Neutrino Theory II: Linkage and Symmetry

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Neutrino Connections

Any choice of a mechanism for neutrino mass is suggestive of new physics connections.

Type I seesaw: N is strongly indicative of $S0(10)$ because the spinorial representation $\underline{16}$ of $S0(10)$ contains exactly one family of quarks and leptons if N is included. However, N is also contained in the fermion $\underline{24}$ of $SU(5)$.

Type II, III seesaws: $(\xi^{++}, \xi^{+}, \xi^{0})$ is contained in the scalar $\underline{15}$ of $SU(5)$ and $(\Sigma^{+}, \Sigma^{0}, \Sigma^{-})$ is contained in the fermion $\underline{24}$ of $SU(5)$.

The mass scale of neutrino mass generation may also be tied to other phenomena, such as new gauge interactions, dark matter, and strong CP.

With three families of leptons and quarks, the theoretical framework for neutrino mass may also be connected to the flavor problem, allowing for flavor symmetries such as A_4.

Possible new neutrino interactions may also be suggestive of how matter and dark matter could be unified, under $SU(6)$ for example.

Strong CP

Neutrino-Axion Linkage in Supersymmetry (2001: Ma):

Add to the minimal supersymmetric standard model (MSSM) 6 neutral singlet superfields $\hat{N}^c_{1,2,3}$ and $\hat{S}_{0,1,2}$.

PQ charges: $\hat{S}_0 \sim -2$, $\hat{S}_1 \sim -1$, $\hat{S}_2 \sim 2$;
$\hat{H}_u = (\hat{h}_u^+, \hat{h}_u^0)$, $\hat{H}_d = (\hat{h}_d^0, \hat{h}_d^-) \sim -1$;
$\hat{Q} = (\hat{u}, \hat{d}), \hat{u}^c, \hat{d}^c, \hat{L} = (\hat{\nu}, \hat{e}), \hat{e}^c, \hat{N}^c \sim 1/2$.

The superpotential is then given by

$$\hat{W} = m_2 \hat{S}_2 \hat{S}_0 + f \hat{S}_2 \hat{S}_1 \hat{S}_1 + h_1 \hat{S}_1 \hat{N}^c \hat{N}^c + h_2 \hat{S}_2 \hat{H}_u \hat{H}_d$$
$$+ h_N \hat{H}_u \hat{L} \hat{N}^c + h_e \hat{H}_d \hat{L} \hat{e}^c + h_u \hat{H}_u \hat{Q} \hat{u}^c + h_d \hat{H}_d \hat{Q} \hat{d}^c.$$

Since $\hat{W}$ does not allow $\hat{L}$ and $\hat{H}_d$ to have the same PQ charge, R parity is automatically conserved.

$\hat{W}$ has only one mass scale m_2. This sets the scale for $U(1)_{PQ}$ breaking as follows:

$$V_{SUSY} = |m_2 S_0 + f S_1^2|^2 + m_2^2 |S_2|^2 + 4 f^2 |S_1|^2 |S_2|^2,$$

so that $v_2 = 0$, $m_2 v_0 + f v_1^2 = 0$ is a supersymmetry preserving minimum. The linear combination $(v_1 \hat{S}_1 + 2 v_0 \hat{S}_0)/\sqrt{|v_1|^2 + 4|v_0|^2}$ is a massless superfield, and $v_{0,1} \sim m_2$. At the same time, $m_N = 2 h_1 v_1$, i.e. the seesaw scale $\sim$ the axion decay constant.

At this stage, $U(1)_{PQ}$ has been broken at the scale m_2 without breaking the supersymmetry. Another mass scale M_{SUSY} must be added, i.e. soft supersymmetry breaking terms, so that

$$v_2 \sim M_{SUSY}, \quad m_2 v_0 + f v_1^2 \sim M_{SUSY}^2.$$

The μ term of the MSSM becomes $h_2 v_2 \sim M_{SUSY}$.
With electroweak symmetry breaking, the axion is then contained in the phases of $S_{0,1,2}$ as well as $h_{u,d}^0$ as in the DFSZ model:
$a = V^{-1}(v_1\theta_1 + 2v_0\theta_0 - 2v_2\theta_2 + v_u\theta_u + v_d\theta_d)$.

The 4×4 neutralino mass matrix of the MSSM now becomes 7×7 with the inclusion of the $\tilde{S}_{0,1,2}$ fermions. Note that $\tilde{S}_2$ and $(2v_0\tilde{S}_1 - v_1\tilde{S}_0)/\sqrt{4|v_0|^2 + |v_1|^2}$ combine to form a Dirac fermion with mass $m_2(1 + 4v_0^2/v_1^2)^{1/2}$. The remaining Majorana fermion is the axino with mass $-2fv_2(1 + 4v_0^2/v_1^2)^{-1}$ which couples very weakly ($\sim v_{u,d}/V$) to the MSSM neutralinos. Even if this axino is a significant component of dark matter, it will not be detected directly in underground experiments. The corresponding saxion also has mass $\sim M_{SUSY}$ and couples just as weakly.

Using the Residual Z_2 from $U(1)_{PQ}$ for Dark Matter and Neutrino Mass (2013: Dasgupta/Ma/Tsumura):

The QCD Lagrangian has a term

$$\mathcal{L} = \bar{\theta}\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu},$$

which violates CP. The parameter $\bar{\theta}$ is given by $\theta_{QCD} + a/F_a$, where θ_{QCD} comes from the SM, and a is the axion field coming from the anomalous global $U(1)_{PQ}$ with F_a its decay constant, i.e. vacuum expectation value divided by domain wall number.

