r(m_t, X)\right]$$

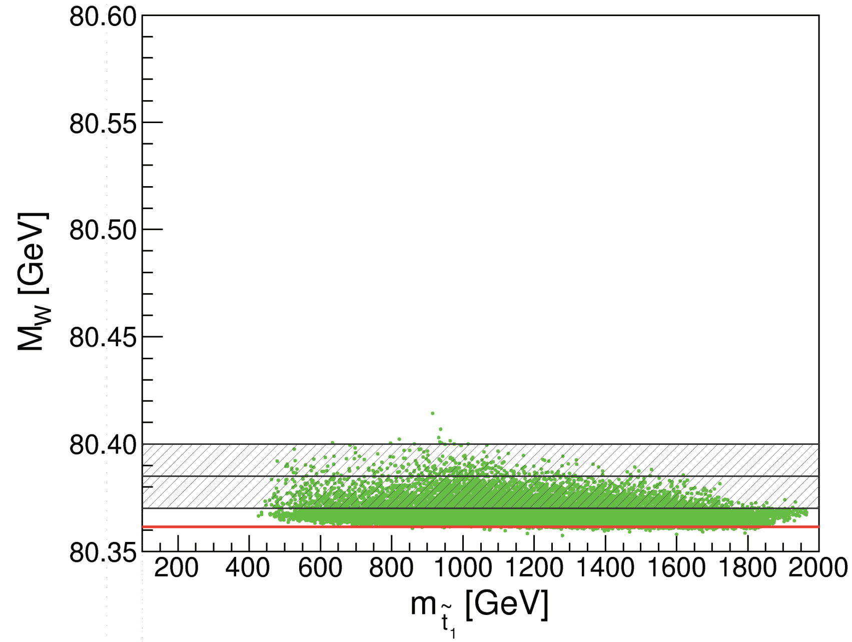
dark: $m_{\tilde{t}}, \, m_{\tilde{b}} > 500\,\mathrm{GeV}$

$m_{\tilde{q}}, \, m_{\tilde{g}} > 1200\,\mathrm{GeV}$

determines W mass

$$M_W = M_W(\alpha, G_F, M_Z, m_t, X)$$





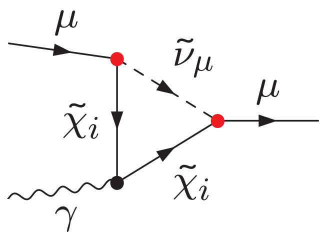
$$m_{\tilde{t}},\, m_{\tilde{b}} > 1000\,\mathrm{GeV}$$

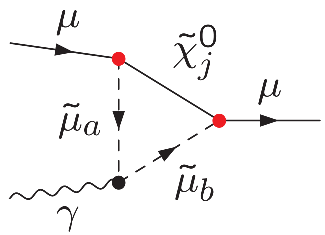
$$(m_{\tilde{q}},\, m_{\tilde{g}} > 1200\,\mathrm{GeV})$$

+ *charginos and sleptons above 500 GeV*

muon $g-2$

new contributions from virtual SUSY partners of $\mu,\ \nu_\mu$
and of $W^\pm,\ Z$





extra terms

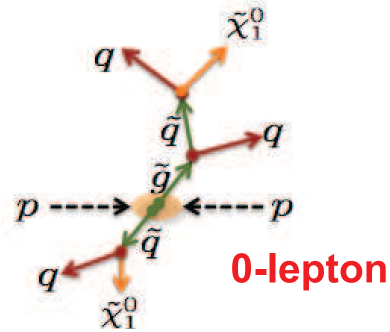
$$+ \quad \frac{\alpha}{\pi}\,\frac{m_\mu^2}{M_{\rm SUSY}^2}\cdot\frac{v_2}{v_1}$$

can provide missing contribution for

$$M_{\rm SUSY} = 200 - 600\,\mathrm{GeV}$$

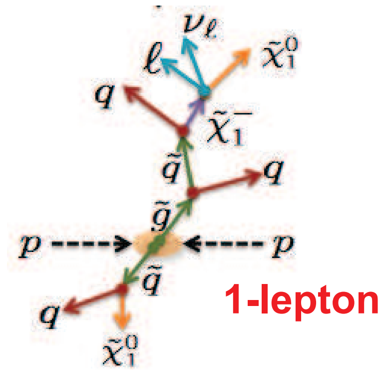
direct: SUSY searches at the LHC

- at the LHC sparticles are pair produced
  - dominantly squarks and gluinos via the strong interaction
  - they decay via cascades into the stable LSP (neutralino or gravitino), assuming R-parity conservation



