Dark Energy Theory

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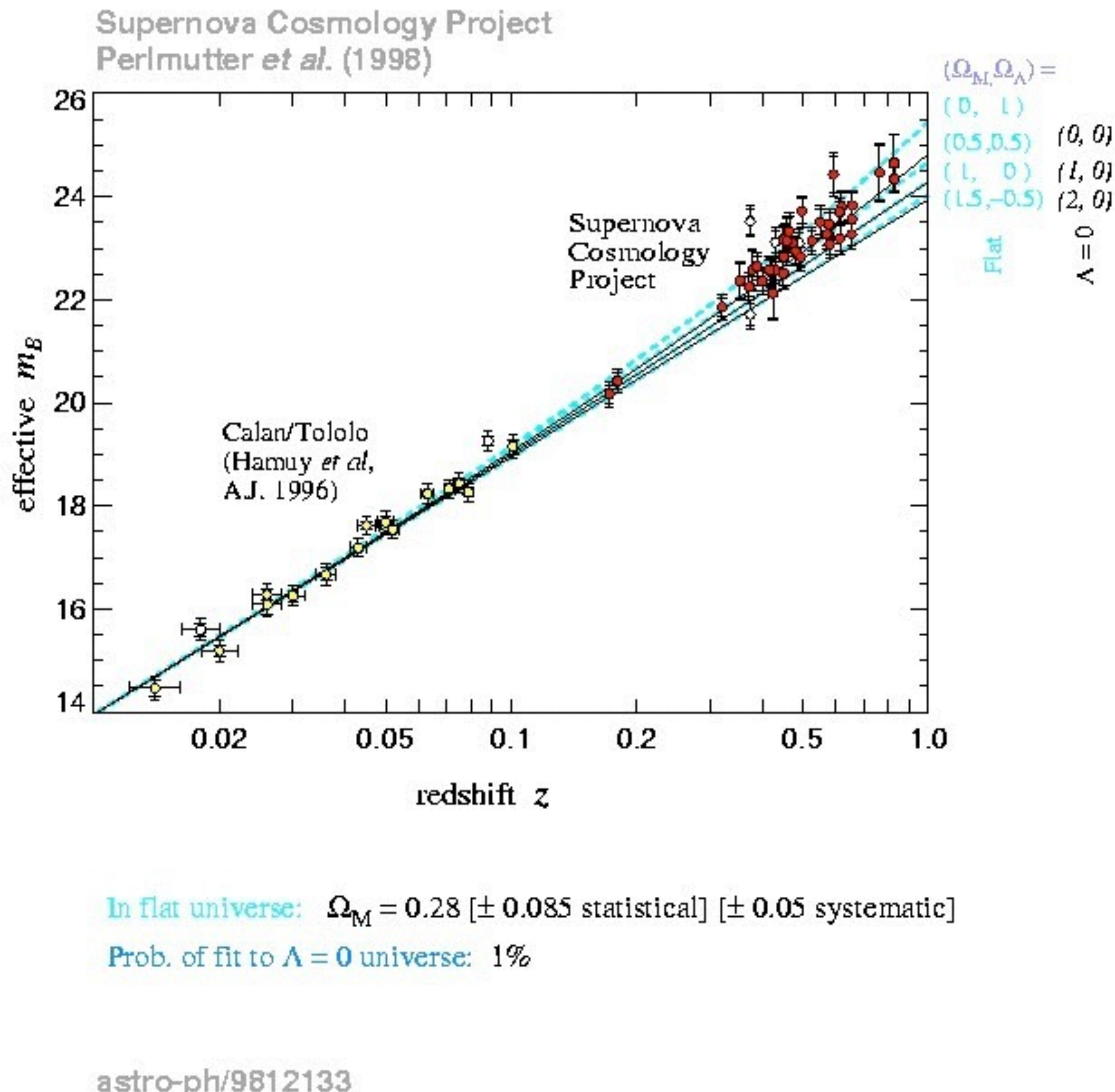
1. Issues with pure Lambda
2. Models of Dark Energy
3. Modified Gravity approaches
4. Testing for and parameterising Dark Energy

Corfu TR33 - Sept 17th 2015



The Universe is accelerating and yet we still really have little idea what is causing this acceleration.

Is it a cosmological constant, an evolving scalar field, evidence of modifications of General Relativity on large scales or something yet to be dreamt up ?

Brief reminder why the cosmological constant is regarded as a problem?

The CC gravitates in General Relativity:

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{16\pi G} - \rho_{\rm vac}\right)$$

$$G_{\mu\nu} = -8\pi G \rho_{\rm vac} g_{\mu\nu}$$

Now:

$$\rho_{\rm vac}^{\rm obs} \ll \rho_{\rm vac}^{\rm theory}$$

Just as well because anything much bigger than we have and the universe would have looked a lot different to what it does look like. In fact structures would not have formed in it.

Estimate what the vacuum energy should be :

$$\rho_{\text{vac}}^{\text{theory}} \sim \rho_{\text{vac}}^{\text{bare}}$$

$$+$$

zero point energies of each particle

$$+$$

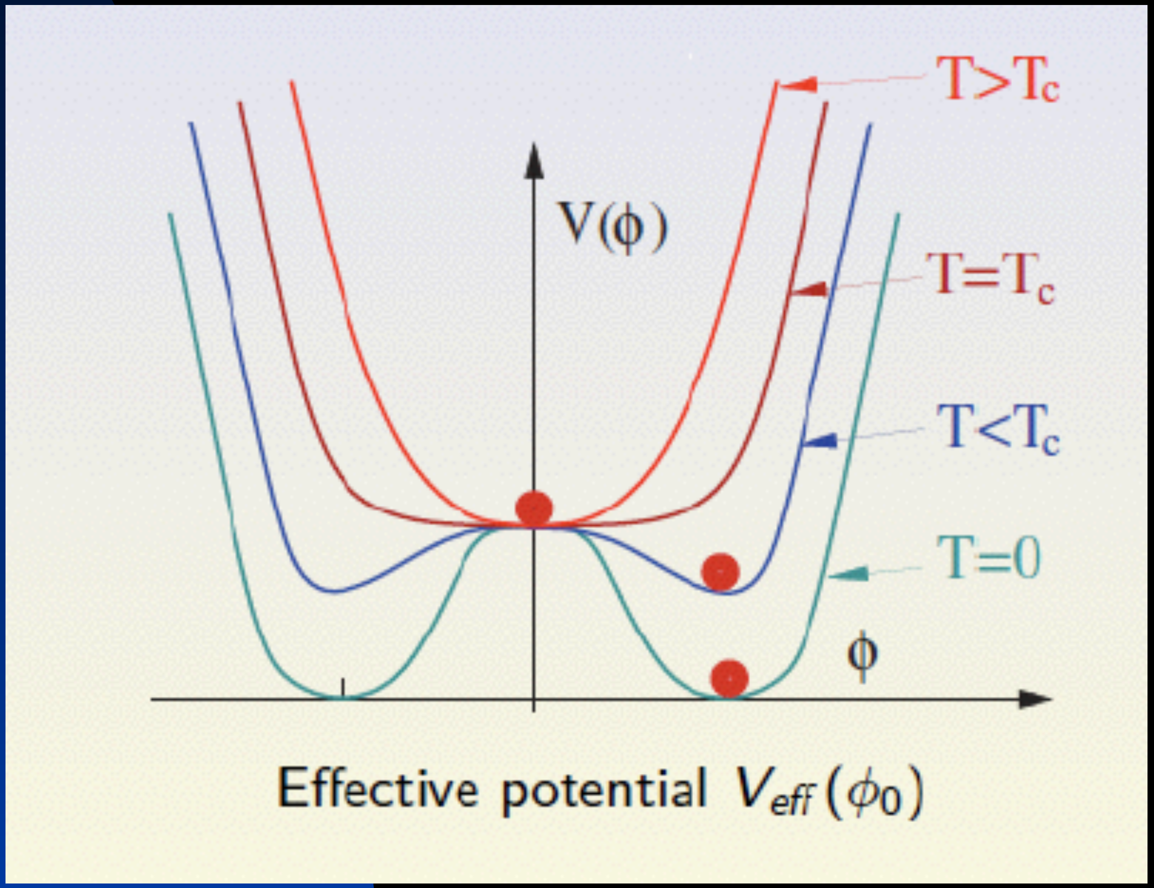
contributions from phase transitions in the early universe zero point energies of each particle