Using the fact that a is a dynamical field with a dynamical potential, Peccei and Quinn showed that it will adjust itself so that $\bar{\theta}$ relaxes to zero, thereby getting rid of this term entirely. The residual excitation (pointed out by Weinberg and Wilczek), i.e. the axion particle, is physical. Its mass is roughly given by $m_a \sim f_\pi m_\pi / F_a \sim 0.6 \times 10^{-3}\ \mathrm{eV}/(F_a/10^{10}\ \mathrm{GeV})$.

Three generic models by what couples to $U(1)_{PQ}$:
(A) KSVZ(1979,80): only new heavy singlet quarks;
(B) DFSZ(1980,81): only the known quarks;
(C) Demir/Ma(2000): only gluinos in supersymmetry.

In each case, as $U(1)_{PQ}$ is spontaneously broken, an exactly conserved residual discrete Z_2 symmetry remains. In (B), it is $(-1)^{3B}$; in (C) it is R parity; and in (A) it is a new symmetry defined only on the new heavy singlet quarks. This latter is tailor-made for having absolutely stable dark matter and realizing the notion that neutrino mass is induced radiatively by dark matter.

Let the new heavy singlet quark of the KSVZ model be Q with charge -1/3. Under $U(1)_{PQ}$, $Q_L \sim 1/2$ and $Q_R \sim -1/2$, hence $f_Q \zeta^0 \bar{Q}_L Q_R + H.c.$ (where $\zeta^0 \sim 1$ under $U(1)_{PQ}$) generates m_Q as well as a.

To exploit the residual discrete Z_2 symmetry for dark matter, a neutral complex scalar $\eta^0 \sim 1/2$ is added. The Lagrangian involving $Q_{L,R}$, ζ^0, and η^0 is then given by

$$\mathcal{L} = \mu_\zeta^2|\zeta|^2 + \frac{1}{2}\lambda_\zeta|\zeta|^4 + \mu_\eta^2|\eta|^2 + \frac{1}{2}\lambda_\eta|\eta|^4 + \lambda'|\zeta|^2|\eta|^2$$
$$+ [f_Q\zeta\bar{Q}_L Q_R + f_d\eta\bar{Q}_L d_R + \epsilon_\eta\zeta^\dagger\eta^0\eta^0 + H.c.]$$

Let $\zeta = (1/\sqrt{2})(v+\sigma)e^{ia/v}$, where $v = \sqrt{-2\mu_\zeta^2/\lambda_\zeta}$, then a is the axion and $v = F_a$.

Further $m_Q = f_Q v/\sqrt{2}$ and for $\eta = (1/\sqrt{2})(\eta_1 + i\eta_2)$, $m_{1,2}^2 = \mu_\eta^2 + \lambda' v^2/2 \pm \epsilon_\eta v\sqrt{2}$.

Since $v > 4 \times 10^8$ GeV from SN1987A, fine tuning is unavoidable for $m_{1,2} \sim$ TeV, just as in the case of the SM Higgs boson in any nonsupersymmetric axion model. On the other hand, $\epsilon_\eta = 0$ corresponds to an extra U(1) symmetry, i.e. $\eta, Q_L, Q_R \sim 1$ independent of $U(1)_{PQ}$.

$\Omega_a h^2 \sim 0.3(F_a/10^{12}\ \mathrm{GeV})^{7/6} \sim 10^{-4}$ for $F_a \sim 10^9$ GeV. This would make axions only 1 percent of dark matter. In that case, $m_Q \sim$ TeV if $f_Q \sim 10^{-5}$. Let $m_1 < m_2$, then the interaction $\eta_1^2(\Phi^\dagger\Phi)$ allows it to have the correct relic abundance and be consistent with direct-search bounds.

Scotogenic Neutrino Mass:

The same symmetry, i.e. Z_2, which enables dark matter to exist may also be responsible for neutrino mass from a radiative mechanism. This may be called 'scotogenic' from the Greek 'scotos' meaning darkness.

(1) One loop: Ma(2006),

(2) Two loops: Ma/Sarkar(2007),

(3) Three loops: Krauss/Nasri/Trodden(2003).

In the present case, add three heavy neutral fermions as usual but with $N_R \sim 1/2$ under $U(1)_{PQ}$ plus one scalar leptoquark $\xi^{-1/3} \sim 0$. As a result, there are the additional couplings: $\zeta^\dagger N_R N_R$, $\zeta^\dagger \eta^0 \eta^0$, as well as $\xi^\dagger(\nu d - l u)$ and $\xi \bar{Q}_L N_R$. As $U(1)_{PQ}$ breaks to Z_2, N_R gets a Majorana mass, and radiative neutrino masses are obtained in three loops in analogy to KNT.

Now N may be dark matter instead of η_1. In fact, unsuppressed NN annihilation into $\xi\xi^\dagger$ is now possible. The predicted scalar leptoquark ξ will decay into ul or $d\nu$, enabling it to be discovered at the Large Hadron Collider (LHC). Currently, the LHC limit on m_ξ is about 850 GeV.

