Matrix model description of gauge theory (and gravity?)

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Quantum Structure of Spacetime

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Motivation

- aim: quantum theory of fundamental interactions incl. gravity
- classical geometry breaks down at Planck scale,
  expect quantum structure of space-time
  how?
- $\begin{cases} \text{quantize gravity d.o.f.} \\ \text{quantize other, pre-geometric dof: “emergent gravity”} \end{cases}$

  (this talk)

emergent NC geometry

- need models with $\left\{ \begin{array}{l} \text{admit dynamical geometry} \\ \text{well-behaved under quantization} \\ \approx \text{QFT on quantum geometries} \end{array} \right.$
- $$\overbrace{\text{NC gauge theory} \qquad \text{dynamical geometry}}^{\text{Matrix Models}}$$

  simple, far-reaching, pre-geometric

  good properties of string theory, clear-cut definition
- generic feature: UV/IR mixing

  $\rightarrow$ ONE model singled out: $\mathcal{N} = 4$ SYM $\equiv$ IKKT model

Matrix Models as fundamental theory

1996: BFSS model, IKTT model proposed as
non-perturbative definition of M-theory / IIB string theory
focus on IKKT: Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[X,\Psi] = -Tr\left([X^a,X^b][X^{a'},X^{b'}]\eta_{aa'}\eta_{bb'} + \bar{\Psi}\gamma_a[X^a,\Psi]\right)$$

$$X^a = X^{a\dagger} \in Mat(N,\mathbb{C}), \qquad a = 0,...,9 \qquad (N\to\infty)$$

gauge symmetry $X^a \to UX^aU^{-1}$, $SO(9,1)$, SUSY

1) nonpert. def. of IIB string theory (on $\mathbb{R}^{10}$) $(IKKT)$
2) $\mathcal{N}=4$ SUSY Yang-Mills gauge thy. on *"noncommutative"* $\mathbb{R}^4_\theta$

governs simultaneously (quantum) space(time) & physics on it
geometry, gauge theory, ... emerge
no need to invent new math!

quantization of matrix model:

$$Z = \int dX^a d\Psi \, e^{-S[X,\Psi]}$$

$$\langle Tr([X,X]...) Tr(...) \rangle = \frac{1}{Z} \int dX^a d\Psi \, Tr([X,X]...) Tr(...) e^{-S[X,\Psi]}$$

gauge invariant, non-perturbative

- integral well-def in Euclidean case Krauth, Staudacher 1998
- proposal for regularization in Minkowski case
  $\rightarrow$ "Monte-Carlo" studies: Kim, Nishimura, Tsuchiya arXiv:1108.1540 ff
  evidence for "expanding universe" behavior, 3+1 dimensions
- includes integral over geometries!
- new techniques:
  eigenvalue distribution $\leftrightarrow$ renormalization, phase trans.
  H.S. hep-th/0501174, A. Polychronakos arXiv:1306.6645, Tekel arXiv:1407.4061
  RG analysis (Grosse-Wulkenhaar), multiscale analysis (Rivasseau, ...)

perturbative approach:

- choose background solution (e.g. $\mathbb{R}^4_\theta$)
- fluctuations around $\mathbb{R}^4_\theta$:
  $\to$ NC gauge theory, Filk rules, (non-)planar diagrams, ...
- most models: strong UV/IR mixing, non-renormaliz.
- ONE model well-behaved (perturbatively finite ?!):
  $\mathcal{N} = 4$ NC SYM on $\mathbb{R}^4_\theta$ $\Leftrightarrow$ (IKKT) model, in 9+1 dimensions

physical meaning of X^a: quantized embedding function

$$X^a \sim x^a: \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$$

consistent with:

- spectrum of X^a ... possible locations in x^a - directions
  $[X^a, X^b] \neq 0 \;\Rightarrow$ non-locality, uncertainty
- $\langle X^a \rangle$ for optimally localized states $\cong$ coherent states

quantized Poisson (symplectic) manifolds

$(\mathcal{M}, \theta^{\mu\nu}(x))$ ... $2n$-dimensional manifold with Poisson structure

Its quantization $\mathcal{M}_\theta$ is NC algebra such that

$$\mathcal{Q}: \ \mathcal{C}(\mathcal{M}) \quad \to \quad \mathcal{A} \subset End(\mathcal{H})$$

such that

$$\begin{aligned} \mathcal{Q}(f)\,\mathcal{Q}(g) &= \mathcal{Q}(fg) \ + O(\theta) \\ [\mathcal{Q}(f), \mathcal{Q}(g)] &= \mathcal{Q}(i\{f,g\}) \ + O(\theta^2) \end{aligned}$$

