Neutrino Theory I:
Mass and Interactions

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Contents

Neutrino Mass in Perspective

Weinberg (1967): Minimal $SU(2)_L \times U(1)_Y$ Standard Model (SM) with one Higgs doublet $(\phi^+, \phi^0) \sim (2, 1/2)$ and lepton content $(\nu_e, e)_L, (\nu_\mu, \mu)_L \sim (2, -1/2)$, $e_R, \mu_R \sim (1, -1)$.

As ϕ^0 acquires a nonzero vacuum expectation value v, the charged leptons acquire masses through $\bar{e}_L e_R \phi^0$ and $\bar{\mu}_L \mu_R \phi^0$. Neutrinos are massless and their interactions are limited to $(g/\sqrt{2})\bar{\nu}_L \gamma^\mu l_L W_\mu^+ + H.c.$.

New particles are $W^\pm, Z^0$ and $h = \sqrt{2}(\phi^0 - v)$.

What about quarks: $(u, d\cos\theta_C + s\sin\theta_C)_L \sim (2, 1/6)$?

Weinberg (1979): If physics beyond the SM occur at mass scales much greater than the electroweak breaking scale v, there is a unique dimension-five operator for Majorana neutrino mass:

$$\mathcal{L}_5 = \frac{f_{\alpha\beta}}{2\Lambda}(\nu_\alpha \phi^0 - l_\alpha \phi^+)(\nu_\beta \phi^0 - l_\beta \phi^+)$$

Hence $\mathcal{M}_\nu = f_{\alpha\beta} v^2/\Lambda$. This shows that seesaw is always obtained for $v << \Lambda$.

Exception: If new physics occurs at scales lower than v, then other possibilities exist.

For example, if there are singlet neutral fermions N_R, often called right-handed neutrinos or sterile neutrinos, then the interaction $f\bar{N}_R\nu_L\phi^0 + H.c.$ is allowed and ν_L pairs up with N_R to form a Dirac fermion of mass $fv << v$.

To forbid the $N_R N_R$ mass terms, global U(1) lepton number L may be imposed. On the other hand, these mass terms could be nonzero thus breaking L to $(-1)^L$. If these terms are also smaller than v, then there is a possible 6×6 neutrino mass matrix to be studied, with many phenomenological implications.

Early Specific Ideas

(1979) Add N_R with m_N very large, then

$$\begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \Rightarrow m_\nu \simeq \frac{-m_D^2}{m_N}.$$

This is the famous original seesaw mechanism (coined by Yanagida), and it implies only the interaction $f\bar{N}_R\nu_L\phi^0$ which generates $m_D = fv$.

(1980) Add (ξ^{++}, ξ^+, ξ^0) with $\langle\xi^0\rangle = u$, then $(f/2)\xi^0\nu\nu \Rightarrow m_\nu = fu$.

This scalar triplet mechanism implies new scalar particles with gauge interactions as well as

$$V = m^2\Phi^\dagger\Phi + M^2\xi^\dagger\xi + \frac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2(\xi^\dagger\xi)^2$$
$$+\lambda_3|2\xi^{++}\xi^0 - \xi^+\xi^+|^2 + \lambda_4(\Phi^\dagger\Phi)(\xi^\dagger\xi)$$
$$+\frac{1}{2}\lambda_5[|\sqrt{2}\xi^{++}\phi^- + \xi^+\bar{\phi}^0|^2 + |\xi^+\phi^- + \sqrt{2}\xi^0\bar{\phi}^0|^2]$$
$$+\mu(\bar{\xi}^0\phi^0\phi^0 + \sqrt{2}\xi^-\phi^0\phi^+ + \xi^{--}\phi^+\phi^+) + H.c.$$

If $\mu = 0$, then ξ may be assigned $L = -2$, and $u \neq 0$ breaks it spontaneously, resulting in a massless Goldstone boson (triplet majoron), ruled out by Z decay.

