

Beyond the Standard Model (after LHC 8TeV)

G. Ross, Corfu, September 2015

Beyond the Standard Model (after LHC 8TeV)

I. Introduction

II. Grand Unification

III. The Higgs era: re-evaluation of the hierarchy problem

IV. "Just" the Standard Model

V. SUSY

(VI. Composite Higgs)

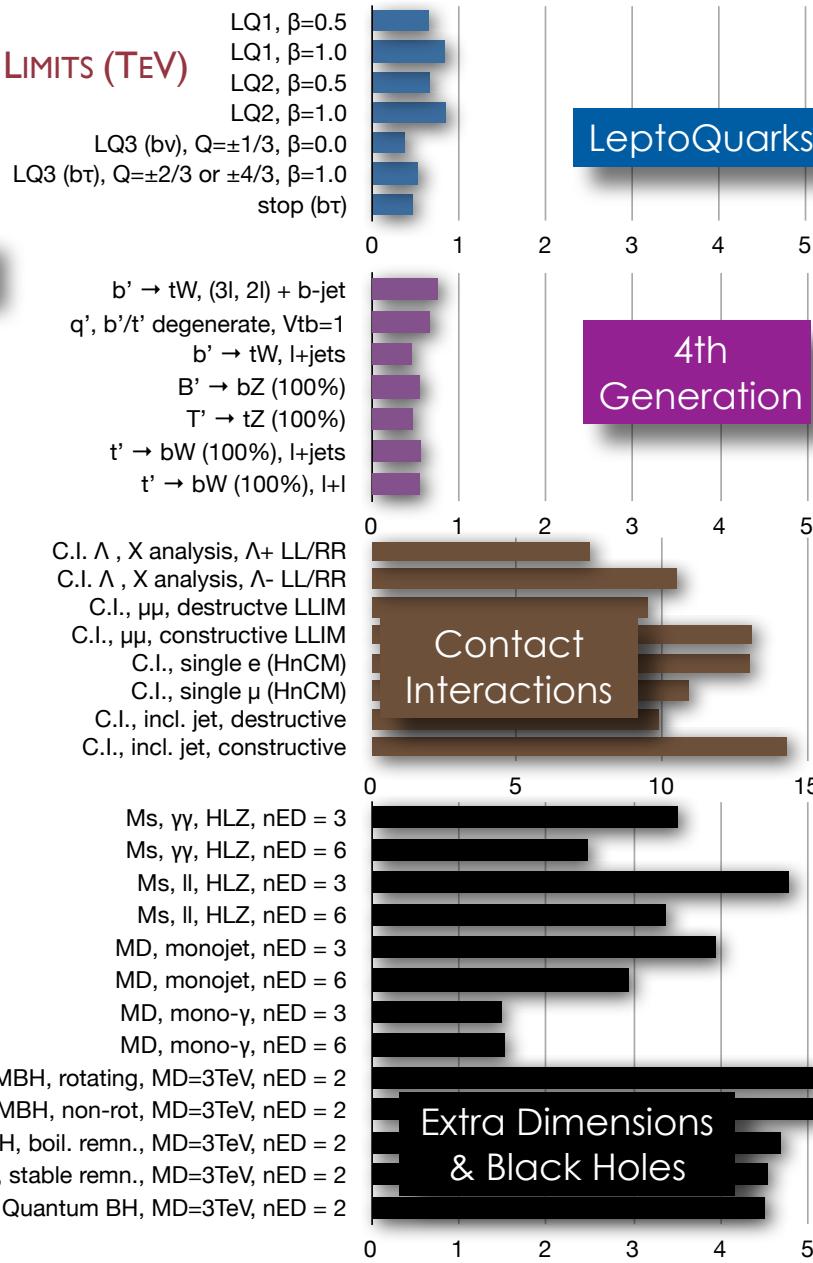
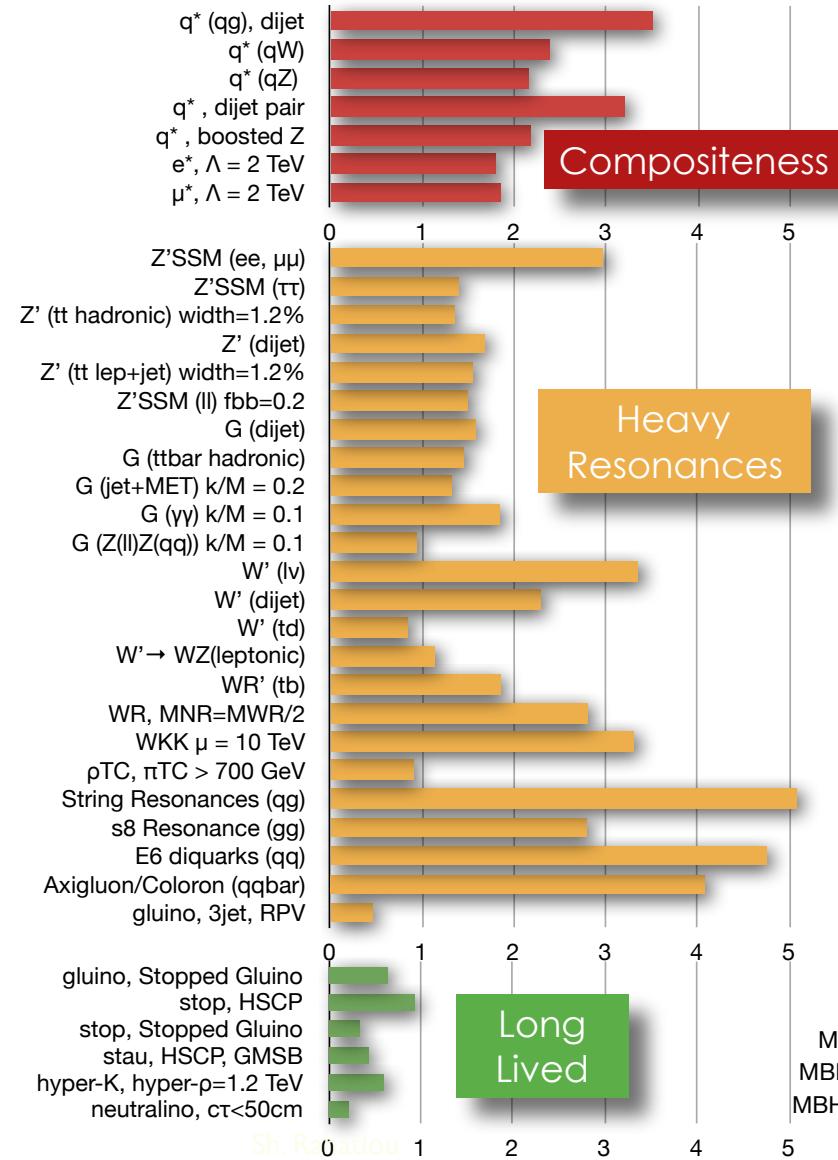
(Lecture notes: <http://goo.gl/eLpaCH>)

I. Introduction

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LHC 8: No evidence (yet) for BSM

CMS EXOTICA 95% CL EXCLUSION LIMITS (TeV)



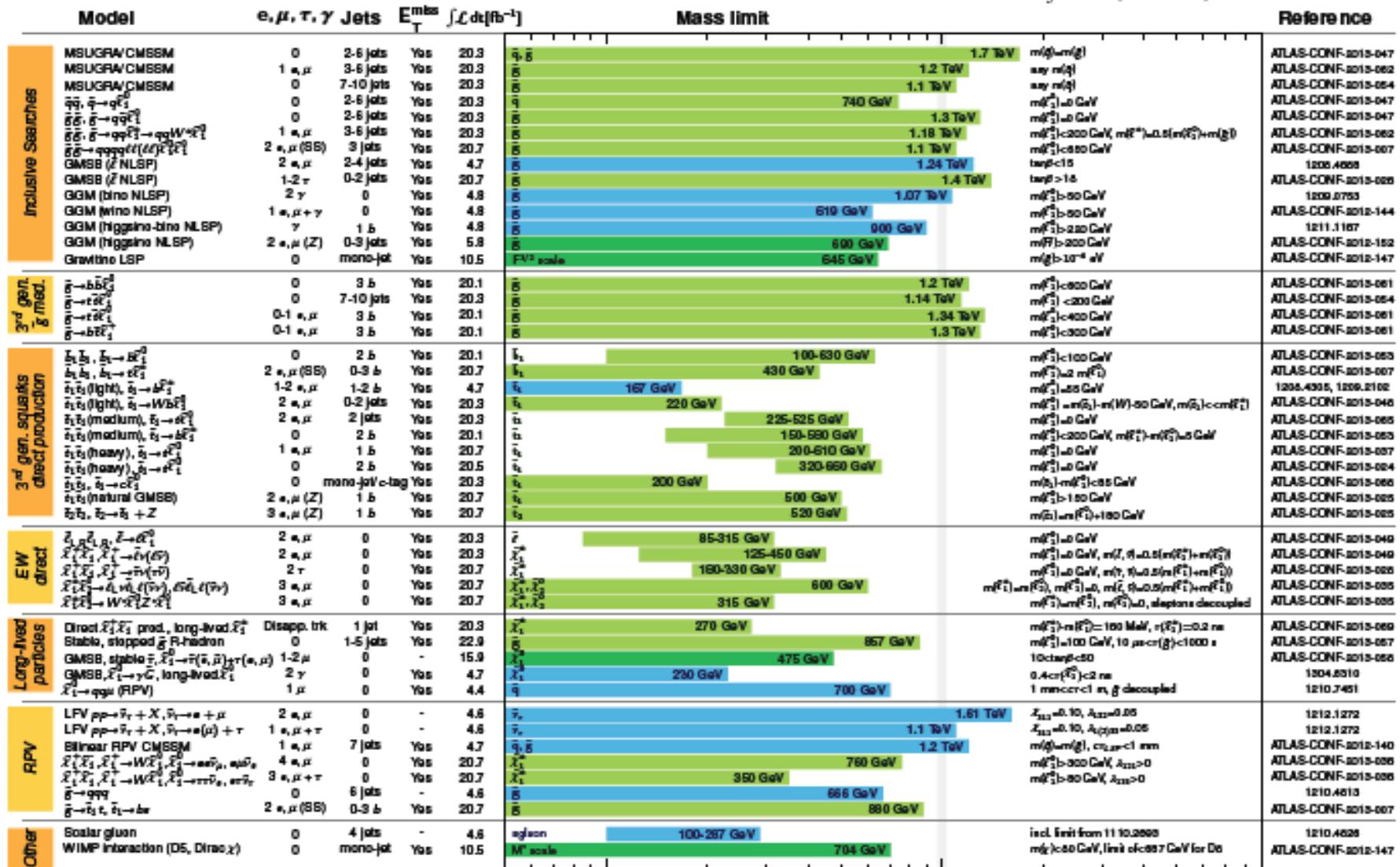
ATLAS Summary

ATLAS SUSY Searches* - 95% CL Lower Limits

Status: EPS 2013

ATLAS Preliminary

$\int \mathcal{L} dt = (4.4 - 22.9) \text{ fb}^{-1}$ $\sqrt{s} = 7, 8 \text{ TeV}$



$\sqrt{s} = 7 \text{ TeV}$
full data

$\sqrt{s} = 8 \text{ TeV}$
partial data

$\sqrt{s} = 8 \text{ TeV}$
full data

10^{-1}

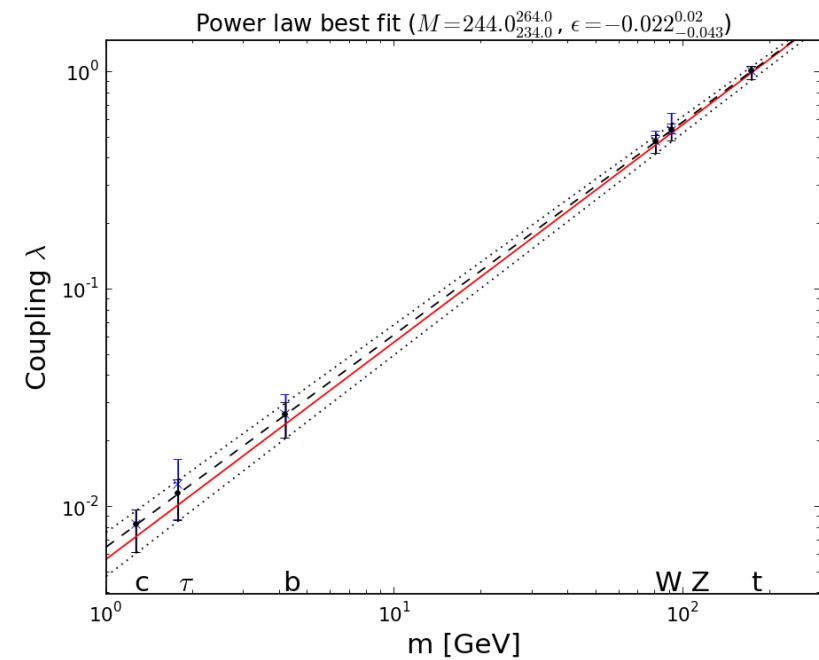
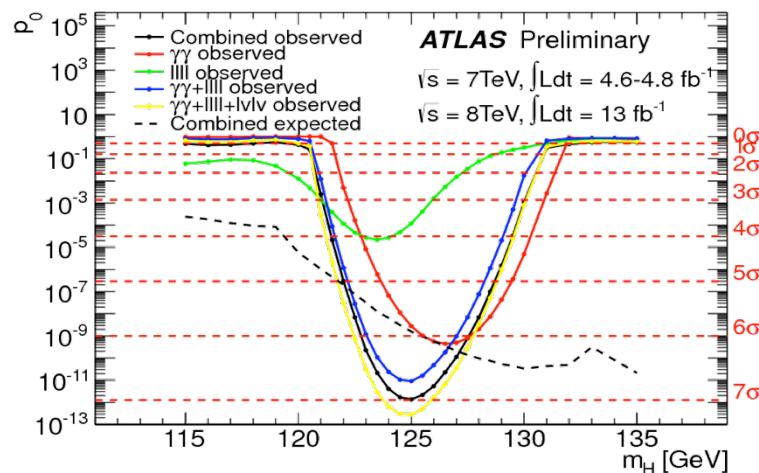
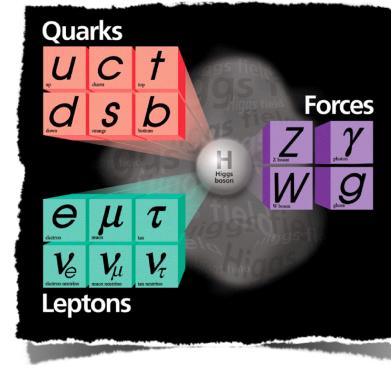
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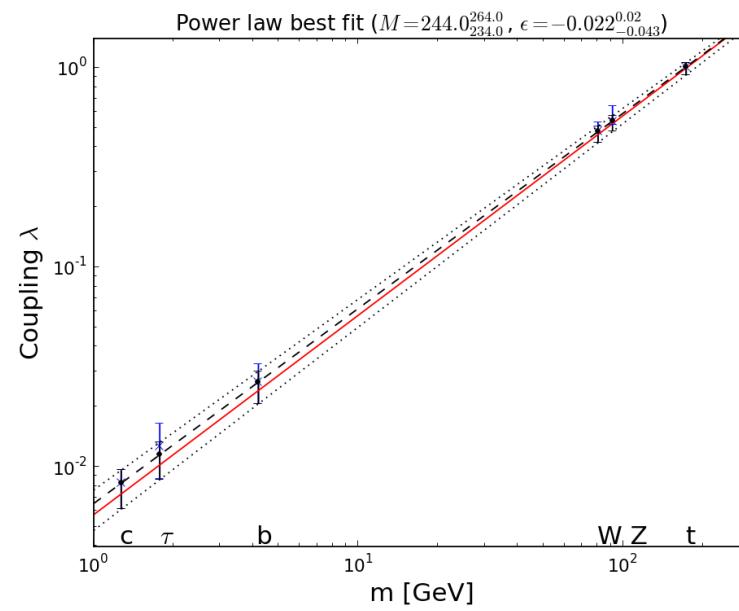
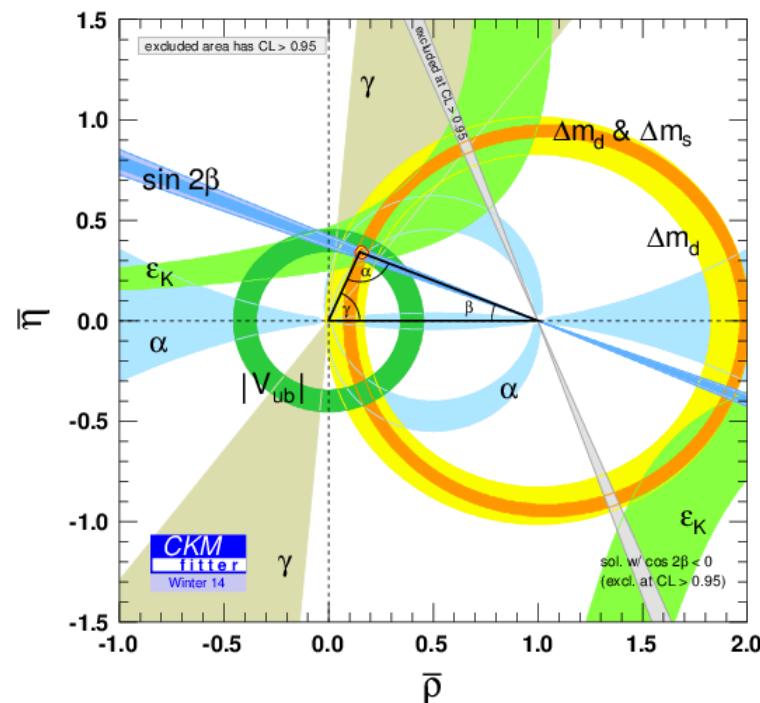
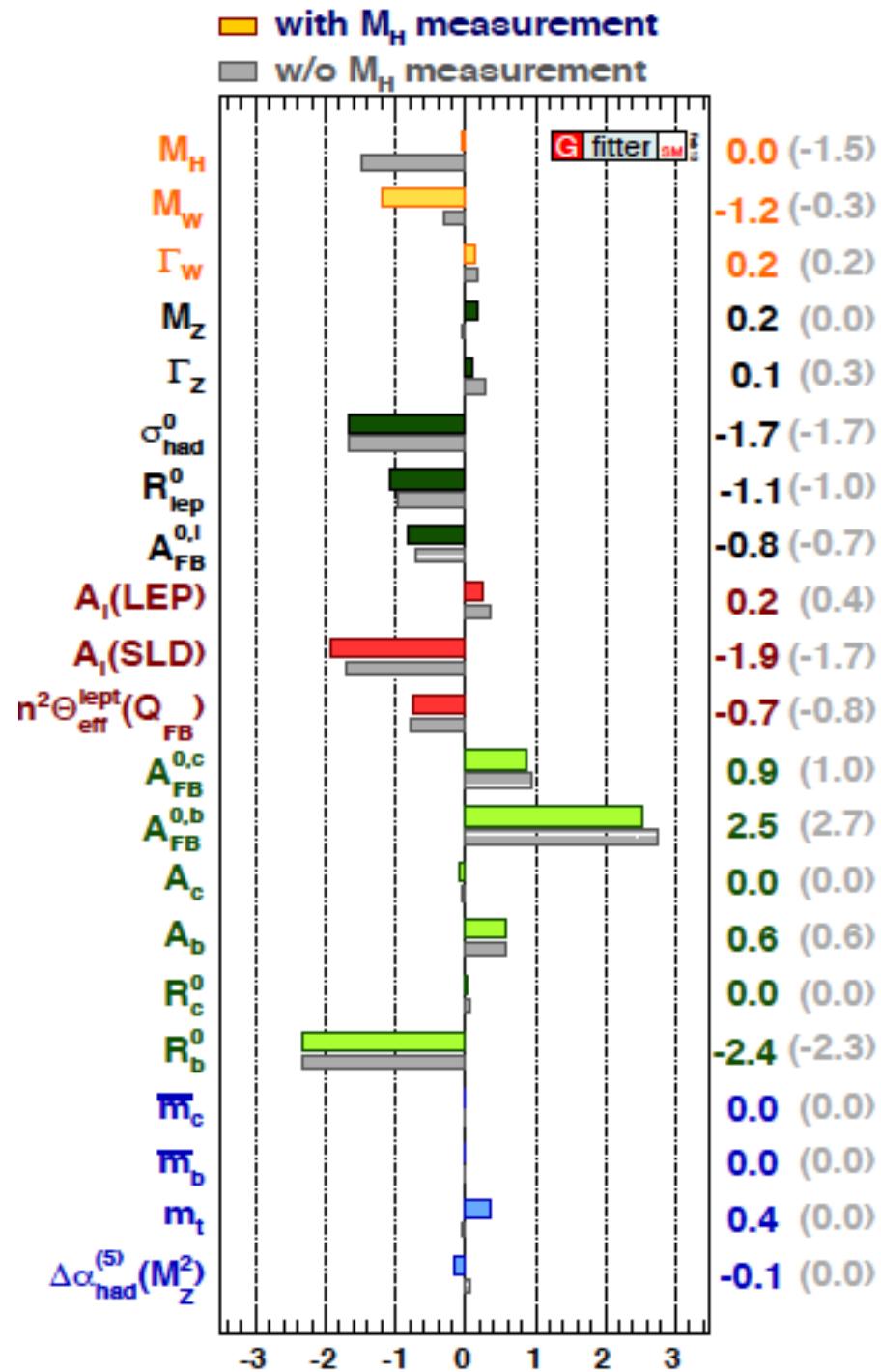
Mass scale [TeV]

LHC 8

No evidence (yet) for BSM

Higgs discovery





LHC 8

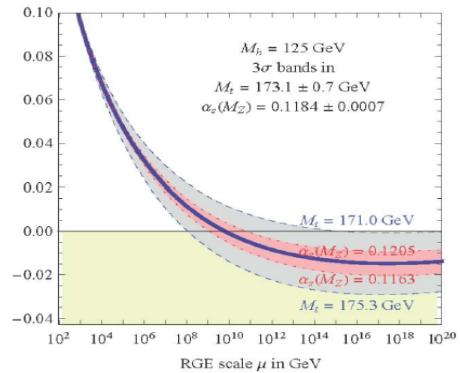
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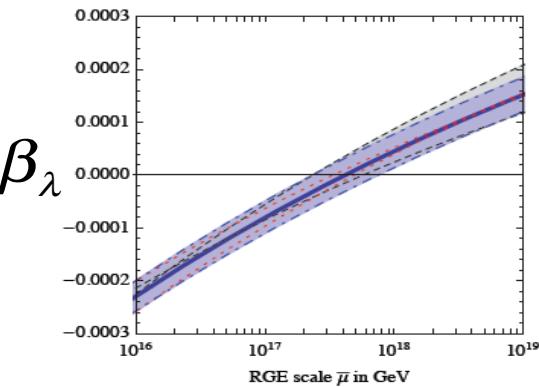
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"Just" the SM (JSM)?

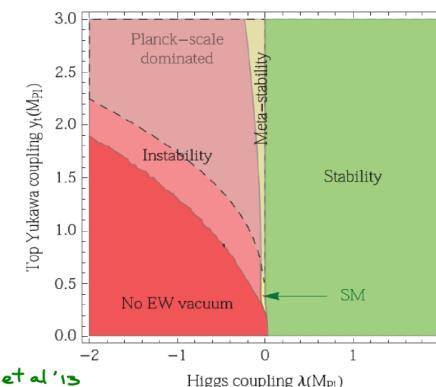
λ



β_λ



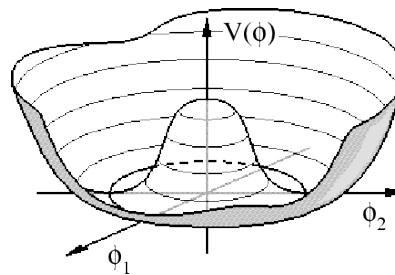
Buttazzo et al '13



DeGrassi et al,...

$$V(H) = -m^2 |\phi|^2 + \lambda |\phi|^4$$

$$m^2 \simeq (89 \text{ GeV}^2), \lambda \simeq 0.13$$



Why go beyond the Standard Model?

