Introduction to Flavourdynamics

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The Standard Model (SM) of the electroweak and strong interactions is based on the gauge-group:

$$G_{SM} = SU(3)_c \times SU(2) \times U(1)$$

# of generators:	8	3	1

G_{SM} has 12 generators and to each one of these generators corresponds a gauge field

The SM is a beautiful generalization of QED

The SM fermion Content

		Representations
Quarks	$q_{iL} \equiv \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L$	$(3, 2, 1/6)$
	u_{iR}	$(3, 1, 2/3)$
	d_{iR}	$(3\ 1, -1/3)$
Leptons	$\ell_{iL} \equiv \begin{pmatrix} \nu_i \\ e^-_i \end{pmatrix}_L$	$(1, 2, -1/2)$
	e^-_{iR}	$(1, 1, -1)$

Gauge fields:

$$SU(3)_C \longrightarrow G_\mu^k \qquad k = 1, \dots 8$$

$$SU(2)_L \longrightarrow W_\mu^j \qquad j = 1, 2, 3$$

$$U(1) \longrightarrow B_\mu$$

The electroweak interactions are mediated by linear combinations of W_μ^a, B_μ

$$W_\mu^a,\ B_\mu \longrightarrow W_\mu^+ W_\mu^-,\ Z_\mu,\ A_\mu$$

Electric Charge Operator :

$$Q = T_3 + Y$$

Input from experiment (Crucial!)

Only evidence for left-handed charged currents (Parity Violation)

$\Downarrow$

- left-handed components of fermion fields are put in doublets
- right-handed components are put in singlets

Gauge interactions are determined by the covariant derivative which is dictated by the transformation properties of the various fields under the gauge group:

$$D_\mu = \partial_\mu - i g_s L^k G_\mu^k - i g T^j W_\mu^j - i g' Y B_\mu$$

T^j are the $SU(2)$ generators,

L^k are the $SU(3)$ generators

$$T^j = \begin{cases} 0 & \text{singlet} \\ \frac{\tau^j}{2} & \text{fundamental} \end{cases} \; ; \; L^k = \begin{cases} 0 & \text{singlet} \\ \frac{\lambda_k}{2} & \text{fundamental} \end{cases}$$

τ_j, λ_k $\rightarrow$ Pauli and Gell-Mann matrices

For the fermions of the SM:

$$D_\mu q_L = \left[\partial_\mu - i\frac{g_s}{2}\lambda_k G^k_\mu - i\frac{g}{2}\tau_j W^j_\mu - i\frac{g'}{6}B_\mu\right) q_L$$

$$D_\mu u_R = \left[\partial_\mu - i\frac{g_s}{2}\lambda_k G^k_\mu - i\frac{2g'}{3}B_\mu\right] u_R$$

$$D_\mu d_R = \left[\partial_\mu - i\frac{g_s}{2}\lambda_k G^k_\mu + i\frac{g'}{3}B_\mu\right] d_R$$

$$D_\mu l_L = \left[\partial_\mu - i\frac{g}{2}\tau_j W^j_\mu + \frac{g'}{2}B_\mu\right] q_L$$

$$D_\mu e^-_R = \left(\partial_\mu + i g B_\mu\right) e^-_R$$

In order to generate masses for the gauge bosons $W_\mu^\pm$, Z_μ, without destroying renormalizability, the gauge symmetry must be spontaneously broken. Simplest possibility:

$$\phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} \sim \left[1,\ 2,\ 1/2\right]$$

complex doublet
Higgs scalar field

This leads to the breaking:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{em}$$

Most general gauge invariant, renormalisable scalar potential :

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$\lambda > 0$ so that the potential is bounded from below

For $\mu^2 = 0$, minimum at $\langle 0 | \Phi | 0 \rangle 0$

For $\mu^2 < 0$, minimum at

$$\langle 0 | \Phi | 0 \rangle = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{bmatrix} \quad ; \qquad v^2 = -\frac{\mu^2}{\lambda}$$

This minimum breaks G_{SM} into:

$$SU(3)_c \times U(1)_{em}$$

One can check that $U(1)_{em}$ remains unbroken:

$$Q = T_3 + Y$$

For the Higgs doublet:

$$Q = \begin{bmatrix} 1/2 & 0 \\ 0 & -1/2 \end{bmatrix} + \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Since $Q \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} = 0$, one has

$$e^{i\alpha Q} \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{bmatrix} = \left[1 + \alpha Q + \cdots\right] \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}$$

Electric charge is automatically conserved in the SM with one Higgs doublet. This is no longer true in the two-Higgs doublets model (2HDM), including the case of SUSY extensions of the SM.

In the general 2HDM, without loss of generality one has

$$\langle 0|\phi_1|0\rangle = \begin{bmatrix} 0 \\ 1/\sqrt{2}\, v_1 \end{bmatrix} \quad ; \quad \langle 0|\phi_2|0\rangle = \begin{bmatrix} \xi \\ 1/\sqrt{2}\, v_2 e^{i\theta} \end{bmatrix}$$

ξ real. In order to conserve electric charge one has to choose a region of parameter space, where $\xi = 0$.

- Comment : Note that the SM does not provide an explanation for the charges of elementary fermions. The values of the hypercharge Y are chosen in such a way that the correct electric charges are obtained.
- There are extensions of the SM, for example $SU(5)$, $SO(10)$ where the electric charges of quarks and leptons are related, with the prediction :

$$Q_{proton} = - Q_{electron}$$

In order to describe spontaneous symmetry breaking in the SM, it is useful to introduce a convenient parametrization of the Higgs doublet ϕ :

$$\Phi = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + H + i\, G_0\right) \end{bmatrix}$$

$G^+ \rightarrow$ charged complex scalar field

$G^0 \rightarrow$ real pseudoscalar field

$H \rightarrow$ real scalar field.

Through the Brout-Englert-Higgs mechanism, G^+, G^0 are absorbed as longitudinal components of $W_\mu^\pm$ and Z_μ, which acquire a mass

Z_μ is a linear combination of B_μ and W_μ^3:

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$$

$$\tan\theta_W \equiv g'/g \quad ; \quad A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$$

photon

$$M_W = \frac{gv}{2} \quad ; \quad M_Z = \sqrt{g^2+g'^2}\,\frac{v}{2} = \frac{M_W}{\cos\theta_W}$$

$$e = \frac{g g'}{\sqrt{g^2+g'^2}} = g\sin\theta_W = g'\cos\theta_W$$

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}$$

Generation of quark masses and mixing

In the SM, one cannot include in the Lagrangian a mass term, because it would violate the gauge symmetry:

$$q_L \qquad q_R$$

doublet $\qquad$ singlet

The SM fermions acquire mass through Yukawa couplings, once the SM group is spontaneously broken

The Yukawa interactions are the most general terms of the Lagrangian allowed by the SM gauge group and renormalizability, involving fermions and the Higgs doublet.

$$-\mathcal{L}_Y = (Y_u)_{ij}\, \bar{q}_{iL}\, \tilde{\phi}\, u_{iR} + (Y_d)_{ij}\, \bar{q}_{iL}\, \phi\, d_{iR}$$

$$+ (Y_\ell)_{ij}\, \bar{\ell}_{iL}\, \phi\, e_{R_j} + \text{h.c.}$$

with $\tilde{\phi} \equiv i \tau_2 \phi^*$. The Yukawa matrices Y_u, Y_d, Y_ℓ are arbitrary complex matrices.