- common signature:
  - multiple, high energetic jets and transverse missing momentum
  - distinguish final states by additional particles

    zero, one, two, .. leptons (e, μ), two photons, …
    b-jets if 3rd generation squarks are lighter than other generation squarks



- incomplete event reconstruction due to LSP

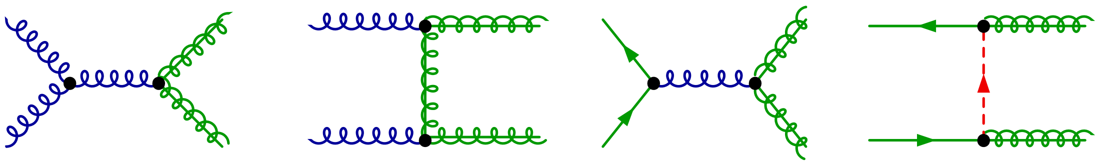
$\rightarrow$ *distributions of jets (and leptons)*

- **searches need predictions for production and decays of SUSY particles**

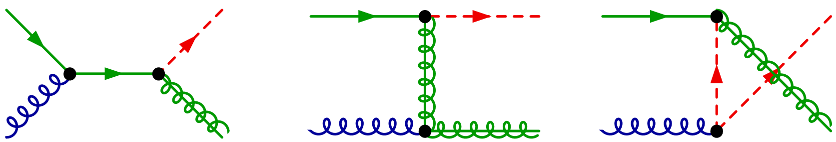
- LHC: LO contributions to squark pair production (QCD tree level)

cross sections depend essentially only on α_s and s-particle masses

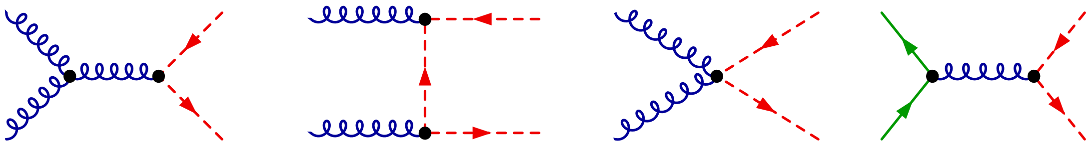
- $\mathcal{O}(\alpha_s^2)$: – $\tilde{g}\tilde{g}$ production



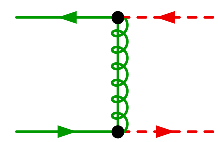
– $\tilde{g}\tilde{q}$ production



– $\tilde{q}\tilde{q}^*$, $\tilde{b}_i\tilde{b}_i^*$, $\tilde{t}_i\tilde{t}_i^*$ production; $\tilde{q}\tilde{q}$ production