For many fields (i.e. leptons, quarks, gauge fields etc...):

$$< \rho > = \frac{1}{2} \sum_{\mathrm{fields}} g_i \int_0^{\Lambda_i} \sqrt{k^2 + m^2} \frac{d^3 k}{(2\pi)^3} \simeq \sum_{\mathrm{fields}} \frac{g_i \Lambda_i^4}{16\pi^2}$$

where g_i are the dof of the field (+ for bosons, - for fermions).

contributions from phase transitions in the early universe



$$\Delta V_{\rm ewk} \sim (200\ \mathrm{GeV})^4$$

$$\Delta V_{\rm QCD} \sim (0.3\ \mathrm{GeV})^4$$

Quantum Gravity cut-off — $(10^{18}\ \text{GeV})^4$ — fine tuning to 120 decimal places

SUSY cut-off — $(\text{TeV})^4$ — fine tuning to 60 decimal places

EWK phase transition — $(200\text{GeV})^4$ — fine tuning to 56 decimal places

QCD phase transition — $(0.3\text{GeV})^4$

Muon — $(100\text{MeV})^4$

fine tuning to 44 decimal places

electron — $(1\ \text{MeV})^4$ — fine tuning to 36 decimal places

$(\text{meV})^4$ — Observed value of the effective cosmological constant today !

Friedmann:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a(t) depends on matter.

Energy density $\rho(t)$: Pressure $p(t)$

Related through : $p = w\rho$

w=1/3 – Rad dom: w=0 – Mat dom: w=-1– Vac dom

$$w(a) \equiv \frac{P}{\rho} = w_0 + (1-a)w_a$$

Typical parameterisation

$$\mathbf{H^2(z) = H_0^2 \left(\Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_{de} \exp \left(3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right) \right)}$$

Dark Energy

Parameterise eos: $$w(a) \equiv \frac{p}{\rho} = w_0 + (1-a)w_a$$

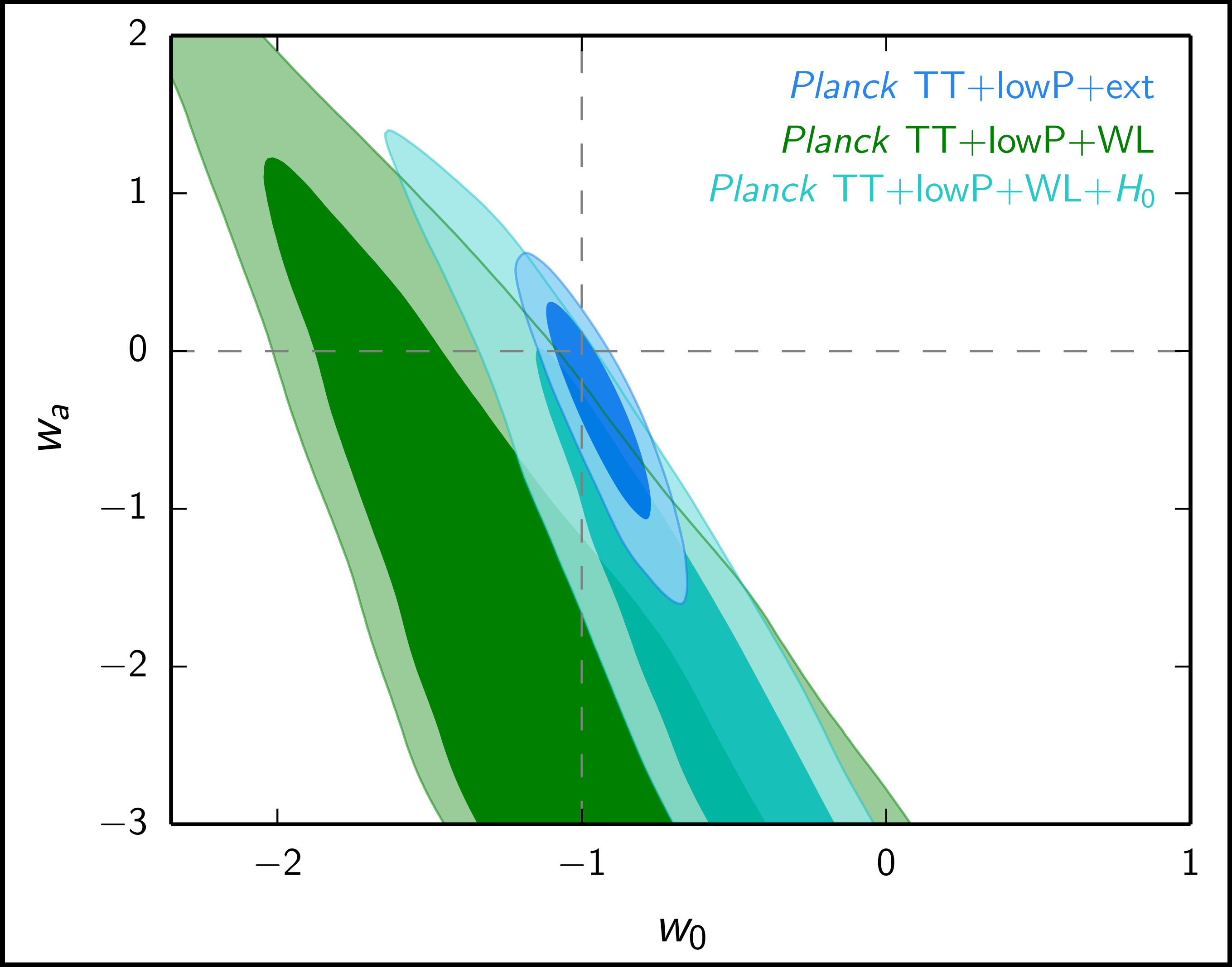
Planck alone weak constraints on DE because of degeneracy of w with H_0:

Break with other probes including lensing, SN, BAO ...