After spontaneous gauge symmetry breaking, one obtains:

$$-\mathcal{L}_Y = \Big\{ (Y_u)_{ij} \frac{1}{\sqrt{2}} \bar{u}^0_{iL} u^0_{jR} H + (Y_d)_{ij} \frac{1}{\sqrt{2}} \bar{d}^0_{iL} d^0_{jR} H + (Y_\ell)_{ij} \frac{1}{\sqrt{2}} \bar{e}^0_{iL} e^0_{Rj} H + \text{h.c.} \Big\}$$

$$\mathcal{L}_{mass} = (m_u)_{ij} \bar{u}^0_{iL} u^0_{jR} + (m_d)_{ij} \bar{d}^0_{iL} d^0_{jR} + (m_\ell)_{ij} \bar{e}^0_{iL} e^0_{Rj}$$

$$m_u \equiv \frac{v}{\sqrt{2}} Y_u \; ; \quad m_d \equiv \frac{v}{\sqrt{2}} Y_d \; ; \quad m_\ell = \frac{v}{\sqrt{2}} Y_\ell$$

$u^0, d^0, e^0 \rightarrow$ fermions in the Weak-basis

- Important feature of the SM:

  fermion mass matrices are proportional to the corresponding Yukawa matrices.

  Fermion mass matrices are diagonalized by bi-unitary transformations:

$$u^0_L = U^u_L \, u_L \, ; \quad d^0_L = U^d_L \, d_L \, ; \quad e^0_L = U^e_L \, e_L$$

$$u^0_R = U^u_R \, u_R \quad d^0_R = U^d_R \, d_R \, ; \quad e^0_R = U^e_R \, e_R$$

$U^{u,d,e}_{L,R}$ are unitary matrices such that:

$$U^{u\,\dagger}_L \, m_u \, U^u_R = \mathrm{diag.}\,(m_u, m_c, m_t)$$

With analogous relations for diagonalization of m_d, m_e

These bi-unitary transformations affect the charged current interactions. In the **Weak-basis**, one had:

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{u}^{0}_{iL}\,\gamma^{\mu}\,d^{0}_{iL} + \overline{\nu}^{0}_{iL}\,\gamma^{\mu}\,e^{0}_{iL}\right]W^{+}_{\mu} + \text{h.c.}$$

In the mass-eigenstate basis:

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{u}_{L}\,\gamma^{\mu}\underbrace{\left(U^{u\dagger}_{L}\,U^{d}_{L}\right)}\,d_{L} + \overline{\nu}^{0}_{L}\,\gamma^{\mu}\,U^{e}_{L}\,e_{L}\right]$$

$$V_{CKM} \equiv \left(U^{u}_{L}\,U^{d}_{L}\right)$$

; Since in the SM neutrinos are massless, one can redefine $\nu^{0}_{L} \to \nu_{L} = U^{e\dagger}\nu^{0}_{L}$

$\Rightarrow$ no leptonic mixing !!

The electromagnetic and neutral current interactions remain invariant when one changes from the weak-basis to the mass eigenstate basis. In the weak-basis one has:

$$J^\mu_{em} = \frac{2}{3}\left[\bar{u}^0_L \gamma^\mu u^0_L + \bar{u}^0_R \gamma^\mu u_R\right] - $$
$$-\frac{1}{3}\left[\bar{d}^0_L \gamma^\mu d^0_L + \bar{d}^0_R \gamma^\mu d^0_R\right] - \left[\bar{e}_L \gamma^\mu e^0_L + \bar{e}^0_R \gamma_\mu e_R\right]$$

In the mass eigenstate basis:

$$J^\mu_{em} = \frac{2}{3}\left[\bar{u}_L \gamma^\mu \overbrace{U^{u\dagger}_L U^u_L}^{1} u_L + \bar{u}_R U^{u\dagger}_R U^u_R u_R\right] -$$
$$-\frac{1}{3}\left[\bar{d}_L U^{d\dagger}_L U^d_L \gamma^\mu d_L + \bar{d}_R U^{d\dagger}_R U^d_R d_R\right] - \cdots$$

For the neutral current interactions:

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W} \cdot \Big[\bar{u}^0_L \gamma^\mu u^0_L - \bar{d}^0_L \gamma^\mu d^0_L + \bar{\nu}^0_L \gamma^\mu \nu^0_L - \bar{e}^0_L \gamma^\mu e^0_L - \sin^2\theta_W \, J^\mu_{em} \Big] Z_\mu$$

In the mass eigenstate basis:

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W} \Big[\bar{u}_L \underbrace{U^+_L U_L}_{1} \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu \nu_L - \bar{e} \gamma^\mu e_L - \sin^2\theta_W J^\mu_{em} \Big] Z_\mu$$

- Flavour-Changing Neutral Currents (FCNC) are naturally absent at tree level in the SM, due to the GIM mechanism. Charm was "invented" in order to achieve this cancellation of FCNC.

- In the SM with one Higgs doublet there are no Higgs-mediated FCNC.

$$m_q \propto Y_q \ (q = d, u) \ ; \quad m_\ell \propto Y_\ell$$

Once m_u, m_d, m_ℓ are diagonalised, Y_u, Y_d, Y_ℓ are also diagonal.

In order to avoid FCNC, Glashow, Weinberg and Paschos proposed the following principle:

- All quarks of fixed charge and helicity must transform according to the same irreducible representation of SU(2) and correspond to the same eigenvalue of T_3
- All quarks should receive their contributions to the quark mass matrix from a single neutral scalar vev.

Question : Can one violate these two "dogmas" in reasonable extensions of the SM?

Answer : Yes!

In the gauge sector the "dogma" can be violated through the introduction of a $Q = -1/3$ or $Q = 2/3$ vector-like, since in this model one has naturally small violations of 3×3 unitarity of V_{CKM}, which leads to Z-mediated FCNC at tree level, which are naturally suppressed

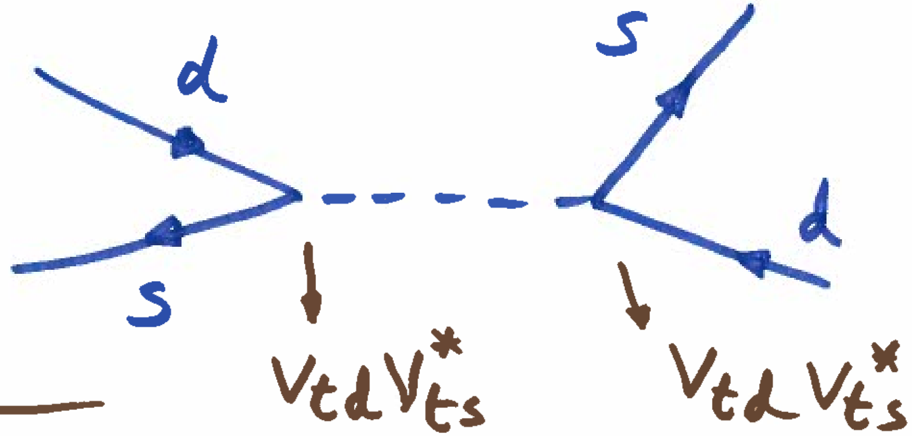
In the scalar sector, the dogma can be violated in a class of 2 Higgs doublet models (2HDM) where there are FCNC at tree level, but naturally suppressed by small CKM matrix elements.