$(\tilde{q} \neq \tilde{t})$
$+$



- *decay modes depend in detail on model parameters and chiralities*

- *simplifying assumptions for experimental analyses*

ATLAS SUSY Searches* - 95% CL Lower Limits

Status: July 2015

ATLAS Preliminary

$\sqrt{s}$ = 7, 8 TeV

$\sqrt{s}$ = 7 TeV | $\sqrt{s}$ = 8 TeV

	Model	e,μ,τ,γ	Jets	E_T^{miss}	$\int\mathcal{L}\,dt[fb^{-1}]$	Mass limit		Reference
Inclusive Searches	MSUGRA/CMSSM	0-3 e,μ/1-2 τ	2-10 jets/3 b	Yes	20.3	$\tilde{q},\tilde{g}$ 1.8 TeV	$m(\tilde{q})=m(\tilde{g})$	1507.05525
	$\tilde{q}\tilde{q}$, $\tilde{q}\to q\tilde{\chi}_1^0$	0	2-6 jets	Yes	20.3	$\tilde{q}$ 850 GeV	$m(\tilde{\chi}_1^0)$=0 GeV, m(1st gen. $\tilde{q}$)=m(2nd gen. $\tilde{q}$)	1405.7875
	$\tilde{q}\tilde{q}$, $\tilde{q}\to q\tilde{\chi}_1^0$ (compressed)	mono-jet	1-3 jets	Yes	20.3	$\tilde{q}$ 100-440 GeV	$m(\tilde{q})-m(\tilde{\chi}_1^0)$<10 GeV	1507.05525
	$\tilde{q}\tilde{q}$, $\tilde{q}\to q(\ell\ell/\ell\nu/\nu\nu)\tilde{\chi}_1^0$	2 e,μ (off-Z)	2 jets	Yes	20.3	$\tilde{q}$ 780 GeV	$m(\tilde{\chi}_1^0)$=0 GeV	1503.03290
	$\tilde{g}\tilde{g}$, $\tilde{g}\to q\bar{q}\tilde{\chi}_1^0$	0	2-6 jets	Yes	20.3	$\tilde{g}$ 1.33 TeV	$m(\tilde{\chi}_1^0)$=0 GeV	1405.7875
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qq\tilde{\chi}_1^{\pm}\to qqW^{\pm}\tilde{\chi}_1^0$	0-1 e,μ	2-6 jets	Yes	20	$\tilde{g}$ 1.26 TeV	$m(\tilde{\chi}_1^0)$<300 GeV, $m(\tilde{\chi}^{\pm})=0.5(m(\tilde{\chi}_1^0)+m(\tilde{g}))$	1507.05525
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qq(\ell\ell/\ell\nu/\nu\nu)\tilde{\chi}_1^0$	2 e,μ	0-3 jets	-	20	$\tilde{g}$ 1.32 TeV	$m(\tilde{\chi}_1^0)$=0 GeV	1501.03555
	GMSB ($\tilde{\ell}$ NLSP)	1-2 τ + 0-1 ℓ	0-2 jets	Yes	20.3	$\tilde{g}$ 1.6 TeV	tanβ >20	1407.0603
	GGM (bino NLSP)	2 γ	-	Yes	20.3	$\tilde{g}$ 1.29 TeV	$c\tau$(NLSP)<0.1 mm	1507.05493
	GGM (higgsino-bino NLSP)	γ	1 b	Yes	20.3	$\tilde{g}$ 1.3 TeV	$m(\tilde{\chi}_1^0)$<900 GeV, $c\tau$(NLSP)<0.1 mm, μ<0	1507.05493
	GGM (higgsino-bino NLSP)	γ	2 jets	Yes	20.3	$\tilde{g}$ 1.25 TeV	$m(\tilde{\chi}_1^0)$<850 GeV, $c\tau$(NLSP)<0.1 mm, μ>0	1507.05493
	GGM (higgsino NLSP)	2 e,μ (Z)	2 jets	Yes	20.3	$\tilde{g}$ 850 GeV	m(NLSP)>430 GeV	1503.03290
	Gravitino LSP	0	mono-jet	Yes	20.3	$F^{1/2}$ scale 865 GeV	$m(\tilde{G})>1.8\times10^{-4}$ eV, $m(\tilde{g})=m(\tilde{q})$=1.5 TeV	1502.01518
