QCD

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2^{nd} Lecture

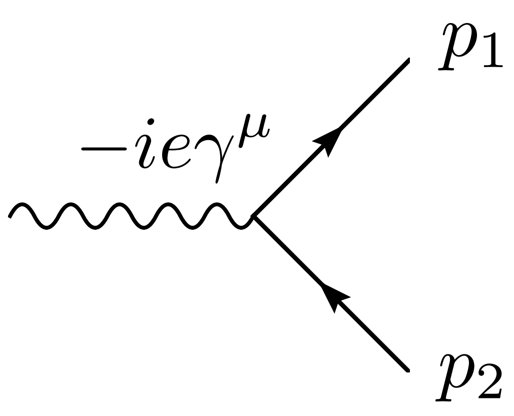
Summer School, Corfu — September 2015

The soft approximation

Let's consider again the R-ratio. This is determined by $\gamma^* \to q\bar{q}$

At leading order:

$$M_0^\mu = \bar{u}(p_1)(-ie\gamma^\mu)v(p_2)$$



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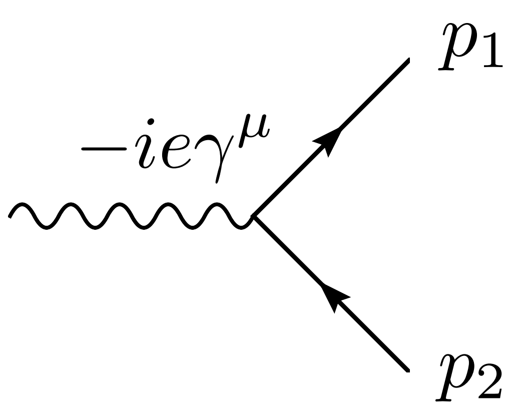
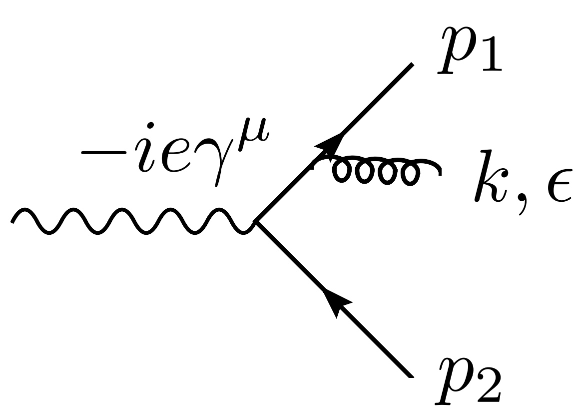


Emit one gluon:

$$\begin{aligned} M^\mu_{q\bar{q}g} &= \bar{u}(p_1)(-ig_s t^a \not{\epsilon})\frac{i(\not{p}_1+\not{k})}{(p_1+k)^2}(-ie\gamma^\mu)v(p_2) \\ &+ \bar{u}(p_1)(-ie\gamma^\mu)\frac{i(\not{p}_2-\not{k})}{(p_2-k)^2}(-ig_s t^a \not{\epsilon})v(p_2) \end{aligned}$$



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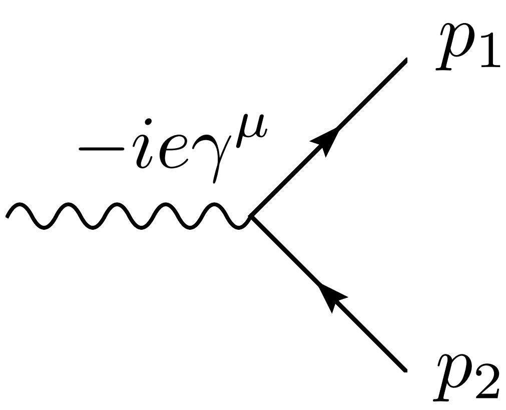

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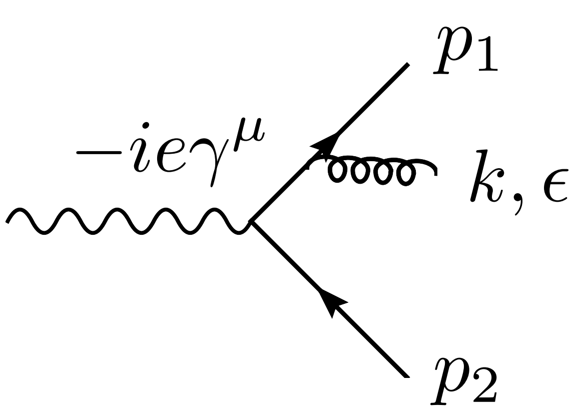

Consider the soft approximation: $k \ll p_1, p_2$

$$M^\mu_{q\bar{q}g} = \bar{u}(p_1)\left((-ie\gamma^\mu)(-ig_s t^a)v(p_2)\right)\left(\frac{p_1\epsilon}{p_1 k} - \frac{p_2\epsilon}{p_2 k}\right)$$

$\Rightarrow$ factorization of soft part (crucial for resummed calculations)

Soft divergences

The squared amplitude becomes

$$|M^{\mu}_{q\bar{q}g}|^2 = \sum_{\text{pol}} \left| \bar{u}(p_1)\left((-ie\gamma^{\mu})(-ig_s t^a)v(p_2)\right)\left(\frac{p_1\epsilon}{p_1 k} - \frac{p_2\epsilon}{p_2 k}\right)\right|^2$$

$$= \quad |M_{q\bar{q}}|^2 C_F g_s^2 \frac{2p_1p_2}{(p_1k)(p_2k)}$$

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= & \quad |M_{q\bar{q}}|^2 C_F g_s^2 \frac{2 p_1 p_2}{(p_1 k)(p_2 k)}
\end{aligned}
$$

Including phase space

$$
\begin{aligned}
d\phi_{q\bar{q}g} |M_{q\bar{q}g}|^2 &= d\phi_{q\bar{q}} |M_{q\bar{q}}|^2 \frac{d^3 k}{2\omega (2\pi)^3} C_F g_s^2 \frac{2 p_1 p_2}{(p_1 k)(p_2 k)} \\
&= d\phi_{q\bar{q}} |M_{q\bar{q}}|^2 \omega d\omega d\cos\theta \frac{d\phi}{2\pi} \frac{2\alpha_s C_F}{\pi} \frac{1}{\omega^2 (1 - \cos^2\theta)}
\end{aligned}
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Including phase space

$$d\phi_{q\bar{q}g}|M_{q\bar{q}g}|^2 \quad = \quad d\phi_{q\bar{q}}|M_{q\bar{q}}|^2 \frac{d^3k}{2\omega(2\pi)^3} C_F g_s^2 \frac{2p_1p_2}{(p_1k)(p_2k)}$$

$$= \quad d\phi_{q\bar{q}}|M_{q\bar{q}}|^2 \omega d\omega d\cos\theta \frac{d\phi}{2\pi}\frac{2\alpha_s C_F}{\pi}\frac{1}{\omega^2(1-\cos^2\theta)}$$

The differential cross section is

$$d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}}\frac{2\alpha_s C_F}{\pi}\frac{d\omega}{\omega}\frac{d\theta}{\sin\theta}\frac{d\phi}{2\pi}$$

Soft & collinear divergences

Cross section for producing a qq̄-pair and a gluon is infinite (IR divergent)

$$d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}} \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

<u>$\omega \to 0$:</u> soft divergence

<u>$\theta \to 0$:</u> collinear divergence

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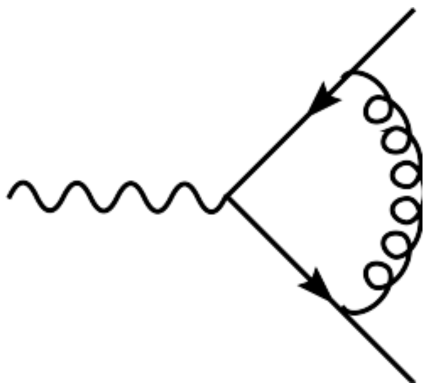
$\omega \to 0$: soft divergence

$\theta \to 0$: collinear divergence

But the full O(α_s) correction to R is finite, because one must include a virtual correction which cancels the divergence of the real radiation

$$d\sigma_{q\bar{q},v} \sim -d\sigma_{q\bar{q}} \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$



NB: here we kept only soft terms, if we do the full calculation one gets a finite correction of α_s/π

Soft & collinear divergences

$\underline{\omega \to 0}$ soft divergence: the four-momentum of the emitted particle approaches zero, typical of gauge theories, even if matter (radiating particle) is massive

$\underline{\theta \to 0}$ collinear divergence: particle emitted collinear to emitter. Divergence present only if all particles involved are massless

NB: the appearance of soft and collinear divergences discussed in the specific contect of $e^+e^- \to qq$ are a general property of QCD

Infrared safety (= finiteness)

So, the R-ratio is an infrared safe quantity.

In perturbation theory one can compute only IR-safe quantities, otherwise get infinities, which can not be renormalized away (why not?)

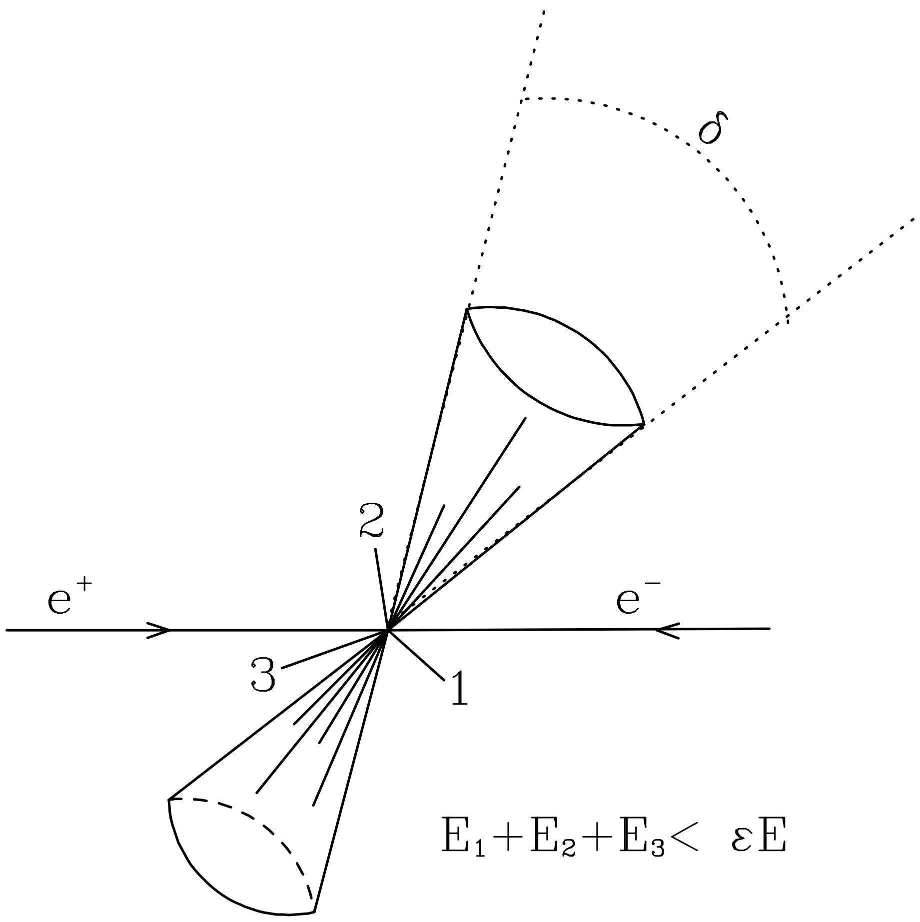
So, the natural questions are:

- are there other IR-safe quantities?
- what property of R guarantees its IR-safety?

Sterman-Weinberg jets

First formulation of cross-sections which are finite in perturbation theory and describe the hadronic final state

Introduce two parameters ε and δ: a pair of Sterman-Weinberg jets are two cones of opening angle δ that contain all the energy of the event excluding at most a fraction ε



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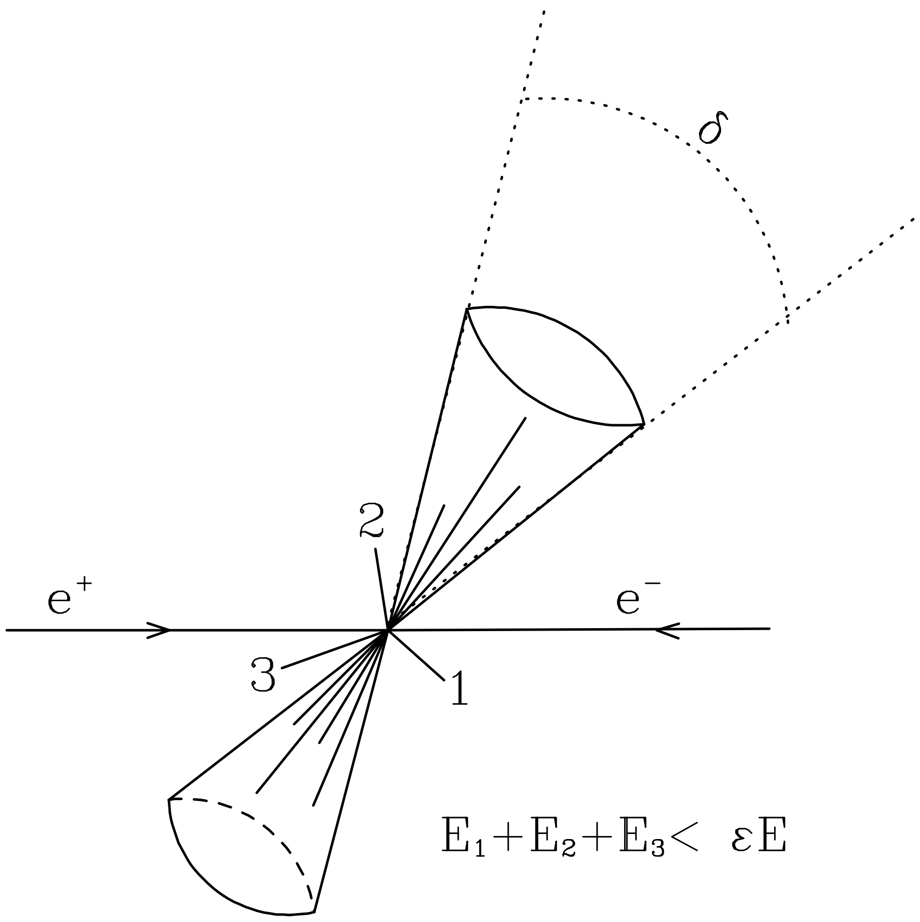