GCB, W. Grimus, L. Lavoura (BGL models)

Example: $\Delta S = 2$ transitions

$|V_{td}| - \lambda^3$

$|V_{ts}| - \lambda^2$



$K^0 - \bar{K}^0$

λ^{10} suppression

Fundamental Properties of the V_{CKM} matrix

$$\mathcal{L}_{CC} = (\bar{u}\ \bar{c}\ \bar{t})_L\, \gamma^\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^+ + h.c.$$

V_{CKM} is complex, but the individual phases of its elements have no meaning, due to the freedom to rephase the mass eigenstate quark fields.

$$u_\alpha = e^{i\varphi_\alpha}\, u'_\alpha \qquad d_k = e^{i\varphi_k}\, d'_k$$

Under rephasing :
$$V'_{\alpha k} = e^{i(\varphi_k - \varphi_\alpha)}\, V_{\alpha k}$$

It is useful to consider rephasing invariant quantities, which do not change under rephasing

Simplest examples: moduli and quartets:

$$Q_{\alpha i \beta j} \equiv V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}$$

with $\alpha \neq \beta$, $i \neq j$

Invariants of higher order can be written as functions of the quartets and moduli.

Example:

$$V_{\alpha i} V_{\beta j} V_{\gamma k} V^*_{\alpha j} V^*_{\beta k} V^*_{\gamma i} = \frac{Q_{\alpha i \beta j} Q_{\beta i \alpha j}}{|V_{\beta i}|^2}$$

Quartets can be easily constructed using the following scheme:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The two quartets:

$$Q_{uscb} \equiv V_{us} V_{cb} V_{ub}^* V_{cs}^* \quad ; \quad Q_{cdts} \equiv V_{cd} V_{ts} V_{td}^* V_{cs}^*$$

Counting of parameters:

In the SM with n_g generations, the CKM matrix is unitary. Therefore it has in general n_g^2 parameters. But some of them can be eliminated by rephasing:

$$N_{\text{parameters}} = n_g^2 - \underbrace{(2n_g - 1)}_{\text{eliminated by rephasing}} = (n_g - 1)^2$$

A $n_g \times n_g$ orthogonal matrix is parametrized by

$$N_{\text{angle}} = \frac{1}{2} n_g (n_g - 1)$$

$$N_{\text{phases}} = N_{\text{par.}} - N_{\text{angle}} = \frac{1}{2}(n_g - 1)(n_g - 2)$$

Therefore, one has in the SM:

$$n_g = 1, 2 \quad \rightarrow \quad N_{phase} = 0$$

$$n_g = 3 \quad \rightarrow \quad N_{phase} = 1$$

One can show that $N_{phase} \neq 0$ is associated to CP violation in the SM. Kobayashi - Maskawa (1973)

Let us consider rephasing invariant quartets (RIQ)

For 2 generations there is only one RIQ:

$$Q_{udcs} \equiv V_{ud} V_{cs} V_{us}^* V_{cd}^*$$

But from orthogonality of V_{CKM}, one has:

$V_{ud} V_{cd}^* + V_{us} V_{cs}^* = 0$; Multiplying by $V_{us}^* V_{cs}$ one obtains

$$Q_{udcs} = -|V_{us}|^2 |V_{cs}|^2 \rightarrow \text{real!}$$

Unitarity is crucial!!

For 3 generations, orthogonality of the first two rows lead to :

$$\boxed{V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0}$$

Multiplying by $V_{us}^* V_{cs}$ and taking imaginary parts, one has :

$$\mathrm{Im}\, Q_{udcs} = -\mathrm{Im}\, Q_{ubcs}$$

For $n_g = 3$, one can show that

$$I \equiv |\mathrm{Im}\, Q|$$

has the same value for all invariant quartets. It gives the strength of CP violation in the SM

CP Violation

In order to study the CP properties of a Lagrangian, it is convenient to separate the $\mathcal{L}$ in two parts:

$$\mathcal{L} = \mathcal{L}_{(CP)} + \mathcal{L}'$$

where $\mathcal{L}_{(CP)}$ is the part of the Lagrangian which one knows that conserves CP. One should allow for the most general CP transformation allowed by $\mathcal{L}_{CP}$.

CP can be investigated in the fermion mass eigenstate basis and in a weak basis. We shall do the analysis in both cases.

Mass-eigenstate basis - Let us consider the SM, after gauge symmetry breaking and after diagonalization of the quark mass matrices:

$$m_u = \mathrm{diag}(m_u, m_c, m_t)\ ; \quad m_d = \mathrm{diag}(m_d, m_s, m_b)$$

Taking into account that the quark masses are non-degenerate, the most general CP transformation is:

$$(CP)\, W^{+\mu}(t,\vec{r})(CP)^{-1} = -e^{i\xi_W} W^{-\mu}(t,-\vec{r})$$

$$(CP)\, W^{-\mu}(t,\vec{r})(CP)^{-1} = -e^{i\xi_W} W^{+\mu}(t,-\vec{r})$$

$$(CP)\, u_\alpha(t,\vec{r})(CP)^{-1} = e^{i\xi_\alpha}\gamma^0 C\, \bar{u}_\alpha^T(t,-\vec{r})$$

$$(CP)\, d_k(t,\vec{r})(CP)^{-1} = e^{i\xi_k}\gamma^0 C\, \bar{u}^T(t,-\vec{r})$$

Invariance of the charged current weak interactions under CP, constrains $V_{\alpha k}$ to satisfy the constraint:
$$V^*_{\alpha k} = e^{i(\xi_W + \xi_k - \xi_\alpha)}\, V_{\alpha k}$$