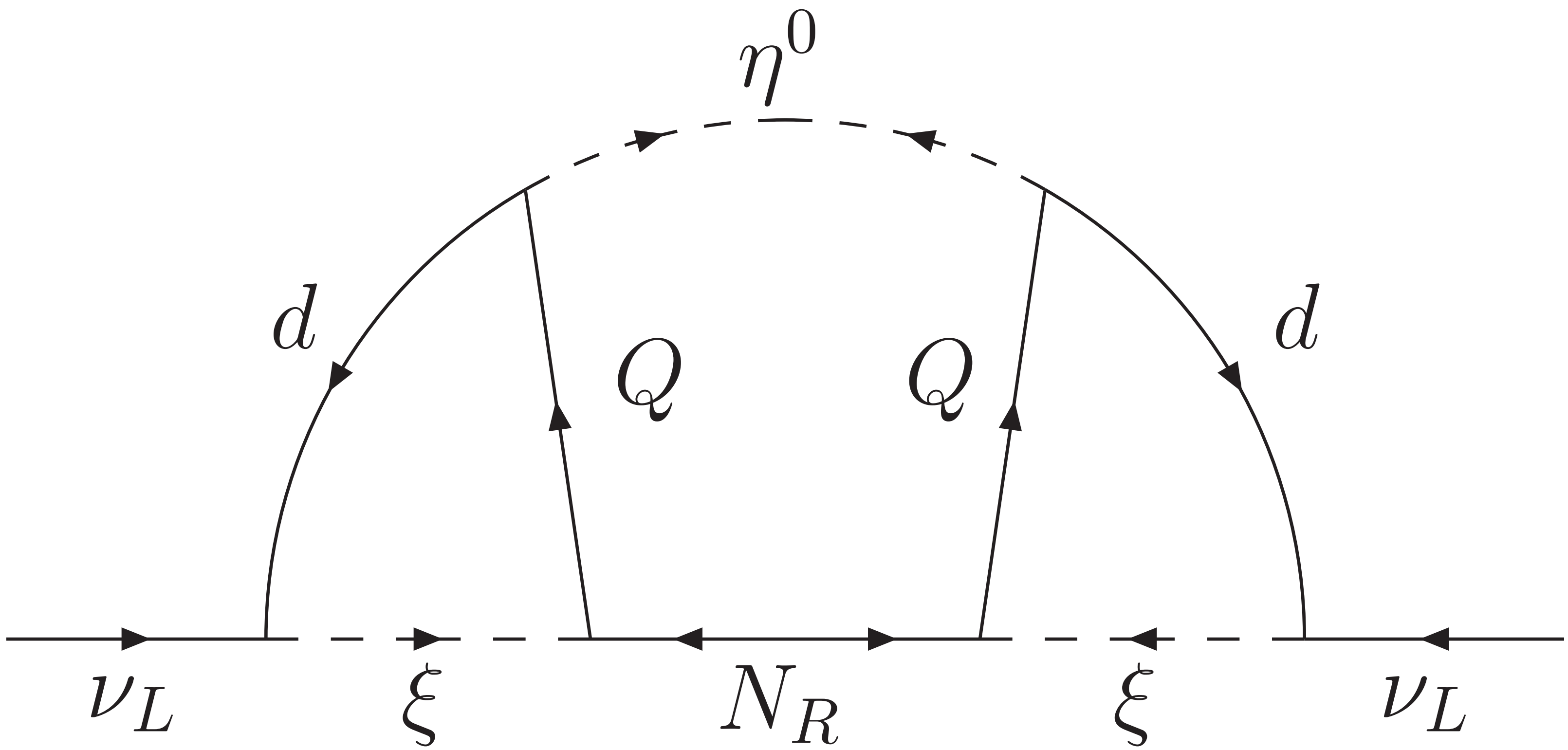


Figure 1: Three-loop neutrino mass with $U(1)_{PQ}$.

Neutrino Flavor Symmetry

Brief History of A_4: In 1978 (37 years ago), soon after the putative discovery of the third family of leptons and quarks, it was conjectured independently by Cabibbo and Wolfenstein:

$$U_{l\nu} = U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. In the PDG convention, this implies $s_{23} = c_{23} = 1/\sqrt{2}$, $s_{12} = c_{12} = 1/\sqrt{2}$, $s_{13} = 1/\sqrt{3}$, $c_{13} = \sqrt{2/3}$, and $\delta = \pi/2$. If $\omega \leftrightarrow \omega^2$, then $\delta = -\pi/2$.

In 2001 (14 years ago), without knowing about Cabibbo and Wolfenstein, U_ω was discovered by Ma and Rajasekaran in the context of A_4.

This non-Abelian discrete symmetry has 12 elements and 4 irreducible representations: $\underline{1}, \underline{1}', \underline{1}'', \underline{3}$. Using

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}.$$

the following decompositions are obtained:

$$\underline{1} = 11 + 22 + 33,$$
$$\underline{1}' = 11 + \omega 22 + \omega^2 33,$$
$$\underline{1}'' = 1 + \omega^2 22 + \omega 33.$$

Let $(\nu, l)_i \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, and $\Phi_i \sim \underline{3}$, then

$$\mathcal{M}_l = \begin{pmatrix} f_e v_1^* & f_\mu v_1^* & f_\tau v_1^* \\ f_e v_2^* & f_\mu \omega v_2^* & f_\tau \omega^2 v_2^* \\ f_e v_3^* & f_\mu \omega^2 v_3^* & f_\tau \omega v_3^* \end{pmatrix}$$

$$= \begin{pmatrix} v_1^* & 0 & 0 \\ 0 & v_2^* & 0 \\ 0 & 0 & v_3^* \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix}.$$

For $v_1 = v_2 = v_3$, a residual Z_3 symmetry exists with $U_\omega^\dagger$ as the link between $\mathcal{M}_l$ and $\mathcal{M}_\nu$.