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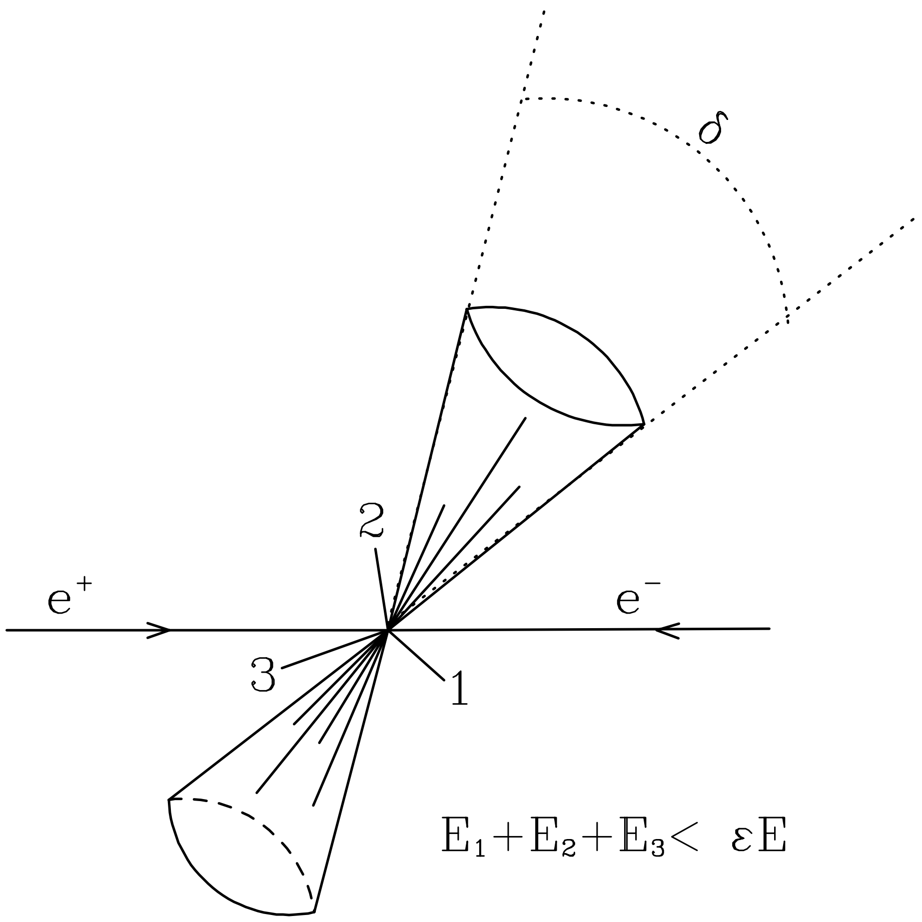

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Kinoshita-Lee-Nauenberg (KLN) theorem:
final-state infrared divergences cancel in measurable quantities (transition probabilities, cross-sections summed over indistinguishable states...)

Sterman-Weinberg jets

The Sterman-Weinberg jet cross-section up to $O(\alpha_s)$ is given by

$$\sigma_1 = \sigma_0 \left(1 + \frac{2\alpha_s C_F}{\pi} \ln\epsilon \ln\delta^2\right)$$

Effective expansion parameter in QCD is often $\alpha_s C_F/\pi$ not α_s

α_s-expansion enhanced by a double log: left-over from real-virtual cancellation

- if more gluons are emitted, one gets for each gluon
  - a power of $\alpha_s C_F/\pi$
  - a soft logarithm $\ln\varepsilon$
  - a collinear logarithm $\ln\delta$
- if ε and/or δ become too small the above result diverges
- if the logs are large, fixed order meaningless, one needs to resum large infrared and collinear logarithms to all orders in the coupling constant

Infrared safety: definition

An observable $\mathcal{O}$ is infrared and collinear safe if

$$\mathcal{O}_{n+1}(k_1, k_2, \ldots, k_i, k_j, \ldots k_n) \to \mathcal{O}_n(k_1, k_2, \ldots k_i + k_j, \ldots k_n)$$

whenever one of the k_i/k_j becomes soft or k_i and k_j are collinear

i.e. the observable is insensitive to emission of soft particles or to collinear splittings

Infrared safety: examples

Infrared safe ?

- energy of the hardest particle in the event
- multiplicity of gluons
- momentum flow into a cone in rapidity and angle
- cross-section for producing one gluon with $E > E_{min}$ and $\theta > \theta_{min}$
- jet cross-sections

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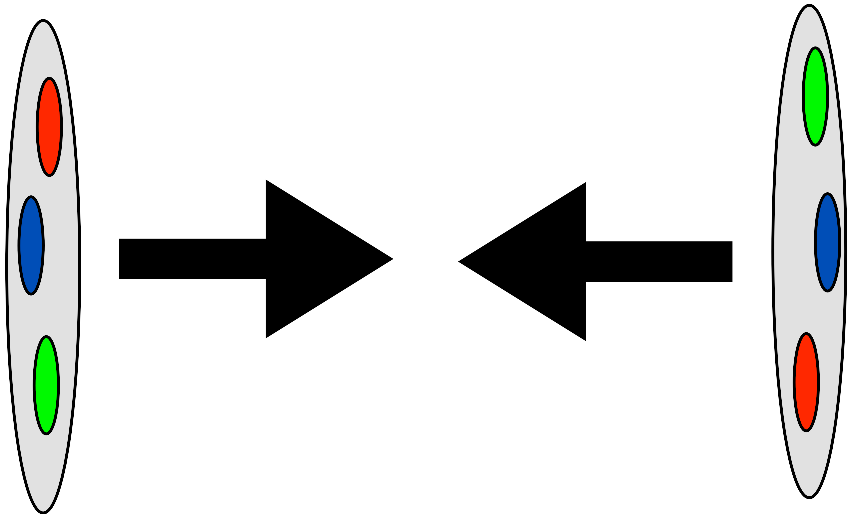
Partons in the initial state

- We talked a lot about final state QCD effects
- This is the only thing to worry about at e^+e^- colliders (LEP)
- Hera/Tevatron/LHC involve protons in the initial state
- Proton are made of QCD constituents

Next we will focus mainly on aspects related to initial state effects



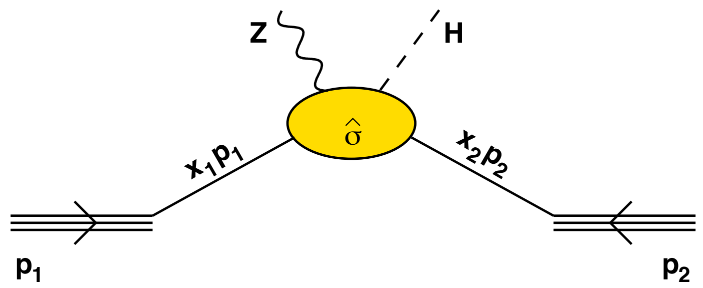
The parton model

Basic idea of the parton model: intuitive picture where in a high transverse momentum scattering partons behave as quasi free in the collision

$\Rightarrow$ cross section is the incoherent sum of all partonic cross-sections

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s) \qquad \hat{s} = x_1 x_2 s$$

NB: This formula is wrong/incomplete (see later)



$f_i^{(P_j)}(x_i)$: parton distribution function (PDF) is the probability to find parton i in hadron j with a fraction x_i of the longitudinal momentum (transverse momentum neglected), extracted from data

$\hat{\sigma}(x_1 x_2 s)$: partonic cross-section for a given scattering process, computed in perturbative QCD

Sum rules

Momentum sum rule: conservation of incoming total momentum

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Conservation of flavour: e.g. for a proton

$$\int_0^1 dx \left(f_u^{(p)}(x) - f_{\bar{u}}^{(p)}(x) \right) = 2$$

$$\int_0^1 dx \left(f_d^{(p)}(x) - f_{\bar{d}}^{(p)}(x) \right) = 1$$

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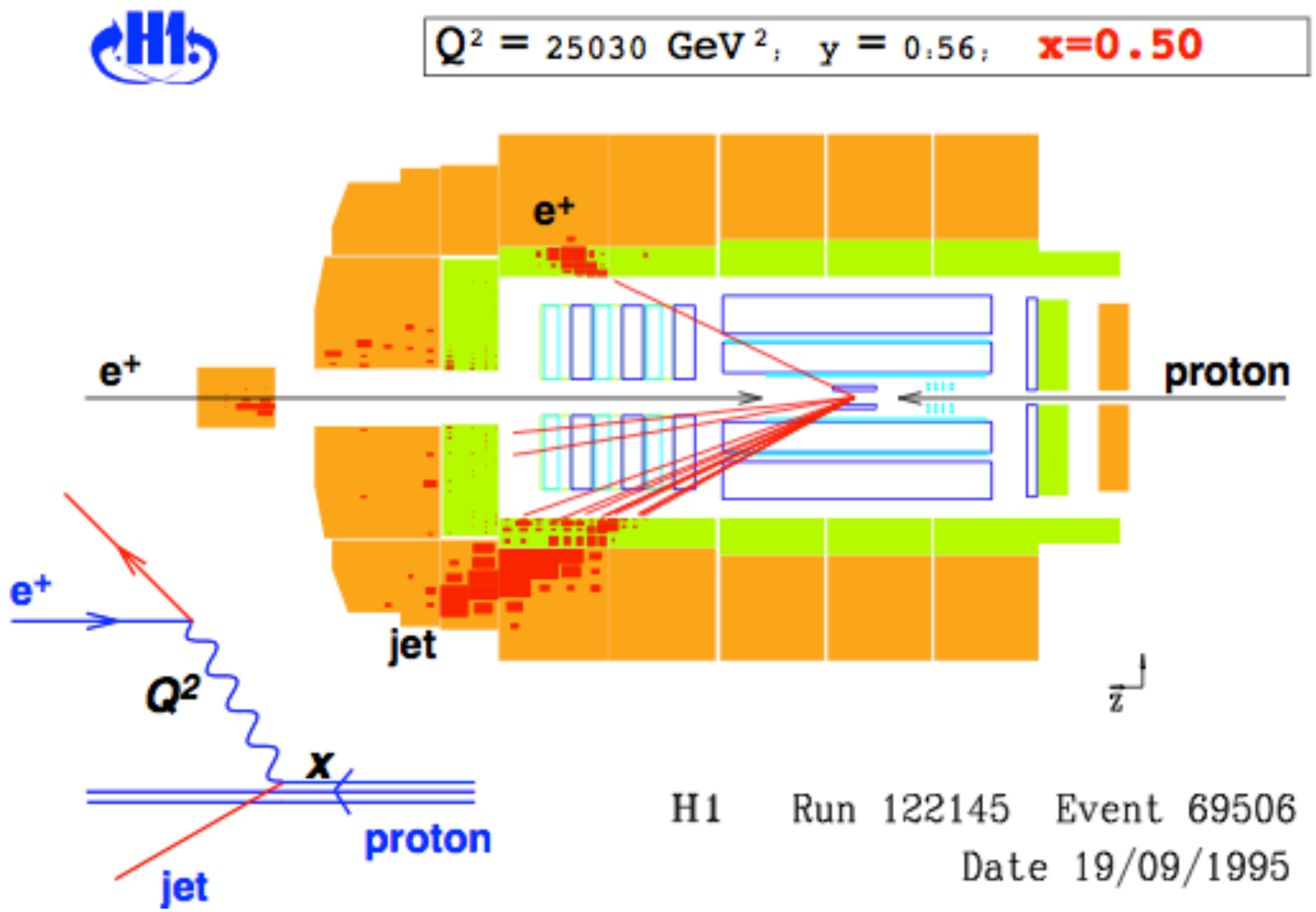
How can parton densities be extracted from data?

Deep inelastic scattering

Easier than processes with two incoming hadrons is the scattering of a lepton on a (anti)-proton



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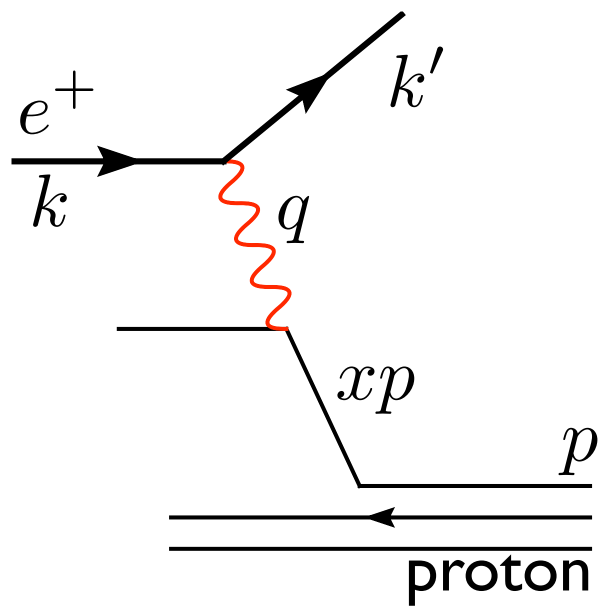

Kinematics:

$$Q^2 = -q^2 \quad s = (k+p)^2 \quad x_{Bj} = \frac{Q^2}{2p\cdot q} \quad y = \frac{p\cdot q}{k\cdot p}$$

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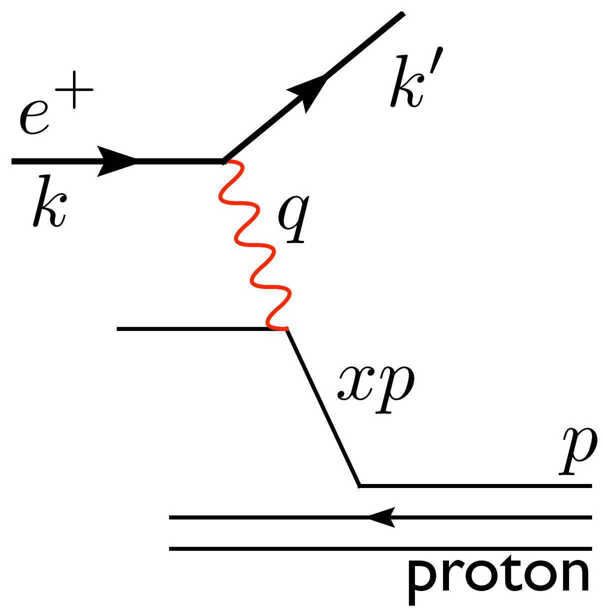