$\Phi = \mathcal{Q}(\phi) \in End(\mathcal{H}) \quad \sim \quad$ quantized function $\phi(x)$ on $\mathcal{M}$

semi-class:

$$(2\pi)^n \mathrm{Tr}\, \mathcal{Q}(\phi) \quad \sim \quad \int \omega^n \, \phi(x)$$

in particular:

$$X^a \ \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$$

Example: the fuzzy sphere S^2_N

classical S^2 :

$$\left.\begin{array}{rcl} x^a : S^2 & \hookrightarrow & \mathbb{R}^3 \\ x^a x^a & = & 1 \end{array}\right\} \Rightarrow \mathcal{A} = \mathcal{C}^\infty(S^2)$$

fuzzy sphere S^2_N : (Hoppe, Madore)

algebra $\mathcal{A} = \mathrm{Mat}(N, \mathbb{C})$ ... alg. of functions on S^2_N

$SO(3)$ action:

$$\begin{array}{rcl} \mathfrak{su}(2) \times \mathcal{A} & \rightarrow & \mathcal{A} \\ (J^a, \phi) & \mapsto & [\pi_N(J^a), \phi] \end{array}$$

decompose $\mathcal{A} = \mathrm{Mat}(N, \mathbb{C})$ into irreps of $SO(3)$:

$$\begin{array}{rcl} \mathcal{A} = \mathrm{Mat}(N, \mathbb{C}) \cong (N) \otimes (\bar{N}) & = & (1) \oplus (3) \oplus \ldots \oplus (2N-1) \\ & = & \{\hat{Y}^0_0\} \oplus \{\hat{Y}^1_m\} \oplus \ldots \oplus \{\hat{Y}^{N-1}_m\}. \end{array}$$

... fuzzy spherical harmonics; UV cutoff

$$X^a = \pi_N(J^a), \qquad X^a X^a = R^2 \mathbf{1}$$

basic solutions of M.M: branes

e.o.m.: $\delta S = 0 \quad \Leftrightarrow$

$$\Box_X X^b \equiv [X_a, [X^a, X^b]] = 0$$

(assume $\Psi = 0$)

basic solutions: (allow $N \to \infty$)

- flat "branes" $\mathbb{R}^{2n}_\theta$ embedded in $\mathbb{R}^{10}$

$$X^a = \begin{pmatrix} X^\mu \\ c^i \mathbf{1} \end{pmatrix} , \qquad \mu = 1, ..., 2n$$

$$[X^\mu, X^\nu] = i\theta^{\mu\nu}\,\mathbf{1}$$

"Moyal-Weyl quantum plane"

... quantized symplectic space $(\mathbb{R}^{2n}, \omega)$

$$\omega = \tfrac{1}{2}\theta^{-1}_{\mu\nu} dx^\mu dx^\nu$$

... Heisenberg algebra, interpreted as space of functions on $\mathbb{R}^4_\theta$

uncertainty relations $\Delta X^\mu \Delta X^\nu \geq |\theta^{\mu\nu}|$

Weyl quantization $e^{ik_\mu x^\mu} \quad \leftrightarrow \quad e^{ik_\mu X^\mu}$

embedding recovered from optimally localized states:

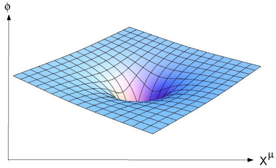
coherent states $|p\rangle$

$$\mathcal{M} = \{x^a = \langle p|X^a|p\rangle\} \quad = \mathbb{R}^{2n} \subset \mathbb{R}^{10}$$

$$\langle p|\sum(\Delta X^\mu)^2|p\rangle \quad \approx |\theta| \approx \min$$

- generic (curved) branes $\mathcal{M}^{2n}$ $=$ “deformed” $\mathbb{R}^{2n}_\theta$

$$X^a \sim x^a = \begin{pmatrix} x^\mu \\ \phi(x^\mu) \end{pmatrix} : \quad \mathcal{M}^{2n} \hookrightarrow \mathbb{R}^{10}$$



... quantized embedding map

$(\mathcal{M}^{2n}, \omega)$ ... quantized symplectic manifold embedded in $\mathbb{R}^{10}$

$$\omega = \frac{1}{2}\theta^{-1}_{\mu\nu}(x) dx^\mu dx^\nu$$

fluctuations

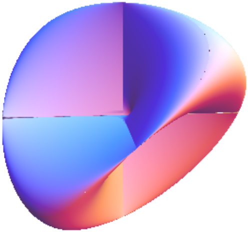
$$X^a = \bar{X}^a + \mathcal{A}^a(X)$$

describe NC gauge theory, dynamical eff. metric

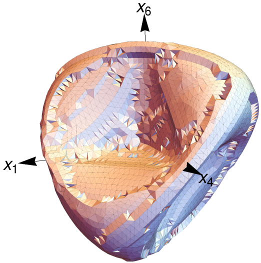
$\sim$ D-brane with B-field in string thy, open string metric less trivial examples:

- squashed fuzzy $\mathbb{C}P^2_N$ (self-intersecting brane)

$X_a = \pi_{(N,0)}(T_a) \sim x^a : \ \mathbb{C}P^2 \hookrightarrow \mathbb{R}^8 \stackrel{\Pi}{\to} \mathbb{R}^6$ ... $SU(3)$ ladder op's



H.S., J. Zahn arXiv:1409.1440



H.S, L. Schneiderbauer

quantized symplectic manifold, degenerate embedding

$\Rightarrow$ strings connecting sheets, stringy geometry

stabilized in M.M. e.g. by cubic potential

- degenerate solutions: fuzzy S^4_N

$X_a = \hat{\Gamma}_a = c^\dagger_\alpha (\Gamma_a)^\alpha_\beta c^\beta$ on $(\mathbb{C}^4)^{\otimes_S N}$ (Γ_a... $SO(5)$ Clifford)

$$X_a X_a = R^2 \mathbb{1}$$

Castellino, Lee, Taylor hep-th/9712105; Ramgoolam, ...

in fact $S^4_N = \mathbb{C}P^3_N / S^2$ Medina,O'Connor hep-th/0212170

$$[X_a, X_b] = M_{ab}, \qquad [M_{ab}, X_c] = i(\delta_{ac} X_b - \delta_{bc} X_a), \qquad \text{etc.}$$

fully covariant under $SO(5)$ (cf. Snyder space)

symplectic structure "averaged away" over fiber S^2

(cf. Doplicher Fredenhagen Roberts 1995)

fluctuations

$$X^a = \bar{X}^a + \mathcal{A}^a(X, M)$$

describe NC higher spin theory (?)

- degenerate solutions: fuzzy S^4_N

$X_a = \hat{\Gamma}_a = c^\dagger_\alpha (\Gamma_a)^\alpha_\beta c^\beta$ on $(\mathbb{C}^4)^{\otimes_S N}$ (Γ_a...$SO(5)$ Clifford)

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fluctuations

$$X^a = \bar{X}^a + \mathcal{A}^a(X, M)$$

describe NC higher spin theory (?)

- classical manifolds as solution of IKKT:

let $(x^\mu, p_\mu = \nabla_\mu)$ ... phase space

$$X^a = \begin{pmatrix} p^\mu \\ 0 \end{pmatrix}$$

... commutative $\mathbb{R}^n$ solutions:

$$[X^a, X^b] = 0$$

fluctuations

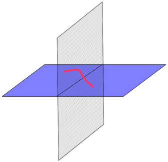
$$X^a = \bar{X}^a + \mathcal{A}^a(x,p)$$

describes some higher derivative / higher spin theory (?)

Hanada, Kawai, Kimura hep-th/0508211; ...

($\mathbb{R}^{2n}$ re-appears, unlike for NC branes!)

stacks of branes in M.M.



- assume $X^a_{(i)}$ ... solutions of e.o.m.

  $\to$ new solution: $X^a = \begin{pmatrix} X^a_{(1)} \mathbf{1}_{n_1} & 0 \\ 0 & X^a_{(2)} \mathbf{1}_{n_2} \end{pmatrix}$

  ... stacks of n_1 & n_2 coincident branes
  breaks $U(N)$ to $U(n_1) \times U(n_2)$
- fermions may connect different branes

  $$\Psi = \begin{pmatrix} 0 & \psi_{(12)} \\ \psi_{(21)} & 0 \end{pmatrix},$$

  $\psi_{(12)}$ transform in bifundamental $(n_1) \otimes (\overline{n}_2)$ $(=$ strings!)
- interaction between $\mathbb{R}^{2n}_\theta$ branes consistent with IIB SUGRA (quantum effect!) (IKKT 1997, Chepelev,Makeenko, Zarembo 1997,...)

$\to$ can get close to particle physics

Chatzistavrakidis, Zoupanos, H.S: arXiv:1107.0265; H.S.: arXiv:1504.05703 etc.

Part two: fluctuations on noncommutative branes

NC gauge theory $\leftrightarrow$ geometric fluctuations

Claim A:

fluctuations on branes $\rightarrow$ noncommutative gauge fields

Claim B:

$U(1)$ fluctuations on branes $\rightarrow$ fluctuations of geometry, “gravity”

- both claims are correct
- 2^{nd} interpretation more useful, explains UV/IR mixing in M.M.
- consistent with string theory
- physical relevance not yet clear

claim A:

fluctuations on a stack of n coincident $\mathbb{R}^4_\theta$ branes in IKKT
$\rightarrow$ noncommutative $U(n)$ $\mathcal{N}=4$ super-Yang-Mills on $\mathbb{R}^4_\theta$

(Aoki, Ishibashi,Iso,Kawai,Kitazawa, Tada 1999)

sketch:

- background solution: stack of n coinciding $\mathbb{R}^4_\theta$ branes

$$X^a = \begin{pmatrix} X^\mu \\ \phi^i \end{pmatrix} = \begin{pmatrix} \bar{X}^\mu \otimes \mathbb{1}_n \\ 0 \end{pmatrix}, \qquad \begin{array}{l} \mu = 0, ..., 3 \\ i = 4, 5, .., 9 \end{array}$$

$[\bar{X}^\mu, \bar{X}^\nu] = i\theta^{\mu\nu}$ ... Heisenberg algebra, generate $\mathcal{A}_\theta \approx End(\mathcal{H})$

- add fluctuations:

$$X^a = \begin{pmatrix} \bar{X}^\mu \otimes \mathbb{1}_n + \theta^{\mu\nu} A_\nu \\ \phi^i \end{pmatrix} \qquad \in \mathcal{A}_\theta \otimes Mat(n, \mathbb{C})$$

$$A_\mu = A_\mu(\bar{X}) = A_{\mu,\alpha}(\bar{X})\lambda_\alpha \ \in End(\mathcal{H}^n) \cong \mathcal{A}_\theta \otimes Mat(n, \mathbb{C})$$

define derivatives as inner derivations:

$$[\bar{X}^\mu, \phi(X)] =: i\theta^{\mu\nu}\partial_\nu\phi(X), \qquad [\partial_\mu, \partial_\nu] = 0$$

thus

$$\begin{aligned}
[X^\mu, \phi(X)] &= i\theta^{\mu\nu} D_\nu \phi(X), \qquad D_\mu = \partial_\mu + i[A_\mu, .] \\
[X^\mu, X^\nu] &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'}\,(\partial_{\mu'}A_{\nu'} - \partial_{\nu'}A_{\mu'} + [A_{\mu'}, A_{\nu'}]) \\
&= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'}\,F_{\mu'\nu'}
\end{aligned}$$

$F_{\mu'\nu'}$ ... Yang-Mills field strength

$S = Tr([X^a, X^b][X_a, X_b])$ is gauge-invariant: $X^a \to U^{-1}X^a U$

$\to$ tangential fluctuations $X^\mu = \bar{X}^\mu + \theta^{\mu\nu}A_\nu$ transform as
$A_\mu \to U^{-1}A_\mu U + iU^{-1}\partial_\mu U$ ...$\mathfrak{u}(n)$ gauge fields!

$\to$ transversal fluctuations $\phi^i \to U^{-1}\phi^i U$ ...$\mathfrak{u}(n)$ scalar fields!

insert in IKKT action:

$$
\begin{aligned}
S &= \Lambda_0^4\, Tr\Big([X^a, X^b][X_a, X_b] + \bar\Psi\Gamma_a[X^a, \Psi]\Big) \\
&= \int d^4x\, \sqrt{G}\, tr_n\Big(\tfrac{1}{4g^2}(\mathcal{F}\mathcal{F})_G + \tfrac{1}{2}G^{\mu\nu}D_\mu\Phi^i D_\nu\Phi_i - \tfrac{1}{4}g^2[\Phi^i, \Phi^j][\Phi_i, \Phi_j] \\
&\qquad + \bar\psi\tilde\gamma^\mu(i\partial_\mu + [\mathcal{A}_\mu, .])\psi + g\bar\psi\Gamma^i[\Phi_i, \psi]\Big) + \int \rho\theta^{ab}\theta_{ab}
\end{aligned}
$$

where

$$
\begin{aligned}
G^{\mu\nu} &= \rho\theta^{\mu\nu'}\theta^{\nu\nu'}\eta_{\mu'\nu'}, \qquad \rho = \sqrt{|\theta^{-1}|} \\
\tilde\gamma^\mu &= \rho^{1/2}\,\theta^{\nu\mu}\gamma_\nu, \\
\frac{1}{4g^2} &= \frac{\Lambda_0^4}{(2\pi)^2}\rho^{-1}
\end{aligned}
$$

IKKT on stack of n branes $\rightarrow$ $U(n)$ $\mathcal{N} = 4$ SYM coupled to $G^{\mu\nu}$

(cf. large N reduction !)

very simple & compelling origin of gauge theory
however, misleading for $U(1)$ sector:

- deformations of branes are obviously geometrical d.o.f.
- cannot disentangle $U(1)$ from $SU(n)$

because $U(1)$ is gravity sector!