Example - if assume $w_a = 0$, 95% CL

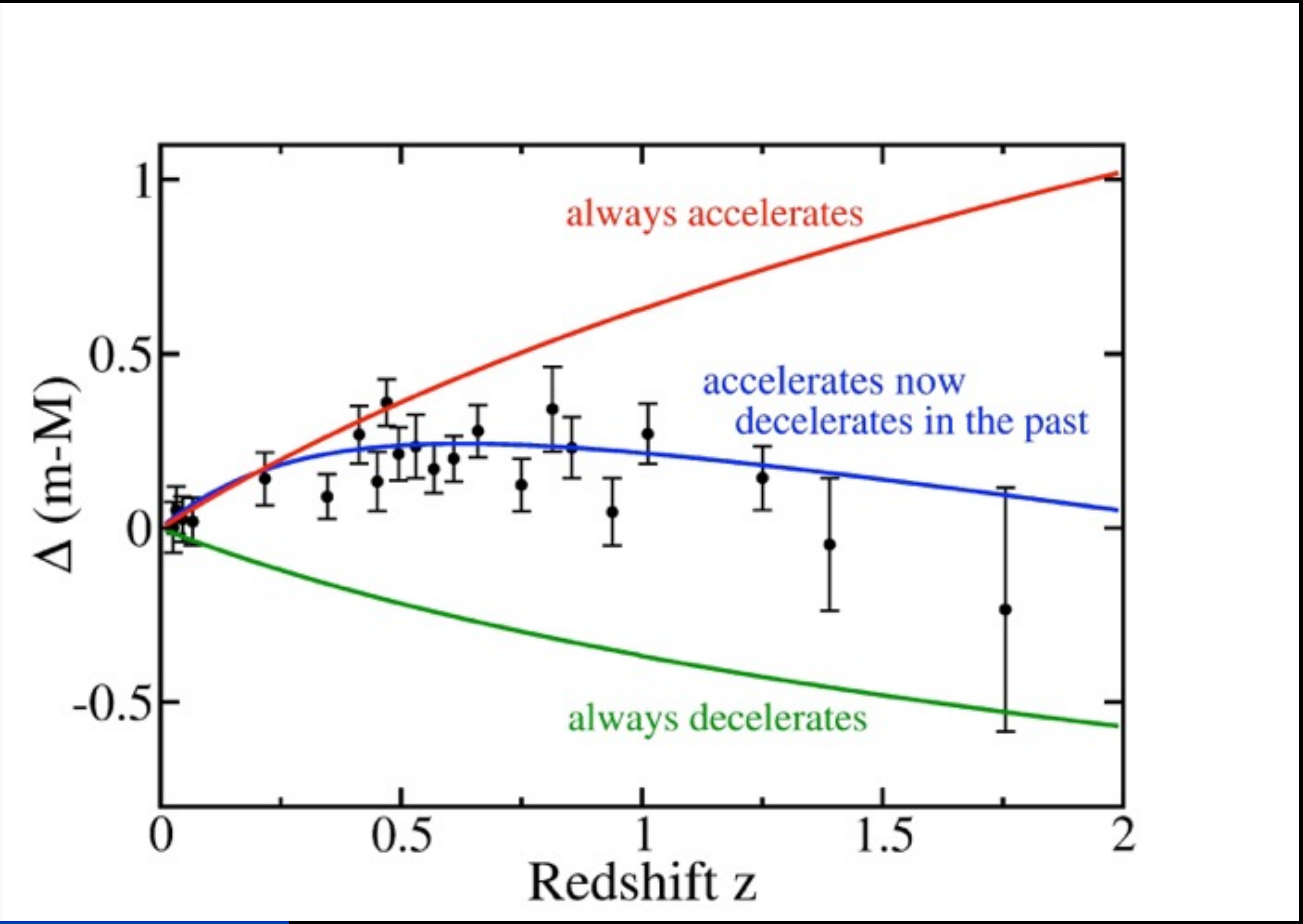
$$w = -1.023^{+0.091}_{-0.096} \quad \textit{Planck}\ \text{TT+lowP+ext}\,; \quad ($$
$$w = -1.006^{+0.085}_{-0.091} \quad \textit{Planck}\ \text{TT+lowP+lensing+ext}\,; \quad ($$
$$w = -1.019^{+0.075}_{-0.080} \quad \textit{Planck}\ \text{TT, TE, EE+lowP+lensing+ext}$$

Planck 2015:



How should we parameterise w?

The acceleration has not been forever -- pinning down the turnover will provide a very useful piece of information.



Huterer 2010

Help address cosmic coincidence problem ! A region hopefully DES and EUCLID will be able to probe

Approaches to Dark Energy:

- A true cosmological constant -- but why this value?
- Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.
- Modifications of Einstein gravity leading to acceleration today.
- Anthropic arguments.
- Perhaps GR but Universe is inhomogeneous.
- Hiding the cosmological constant -- its there all the time but just doesn't gravitate
- Yet to be proposed ...

String - theory -- where are the realistic models?

\`No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

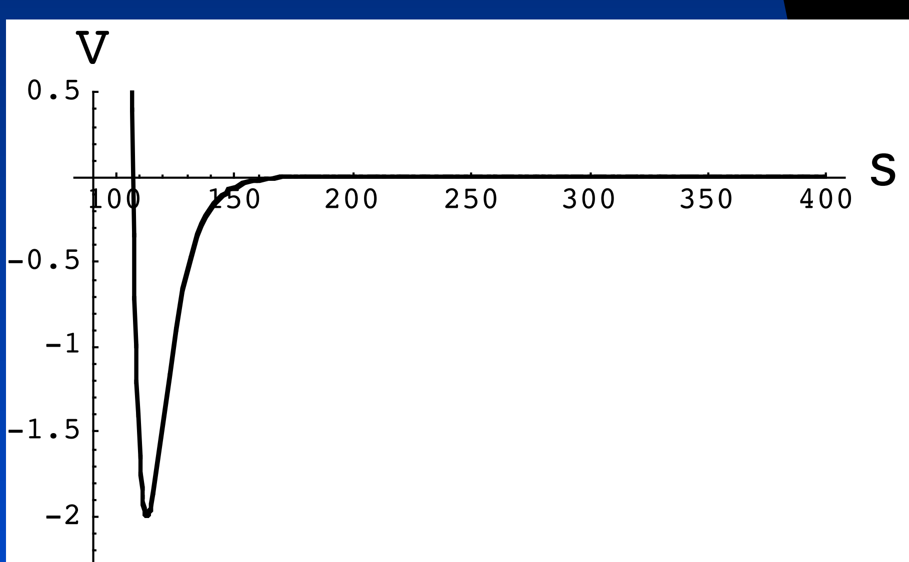
Avoid no-go theorem by relaxing conditions of the theorem.