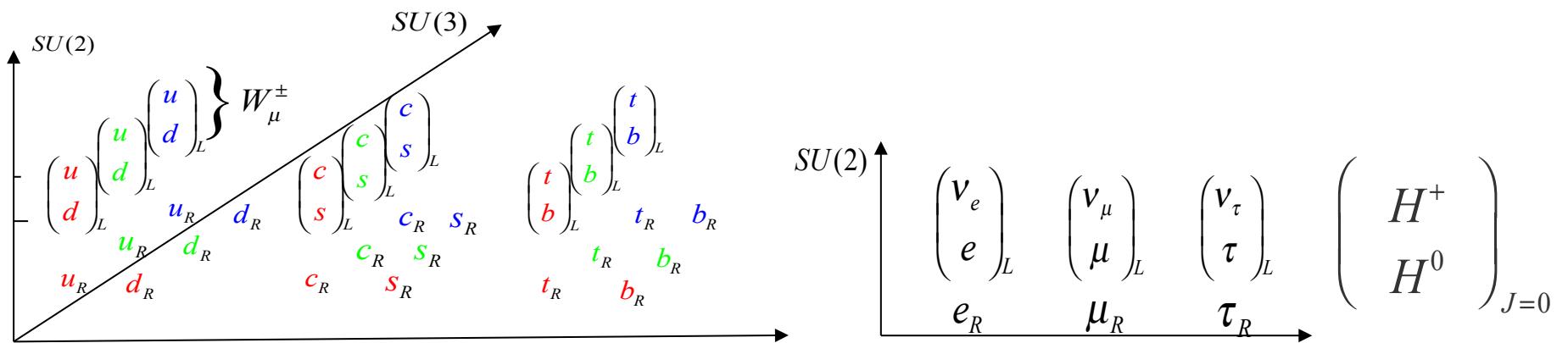
The Standard Model - unanswered questions

- Complicated choice of multiplets
- Fractional and integral charges?
- Neutrino masses?
- Many parameters 19 (28)
- Only partial unification $A_\mu^\gamma = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$
- The hierarchy problem
- Strong CP problem
- Dark matter, baryogenesis, inflation.....

II. Grand Unification

● Unification incomplete:

$$SU(3) \times SU(2) \times U(1)$$



19 (28) parameters: $g_i, m_i, \theta_i, \delta_i, M_{W,H}, \theta_{QCD}$

Gravity?

II. Grand Unification $SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$

Georgi Glashow

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Group of 5×5 complex unitary matrices with determinant 1

$50 - 25 - 1 = 24$ independent matrices - adjoint representation

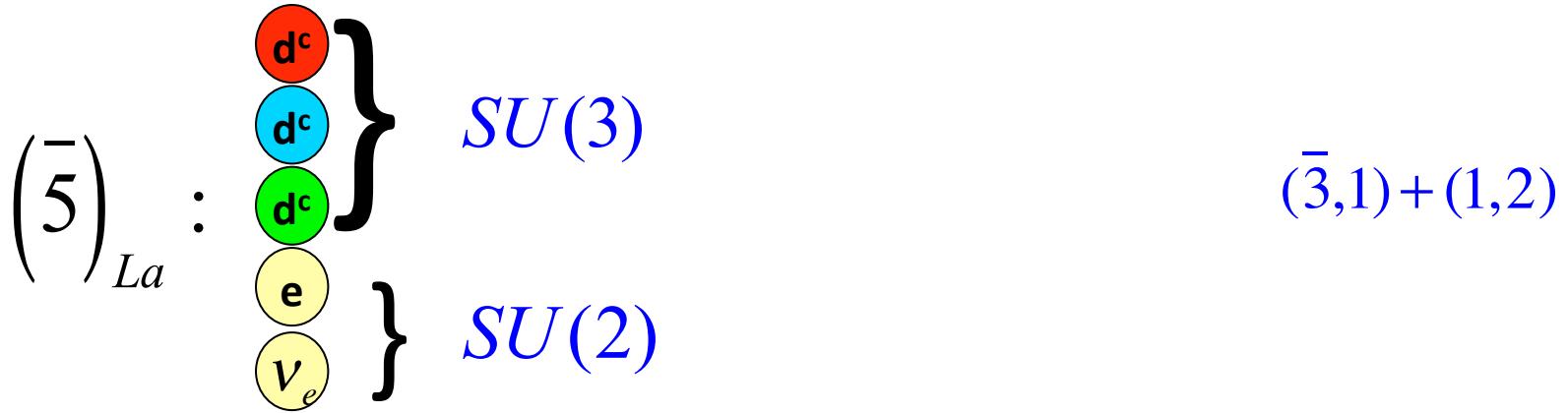
$$U = \exp\left(-i \sum_{i=1}^{24} \beta^i L^i\right), \quad U^\dagger U = 1 \Rightarrow L^i \text{ Hermitian generators}$$

Covariant derivative: Gauge bosons V_μ^a (3,1)+(1,2)

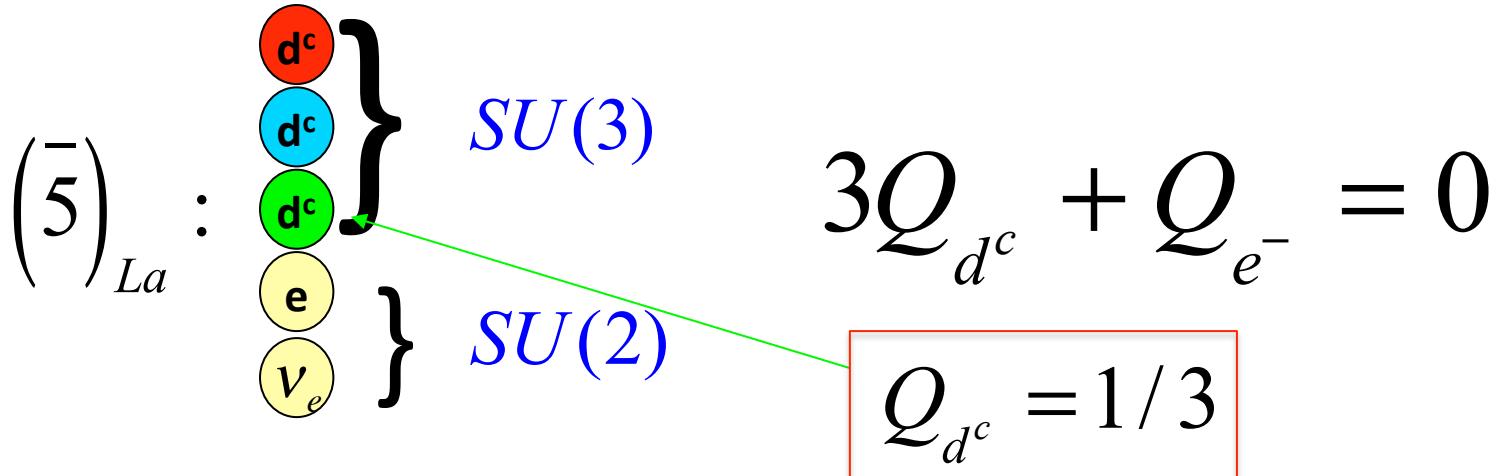
$$\text{Define } \frac{1}{\sqrt{2}} V_\mu \equiv \frac{1}{2} \sum_{a=1}^{24} V_\mu^a L^a, \quad (D_\mu \psi_5)^i = \left[\delta_j^i \partial_\mu - \frac{ig}{2} \sum_{a=1}^{24} V_\mu^a (L^a)_j^i \right] \psi_5^j$$

$$V_\mu = \begin{bmatrix} G_1^1 - \frac{2B}{\sqrt{30}} & G_2^1 & G_2^1 & \bar{X}_1 & \bar{Y}_1 \\ G_1^2 & G_2^2 - \frac{2B}{\sqrt{30}} & G_3^2 & \bar{X}_2 & \bar{Y}_2 \\ G_1^3 & G_2^3 & G_3^3 - \frac{2B}{\sqrt{30}} & \bar{X}_3 & \bar{Y}_3 \\ X_1 & X_2 & X_3 & \frac{W_\mu^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{W_\mu^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} \end{bmatrix},$$

II. Grand Unification $SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$



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$$\left(\bar{5}\right)_{La} : \begin{array}{c} \text{d}^c \\ \text{d}^c \\ \text{d}^c \\ \text{e} \\ \nu_e \end{array} \left. \begin{array}{c} \} \\ \} \\ \} \\ \} \end{array} \right. \begin{array}{c} SU(3) \\ \\ \\ SU(2) \end{array}$$

$$3Q_{d^c} + Q_{e^-} = 0$$

$Q_{d^c} = 1/3$

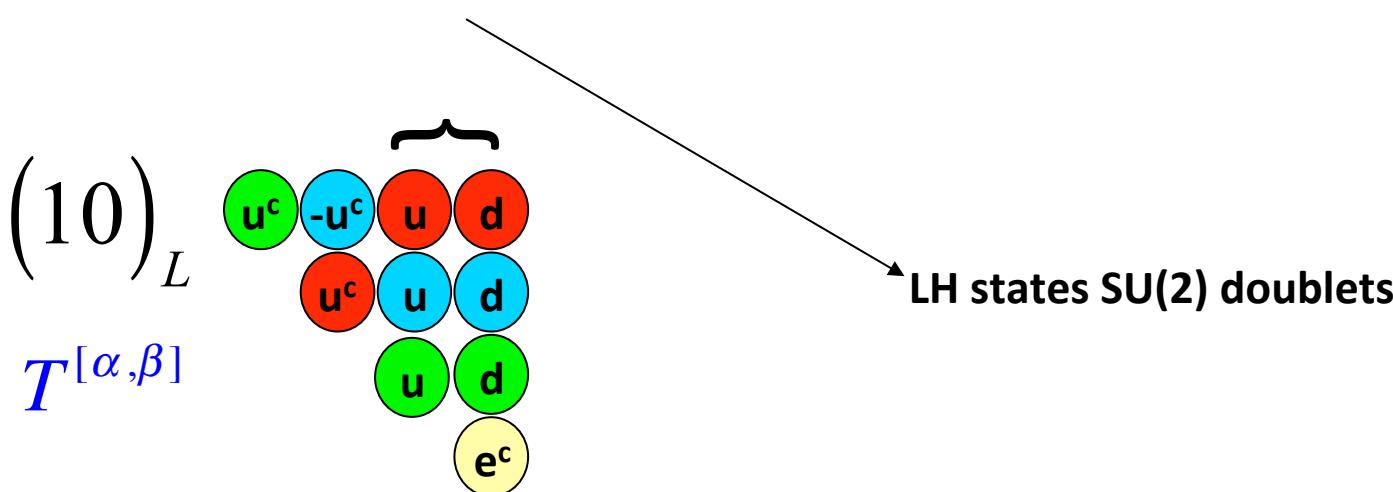
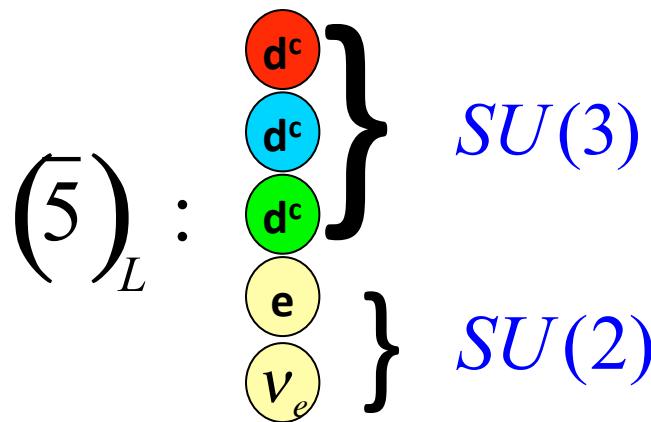
Remaining 10 states?

$$T^{[\alpha, \beta]}$$

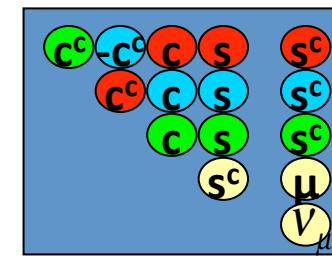
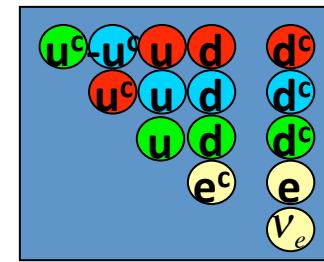
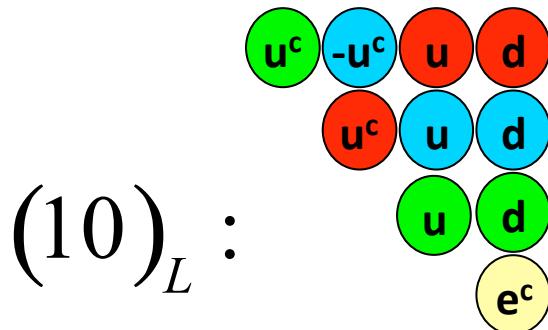
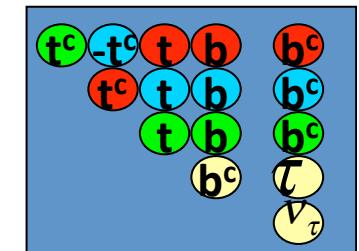
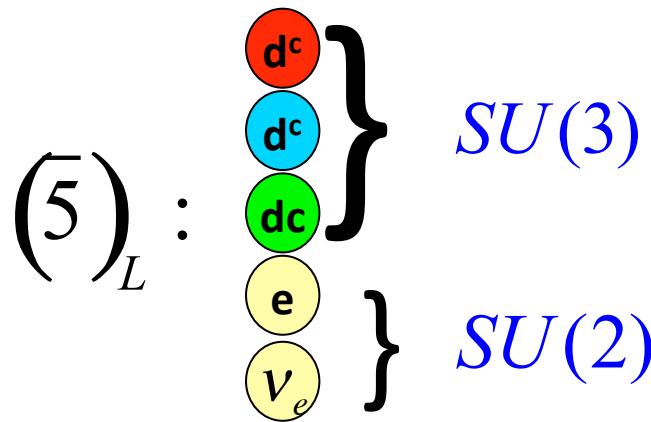


$$\frac{n(n-1)}{1 \times 2} = 10$$

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Generations ?

II. Grand Unification $SO(10) \supset SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$

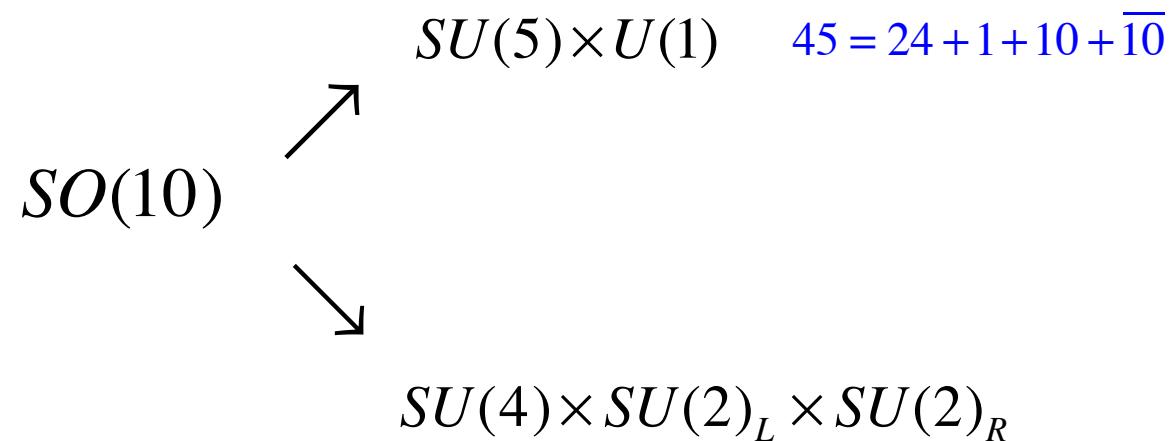
Anomaly free

$SO(10)$: Group of matrices R that leave invariant length of 10-dim vector

$$R^T R = RR^T = 1 \quad \text{Adjoint representation} \quad SO(n): n^2 - (n^2 + n)/2 = n(n-1)/2$$

$SO(10) \quad 45 \text{ gauge bosons}$

Rank 5



$SO(10)$: Group of matrices R that leave invariant length of 10-dim vector

$$R^T R = RR^T = 1 \quad \stackrel{\dagger}{\text{Adjoint representation}} \quad O(n): n^2 - (n^2 + n)/2 = n(n-1)/2$$

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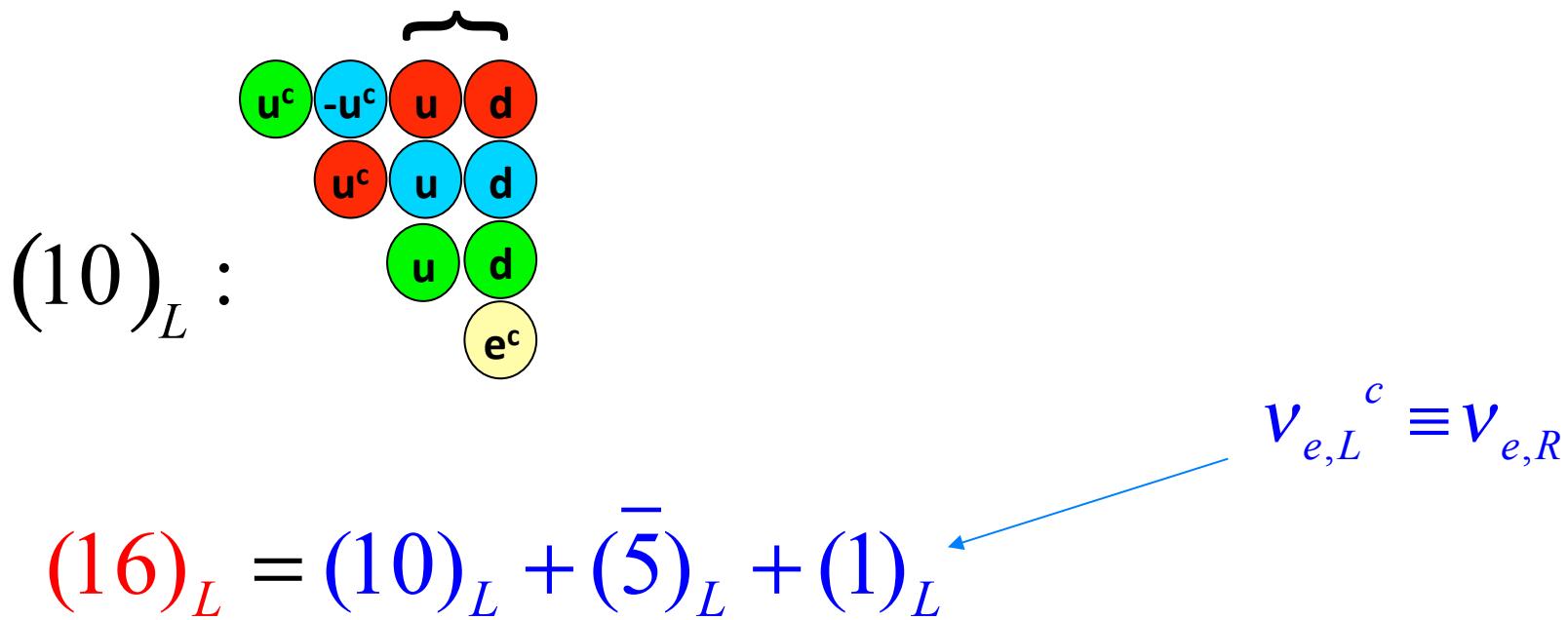
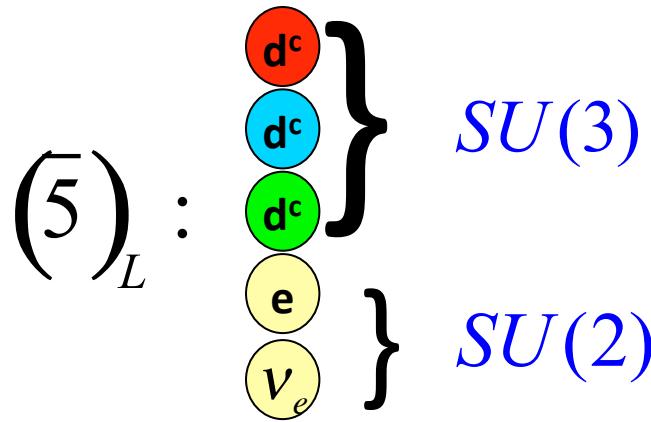
Spinorial (16 dim) representation:

$$c.f. \quad SO(3) \sim SU(2) \quad \psi_{\alpha=1,2}, \quad R = e^{i\omega^{ab}\sigma_{ab}}, \quad \sigma_{ab} = \frac{1}{2}\epsilon_{abc}\sigma_c \equiv \frac{i}{2}[\sigma_a, \sigma_b]$$

$$SO(10) \quad \chi_{16}^{\pm} = \psi_1 \times \psi_2 \times \psi_3 \times \psi_4 \times \psi_5 \quad \text{with} \quad \sum_{i=1}^5 \sigma_3^i = \pm 1$$

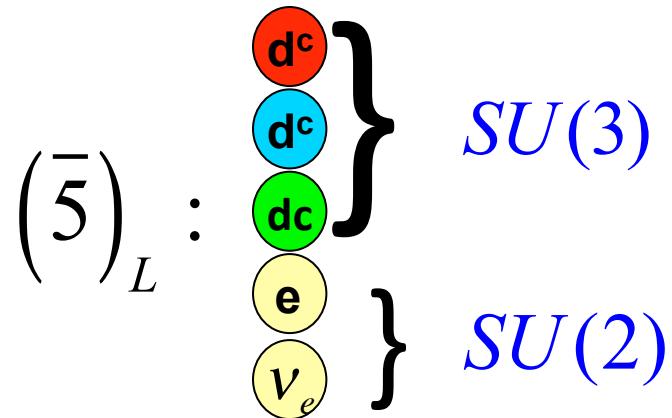
\uparrow
 2^4

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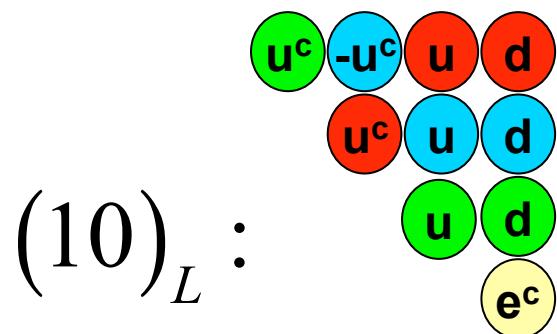
Gauge Couplings

$$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$



$$M_X$$

$$g_5 \supset g_3 \qquad \qquad g_2 \qquad \qquad g_1$$



$$g_1(M_X) = g_2(M_X) = g_3(M_X) = g_5$$

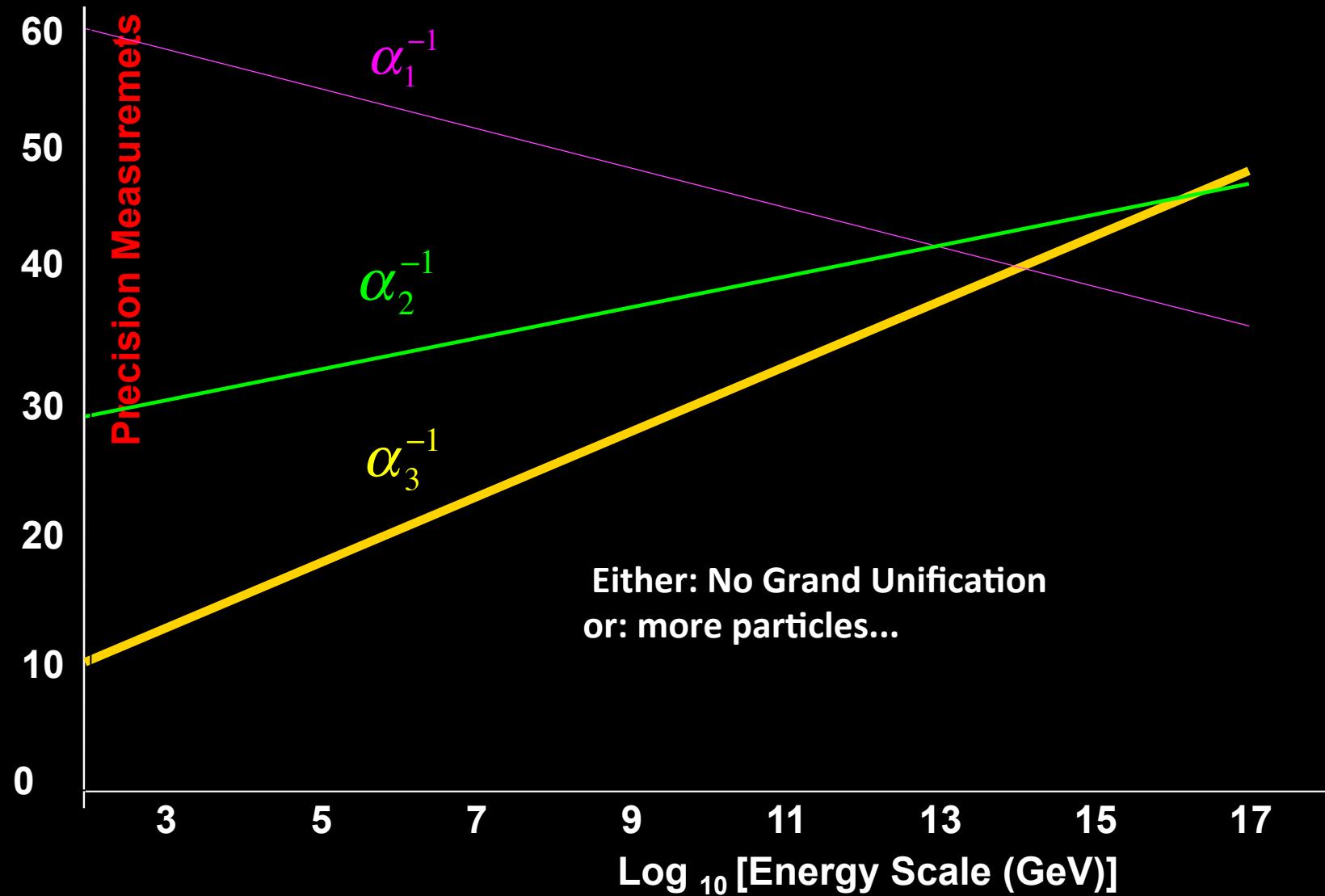
Gauge Couplings

SM evolution of gauge couplings

$$\alpha_i^{-1}(\mu) = \alpha^{-1}(M_X) + \frac{1}{2\pi} b_i \ln\left(\frac{M_X}{\mu}\right) + ..$$

$$b_i^{SM} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_g \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + H \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix}$$

$$M_X \sim 10^{14} \text{ GeV}$$



Fermion masses

$$\bar{5} \times 10 = \textcolor{red}{5} + \overline{45}$$

$$10 \times 10 = \bar{5} + 45 + 50$$

$$\bar{5} \times \bar{5} = \overline{10} + \overline{15}$$

$$L^5_{Yukawa} = \left(\psi_{Ri\alpha}^\dagger \right) m_{ij}^D \chi_{Lj}^{\alpha\beta} \textcolor{red}{H}_\beta^\dagger - \tfrac{1}{4} \epsilon_{\alpha\beta\gamma\delta\varepsilon} \left(\chi^T \right)_{Li}^{\alpha\beta} \sigma^2 m_{ij}^U \chi_{Lj}^{\gamma\delta} \textcolor{red}{H}^\varepsilon + h.c.$$