It can be easily shown that this constrains all rephasing invariant functions of V_{CKM} to be real

$$\mathrm{Im}\, Q \neq 0 \;\Rightarrow\; \text{CP violation}$$

Unitarity triangles

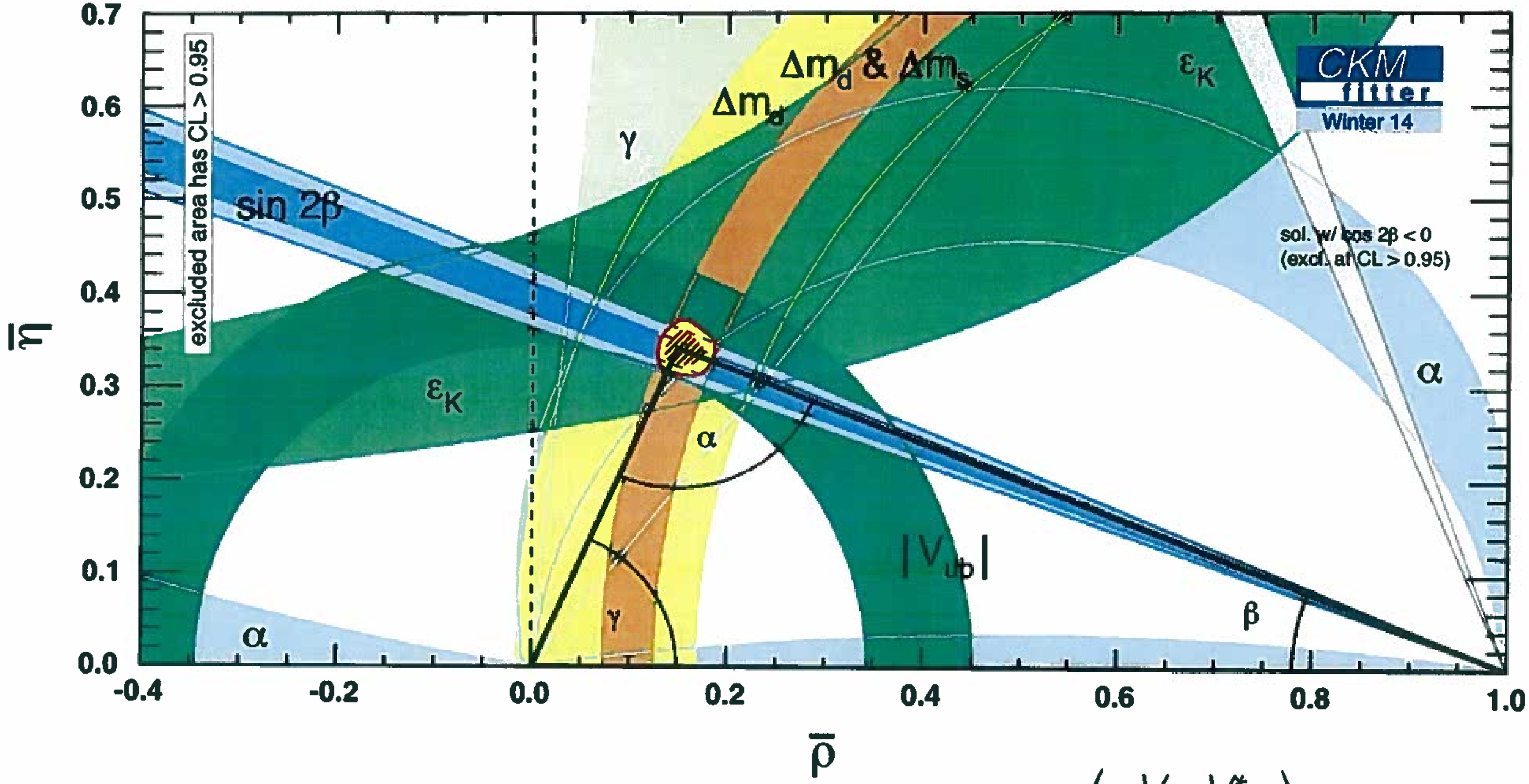
Consider orthogonality of the first and third columns of V_{CKM} :

$$\underbrace{V_{ud} V_{ub}^*}_{\lambda^3} + \underbrace{V_{cd} V_{cb}^*}_{\lambda^3} + \underbrace{V_{td} V_{tb}^*}_{\lambda^3} = 0$$

$$|V_{CKM}| \approx \begin{bmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix}$$

See Guido Martinelli's lectures at CORFU 2015

CP violation in the SM quark sector



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \qquad \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

- current world average: $\sin 2\beta = 0.679 \pm 0.020$

The internal angles of the triangle are rephasing invariant:

$$\alpha \equiv \arg\left[-V_{td} V_{ub} V_{ud}^* V_{tb}^*\right]$$

$$\beta \equiv \arg\left[-V_{cd} V_{tb} V_{cb}^* V_{td}^*\right]$$

$$\gamma \equiv \arg\left[-V_{ud} V_{cb} V_{ub}^* V_{cd}^*\right]$$

one gets then:

$$\boxed{\alpha + \beta + \gamma = \pi}$$

This is true by definition and no test of unitarity !!

The strength of CP violation in the SM is small due to the smallness of some $|V_{ij}|$ like V_{ub}, V_{cb}.

$$|\mathrm{Im}\, Q| = |V_{ud} \underbrace{V_{ub}}_{\lambda^3} \underbrace{V_{cd}}_{\lambda} \underbrace{V_{cb}}_{\lambda^2}| \sin\gamma$$

Therefore $\boxed{|\mathrm{Im}\, Q| \cong \lambda^6 \sin\gamma}$

What would be the maximal possible value for $\mathrm{Im}\, Q$?

It corresponds to:

$$V = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^* \\ 1 & \omega^* & \omega \end{bmatrix}$$

$$\mathrm{Im}\, Q = \frac{1}{6\sqrt{3}} \approx 0.096.$$

A convenient parametrization of V_{CKM} is:

$$V = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$$s_{13} = |V_{ub}| \quad ; \quad s_{12} = \frac{|V_{u\ }|}{\sqrt{1-|V_{ub}|^2}} \quad ; \quad s_{23} = \frac{|V_{cb}|}{\sqrt{1-|V_{ub}|^2}}$$

Once s_{ij} are fixed, all other experimental data has to be fit with a single parameter δ.

The SM is very predictive !!

Invariant Approach to CP Violation

Let us consider the $\mathcal{L}^{SM}$, written in a weak-basis where all gauge currents are flavour diagonal. The most general CP transformation which leaves $\mathcal{L}_{CP}$ invariant is:

$$(CP)\, u_L^0 \,(CP)^{-1} = e^{i\xi_W} K_L \gamma^0 C\, \bar{u}_L^{0\,T}$$

$$(CP)\, d_L^0 \,(CP)^{-1} = K_L \gamma^0 C\, \bar{d}_L^{0\,T}$$

$$(CP)\, u_R^0 \,(CP)^{-1} = K_R^u \gamma^0 C\, \bar{u}_R^{0\,T}$$

$$(CP)\, d_R^0 \,(CP)^{-1} = K_R^d \gamma^0 C\, \bar{d}_R^{0\,T}$$

K_L, K_R^u, K_R^d are unitary matrices acting in flavour space

It can be readily shown that in order for the Yukawa interactions (or equivalently the mass terms in m_u, m_d) to be invariant the following relations have to be satisfied: J. Bernabéu, GCB and M. Gronau

$$K_L^\dagger m_u K_R^u = m_u^* \quad ; \quad K_L^\dagger m_d K_R^d = m_d^*$$

The existence of the matrices $K_L\ K_R^u, K_R^d$ is a necessary and sufficient condition for CP invariance in the SM, for any number of generations! From the above Eqs one derives:

$$K_L^\dagger H_u K_L = H_u^* = H_u^T \quad ; \quad K_L^\dagger H_d K_L = H_d^T \ ; \quad H_{d,u} = M_{d,u} M_{d,u}^\dagger$$

So: $$K_L^\dagger [H_u, H_d] K_L = [H_u^T, H_d^T] = -[H_u, H_d]^T$$

and one obtains $\boxed{\operatorname{tr}[H_u, H_d]^r = 0 \quad (r \text{ odd})}$

The minimal non-trivial case corresponds to $r=3$.