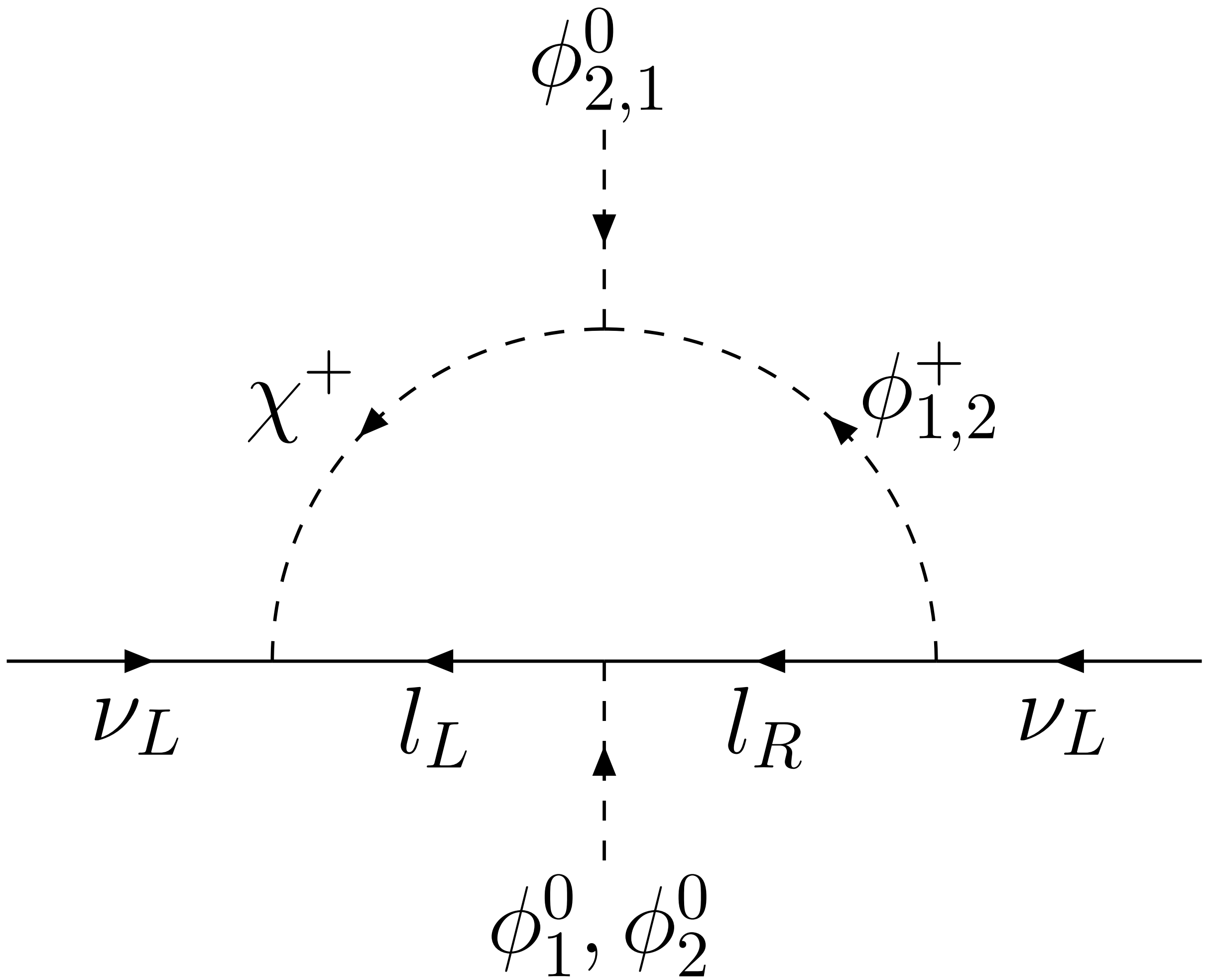
The minimum of V is given by

$$m^2 + \lambda_1 v^2 + (\lambda_4 + \lambda_5)u^2 + 2\mu u = 0,$$

$$u[M^2 + \lambda_2 u^2 + (\lambda_4 + \lambda_5)v^2] + \mu v^2 = 0.$$

For $\mu \neq 0$, $u \simeq -\mu v^2/[M^2 + (\lambda_4 + \lambda_5)v^2]$, where $v^2 \simeq -m^2/\lambda_1$. Note that this is also a seesaw formula for $v^2 << M^2$.

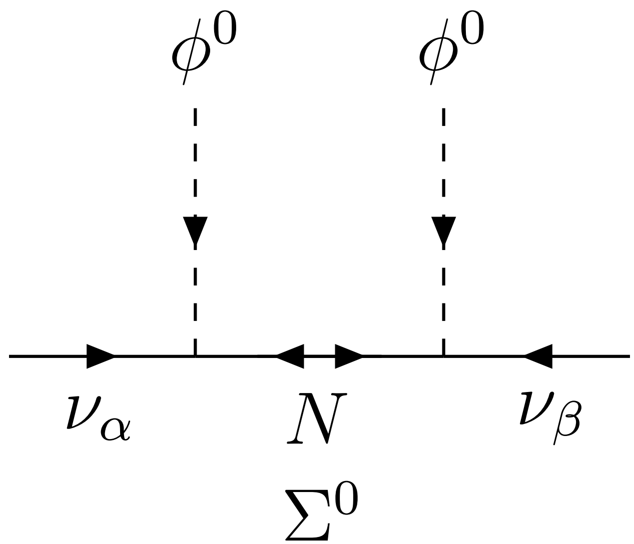
(1980): Add charged singlet scalar χ^+ and second scalar doublet (ϕ_2^+, ϕ_2^0), then a one-loop m_ν is generated. This also implies new gauge and other interactions.

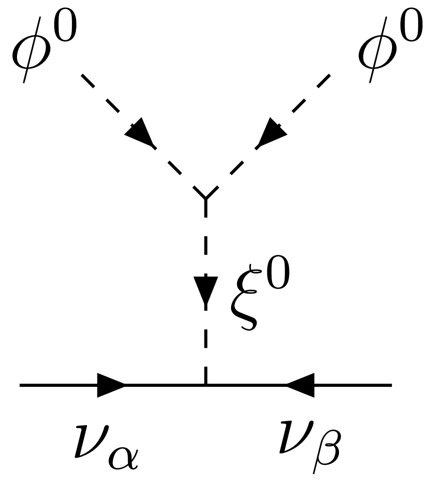


$\phi^0_{2,1}$
χ^+
$\phi^+_{1,2}$
ν_L
l_L
l_R
ν_L
ϕ^0_1, ϕ^0_2

(1989): Add $(\Sigma^+, \Sigma^0, \Sigma^-)_R$, then $\bar{\Sigma}^0_R \nu_L \phi^0 \Rightarrow m_D$, and $m_\nu \simeq -m_D^2/m_\Sigma$. Gauge interactions are implied.

(1998): [PRL 81, 1171 (1998)]
$\mathcal{L}_5$ has three and only three tree-level realizations:
(I) fermion singlet N (1979),
(II) scalar triplet (ξ^{++}, ξ^+, ξ^0) (1980),
(III) fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$ (1989);
and three generic one-loop realizations:
(R1) (Zee, 1980), (R2) (Ma, 2006), (R3) (Fraser/Ma/Popov, 2014). The nomenclature of Type I, II, III seesaw is now well established.