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After diagonalising down quark mass matrix:

$$m_d = m_e \quad \text{✗}$$

$$m_s = m_\mu \quad \text{✗}$$

$$m_b = m_\tau \quad \text{✓?}$$

Fermion masses

$$\bar{5} \times 10 = 5 + \overline{45}$$

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$$L_{Yukawa}^5 = (\psi_{Ri\alpha}^\dagger) m_{ij}^D \chi_{Lj}^{\alpha\beta} H_\beta^\dagger - \frac{1}{4} \epsilon_{\alpha\beta\gamma\delta\varepsilon} (\chi^T)_{Li}^{\alpha\beta} \sigma^2 m_{ij}^U \chi_{Lj}^{\gamma\delta} H^\varepsilon + h.c.$$

$$L_Y^{45} = (\psi_{Ri\alpha}^\dagger) m_{ij}^d \chi_{Lj}^{\beta\gamma} H_{\beta\gamma}^{\dagger\alpha} + \epsilon_{\alpha\beta\gamma\rho\tau} (\chi^T)_{Li}^{\alpha\beta} \sigma^2 m_{ij}^u \psi_{Lj}^{\gamma\delta} H_\delta^{\rho\tau} + h.c.$$

$$-3m_d = m_e^x$$

$$-3m_s = m_\mu^v$$

$$-3m_b = m_\tau^x$$

$$\langle H_a^{b5} \rangle = v_{45} (\delta_a^b - 4\delta_a^4 \delta_4^b), a,b = 1..4$$

SU(2)XU(1) invariant component

Fermion masses

$$\text{Georgi-Jarlskog} \quad \left(L^5\right)_{33+12+21} + \left(L^{45}\right)_{22}$$

$$Det(M^l) = Det(M^d)|_{M_X}$$

$$\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$$

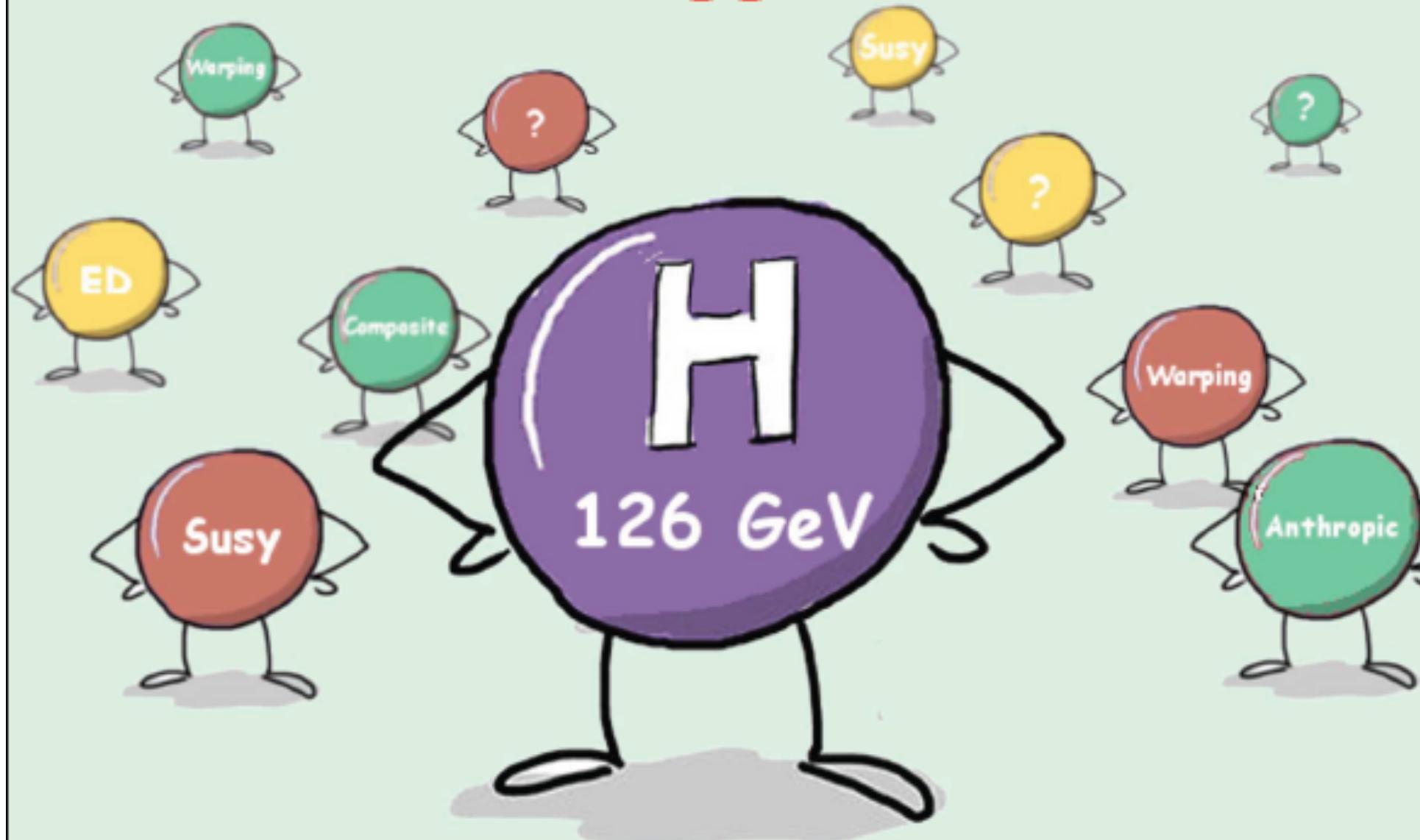
$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} 0 & \varepsilon^3 & 0 \\ \varepsilon^3 & a\varepsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{aligned} \frac{m_b}{m_\tau}(M_X) &= 1 \\ \varepsilon^d &= 0.15, \quad a_{45}^s = 1 \\ \varepsilon^l &= 0.15, \quad a_{45}^\mu = -3 \end{aligned}$$

$$m_b = 3m_\tau \quad \checkmark$$

$$m_s = 3 \cdot \frac{1}{3} \cdot m_\mu \quad \checkmark$$

$$m_d = 3 \cdot 3 \cdot m_e \quad \checkmark$$

The Higgs Era



III. The Hierarchy Problem

The Standard model as an effective field theory...

A renormalisable, spontaneously broken, local gauge quantum field theory

$$L_{\text{eff}}(\phi_{\text{light}}, \psi_{\text{heavy}}, M, E) \xrightarrow{E \ll M} L_{\text{eff}}(\phi_{\text{light}}, E) + O\left(\frac{1}{M}\right)$$

- Renormalisable $D \leq 4 + O(1/M)$ ✓

$$L_{\text{effective}}^{\text{SM}} \supset M_A \cancel{A^\mu} + m_f \cancel{\bar{f}_L f_R}$$

$$M_A, m_f \ll M_X, M_{\text{Planck}}$$

- Fermions chiral ✓

- Vector gauge bosons ✓

(Massless - vectorlike couplings; massive - chiral couplings)

- Light Higgs ? $L_{\text{effective}}^{\text{SM}} \supset M_H^2 |H|^2$

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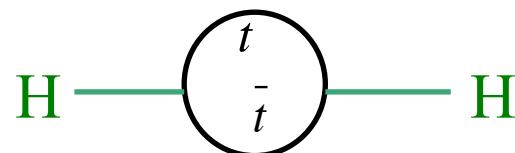
$$M_A, m_f \ll M_X, M_{\text{Planck}}$$

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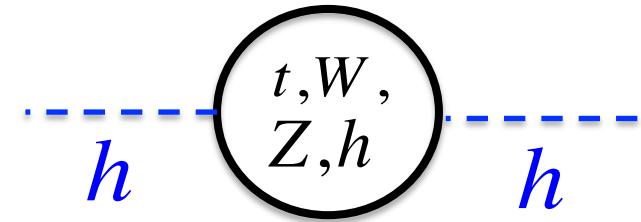
(Massless - vectorlike couplings; massive - chiral couplings)

- Light Higgs X **The Hierarchy problem** $\Lambda \leq 1 \text{TeV}??$



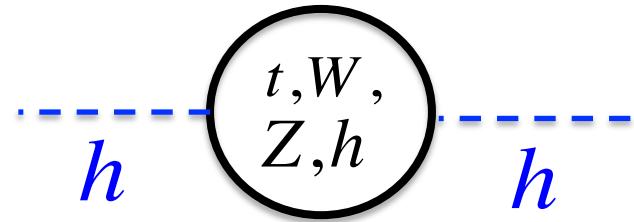
$$\delta^t M_H^2 \simeq -\frac{h_t^2}{8\pi^2} \int_0^{\Lambda^2} dk^2 = \frac{h_t^2}{8\pi^2} \Lambda^2 + O(m_t^2 \ln(\frac{m_t}{\Lambda}))$$

Hierarchy problem?



$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2} (4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2) \Lambda^2 = \left(\frac{\Lambda}{500GeV} \right)^2$$

Hierarchy problem?

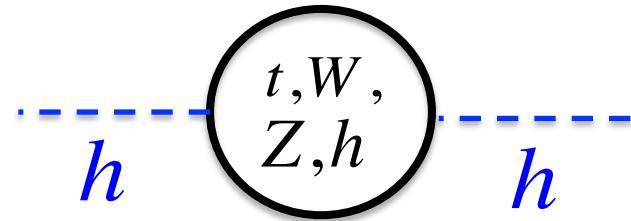


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Field theory: δm^2 not measureable

...only $m^2 = m_0^2 + \delta m^2$ "physical"

Hierarchy problem?



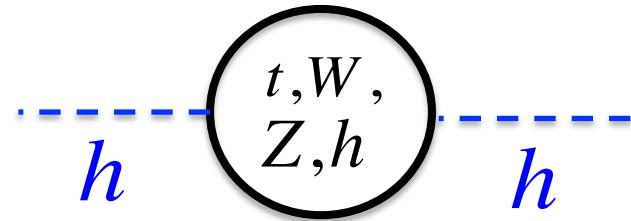
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Only $m^2 = 0$ special ("classical" scale invariance)

$$\Rightarrow \frac{d m_H^2}{d \ln \mu} = \frac{3m_H^2}{8\pi^2} \left(2\lambda + y_t^2 - \frac{3g_2^2}{4} - \frac{3g_1^2}{20} \right)$$

Hierarchy problem?



$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2} (4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2) \Lambda^2 = \left(\frac{\Lambda}{500\text{GeV}} \right)^2$$

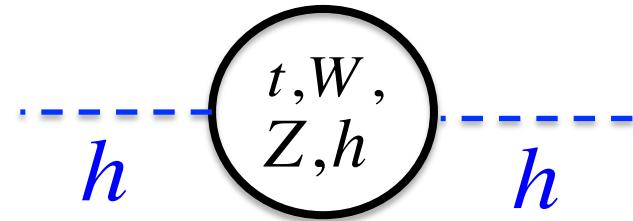
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... but is the SM all there is?

Hierarchy problem?



$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2} (4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2) \Lambda^2 = \left(\frac{\Lambda}{500\text{GeV}} \right)^2$$

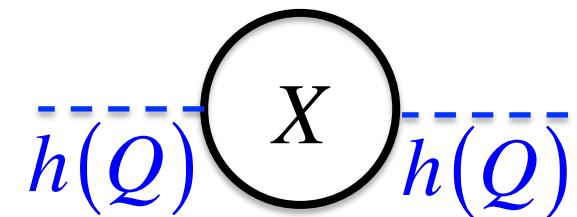
Field theory: δm^2 not measureable

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GUTS:

$$\delta m_h^2 \propto M_X^2 \ln \left(\frac{Q^2 + M_X^2}{\Lambda^2} \right)$$

- "real hierarchy problem"



Solutions to the hierarchy problem

$\Lambda \leq 1 TeV ??$

- Just the (scale invariant) Standard Model...**no heavy states**

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$\Lambda \leq 1 TeV ??$

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- Scanning....Higgs mass minimised when minimising scalar potential

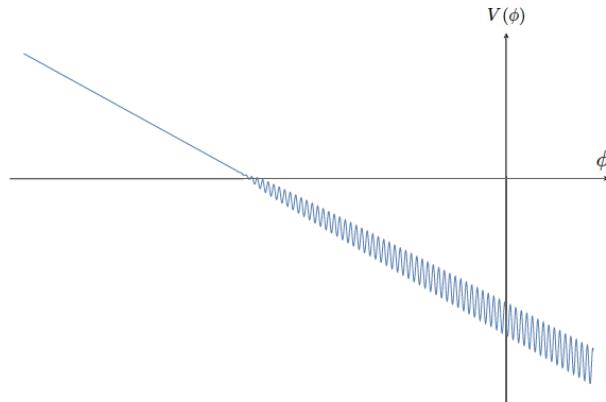
Solutions to the hierarchy problem

$\Lambda \leq 1\text{TeV} ??$

- Just the (scale invariant) Standard Model...no heavy states



- Scanning....Higgs mass minimised when minimising scalar potential



Slow roll - inflation

$$(-M^2 + g\phi)|h|^2 + (gM^2\phi + g^2\phi^2 + \dots) + \Lambda^3 h \cos(\phi/f)$$

hierarchy problem term

QCD axion

$$m_h^2 \sim g f$$

g small controlled by shift symmetry: $\phi \rightarrow \phi + c$

Solutions to the hierarchy problem

$\Lambda \leq 1 TeV ??$

- Just the (scale invariant) Standard Model...**no heavy states**
- Scanning....**Higgs mass minimised when minimising scalar potential**
- Composite models: **e.g. technicolour (no light Higgs)**

Solutions to the hierarchy problem

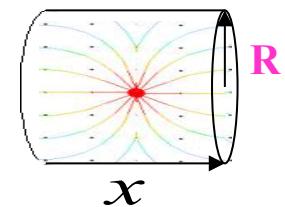
$\Lambda \leq 1\text{TeV}??$

- Just the (scale invariant) Standard Model...**no heavy states**
- Scanning....**Higgs mass minimised when minimising scalar potential**
- Composite models: **e.g. technicolour (no light Higgs)**
- $\Lambda_{\text{fundamental}} \sim 1\text{TeV}!$

Xtra dimensions

$$V(r) = \frac{1}{M_*^{2+d} R^d} \frac{m_1 m_2}{r}, \quad D = 4 + d, \quad r \ll R$$

$$M_{\text{Planck}}^2 = M_*^2 (M_* R)^d$$



(or warped extra dimensions)

... covered in following lectures by Professor Antoniadis

Solutions to the hierarchy problem

$\Lambda \leq 1 TeV ??$

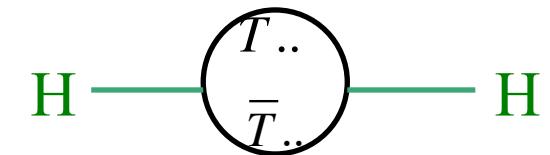
- Symmetry protection

e.g. $SU(3) \rightarrow SU(2)$

$8 \rightarrow 3$ 5 Goldstone bosons

$$\begin{pmatrix} & & H^{*+} \\ & & H^{*0} \\ & & . \\ & & H^0 \\ H^- & H^0 & . \end{pmatrix}$$

Nambu - Goldstone



Symmetry broken by gauge interactions -
pseudo Goldstone bosons

...addresses little hierarchy problem only
....little Higgs

Solutions to the hierarchy problem

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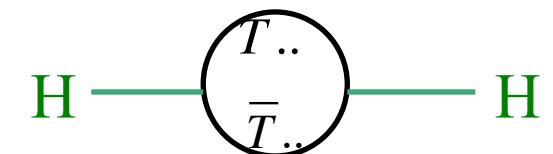
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(AdS/CFT: Composite $\Leftrightarrow \chi$ dimensions)

Solutions to the hierarchy problem

$\Lambda \leq 1 TeV ??$

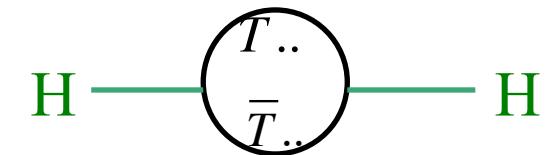
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Symmetry broken by gauge interactions -
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- Symmetry protection - supersymmetry

Radiative corrections

$$\delta^{\tilde{t}} M_H^2 \simeq \frac{\lambda_s}{8\pi^2} \left(\Lambda^2 - m_t^2 \ln\left(\frac{\Lambda^2}{m_t^2}\right) \right)$$

$$\psi_H = \begin{pmatrix} \tilde{H} \\ H \end{pmatrix} \quad \text{chiral symmetry}$$

$$m_{\tilde{t}} \leq 1 TeV \quad \cancel{\bar{\psi}_H \psi_H}, \quad t_{L,R}$$

$$H \quad \text{---} \quad H$$

$$\delta^{t+\tilde{t}} M_H^2 = \frac{h_t^2}{8\pi^2} \left(m_t^2 \ln\left(\frac{\Lambda^2}{m_t^2}\right) - m_t^2 \ln\left(\frac{\Lambda^2}{m_{\tilde{t}}^2}\right) \right)$$

Solutions to the hierarchy problem

$\Lambda \leq 1\text{TeV}??$

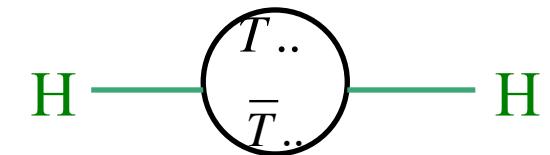
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Symmetry broken by gauge interactions -
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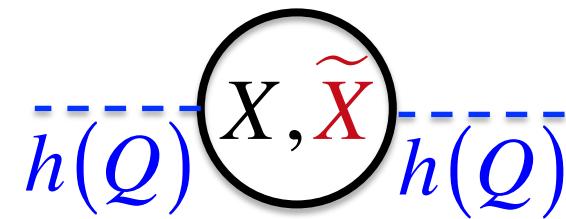
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- Symmetry protection - supersymmetry

$$\psi_H = \begin{pmatrix} \tilde{H} \\ H \end{pmatrix} \quad \cancel{\bar{\psi}_H \psi_H}, \quad \text{chiral symmetry}$$

Radiative corrections

$GUTs \Rightarrow SUSY - GUTs$



Twin Higgs

- Tree-level Higgs scalar potential (fully $SU(4)$ -symmetric):

8 real dof

$$\Phi = \begin{pmatrix} H \\ H' \end{pmatrix} \quad \text{transforms as fundamental of global } SU(4)$$

$$\begin{aligned} V_{tree}(\Phi) &= -m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \\ &= -m^2 (|H|^2 + |H'|^2) + \lambda (|H|^4 + |H'|^4 + 2|H|^2|H'|^2) \end{aligned}$$

$\text{SU}(4)\text{-invariant quartic}$

$$\Rightarrow \langle |\Phi|^2 \rangle = \langle |H|^2 \rangle + \langle |H'|^2 \rangle = \frac{m^2}{2\lambda} \equiv \left(\frac{f}{\sqrt{2}} \right)^2$$

$$\begin{array}{c} \text{Su(4)} \longrightarrow \text{Su(3)} \\ (\text{rather: O(8)} \longrightarrow \text{O(7)}) \end{array}$$

2 real scalars remain:

- Goldstone:

$$\hat{m}^2 = 0$$

- 'Normal' Higgs:

$$\hat{M}^2 = 2\lambda f^2$$

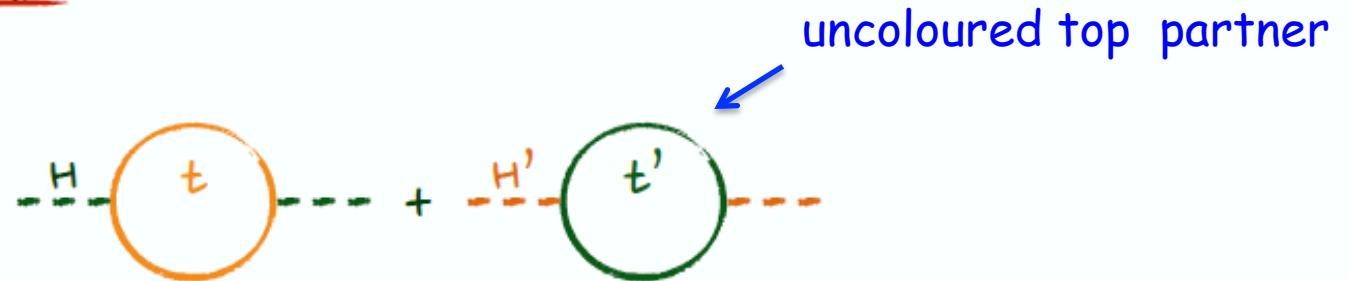
\rightarrow 7 Goldstone bosons!

3 eaten by $SU(2)$
3 eaten by $SU(2)'$
1 left: the Higgs we've seen!