1. Allow internal space to be time-dependent scalar fields (radion)

2. Brane world set up require uplifting terms to achieve de Sitter vacua hence accn

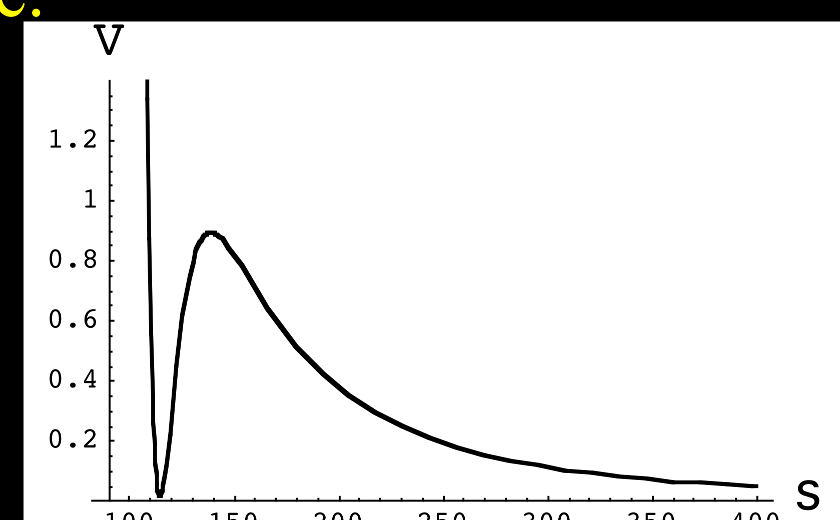
Example of stabilised scenario: Metastable de Sitter string vacua in TypeIIB string theory, based on stable highly warped IIB compactifications with NS and RR three-form fluxes. [Kachru, Kallosh, Linde and Trivedi 2003]

Metastable minima arises from adding positive energy of anti-D3 brane in warped Calabi-Yau space.



AdS minimum

$$V_{\mathrm{KKLT}} = V_{\mathrm{AdS}} + \frac{D}{\sigma^2}$$



Metastable dS minimum

The String Landscape approach

Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes. Assumes natural AdS vacuum uplifted to de Sitter vacuum through additional fluxes !

Many many vacua ~ 10^{500} ! Typical separation ~ $10^{-500}\ \Lambda_{pl}$

Assume randomly distributed, tunnelling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua < $10^{-118}\ \Lambda_{pl}$ [Weinberg] Most likely to find values not equal to zero!

Landscape gives a realisation of the multiverse picture.

There isn't one true vacuum but many so that makes it almost impossible to find our vacuum in such a Universe which is really a multiverse.

So how can we hope to understand or predict why we have our particular particle content and couplings when there are so many choices in different parts of the universe, none of them special ?

SUSY large extra dimensions and Lambda - Burgess et al 2013, 2015

Soln to 6D Einstein-Maxwell-scalar with chiral gauged sugr.

In more than 4D, the 4D vac energy can curve the extra dimensions.

Proposal: Physics is 6D above 0.01eV scale with SUSY bulk. We live in 4D brane with 2 extra dim. 4D vac energy cancelled by Bulk contributions - quintessence like potential generated by Qu corrections leading to late time accn.

Sequestering Lambda - Kaloper and Padilla 2013,14,15

IR soln to the problem - initial version adds a global term to Einstein action

Introduce global dynamical variables Λ, λ

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \Lambda - \lambda^4 \mathcal{L}(\lambda^{-2}g^{\mu\nu}, \Psi)\right] + \sigma\left(\frac{\Lambda}{\lambda^4\mu^4}\right)$$

λ sets the hierarchy between matter scales and M_{pl}

$$\frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}}$$

Padilla 2015

Eq of motion:

$$M_{pl}^2 G^\mu{}_\nu = \tau^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu \langle \tau^\alpha{}_\alpha \rangle$$

$$T^\mu_\nu = -V_{vac}\delta^\mu_\nu + \tau^\mu_\nu$$

where: $\Lambda = \frac{1}{4}\langle T^\alpha{}_\alpha \rangle, \qquad \langle Q \rangle = \frac{\int d^4x Q\sqrt{g}}{\int d^4x \sqrt{g}}$ spacetime volume must be finite

Vacuum energy drops out at each and every loop order

Universe has finite spacetime volume

Ends in a crunch
w=-1 is transient
$\Omega_k>0$

collapse triggered by dominating dark energy

Padilla 2015

Linear potential $V=m^3\varphi$

form protected by shift symmetry, size of m^3 technically natural

Self tuning - with the Fab Four

with Charmousis, Padilla and Saffin

PRL 108 (2012) 051101; PRD 85 (2012) 104040

In GR the vacuum energy gravitates, and the theoretical estimate suggests that it gravitates too much.

Basic idea is to use self tuning to prevent the vacuum energy gravitating at all.

The cosmological constant is there all the time but is being dealt with by the evolving scalar field.

Most general scalar-tensor theory with second order field equations:

[G.W. Horndeski, Int. Jour. Theor. Phys. 10 (1974) 363-384]

The action which leads to required self tuning solutions :

$$
\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g}V_{john}(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\
\mathcal{L}_{paul} &= \sqrt{-g}V_{paul}(\phi)P^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi \\
\mathcal{L}_{george} &= \sqrt{-g}V_{george}(\phi)R \\
\mathcal{L}_{ringo} &= \sqrt{-g}V_{ringo}(\phi)\hat{G}
\end{aligned}
$$

In other words it can be seen to reside in terms of the four arbitrary potential functions of ϕ coupled to the curvature terms.

Covers most scalar field related modified gravity models studied to date.

Possible to have a self tuning `classical' solution in which the system adjusts itself to the Minkowski vacuum irrespective of the magnitude of the cosmological constant and whether it changes. It relies on breaking the assumption of Poincare invariance demanded by Weinberg in his original no-go theorem. In particular we have to have the scalar field evolving in time.

In general system is complicated to solve.

