Corrections to black hole entropy and the GUP

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- C.A. Mead was the first to point out the role of gravity on the existence of a fundamental measurable length, Phys. Rev. **135** B849 (1964).
- Since 1989, Amati, Ciafaloni, and Veneziano Phys. Lett. **B216** 41 (1989) a lot of effort and times been devoted to study the modification of Heisenberg Uncertainty Principle, known as Generalized Uncertainty Principle (GUP).
- GUP has been argued in String Theory, LQG, Non-Commutative Theories, Black Hole Physics.
- Since there is a plethora of different forms of GUP, the phenomenological implications of GUP are numerous.

- There has been a very recent interest in a different version of the GUP introduced by Das, ECV, and A.F. Ali Phys. Lett. **B678** 497 (2009), Phys. Lett. **B690** 407 (2010), Phys. Rev. **D84** 044013 (2011) which predicts not only a minimum length but also a maximum momentum introduced by Magueijo and Smolin Phys. Rev. Lett. **88** 190403 (2002), Phys. Rev. **D67** 044017 (2003), Phys. Rev. **D71** 026010 (2005).

- It was shown by Das and ECV Phys. Rev. Lett. **101** 221301 (2008) that the effects of the specific GUP can be implemented both in classical and quantum systems by defining deformed commutation relations by means of

$$
\begin{aligned}
x_i &= x_{0i} \\
p_i &= p_{0i}\left(1 - \alpha p_0 + 2\alpha^2 p_0^2\right)
\end{aligned}
$$

where $[x_{0i}, p_{0j}] = i\hbar\delta_{ij}$ and $p_0^2 = \sum_{j=1}^{3} p_{0j}p_{0j}$ and $\alpha = \alpha_0/m_p c$, α_0 being a dimensionless constant.

- Among all quantum gravitational effects one can think of, black hole (BH) entropy can be considered as the paradigmatic one.

- From the realization that BHs are thermodynamic objects which radiate, the entropy of a Schwarzschild BH is given by the Bekenstein–Hawking relation

$$S_{BH} = \frac{A_{BH}}{4l_p^2}$$

where A_{BH}: the area of the BH horizon and $l_p = \sqrt{\frac{G\hbar}{c^3}}$: the Planck length.

- After these findings, several approaches to quantum gravity (QG) have predicted the following form for the QG-corrected BH entropy

$$\frac{S_{QG}}{k} = \frac{A_{BH}}{4l_p^2} + c_0 \ln\left(\frac{A_{BH}}{4l_p^2}\right) + \sum_{n=1}^{\infty} c_n \left(\frac{A_{BH}}{4l_p^2}\right)^{-n}$$

where the c_n coefficients are model–dependent parameters.

- Specifically, LQG calculations are used to fix $c_0 = -1/2$.

- The deformed commutation relations previously presented have been widely used to compute the effects of the GUP on the BH entropy from different perspectives, (for instance Medved and ECV, Phys. Rev. D **70**, 124021 (2004); G. Amelino–Camelia, Arzano, Ling, and Mandanici, Class. Quant. Grav. **23**, 2585 (2006) , which reads

$$\frac{S_{GUP}}{k} = \frac{A_{BH}}{4l_p^2} + \frac{\sqrt{\pi}\alpha_0}{4}\sqrt{\frac{A_{BH}}{4l_p^2}} - \frac{\pi\alpha_0^2}{64}\ln\left(\frac{A_{BH}}{4l_p^2}\right) + \mathcal{O}(l_p^3).$$

- Therefore, both the logarithmic correction and a new term which goes as $\sqrt{A_{BH}}$ can be derived employing GUP.

- Following Mäkelä and Repo, Phys. Rev. **D57**, 4899 (1998) , a Schwarzschild BH can be described as a canonical system by a Hamiltonian with $H = m$ and a new canonical pair (a, p_a)

$$H = \frac{p_a^2}{2a} + \frac{a}{2}$$

- Following Obregón, Sabido, and Tkach, Gen. Relativ. Gravit. **33**, 913 (2001) within the canonical quantization framework, the corresponding Wheeler–DeWitt eqn in the form of a Schrödinger eqn for a quantum harmonic oscillator

$$\left(-\frac{1}{2} l_p^2 E_p \frac{d^2}{dx^2} + \frac{E_p}{2 l_p^2} x^2\right) U(x) = \frac{R_s}{4 l_p} E_s U(x)$$

$E_p = \sqrt{\frac{\hbar c^5}{G}}$: Planck energy, $E_s = Mc^2$: BH ADM energy, $R_s = \frac{2GM}{c^2}$: Schwarzschild radius

- The function $U(x)$ is related to the BH wavefunction $\Psi(x)$

$$U(x) = a^{-1}\Psi(x)$$

where $x = (a - R_s)$.

- As shown by Obregón, Sabido, and Tkach, Gen. Relativ. Gravit. **33**, 913 (2001) quantum effects on the thermodynamics of the Schwarzschild BH can be introduced by means of the path integral method applied to the harmonic oscillator (see Feynman and Hibbs, *Quantum Mechanics and Path Integrals*).