For many years, theoretical effort was focused on obtaining a specific form of $\mathcal{M}_\nu$ so that tribimaximal neutrino mixing is realized:

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} =$$

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix}.$$

This means that

$$\mathcal{M}_\nu = \begin{pmatrix} m_2 & 0 & 0 \\ 0 & (m_1 - m_3)/2 & (m_1 + m_3)/2 \\ 0 & (m_1 + m_3)/2 & (m_1 - m_3)/2 \end{pmatrix}.$$

Pioneer A_4 papers: Ma/Rajasekaran(2001), Ma(2002), Babu/Ma/Valle(2003), Ma(2004), Altarelli/Feruglio(2005), Babu/He(2005).

This $\mathcal{M}_\nu$ is very hard to obtain in the context of a four-dimensional renormalizable field theory, because of the basic clash (or misalignment) of the residual symmetries (Z_3 for $\mathcal{M}_l$ and Z_2 for $\mathcal{M}_\nu$) [Lam]

On March 8, 2012, Daya Bay announced that θ_{13} had been measured at 8.8°, thus ending tribimaximal mixing.

The 2014 PDG values are: $\sin^2(2\theta_{12}) = 0.846 \pm 0.021$,
$\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2$,
$\sin^2(2\theta_{23}) = 0.999\ (+0.001/-0.018)$,
$\Delta m^2_{32} = (2.44 \pm 0.06) \times 10^{-3}\ \mathrm{eV}^2$ (normal),
$\sin^2(2\theta_{23}) = 1.000\ (+0.000/-0.017)$,
$\Delta m^2_{32} = (2.52 \pm 0.07) \times 10^{-3}\ \mathrm{eV}^2$ (inverted),
$\sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2}$.

In retrospect, the $Z_3 - Z_2$ clash should have been a warning against tribimaximal mixing.

Special Form of $\mathcal{M}_\nu$: Ma(2002), Babu/Ma/Valle(2003), Grimus/Lavoura(2004):

A special form of the neutrino mass matrix (in the basis where the charged-lepton mass matrix is diagonal) was written down 13 years ago, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},$$

where A, B are real.

θ_{13} and θ_{12} are determined by $s_{13}/c_{13} = -D_I/\sqrt{2}C_R$, $s_{13}c_{13}/(c_{13}^2 - s_{13}^2) = \sqrt{2}C_I/(A - B + D_R)$, and

$$\frac{s_{12}c_{12}}{c_{12}^2 - s_{12}^2} = \frac{-\sqrt{2}(c_{13}^2 - s_{13}^2)C_R}{c_{13}[c_{13}^2(A - B - D_R) + 2s_{13}^2 D_R]}.$$

The three neutrino masses are determined by

$$m_2 + m_1 \simeq A + B + D_R + s_{13}^2(A - B + D_R),$$

$$(c_{12}^2 - s_{12}^2)(m_2 - m_1) \simeq -A + B + D_R - s_{13}^2(A - B + D_R),$$

$$m_3 \simeq -B + D_R + s_{13}^2(A - B + D_R).$$

This allows $\theta_{13} \neq 0$ and yet $\theta_{23} = \pi/4$ is maintained, together with the prediction that $\delta_{CP} = \pm\pi/2$. This pattern is protected by a symmetry, i.e. $e \to e$ and $\mu \leftrightarrow \tau$ exchange with CP conjugation. Present T2K data with input from reactor data indicate a preference for $\delta_{CP} = -\pi/2$.

Note that this special form predicts that $|U_{\mu i}| = |U_{\tau i}|$. This harkens back to the original U_ω of 1978, where indeed this is satisfied. It is strongly suggestive that U_ω itself must have something to do with the realization of this special form of $\mathcal{M}_\nu$.

Since 2012, many authors have incorporated this generalized CP transformation into non-Abelian discrete symmetries (some rather complicated) to pin down the other angles, i.e. θ_{12} and θ_{13}.

See for example: Mohapatra/Nishi(2012),
Holthausen/Lindner/Schmidt(2013),
Feruglio/Hagedorn/Ziegler(2013, 2014),
Chen/Fallbacher/Mahanthappa/Ratz/Trautner(2014),
Hagedorn/Meroni/Molinaro(2014),
Ding/King/Neder(2014).

Typical result links θ_{12} with θ_{13}.

A_4 Bounces Back !!

Whereas tribimaximal mixing is dead, A_4 is not.
In fact, two papers appeared this year:

X.-G. He, arXiv:1504.01560; E. Ma, arXiv:1504.02086;

which use A_4 to obtain $\theta_{23} = \pi/4$ and $\delta_{CP} = -\pi/2$, and in the case of the former, also $\sin^2\theta_{12} \simeq 1/3$.

I will discuss the latter paper which also addresses the issue of dark matter and the apparent one Higgs boson of electroweak symmetry breaking.

The simple yet crucial observation is that if $U_{l\nu} = U_\omega{}^\dagger \mathcal{O}$, where $\mathcal{O}$ is orthogonal, then $U_{2i}^* = U_{3i}$ for $i = 1, 2, 3$. Compared this to the PDG form of $U_{l\nu}$, i.e.

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

it is obvious that after rotating the phases of the third column and the second and third rows, the two matrices are identical if and only if $s_{23} = c_{23}$ and $\cos\delta = 0$, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$. [F/M/T/Y(2000)]

Obviously $\mathcal{O}$ would come from diagonalizing a real mass matrix. So if $\mathcal{M}_\nu$ is somehow purely real in the A_4 basis, then

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = U_\omega^\dagger \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix} U_\omega^* = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},$$

where $A = (a + 2b + 2c + d + 2e + f)/3$,
$B = (a - b - c + d - e + f)/3$,
$C = (a - b - \omega c + \omega^2 d - \omega^2 e + \omega f)/3$,
$D = (a + 2b + 2\omega c + \omega^2 d + 2\omega^2 e + \omega f)/3$.
The special form of $\mathcal{M}_\nu$ is thus automatically obtained.