3rd gen. $\tilde{g}$ med.	$\tilde{g}\tilde{g}$, $\tilde{g}\to b\bar{b}\tilde{\chi}_1^0$	0	3 b	Yes	20.1	$\tilde{g}$ 1.25 TeV	$m(\tilde{\chi}_1^0)$<400 GeV	1407.0600
	$\tilde{g}\tilde{g}$, $\tilde{g}\to t\bar{t}\tilde{\chi}_1^0$	0	7-10 jets	Yes	20.3	$\tilde{g}$ 1.1 TeV	$m(\tilde{\chi}_1^0)$ <350 GeV	1308.1841
	$\tilde{g}\tilde{g}$, $\tilde{g}\to t\bar{t}\tilde{\chi}_1^0$	0-1 e,μ	3 b	Yes	20.1	$\tilde{g}$ 1.34 TeV	$m(\tilde{\chi}_1^0)$<400 GeV	1407.0600
	$\tilde{g}\tilde{g}$, $\tilde{g}\to b\bar{t}\tilde{\chi}_1^+$	0-1 e,μ	3 b	Yes	20.1	$\tilde{g}$ 1.3 TeV	$m(\tilde{\chi}_1^0)$<300 GeV	1407.0600
3rd gen. squarks direct production	$\tilde{b}_1\tilde{b}_1$, $\tilde{b}_1\to b\tilde{\chi}_1^0$	0	2 b	Yes	20.1	$\tilde{b}_1$ 100-620 GeV	$m(\tilde{\chi}_1^0)$<90 GeV	1308.2631
	$\tilde{b}_1\tilde{b}_1$, $\tilde{b}_1\to t\tilde{\chi}_1^{\pm}$	2 e,μ (SS)	0-3 b	Yes	20.3	$\tilde{b}_1$ 275-440 GeV	$m(\tilde{\chi}_1^{\pm})$=2 $m(\tilde{\chi}_1^0)$	1404.2500
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to b\tilde{\chi}_1^{\pm}$	1-2 e,μ	1-2 b	Yes	4.7/20.3	$\tilde{t}_1$ 110-167 GeV; 230-460 GeV	$m(\tilde{\chi}_1^{\pm})$ = 2$m(\tilde{\chi}_1^0)$, $m(\tilde{\chi}_1^0)$=55 GeV	1209.2102, 1407.0583
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to Wb\tilde{\chi}_1^0$ or $t\tilde{\chi}_1^0$	0-2 e,μ	0-2 jets/1-2 b	Yes	20.3	$\tilde{t}_1$ 90-191 GeV; 210-700 GeV	$m(\tilde{\chi}_1^0)$=1 GeV	1506.08616
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to c\tilde{\chi}_1^0$	0	mono-jet/c-tag	Yes	20.3	$\tilde{t}_1$ 90-240 GeV	$m(\tilde{t}_1)-m(\tilde{\chi}_1^0)$<85 GeV	1407.0608
	$\tilde{t}_1\tilde{t}_1$(natural GMSB)	2 e,μ (Z)	1 b	Yes	20.3	$\tilde{t}_1$ 150-580 GeV	$m(\tilde{\chi}_1^0)$>150 GeV	1403.5222
	$\tilde{t}_2\tilde{t}_2$, $\tilde{t}_2\to\tilde{t}_1+Z$	3 e,μ (Z)	1 b	Yes	20.3	$\tilde{t}_2$ 290-600 GeV	$m(\tilde{\chi}_1^0)$<200 GeV	1403.5222
EW direct	$\tilde{\ell}_{L,R}\tilde{\ell}_{L,R}$, $\tilde{\ell}\to\ell\tilde{\chi}_1^0$	2 e,μ	0	Yes	20.3	$\tilde{\ell}$ 90-325 GeV	$m(\tilde{\chi}_1^0)$=0 GeV	1403.5294
	$\tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^+\to\tilde{\ell}\nu(\ell\tilde{\nu})$	2 e,μ	0	Yes	20.3	$\tilde{\chi}_1^{\pm}$ 140-465 GeV	$m(\tilde{\chi}_1^0)$=0 GeV, $m(\tilde{\ell},\tilde{\nu})=0.5(m(\tilde{\chi}_1^{\pm})+m(\tilde{\chi}_1^0))$	1403.5294
	$\tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^+\to\tilde{\tau}\nu(\tau\tilde{\nu})$	2 τ	-	Yes	20.3	$\tilde{\chi}_1^{\pm}$ 100-350 GeV	$m(\tilde{\chi}_1^0)$=0 GeV, $m(\tilde{\tau},\tilde{\nu})=0.5(m(\tilde{\chi}_1^{\pm})+m(\tilde{\chi}_1^0))$	1407.0350