ϕ^0
ϕ^0
ν_α
N
Σ^0
ν_β


ϕ^0
ϕ^0
ξ^0
ν_α
ν_β

(I) $(\phi^0\nu_i - \phi^+ l_i)(\phi^0\nu_j - \phi^+ l_j)$,

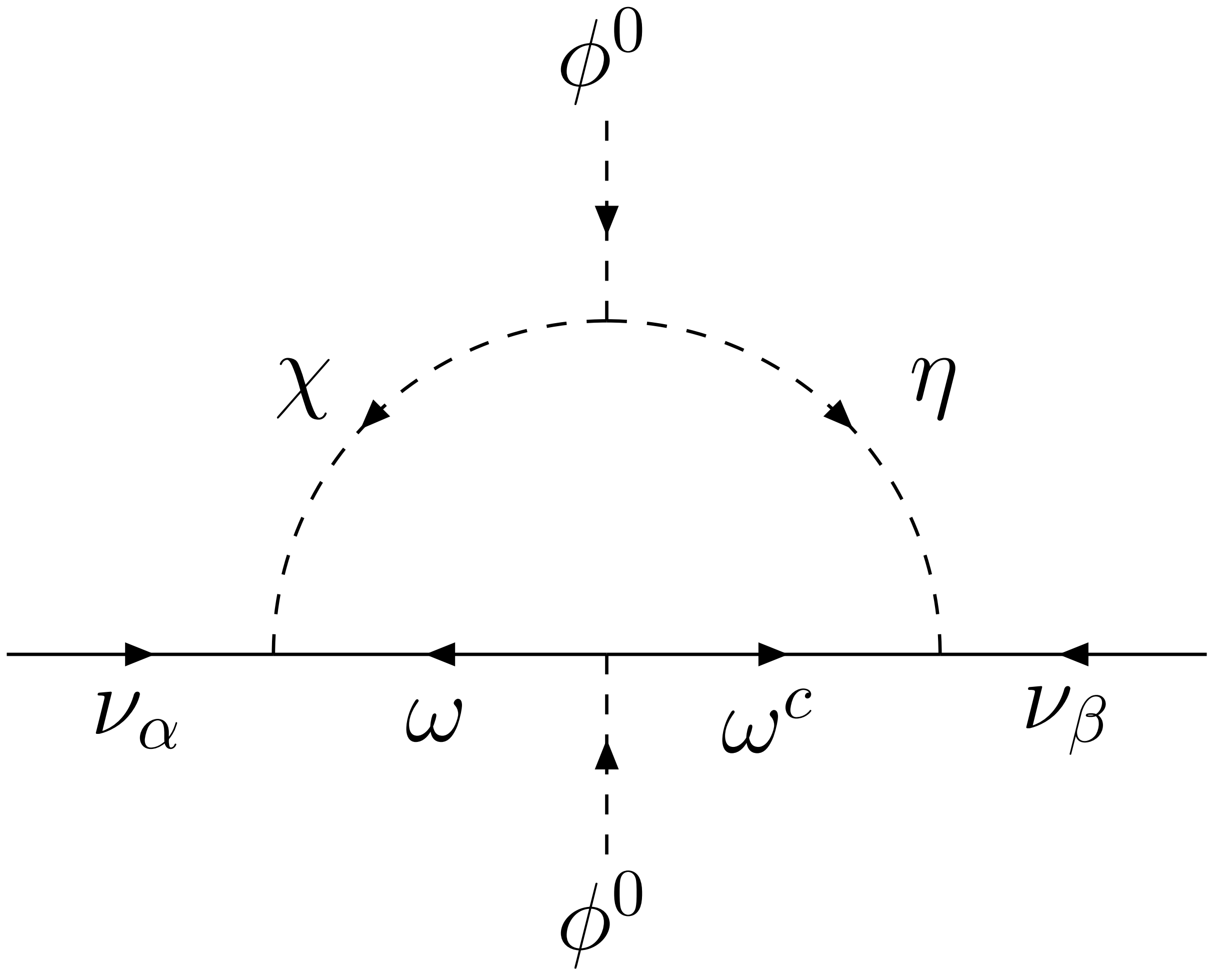
(II) $\phi^0\phi^0\nu_i\nu_j - \phi^+\phi^0(\nu_i l_j + l_i\nu_j) + \phi^+\phi^+ l_i l_j$,

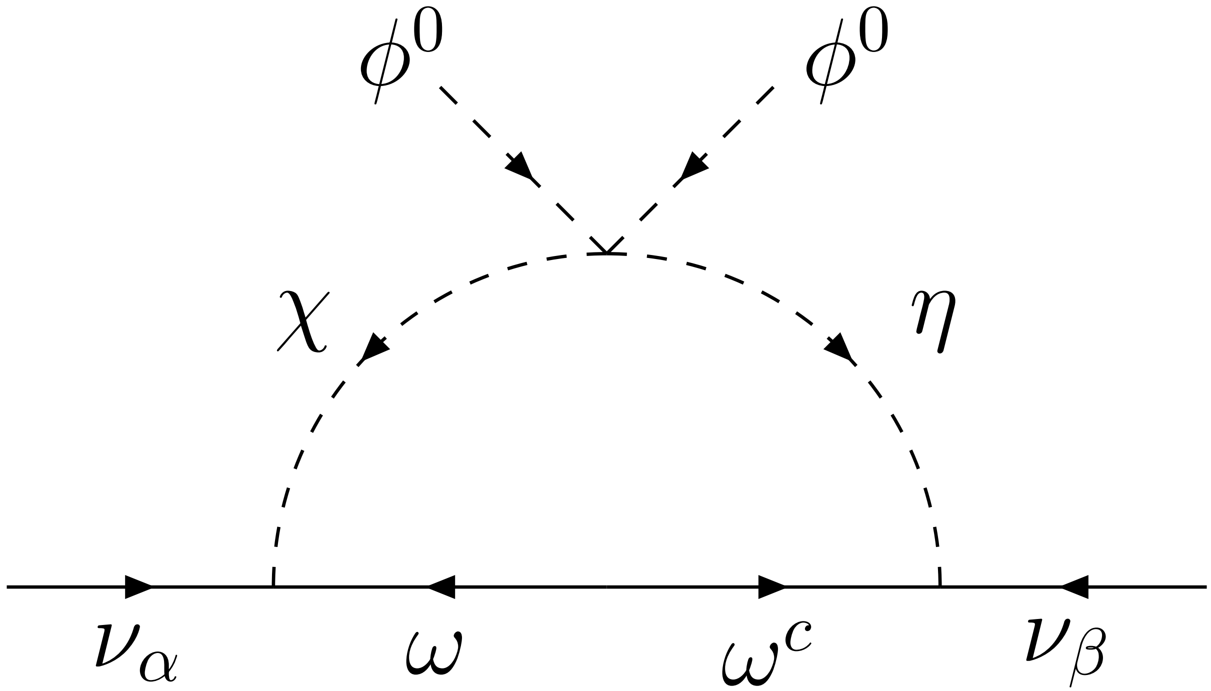
(III) $(\phi^0\nu_i + \phi^+ l_i)(\phi^0\nu_j + \phi^+ l_j) - 2\phi^+\nu_i\phi^0 l_j - 2\phi^0 l_i\phi^+\nu_j$.

Radiative mechanism: The two external neutrino lines are connected in one loop by an internal fermion line and an internal scalar line.

(R1) The two external Higgs lines are attached one to the scalar line and one to the fermion line.

(R2, R3) The two Higgs lines are both attached to the (scalar, fermion) line.