The Majorana neutrino mass matrix is in general complex, so how does one guarantee it to be real? The answer was already there in a radiative inverse seesaw model of neutrino mass [Fraser/Ma/Popov(2014), Ma/Natale/Popov(2015)], where the origin of the neutrino mass matrix is that of a real scalar mass-squared matrix.

Actually the neutrino mass eigenvalues may pick up phases from the parameters involved in the loop calculation, but to obtain $|U_{\mu i}| = |U_{\tau i}|$, all that is required is for $\mathcal{M}_\nu$ to be diagonalized by $\mathcal{O}$.

Radiative Inverse Seesaw

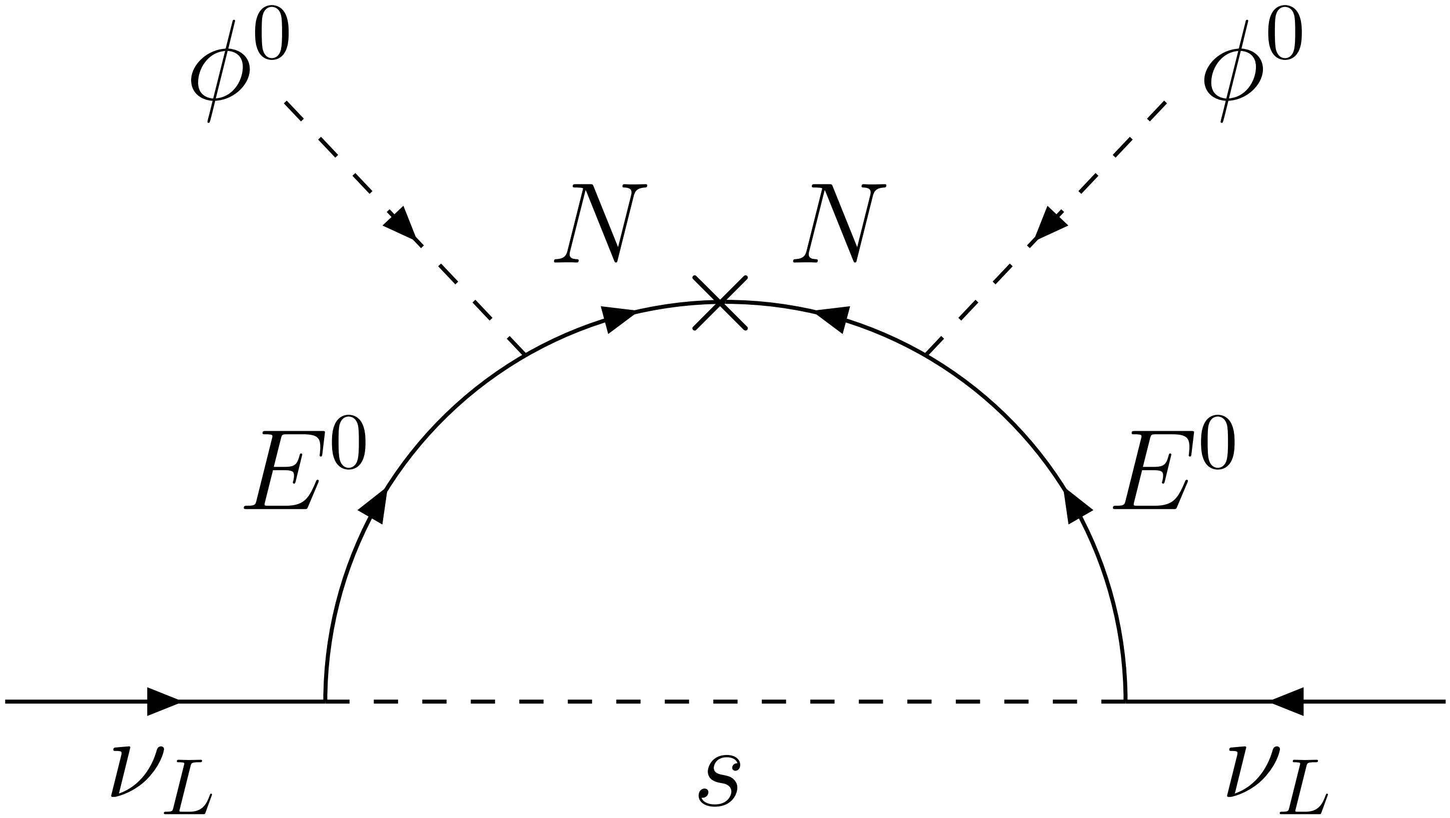
Under A_4, let the three families of leptons transform as

$$(\nu_i, l_i)_L \sim \underline{3}, \quad l_{iR} \sim \underline{1}, \underline{1}', \underline{1}''.$$

Add the following new particles, all assumed odd under an exactly conserved discrete Z_2 (dark) symmetry, whereas all SM particles are even:

$$(E^0, E^-)_{L,R} \sim \underline{1}, \quad N_{L,R} \sim \underline{1}, \quad s_i \sim \underline{3},$$

where (E^0, E^-) is a fermion doublet, N a neutral fermion singlet, and $s_{1,2,3}$ are real neutral scalar singlets.