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Partonic variables:

$$\hat{p} = xp \quad \hat{s} = (k+\hat{p})^2 = 2k\cdot\hat{p} \quad \hat{y} = \frac{\hat{p}\cdot q}{k\cdot\hat{p}} = y \quad (\hat{p}+q)^2 = 2\hat{p}\cdot q - Q^2 = 0$$

$$\Rightarrow x = x_{Bj}$$

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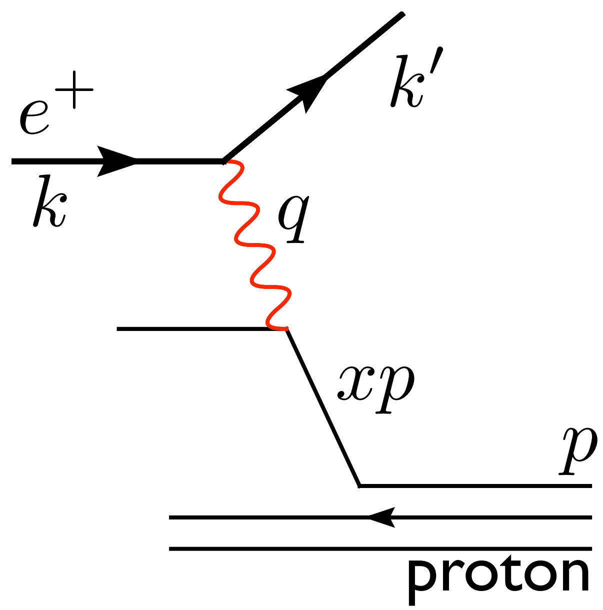

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Partonic cross section:

(just apply QED Feynman rules and add phase space)

$$\boxed{\frac{d\hat{\sigma}}{d\hat{y}} = q_l^2 \frac{\hat{s}}{Q^4} 2\,\pi\,\alpha_{em}\left(1+(1-\hat{y})^2\right)}$$

Deep inelastic scattering

Hadronic cross section:

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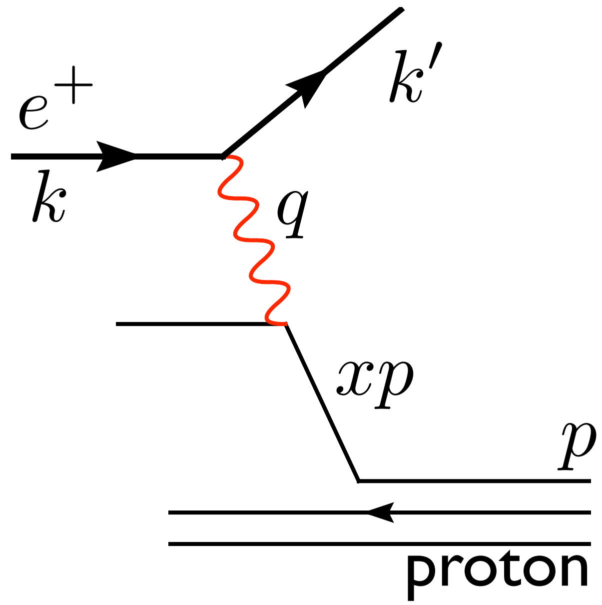
$$\frac{d\sigma}{dy\,dx_{Bj}} = \sum_l f_l^{(p)}(x) \frac{d\hat{\sigma}}{d\hat{y}}$$
$$= \frac{2\pi\,\alpha_{em}^2 s x_{Bj}}{Q^4} \left(1 + (1-y)^2\right) \sum_l q_l^2 f_l^{(p)}(x_{Bj})$$



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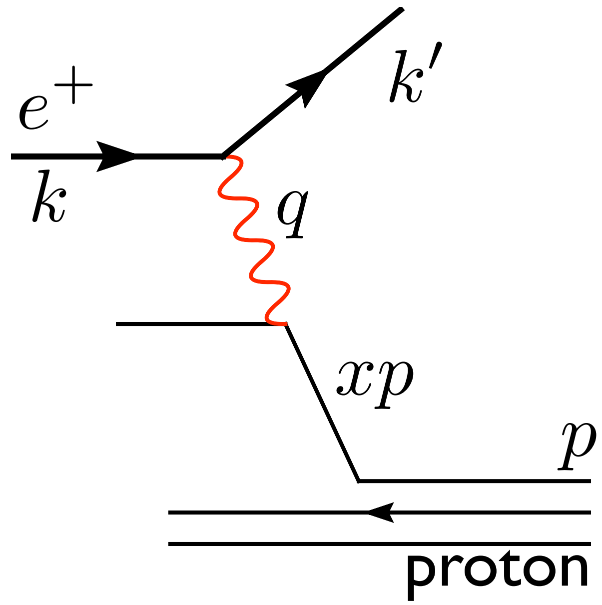

1. at fixed x_{Bj} and y the cross-section scales with s
2. the y-dependence of the cross-section is fully predicted and is typical of vector interaction with fermions ⇒Callan-Gross relation
3. can access (sums of) parton distribution functions
4. Bjorken scaling: pdfs depend on x and not on Q^2

The structure function F_2

$$\frac{d\sigma}{dydx} = \frac{2\pi\alpha_{em}^2 s}{Q^4}\left(1 + (1-y^2\right) F_2(x) \qquad F_2(x) = \sum_l x q_l^2 f_l^{(p)}(x)$$

F_2 is called structure function (describes structure/constituents of nucleus)

For electron scattering on proton

$$F_2(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

NB: use perturbative language of quarks and gluons despite the fact that parton distribution are non-perturbative

Question: F_2 gives only a linear combination of u and d. How can they be extracted separately?

Isospin

Neutron is like a proton with u & d exchanged

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$$F_2^p(x) = x\left(\frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)\right)$$

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For electron scattering on a neutron

$$F_2^n(x) = x\left(\frac{1}{9}d_n(x) + \frac{4}{9}u_n(x)\right) = x\left(\frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)\right)$$

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F_2^n and F_2^p allow determination of u_p and d_p separately

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NB: experimentally get F_2^n from deuteron: $F_2^d(x) = F_2^p(x) + F_2^n(x)$

Sea quark distributions

Inside the proton there are fluctuations, and pairs of $u\bar{u}$,$d\bar{d}$,$c\bar{c}$,$s\bar{s}$ ... can be created

An infinite number of pairs can be created as long as they have very low momentum, because of the momentum sum rules.

We saw before that when we say that the proton is made of uud what we mean is

$$\int_0^1 dx\,(u_p(x) - \bar{u}_p(x)) = 2 \qquad \int_0^1 dx\,\left(d_p(x) - \bar{d}_p(x)\right) = 1$$

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Photons interact in the same way with u(d) and $\bar{u}$($\bar{d}$)

How can one measure the difference?

Question: What interacts differently with particle and antiparticle?

Sea quark distributions

Inside the proton there are fluctuations, and pairs of $u\bar{u}$,$d\bar{d}$,$c\bar{c}$,$s\bar{s}$ ... can be created

An infinite number of pairs can be created as long as they have very low momentum, because of the momentum sum rules.

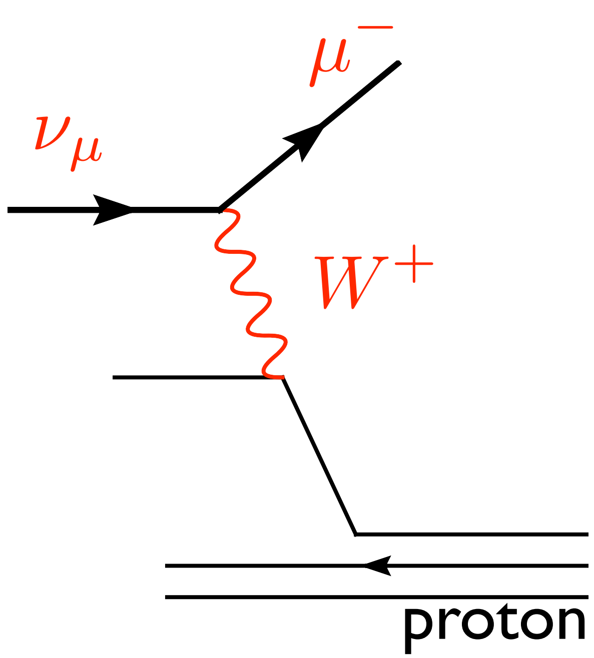
We saw before that when we say that the proton is made of uud what we mean is

$$\int_0^1 dx\,(u_p(x) - \bar{u}_p(x)) = 2 \qquad \int_0^1 dx\,\left(d_p(x) - \bar{d}_p(x)\right) = 1$$

Photons interact in the same way with u(d) and $\bar{u}$($\bar{d}$)

How can one measure the difference?

Question: What interacts differently with particle and antiparticle? W^+/W^- from neutrino scattering



Check of the momentum sum rule

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

u_v	0.267
d_v	0.111
u_s	0.066
d_s	0.053
s_s	0.033
c_c	0.016
total	0.546

➟ *half of the longitudinal momentum carried by gluons*

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$\gamma/W^{+/-}$ don't interact with gluons

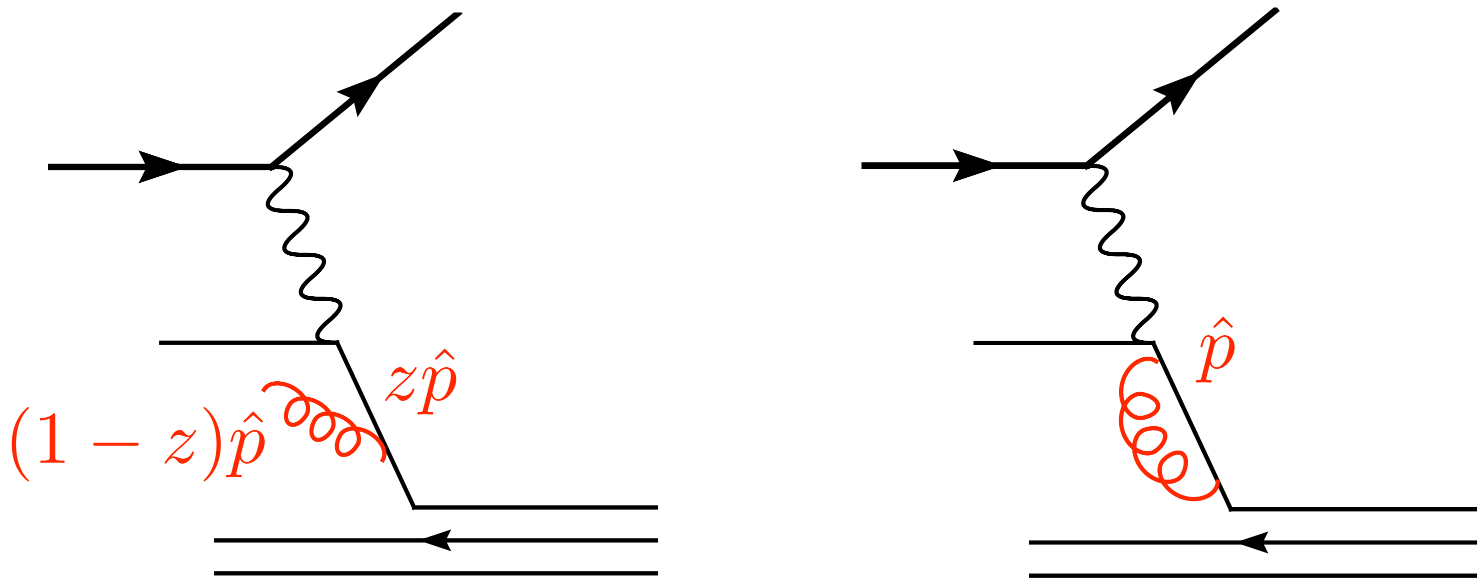
How can one measure gluon parton densities?

We need to discuss radiative effects first

Radiative corrections

To first order in the coupling:

need to consider the emission of one real gluon and a virtual one



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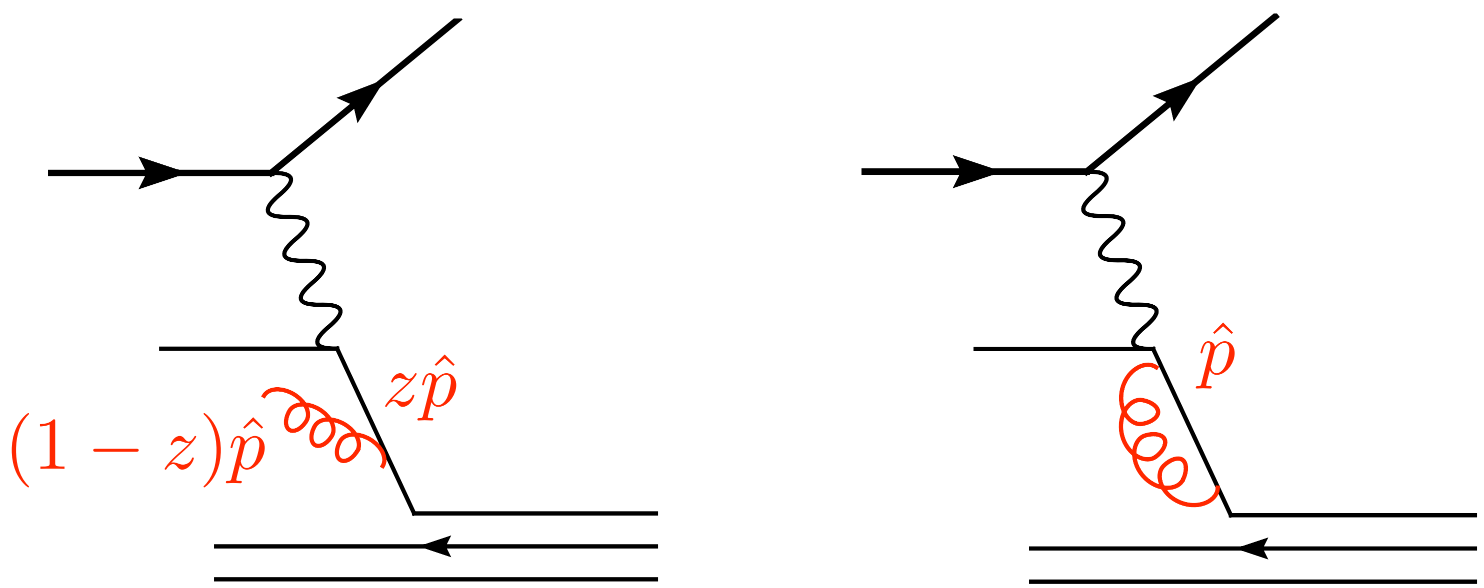

Adding real and virtual contributions, the partonic cross-section reads

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int dz \frac{dk_\perp^2}{k_\perp^2} \frac{1+z^2}{1-z} \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Partial cancellation between real (positive), virtual (negative), but real gluon changes the energy entering the scattering, the virtual does not

Radiative corrections

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int dz \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right) , \quad P(z) = C_F \frac{1+z^2}{1-z}$$

Soft limit: singularity at z=1 cancels between real and virtual terms

Collinear singularity: $k_\perp \to 0$ with finite z. Collinear singularity does not cancel because partonic scatterings occur at different energies

Radiative corrections

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$\Rightarrow$ naive parton model does not survive radiative corrections

Radiative corrections

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$\Rightarrow$ naive parton model does not survive radiative corrections

Similarly to what is done when renormalizing UV divergences, collinear divergences from initial state emissions are absorbed into parton distribution functions

The plus prescription

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

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Plus prescription makes the universal cancelation of singularities explicit

$$\int_0^1 dz f_+(z) g(z) \equiv \int_0^1 f(z) \left(g(z) - g(1) \right)$$

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$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int dz \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} P_+(z) \sigma^{(0)}(z\hat{p}) \,, \quad P(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

Collinear singularities still there, but they factorize.