ϕ^0
χ
η
ν_α
ω
ω^c
ν_β
ϕ^0


ϕ^0
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χ
η
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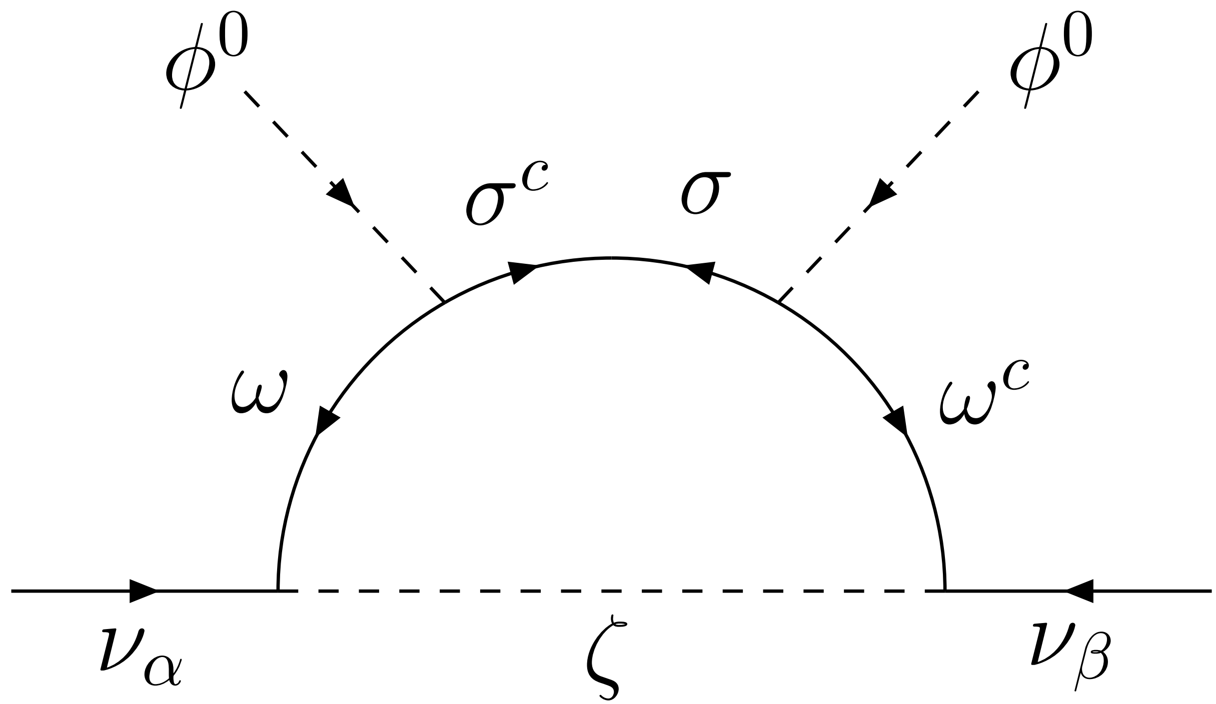

ϕ^0
ϕ^0
σ^c
σ
ω
ω^c
ν_α
ζ
ν_β

Super-K (June, 1998): [Neutrino 98 Conference, Japan] Atmospheric neutrino oscillations established.

Headline news around the world: Neutrinos Have Mass!!

SNO (2002): Solar neutrino oscillations established.

Absent of other information, how neutrinos get their mass is still unknown. In fact, we still do not know if its mass is Dirac or Majorana, without a positive signal from neutrinoless double beta decay.

(2012): The 125 GeV particle was discovered at CERN, but no other new particle since then. It is presumably the SM Higgs boson h, with important implications.

Seesaw Variants

Neutrino mass literature used to be almost exclusively dominated by the Type I seesaw, but after 1998 and more so after 2006, other ideas are being discussed with greater frequency.

Nevertheless, Type I seesaw is the simplest idea, but basically impossible to prove. On the other hand, there are variants which may offer some hope of experimental verification. Neutrino mass generation may also be connected with other phenomena, such as dark matter, flavor, and the strong CP problem.

With 1 doublet neutrino ν and 1 singlet neutrino N, their 2×2 mass matrix is the well-known

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix},$$

resulting in the famous seesaw formula $m_\nu \simeq -{m_D}^2/m_N$. Hence $\nu - N$ mixing $\simeq m_D/m_N \simeq \sqrt{m_\nu/m_N} < 10^{-6}$, for $m_\nu < 1$ eV and $m_N > 1$ TeV.

Since N has no other interaction, the only way that it can be produced is through its mixing with ν. It is basically hopeless in the case of Type I seesaw.

Consider now 1 ν and 2 singlets: $N_{1,2}$. Their 3×3 mass matrix is then

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & m_1 & m_N \\ 0 & m_N & m_2 \end{pmatrix},$$

resulting in $m_\nu \simeq {m_D}^2 m_2/({m_N}^2 - m_1 m_2)$. Since the limit $m_1 = 0$ and $m_2 = 0$ corresponds to lepton number conservation ($L = 1$ for ν and N_2, $L = -1$ for N_1), their smallness is natural $\Rightarrow$ the inverse seesaw [Mohapatra/Valle(1986)].

Here $\nu - N_1$ mixing $\sim m_\nu / m_D$,
whereas $\nu - N_2$ mixing $\sim m_D / m_N$,
and they are not constrained to be the same
as was in the canonical seesaw.

For example, let $m_D \sim 10$ GeV, $m_N \sim 1$ TeV, $m_2 \sim 10$ keV, then $m_\nu \sim 1$ eV,
and $\nu - N_1$ mixing $\sim 10^{-10}$ is very small,
but $\nu - N_2$ mixing $\sim 10^{-2}$ is large enough to be observable.
Note that the geometric mean of the two mixings is again 10^{-6} as before.

Linear Seesaw:

[Barr(2004), Malinsky/Romao/Valle(2005)]

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D & m'_D \\ m_D & 0 & m_N \\ m'_D & m_N & 0 \end{pmatrix}$$

$\Rightarrow m_\nu \simeq -2 m_D m'_D / m_N$, which is only linear in m_D.

Ma(2009): For the linear seesaw to work, m'_D must be very small. In the limit it is zero, $\mathcal{M}_{\nu N}$ is the same as that of the inverse seesaw in the same limit, so they must have the same origin.

To prove this, let $m'_D/m_D = \tan\theta$, then rotate the (N_1, N_2) basis by θ, we get

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D/c & 0 \\ m_D/c & m_N s_2 & m_N c_2 \\ 0 & m_N c_2 & -m_N s_2 \end{pmatrix}$$

where $c = \cos\theta$, $c_2 = \cos 2\theta$, and $s_2 = \sin 2\theta$.

For small θ, this is just the inverse seesaw, with

$$m_\nu \simeq \frac{-2 m_N m'_D}{m_D} \left(\frac{{m_D}^2}{{m_N}^2} \right) = \frac{-2 m_D m'_D}{m_N}.$$

Zero Neutrino Mass from Cancellation

If the inverse or linear seesaw mechanisms are invoked to get large $\nu - N$ mixing for each family, we will need 2 singlet neutrinos for each doublet neutrino, thus implying a 9×9 mass matrix. Is that really necessary?