Try dynamical systems approach to find scaling solutions.

$$N = \ln(a); \quad x = H^{\alpha}\phi'; \quad y_n = H^{\beta_n} V_n; \quad \sigma = \frac{\sqrt{-k}}{Ha}$$

$$\lambda_n = H^{\gamma_n}\frac{V_n'}{V_n}; \quad h = \ln(H)' \quad \mu_n = \frac{V_n V_n''}{(V_n')^2}$$

For $\mu = \text{const} \rightarrow V \sim \phi^{\frac{1}{1-\mu}},\ e^{A\phi}$

$$x' = \ldots,\ y' = \ldots, \lambda' = \ldots, \mu' = \ldots, \sigma' = \ldots, h' = \ldots$$

fab four cosmology

TABLE I: Examples of interesting cosmological behaviour for various fixed points with $\sigma = 0$.

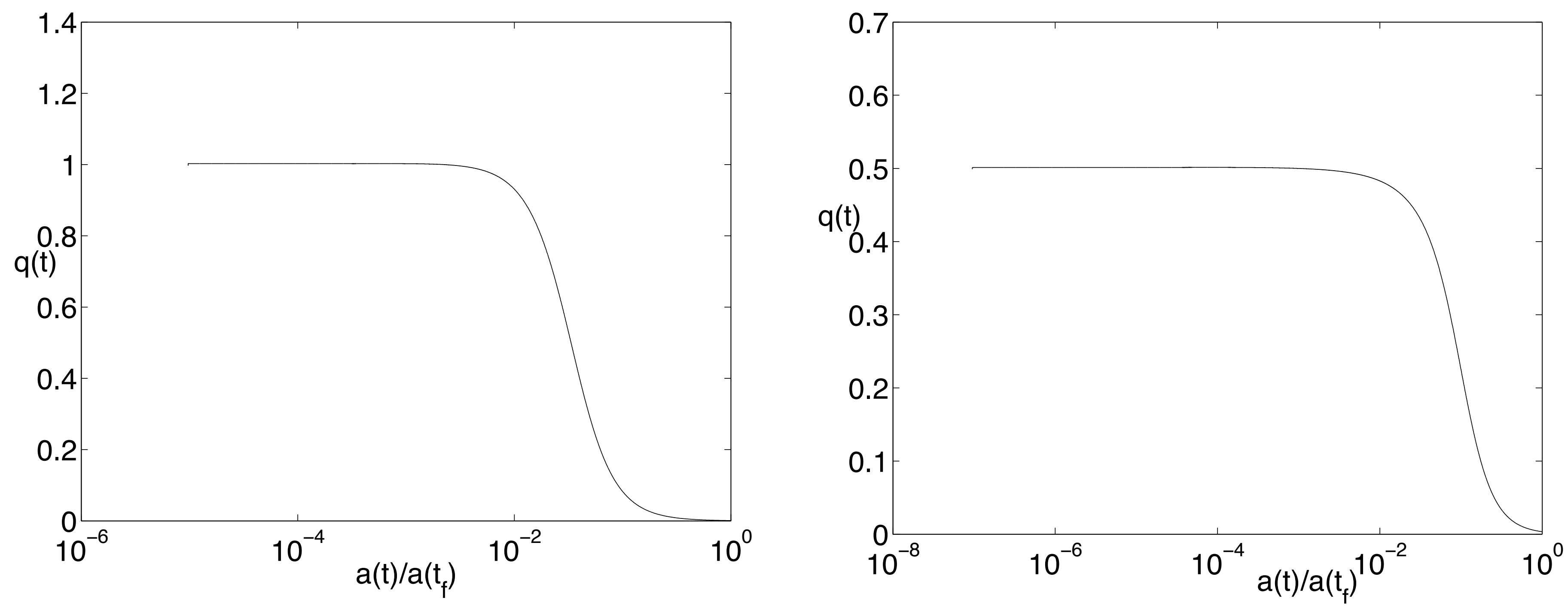
Case	cosmological behaviour	$V_j(\phi)$	$V_p(\phi)$	$V_g(\phi)$	$V_r(\phi)$
Stiff fluid	$H^2 \propto 1/a^6$	$c_1\phi^{\frac{4}{\alpha}-2}$	$c_2\phi^{\frac{6}{\alpha}-3}$	0	0
Radiation	$H^2 \propto 1/a^4$	$c_1\phi^{\frac{4}{\alpha}-2}$	0	$c_2\phi^{\frac{2}{\alpha}}$	$-\frac{\alpha^2}{8}c_1\phi^{\frac{4}{\alpha}}$
Curvature	$H^2 \propto 1/a^2$	0	0	0	$c_1\phi^{\frac{4}{\alpha}}$
Arbitrary	$H^2 \propto a^{2h}, \quad h \neq 0$	$c_1(1+h)\phi^{\frac{4}{\alpha}-2}$	0	0	$-\frac{\alpha^2}{16}h(3+h)c_1\phi^{\frac{4}{\alpha}}$

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

$$a \sim t^p \sim t^{-1/h}$$

$$q = -\frac{p(p-1)}{p^2} = -(1+h)$$

"radiation" "matter"



See also:

Appleby et al JCAP 1210 (2012) 060; Amendola et al PRD 87 (2013) 2, 023501; Martin-Moruno et al PRD 91 (2015) 8, 084029; Babichev et al arXiv:1507.05942 [gr-qc] ,