For two generations the invariant automatically vanishes

For 3 generations one obtains:

$$\mathrm{tr}\,[H_u, H_d]^3 = 6i\,(m_t^2 - m_c^2)(m_t^2 - m_u^2) \times (m_c^2 - m_u^2) \times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\,\mathrm{Im}\,Q$$

$\mathrm{tr}\,[H_u, H_d]^3$ is a necessary condition for CP invariance for any number of generations. For $n_g = 3$, it is a necessary and sufficient condition for CP invariance. For $n_g = 3$ one has also:

$$\mathrm{tr}\,[H_u, H_d]^3 = 3 \det\,[H_u, H_d] \quad \text{Jarlskog}$$

The question of Neutrino mass

In the SM neutrinos are strictly massless

- Neutrinos cannot have a Dirac mass because ν_R is not introduced in the SM
- Neutrinos in the SM cannot have a Direct mass because it is not gauge invariant:

$$\nu_L^T C M_L \nu_L$$

Also at tree-level no mass, since no Higgs triplet is introduced

- A non-vanishing neutrino mass cannot be generated in the SM in higher orders of perturbation or through non-perturbative effects due to exact B-L conservation

Therefore, the discovery of neutrino masses and oscillations, rules out the SM!!

(Un)Fortunately a minimal extension of the SM solves the problem

Minimal extension of the SM:

$$SM \to \nu SM$$

Just add ν_R to the spectrum of the SM and write the most general Lagrangian consistent with gauge invariance. This includes the term:

$$\nu_R^T C M_R \nu_R$$

Since ν_R is a singlet of $SU(2)\times(1)$, this term is gauge invariant.

Furthermore, one expects M_R to be large, since it is not "protected" by gauge invariance. This automatically leads to the seesaw mechanism:

3 light neutrinos which enter in neutrino oscillations with a mass

$$\boxed{m_\nu \approx \frac{m_D^2}{M_R}}$$

3 have neutrinos with mass of order M_R

Why Glashow, Weinberg and Salam did not introduce ν_R?

Probable reason: They wanted to avoid non-vanishing neutrino masses! Seesaw mechanism was not known. The mechanism was introduced in more ambitions extensions of the SM, like

$$SO(10)\ ; \quad SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)$$

etc, where ν_R could not be avoided.

CP Violation in the leptonic sector

If neutrinos are Dirac particles, there is essential no modification with respect to the quark sector. If neutrinos are Majorana particles, some subtleties arise.

Let us assume that nature chooses Majorana Neutrinos. Without loss of generality, one can choose to work in a weak-basis where the charged-lepton mass matrix is diagonal:

$$m_\ell = d_\ell = \text{diag.}(m_e, m_\mu, m_\tau)$$

In this basis, the Majorana neutrino mass matrix is a 3×3 complex symmetric mass matrix. One can still make the rephasing:

$$\ell'_{L,R} = K_L \, \ell_{L,R} \qquad \nu'_L = K_L \, \nu_L$$

with $K_L = \text{diag.}\left(e^{i\varphi_1} \; e^{i\varphi_2} \; e^{i\varphi_3}\right)$. Under this rephasing:

$$(m'_\nu)_{ij} = e^{i(\varphi_i + \varphi_j)} (m_\nu)_{ij}$$

Through this rephasing one can eliminate n phases from m_ν.

The total number of phases in m_ν is

$$N_{\text{phases}} = \underbrace{\tfrac{1}{2}n(n+1)}_{\substack{\text{number of phases} \\ \text{in a complex symmetric matrix}}} - n = \tfrac{1}{2}n(n-1)$$

So for $n = 3$, one has 3 CP violating phases in m_ν. The individual phases of m_ν have no physical meaning. But one can construct rephasing invariant polynomials:

$$P_1 \equiv (m_\nu^*)_{11} (m_\nu^*)_{22} (m_{12}^2)$$

$$P_2 = (m_\nu^*)_{11} (m_\nu^*)_{33} (m_\nu)_{13}^2$$

$$P_3 = (m_\nu^*)_{33} (m_\nu^*)_{12} (m_\nu)_{13} (m_\nu)_{23}$$

In the mass eigenstate basis the charged currents interactions can be written:

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} (\bar{e}\ \bar{\mu}\ \bar{\tau}) \gamma^\mu \underbrace{\begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}}_{U^{PMNS}} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

How many CP violating phases. Recall that in V^{CKM} one could eliminate 5 phases through rephasing. In this case one can only rephase the 3 charged lepton fields, so one can only eliminate 3 phases. So altogether one has 3 CP violating phases

So one can write

$$\overset{\text{PMNS}}{U} = \underbrace{V}_{\text{one phase}} \; K \longrightarrow \text{Majorana type phases}$$

$$K = \mathrm{diag}\left(1, e^{i\alpha_1}\, e^{i\alpha_2}\right)$$

Recall that in V^{CKM} the only physically meaning-full phases were the arguments of quartets.
A novel feature in the leptonic sector with Majorana neutrinos: one has rephasing invariant bi-linears:

$$\arg\left(U_{\ell\alpha} U^*_{\ell\beta}\right)$$ Majorana-type phases

GCB, M.N. Rebelo

Invariant approach to Majorana type CP violation.

One can derive CP-odd WB invariants sensitive to Majorana type CP violation:

$$I^{CP}_{\text{Majorana}} = \operatorname{Im}\operatorname{Tr}\left(m_\ell m_\ell^\dagger m_\nu^* m_\nu m^* m_\ell^T m_\ell^* m_\nu\right)$$

In the case of $n_g = 2$

$$I^{CP}_{\text{Majorana}} = \frac{1}{4} m_1 m_2 \Delta m_{21}^2 \left(m_\mu^2 - m_e^2\right) \times \sin^2\theta \sin 2\gamma$$

where PMNS matrix is:

$$U = \begin{bmatrix} \cos\theta & -\sin\theta e^{i\gamma} \\ \sin\theta e^{-i\gamma} & \cos\theta \end{bmatrix}$$

$\gamma = \pi/2$ CP invariant

Is there any motivation to have

- New sources of CP violation, beyond the KM mechanism present in the SM? Yes !!