ϕ^0
ϕ^0
N
N
E^0
E^0
ν_L
s
ν_L

The mass matrix linking $(\bar{N}_L, \bar{E}^0_L)$ to (N_R, E^0_R) is given by

$$\mathcal{M}_{N,E} = \begin{pmatrix} m_N & m_D \\ m_F & m_E \end{pmatrix},$$

where m_N, m_E are invariant mass terms, and m_D, m_F come from the respective Higgs couplings with $\langle \phi^0 \rangle = v/\sqrt{2}$. As a result, N and E^0 mix to form two Dirac fermions of masses $m_{1,2}$ with mixing angles

$$m_D m_E + m_F m_N = \sin\theta_L \cos\theta_L (m_1^2 - m_2^2),$$

$$m_D m_N + m_F m_E = \sin\theta_R \cos\theta_R (m_1^2 - m_2^2).$$

The mass terms $(m_{L,R}/2) N_{L,R} N_{L,R}$ also exist.

$$m_\nu = f^2 m_R s_R^2 c_R^2 (m_1^2 - m_2^2)^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)^2} \frac{1}{(k^2 - m_2^2)^2}$$

$$+ f^2 m_L m_1^2 s_R^2 c_L^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)^2}$$

$$+ f^2 m_L m_2^2 s_L^2 c_R^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_2^2)^2}$$

$$-2 f^2 m_L m_1 m_2 s_L s_R c_L c_R \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)} \frac{1}{(k^2 - m_2^2)},$$

where s is a mass eigenstate. If A_4 is unbroken, then $\mathcal{M}_\nu$ is proportional to the identity matrix. However, if A_4 is softly broken by the necessarily real $s_i s_j$ terms, then

$$\mathcal{M}_\nu = \mathcal{O} \begin{pmatrix} m_{\nu 1} & 0 & 0 \\ 0 & m_{\nu 2} & 0 \\ 0 & 0 & m_{\nu 3} \end{pmatrix} \mathcal{O}^T,$$

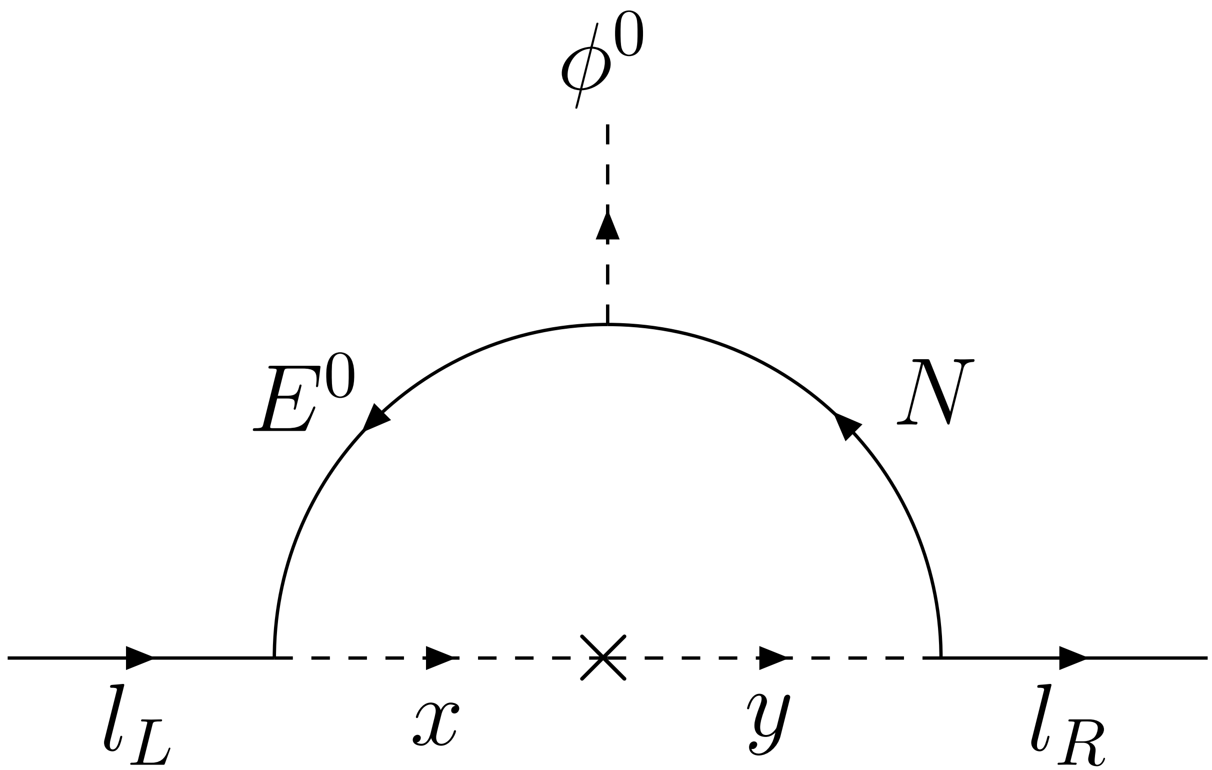
where $\mathcal{O}$ is an orthogonal matrix. Whereas f, m_L, m_R may be complex, only the relative phase between m_L and m_R appears in the two relative intrinsic Majorana phases of the neutrino mass eigenstates from the different s masses. Thus the desired form of $U_{l\nu}$ is obtained with $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$, once U_ω is applied.

Radiative Lepton Mass with Dark Matter:

Instead of using three Higgs doublets $\Phi_i \sim \underline{3}$ to obtain U_ω in the charged-lepton sector as in the original A_4 model of 2001, a radiative model of lepton mass is proposed. Again the fermion doublet (E^0, E^-) and singlet N are used, but now in conjunction of two sets of charged scalars which are also odd under dark Z_2, i.e.

$$x_i^- \sim \underline{3}, \quad y_i^- \sim \underline{1}, \underline{1}', \underline{1}''.$$

To connect x with y, the soft scalar term $x_i y_j^*$ is assumed to break A_4 to Z_3.