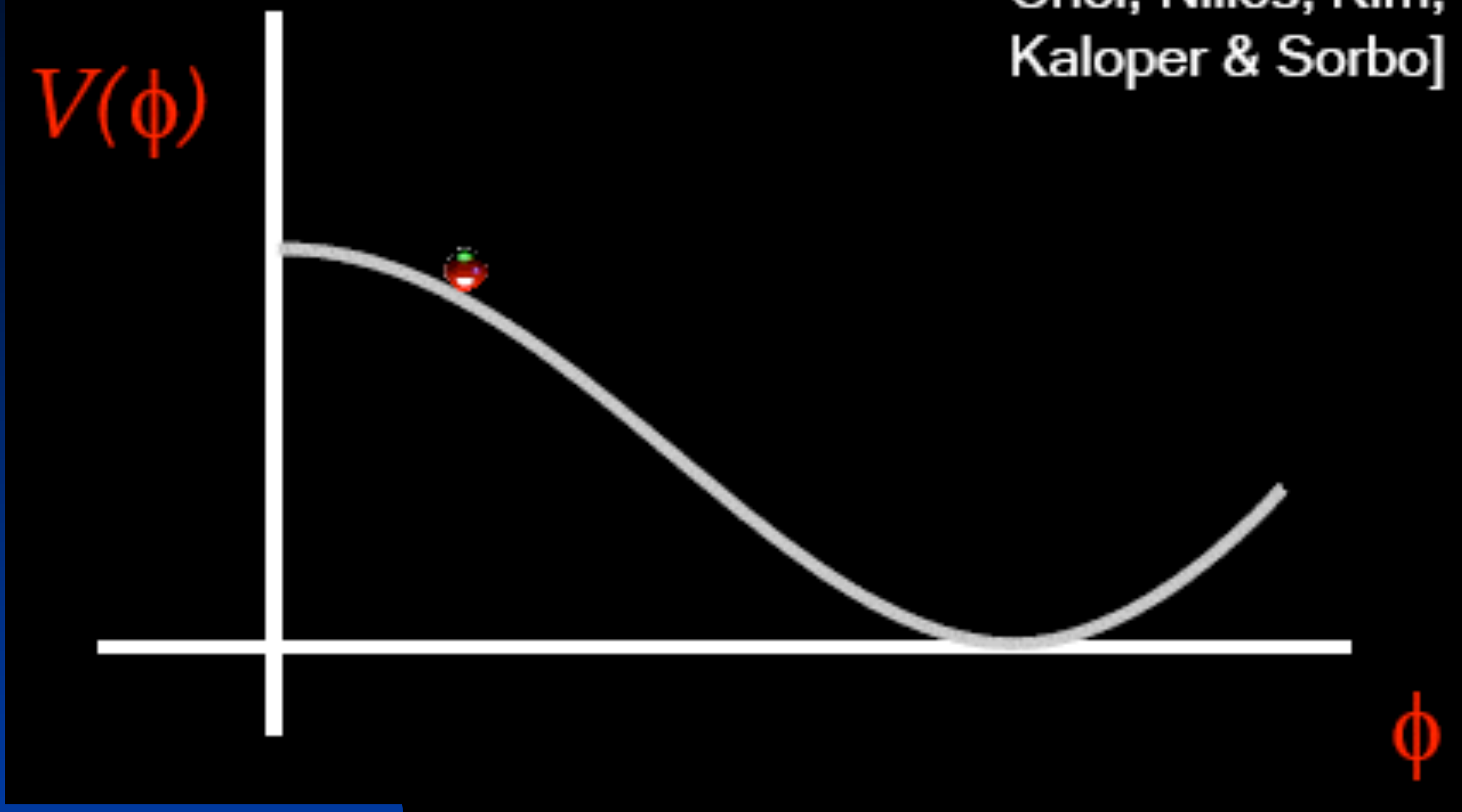
Particle physics inspired models?

Pseudo-Goldstone Bosons -- approx sym $\phi \to \phi + \text{const.}$

Leads to naturally small masses, naturally small couplings

[Hill, Freiman, et al; Choi; Nilles; Kim; Kaloper & Sorbo]

Barbieri et al



$$V(\phi) = \lambda^4 (1 + \cos(\phi / F_a))$$

Axions could be useful for strong CP problem, dark matter and dark energy.

Axions could be useful for strong CP problem, dark matter and dark energy.

Strong CP problem intro axion : $$m_a = \frac{\Lambda^2_{\rm QCD}}{F_a};\ F_a\ -\ \text{decay constant}$$

PQ axion ruled out but invisible axion still allowed: $$10^9\ \text{GeV} \le F_a \le 10^{12}\ \text{GeV}$$

Sun stability — CDM constraint

String theory has lots of antisymmetric tensor fields in 10d, hence many light axion candidates.

Can have $F_a \sim 10^{17}$-10^{18} GeV

Quintessential axion -- dark energy candidate [Kim & Nilles].

Requires $F_a \sim 10^{18}$ GeV which can give:

$$E_{\rm vac} = (10^{-3}\ \text{eV})^4 \to m_{\rm axion} \sim 10^{-33}\ \text{eV}$$

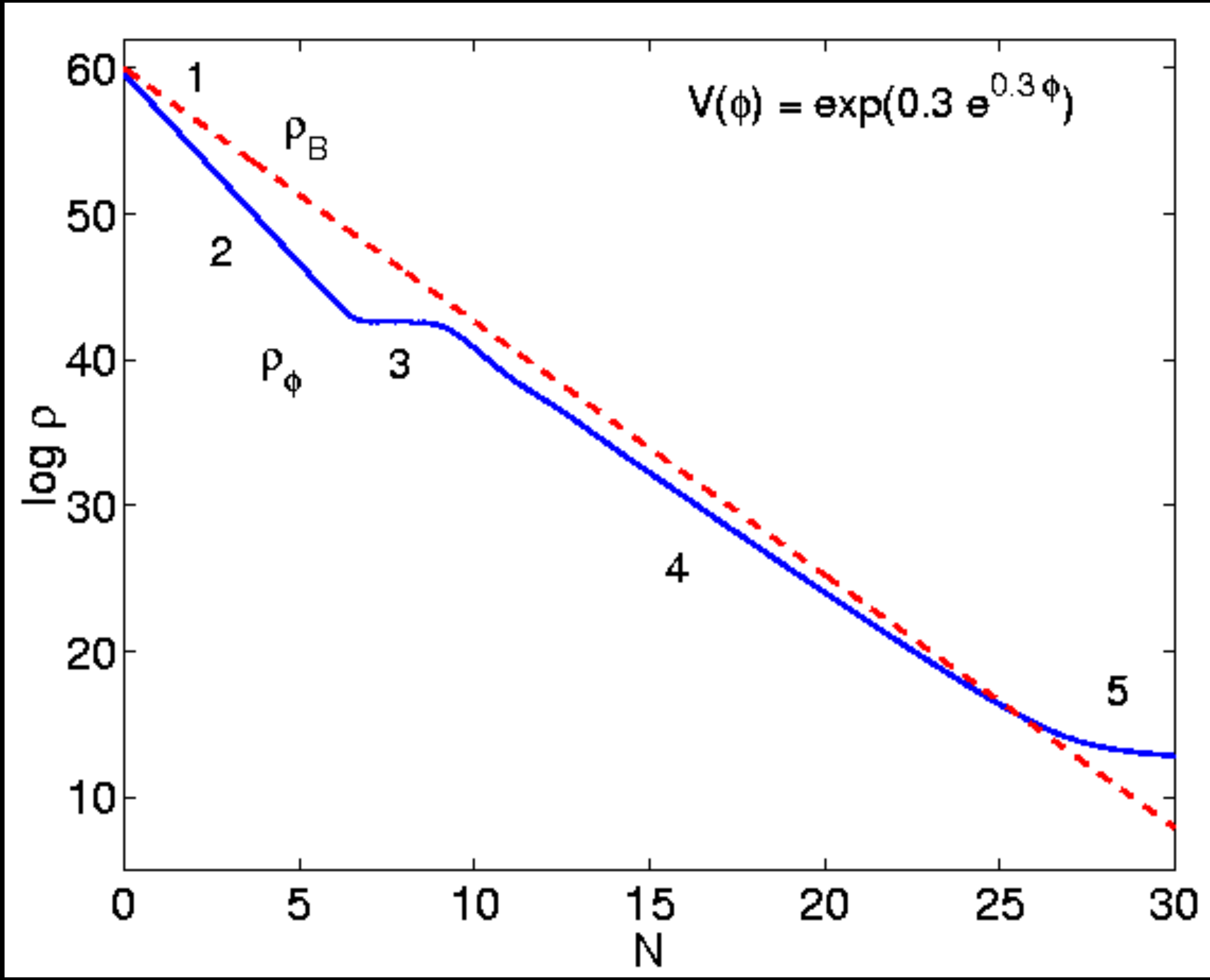
Because axion is pseudoscalar -- mass is protected, hence avoids fifth force constraints

Slowly rolling scalar fields -- Quintessence

As of 14 Mar 2013, can really use this language !