Slide courtesy of Isabel Garcia Garcia
c.f. talk by S. Pokorsky here

Twin Higgs

But, crucially:

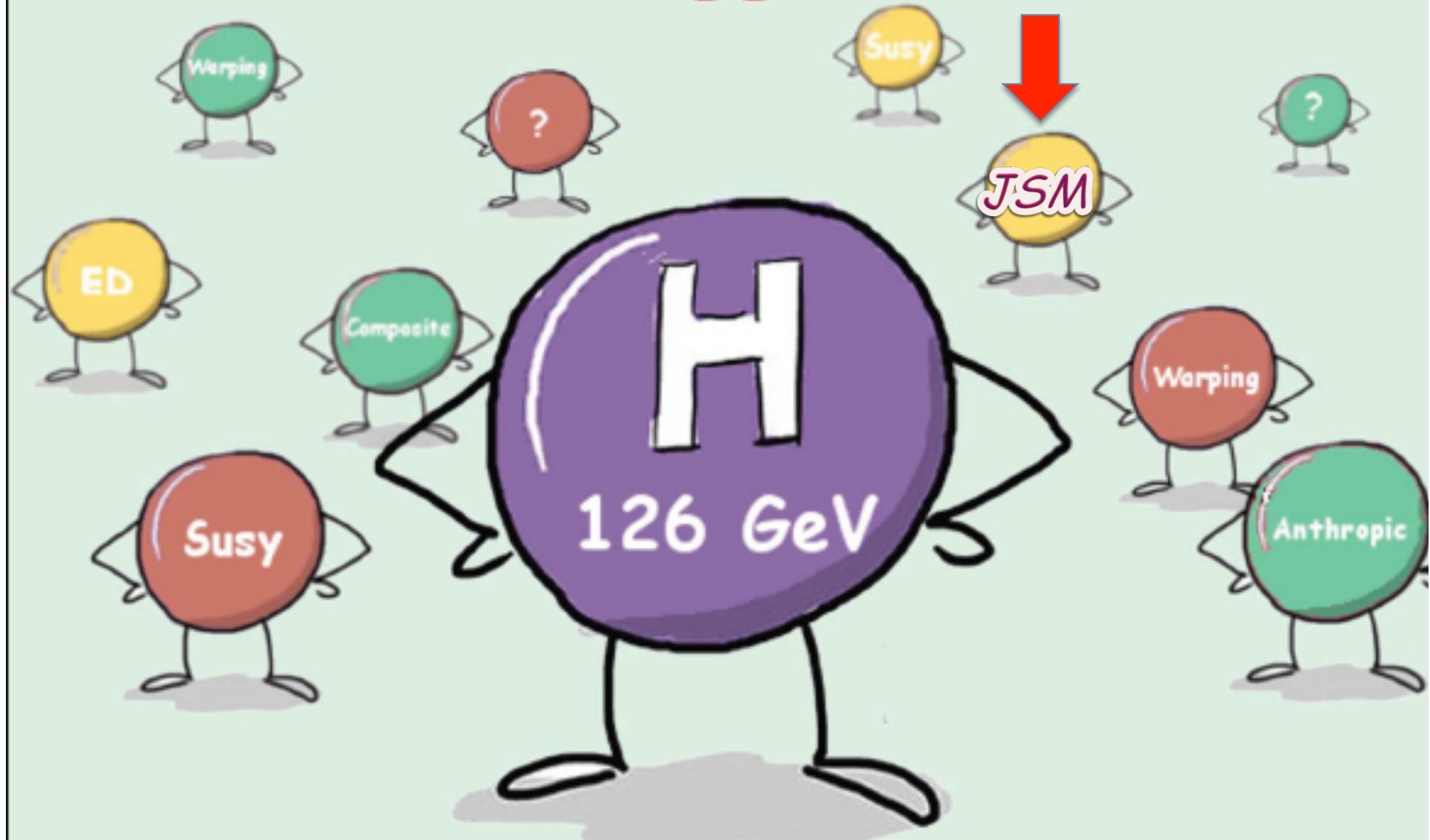


$$\Rightarrow \delta V \sim -\frac{3y_t^2}{16\pi^2}\Lambda^2|H|^2 - \frac{3y_t'^2}{16\pi^2}\Lambda^2|H'|^2 = -\frac{3y_t^2}{16\pi^2}\Lambda^2(|H|^2 + |H'|^2)$$

$\underbrace{}_{|\Phi|^2}$

No correction to the (Pseudo-Goldstone) Higgs!

The Higgs Era



IV. "Just" the Standard Model

Implications of a 125 GeV Higgs

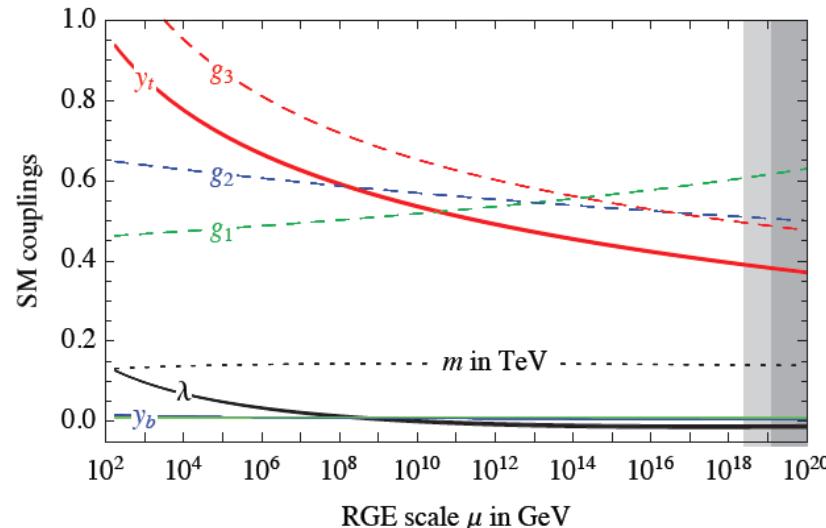
RG equations:

$$\beta_1^{\text{SM}} = \frac{41}{96\pi^2} g_1^3, \quad \beta_2^{\text{SM}} = -\frac{19}{96\pi^2} g_2^3, \quad \beta_3^{\text{SM}} = -\frac{7}{16\pi^2} g_3^3$$

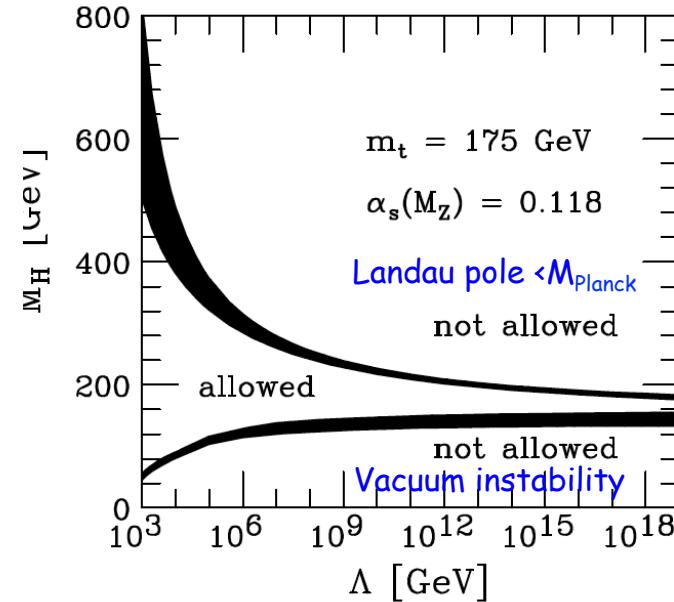
$$\beta_h^{\text{SM}} = \frac{1}{16\pi^2} \left[\frac{9}{2} h^3 - 8g_3^2 h - \frac{9}{4} g_2^2 h - \frac{17}{12} g_1^2 h \right]$$

$$\begin{aligned} \beta_\lambda^{\text{SM}} = & \frac{1}{16\pi^2} \left[24\lambda^2 + 12\lambda h^2 - 9\lambda (g_2^2 + \frac{1}{3} g_1^2) \right. \\ & \left. - 6h^4 + \frac{9}{8} g_2^4 + \frac{3}{8} g_1^4 + \frac{3}{4} g_2^2 g_1^2 \right]. \end{aligned}$$

Implications of a 125 GeV Higgs



RGE - just the Standard Model



Higgs coupling small

Hambye, Riesselmann

$$V_0 = -\frac{m_0^2}{2}|H_0|^2 + \lambda_0|H_0|^4$$

LHC 8

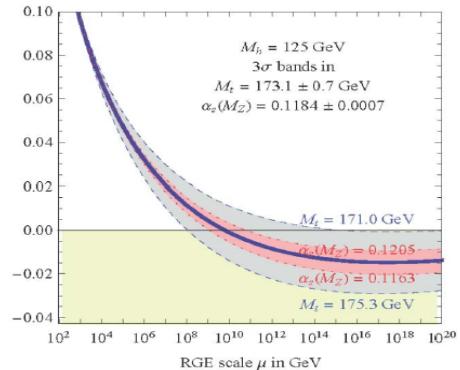
No evidence (yet) for BSM

Higgs discovery

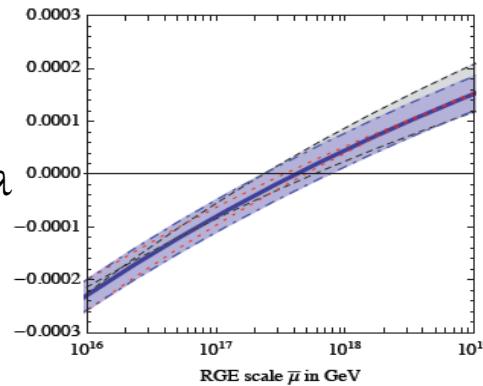
}

"Just" the SM (JSM)?

λ



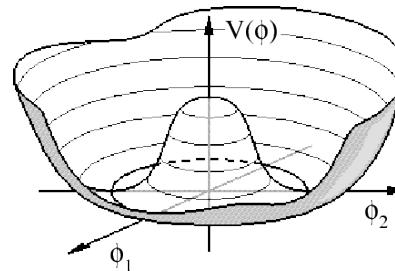
β_λ



DeGrassi et al,...

$$V(H) = -m^2 |\phi|^2 + \lambda |\phi|^4$$

$$m^2 \simeq (89 \text{ GeV}^2), \lambda \simeq 0.13$$



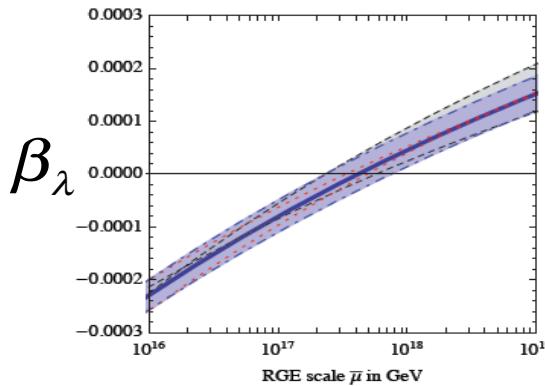
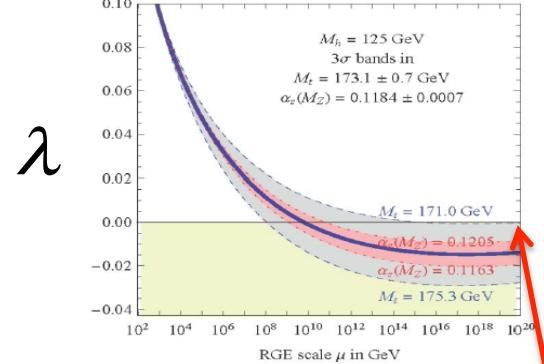
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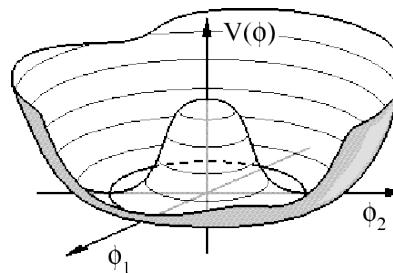


DeGrassi et al,...

IR structure???

$$V(H) = -m^2 |\phi|^2 + \lambda |\phi|^4$$

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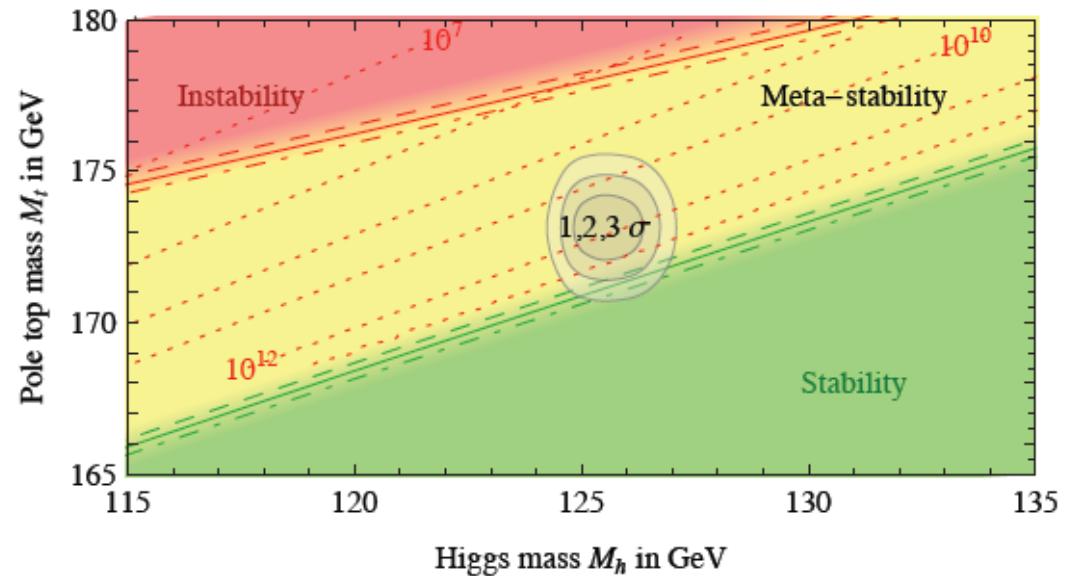
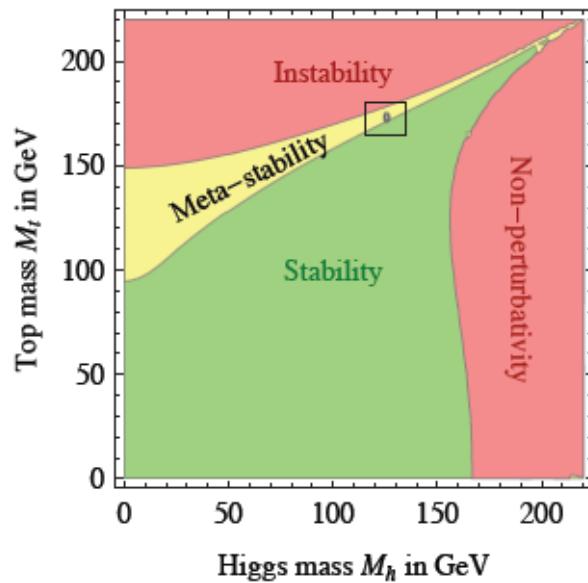
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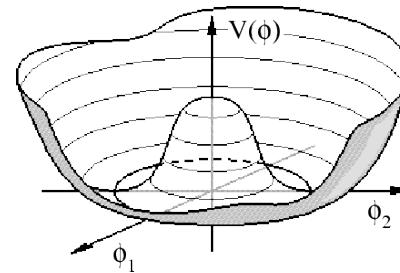
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"Just" the SM (JSM)?



$$V(H) = -m^2 |\phi|^2 + \lambda |\phi|^4$$

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"Just" the Standard Model

$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2} (4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2) \Lambda^2 = \left(\frac{\Lambda}{500\text{GeV}} \right)^2$$

δm^2 not measureable ...only $m^2 = m_0^2 + \delta m^2$ "physical"

Classical scale invariance, $m_h = 0$... origin of EW breaking?

"Just" the Standard Model

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Classical scale invariance, $m_h = 0$... origin of EW breaking?

Coleman-Weinberg - dynamical symmetry breaking :

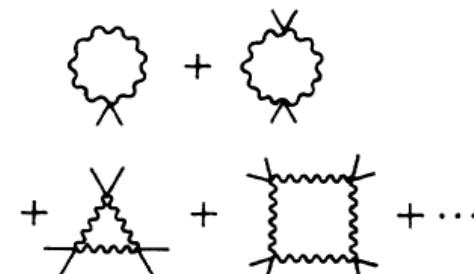
e.g. scalar electrodynamics

$$V = \left\{ \frac{\lambda}{4!} \phi^4 + \frac{3e^4}{64\pi^2} \phi^4 \ln \frac{\phi^2}{M^2} \right\}$$

$$= \frac{3e^4}{64\pi^2} \phi^4 \left(\ln \frac{\phi^2}{\langle \phi \rangle^2} - \frac{1}{2} \right)$$

$$m_\phi^2 = \frac{3e_\phi^2}{8\pi^2} m_X^2 \ll m_X^2$$

scale invariance broken by Trace anomaly



"real" hierarchy problem

..... many models with new Higgs interactions + no heavy states

No heavy states?

- Neutrino masses?
- Baryogenesis?
- Strong CP problem?
- Gravity?

No heavy thresholds?

- Neutrino masses?
- Baryogenesis?
- Strong CP problem?
- Gravity?

Neutrino masses:

Add singlet neutrinos ν_{Ra}

$$L_{mass} = h_a \bar{l}_a \nu_{Ra} H + \frac{M_{ab}}{2} \nu_{Ra}^T C \nu_{Rb}$$

e.g. $h_a^2 = 5 \cdot 10^{-14}$, $h_b^2 = 5 \cdot 10^{-15}$, $M_a = 20 \text{ GeV}$

Ultra-weak:
Natural due to
chiral symmetry

$$m_a \simeq 0.1 \text{ eV}, \quad m_b \simeq 0.01 \text{ eV}$$

Baryogenesis

$$L_{mass} = h_a \bar{l}_a v_{Ra} H + \frac{M_{ab}}{2} v_{Ra}^T C v_{Rb}$$

- v_{Ra} produced via Yukawa interactions

$$L_A = L_B = L_C = 0$$

Baryogenesis

$$L_{mass} = h_a \bar{l}_a v_{Ra} H + \frac{M_{ab}}{2} v_{Ra}^T C v_{Rb}$$

- v_{Ra} produced via Yukawa interactions $L_A = L_B = L_C = 0$
- v_{Ra} oscillate $\mathcal{CP}, \quad L_{A,B,C} \neq 0, \quad L_A + L_B + L_C = 0$

Baryogenesis

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- v_{Ra} oscillate $\mathcal{CP}, \quad L_{A,B,C} \neq 0, \quad L_A + L_B + L_C = 0$
- Only
- $v_{Ra,b}$ in thermal equilibrium by t_{EW} when sphalerons inoperative

$$\Delta_{L_{AB}} = L_A + L_B \xrightarrow{\text{Sphalerons}} \Delta B = \Delta_{L_{AB}} / 2 \quad \checkmark$$

Akhmedov, Rubakov, Smirnov

Strongly constrained if demand v_{Rc} is dark matter

Shaposhnikov et al

Strong CP problem:

$$\frac{\theta}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \quad \theta \leq 10^{-10} ??$$

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$$\frac{\theta}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \quad \theta \leq 10^{-10} ??$$

Make θ a dynamical variable the axion, $a \dots \theta=0$ at minimum of its potential

Vafa, Witten

... complex scalar field, S

$$S = (|S| + f_a) e^{i \frac{a}{f_a}}, \quad 10^{10} \text{GeV} \leq f_a \leq 10^{12} \text{GeV}$$

Strong CP problem:

$$\frac{\theta}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \quad \theta \leq 10^{-10} ??$$

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DFSZ axion: 2 Higgs doublets $H_{1,2}$, complex singlet, S

$$\begin{aligned} V(H_1, H_2) = & \frac{\lambda_1}{2} |H_1|^4 + \frac{\lambda_2}{2} |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 \\ & + \lambda_4 |H_1^\dagger H_2|^2 + \zeta_1 |S|^2 |H_1|^2 + \zeta_2 |S|^2 |H_2|^2 \\ & + \zeta_3 S^2 H_1 H_2 + h.c. + \zeta_4 |S|^4 \end{aligned}$$

PQ symmetry: $H_1 \rightarrow H_1 e^{i\alpha}, \quad H_2 \rightarrow H_2 e^{i\beta}, \quad S \rightarrow S e^{-i(\alpha+\beta)/2}$

Axion, $\textcolor{red}{a}$: Pseudo Goldstone boson of spontaneously broken PQ

Strong CP problem:

$$\frac{\theta}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \quad \theta \leq 10^{-10} ??$$

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Axion, a : Pseudo Goldstone boson of spontaneously broken PQ

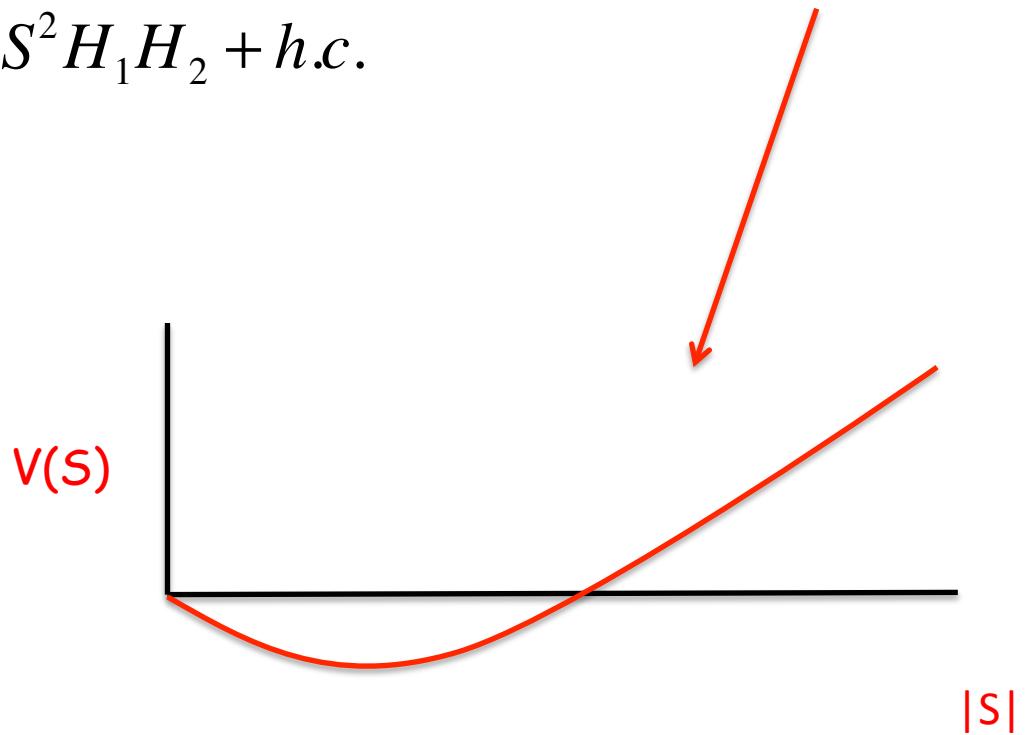
Ultra weak sector:

$$\zeta_{1,2,3} \leq 10^{-20} \left(\frac{10^{12} \text{GeV}}{f_a} \right)^2 \quad \begin{array}{l} \text{shift symmetry} \\ S \rightarrow S + \delta \end{array}$$

Coleman Weinberg in DFSZ model

$$V_{DFSZ}(H_1, H_2, S) \approx \frac{\lambda_1}{2} \left(|H_1|^2 + \frac{\zeta_1}{\lambda_1} |S|^2 \right)^2 + \frac{1}{64\pi^2} (\zeta_2 |S|^2)^2 \left(-\frac{1}{2} + \ln \frac{|S|^2}{f_a^2} \right) + \frac{\lambda_2}{2} |H_2|^4 + \zeta_3 S^2 H_1 H_2 + h.c.$$

$\zeta_2 |S|^2 |H|^2$



$$\langle H_1^2 \rangle = -\frac{\zeta_1}{\lambda_1} \langle S^2 \rangle \text{ triggers EW breaking}$$

Coleman Weinberg in DFSZ model

$$V_{DFSZ}(H_1, H_2, S) \simeq \frac{\lambda_1}{2} \left(|H_1|^2 + \frac{\zeta_1}{\lambda_1} |S|^2 \right)^2 + \frac{1}{64\pi^2} (\zeta_2 |S|^2)^2 \left(-\frac{1}{2} + \ln \frac{|S|^2}{f_a^2} \right)$$

$$+ \frac{\lambda_2}{2} |H_2|^4 + \zeta_3 S^2 H_1 H_2 + h.c. \quad (\zeta_2 > \zeta_1 > \zeta_3 \text{ assumed})$$

$$v_S = f_a, \quad v_{H_1} = \frac{\zeta_1}{\lambda_1} f_a, \quad v_{H_2} = \frac{\zeta_3}{2\zeta_2} v_{H_1}$$

$$m_{H_2^0}^2 = m_{H^\pm}^2 = m_X^2 = -\frac{\zeta_2}{2\zeta_1} m_h^2$$

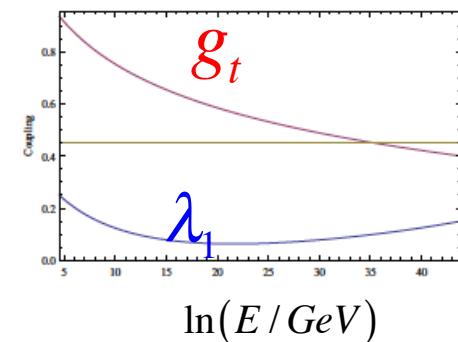
$$m_{|S|}^2 = - \left(\frac{\zeta_2^2}{32\pi^2 \zeta_1} \right)^2 m_h^2 \simeq 13 \left(\frac{10^{12} GeV}{v_S} \right)^2 \left(\frac{m_{H_2}}{m_h} \right)^4 eV^2$$