	$\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0\to\tilde{\ell}_L\nu\tilde{\ell}_L\ell(\tilde{\nu}\nu)$, $\ell\tilde{\nu}\tilde{\ell}_L\ell(\tilde{\nu}\nu)$	3 e,μ	0	Yes	20.3	$\tilde{\chi}_1^{\pm},\tilde{\chi}_2^0$ 700 GeV	$m(\tilde{\chi}_1^{\pm})=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)$=0, $m(\tilde{\ell},\tilde{\nu})=0.5(m(\tilde{\chi}_1^{\pm})+m(\tilde{\chi}_1^0))$	1402.7029
	$\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0\to W\tilde{\chi}_1^0Z\tilde{\chi}_1^0$	2-3 e,μ	0-2 jets	Yes	20.3	$\tilde{\chi}_1^{\pm},\tilde{\chi}_2^0$ 420 GeV	$m(\tilde{\chi}_1^{\pm})=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)$=0, sleptons decoupled	1403.5294, 1402.7029
	$\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0\to W\tilde{\chi}_1^0h\tilde{\chi}_1^0$, $h\to b\bar{b}/WW/\tau\tau/\gamma\gamma$	e,μ,γ	0-2 b	Yes	20.3	$\tilde{\chi}_1^{\pm},\tilde{\chi}_2^0$ 250 GeV	$m(\tilde{\chi}_1^{\pm})=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)$=0, sleptons decoupled	1501.07110
	$\tilde{\chi}_2^0\tilde{\chi}_3^0$, $\tilde{\chi}_{2,3}^0\to\tilde{\ell}_R\ell$	4 e,μ	0	Yes	20.3	$\tilde{\chi}_{2,3}^0$ 620 GeV	$m(\tilde{\chi}_2^0)=m(\tilde{\chi}_3^0)$, $m(\tilde{\chi}_1^0)$=0, $m(\tilde{\ell},\tilde{\nu})=0.5(m(\tilde{\chi}_2^0)+m(\tilde{\chi}_1^0))$	1405.5086
	GGM (wino NLSP) weak prod.	1 $e,\mu+\gamma$	-	Yes	20.3	$\tilde{W}$ 124-361 GeV	$c\tau$<1 mm	1507.05493
Long-lived particles	Direct $\tilde{\chi}_1^+\tilde{\chi}_1^-$ prod., long-lived $\tilde{\chi}_1^{\pm}$	Disapp. trk	1 jet	Yes	20.3	$\tilde{\chi}_1^{\pm}$ 270 GeV	$m(\tilde{\chi}_1^{\pm})-m(\tilde{\chi}_1^0)$~160 MeV, $\tau(\tilde{\chi}_1^{\pm})$=0.2 ns	1310.3675
	Direct $\tilde{\chi}_1^+\tilde{\chi}_1^-$ prod., long-lived $\tilde{\chi}_1^{\pm}$	dE/dx trk	-	Yes	18.4	$\tilde{\chi}_1^{\pm}$ 482 GeV	$m(\tilde{\chi}_1^{\pm})-m(\tilde{\chi}_1^0)$~160 MeV, $\tau(\tilde{\chi}_1^{\pm})$<15 ns	1506.05332
	Stable, stopped $\tilde{g}$ R-hadron	0	1-5 jets	Yes	27.9	$\tilde{g}$ 832 GeV	$m(\tilde{\chi}_1^0)$=100 GeV, 10 μs<$\tau(\tilde{g})$<1000 s	1310.6584
	Stable $\tilde{g}$ R-hadron	trk	-	-	19.1	$\tilde{g}$ 1.27 TeV		1411.6795
	GMSB, stable $\tilde{\tau}$, $\tilde{\chi}_1^0\to\tilde{\tau}(\tilde{e},\tilde{\mu})+\tau(e,\mu)$	1-2 μ	-	-	19.1	$\tilde{\chi}_1^0$ 537 GeV	10<tanβ<50	1411.6795