Factorization scale

Schematically use $$\ln\frac{Q^2}{\lambda^2} = \ln\frac{Q^2}{\mu_F^2} + \ln\frac{\mu_F^2}{\lambda^2}$$

$$\sigma = \sigma^{(0)} + \sigma^{(1)} = \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{\mu_F^2}{\lambda^2}P_+\right) \times \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu_F^2}P_+\right)\sigma^{(0)}$$

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So we define

$$f_q(x,\mu_F) = f_q(x) \times \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{\mu_F^2}{\lambda^2}P_{qq}^{(0)}\right) \qquad \hat{\sigma}(p,\mu_F) = \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu_F^2}P_{qq}^{(0)}\right)\sigma^{(0)}(p)$$

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NB:

- universality, i.e. the PDF redefinition does not depend on the process
- choice of $\mu_F \sim Q$ avoids large logarithms in partonic cross-sections
- PDFs and hard cross-sections don't evolve independently
- the factorization scale acts as a cut-off, it allows to move the divergent contribution into non-pertubative parton distribution functions

Improved parton model

Naive parton model:

$$\xcancel{\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s)} \qquad \hat{s} = x_1 x_2 s$$

After radiative corrections:

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1, \mu^2) f_2^{(P_2)}(x_2, \mu^2) \hat{\sigma}(x_1 x_2 s, \mu^2)$$

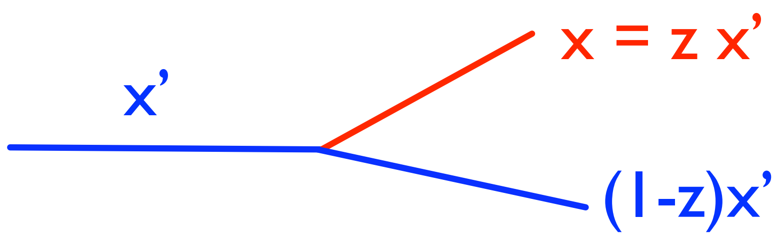
Intermediate recap

- With initial state parton collinear singularities don't cancel
- Initial state emissions with $k_\perp$ below a given scale are included in PDFs
- This procedure introduces a scale μ_F, the so-called factorization scale which factorizes the low energy (non-perturbative) dynamics from the perturbative hard cross-section
- As for the renormalization scale, the dependence of cross-sections on μ_F is due to the fact that the perturbative expansion has been truncated
- The dependence on μ_F becomes milder when including higher orders

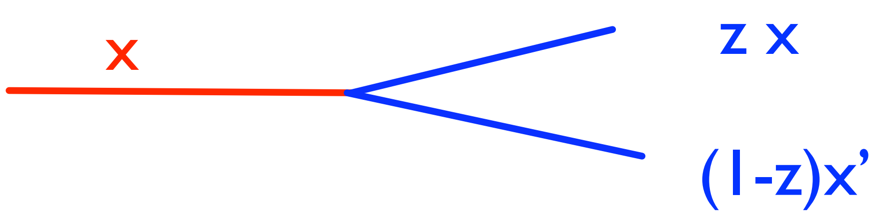
Evolution of PDFs

A parton distribution changes when

- a different parton splits and produces it



- the parton itself splits



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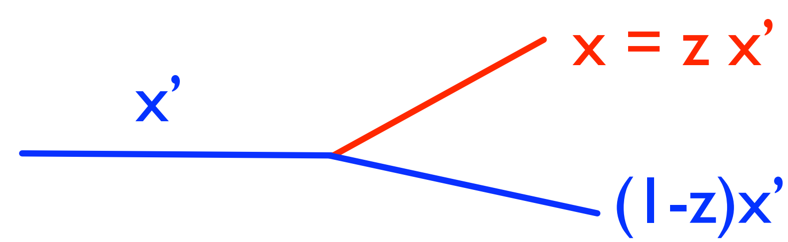

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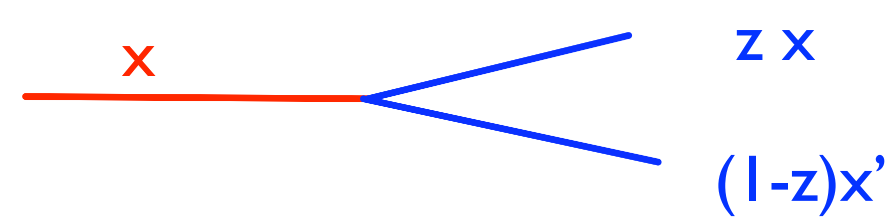


$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_0^1 dx' \int_x^1 dz \frac{\alpha_s}{2\pi} P(z) f(x',\mu^2) \delta(zx'-x) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f(x,\mu^2)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f\left(x,\mu^2\right)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z},\mu^2\right)$$

The plus prescription $\quad \int_0^1 dz f_+(z) g(z) \equiv \int_0^1 dz f(z) \left(g(z) - g(1)\right)$

DGLAP equation

$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right)$$

Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

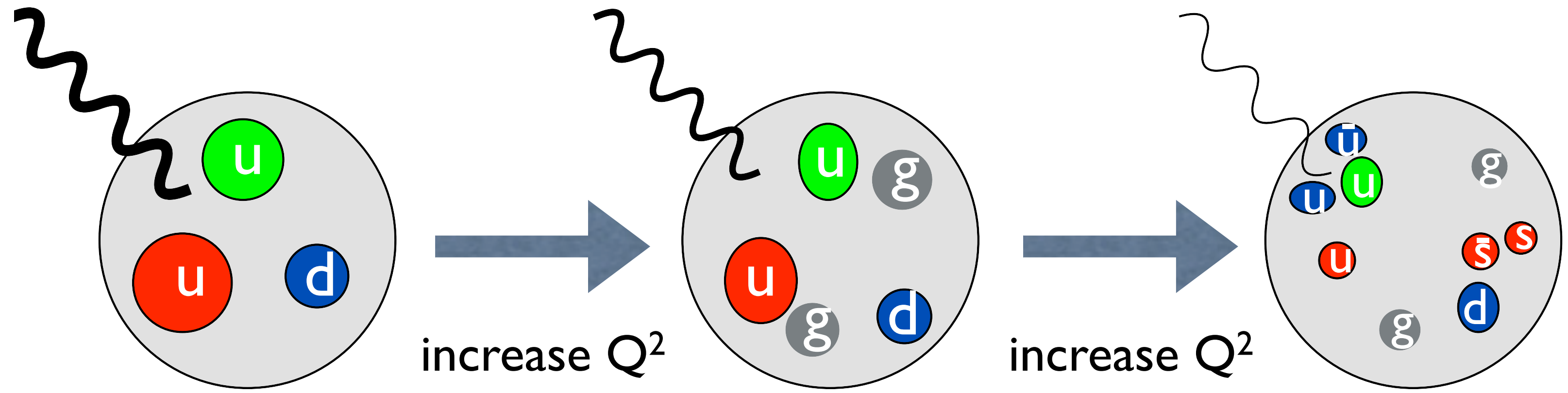
Master equation of QCD: we can not compute parton densities, but we can predict how they evolve from one scale to another

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

Evolution

So, in perturbative QCD we can not predict values for

- the coupling
- the masses
- the parton densities
- ...



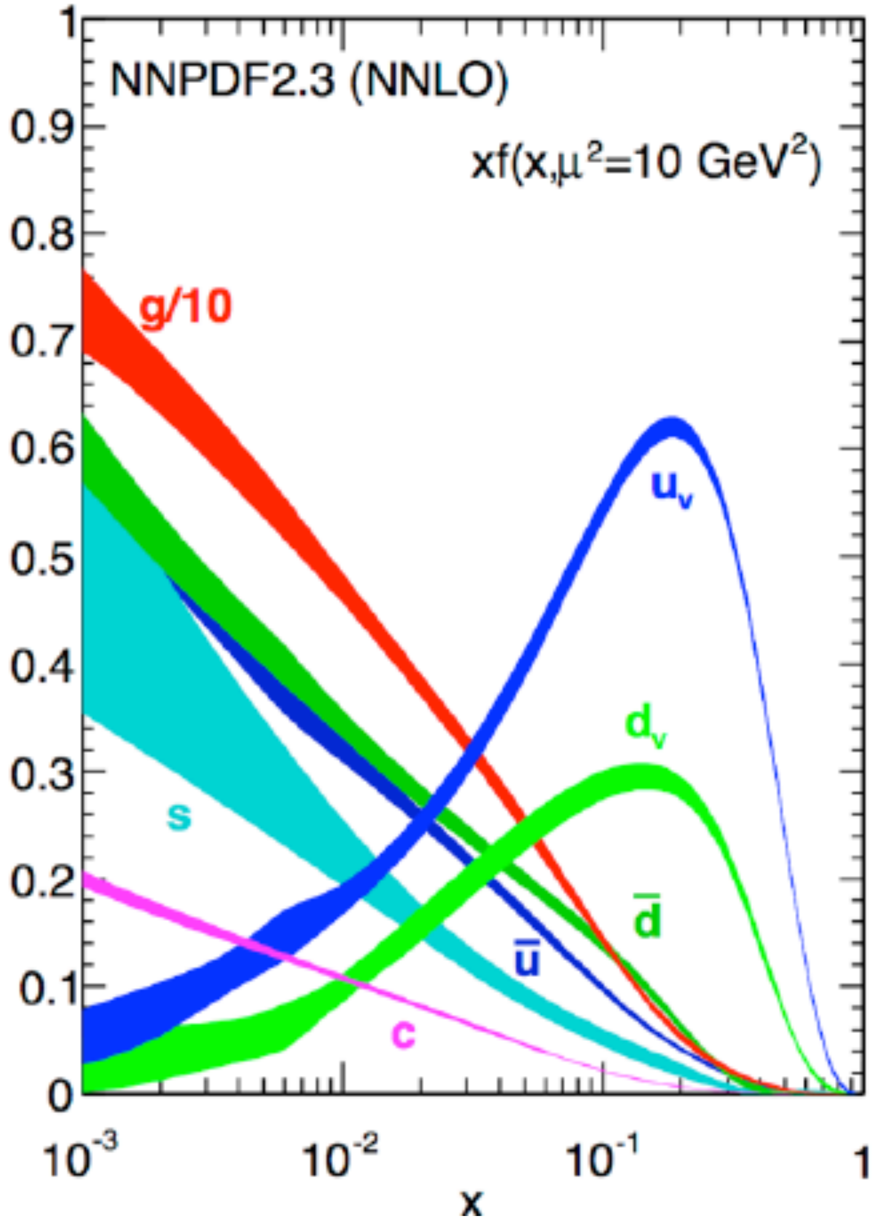
What we can predict is the evolution with the Q^2 of those quantities. These quantities must be extracted at some scale from data.

- not only is the coupling scale-dependent, but partons have a scale dependent sub-structure
- we started with the question of how one can access the gluon pdf: In DIS: because of the DGLAP evolution, we can access the gluon pdf indirectly, through the way it changes the evolution of quark pdfs. Today also direct measurements using Tevatron jet data and LHC tt production

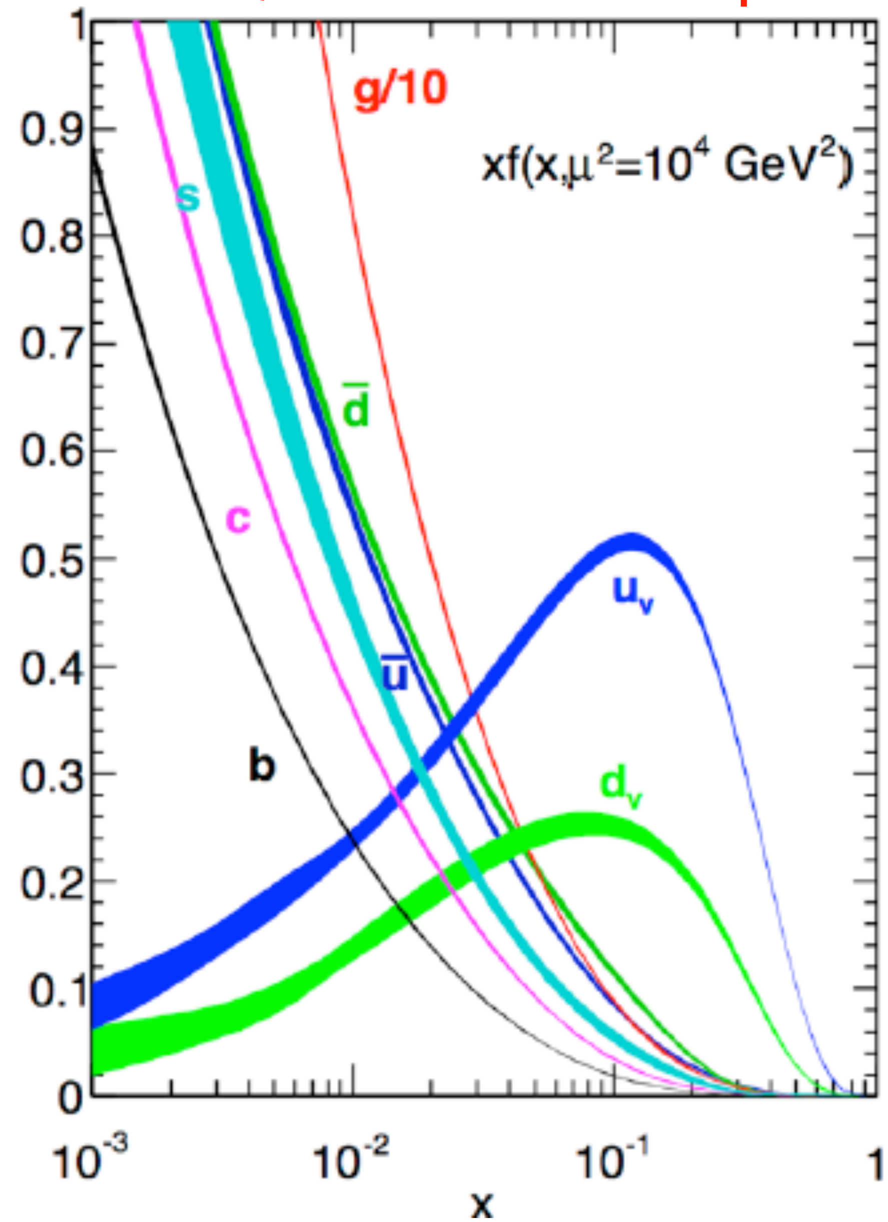
DGLAP Evolution

The DGLAP evolution is a key to precision LHC phenomenology: it allows to measure PDFs at some scale (say in DIS) and evolve upwards to make LHC (7, 8, 13, 14, 33, 100.... TeV) predictions