$|S|$ Pseudo-dilaton

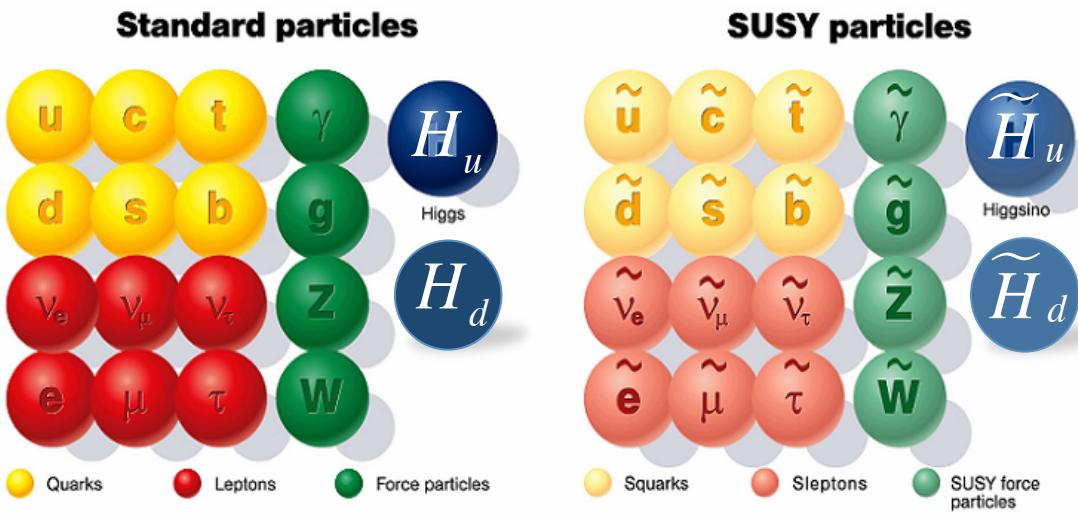
K. Allison, C.Hill, GGR

Summary - IV

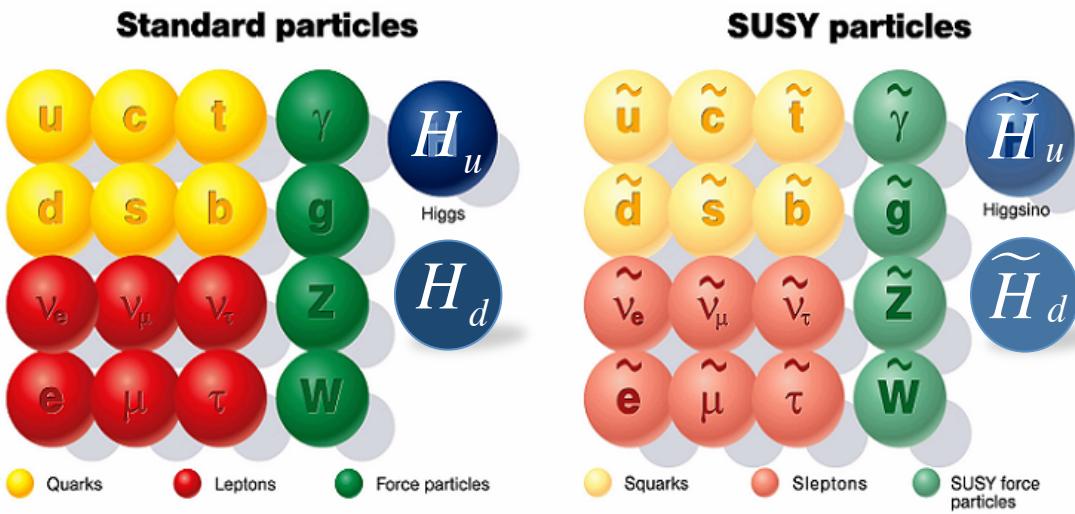
- “JSM” requires ultra-weak sectors - chiral and shift symmetries
- DFSZ axion + dimensional trasmutation $\Rightarrow f_a$
...consistent with classical scale invariance (not KSVZ model)
- Requires two Higgs doublets (type II couplings), light pseudo-dilaton
 - $m_{H_2^0}^2 = m_{H^\pm}^2 = m_X^2 = R^2 m_h^2$
 - $h \approx \text{SM Higgs}$
 - $m_{\text{ISI}} \simeq 0.9 \left(\frac{10^{12} \text{GeV}}{f_a} \right) R^2 eV$
 - Direct (axion-like) searches?
- Stable vacuum but loses simplicity of SM
... and no unification



V. SUSY



V(i). SUSY GUTs



$$G_{GUT} \times G_{Flavour} \times (N=1 \text{ SUSY})$$

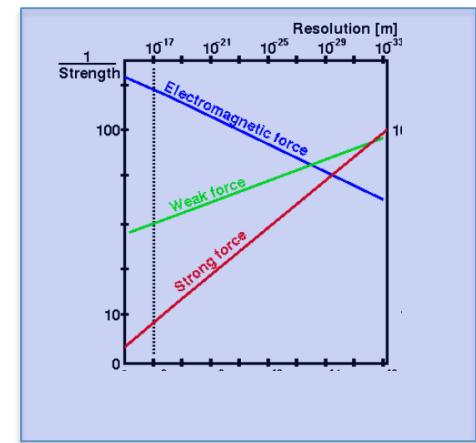
Supermultiplets

SO(10): V_{45} Vector + 3 φ_{16} chiral + H_{10} chiral + ...

SUSY gauge coupling unification

$$\alpha_i^{-1}(\mu) = \alpha^{-1}(M_X) + \frac{1}{2\pi} b_i \ln\left(\frac{M_X}{\mu}\right) + ..$$

$$b_i^{SM} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_g \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + H \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix}$$

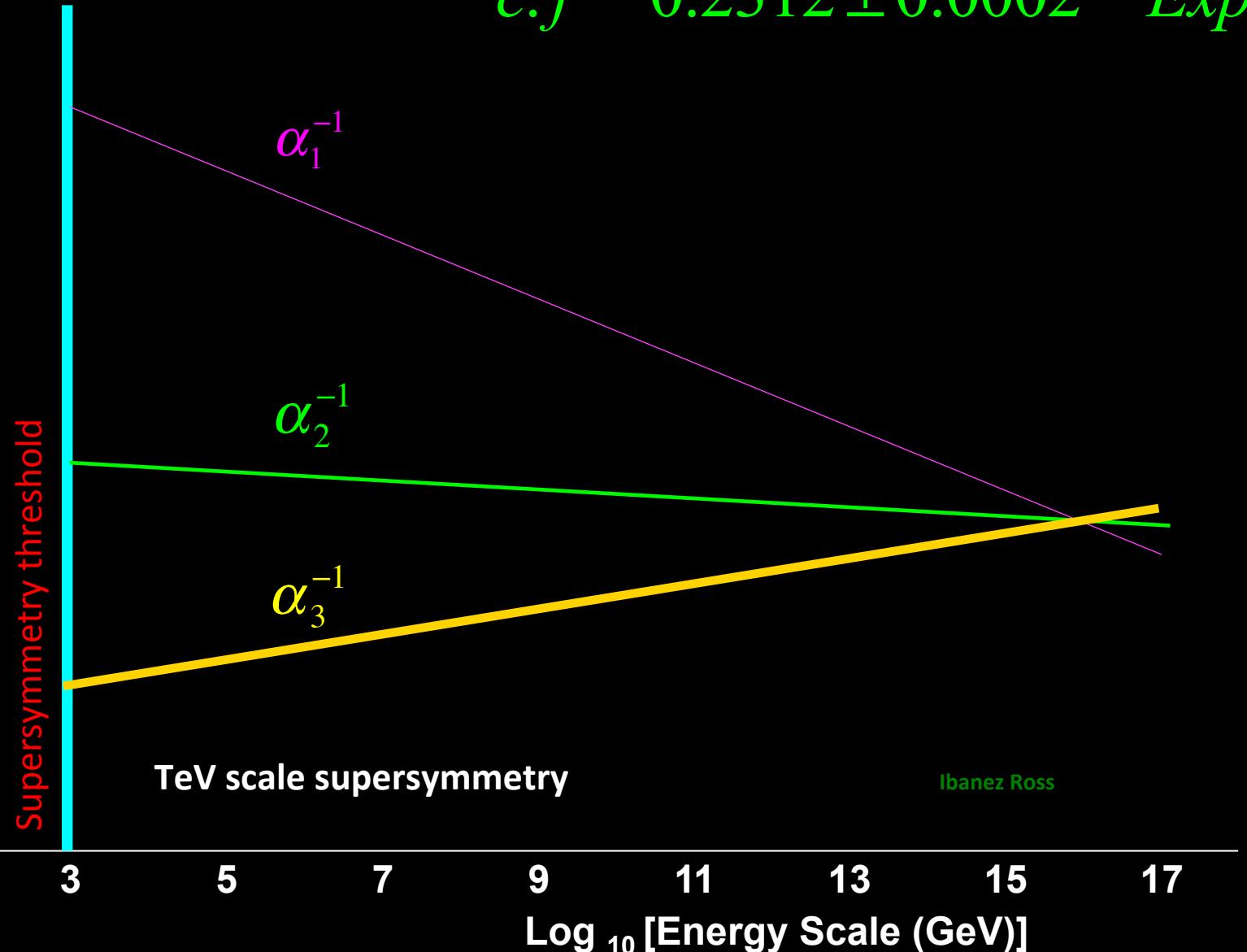


$$b_i^{MSSM} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_g \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + H \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

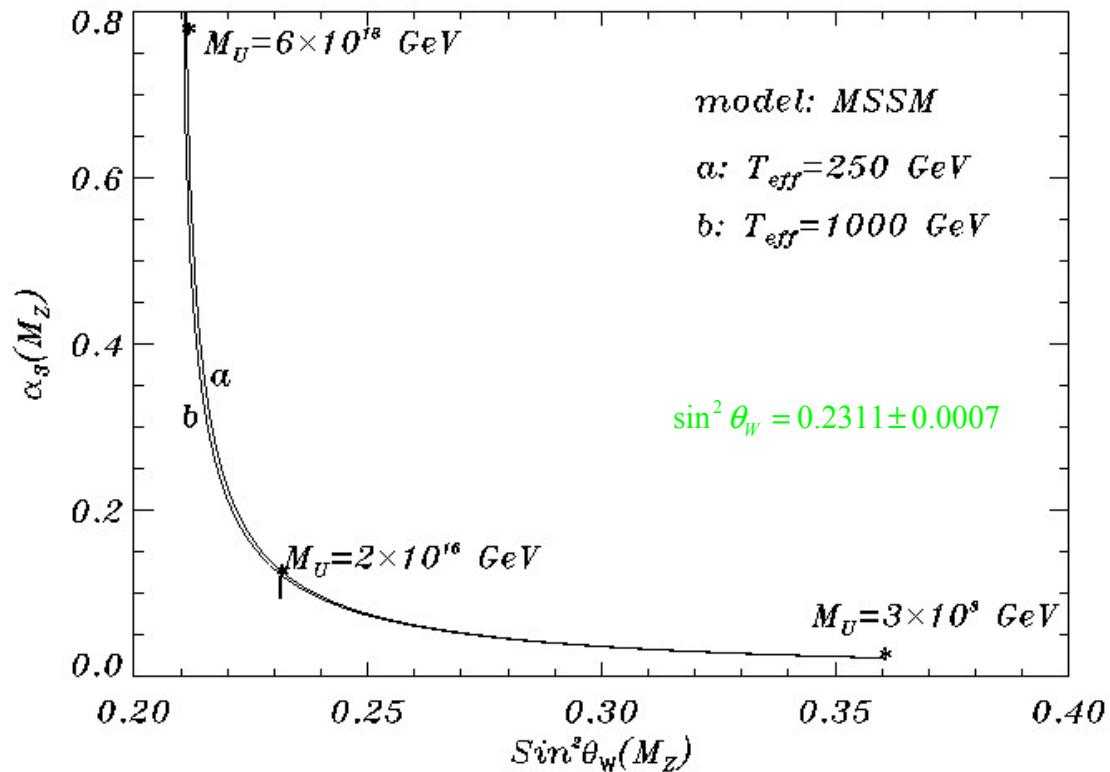
?

$$\sin^2 \theta_W = 0.2337 \pm 0.0015$$

c.f 0.2312 ± 0.0002 *Expt*



SUSY gauge coupling unification

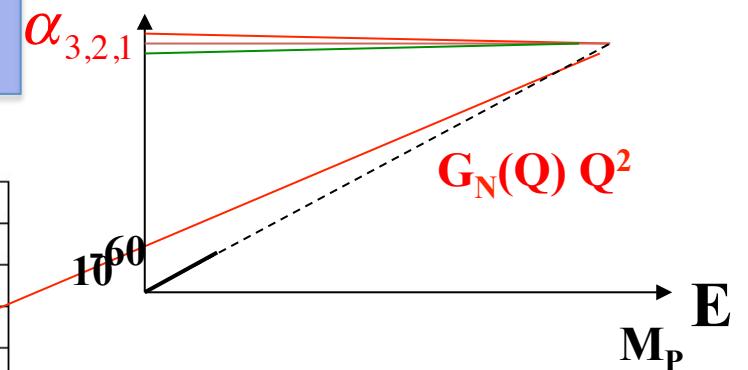
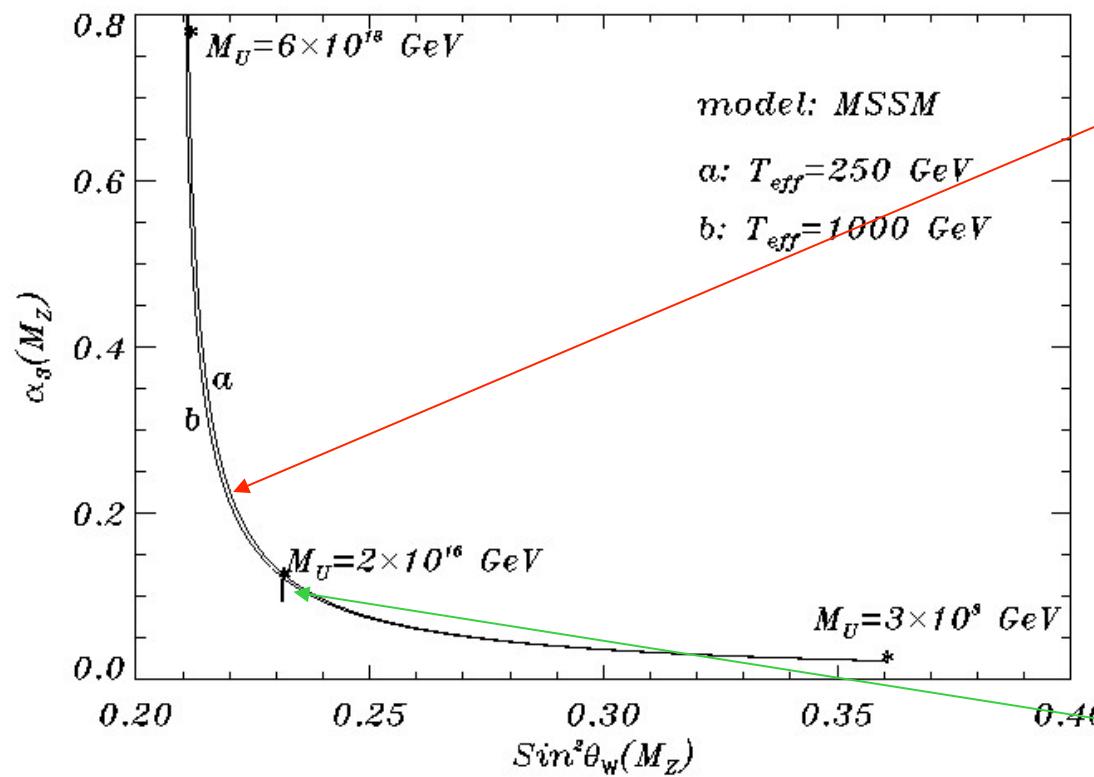


$$\sin^2 \theta_W = 0.2334 \pm 0.0025 - 0.25(\alpha_s - 0.119) = 0.2311 \pm 0.0007 \quad (\text{Expt})$$

$$\alpha_s = 0.134 \pm 0.01 - 4(\sin^2 \theta_W - 0.2334) = 0.119 \pm 0.01 \quad (\text{Expt})$$

SUSY gauge coupling unification

Unification with gravity?



$$M_U = (2.6 \pm 2) \cdot 10^{16} \text{ GeV}$$

$$\sin^2 \theta_W = 0.23116(12) \quad (\text{Expt})$$

$$\alpha_s = 0.134 \pm 0.01 - 4(\sin^2 \theta_W - 0.23116)$$

$$c.f. \quad 0.1184(7) \quad (\text{Expt})$$

Gauge unification - Heterotic String

$$L_{eff}^{HS} = \int d^{\textcolor{red}{10}}x \sqrt{g} e^{-\phi} \left(\frac{4}{\alpha'^4} R + \frac{k_i}{\alpha'^3} Tr F_i^2 + \dots \right)$$

$$\int d^4x V \quad \textcolor{blue}{\alpha_{10}^{-1}}$$

$\alpha' = 1/M_{string}^2$ only scale




$$G_N = \frac{\alpha_{10}\alpha'^4}{64\pi V}, \quad \alpha_{String} = \frac{\alpha_{10}\alpha'^3}{16\pi V} \quad \rightarrow \quad G_N = \frac{\alpha_{String}\alpha'}{4}$$

$$\boxed{\frac{1}{g_i^2(M_Z)} = \frac{k_i}{g_{string}^2} + b_i \ln \left(\frac{M_{string}}{M_Z} \right) + \Delta_i}$$

$$M_{string} = g_{string} \cdot M_{Planck} = 3.6 \times 10^{17} GeV \quad c.f. M_U^{\text{"expt"}} = (2.6 \pm 2) \cdot 10^{16} GeV$$

Spontaneous symmetry breaking

$$SU(5) \xrightarrow[\Sigma_{24}]{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow[H_{\bar{5}}]{M_W} SU(3) \times U(1)$$

Spontaneous symmetry breaking

$$SU(5) \xrightarrow[\Sigma_{24}]{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow[H_{\bar{5}}]{M_W} SU(3) \times U(1)$$

$$P = \frac{\beta_2}{2} M \operatorname{Tr}(\Sigma^2) + \frac{\beta_3}{3} \operatorname{Tr}(\Sigma^3) \quad \text{superpotential}$$

$$V(\Sigma) = \sum_a \left| \frac{\partial P}{\partial \Sigma^a} \right|^2 = \operatorname{Tr} \left| \beta_3 \Sigma^2 + \beta_2 M \Sigma - I \frac{\beta_3}{5} \operatorname{Tr}(\Sigma^2) \right|^2$$

$$\left(\frac{\partial P}{\partial \Sigma^a} \rightarrow \frac{\partial P}{\partial \Sigma^i} - \frac{1}{N} \delta_j^i \operatorname{Tr} \left(\frac{\partial P}{\partial \Sigma} \right), \quad i, j = 1..5, \quad a = 1..24 \right)$$

$$\langle \Sigma \rangle = 0$$

$$\langle \Sigma \rangle = v_4 \operatorname{Diagonal}(1,1,1,1,-4)$$

$$\langle \Sigma \rangle = v_3 \operatorname{Diagonal}(2,2,2,-3,-3)$$

} Degenerate

SUGRA

Radiative
corrections

Spontaneous symmetry breaking

$$SU(5) \xrightarrow[\Sigma_{24}]{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow[H_5]{M_W} SU(3) \times U(1)$$

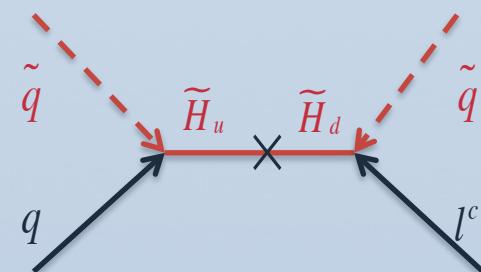
$$P_{Higgs} = \mu H_u H_d + \lambda H_u \Sigma H_d$$

X

$$V = \left(|\mu H_u + \lambda H_u \Sigma|^2 + |\mu H_d + \lambda \Sigma H_d|^2 \right) + \left| H_u H_d - \frac{1}{5} (H_u H_d) \right|^2$$

Must forbid these terms by symmetry

+ doublet- triplet splitting



D=5 proton decay amplitude

$$A_{p\,decay} \propto \frac{1}{M_{\tilde{H}}} c.f. \cdot \frac{1}{M_{X,Y}^2}$$

Doublet -triplet splitting

Missing doublet mechanism

No (1,2) component

$$\Theta_{50} = (8,2) + (6,3) + (\bar{6},1) + (3,2) + (\bar{3},1) + (1,1)$$

$$P_{MD} = b \Theta \Sigma_{75} H_u + b' \bar{\Theta} \Sigma_{75} H_d + \tilde{M} \bar{\Theta} \Theta$$

$\langle \Sigma_{75} \rangle \propto M$ breaks SU(5) to SM

$$P_{MD} \supset b M \Theta_3 H_{uT} + b' M \bar{\Theta}_3 H_{dT} + \tilde{M} \bar{\Theta}_3 \Theta_3$$

Triplets get mass $\frac{M^2}{\tilde{M}}$ (Still need to drive SSB)

Doublet -triplet splitting

Higher dimensions (String unification)

Compactification:

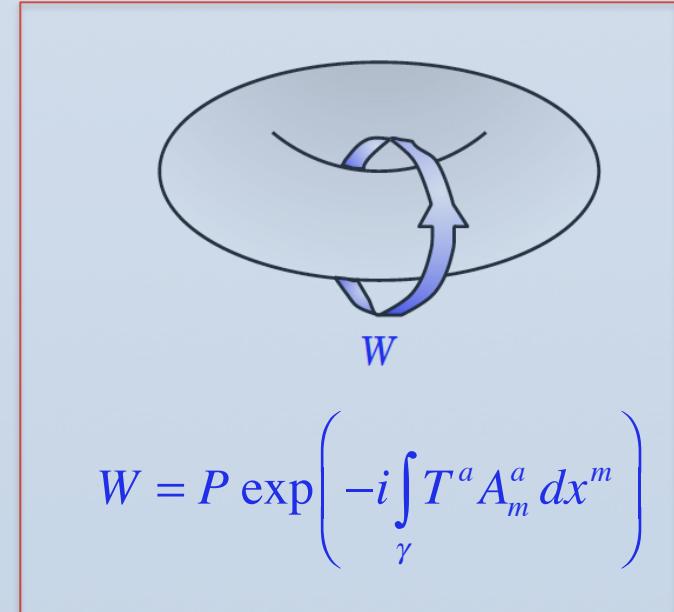
$$K = K_0 / H$$

↑
freely acting discrete group

Wilson line breaking: $W : \overline{H} \subset G$

↑
embedding of H into gauge group G

Massless states: $H \otimes \overline{H}$ singlets



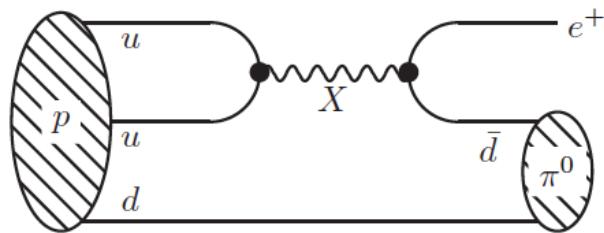
Breit, Ovrut, Segre

e.g. $SU(5)$: $H = Z_3$, $\overline{H} = \text{Diag}(\alpha, \alpha, \alpha, 1, 1)$, $\alpha = e^{2i\pi/3}$

$$(R \otimes \overline{R}) : (1 \otimes \overline{5}) \rightarrow \begin{pmatrix} H^- \\ \overline{H}^0 \end{pmatrix}_1, \quad (3, \overline{5}) \rightarrow \begin{pmatrix} e \\ v_e \end{pmatrix}_1 \oplus \begin{pmatrix} d^c \\ d^c \\ d^c \end{pmatrix}_{\alpha^2}, \quad \text{Matter} \rightarrow (3, \overline{5} + 10)$$