- UV/IR mixing $\rightarrow$ different physics

claim B:

fluctuations on a stack of n coincident $\mathbb{R}^4_\theta$ branes in IKKT
$U(1)_{\rm tr} \to$ dynamical $G^{\mu\nu}(x)$, $SU(n)$ SYM coupled to $G^{\mu\nu}(x)$

H.S., JHEP 0712:049 (2007)
(review: JHEP 0902:044,(2009), Class.Quant.Grav. 27 (2010) 133001)

analogous for finite matrix geometries, $\mathcal{A} = Mat(N, \mathbb{C})$

explains UV/IR mixing (quantitatively!)

metric structure on branes:

fluctuations governed by matrix Laplacian

$$S[\varphi] = -Tr\,[X^a,\varphi][X^b,\varphi]\,\eta_{ab} = Tr\,\varphi\Box\varphi$$

$$\Box\varphi \equiv \eta_{ab}[X^a,[X^b,\varphi]]$$

encodes metric!

e.g. on S^2_N:

$$\Box\phi = \frac{1}{C_N}\,J^aJ^a\phi$$

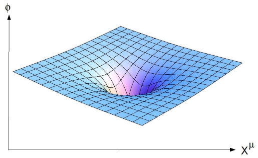
$SO(3)$ invariant $\Rightarrow$ $\boxed{\Box\,\hat{Y}^l_m = \frac{1}{C_N}\,l(l+1)\,\hat{Y}^l_m}$

spectrum identical with classical case $\Delta_g\phi = \frac{1}{\sqrt{|g|}}\partial_\mu(\sqrt{|g|}g^{\mu\nu}\partial_\nu\phi)$

$\Rightarrow$ effective metric = round metric on S^2

geometry of generic NC branes:

$X^a \sim x^a: \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$



Lemma: assume $\dim \mathcal{M} > 2$. Then

$$\Box f(X) \sim -\eta_{ab}\{x^a, \{x^b, f(x)\}\} = -e^\sigma \Box_G f(x)$$

... Matrix Laplace- operator, effective metric

$$G^{\mu\nu}(x) = e^{-\sigma}\theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\, g_{\mu'\nu'}(x) \quad \text{effective metric (cf. open string m.)}$$

$$g_{\mu\nu}(x) = \partial_\mu x^a \partial_\nu x^b \eta_{ab} \quad \text{induced metric on } \mathcal{M}^4_\theta \text{ (cf. closed string m.)}$$

$$e^{-2\sigma} = \frac{|\theta^{-1}_{\mu\nu}|}{|g_{\mu\nu}|}$$

(H.S. Nucl.Phys. B810 (2009))

follows by coupling to scalar field φ:

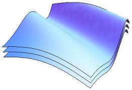
$$S[\varphi] = Tr\,[X^a, \varphi][X^b, \varphi]\, g_{ab}$$

$$\sim \int d^{2n}x \sqrt{|G|}\, G^{\mu\nu}(x)\, \partial_\mu\varphi\partial_\nu\varphi = \int d\varphi \wedge \star_G d\varphi$$

stack of coincident curved branes $\to$ $\mathfrak{su}(n)$ gauge thy

generic background branes

$$X^a = \begin{pmatrix} \bar{X}^\mu \otimes \mathbf{1}_n \\ \bar{\phi}^i \otimes \mathbf{1}_n \end{pmatrix}$$



general CR $[\bar{X}^\mu, \bar{X}^\nu] = i\theta^{\mu\nu}(\bar{X})$

fluctuations:

$$X^a = \begin{pmatrix} \bar{X}^\mu \otimes \mathbf{1}_N + \mathcal{A}^\mu \\ \bar{\phi}^i \otimes \mathbf{1}_N + \Phi^i \end{pmatrix}$$

$\mathcal{A}^\mu, \Phi^i \sim \mathbf{1}_n$ d.o.f. change background $\bar{X}^a$, geometrical d.o.f. $\theta^{\mu\nu}, g_{\mu\nu}$

write $\mathcal{A}^\mu = \theta^{\mu\nu} A_\nu$, note $[\bar{X}^\mu, f] \sim i\theta^{\mu\nu}\partial_\nu f$

$$\begin{aligned} [X^\mu, X^\nu] &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'}(\partial_{\mu'}A_{\nu'} - \partial_{\nu'}A_{\mu'} + [A_{\mu'}, A_{\nu'}]) \\ &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'} F_{\mu'\nu'} \qquad \text{field strength} \end{aligned}$$

$\Rightarrow$ effective action on $\mathcal{M}^4_\theta$ (semi-classical):

$$S = \Lambda_0^4\, Tr\Big([X^a, X^b][X_a, X_b] + \bar\Psi\Gamma_a[X^a,\Psi]\Big)$$
$$\sim \int d^4x\,\sqrt{G}\, tr_n\Big(\tfrac{1}{4g^2}(\mathcal{F}\mathcal{F})_G + \tfrac{1}{2}(D\Phi^i D\Phi_i)_G - \tfrac{1}{4}g^2[\Phi^i,\Phi^j][\Phi_i,\Phi_j]$$
$$+\bar\psi\tilde\gamma^\mu(i\partial_\mu + [\mathcal{A}_\mu, .])\psi + g\bar\psi\Gamma^i[\Phi_i,\psi]\Big) + \int 2\eta(\theta\wedge\theta + tr_n F\wedge F)$$

where

$$G^{\mu\nu}(x) = \rho\theta^{\mu\nu'}(x)\theta^{\nu\nu'}(x)g_{\mu'\nu'}(x), \qquad \rho = \sqrt{|\theta^{-1}|}$$
$$\tilde\gamma^\mu(x) = \rho^{1/2}\,\theta^{\nu\mu}(x)\gamma_\nu, \qquad \eta = Gg$$
$$\frac{1}{4g^2} = \frac{\Lambda_0^4}{(2\pi)^2}\rho^{-1}$$