Measure PDFs at 10 GeV



Evolve in Q^2 and make LHC predictions



Different PDFs evolve in different ways (different equations + unitarity constraint)

Progress in PDFs

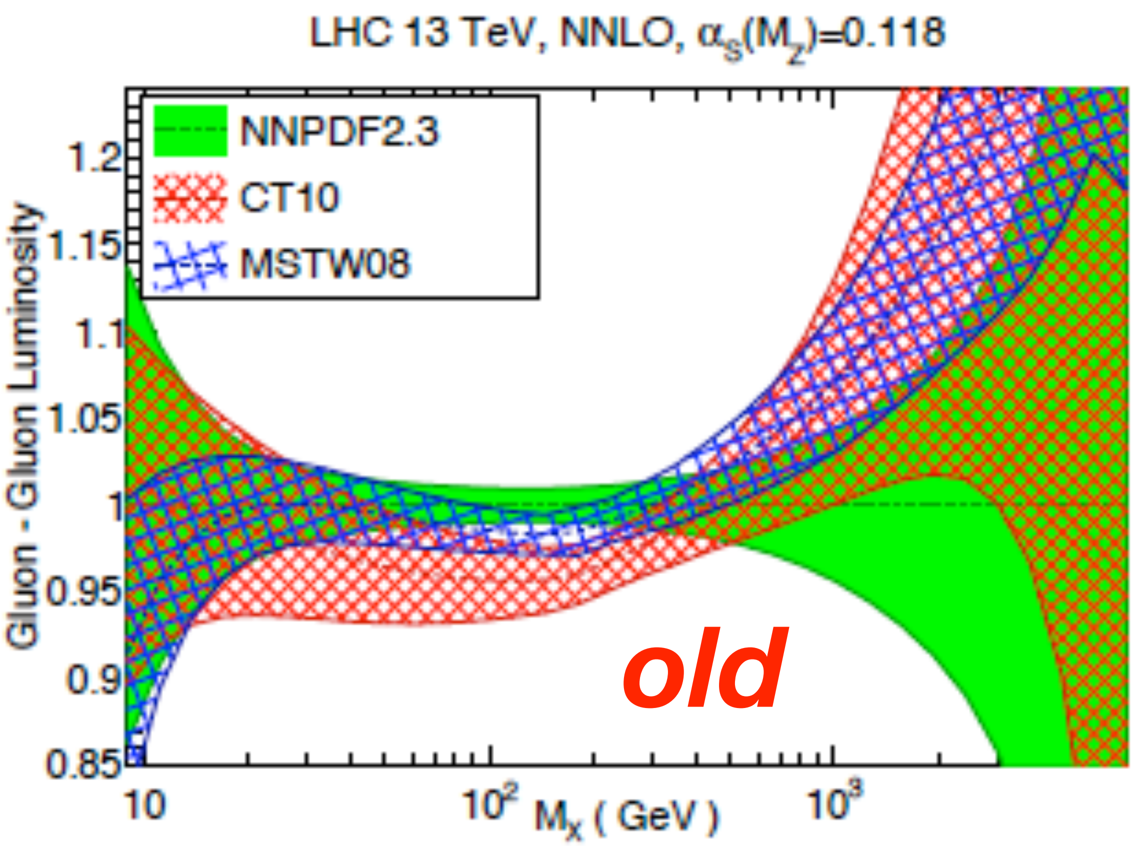
PDFs are an essential ingredient for the LHC program.

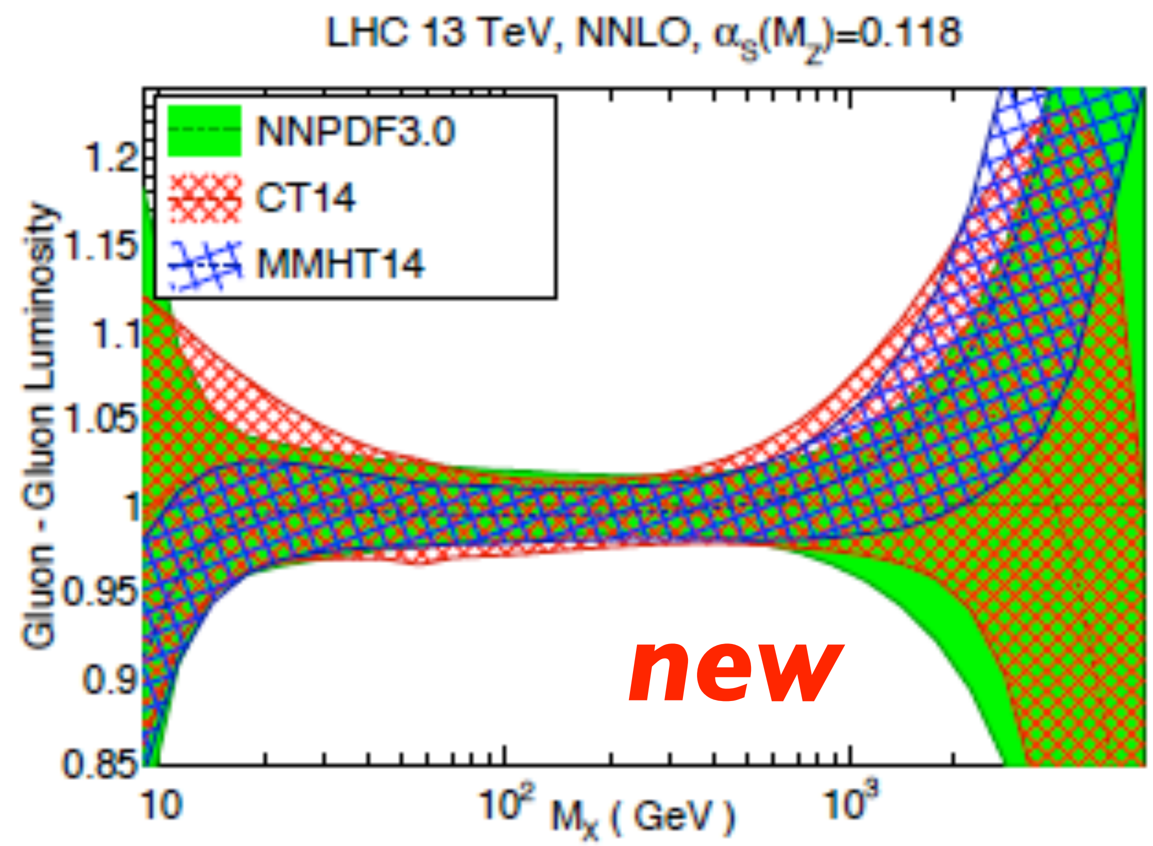
Recent progress includes

- better assessment of uncertainties (e.g. different groups now agree at the 1σ level where data is available)
- exploit wealth of new information from LHC Run I measurements
- progress in tools and methods to include these data in the fits

Progress in PDFs: gluon luminosity

<u>Example</u>: gluon-gluon luminosity as needed for Higgs measurements



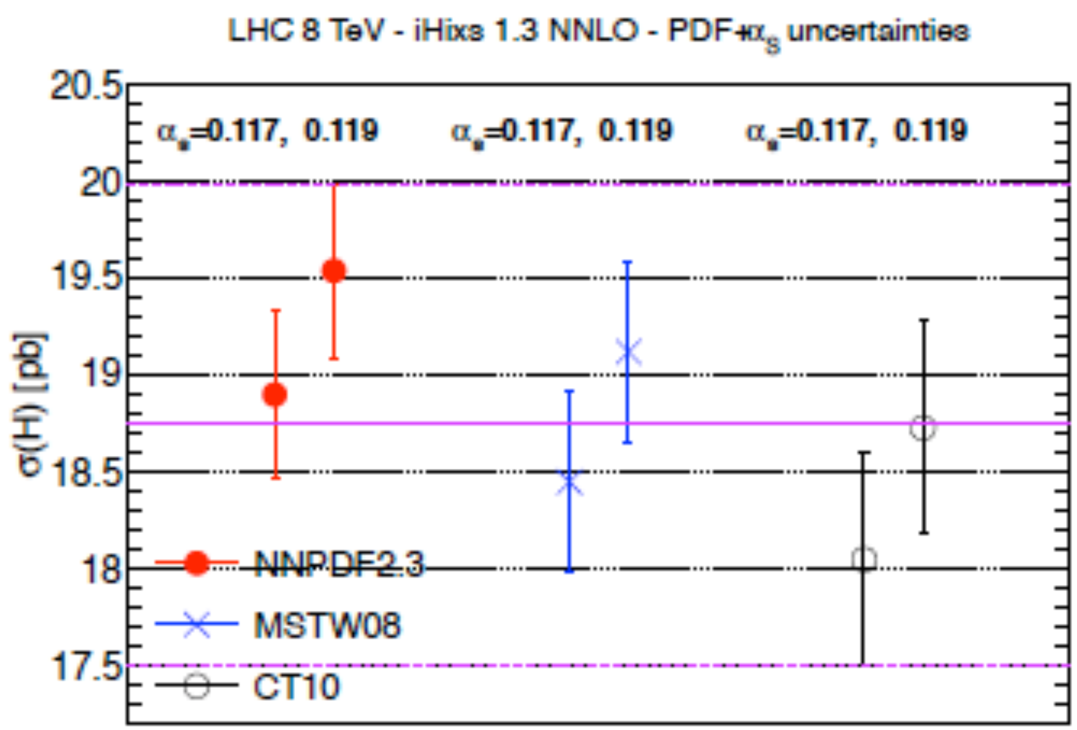


- obvious improvement from older sets to newer ones
- agreement at 1σ between different PDFs in the intermediate mass region relevant for Higgs studies (but larger differences at large M, key-region for NP searches)

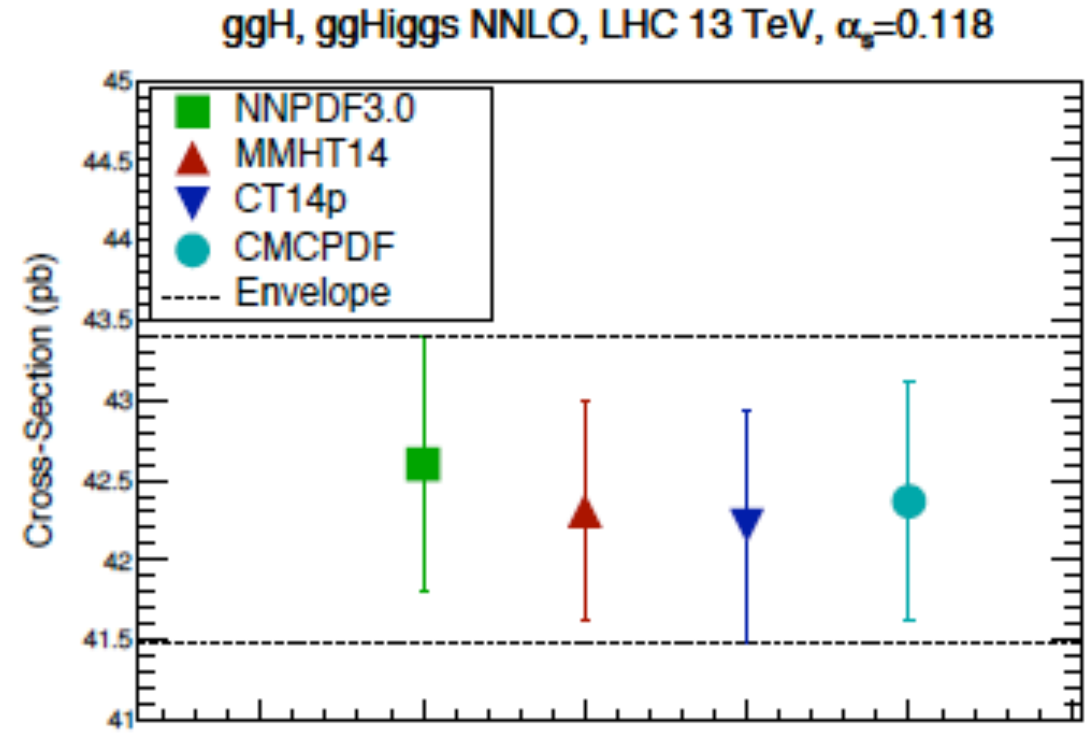
Progress in PDFs: Higgs case

Improved control on gluon distributions results in more consistent Higgs production cross-sections

2012



2015



- PDF uncertainty in the Higgs cross-section down to about 2-3%
- envelope of 3 PDFs (previous recommendation) no longer needed

Perturbative calculations

Perturbative calculations rely on the idea of an order-by-order expansion in the small coupling

$$\sigma \sim A + B\alpha_s + C\alpha_s^2 + D\alpha_s^3 + \ldots$$

LO NLO NNLO NNNLO

- Perturbative calculations are possible because the coupling is small at high energy
- In QCD (or in a generic QFT) the coupling depends on the energy (renormalization scale)
- So changing scale the result changes. By how much? What does this dependence mean?
- Let's consider some examples

Leading order n-jet cross-section

- Consider the cross-section to produce n jets. The leading order result at scale μ result will be

$$\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \ldots)$$

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- Instead, choosing a scale μ' one gets

$$\sigma^{\rm LO}_{\rm njets}(\mu') = \alpha_s(\mu')^n A(p_i, \epsilon_i, \ldots) = \alpha_s(\mu)^n \left(1 + n\, b_0\, \alpha_s(\mu) \ln \frac{\mu^2}{\mu'^2} + \ldots\right) A(p_i, \epsilon_i, \ldots)$$

So the change of scale is a NLO effect ($\propto\alpha_s$), but this becomes more important when the number of jets increases ($\propto$n)

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So the change of scale is a NLO effect ($\propto \alpha_s$), but this becomes more important when the number of jets increases ($\propto$n)