- Is there any "experimental" evidence for New Sources of CP violation? Yes !!

Generation of the Baryon-Asymmetry of the Universe (BAU)

The ingredients to dynamically generate BAU from an initial state with zero BA, were formulated by Sakharov a longtime ago (1967)

(i) Baryon number Violation

(ii) C and CP Violation

(iii) Departure from thermal equilibrium

All these ingredients exist in the SM but it has been established that in the SM one cannot generate the observed

BAU :

$$\boxed{\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.20 \pm .15) \times 10^{-10}}$$

n_B, $n_{\bar{B}}$, n_γ number density of baryons anti-baryons and photons at present time

Reasons why the SM cannot generate sufficient BAU:

(i) CP violation in the SM is too small:

$$\frac{\mathrm{tr}\left[H_u, H_d\right]^3}{T_{ew}^{12}} \approx 10^{-20}$$

(ii) Successful baryogenesis needs a strongly first order phase transition which would require a light Higgs mass:

$$m_H \lesssim 70\,\mathrm{GeV}$$

Open Questions in CP Violation

- What is the Origin of CP Violation?
  - Explicitly broken as in the Standard Model through the Kobayashi-Maskawa mechanism (1973)

    or

  - Spontaneously broken as suggested by T. D. Lee (1973)
- Pure gauge interactions conserve CP

  W. Grimus, M.N. Rebelo

- How to conceive an experiment that could distinguish between spontaneous and explicit CP violation?

An important but very difficult question!

Some interesting work was done using CP-odd invariants relevant for the scalar sector

G.C.B, M.N. Rebelo, J. Silva-Marcos

F. Gunion and H. Haber

B. Grzadkowski, O.M. Ogreid, P. Osland

M. Krawczyk, D. Sokolowska

- What is the connection between CP Violation and Family Symmetries?

W. Grimus, G. Ecker

- Can one have Geometrical CP Violation?

GCB, J.M. Gérard, W. Grimus (1984)

Recent developments:

I. de Medeiros Varzielas
S. King
M. Lindner, M. Holthausen, M. Lindner, M. Schmidt
I. P. Ivanov, L. Lavoura
F. Feruglio
M.C. Chen, M. Fallbacher, K. Mahanthappa, M. Ratz
etc

- Are there New Sources of CP Violation beyond those present in the K. M. mechanism?

The answer is Yes!, since the SM cannot generate sufficient BAU.

But then the next question is:

- How to generate adequate BAU? Leptogenesis? Extended Higgs sector? Many scenarios have been proposed.
- How to test, experimentally, Leptogenesis?
- How relate CP violation relevant for Leptogenesis from leptonic CP violation, detectable at low energies through neutrino oscillations?

Answer: In general, it is not possible to establish this connection. It is conceivable to establish the connection, if leptonic flavour symmetries are introduced.

- How to solve the Strong CP problem? Peccei-Quinn provided an elegant solution ... but Axions have not been found. Are there other plausible solutions?

On the relationship between SCPV and Family Symmetries.

Let us consider the original Lee Model, with 3 quark generations: 2 Higgs doublets, no extra symmetry

As a result: down quarks (or up quarks) receive masses from both ϕ_1, ϕ_2. In the Higgs potential, there are terms like:

$$(\phi_i^\dagger \phi_i)(\phi_1^\dagger \phi_2) + h.c. \; ; \; (\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_2) + h.c. \; , \text{etc}$$

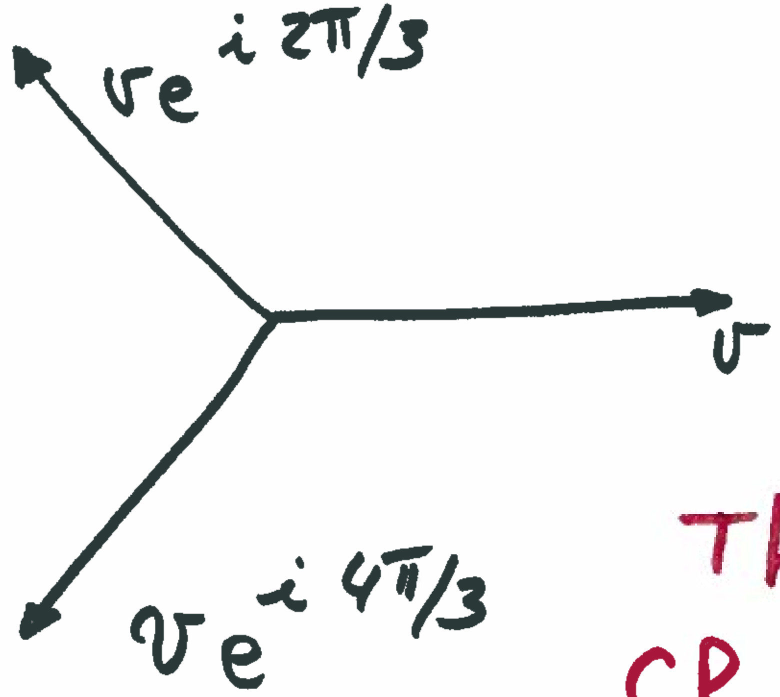
which see relative phases. There is a region of the parameter of the potential where the minimum is at

$$\langle \phi_1 \rangle = \begin{bmatrix} 0 \\ v_1 \end{bmatrix} \; ; \; \langle \phi_2 \rangle = \begin{bmatrix} 0 \\ v_2 e^{i\theta} \end{bmatrix} \Rightarrow \text{SCPV}$$

In general θ has an arbitrary value and for $\theta \neq 0, \pi$ one has spontaneous CP violation. This means that the $\mathcal{L}$agrangian is CP invariant but the vaccuum is not CP invariant. In multi-Higgs models, in presence of symmetries, the $\langle \phi_i \rangle$ may have "geometrical values".

Example: Three Higgs doublets with a S_3 symmetry

For a region of the parameters of the potential, the minimum is at:



This vacuum is CP invariant!

At present there is clear evidence that V_{CKM} is complex, independently of the possible presence of New Physics contributing to CP Violation. Does this exclude the possibility of having spontaneous CP Violation, with real Yukawa couplings? NO!!

Actually the original T. D. Lee Model with 2 Higgs doublets generates a complex V_{CKM} :

$$M_d = Y_1^d \frac{v_1}{\sqrt{2}} + Y_2^d \frac{v_2}{\sqrt{2}} e^{i\theta}$$

$$M_d M_d^{+} = \frac{1}{2}\Big\{ v_1^2 (Y_1 Y_1^{+}) + v_2^2 (Y_2 Y_2^{+}) + 2 v_1 v_2 (Y_1 Y_2^{+} + Y_2^{+} Y_1) c_\theta$$
$$+ 2 i v_1 v_2 \sin\theta \,(Y_2 Y_1^{+} - Y_1 Y_2^{+}) \Big\}$$

Similar for $M_u M_u^{+}$

In general this leads to a complex V^{CKM}.

But too large FCNC !!