IKKT on stack of branes $\rightarrow$ $SU(n)$ $\mathcal{N}=4$ SYM coupled to $G^{\mu\nu}$

dynamical $G^{\mu\nu}(x)$! ($\rightarrow$ gravity ?!)

H.S., JHEP 0712:049 (2007), JHEP 0902:044,(2009), Class.Quant.Grav. 27 (2010)

fermions

Ψ ... $\mathcal{A}$ - valued Majorana-Weyl spinor of $SO(9,1)$

$$
\begin{aligned}
S[\Psi] &= \mathrm{Tr}\, \overline{\Psi}\Gamma_a[X^a, \Psi] \equiv \mathrm{Tr}\, \overline{\Psi} \not{D} \Psi \\
&\sim \int d^4x \sqrt{\theta^{-1}}\, \overline{\Psi} i \tilde{\gamma}^\mu (\partial_\mu + [A_\mu, .])\Psi,
\end{aligned}
$$

with

$$
\begin{aligned}
\tilde{\gamma}^\mu &= \rho^{1/2}\, \Gamma_a \theta^{\nu\mu} \partial_\nu x^a \\
\{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} &= 2\, G^{\mu\nu}(x)
\end{aligned}
$$

Ψ decomposes into 4 Weyl fermions $\rightarrow$ $\mathcal{N} = 4$ SYM result:

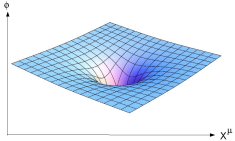
- trace-$U(1)$ sector defines geometry $\mathcal{M}^{2n} \subset \mathbb{R}^{10}$
- $SU(n)$ fluctuations of matrices X^a, Ψ

  $\to$ gauge fields, scalar fields, fermions on $\mathcal{M}^{2n}$ (NOT 10 dim!)

all fields couple to metric $G^{\mu\nu}(x)$
determined by $\theta^{\mu\nu}(x)$, embedding
dynamical $\Rightarrow$ ("emergent") gravity

matrix e.o.m $[X^a, [X^{a'}, X^b]]\eta_{aa'} = 0 \quad \Longleftrightarrow$

$$\Box_G x^a = 0, \qquad \text{"minimal surface"}$$
$$\nabla^\mu (e^\sigma \theta^{-1}_{\mu\nu}) = e^{-\sigma}\, G_{\rho\nu}\theta^{\rho\mu}\partial_\mu \eta$$
$$\eta \sim G^{\mu\nu} g_{\mu\nu}$$



covariant formulation in semi-classical limit (H.S. Nucl.Phys. B810 (2009))

$\Rightarrow$ 2 interpretations for quantization:

$$Z = \int dX^a d\Psi\, e^{-S[X]-S[\Psi]}$$

1. on $\mathbb{R}^4_\theta$: $X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu$, $\bar{X}^\mu$...Moyal-Weyl

   $\rightarrow$ NC gauge theory on $\mathbb{R}^4_\theta$, UV/IR mixing in $U(1)$ sector

   IKKT model: $\mathcal{N} = 4$ SYM, perturb. finite !(?)

2. on $\mathcal{M}^4 \subset \mathbb{R}^{10}$: $U(1)$ absorbed in $\theta^{\mu\nu}(x)$, $g_{\mu\nu}$

   $\rightarrow$ quantized gravity, induced E-H. action

$$S_{eff} \sim \int d^4x \sqrt{|G|}\, (\Lambda^4 + c\Lambda_4^2\, R[G] + ...)$$

- explanation for UV/IR mixing & $U(1)$ entanglement
- good quantization for theory with dynamical geometry

$\Rightarrow$ 2 interpretations for quantization:

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1. on $\mathbb{R}^4_\theta$: $X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu,$ $\bar{X}^\mu$...Moyal-Weyl

   $\rightarrow$ NC gauge theory on $\mathbb{R}^4_\theta$, UV/IR mixing in $U(1)$ sector

   IKKT model: $\mathcal{N} = 4$ SYM, perturb. finite !(?)