Buchmuller/Wyler(1990), Pilaftsis(1992,2005), Kersten/Smirnov(2007): Consider first two families: $\nu_{1,2}$ and $N_{1,2}$, with $\mathcal{M}_N = \text{diag}(M_1', M_2')$, where

$$\mathcal{M}_D = \begin{pmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \begin{pmatrix} b_1 & b_2 \end{pmatrix}.$$

One neutrino mass is zero because the determinant of $\mathcal{M}_D$ is zero by construction. If we now also impose the arbitrary condition $b_1^2/M_1' + b_2^2/M_2' = 0$, then the other neutrino mass is zero as well. However, $\nu - N$ mixing may be large, because $a_i b_j$ need not be small. In other words, zero mixing is now not the limit of zero neutrino mass as in the previous seesaw formulas.

If small deviations from this texture are present, small neutrino masses will appear, but the large $\nu - N$ mixing will remain. This appears to be fine tuning, but a lot of phenomenological studies have been done in this context.

Observable Nonunitary Neutrino Mixing Matrix

The addition of heavy singlet fermions N_i to the Standard Model induces both the well-known dimension-five operator

$$\Lambda^{-1} f_{\alpha\beta} (L_\alpha \Phi)(L_\beta \Phi) \quad [\text{mass mixing}]$$

and the less-known dimension-six operator

$$\Lambda^{-2} f_{\alpha\beta} (\Phi^\dagger \bar{L}_\alpha) i\partial^\mu \gamma_\mu (L_\beta \Phi) \quad [\text{kinetic mixing}].$$

Both operators will probe m_D/m_N, which is of course too small in the canonical seesaw, but if the inverse seesaw or linear seesaw or the texture hypothesis is used, then they will provide useful phenomenological constraints.

Antusch/Biggio/Fernandez-Martinez/Gavela/Lopez-Pavon(2006),
Antusch/Baumann/Fernandez-Martinez(2009):
Let the neutrino mixing matrix be $(1+\eta)U$, then

$$|\eta| < \begin{pmatrix} 2.0\times 10^{-3} & 6.0\times 10^{-5} & 1.6\times 10^{-3} \\ \sim & 8.0\times 10^{-4} & 1.1\times 10^{-3} \\ \sim & \sim & 2.7\times 10^{-3} \end{pmatrix}.$$

Ohlsson/Popa/Zhang(2010):

For the texture hypothesis, i.e.

$$\mathcal{M}_D = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}$$

with the condition $b_1^2/M_1' + b_2^2/M_2' + b_3^2/M_3' = 0$, the parameters $\eta_{e\tau}$ and $\eta_{\mu\tau}$ are correlated:

$$\left|\frac{\eta_{e\tau}}{1.6 \times 10^{-3}}\right|^2 + \left|\frac{\eta_{\mu\tau}}{1.1 \times 10^{-3}}\right|^2 < 1.$$

Symmetry Origin of the Texture Hypothesis

He/Ma(2009): To understand the mechanism and symmetry of the texture hypothesis, change the neutrino basis to

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \\ m_1 & 0 & M_1 & M_3 \\ 0 & m_2 & M_3 & M_2 \end{pmatrix}.$$

Note that $m_1 = M_1 = 0$ would result in two massless neutrinos.

If $\mathcal{M}_D$ of the texture hypothesis is now rotated on the left with $-\tan^{-1}(a_1/a_2)$ and on the right with $\tan^{-1}(b_1/b_2)$, then $m_1 = 0$ automatically. Furthermore, $M_1 = \cos^2\theta_R M_1' + \sin^2\theta_R M_2' = (b_1^2/M_1' + b_2^2/M_2')M_1'M_2'/(b_1^2 + b_2^2) = 0$ as well. Hence ν_1 and $\nu_2' = (M_3\nu_2 - m_2 N_1)/\sqrt{M_3^2 + m_2^2}$ are massless, the latter showing also how the large mixing occurs between ν_2 and N_1 , i.e. through the inverse seesaw mechanism. However, lepton number conservation would not only forbid M_1 but also M_2, which is arbitrary here. Where is the symmetry which does this?

Let $\nu_{1,2}$, $N_{1,2}$ have $L = 1, 1, 3, -1$.

Add the usual Higgs doublet (ϕ_1^+, ϕ_1^0) with $L = 0$ and the Higgs singlet χ_2 with $L = 2$. Then m_2 comes from $\langle \phi_1^0 \rangle$, M_2 from $\langle \chi_2 \rangle$, and M_3 from $\langle {\chi_2}^\dagger \rangle$.

$m_1 = 0$ at tree level, because there is no Higgs doublet with $L = -4$, and $M_1 = 0$ at tree level, because there is no Higgs singlet with $L = \pm 6$.

In one loop, M_1 will be induced, thus giving ν_2' an inverse seesaw mass $= M_1 m_2^2 / M_3^2$. Once ν_2 is massive, ν_1 also gets a two-loop radiative mass from the exchange of two W's [Babu/Ma(1988)].

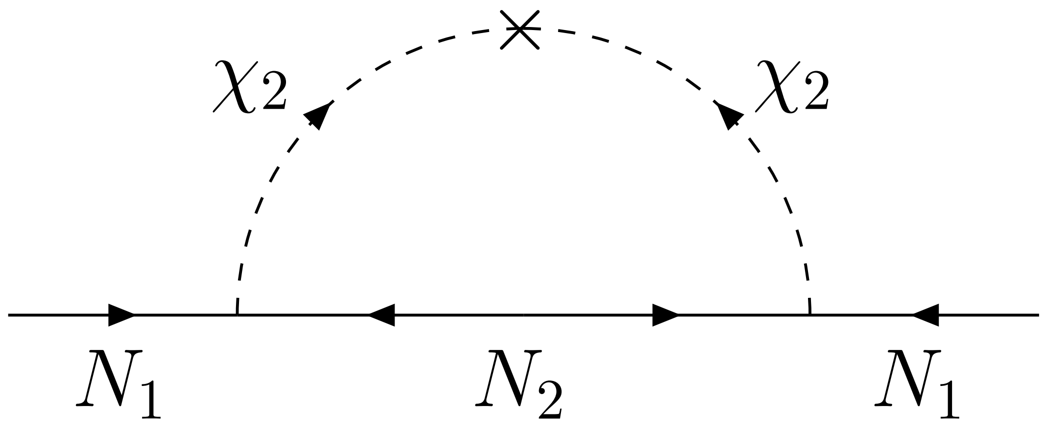


Figure 1: One-loop generation of M_1.