ϕ^0
E^0
N
l_L
x
y
l_R

Each lepton mass is then given by

$$m_l = f' f_l \mu_l^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_{1l}^2)(k^2 - m_{2l}^2)} \left[\frac{m_1 c_R s_L}{k^2 - m_1^2} - \frac{m_2 c_L s_R}{k^2 - m_2^2} \right],$$

where f' is the $E_L^0 l_L x^*$ Yukawa coupling, f_l is the $N_R l_R y^*$ Yukawa coupling, μ_l^2 is the scalar xy^* mass-squared term, and $m_{1l,2l}$ are the mass eigenvalues of the 2×2 mass-squared matrix

$$\mathcal{M}_{xy}^2 = \begin{pmatrix} m_x^2 & \mu_l^2 \\ \mu_l^2 & m_y^2 \end{pmatrix},$$

with $\mu_l^2 = \sin\theta_l \cos\theta_l (m_{1l}^2 - m_{2l}^2)$. This means that the $h\bar{l}l$ coupling will differ from $m_l/(246 \text{ GeV})$ without the usual $16\pi^2$ suppression.

There is a one-to-one correlation of the neutrino mass eigenstates to the $s_{1,2,3}$ mass eigenstates, the lightest of which is dark matter.

It is also clear that all three neutrino masses are expected to be of the same order of magnitude and their mass-squared differences are related to the scalar mass differences. Using the most recent cosmological data

$$\sum m_\nu < 0.23 \text{ eV},$$

the effective neutrino mass m_{ee} in neutrinoless double beta decay is bounded below 0.07 eV for normal ordering and 0.08 eV for inverted ordering.

Ma(2015):

The dark matter parity of this model is also derivable from lepton parity.

Under lepton parity, let the new particles $(E^0, E^-), N$ be even and s, x, y be odd, then the same Lagrangian is obtained. As a result, dark parity is simply given by $(-1)^{L+2j}$, which is odd for all the new particles and even for all the SM particles.

Note that the tree-level Yukawa coupling $\bar{l}_L l_R \phi^0$ would be allowed by lepton parity alone, but is forbidden here because of the A_4 symmetry.

The radiative lepton mass matrix is diagonal because of the Z_3 residual symmetry. This means that the muon anomalous magnetic moment Δa_μ gets a significant contribution from xy exchange, but not $\mu \to e\gamma$. Because Δa_μ is now of order m_μ^2/m_E^2 instead of the usual $(16\pi^2)^{-1} m_\mu^2/m_E^2$, a large $m_E \sim 1$ TeV is possible for the explanation of the experimental-theoretical discrepancy instead of the usual $m_E \sim 200$ GeV.

As for $\mu \to e\gamma$, it will come from s exchange in the analog diagram to radiative neutrino mass. For $m_E \sim 1$ TeV, it will be suitably suppressed.

Unification of Matter with Dark Matter

To unify matter and dark matter, the well-known $SU(5)$ grand unification of quarks and leptons may be simply extended to $SU(6)$. The fundamental $\underline{5}^*_F = (d^c, d^c, d^c, e, \nu)$ of $SU(5)$ is extended to the $\underline{6}^*_F = (d^c, d^c, d^c, e, \nu, N)$ of $SU(6)$. The $\underline{10}_F$ of $SU(5)$ is extended by a heavy $\underline{5}_F$ to form a $\underline{15}_F$ of $SU(6)$. Together with the similar extensions of the scalar multiplets, the $SU(5)$ Yukawa terms

$$\underline{5}^*_F \times \underline{10}_F \times \underline{5}^*_S, \quad \underline{10}_F \times \underline{10}_F \times \underline{5}_S,$$

are extended to

$$\underline{6}^*_F \times \underline{15}_F \times \underline{6}^*_S, \quad \underline{15}_F \times \underline{15}_F \times \underline{15}_S.$$