2. on $\mathcal{M}^4 \subset \mathbb{R}^{10}$: $U(1)$ absorbed in $\theta^{\mu\nu}(x)$, $g_{\mu\nu}$

   $\rightarrow$ quantized gravity, induced E-H. action

$$S_{eff} \sim \int d^4x \sqrt{|G|}\, (\Lambda^4 + c\Lambda_4^2 R[G] + \ldots)$$

- explanation for UV/IR mixing & $U(1)$ entanglement
- good quantization for theory with dynamical geometry

semi-classical limit of UV/IR mixing:

interaction of two scalar field components

$$S_{int} \ni Tr([\phi_1,\phi_2][\phi_1,\phi_2]) = 2\,Tr(\phi_1\phi_2\phi_1\phi_2 - \phi_1^2\phi_2^2)$$

integrate out $A \equiv \phi_2$ $\Rightarrow$ eff. action for $\phi \equiv \phi_1$

phase factors for non-planar diagrams, $e^{ikX}e^{ilX} = e^{ik\theta l}e^{ilX}e^{ikX}$

(planar – non-planar diagram) $\sim \Lambda^2\left(1 - \frac{1}{1-\frac{p^2\Lambda^2}{\Lambda_{NC}^4}}\right)$

usual treatment: high UV cutoff $\Lambda \gg \Lambda_{NC}$

$\Rightarrow$ IR divergence $\sim \frac{1}{p^2}$, accumulates

<u>different limit:</u> low UV cutoff $\frac{p^2\Lambda^2}{\Lambda_{NC}^4} \ll 1$ (max. SUSY !)

$$\Lambda^2\left(1 - \frac{1}{1-\frac{p^2\Lambda^2}{\Lambda_{NC}^4}}\right) = \frac{p^2\Lambda^4}{\Lambda_{NC}^4} + O(p^4\Lambda^6)$$

phase factors $[e^{ikX}, e^{ilX}] = 2i\sin(\frac{k\theta l}{2})e^{i(l+k)X}$ can be understood semi-classically:

$$[\phi, A] \sim \{\phi, A\} = \theta^{\mu\nu}\partial_\mu\phi\partial_\nu A$$

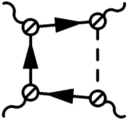
integrate out A

$$\langle[\phi, A][\phi, A]\rangle_A \sim \theta^{\mu\mu'}\theta^{\nu\nu'}\underbrace{(\partial_\mu \overbrace{A\partial_\nu A})}_{\sim\Lambda^4 G_{\mu\nu}}\partial_{\mu'}\phi\partial_{\nu'}\phi$$

$\Rightarrow$ 1-loop correction to kinetic term (metric!) of ϕ:

$$\delta S_{kin}[\phi] \ni \langle[A, \phi][A, \phi]\rangle_A \sim \Lambda^4\theta^{\mu\mu'}\theta^{\nu\nu'}G_{\mu'\nu'}\partial_\mu\phi\partial_\nu\phi \sim \Lambda^4 g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

careful treatment: 1-loop eff. action due to fermion loops:
(all terms of dim ≤ 6):



$$\begin{aligned}\Gamma_{\rm eff} &= \frac{\Lambda^4}{\Lambda_{\rm NC}^4}\int\frac{d^4x}{(2\pi)^2}\Big(g^{\alpha\beta}D_\alpha\varphi^i D_\beta\varphi_i \\ &\quad -\tfrac{1}{2}\Lambda_{\rm NC}^4\big(\bar\theta^{\mu\nu}F_{\nu\mu}\bar\theta^{\rho\sigma}F_{\sigma\rho}+(\bar\theta^{\sigma\sigma'}F_{\sigma\sigma'})(F\bar\theta F\bar\theta)\big) \\ &\quad -2\bar\theta^{\nu\mu}F_{\mu\alpha}g^{\alpha\beta}\partial_\nu\varphi^i\partial_\beta\varphi_i+\tfrac{1}{2}(\bar\theta^{\mu\nu}F_{\mu\nu})g^{\alpha\beta}\partial_\beta\varphi^i\partial_\alpha\varphi_i+\text{h.o.}\Big) \\ &\quad +\frac{\Lambda^2}{\Lambda_{\rm NC}^4}\int\frac{d^4x}{(2\pi)^2}\Big(-\tfrac{11}{2}F_{\rho\eta}\Box_g F_{\sigma\tau}\bar G^{\rho\sigma}\bar G^{\eta\tau}-12\Box_g\varphi^i\Box\varphi_i \\ &\quad +\tfrac{1}{2}\Lambda_{\rm NC}^4(\bar\theta^{\mu\nu}F_{\mu\nu})\bar\Box_G(\bar\theta^{\rho\sigma}F_{\rho\sigma}) \quad +\ldots\Big) \\ &\quad +\frac{\Lambda^6}{\Lambda_{\rm NC}^8}\int\frac{d^4x}{(2\pi)^2}(\ldots)+\ldots\end{aligned}$$

(all of this is due to UV/IR mixing, low cutoff, $U(1)$ only)