- Starting from the partition function

$$Z = h^{-1} \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dx e^{-\beta H_Q}$$

where the quantum-corrected Hamiltonian is

$$H_Q = \frac{p^2}{2m_p} + \frac{m_p \omega^2 x^2}{2} + \frac{\beta_Q E_p^2}{16\pi}$$

and equating the average (thermodynamical) energy to the internal grav. energy of the BH, $\bar{E} = Mc^2$, the quantum-corrected inverse BH temperature is

$$\beta_Q = \beta_H \left[1 - \frac{1}{\beta_H Mc^2} + \mathcal{O}\left(\frac{E_p}{Mc^2}\right)^4 \right]$$

with $\beta_H = \frac{8\pi Mc^2}{E_p^2}$: the inverse of Hawking's temperature.

Therefore, the quantum-corrected entropy of the BH can be written as

$$\frac{S_Q}{k} = \frac{A_Q}{4l_p^2} - \frac{1}{2}\ln\left(\frac{A_Q}{4l_p^2}\right) - \frac{1}{2}\ln(24) + 1 + \mathcal{O}\left(\frac{E_p}{Mc^2}\right)^6$$

where

$$A_Q = A_{BH}\left[1 - \frac{1}{8\pi}\left(\frac{E_p}{Mc^2}\right)^2\right]^2$$

is a modified BH area which includes quantum corrections with the BH horizon area, $A_{BH} = 4\pi R_s^2$.

- By means of the GUP-induced deformed canonical commutator, a general non–relativistic Hamiltonian of the form

$$H = \frac{p_0^2}{2m} + V(x)$$

transforms into

$$H_{GUP} = \frac{p_0^2}{2m} + V(x) - \frac{\alpha}{m} p_0^3 + \frac{5\alpha^2}{2m} p_0^4 + \mathcal{O}(\alpha^3)$$

- Thus, the Hamiltonian of the Schwarzschild BH which includes GUP and quantum corrections now reads

$$H_{GUP+Q} = \frac{p^2}{2m_p} + \frac{3m_p E_p^2 x^2}{4\pi\hbar^2} + \frac{5\alpha^2}{2m_p} p^4 + \frac{\beta E_p^2}{16\pi}$$

where p stands for p_0 to simplify the notation, and only the quadratic GUP modification is considered.

- It is easy now to write down the partition function

$$
\begin{aligned}
Z_{GUP+Q} &= \frac{1}{2\alpha E_p\sqrt{15\beta m_p}} e^{\frac{\beta_Q(\pi - 5\alpha^2\beta E_p^2 m_p)}{80\alpha^2 m_p \pi}} K_{1/4}\left(\frac{\beta}{80\alpha^2 m_p}\right) \\
&= \sqrt{\frac{2\pi}{3}} \frac{1}{\beta E_p} e^{-\frac{\beta^2 E_p^2}{16\pi}} (1 - \frac{15 m_p}{2\beta}\alpha^2 + \mathcal{O}(\alpha^4))
\end{aligned}
$$

where $K_i(x)$ stands for the second modified Bessel function of order i.

- The entropy is given by the standard formula

$$S = k(\ln Z) - \beta \frac{\partial \ln(Z)}{\partial \beta}$$

- So the entropy of the Schwarzschild BH will be

$$\frac{S_{GUP+Q}}{k} = 1 + \frac{\beta^2 E_p^2}{16\pi} - \ln\left(\sqrt{\frac{3}{2\pi}}\beta E_p\right) - \frac{15 m_p}{\beta}\alpha^2 + \mathcal{O}(\alpha^4)$$

- After a long but straightforward calculation, the GUP-corrected quantum entropy can be written as

$$\frac{S_{GUP+Q}}{k} = \frac{A_{GUP+Q}}{4l_p^2} - \frac{1}{2}\ln\left(\frac{A_{GUP+Q}}{4l_p^2}\right) - \frac{1}{2}\ln(24) + 1 + \frac{15}{2\sqrt{\pi}}\alpha_0^2\left(\frac{l_p^2}{A_{BH}}\right)^{1/2}$$

where the GUP-corrected quantum BH area is of the form

$$A_{GUP+Q} = A_{BH}\left[1 - \frac{2l_p^2}{A_{BH}} + \frac{15\pi\alpha_0^2}{2}\left(\frac{l_p^2}{A_{BH}}\right)^{3/2}\right]^2$$

- When $\alpha_0 = 0$ the quantum BH entropy is recovered.
- The entropy contains a GUP-dependent term not only in the usual area and logarithmic terms.

1. In the framework of a canonical description of the BH, employing the path integral method, the corrected inverse temperature and the entropy of a Schwarzschild BH were computed when the quantum effects as well as the GUP effects are present.
2. The logarithmic correction in the expression for the corrected entropy of the Schwarzschild BH is also obtained.
3. When the GUP parameter is zero, i.e., $\alpha = 0$, expressions for the quantum corrected inverse temperature and entropy of the Schwarzschild BH that already exist in the literature are obtained.


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