SUSY GUTS - Nucleon decay

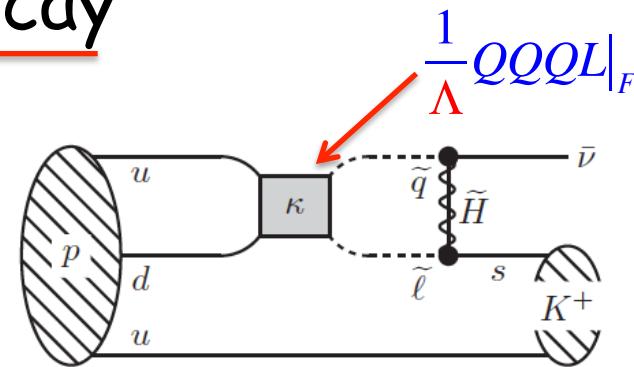
SUSY GUTS - Nucleon decay



(a) Dimension 6.

$$p \rightarrow \pi^0 + e^+$$

$$\tau_{p \rightarrow e^+ \pi^0} > 1 \times 10^{34} \text{ yrs}, M_X > 10^{16} \text{ GeV}$$

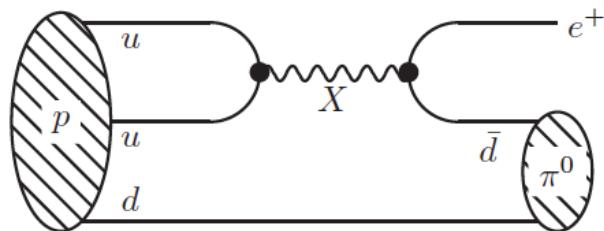


(b) Dimension 5.

$$p \rightarrow K^+ + \bar{\nu}$$

$$\tau_{p \rightarrow K^+ \bar{\nu}} > 3.3 \times 10^{33} \text{ yrs}$$

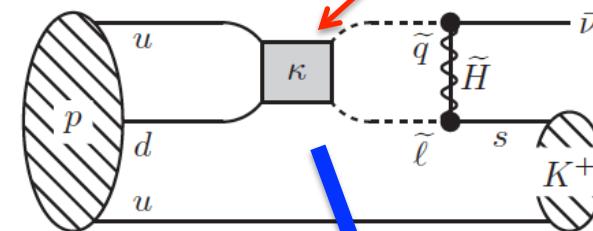
SUSY GUTS - Nucleon decay



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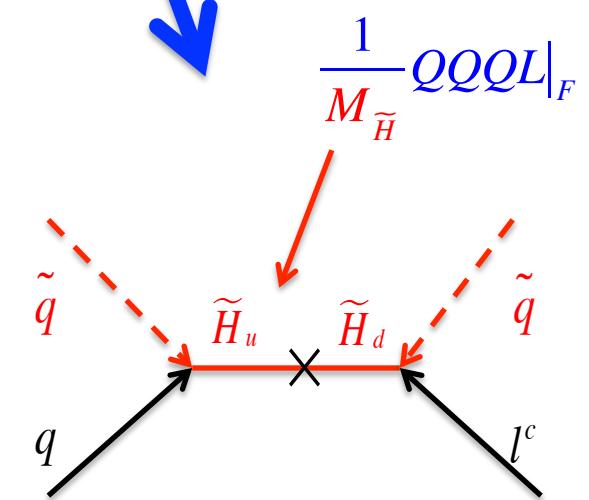


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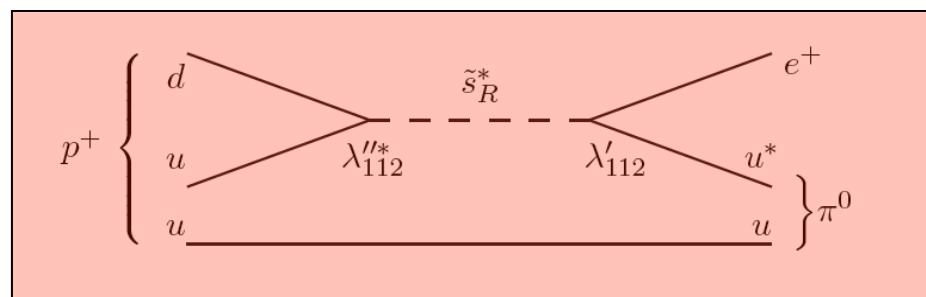
$$\underline{M_{\tilde{H}} > 10^{27} \text{ GeV}, 10^9 M_{\text{Planck}} ???}$$



D=5 proton decay amplitude

SUSY extensions of the Standard Model

$$\begin{aligned} W = & h^E L H_d \bar{E} + h^D Q H_d \bar{D} + h^U Q H_u \bar{U} + \mu H_d H_u \\ & + \lambda L L \bar{E} + \lambda' L Q \bar{D} + \kappa L H_u + \lambda'' \bar{U} \bar{D} \bar{D} \\ & + \frac{1}{M} (Q Q Q L + Q Q Q H_d + Q \bar{U} \bar{E} H_d + \dots (L)) \end{aligned}$$



SUSY extensions of the Standard Model

$$\begin{aligned} W = & h^E L H_d \bar{E} + h^D Q H_d \bar{D} + h^U Q H_u \bar{U} + \mu H_d H_u \\ & + \lambda L L \bar{E} + \lambda' L Q \bar{D} + \kappa L H_u + \lambda'' \bar{U} \bar{D} \bar{D} \\ & + \frac{1}{M} (Q Q Q L + Q Q Q H_d + Q \bar{U} \bar{E} H_d + \dots (L)) \end{aligned}$$

R-parity: Z_2

SUSY states odd

SUSY extensions of the Standard Model

$$W = h^E L H_d \bar{E} + h^D Q H_d \bar{D} + h^U Q H_u \bar{U} + \mu H_d H_u \\ + \lambda L L \bar{E} + \lambda' L Q \bar{D} + \kappa L H_u + \lambda'' \bar{U} \bar{D} \bar{D} \\ + \frac{1}{M} (Q Q Q L + Q Q Q H_d + Q \bar{U} \bar{E} H_d + \dots (\cancel{L}))$$

R-parity: Z_2 SUSY states odd

Z_N^R R-symmetry $\overset{Q_W^R = 2}{\leftarrow}$ $N=4,6,8,12,24$ LSP stable

Z_4^R special:

MSSM spectrum
No perturbative μ term
Commutes with $SO(10)$
Anomaly cancellation

N	q_{10}	$q_{\bar{5}}$	q_{H_u}	q_{H_d}	q_N
4	1	1	0	0	2

Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg, Vaudrevange
Babu, Gogoladze, Wang

Nucleon decay outlook

- Nucleon decay D=6 operators

$$\tau(p \rightarrow \pi^0 e^+) = \left(\frac{M_{\text{GUT}}}{10^{16} \text{ GeV}} \right)^4 \left(\frac{1/35}{\alpha_{\text{GUT}}} \right)^2 \left(\frac{0.015 \text{ GeV}^3}{\alpha_N} \right)^2 \left(\frac{5}{A_L} \right)^2 4.4 \times 10^{34} \text{ yr.}$$

Hadronic matrix element

Operator renormalisation

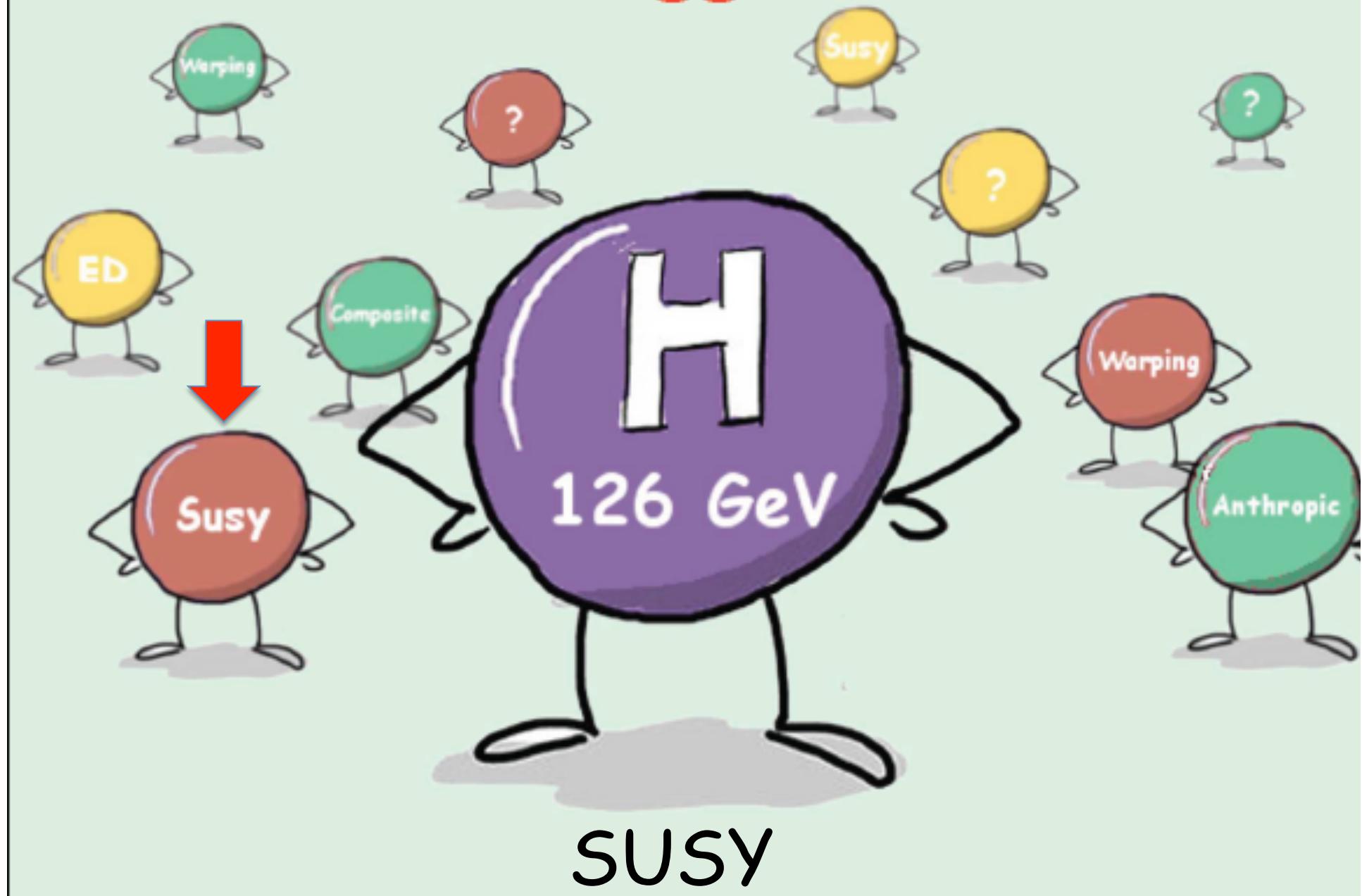
$$\tau_{p \rightarrow e^+ \pi^0}^{\text{SuperK}} > 1 \times 10^{34} \text{ yrs}$$

Giudice, Romanino

$$M_{\text{GUT}} > \left(\frac{\alpha_{\text{GUT}}}{1/35} \right)^{1/2} \left(\frac{\alpha_N}{0.015 \text{ GeV}^3} \right)^{1/2} \left(\frac{A_L}{5} \right)^{1/2} 6 \times 10^{15} \text{ GeV}$$

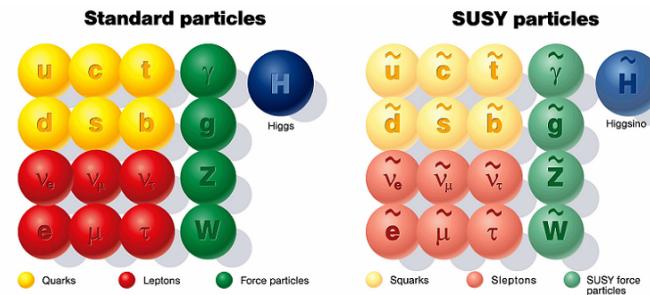
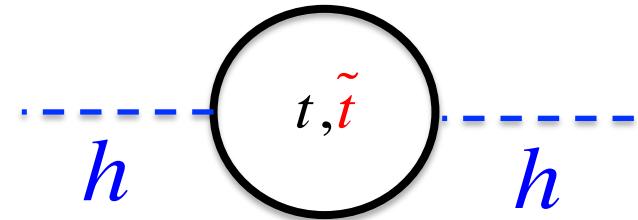
$$c.f. M_U = (2.5 \pm 2) \cdot 10^{16} \text{ GeV}$$

The Higgs Era



V(ii): The Little hierarchy problem in SUSY

Low scale SUSY



$$m_h^2 = M_Z^2 + \frac{3m_t^2 h_t^2}{4\pi^2} \left(\ln \left(\frac{m_{stop}^2}{m_t^2} \right) + \delta_t \right) + \dots \simeq 126 \text{ GeV}$$

$$\delta m_{H_u}^2 \simeq -\frac{3y_t^2}{4\pi^2} \left(m_{stop}^2 + \frac{g_s^2}{3\pi^2} m_{gluino}^2 \log \left(\frac{\Lambda}{m_{gluino}} \right) \right) \log \left(\frac{\Lambda}{m_{stop}} \right)$$

Gauge coupling unification
 $\Lambda \sim M_{GUT}$

$m_{\tilde{t}, \tilde{g}} < 1 \text{ TeV} ??$

Little hierarchy problem

e.g. **MSSM**: 105 +(19) Parameters

$$M_Z^2 = \sum_{\tilde{q}, \tilde{l}} a_i \tilde{m}_i^2 + \sum_{\tilde{g}, \tilde{W}, \tilde{B}} b_i \tilde{M}_i^2 + \dots$$

$$m_{\tilde{q}} > 0.6 - 1 \text{TeV} \Rightarrow \Delta > a \frac{\tilde{m}^2}{M_Z^2} \sim 100 \quad (\text{Unless light stop } m_{\tilde{t},LHC} > 250 \text{ GeV})$$

⇒ Correlations between SUSY breaking parameters
and/or additional low-scale states

Little hierarchy problem

e.g. **MSSM**: 105 +(19) Parameters

$$M_Z^2 = \sum_{\tilde{q}, \tilde{l}} \tilde{a}_i \tilde{m}_i^2 + \sum_{\tilde{g}, \tilde{W}, \tilde{B}} \tilde{b}_i \tilde{M}_i^2 + \dots$$

$$m_{\tilde{q}} > 0.6 - 1 \text{TeV} \Rightarrow \Delta > a \frac{\tilde{m}^2}{M_Z^2} \sim 100 \quad (\text{Unless light stop } m_{\tilde{t}, LHC} > 250 \text{ GeV})$$

⇒ Correlations between SUSY breaking parameters
and/or additional low-scale states

Fine Tuning measure:

$$\Delta(\gamma_i) = \left| \frac{\gamma_i}{M_Z} \frac{\partial M_Z}{\partial \gamma_i} \right|,$$

$$\Delta_m = \text{Max}_{\gamma_i} \Delta(\gamma_i), \quad \Delta_q = \left(\sum \Delta_{\gamma_i}^2 \right)^{1/2}$$

$$\gamma_i = \tilde{m}_i, \tilde{M}_i, \dots$$

Ellis, Enquist, Nanopoulos, Zwirner
Barbieri, Giudice

Fine tuning from a likelihood fit:

“Nuisance” variable

$$L(\text{data} \mid \gamma_i) \propto \int d\mathbf{v} \delta(m_z - m_z^0) \delta\left(\mathbf{v} - \left(-\frac{\mathbf{m}^2}{\lambda}\right)^{1/2}\right) L(\text{data} \mid \gamma_i; \mathbf{v})$$
$$= \frac{1}{\Delta_q} \delta(n_q (\ln \gamma_i - \ln \gamma_i^S)) L(\text{data} \mid \gamma_i; \mathbf{v}_0)$$

Fine tuning not optional!

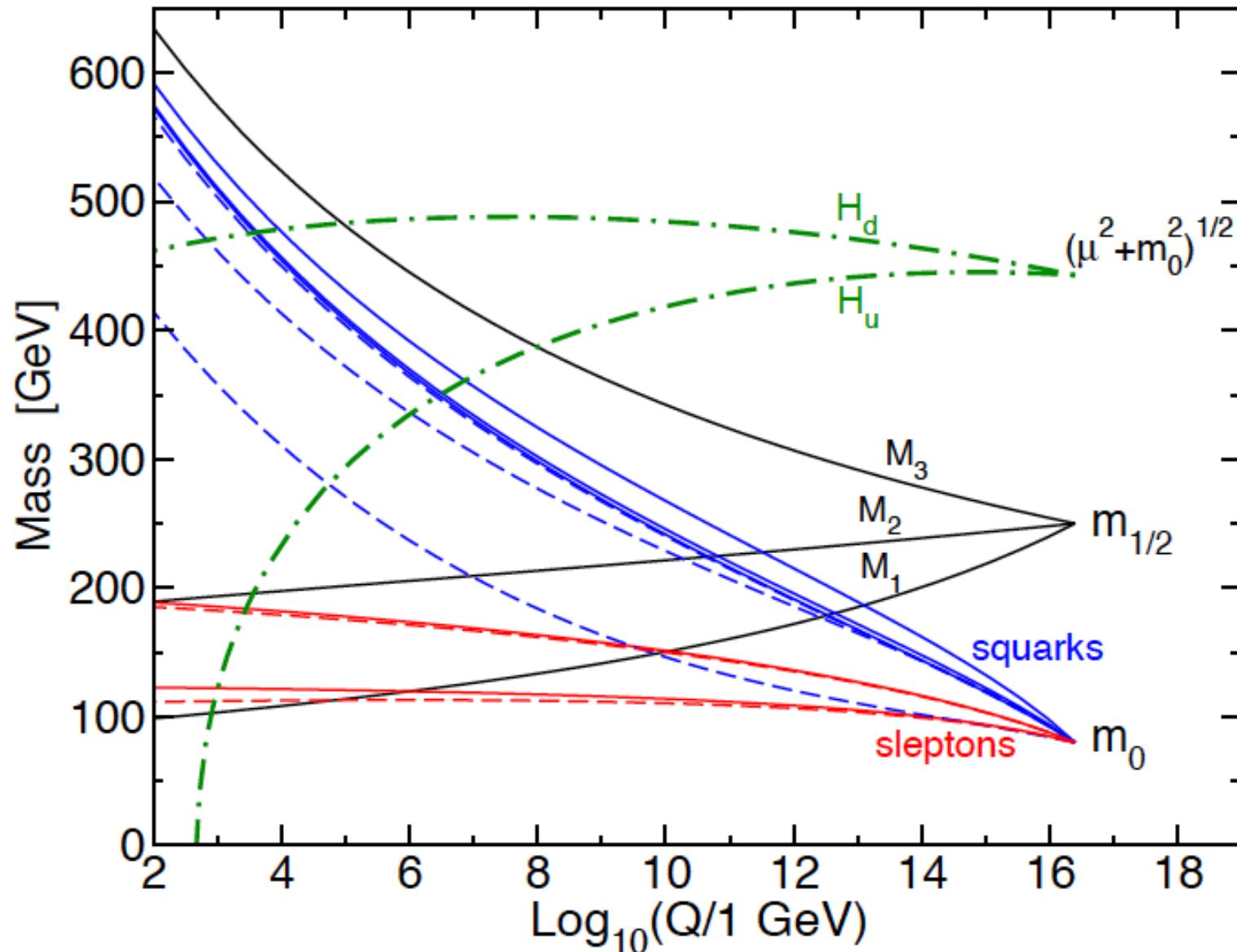
Ghilencea, GGR
Casas et al

Probabilistic interpretation:

$$\chi_{new}^2 = \chi_{old}^2 + 2 \ln \Delta_q \quad \Delta_q \ll 100$$

CMSSM:

$$\gamma_i = \mu_0, m_0, m_{1/2}, A_0, B_0$$



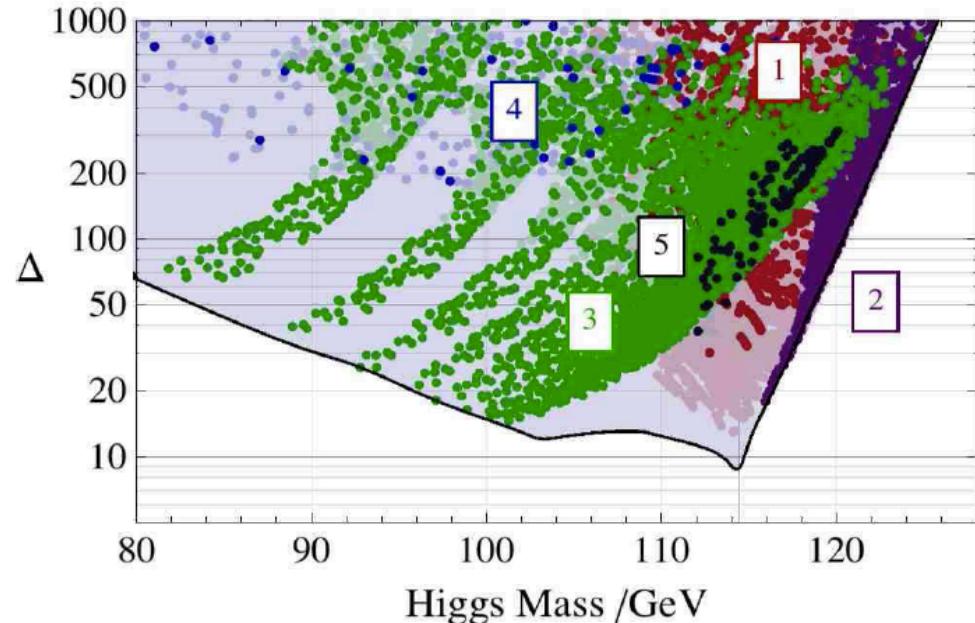
CMSSM: pre Higgs

$$\gamma_i = \mu_0, m_0, m_{1/2}, A_0, B_0$$

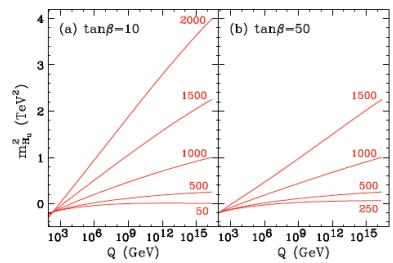
Gauge unification required

Relic density restricted

- 1 h^0 resonant annihilation
- 2 \tilde{h} t-channel exchange
- 3 $\tilde{\tau}$ co-annihilation
- 4 \tilde{t} co-annihilation
- 5 A^0 / H^0 resonant annihilation



Focus point



$$m_{H_u}^2(Q^2) = m_{H_u}^2(M_P^2) + \frac{1}{2} \left(m_{H_u}^2(M_P^2) + m_{Q_3}^2(M_P^2) + m_{u_3}^2(M_P^2) \right) \left[\left(\frac{Q^2}{M_P^2} \right)^{\frac{3y_t^2}{4\pi^2}} - 1 \right]$$

$\simeq -\frac{2}{3}, Q^2 \simeq M_Z^2$

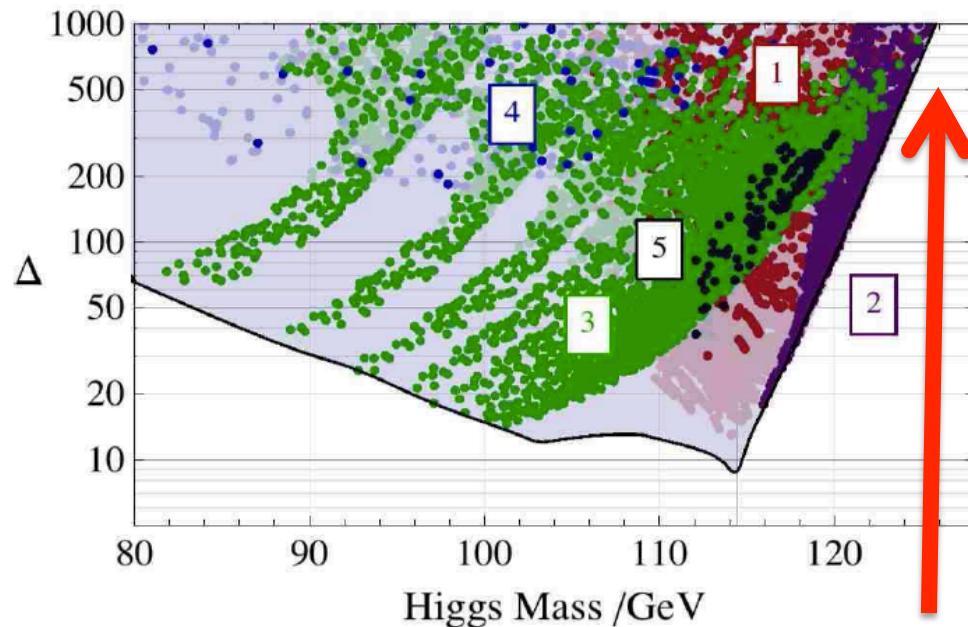
CMSSM: post Higgs

$$\gamma_i = \mu_0, m_0, m_{1/2}, A_0, B_0$$

Gauge unification required

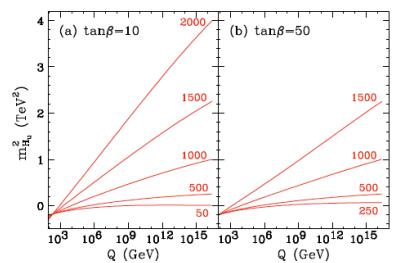
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- 1 h^0 resonant annihilation
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- 3 $\tilde{\tau}$ co-annihilation
- 4 \tilde{t} co-annihilation
- 5 A^0 / H^0 resonant annihilation



$$\Delta_{Min} > 350, \quad m_h = 125.6 \pm 3 \text{ GeV}$$

Focus point
 $m_{H_u} = m_{\tilde{Q}_3} = m_{\tilde{u}_3}$



$$m_{H_u}^2(Q^2) = m_{H_u}^2(M_P^2) + \frac{1}{2} \left(m_{H_u}^2(M_P^2) + m_{\tilde{Q}_3}^2(M_P^2) + m_{\tilde{u}_3}^2(M_P^2) \right) \left[\left(\frac{Q^2}{M_P^2} \right)^{\frac{3y_t^2}{4\pi^2}} - 1 \right]$$