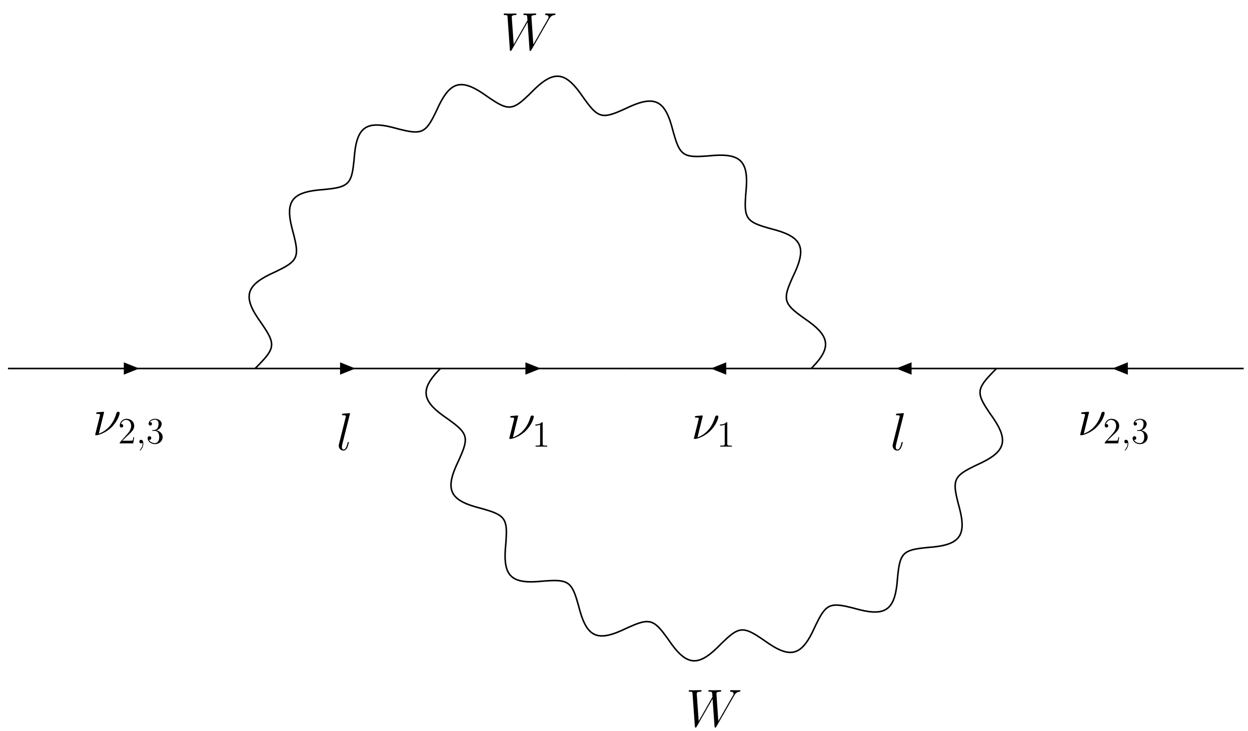


Figure 1: Two-W generation of neutrino mass.

With small m_1 and M_1, the 4×4 neutrino mass matrix $\mathcal{M}_{\nu N}$ is reduced to the 2×2

$$\mathcal{M}_\nu \simeq \begin{pmatrix} {m_1}^2 M_2/M_3^2 & -m_1 m_2/M_3 \\ -m_1 m_2/M_3 & M_1 m_2^2/M_3^2 \end{pmatrix}.$$

Since $M_2 \sim M_3$ in this hypothesis, the (1,1) entry is a canonical seesaw, whereas the (2,2) entry is an inverse seesaw and the (1,2) or (2,1) entry is a linear seesaw. In this basis, only ν_2 has possible large mixing with N. Similarly, if 3 families are considered with 3 N, only one linear combination of the 3 ν may mix significantly with N.

For three families, let

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_3 \\ m_1 & 0 & 0 & M_1 & M_4 & M_5 \\ 0 & m_2 & 0 & M_4 & M_2 & M_6 \\ 0 & 0 & m_3 & M_5 & M_6 & M_3 \end{pmatrix}.$$

The texture hypothesis is equivalent to $m_1 = m_2 = 0$ and $M_1 = M_4 = 0$.

To enforce this pattern at tree level, use $L = 1$ for $\nu_{1,2,3}$ as usual, but $L = 3, -2, -1$ for $N_{1,2,3}$.

Add one Higgs doublet with $L = 0$ and three Higgs singlets with $L = 2, 3, 4$. Then $M_1 = M_4 = 0$, because there is no Higgs singlet with $L = \pm 6$ or $L = \pm 5$, but will become nonzero in one loop, whereas $m_1 = m_2 = 0$ to all orders.

To obtain nonzero $m_{1,2}$, a second Higgs doublet with $L = -4, 1$ may be added.
$\nu_{1,2}$ is then massive in one loop, and $\nu_{2,1}$ becomes massive in two loops.

Lepton number has been used as a global U(1) symmetry, but a discrete version (e.g. Z_7) also works.

The global U(1) may also be gauged, using either $U(1)_{B-L}$ or $U(1)_\chi$ from E_6 where $Q_\chi = 5(B-L) - 4Y$. Many possible signatures of these extensions may be looked for at the LHC.

For example, with $U(1)_\chi$, it is possible to produce N in pairs through $q\bar{q} \to Z' \to N\bar{N}$, if kinematically allowed. The final states to be analyzed are $l^\pm l^\mp W^\pm W^\mp$ and $l^\pm l^\pm W^\mp W^\mp$. Without new physics, N is not likely to be observable. [Ibarra/Molinaro/Petcov(2010).]

Leptogenesis

In this talk, I assume m_ν is Majorana and $(-)^L$ is conserved. What is the mass scale of the new physics using $\mathcal{L}_5$?

$$m_\nu \sim \frac{f^2 v^2}{\Lambda} \sim 1 \text{ eV}$$

$$\Rightarrow \frac{\Lambda}{f^2} \sim \frac{(100 \text{ GeV})^2}{1 \text{ eV}} \sim 10^{13} \text{ GeV}.$$