The simplest way of avoiding Higgs mediated FCNC consists of introducing a Z_2 symmetry under which:

$$\phi_1 \longrightarrow -\phi_1 \;\; ; \;\; \phi_2 \longrightarrow \phi_2 \;\; ; \;\; d_R \longrightarrow -d_R$$

This is sufficient to guarantee Natural Flavour Conservation in the Higgs sector.

S. Glashow, and
S. Weinberg
M. Paschos

Due to the presence of the Z_2 symmetry, the only term in the scalar potential which is sensitive to relative phases is:

$$\lambda\,(\phi_1^{\dagger}\phi_2)(\phi_1^{\dagger}\phi_2) + h.c.$$

For λ positive, the minimum of the potential is at:

$$\langle 0 | \phi_1^0 | 0 \rangle = v_1 \exp(i\pi/2)$$

$$\langle 0 | \phi_2^0 | 0 \rangle = v_2$$

Does this lead to maximal Spontaneous CP violation? NO!

The minimum is CP invariant

Justification: Let us consider an extension of the SM, where n $SU(2)\times U(1)$ scalar doublets are introduced. Let us consider the **most general CP transformation** which leaves invariant the kinetic energy terms of the scalar doublets:

$$CP\,\phi_i\,CP^\dagger = \sum_{j=1}^{n} U_{ij}\,\phi_j^*$$

$U \longrightarrow$ unitary matrix, acting in scalar doublet space.

Let us assume that the vacuum is CP invariant, meaning that :

$$CP|0\rangle = |0\rangle$$

One can derive the following condition :

$$\sum_{j=1}^{n} U_{ij} \langle 0|\phi_j|0\rangle^* = \langle 0|\phi_i|0\rangle$$

If the vacuum is such that none of the symmetries allowed by the Lagrangian satisfy the above condition, this means that CP is spontaneously broken!

In the case of :

$$\langle 0 | \phi_1^0 | 0 \rangle = v_1 \exp(i\pi/2)$$

$$\langle 0 | \phi_2^0 | 0 \rangle = v_2$$

The minimum is CP invariant, and satisfies the condition

$$\langle 0 | \phi_i^0 | 0 \rangle = U_{ij}^{CP} \langle 0 | \phi_j^* | 0 \rangle$$

for the following choice of U^{CP} :

$$U^{CP} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

This is a "generic" phenomena, often (but not always!) the presence of "family symmetries" makes it impossible to generate SCPV in frameworks where, in principle, one can have SCPV. Can one conclude that all "geometrical" vacua are CP invariant?

No, an exception was found in 1984 (G.C.B., J.M.Gérard, W. Grimus)

and other examples were found in the past two years

I. de Medeiros Varzielas, S. King; M. Lindner, etc

see beginning of talk

Question : Is it possible to construct a realistic model, with SCPV, which avoids too large FCNC, while at the same time generating a complex CKM matrix?

Answer Yes, the simplest framework involves the introduction of vector-like quarks.

Vector-like quarks arise in many extensions of the SM : E_6 grand-unified theories

Extra-dimensions models

etc

Seven reasons to consider vector-like quarks

1. They provide a self-consistent framework with naturally small violations of 3×3 unitarity of V^{CKM}.

2. Lead to naturally small Flavour Changing Neutral currents (FCNC) mediated by Z_μ

New Physics in $\begin{cases} B_d - \bar{B}_d \text{ mixing} \\ B_s - \bar{B}_s \text{ mixing} \\ K^0 - \bar{K}^0 \text{ mixing} \end{cases}$

3. Provide the simplest framework to have Spontaneous CP Violation, with a vacuum phase generating a <u>non-trivial CKM phase</u>.

4. Provide New Physics contributions to $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings

5. Provide a simple solution to the Strong CP problem, which <u>does not require Axions</u>

6. May contribute to the understanding of the observed pattern of fermion masses and mixing.

7. Provide a framework where there is a common origin for all CP violations:
(GCB, M.N.Rebelo, P. Parada)

(i) CP violation in the Quark Sector

(ii) CP Violation in the Lepton Sector, detectable through neutrino oscillations

$U_{e3} \neq 0 \longrightarrow$ Great News

(iii) CP violation needed to generate the Baryon Asymmetry of the Universe (BAU) through Leptogenesis.

Comment :

There is nothing "strange" in having deviations of 3×3 unitarity. The PMNS matrix in the leptonic sector in the context of type-one seesaw (νSM) is not 3×3 unitary !!

Minimal realistic model with Spontaneous CP Violation

$$SM + 3\,\nu_R + \underbrace{D_L, D_R}, \; S$$

vector like, singlet under SU(2).

complex scalar, singlet under SU(2)

Instead of a $Q = -1/3$ down-type vectorlike quark, one could have, of course, a $Q = 2/3$ up type vector like quark, or a combination of the two.

- Introduce CP invariance at the Lagrangian level : all Yukawa couplings are real. CP is spontaneously broken.
- Introduce a Z_4 symmetry on the Lagrangian, under which :

$$\psi_L^0 \equiv \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L \to i\,\psi_L^0 \; ; \quad \ell_{Rj}^0 \to i\,\ell_{Rj}^0 \; ; \quad \nu_{Rj}^0 \to i\,\nu_{Rj}^0$$

$$D^0 \to -D^0 \quad ; \quad S \to -S$$

All other fields are invariant under Z_4.

CP is spontaneously broken by the vacuum. The scalar potential contains various terms which do not have phase dependence, but there are terms with phase dependence:

$$V_{phase} = \left(\mu^2 + S^* S + \lambda_2 \phi^\dagger \phi\right)\left(S^2 + S^{*2}\right) + \lambda_3 \left(S^4 + S^{*4}\right)$$

There is a range of the parameters of the Higgs potencial, where the minimum is at:

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}} \qquad \langle S \rangle = \frac{V}{\sqrt{2}} e^{i\theta}$$

This vacuum breaks CP spontaneously.

Remarkable feature.

In general, the phase θ leads to a complex V^{CKM} !!