$\simeq -\frac{2}{3}, Q^2 \simeq M_Z^2$

Beyond the CMSSM

- New states and interactions
(additional contributions to Higgs mass)
- Further
- Correlations between SUSY breaking parameters
Λ

- New (heavy) states- Singlet extensions

$$W = W_{\text{Yukawa}} + \lambda S H_u H_d + \frac{\kappa}{3} S^3 \quad \text{NMSSM}$$

$$\delta V = \lambda^2 |H_u H_d|^2$$

$$W = W_{\text{Yukawa}} + (\mu + \lambda S) H_u H_d + \frac{\mu_S}{2} S^2 + \frac{\kappa}{3} S^3 + \xi S \quad \text{GNMSSM}$$

$$\delta V = \frac{\mu}{\mu_s} \left(|H_u|^2 + |H_d|^2 \right) H_u H_d \quad \mu, \mu_s = O(m_{3/2}), \quad Z_{4,8R}$$

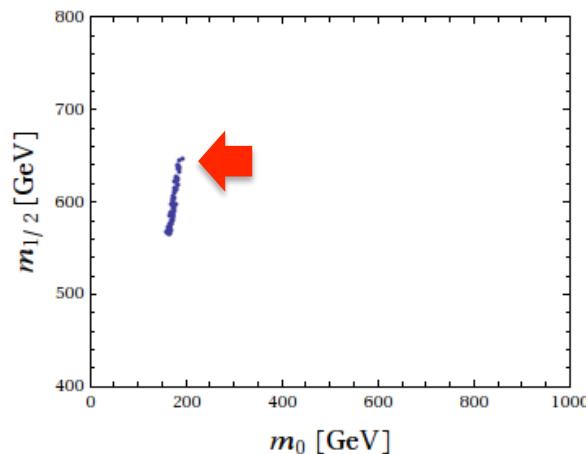
Fine tuning in the CGNMSSM ($\lambda \leq 0.7$)

$$\Delta_{Min} = 60 (500), \quad m_h = 125.6 \pm 3 \text{ GeV}$$

LHC8 SUSY bounds X

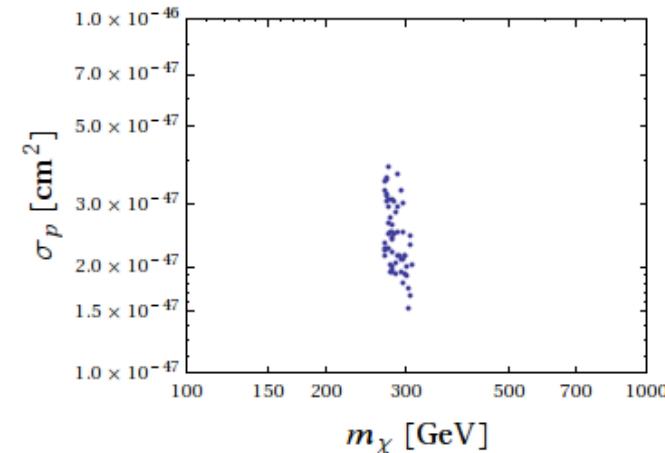
DM relic abundance ✓

DM searches ✓



LSP~Bino

Stau co-annihilation



DM searches insensitive

GGR, Schmidt-Hoberg , Staub

● Correlation between SUSY breaking parameters

...non-universal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3 \left(2 |y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) + 2 |a_t|^2 \right) - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$



New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \approx |M_3|^2$ at M_{SUSY}

Horton, GGR

(Also improves precision of gauge coupling unification)

Shifman, Roszkowski
Krippendorf, Nilles, Ratz, Winkler

Natural ratios? e.g.:

GUT: $SU(5): \Phi^N \subset (24 \times 24)_{symm} = 1 + 24 + 75 + 200; SO(10): (45 \times 45)_{symm} = 1 + 54 + 210 + 770$

$$\eta_3 : 1 : \eta_1$$

$$2.7\eta_3 : 1 : 0.5\eta_1$$

Representation	$M_3 : M_2 : M_1$ at M_{GUT}	$M_3 : M_2 : M_1$ at M_{EWSB}
1	1:1:1	6:2:1
24	2:(-3):(-1)	12:(-6):(-1)
75	1:3:(-5)	6:6:(-5)
200	1:2:10	6:4:10

Fine tuning in the (C)MSSM

Non-universal gaugino masses ✓

$$\Delta_{Min} = 60 \text{ (500)}, \quad m_h = 125.6 \pm 3 \text{ GeV}$$

LHC8 SUSY bounds ✓

DM relic abundance ✓

DM searches ✓

Fine tuning in the (C)GNMSSM

$$\Delta_{Min} = 20, \quad m_h = 125.6 \pm 3 \text{ GeV}$$

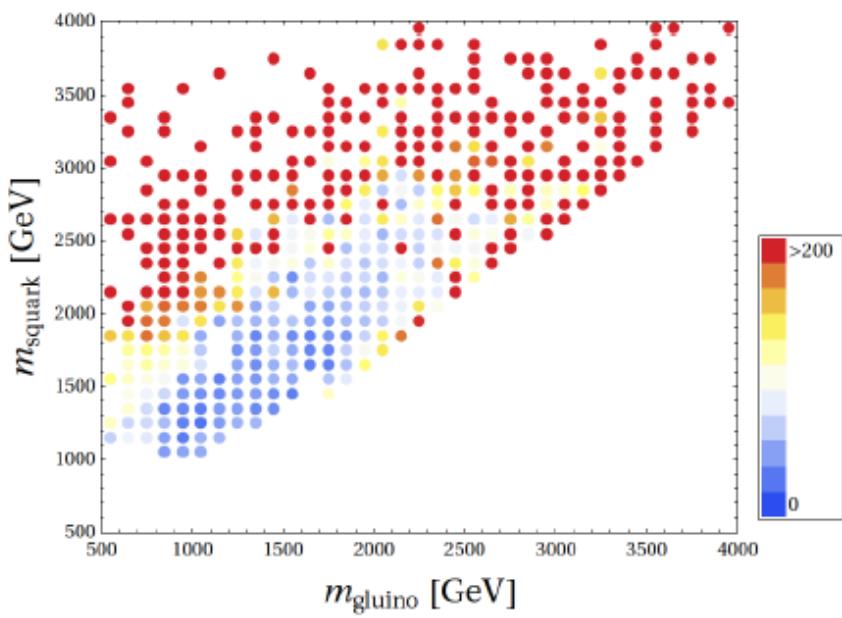
LHC8 SUSY bounds ✓

DM relic abundance ✓

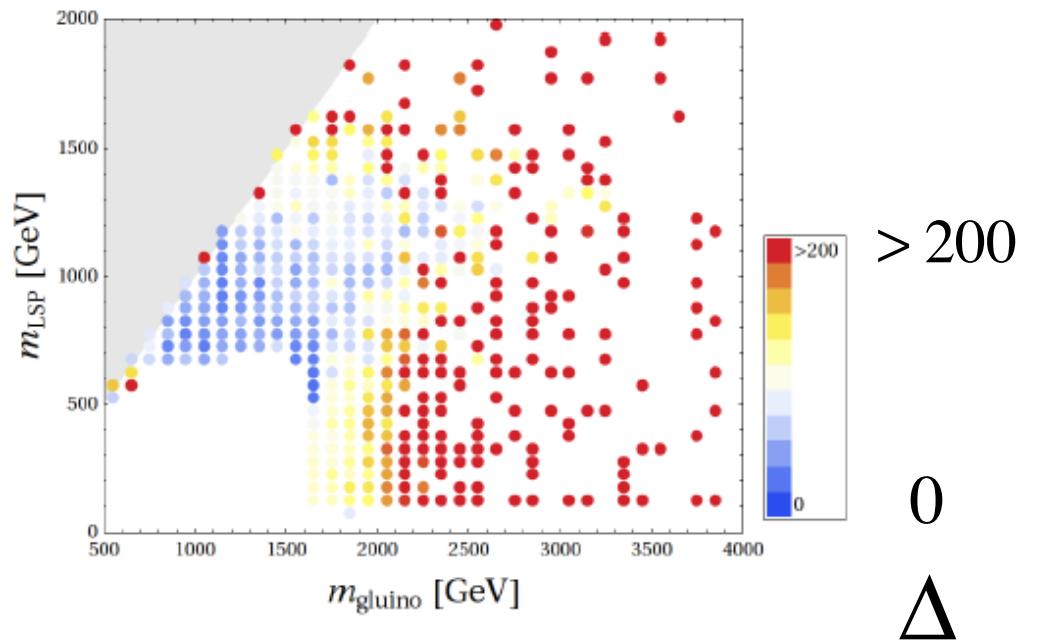
DM searches ✓

Masses v/s fine tuning

m_{squark}



m_{LSP}



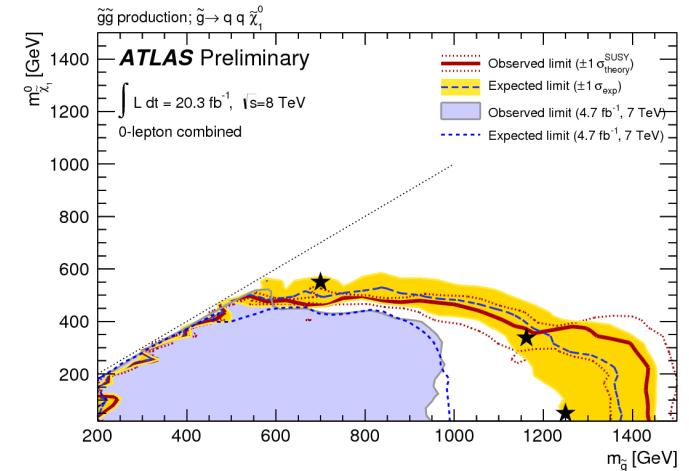
> 200

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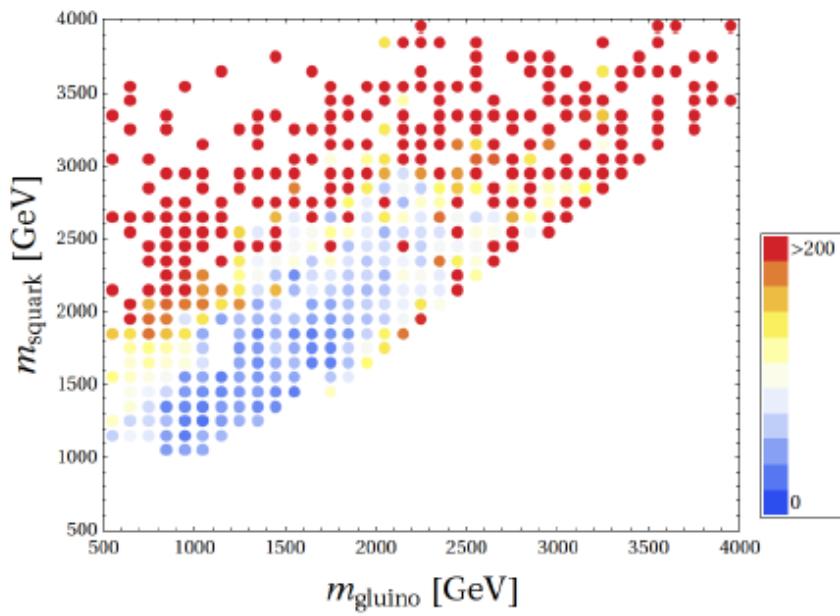
Δ

M_{gluino}

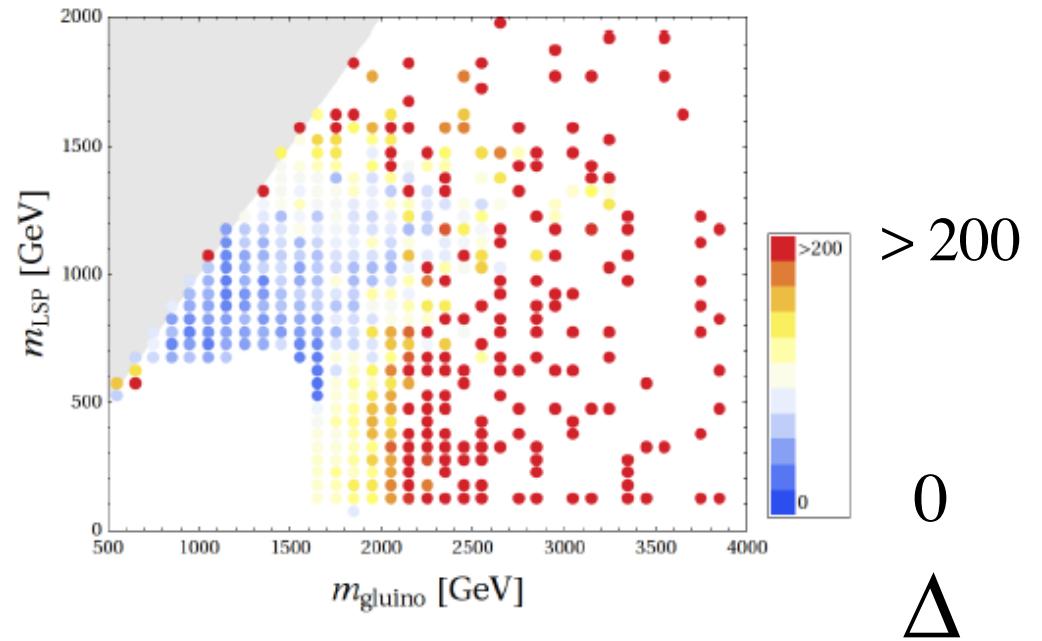
Masses v/s fine tuning



m_{squark}

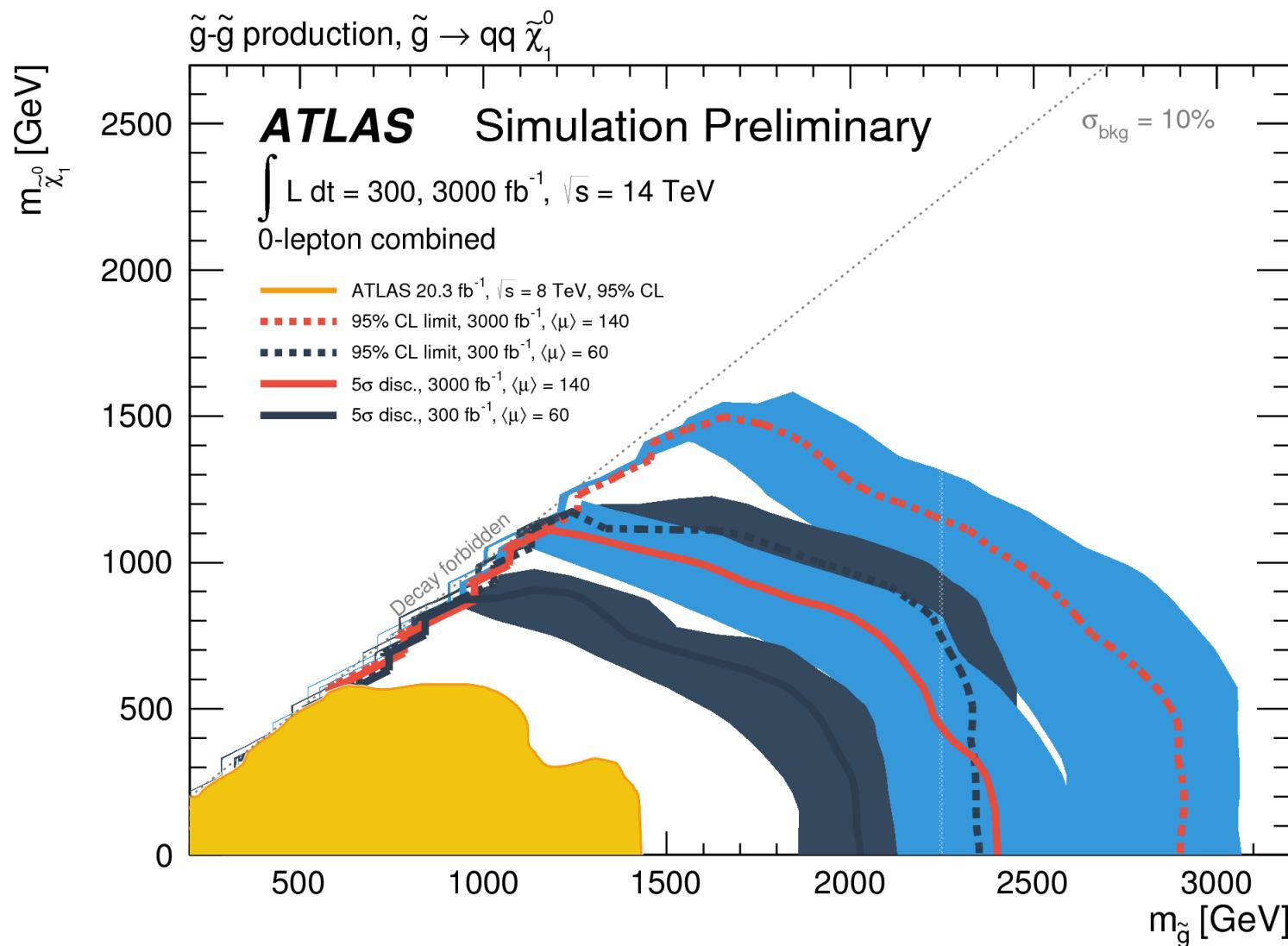


m_{LSP}

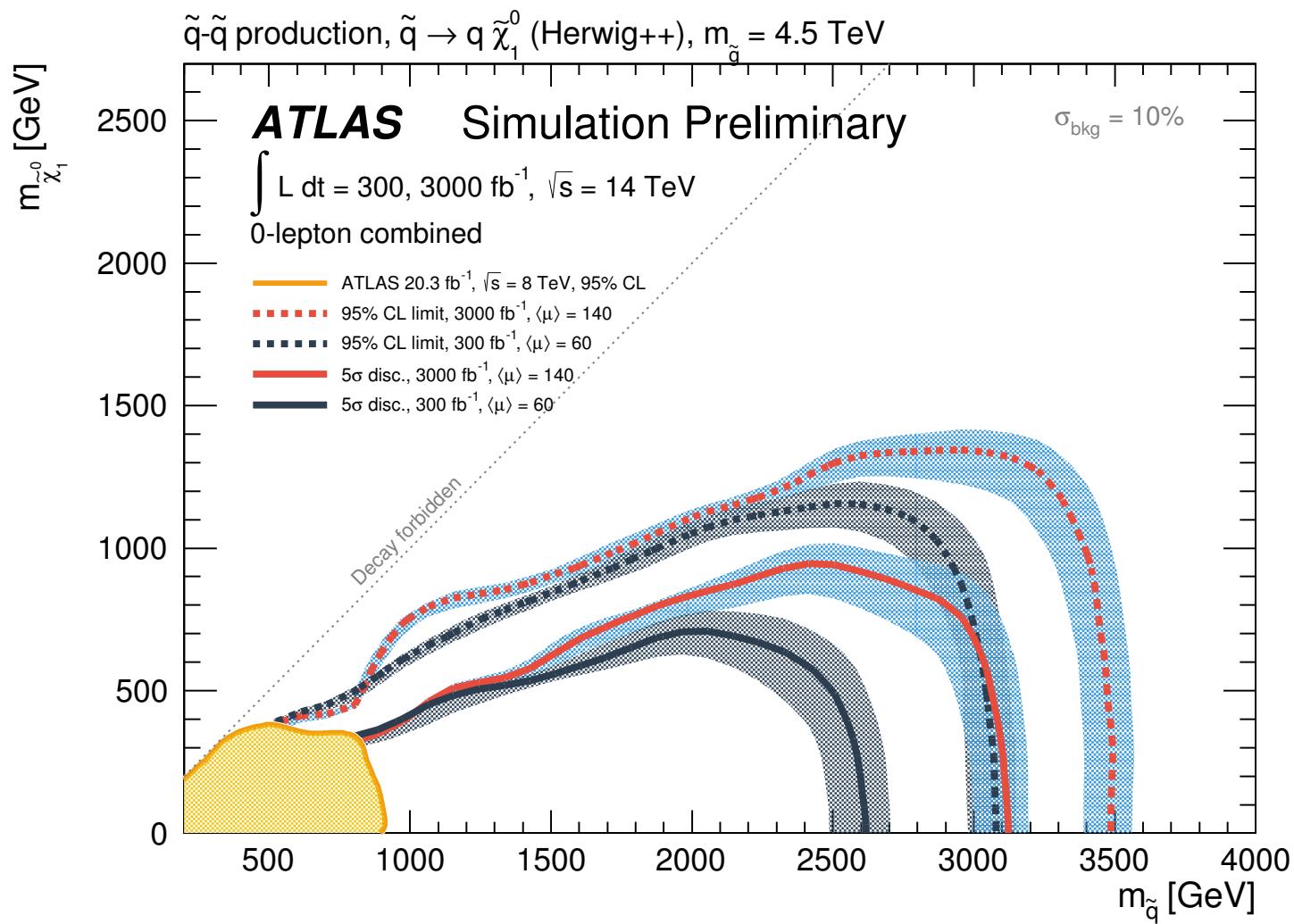


M_{gluino}

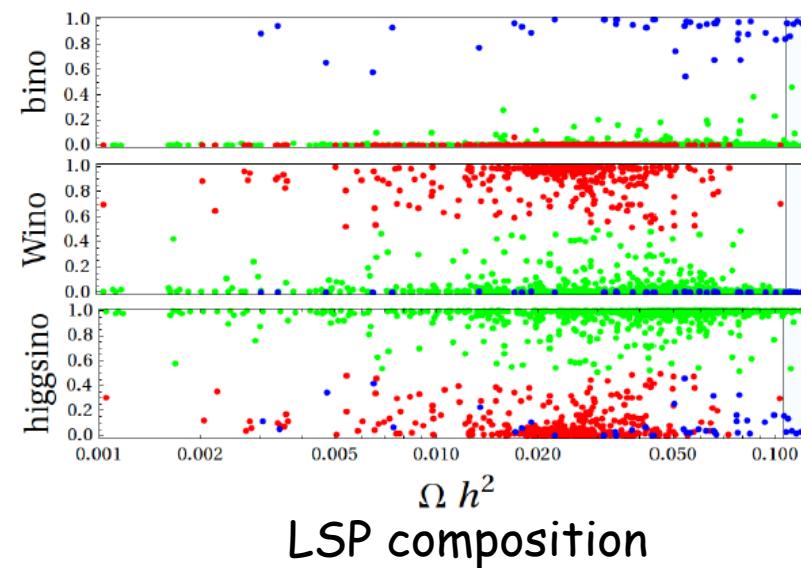
Heavy LSP reach



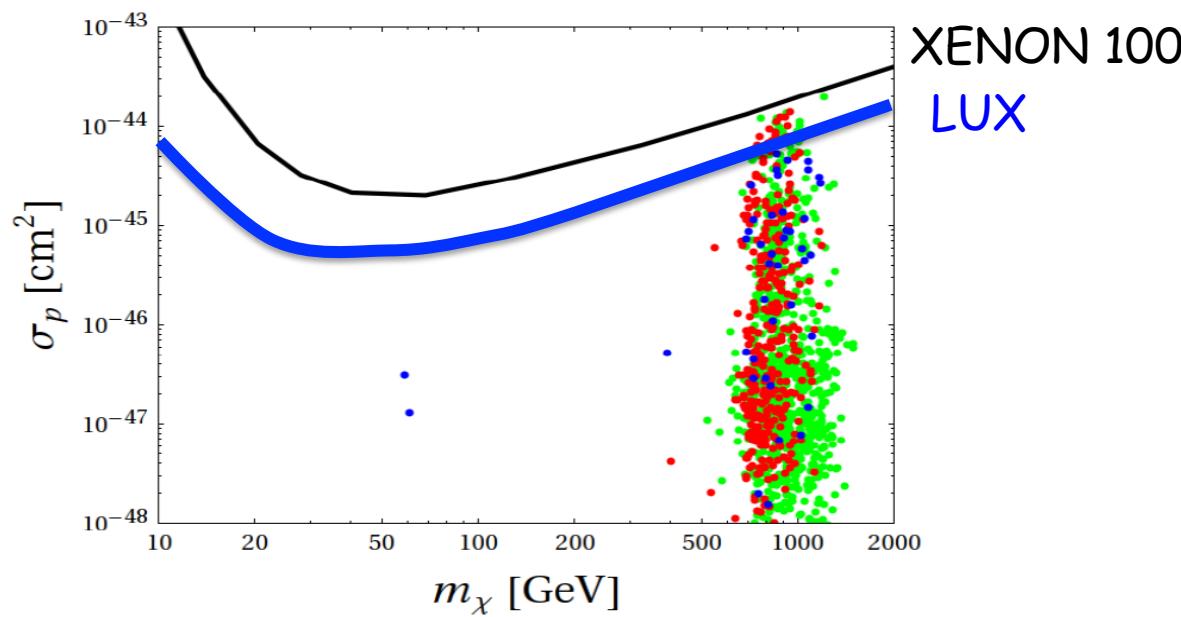
Heavy LSP reach



Dark matter



LSP composition



Direct DM searches

Summary - V

- GUTs $\xrightarrow{\text{SUSY-GUTS}}$ (hierarchy problem)
Gauge coupling unification ✓