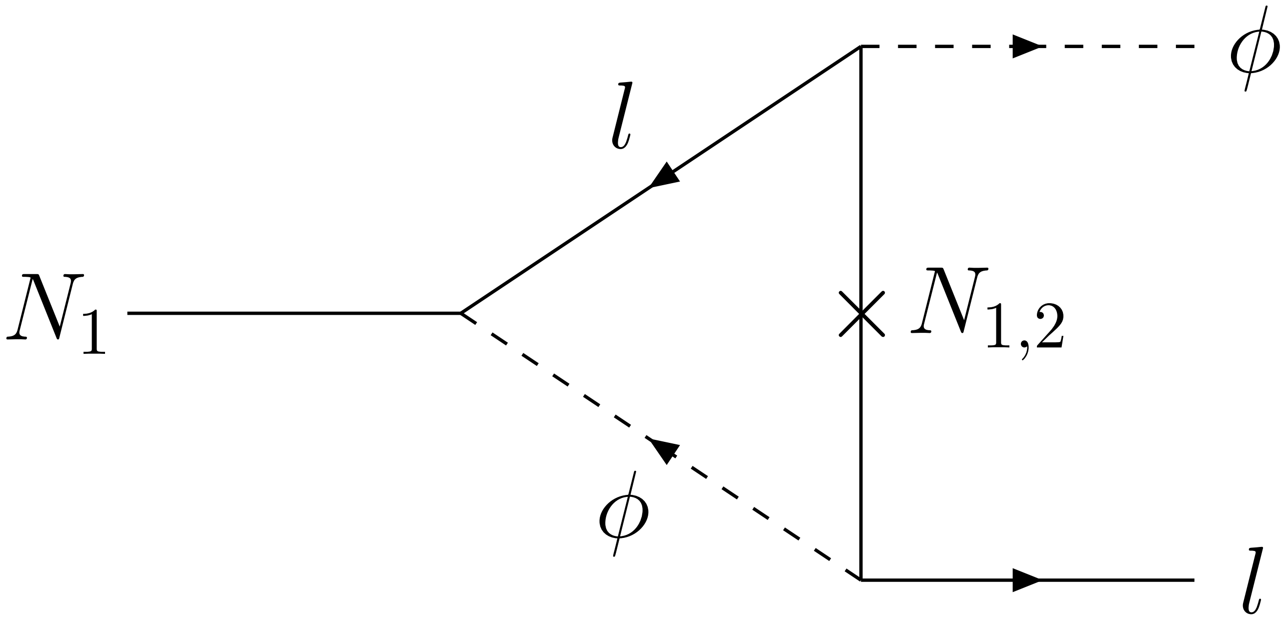
If $f \sim 1$, then $\Lambda \sim 10^{13}$ GeV. This high scale is suitable for leptogenesis.

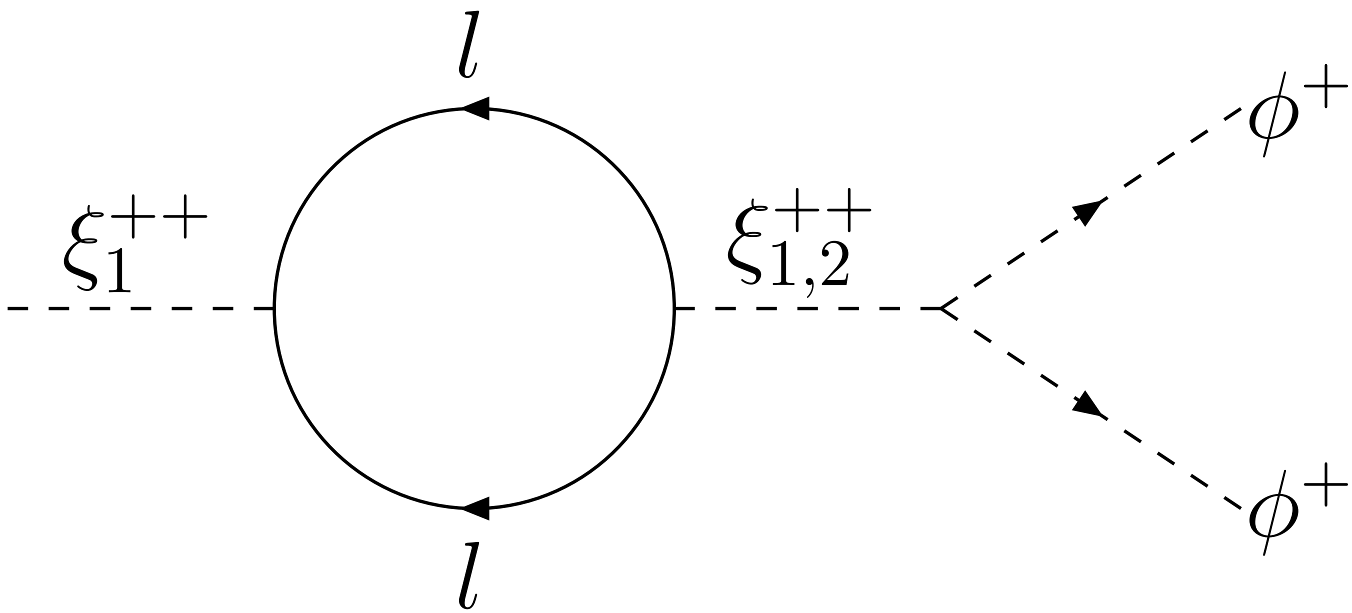
For every seesaw mechanism, there is a leptogenesis scenario for the baryon asymmetry of the Universe.

Type (I) [Fukugita/Yanagida(1986)] $N_1 \to l^\pm \phi^\mp$ at tree level interfering with one-loop (vertex and self-energy) amplitudes involving N_2 with CP violation.

Type (II) [Ma/Sarkar(1998)] $\xi_1^{\pm\pm} \to l^\pm l^\pm, \phi^\pm \phi^\pm$ at tree level interfering with a one-loop (self-energy) amplitude involving ξ_2 with CP violation.

The lepton asymmetry generated is converted by sphalerons during the electroweak phase transition to the present observed baryon asymmetry of the Universe.


N_1
l
ϕ
$N_{1,2}$
ϕ
l



l
ξ_1^{++}
$\xi_{1,2}^{++}$
ϕ^+
ϕ^+
l

Cofactor Zeros

If m_N is indeed very heavy, is there anyway to know that Type I seesaw is the true origin of neutrino mass?

A possible answer is the existence of cofactor zeros in the observed neutrino mass matrix $\mathcal{M}_\nu$, which would be the consequence of zeros in $\mathcal{M}_N$ provided that $\mathcal{M}_D$ is diagonal. [Ma(2005)]

There is now enough experimental data to know that three patterns of cofactor zeros exist which are consistent with best-fit neutrino-oscillation parameter values. [Liao/Marfatia/Whisnant(2014)]

Consider the anomaly-free family gauge symmetry
$\sum_i x_i L_i$ with $\sum_i x_i = 0$ or
$B - \sum_i y_i L_i$ with $\sum_i y_i = 3$.
[Ma/Pollard/Srivastava/Zakeri(2015)]
For example, let $x_i = (1, 2, -3)$, then $\mathcal{M}_D$ is diagonal, and $\mathcal{M}_N$ has the $(x_i x_j)$ structure

$$\begin{pmatrix} 2 & 3 & -2 \\ 3 & 4 & -1 \\ -2 & -1 & -6 \end{pmatrix}$$

If singlet scalars transforming as (2,3,4) are used, then the (23),(32),(3,3) entries are zero.