- Notice that at Leading Order the normalization is not under control:

$$\frac{\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu)}{\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu')} = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu')}\right)^n$$

NLO n-jet cross-section

Now consider n-jet cross-section at NLO. At scale μ the result reads

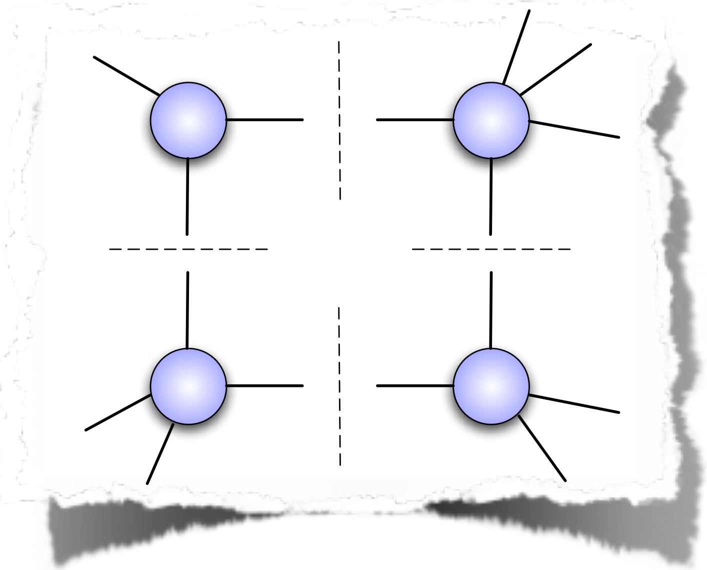
$$\sigma^{\rm NLO}_{\rm njets}(\mu) = \alpha_s(\mu)^n A(p_i,\epsilon_i,\dots) + \alpha_s(\mu)^{n+1}\left(B(p_i,\epsilon_i,\dots) - n b_0 \ln\frac{\mu^2}{Q_0^2}\right) + \dots$$

- So the NLO result compensates the LO scale dependence. The residual dependence is NNLO
- Scale dependence and normalization start being under control only at NLO, since a compensation mechanism kicks in
- Notice also that a good scale choice automatically resums large logarithms to all orders, while a bad one spuriously introduces large logs and ruins the PT expansion
- Scale variation is conventionally used to estimate the theory uncertainty, but the validity of this procedure should not be overrated

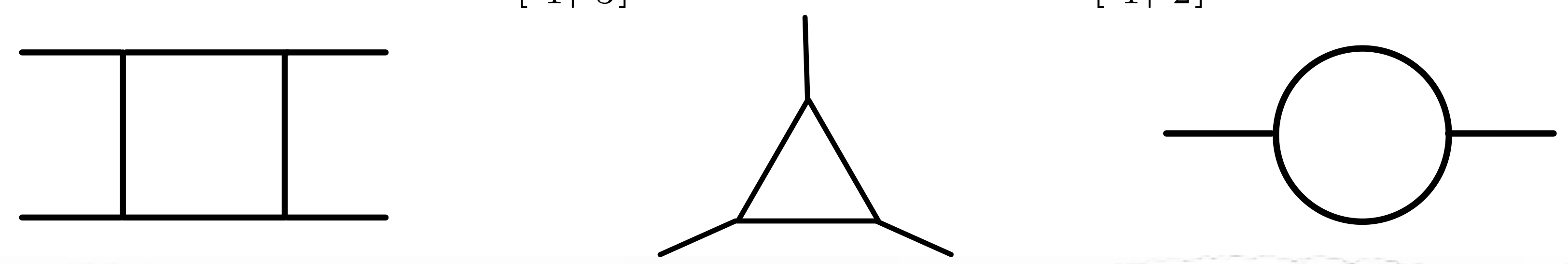
NLO "revolution"

A number of breakthrough ideas developed in the last 10 years, most notably

- sew together tree level amplitudes to compute loop amplitudes [on-shell intermediate states, cuts, generalized unitarity ...]
- OPP: extract coefficients of master integrals by evaluating the amplitudes at specific values of the loop momentum [algebraic method]



$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right) + \mathcal{R}$$



Bern, Dixon, Kosower; Britto, Cachazo, Feng; Ossola, Pittau, Papadopoulos; Ellis, Giele, Kunszt, Melnikov; ....

NLO automation

Various tools developed: Blackhat+Sherpa, GoSam+Sherpa, Helac-NLO, Madgraph5_aMC@NLO, NJet, OpenLoops+Sherpa, Samurai, Recola ...

- the automation of NLO QCD corrections is mostly considered a solved problem
- high-multiplicity processes still difficult (long run-time on clusters to obtain stable distributions, numerical instabilities).
  Edge: 4 to 6 particles in the final state, depends on the process
- also loop-induced processes automated (enhanced by gluon PDF)
  Hirschi, Mattelaer '15
- comparison to NLO is now the standard in most physics analysis

NLO automation: example

Hirschi, Frederix, Garzelli, Maltoni, Pittau 1103.0621

<u>Example</u>: heavy quarks and jets at NLO

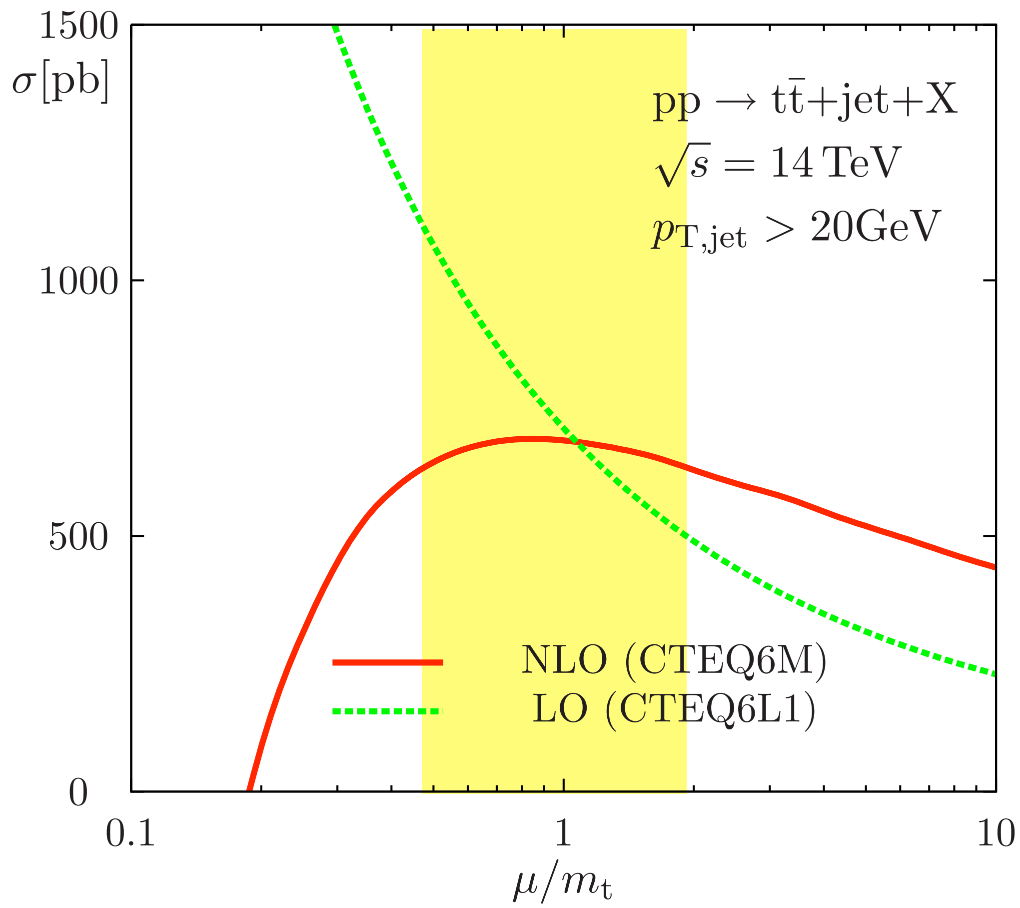
	Process	Syntax	Cross section (pb)			
Heavy quarks+vector bosons			LO 13 TeV		NLO 13 TeV	
e.1	$pp \to W^{\pm} b\bar{b}$ (4f)	`p p > wpm b b~`	$3.074 \pm 0.002 \cdot 10^{2}$	$^{+42.3\%}_{-29.2\%}$ $^{+2.0\%}_{-1.6\%}$	$8.162 \pm 0.034 \cdot 10^{2}$	$^{+29.8\%}_{-23.6\%}$ $^{+1.5\%}_{-1.2\%}$
e.2	$pp \to Z b\bar{b}$ (4f)	`p p > z b b~`	$6.993 \pm 0.003 \cdot 10^{2}$	$^{+33.5\%}_{-24.4\%}$ $^{+1.0\%}_{-1.4\%}$	$1.235 \pm 0.004 \cdot 10^{3}$	$^{+19.9\%}_{-17.4\%}$ $^{+1.0\%}_{-1.4\%}$
e.3	$pp \to \gamma b\bar{b}$ (4f)	`p p > a b b~`	$1.731 \pm 0.001 \cdot 10^{3}$	$^{+51.9\%}_{-34.8\%}$ $^{+1.6\%}_{-2.1\%}$	$4.171 \pm 0.015 \cdot 10^{3}$	$^{+33.7\%}_{-27.1\%}$ $^{+1.4\%}_{-1.9\%}$
e.4*	$pp \to W^{\pm} b\bar{b} j$ (4f)	`p p > wpm b b~ j`	$1.861 \pm 0.003 \cdot 10^{2}$	$^{+42.5\%}_{-27.7\%}$ $^{+0.7\%}_{-0.7\%}$	$3.957 \pm 0.013 \cdot 10^{2}$	$^{+27.0\%}_{-21.0\%}$ $^{+0.7\%}_{-0.6\%}$
e.5*	$pp \to Z b\bar{b} j$ (4f)	`p p > z b b~ j`	$1.604 \pm 0.001 \cdot 10^{2}$	$^{+42.4\%}_{-27.6\%}$ $^{+0.9\%}_{-1.1\%}$	$2.805 \pm 0.009 \cdot 10^{2}$	$^{+21.0\%}_{-17.6\%}$ $^{+0.8\%}_{-1.0\%}$
e.6*	$pp \to \gamma b\bar{b} j$ (4f)	`p p > a b b~ j`	$7.812 \pm 0.017 \cdot 10^{2}$	$^{+51.2\%}_{-32.0\%}$ $^{+1.0\%}_{-1.5\%}$	$1.233 \pm 0.004 \cdot 10^{3}$	$^{+18.9\%}_{-19.9\%}$ $^{+1.0\%}_{-1.5\%}$
e.7	$pp \to t\bar{t} W^{\pm}$	`p p > t t~ wpm`	$3.777 \pm 0.003 \cdot 10^{-1}$	$^{+23.9\%}_{-18.0\%}$ $^{+2.1\%}_{-1.6\%}$	$5.662 \pm 0.021 \cdot 10^{-1}$	$^{+11.2\%}_{-10.6\%}$ $^{+1.7\%}_{-1.3\%}$
e.8	$pp \to t\bar{t} Z$	`p p > t t~ z`	$5.273 \pm 0.004 \cdot 10^{-1}$	$^{+30.5\%}_{-21.8\%}$ $^{+1.8\%}_{-2.1\%}$	$7.598 \pm 0.026 \cdot 10^{-1}$	$^{+9.7\%}_{-11.1\%}$ $^{+1.9\%}_{-2.2\%}$
e.9	$pp \to t\bar{t} \gamma$	`p p > t t~ a`	$1.204 \pm 0.001 \cdot 10^{0}$	$^{+29.6\%}_{-21.3\%}$ $^{+1.6\%}_{-1.8\%}$	$1.744 \pm 0.005 \cdot 10^{0}$	$^{+9.8\%}_{-11.0\%}$ $^{+1.7\%}_{-2.0\%}$
e.10*	$pp \to t\bar{t} W^{\pm} j$	`p p > t t~ wpm j`	$2.352 \pm 0.002 \cdot 10^{-1}$	$^{+40.9\%}_{-27.1\%}$ $^{+1.3\%}_{-1.0\%}$	$3.404 \pm 0.011 \cdot 10^{-1}$	$^{+11.2\%}_{-14.0\%}$ $^{+1.2\%}_{-0.9\%}$
e.11*	$pp \to t\bar{t} Z j$	`p p > t t~ z j`	$3.953 \pm 0.004 \cdot 10^{-1}$	$^{+46.2\%}_{-29.5\%}$ $^{+2.7\%}_{-3.0\%}$	$5.074 \pm 0.016 \cdot 10^{-1}$	$^{+7.0\%}_{-12.3\%}$ $^{+2.5\%}_{-2.9\%}$
e.12*	$pp \to t\bar{t} \gamma j$	`p p > t t~ a j`	$8.726 \pm 0.010 \cdot 10^{-1}$	$^{+45.4\%}_{-29.1\%}$ $^{+2.3\%}_{-2.6\%}$	$1.135 \pm 0.004 \cdot 10^{0}$	$^{+7.5\%}_{-12.2\%}$ $^{+2.2\%}_{-2.5\%}$
e.13*	$pp \to t\bar{t} W^{-} W^{+}$ (4f)	`p p > t t~ w+ w-`	$6.675 \pm 0.006 \cdot 10^{-3}$	$^{+30.9\%}_{-21.9\%}$ $^{+2.1\%}_{-2.0\%}$	$9.904 \pm 0.026 \cdot 10^{-3}$	$^{+10.9\%}_{-11.8\%}$ $^{+2.1\%}_{-2.1\%}$
e.14*	$pp \to t\bar{t} W^{\pm} Z$	`p p > t t~ wpm z`	$2.404 \pm 0.002 \cdot 10^{-3}$	$^{+26.6\%}_{-19.6\%}$ $^{+2.5\%}_{-1.8\%}$	$3.525 \pm 0.010 \cdot 10^{-3}$	$^{+10.6\%}_{-10.8\%}$ $^{+2.3\%}_{-1.6\%}$
e.15*	$pp \to t\bar{t} W^{\pm} \gamma$	`p p > t t~ wpm a`	$2.718 \pm 0.003 \cdot 10^{-3}$	$^{+25.4\%}_{-18.9\%}$ $^{+2.3\%}_{-1.8\%}$	$3.927 \pm 0.013 \cdot 10^{-3}$	$^{+10.3\%}_{-10.4\%}$ $^{+2.0\%}_{-1.5\%}$
e.16*	$pp \to t\bar{t} ZZ$	`p p > t t~ z z`	$1.349 \pm 0.014 \cdot 10^{-3}$	$^{+29.3\%}_{-21.1\%}$ $^{+1.7\%}_{-1.5\%}$	$1.840 \pm 0.007 \cdot 10^{-3}$	$^{+7.9\%}_{-9.9\%}$ $^{+1.7\%}_{-1.5\%}$
e.17*	$pp \to t\bar{t} Z\gamma$	`p p > t t~ z a`	$2.548 \pm 0.003 \cdot 10^{-3}$	$^{+30.1\%}_{-21.5\%}$ $^{+1.7\%}_{-1.6\%}$	$3.656 \pm 0.012 \cdot 10^{-3}$	$^{+9.7\%}_{-11.0\%}$ $^{+1.8\%}_{-1.9\%}$
e.18*	$pp \to t\bar{t} \gamma\gamma$	`p p > t t~ a a`	$3.272 \pm 0.006 \cdot 10^{-3}$	$^{+28.4\%}_{-20.6\%}$ $^{+1.3\%}_{-1.1\%}$	$4.402 \pm 0.015 \cdot 10^{-3}$	$^{+7.8\%}_{-9.7\%}$ $^{+1.4\%}_{-1.4\%}$