- Fine tuning sensitive to SUSY spectrum

...scalar and gaugino focus points

$$\Delta^{CMSSM} > 350 \quad \times$$

$$\Delta^{(C)MSSM} > 60 \quad \checkmark$$

$$\Delta^{CGMSSM} > 60 \quad \times$$

$$\Delta^{(C)GNMMS} > 20 \quad \checkmark$$

c.f. $\Delta_{\text{Low scale}}^{CMSSM} = (10 - 30)$, $m_{\tilde{t}} = (1 - 5)TeV$

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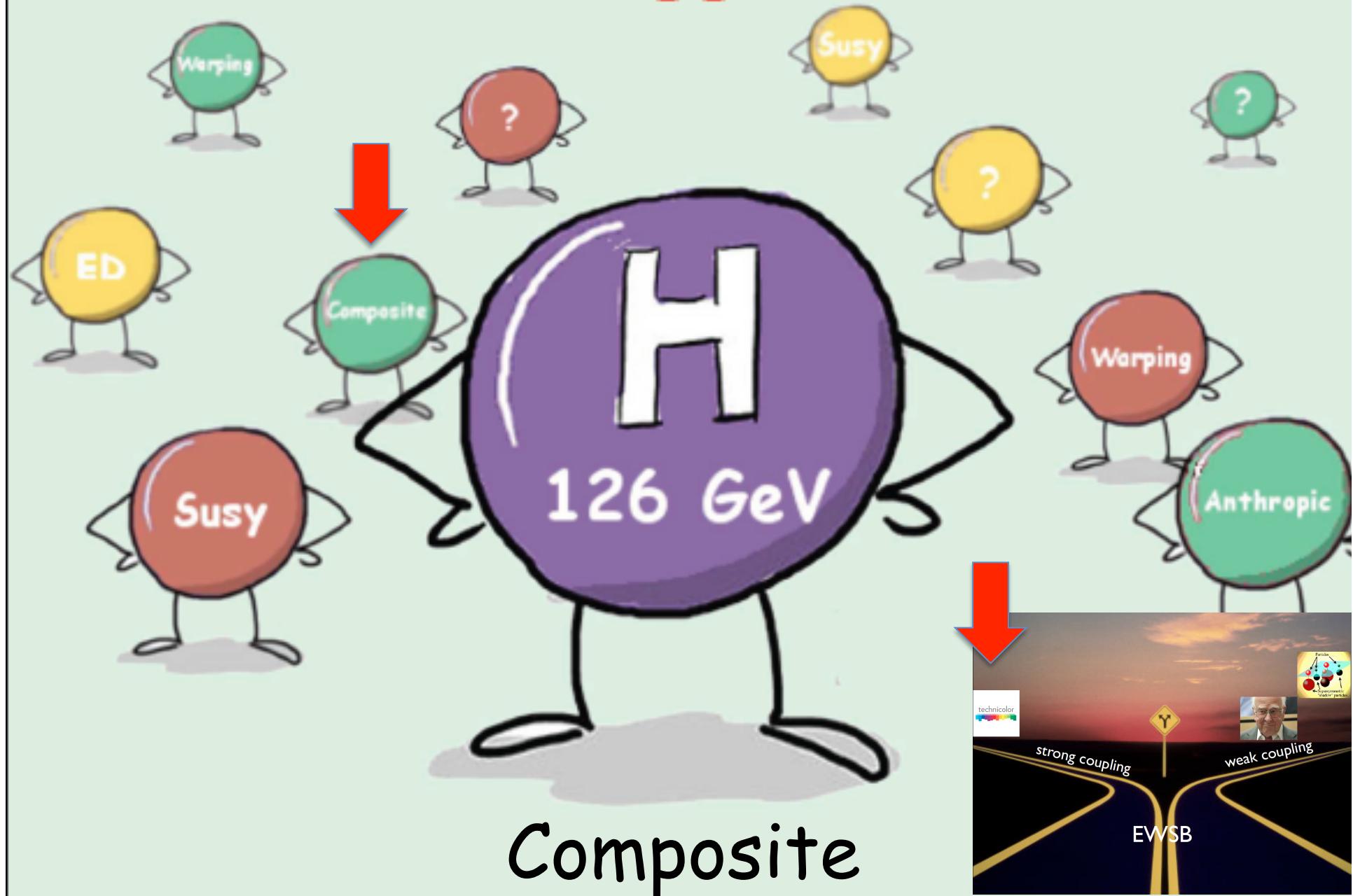
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- Whither SUSY?

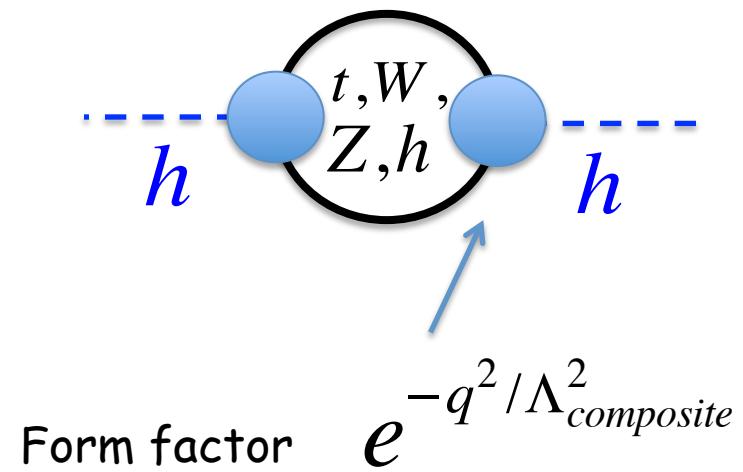
....well motivated SUSY models remain to be tested

Compressed spectra, TeV squarks and gluinos LHC14?
Natural SUSY

The Higgs Era



VI. Composite models



Technicolour - modeled on QCD

$$QCD: \quad SU(2)_L \times SU(2)_R \times U(1)_B \xrightarrow[QCD]{\langle \bar{q}q \rangle} SU(2)_V \times U(1)_B$$

Goldstone modes, $\pi^{a=1,2,3}$ $\Sigma(x) = \exp(i\sigma^a \pi^a(x)/f_\pi)$ $f_\pi = 92 \text{ MeV}$

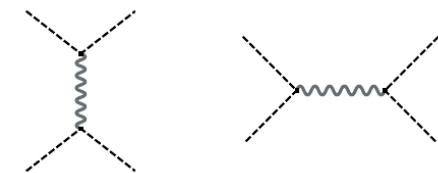
QCD breaks the $SU(2) \times U(1)$ symmetry giving W, Z mass

with EM ($Q = T_{3L} + T_{3R} + B/2$) unbroken

$$m_w = \frac{gf_\pi}{2} \simeq 29 \text{ MeV}$$

$SU(2)_V$ acts as custodial symmetry

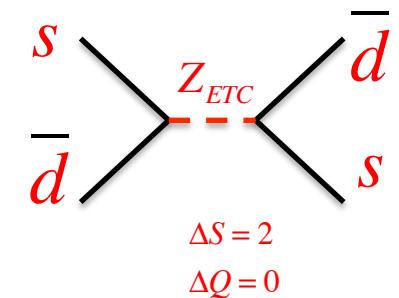
NO LIGHT SCALAR RESONANCE (unitarity enforced by tower of heavier resonances, $\rho\dots$)



For a general review see: Contino 1005.4269

Technicolour models

- Avoids (large) hierarchy problem
- Light Higgs?
- Precision tests?
- Fermion masses?
(FCNC \times)



\Rightarrow Make Higgs a Pseudo Goldstone boson

Higgs (Standard) Model

$$H(x) = \frac{1}{\sqrt{2}} e^{i\sigma^a \chi^a(x)/v} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

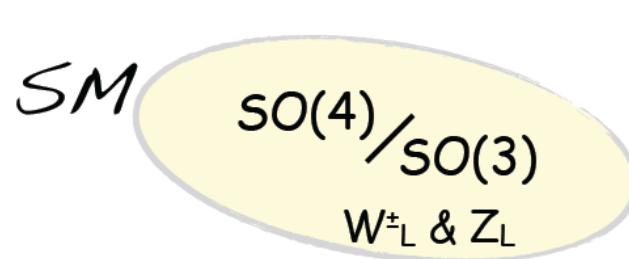
$$\frac{1}{2} (D_\mu H)^\dagger (D^\mu H) = \frac{v^2}{4} (D_\mu \Sigma)^\dagger (D^\mu \Sigma) \left(1 + \frac{h}{v}\right)^2 = \frac{v^2}{4} (D_\mu \Sigma)^\dagger (D^\mu \Sigma) \left(1 + 2\frac{h}{v} + \frac{h^2}{v^2}\right) \dots$$

Custodial symmetry

χ^a, h linear rep of $SU(2)_L \times SU(2)_R$

$$H^\dagger H = \sum_i h_i^2 \quad SO(4) \sim SU(2)_L \times SU(2)_R \xrightarrow[v]{} SO(3) \sim SU(2)_C$$

Goldstone modes:



The Higgs as a Pseudo Goldstone boson



e.g.

$$SO(5)/SO(3):W,Z,h$$

$$SO(6)/SO(3):W,Z,h,a$$

Fermion masses

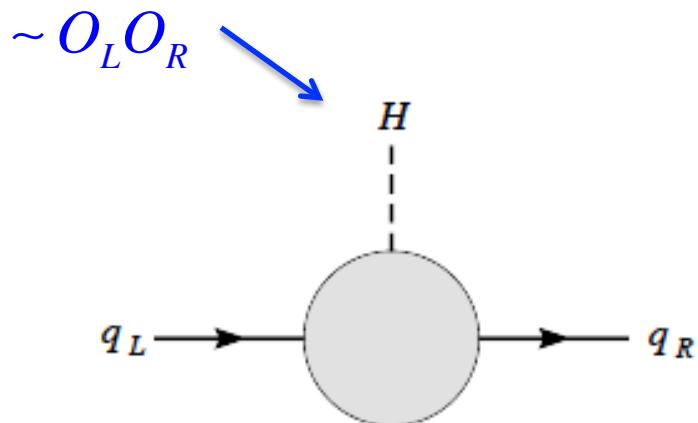
- ETC **X**
- $\mathcal{L}_{\text{mix}} = f q_L^\alpha (\hat{\lambda}_L)_I^\alpha \mathcal{O}_L^{qI} + f t_R^\alpha (\hat{\lambda}_R)_I^\alpha \mathcal{O}_R^{tI} + \text{h.c.}$

composite techni-fermion resonances, m_ψ

Partial compositeness:

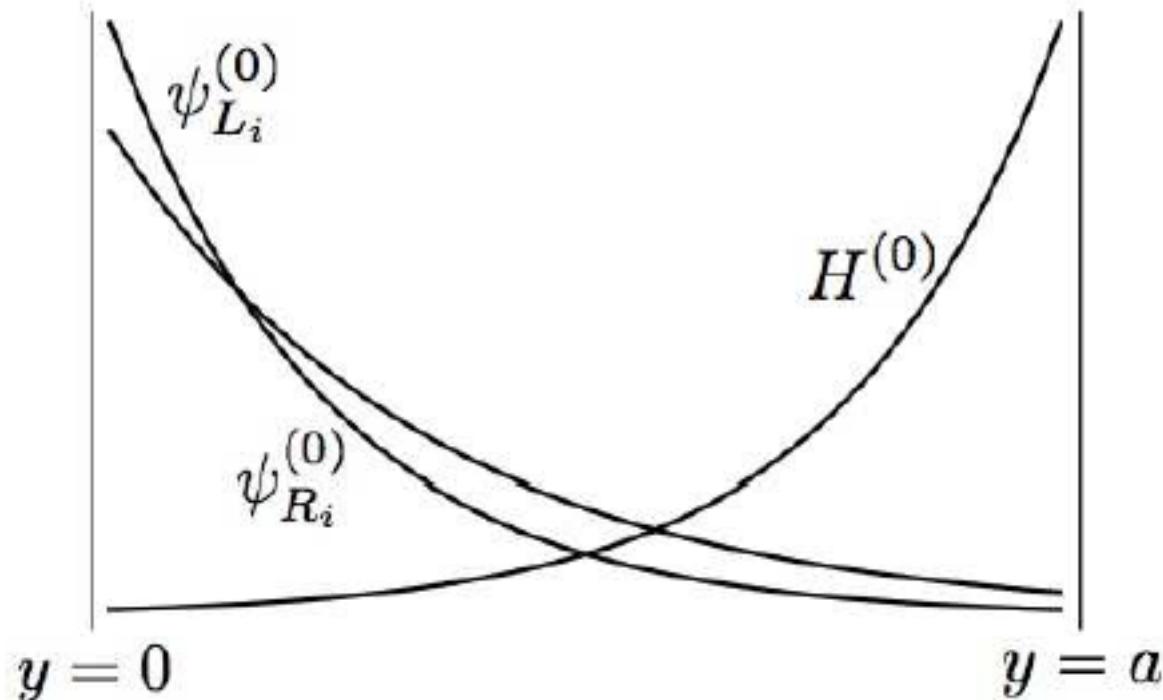
$$t_{L,R}^{\text{mass eigenstate}} \approx t_{L,R} + \epsilon_{L,R} O_{L,R}$$

$$\epsilon_{L,R} = \frac{\lambda_{L,R} f}{m_\psi} \equiv \frac{\lambda_{L,R}}{g_\psi}$$

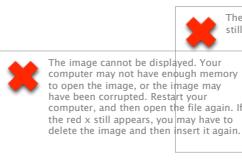


$$h_{q_i} \propto \epsilon_L^{q_i} \epsilon_R^{q_i}$$

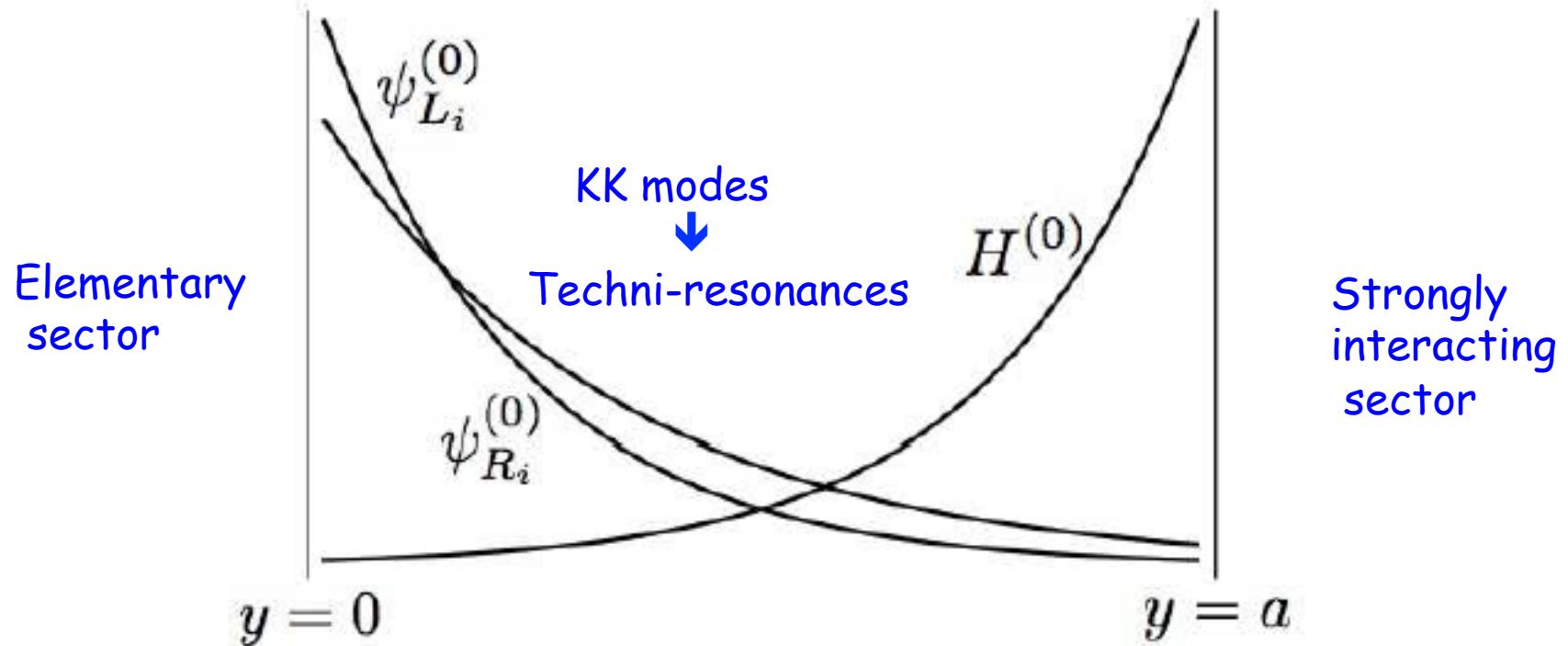
5D Analogue of partial compositeness



$$(\square_{5D} + M^2)\phi^{(0)} e^{ip \cdot x} = e^{ip \cdot x} (-\partial_y^2 + M^2)\phi^{(0)} = 0$$



5D Analogue of partial compositeness

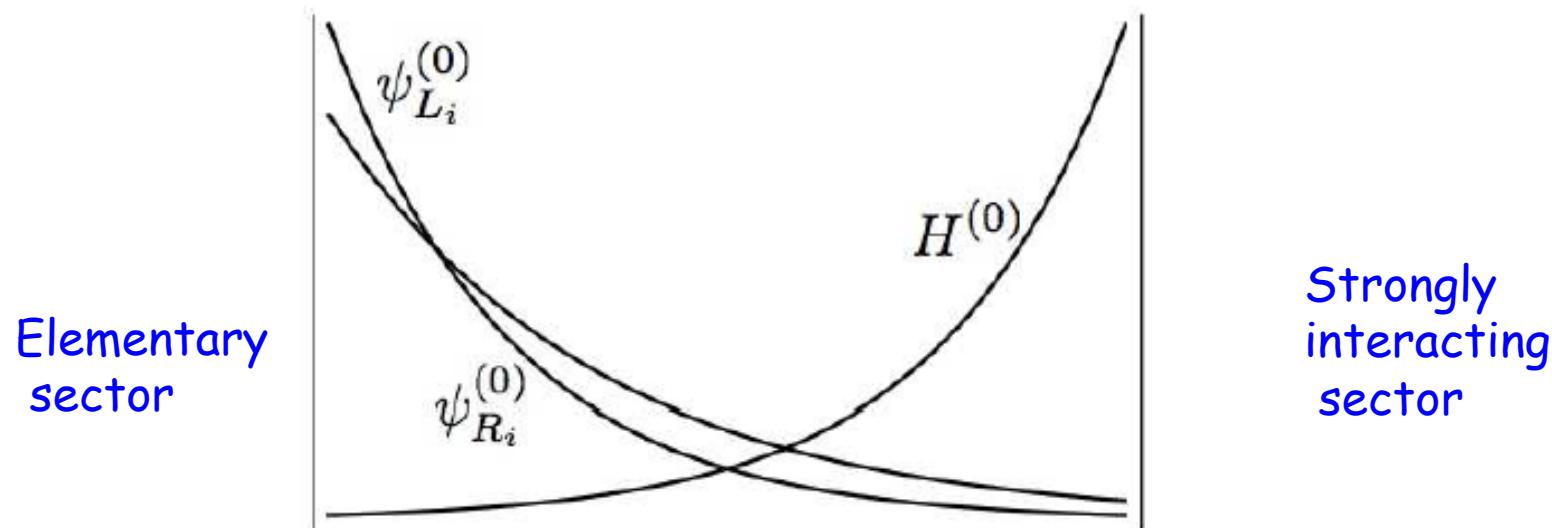


Heavier SM states - more composite

Light quarks and leptons almost elementary - evade composite tests

H, W_L, Z_L composite; W_T, Z_T partially composite

Exponential hierarchies of fermion masses



$$Y_{4D,ij} \sim \int_0^a dy Y_{5D,ij}(y) e^{-(M_{L_i} + M_{R_j})y + M_H(y-a)}$$

$$(M_{L_i} + M_{R_j} > M_H) \quad \swarrow \quad (M_{L_i} + M_{R_j} < M_H)$$

$$\sim \tilde{Y}_{0,ij} e^{-M_H a}$$

\ll

$$\sim \tilde{Y}_{a,ij} e^{-(M_{L_i} + M_{R_j})a}$$

No flavour hierarchies in m, θ
- Dirac neutrinos?

Charged fermion hierarchies

The hierarchy problem and light top quark partners

$$\delta m_h^2 = \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 \Lambda^2 \Rightarrow \Delta \geq \frac{\delta m_h^2}{m_h^2} = \left(\frac{\Lambda}{400 \text{ GeV}} \right)^2 \left(\frac{125 \text{ GeV}}{m_h} \right)^2$$

New states needed 1 TeV



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light vectorlike top quark < 1TeV


but $m_\rho > 2.5 \text{ TeV}$ (S-parameter)

Hence low cut-off for low Δ must be due to top-quark form factor

\Rightarrow Technifermion top quark resonance < 1TeV

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⇒ Technifermion top quark resonance < 1TeV

Fine tuning Sensitive to top resonance representations...minimum

$$\Delta \sim \xi^{-1} \geq 10 \quad \text{for } SO(4) \text{ 9-plet top quark} \quad \left(\xi = \frac{v^2}{f^2} \right)$$

Top techni-resonance phenomenology

QCD pair production : $\sigma_{m_t=500\text{GeV}} = 570\text{fb}$, $\sigma_{m_t=1\text{TeV}} = 1.3\text{fb}$ (8TeV CM)

$$9 \sim 3_{5/3} \oplus 3_{2/3} \oplus 3_{-1/3} \quad (SU(2)_L \times U(1)_Y) \supset 2 \times Q_{5/3} + Q_{8/3} \downarrow \\ 3W^+ + b + \dots$$

$$BR(Q_{5/3(8/3)} \rightarrow l^+l^+..) = 5(6)\%, \quad BR(Q_{5/3(8/3)} \rightarrow lll..) = 3(6.5)\%$$

LHC_{8:} $m_t > 770\text{GeV}$ (95%)

Panico et al 1201.7114
Pappadopulo et al 1303.3062

Summary BSM after Higgs

- Need symmetry to keep Higgs light
 - Scale, SUSY, Nambu-Goldstone
- Require new states
 - More Higgs and/or Higgs interactions,
 - SUSY partners,
 - Top quark partners
- Fine tuning limits \Rightarrow LHC 13/14 discovery!
- Grand Unification still viable - but must find SUSY