Note that $L_e - L_\mu$, $L_e - L_\tau$, $L_\mu - L_\tau$, as well as $B - L_e - L_\mu - L_\tau$, $B - 3L_e$, $B - 3L_\mu$, $B - 3L_\tau$ have all been studied in the past.

The U(1) gauge boson Z' corresponding to $\sum_i x_i L_i$ or $B - \sum_i y_i L_i$ has good discovery reach at the Large Hadron Collider and is being studied.

In fact, the Type I and III seesaw mechanisms would be easier to verify if N or $(\Sigma^+, \Sigma^0, \Sigma^-)$ couple to Z', i.e. $B - L$ or an unusual anomaly-free U(1). [B/B/B(1986)] [Ma(2002)]: One example out of a class of solutions: $e_R \sim -1$ and all others $\sim +1$.

Radiative Seesaw from Dark Matter

Deshpande/Ma(1978): Add to the SM a second scalar doublet (η^+, η^0) which is odd under a new exactly conserved Z_2 discrete symmetry, then η^0_R or η^0_I is absolutely stable. This simple idea lay dormant for almost thirty years until Ma, Phys. Rev. D 73, 077301 (2006). It was then studied seriously in Barbieri/Hall/Rychkov(2006), Lopez Honorez/Nezri/Oliver/Tytgat(2007), Gustafsson/Lundstrom/Bergstrom/Edsjo(2007), and Cao/Ma/Rajasekaran, Phys. Rev. D 76, 095011 (2007).

Radiative Neutrino Mass:

Zee(1980): (R1)

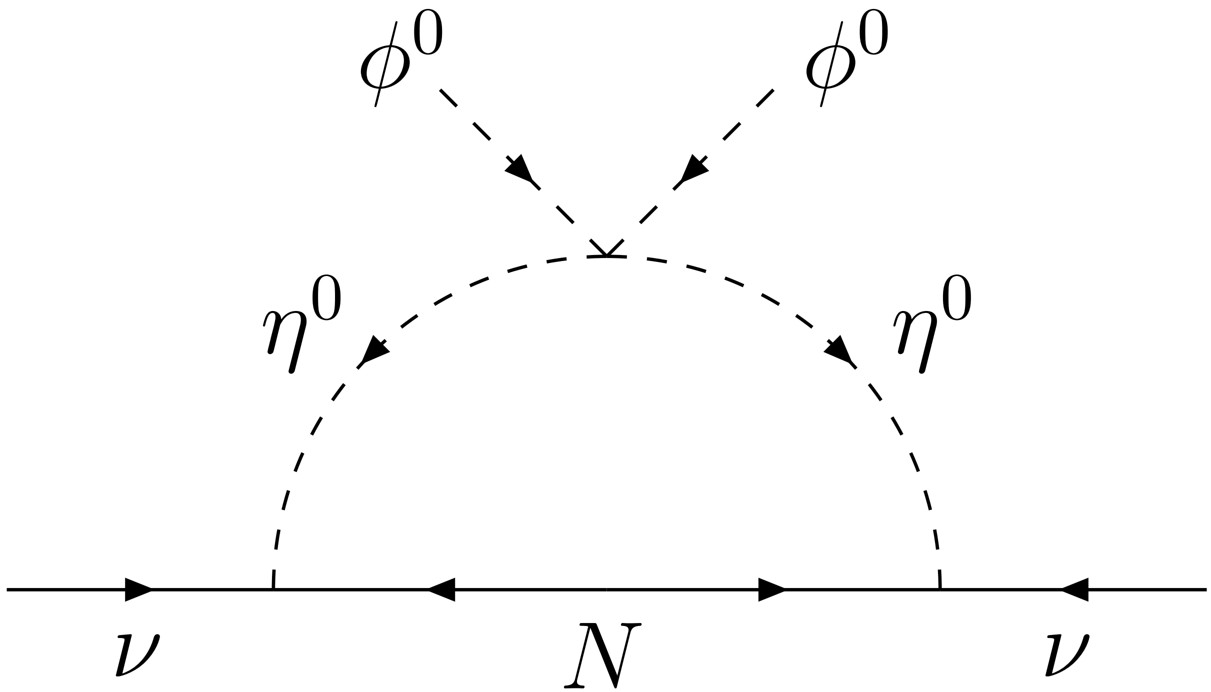
$\omega = (\nu, l), \omega^c = l^c$, $\chi = \chi^+, \eta = (\phi^+_{1,2}, \phi^0_{1,2}), \langle \phi^0_{1,2} \rangle \neq 0$.

Ma(2006): (R2) [scotogenic = caused by darkness]

$\omega = \omega^c = N$ or Σ, $\chi = \eta = (\eta^+, \eta^0), \langle \eta^0 \rangle = 0$.

N or Σ interacts with ν, but they are not Dirac mass partners, because of the exactly conserved Z_2 symmetry, under which N or Σ and (η^+, η^0) are odd, and all SM particles are even. Using $f(x) = -\ln x/(1-x)$,

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2} [f(M_i^2/m_R^2) - f(M_i^2/m_I^2)].$$


ϕ^0
ϕ^0
η^0
η^0
ν
N
ν

Note that $(1/2)\lambda_5(\Phi^\dagger\eta)^2 + H.c.$ splits the mass of $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$, so that $m_R^2 - m_I^2 = 2\lambda_5 v^2$, which makes m_ν finite.

It also solves the problem of the direct detection of $\eta_{R,I}$ through Z exchange with nuclei. Since Z_μ couples to $\eta_R\partial^\mu\eta_I - \eta_I\partial^\mu\eta_R$, a mass gap of just a few hundred keV is enough to forbid its elastic scattering in underground dark-matter search experiments using nucleus recoil.

However, the $h(\eta_R^2 + \eta_I^2)$ coupling remains. It will allow $\eta_{R,I}$ to be discovered in the next generation of detectors.

The linkage of neutrino mass to dark matter provides an important clue to the scale of new physics. It is a possible answer to the **Question**: Is the new physics responsible for neutrino mass also responsible for some other phenomenon in particle physics and astrophysics? Here the answer is yes, and it is dark matter. Since dark matter is mostly assumed to be a Weakly Interacting Massive Particle (WIMP), its mass scale is reasonably set at 1 TeV. This is the crucial missing piece of information which allows us to expect observable new physics related to both dark matter and neutrino mass at the LHC.