	GMSB, $\tilde{\chi}_1^0\to\gamma\tilde{G}$, long-lived $\tilde{\chi}_1^0$	2 γ	-	Yes	20.3	$\tilde{\chi}_1^0$ 435 GeV	2<$\tau(\tilde{\chi}_1^0)$<3 ns, SPS8 model	1409.5542
	$\tilde{g}\tilde{g}$, $\tilde{\chi}_1^0\to ee\nu/e\mu\nu/\mu\mu\nu$	displ. $ee/e\mu/\mu\mu$	-	-	20.3	$\tilde{\chi}_1^0$ 1.0 TeV	7 <$c\tau(\tilde{\chi}_1^0)$< 740 mm, $m(\tilde{g})$=1.3 TeV	1504.05162
	GGM $\tilde{g}\tilde{g}$, $\tilde{\chi}_1^0\to Z\tilde{G}$	displ. vtx + jets	-	-	20.3	$\tilde{\chi}_1^0$ 1.0 TeV	6 <$c\tau(\tilde{\chi}_1^0)$< 480 mm, $m(\tilde{g})$=1.1 TeV	1504.05162
RPV	LFV $pp\to\tilde{\nu}_\tau+X$, $\tilde{\nu}_\tau\to e\mu/e\tau/\mu\tau$	$e\mu,e\tau,\mu\tau$	-	-	20.3	$\tilde{\nu}_\tau$ 1.7 TeV	λ'_{311}=0.11, $\lambda_{132/133/233}$=0.07	1503.04430
	Bilinear RPV CMSSM	2 e,μ (SS)	0-3 b	Yes	20.3	$\tilde{q},\tilde{g}$ 1.35 TeV	$m(\tilde{q})=m(\tilde{g})$, $c\tau_{LSP}$<1 mm	1404.2500
	$\tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^+\to W\tilde{\chi}_1^0$, $\tilde{\chi}_1^0\to ee\tilde{\nu}_\mu, e\mu\tilde{\nu}_e$	4 e,μ	-	Yes	20.3	$\tilde{\chi}_1^{\pm}$ 750 GeV	$m(\tilde{\chi}_1^0)>0.2\times m(\tilde{\chi}_1^{\pm})$, $\lambda_{121}\neq0$	1405.5086
	$\tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^+\to W\tilde{\chi}_1^0$, $\tilde{\chi}_1^0\to\tau\tau\tilde{\nu}_e, e\tau\tilde{\nu}_\tau$	3 $e,\mu+\tau$	-	Yes	20.3	$\tilde{\chi}_1^{\pm}$ 450 GeV	$m(\tilde{\chi}_1^0)>0.2\times m(\tilde{\chi}_1^{\pm})$, $\lambda_{133}\neq0$	1405.5086
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qqq$	0	6-7 jets	-	20.3	$\tilde{g}$ 917 GeV	BR(t)=BR(b)=BR(c)=0%	1502.05686
	$\tilde{g}\tilde{g}$, $\tilde{g}\to q\tilde{\chi}_1^0$, $\tilde{\chi}_1^0\to qqq$	0	6-7 jets	-	20.3	$\tilde{g}$ 870 GeV	$m(\tilde{\chi}_1^0)$=600 GeV	1502.05686
	$\tilde{g}\tilde{g}$, $\tilde{g}\to\tilde{t}_1t$, $\tilde{t}_1\to bs$	2 e,μ (SS)	0-3 b	Yes	20.3	$\tilde{g}$ 850 GeV		1404.250
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to bs$	0	2 jets + 2 b	-	20.3	$\tilde{t}_1$ 100-308 GeV		ATLAS-CONF-2015-026
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to b\ell$	2 e,μ	2 b	-	20.3	$\tilde{t}_1$ 0.4-1.0 TeV	BR($\tilde{t}_1\to be/\mu$)>20%	ATLAS-CONF-2015-015
Other	Scalar charm, $\tilde{c}\to c\tilde{\chi}_1^0$	0	2 c	Yes	20.3	$\tilde{c}$ 490 GeV	$m(\tilde{\chi}_1^0)$<200 GeV	1501.01325

10^{-1} 1 **Mass scale [TeV]**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