(D. Blaschke, H.S., M. Wohlgenannt JHEP 1103 (2011))

summarized in effective generalized matrix model:

re-assemble effective action: $X^a = \begin{pmatrix} \bar{X}^\mu \\ 0 \end{pmatrix} + \begin{pmatrix} -\bar{\theta}^{\mu\nu} A_\nu \\ \phi^i \end{pmatrix}$

$$\Gamma_L[X] = \mathrm{Tr} \frac{L^4}{\sqrt{\frac{1}{2}H^2 - H^{ab}H_{ab} + \frac{1}{L^2}\mathcal{L}_{10,\mathrm{curv}}[X] + \ldots}} \sim \int d^4x\, \Lambda^4(x)\sqrt{g(x)}$$

$$\mathcal{L}_{10,\mathrm{curv}}[X] = c_1[X^c, H^{ab}][X_c, H_{ab}] + c_2 H^{cd}[X_c,[X^a,X^b]][X_d,[X_a,X_b]] + \ldots$$

$$H^{ab} = [X^a, X^c][X^b, X_c] + (a \leftrightarrow b), \qquad H = H^{ab}\eta_{ab}$$

(D. Blaschke, H.S. M. Wohlgenannt arXiv:1012.4344)

$SO(D)$ manifest, broken by background (e.g. $\mathbb{R}^4_\theta$)

$\Rightarrow$ highly non-trivial predictions for (NC) gauge theory

expect generalization to nonabelian $\mathcal{N} = 4$ SYM: full $SO(9,1)$!

effective generalized matrix model
= powerful new tool for (NC) gauge theory and (emergent) gravity

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higher-order terms, curvature

$$
\begin{aligned}
H^{ab} &:= \tfrac{1}{2}[[X^a, X^c], [X^b, X_c]]_+ \\
T^{ab} &:= H^{ab} - \tfrac{1}{4}\eta^{ab} H, \qquad H := H^{ab}\eta_{ab} = [X^c, X^d][X_c, X_d]\,, \\
\Box X &:= [X^b, [X_b, X]]
\end{aligned}
$$

result:

for 4-dim. $\mathcal{M} \subset \mathbb{R}^D$ with $g_{\mu\nu} = G_{\mu\nu}$:

$$
Tr\left(2T^{ab}\Box X_a \Box X_b - T^{ab}\Box H_{ab}\right) \sim \frac{2}{(2\pi)^2}\int d^4x \sqrt{g}\, e^{2\sigma} R
$$

$$
Tr([[X^a, X^c], [X_c, X^b]][X_a, X_b] - 2\Box X^a \Box X^a)
$$

$$
\sim \frac{1}{(2\pi)^2}\int d^4x \sqrt{g}\, e^{\sigma}\left(\tfrac{1}{2}e^{-\sigma}\theta^{\mu\eta}\theta^{\rho\alpha}R_{\mu\eta\rho\alpha} - 2R + \partial^\mu\sigma\partial_\mu\sigma\right)
$$

(Blaschke, H.S. arXiv:1003.4132)

(cf. Arnlind, Hoppe, Huisken arXiv:1001.2223)

$\Rightarrow$ contains Einstein-Hilbert- type action from matrix model
pre-geometric origin, background indep.

gravity from $U(1)$ on branes ?

- + good quantum theory of geometry
- + E-H action induced
- + $\theta^{\mu\nu}$ invisible to scalar fields, gauge fields
- – $\theta^{\mu\nu}$ couples to $U(1)$ d.o.f., **Lorentz-breaking effects**
  $\Rightarrow$ $R_{abcd}\theta^{ab}\theta^{cd} \in$ 1-loop induced action (D. Klammer H.S. 2009)
- + NC gauge field $F_{\mu\nu}$ $\Rightarrow$ 2 propagating Ricci-flat dof
  (Rivelles 2003; cf. Yang 2004)
- – grav. field on $\mathbb{R}^4$ couples to **derivative** of $T_{\mu\nu}$ (class.)
- + non-deriv. coupling to $T_{\mu\nu}$ in presence of extrinsic curvature
  (H.S. 2009 ff)

maybe better: *covariant quantum spaces* e.g. S^4_N, manifest Lorentz/ Euclidean inv.

summary, conclusion

- matrix-models $Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'} \;+$ fermions

  dynamical NC branes $\leftrightarrow$ "emergent" geometry, gravity?

- fluctuations of matrices $\rightarrow$ gauge theory on brane

  all ingredients for physics

- rich solutions of IKKT model with $\mathcal{M}^4 \times \mathcal{K}$

  building blocks for intersecting branes ($\rightarrow$ standard model ?)

- need better understanding of quantum effects
  - perturbative: effective quantum matrix model action
  - new, adapted methods:
    eigenvalue distribution, localization, ...

- identify appropriate background

- ... very rich model, more to be discovered

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