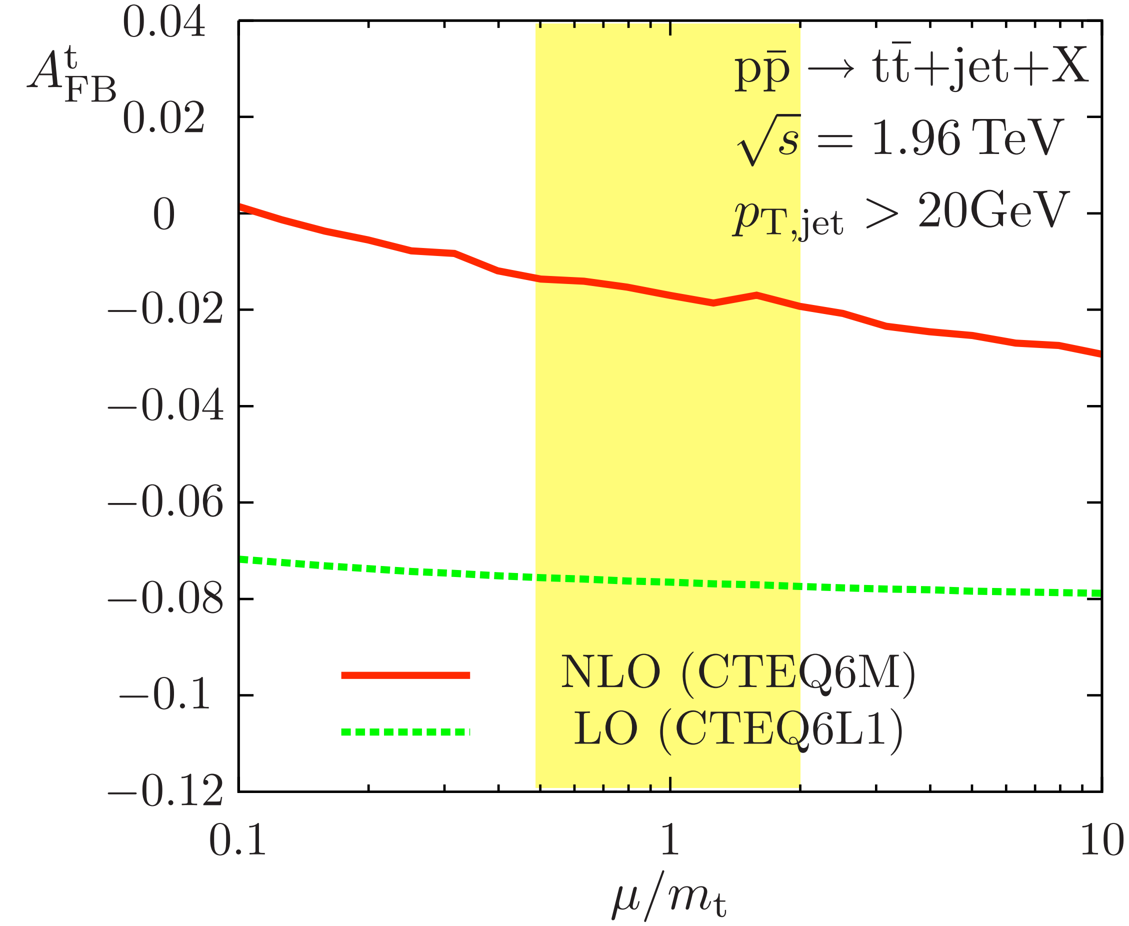
Similar tables for

- boson+jets
- diboson+jets
- triboson+jets
- four bosons
- heavy quarks + jets
- heavy quarks + bosons
- single top
- single Higgs
- Higgs pair
- ...

1. Example of NLO: tt+1jet

Dittmaier, Kallweit, Uwer '08



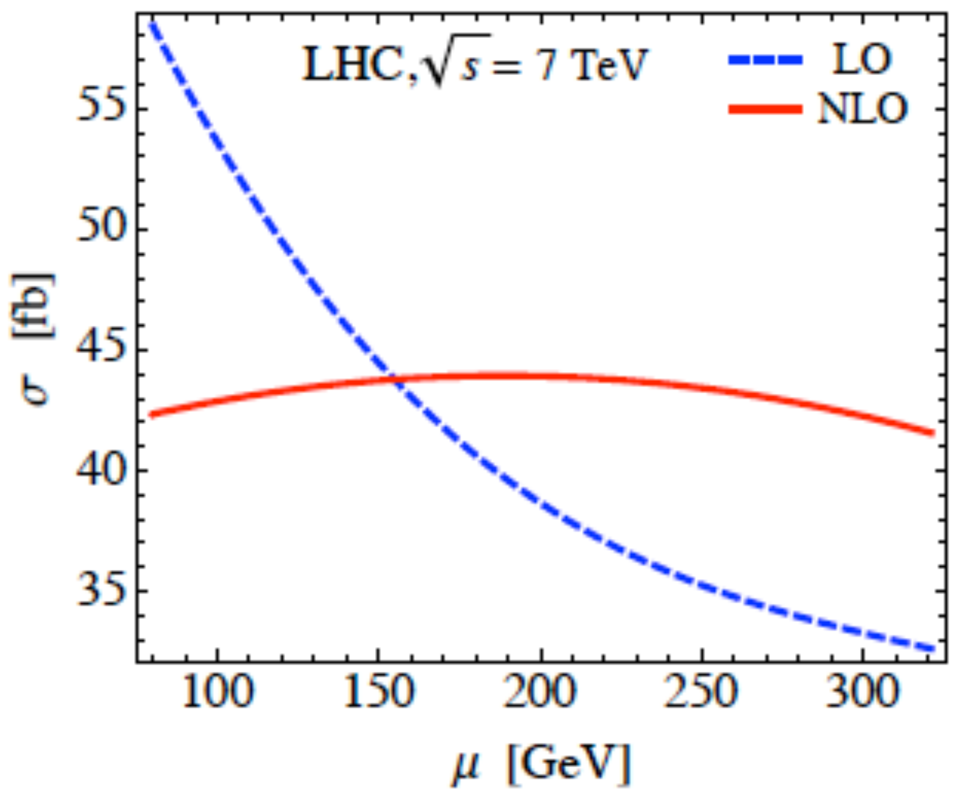


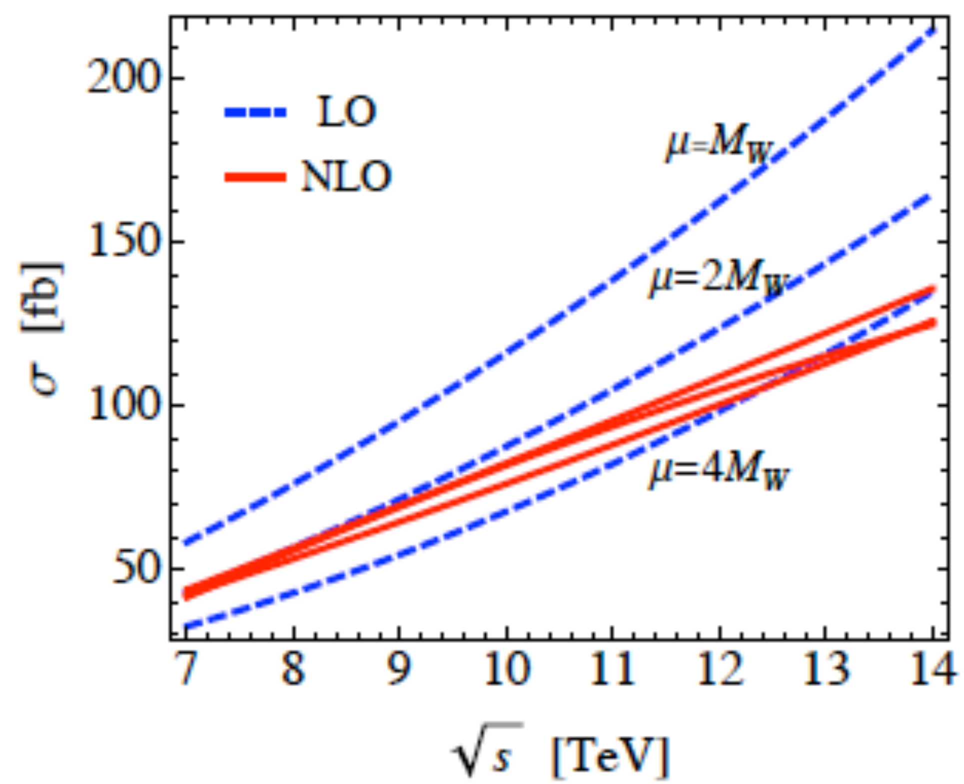
- improved stability of NLO result [but no decays]
- forward-backward asymmetry at the Tevatron compatible with zero
- LO scale uncertainty underestimates shift to NLO for the asymmetry

2. Example of NLO: WW+2jets

LO calculations: very large theoretical uncertainties

<u>Example</u>: cross-section for W^+W^- + 2 jet production at the LHC

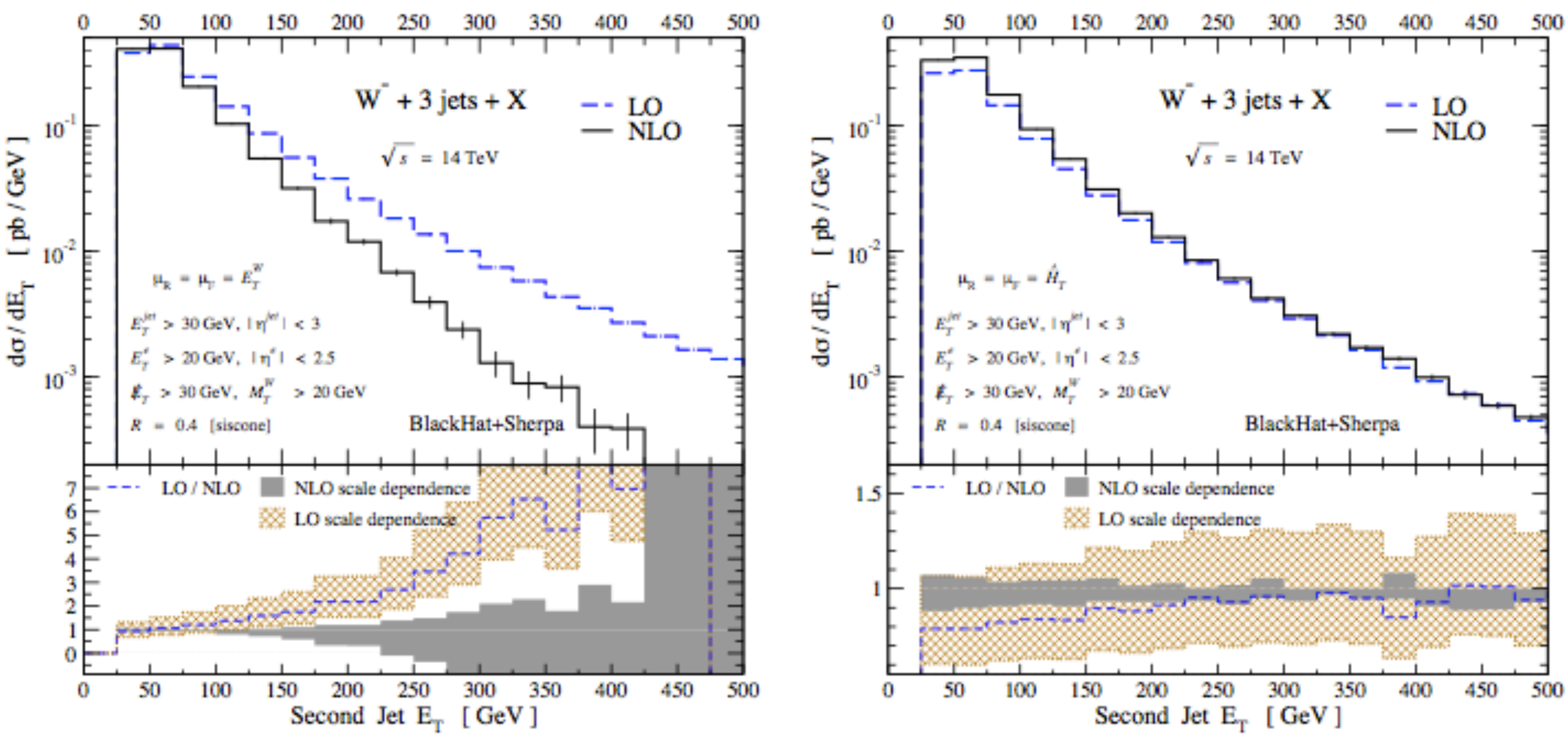




Melia, et al. '11

3. Example of NLO: W+3jets

Scale choice: example of W+3 jets (problem more severe with more jets)



... large logarithms can appear in some distributions, invalidating even an NLO prediction.

Bern et al. '09

NNLO

NNLO is one of the most active areas in QCD now

After pioneering calculations for Higgs and Drell Yan more than 10 years ago, only recently many $2 \to 2$ processes computed at NNLO

NNLO most important in three different situations

Benchmark processes (measured with highest accuracy)

- $Z \to l^+l^-$
- $W \to l$□
- ...

Input to PDFs fits + backgrounds to Higgs studies

- Diboson
- Boson + jet
- top-pairs
- ...

Very large NLO corrections (moderate precision needs NNLO)

- Higgs
- Higgs + jet
- ...

Plus more reliable estimate of theory uncertainty

Still early days, but in the few cases examined (e.g. Higgs and Drell Yan, WW, ZZ, top ...), better agreement with data at NNLO

NNLO

While at NLO the bottleneck has been for a long time the calculation of virtual (one-loop) amplitudes, at NNLO the bottleneck comes mostly from finding a method to cancel divergences before numerical integration.