Peebles and Ratra; Wetterich; Ferreira and Joyce

Zlatev, Wang and Steinhardt



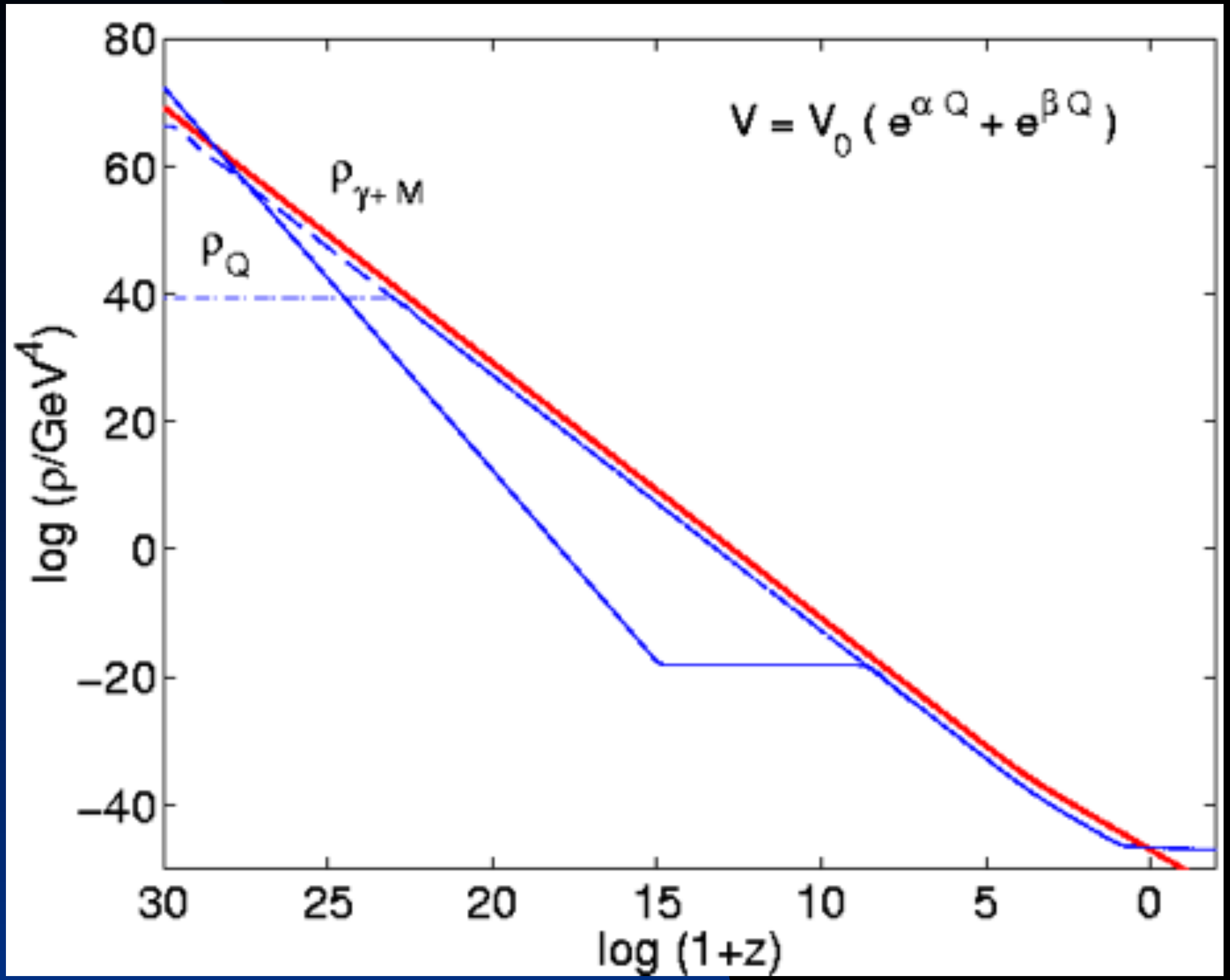
Dashed line - radiation and matter

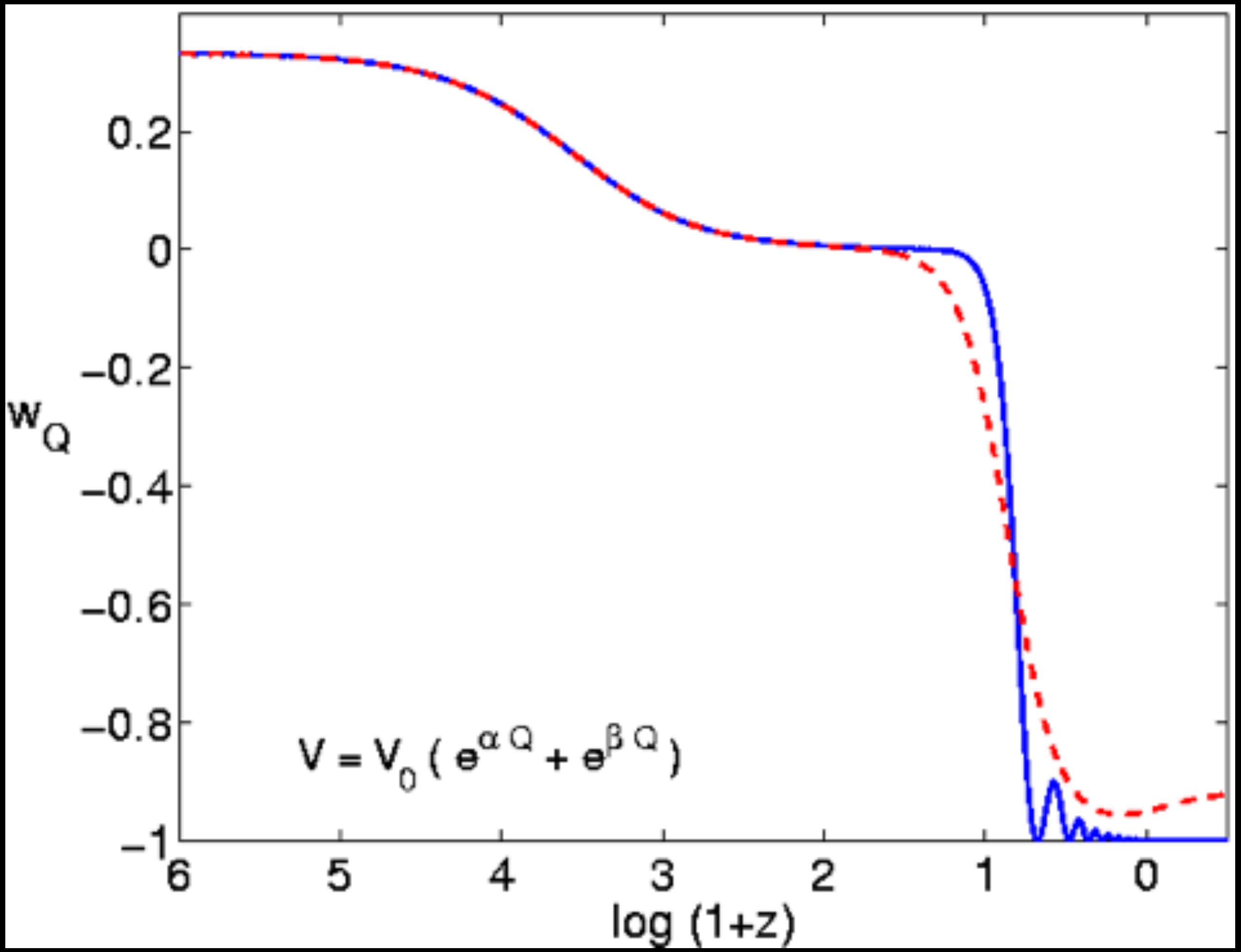
Solid line - Quintessence enters tracking regime (4) and dominates (5)