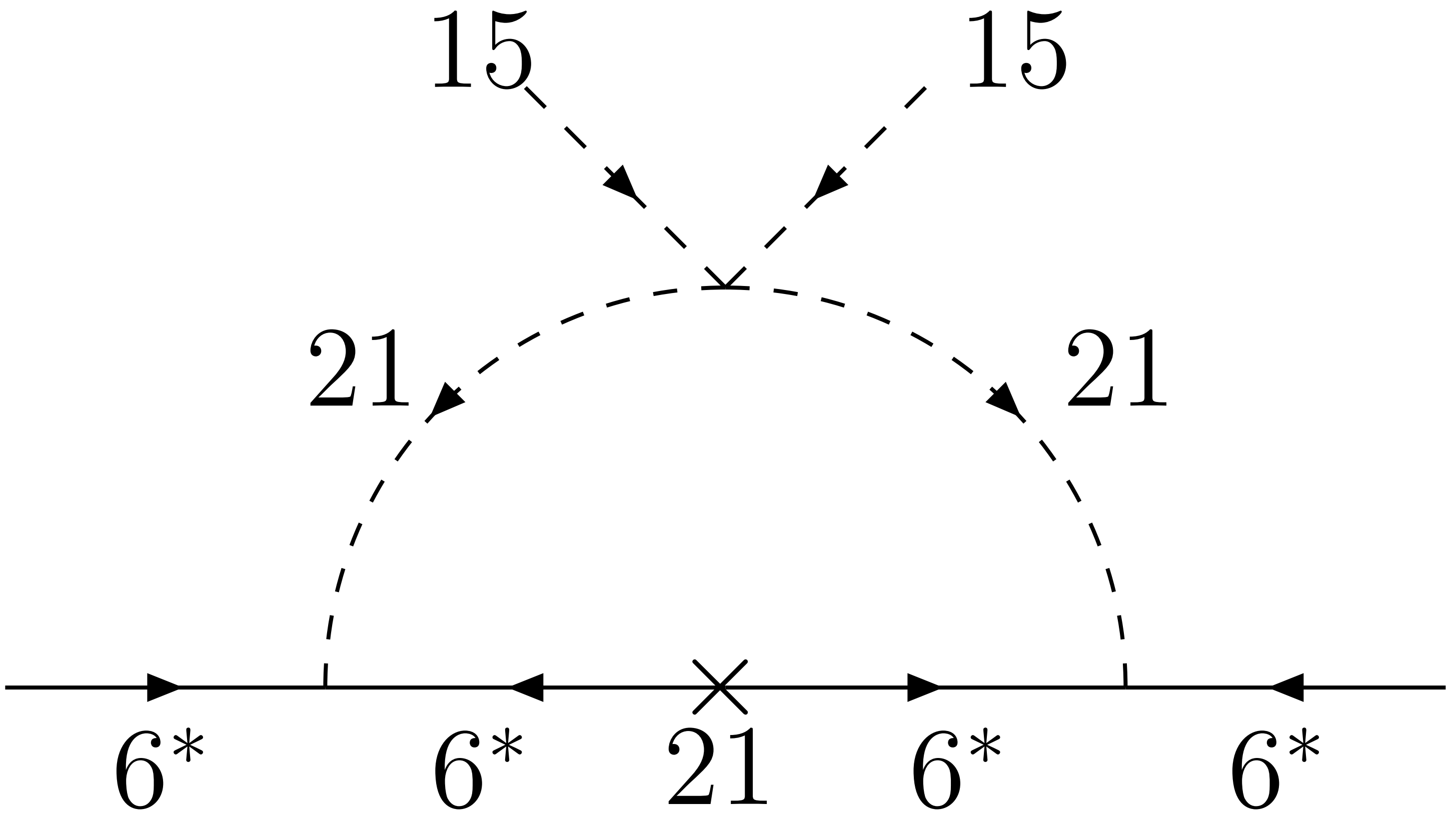
Hence two different Higgs doublets are needed for quark and lepton masses.

Whereas $\underline{5}^*_F + \underline{10}_F$ is anomaly-free in $SU(5)$, the corresponding combination in $SU(6)$ is $\underline{6}^*_F + \underline{6}^*_F + \underline{15}_F$. The $\underline{5}^*_F$ contained in the extra $\underline{6}^*_F$ is heavy and pairs up with the $\underline{5}_F$ contained in the $\underline{15}_F$ through the $\underline{1}_S$ of $\underline{6}^*_S$.

Consider now the $SU(6)$ scalar multiplet $\underline{21}_S$. It decomposes into $\underline{15}_S + \underline{5}_S + \underline{1}_S$ of $SU(5)$. It has then the second scalar doublet (η^+, η^0) and the interactions

$$\underline{6}^*_F \times \underline{6}^*_F \times \underline{21}_S, \quad \underline{15}^*_S \times \underline{15}^*_S \times \underline{21}_S \times \underline{21}_S.$$

Hence N gets a Majorana mass through the $\underline{1}_S$ of $\underline{21}_S$. The scotogenic interaction $(\nu\eta^0 - e\eta^+)N$ is now possible as well as the quartic $(\Phi^\dagger\eta)^2$ interaction. These three terms support a Z_2 symmetry as desired, but it is not absolute. Just as the proton is unstable at the scale of quark-lepton unification, dark matter is expected to be unstable at a similar scale. Matter and dark matter are thus unified through scotogenic neutrino mass.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The heavy color triplet gauge bosons contained in the adjoint $\underline{24}_V$ of $SU(5)$ mediate proton decay.

The extra heavy color triplet gauge bosons contained in the adjoint $\underline{35}_V$ of $SU(6)$ will connect N to the quarks, resulting in $N \to p\pi^-, n\pi^0$.

Another possibility is the mixing of the heavy scalar color triplet $\zeta^{-1/3}$ in $\underline{21}_S$ with its counterpart $\xi^{-1/3}$ in $\underline{15}_S$ through the adjoint $\underline{35}_S$ of $SU(6)$.

The dark Z_2 is thus broken at a high scale, just as baryon number is supposed to be broken in grand unified theories.

In the breaking of $SU(6)$ through the adjoint $\underline{35}_S$, let the 6×6 matrix representing $\underline{35}_S$ develop diagonal vacuum expectation values proportional to $(1,1,1,0,0,-3)$; then the residual symmetry is
$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$.
The dark $U(1)_N$ is broken by $\underline{21}_S$ at a lower energy scale for N to acquire a Majorana mass.
This choice of $\underline{35}_S$ breaking mixes ζ with ξ, but not η with Φ. It is only possible here because of $SU(6)$. For $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$, it must be along the $(1,1,1,-3/2,-3/2)$ direction.

Personal Remarks

Neutrino theory attempts to answer several fundamental questions, foremost is the scale of new physics responsible for neutrino mass and mixing. A possible hint is that they may also be connected to dark matter at the mass scale of 1 TeV. In the scotogenic framework, the A_4 transformation U_ω may be used to obtain a desirable form of $U_{l\nu}$, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$, automatically if the origin of the neutrino mass matrix is a set of real scalars $s_i \sim \underline{3}$ under A_4. Only one Higgs doublet with $\langle\phi^0\rangle$ accounting for all of electroweak symmetry breaking is required. New particles $(E^0, E^-), N, x, y, s$ are predicted and may be observed.