Two main approaches

Slicing:
partition the phase space with a (small) slicing parameter so that divergences are all below the slicing cut. In the divergent region use an approximate expression, neglecting finite terms, above use the exact (finite) integrand.

Subtraction:
since IR singularities of amplitudes are knows, add and subtract counterterms so as to make integrals finite. "Easy" at NLO, but complicated at NNLO due to the more intricate structure of (overlapping) singularities

NNLO

Different practical realizations:

- antenna subtraction
- q_T subtraction (slicing)
- colorful subtraction
- sector improved residue subtraction scheme
- N-jettiness subtraction/slicing

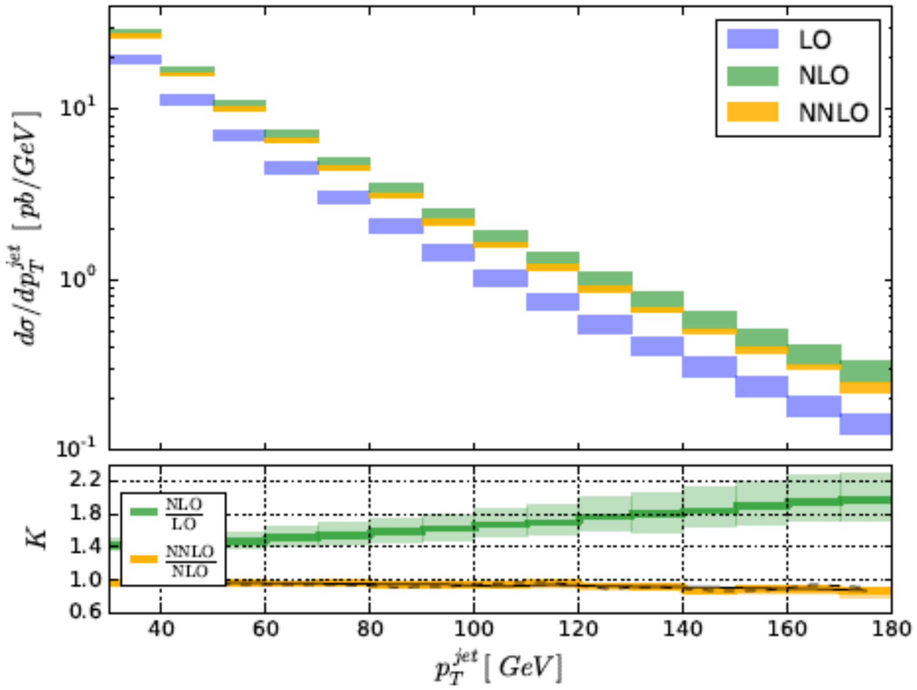
☜ *new kid in town*

Obviously, two-loop integrals are also needed. Lots of progress here too. I will not discuss this here, only mention Henn's conjecture to compute integrals using differential equations

NNLO: V+1jet

W+1jet

1504.02131



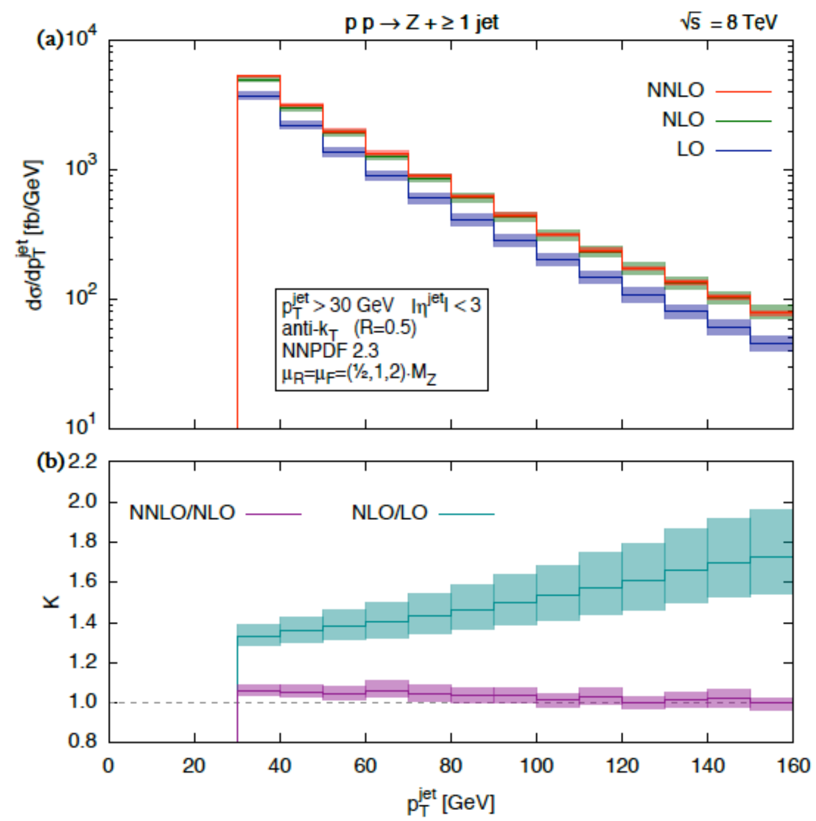
$p_T^{jet} > 30$ GeV, $\lvert\eta_{jet}\rvert < 2.4$	
Leading order:	533^{+39}_{-38} pb
Next-to-leading order:	797^{+63}_{-49} pb
Next-to-next-to-leading order:	787^{+0}_{-8} pb

- flat K-factor (≈1)
- huge reduction of theory error

Z+1jet

1507.02850

$$\sigma_{LO} = 103.6^{+7.7}_{-7.5} \text{ pb}$$
$$\sigma_{NLO} = 144.4^{+9.0}_{-7.2} \text{ pb}$$
$$\sigma_{NNLO} = 151.0^{+4.9}_{-3.6} \text{ pb}$$

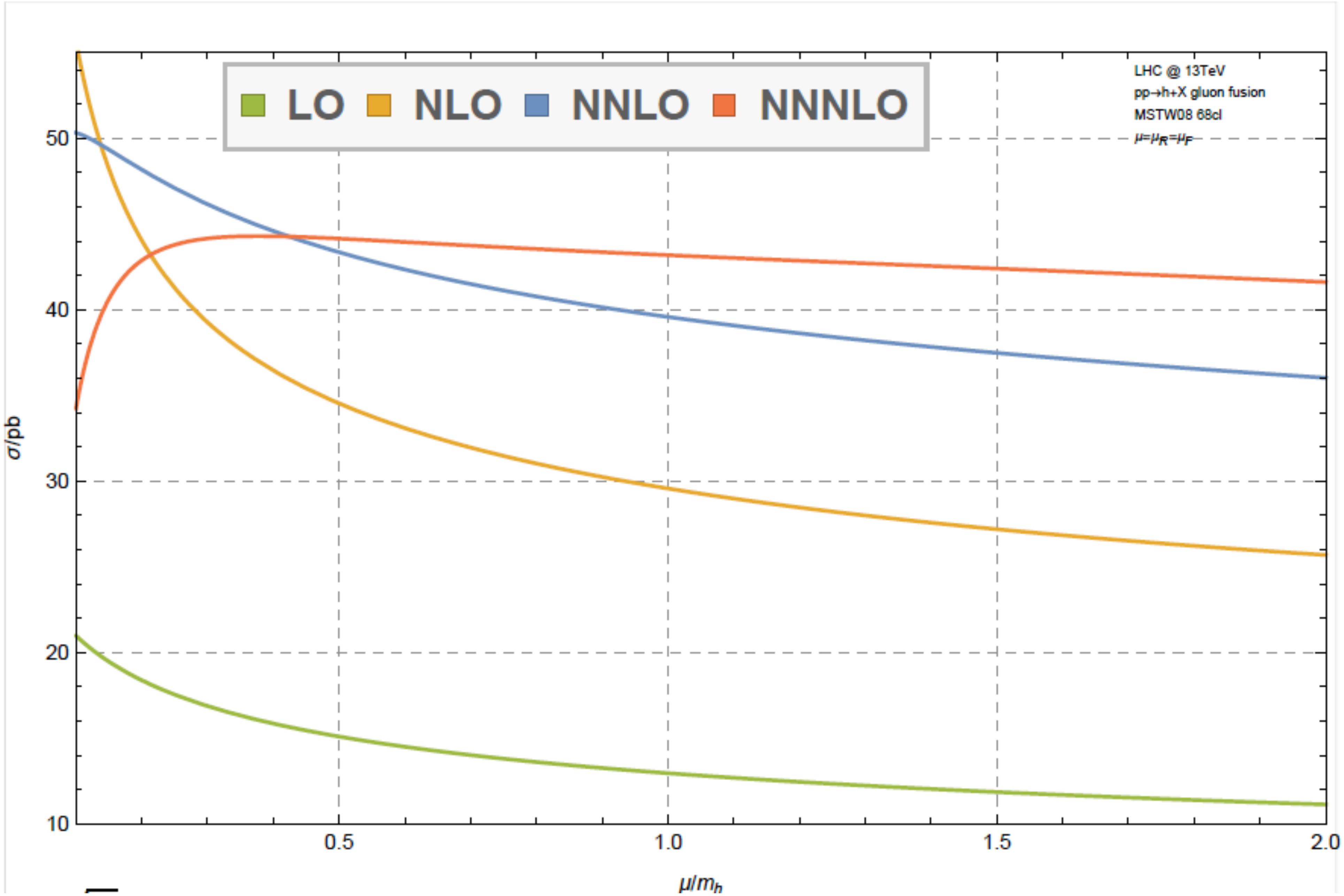


- similar features in Z+jet
- other observables ($p_{t,Z}$, y_Z, ...) non-trivial K-factor

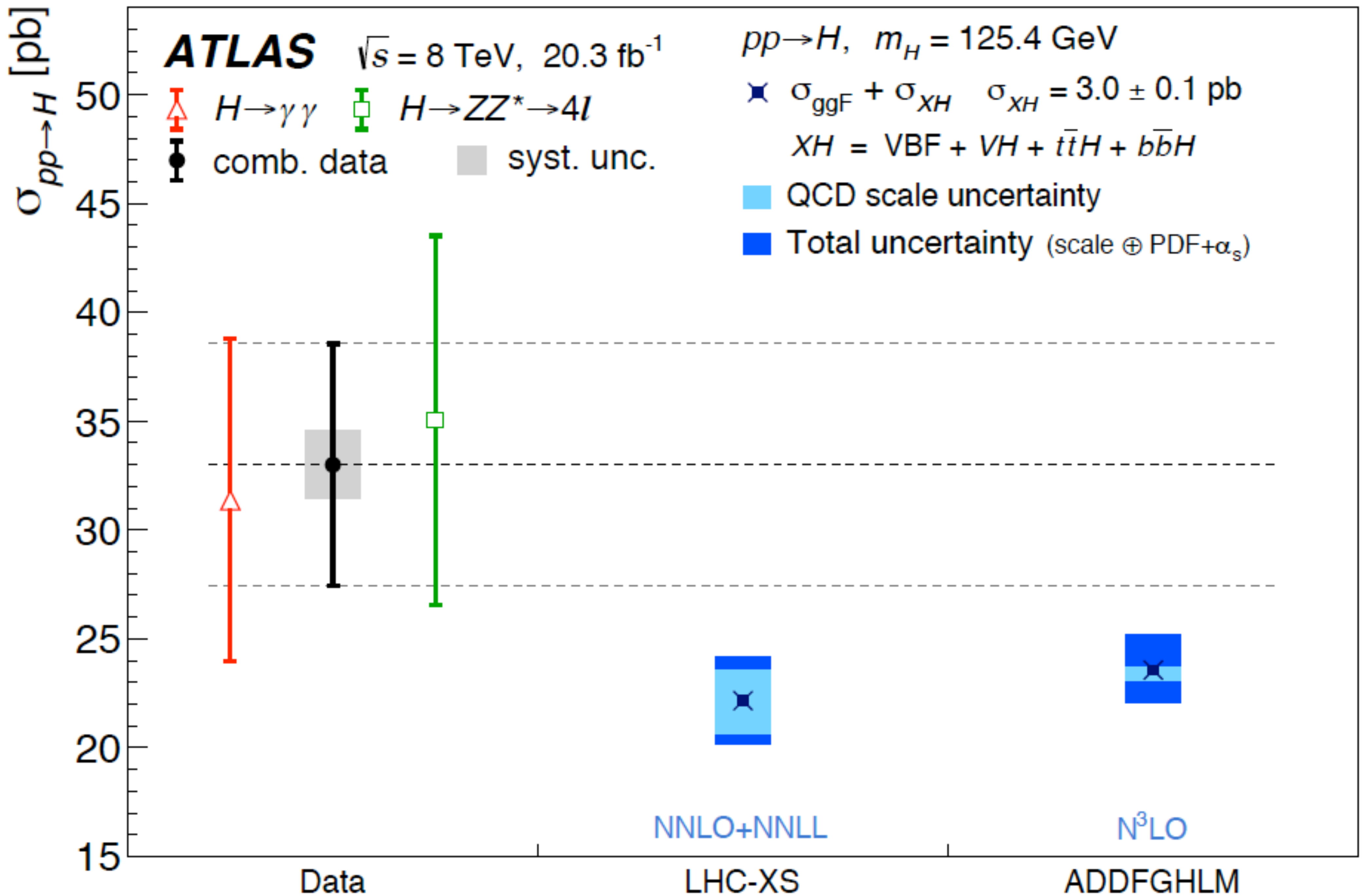
Summary of perturbative calculations

- LO: fully automated. Edge: 10-12 particles in the final state
- NLO: also automated. Edge: 4-6 particles in the final state
- NNLO: the new frontier. Lots of new $2 \to 2$ processes in the last year ($2 \to 1$ more than10 years old). Currently no $2 \to 3$ calculation for the LHC
- NNNLO: fully inclusive Higgs production *(new in 2015)*

Higgs production at N3LO



Higgs production: theory vs data



Conclusions

QCD is a field very active

- NLO revolution belongs already to the past, NNLO the current hottest field.
  Only in the last few months: H+1jet, Z+1jet, W+1jet, VBF Higgs, VV, dijets at NNLO and even Higgs at N3LO

- many other important theoretical and phenomenological developments (NLO multi-jet merging, matching, inclusion of EW corrections, resummations ...)

- tools getting more and more refined. Drastic improvement in theory uncertainties and more attention paid towards a solid estimate

Very exciting to work on QCD as new ideas/calculations are promptly used in LHC analyses. Thrilling times ahead, but also time to start thinking beyond the LHC