The most general $SU(2)\times U(1)\times SU(3)_c \times Z_4$ invariant Yukawa couplings in the quark sector :

real !

$$\mathcal{L}_Y = -\left(\bar{u}^0\ \bar{d}^0\right)_{L_i}\left[g_{ij}\,\phi\, d^0_{R_j} + h_{ij}\,\tilde{\phi}\, u^0_{R_j}\right] - \bar{M}\left(D^0_L D^0_R\right) - \left(f_j S + f'_j S^*\right)\bar{D}^0_L\, d^0_{R_j} + \text{h.c.}$$

no $\bar{d}^0_{iL} D_{R_j}\,\phi \rightarrow$ forbidden by the Z_4 symmetry

Quark mass-matrix for down-type quarks:

3x3, real

$$\begin{bmatrix} d^0_{1L} & d^0_{2L} & d^0_{3L} & D^0_L \end{bmatrix} \underbrace{\left[\begin{array}{ccc|c} & m_d & & 0 \\ & & & 0 \\ & & & 0 \\ \hline M_1 & M_2 & M_3 & \bar{M} \end{array}\right]}_{\mathcal{M}} \begin{bmatrix} d^0_{1R} \\ d^0_{2R} \\ d^0_{3R} \\ D^0_R \end{bmatrix}$$

$$M_j = f_j V e^{i\theta} + f_j' V e^{-i\theta} \; ; \quad \mathcal{M} \to 4\times 4 \text{ matrix}$$

$$\mathcal{U}_L^\dagger \mathcal{M} \mathcal{M}^\dagger \mathcal{U}_L = \begin{bmatrix} d^2 & \\ & D^2 \end{bmatrix}$$

$$d^2 \equiv \overset{\text{diag.}}{} (m_d^2, m_s^2, m_b^2) \quad ; \quad \mathcal{U} = \begin{bmatrix} K & R \\ S & T \end{bmatrix}$$

$$K^{-1} \left[m_d m_d^\dagger - \frac{m_d M^\dagger M m_d^\dagger}{M M^\dagger + \bar{M}^2} \right] K = d^2$$

The phase θ arising from $\langle S \rangle$ generates a non-trivial CKM phase provided $|M_j|$ and $\overline{M}$ are of the same order of magnitude, which is entirely natural.

$$K^{-1} m_{eff} m_{eff}^\dagger K = \text{diag.}\left(m_d^2, m_s^2, m_b^2\right)$$

$$m_{eff} m_{eff}^\dagger = m_d m_d^\dagger - \frac{m_d M^\dagger M m_d^\dagger}{\left(M M^\dagger + \overline{M}^2\right)}$$

$$M_j = \left[f_j V e^{i\theta} + f_j' V e^{-i\theta}\right]$$

Weak Charged Current Interactions

$$(\bar{u}\ \bar{c}\ \bar{t})\begin{bmatrix} V_{ud} & V_{us} & V_{ub} & V_{uD} \\ V_{cd} & V_{cs} & V_{cb} & V_{cD} \\ V_{td} & V_{ts} & V_{tb} & V_{tD} \end{bmatrix}\begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix}$$

There are naturally small violations of 3×3 V^{CKM} unitarity:

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = Z_{bd}$$

$$Z_{bd} = O\left(m^2/M^2\right)$$

Suppose that one drops the requirement of 3×3 V^{CKM} unitarity. How many independent parameters are there in $(V^{CKM})_{3\times 3}$?

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} & \cdots \\ V_{cd} & V_{cs} & V_{cb} & \cdots \\ V_{td} & V_{ts} & V_{tb} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

9 moduli + 4 rephasing invariant phases

= 13 parameters

The 4 rephasing invariant phases can be chosen:

$$\beta \equiv \arg\left(-V_{cd} V_{tb} V_{cb}^* V_{td}^*\right)$$

$$\gamma \equiv \arg\left(-V_{ud} V_{cb} V_{ub}^* V_{cd}^*\right)$$

$$\chi \equiv \beta_s \equiv \arg\left(-V_{cb} V_{ts} V_{cs}^* V_{tb}^*\right)$$

$$\chi' \equiv \arg\left(-V_{us} V_{cd} V_{ud}^* V_{cs}^*\right)$$

The SM with 3 generations predicts a series of exact relations among these 13 measurable quantities.

Violations of any of these exact relations signals the presence of New Physics which may involve deviations of 3×3 unitarity or not. Crucial point:

The presence of New Physics contributions to $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings affects the extraction of $|V_{td}|$, $|V_{ts}|$ from the data even in the framework of New Physics which respects 3×3 V_{CKM} unitarity.

Example: SUSY extensions of the SM.

In many of the extensions of the SM, the dominant effect of New Physics arises from new contributions to B_d-$\overline{B}_d$ and B_s-$\overline{B}_s$ mixings which may be parametrized as:

$$M_{12}^q = (M_{12}^q)^{SM} \, r_q \, e^{2i\theta_q} \quad ; \quad q = d, s$$

$$\Delta M_{B_d} = r_d^2 \, (\Delta M_{B_d})^{SM} \longrightarrow$$ Affects the extraction of $|V_{td}|$ from ΔM_{B_d}

$$\Delta M_{B_s} = r_s^2 \, (\Delta M_{B_s})^{SM} \longrightarrow$$ Affects the extraction of $|V_{ts}|$ from ΔM_{B_s}

Furthermore :

$$S_{J/\psi K_S} = \sin\left(2\beta + 2\theta_d\right) = \sin\left(2\bar{\beta}\right)$$

How to detect the presence of New Physics ?

Answer : Use the exact relations predicted by the SM :

(db) $$|V_{ub}| = \frac{V_{cd}\,V_{cb}}{|V_{ud}|}\,\frac{\sin\beta}{\sin(\gamma+\beta)} \rightarrow \text{extraction of } \theta_d$$

(sb) $$\sin\beta_s = \frac{|V_{us}|\,|V_{ub}|}{|V_{cs}|\,|V_{cb}|}\,\sin\left(\gamma - \beta_s + \chi'\right)$$

$\rightarrow$ extraction of θ_s

Other interesting unitarity relations:

$$\sin\beta_s = \frac{|V_{us}||V_{cd}||V_{cb}|}{|V_{cs}||V_{tb}||V_{ud}|} \frac{\sin\beta \sin(\gamma+\chi')}{\sin(\gamma+\beta)}$$

well approximated by: $\sin\beta_s = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{\sin\beta \sin\gamma}{\sin(\beta+\chi)}$

Silva, Wolfenstein

$$\boxed{\sin\beta_s = \frac{|V_{td}|}{|V_{ts}|} \frac{|V_{cd}|}{|V_{cs}|} \sin\beta}$$

Extraction of θ_d :

$$\tan\theta_d = \frac{R_u \sin(\gamma+\bar{\beta}) - \sin\bar{\beta}}{\cos\bar{\beta} - R_u \cos(\gamma+\bar{\beta})}$$

where

$$R_u = \frac{|V_{ud}||V_{ub}|}{|V_{cd}||V_{cb}|}$$

Extraction of θ_s :

$\bar{\beta}_s$

$$\tan\theta_s = \frac{\sin\bar{\chi} - C\sin(\gamma-\bar{\beta}_s)}{C\cos(\gamma-\bar{\beta}_s) + \cos\bar{\beta}_s}$$

$$C = \frac{|V_{us}||V_{ub}|}{|V_{cs}||V_{cb}|}$$

There is Strong motivation to have New Physics with potential implications for CP Violation.

A multi-billion reais (or CH, or €, or \$)

Question : At what scale should one expect New Physics to be manifest?

Answer : Nobody knows.

A situation very different from what one encountered in the case of :

Neutral currents, charm, etc

Conclusions

(i) The Origin of CP Violation is a deep, open question in Particle Physics.

(ii) It is crucial to continue testing the SM and its CKM mechanism of mixing and CP violation.

(iii) Remember Neutrinos!!

In the SM neutrinos are strictly massless!