Nunes

Attractors make initial conditions less important

$$V(\phi) = V_1 + V_2 = V_{01} e^{-\kappa\lambda_1\phi} + V_{02} e^{-\kappa\lambda_2\phi}$$





$\alpha = 20; \beta = 0.5$

Scaling for wide range of i.c.

Fine tuning: $V_0 \approx \rho_\phi \approx 10^{-47}\ \mathrm{GeV}^4 \approx (10^{-3}\ \mathrm{eV})^4$

Mass: $m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \mathrm{eV}$

Generic issue Fifth force - require screening mechanism!

1. Chameleon fields [Khoury and Weltman (2003) ...]

Non-minimal coupling of scalar to matter in order to avoid fifth force type constraints on Quintessence models: the effective mass of the field depends on the local matter density, so it is massive in high density regions and light (m~H) in low density regions (cosmological scales).

2. K-essence [Armendariz-Picon et al ...]

Scalar fields with non-canonical kinetic terms. Includes models with derivative self-couplings which become important in vicinity of massive sources. The strong coupling boosts the kinetic terms so after canonical normalisation the coupling of fluctuations to matter is weakened -- screening via Vainshtein mechanism

Similar fine tuning to Quintessence -- vital in brane-world modifications of gravity, massive gravity, degravitation models, DBI model, Gallileons, ...

3. Symmetron fields [Hinterbichler and Khoury 2010 ...]

vev of scalar field depends on local mass density: vev large in low density regions and small in high density regions. Also coupling of scalar to matter is prop to vev, so couples with grav strength in low density regions but decoupled and screened in high density regions.

4. Interacting Dark Energy

[Kodama & Sasaki (1985), Wetterich (1995), Amendola (2000) + many others…]

Ex: Including neutrinos -- 2 distinct DM families -- resolve coincidence problem

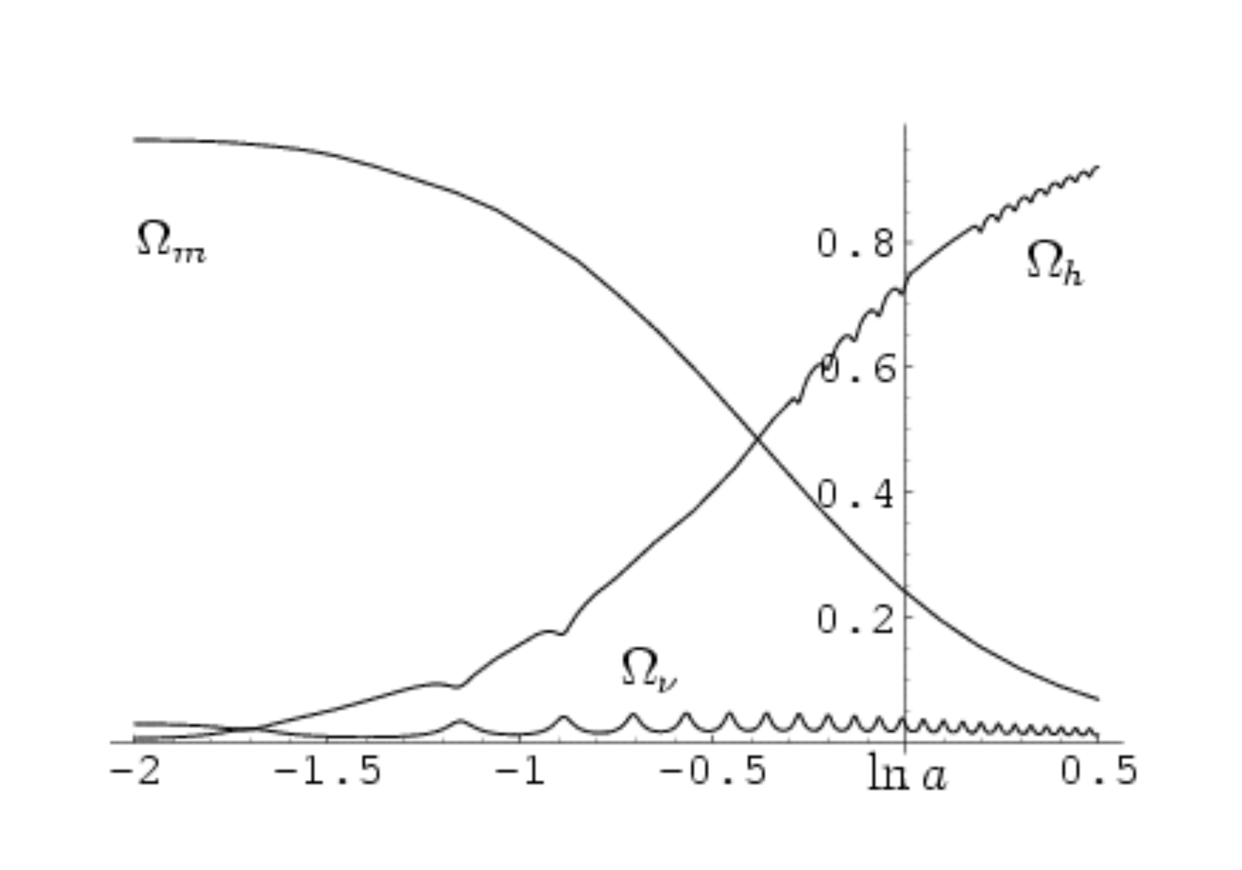
Amendola et al (2007)

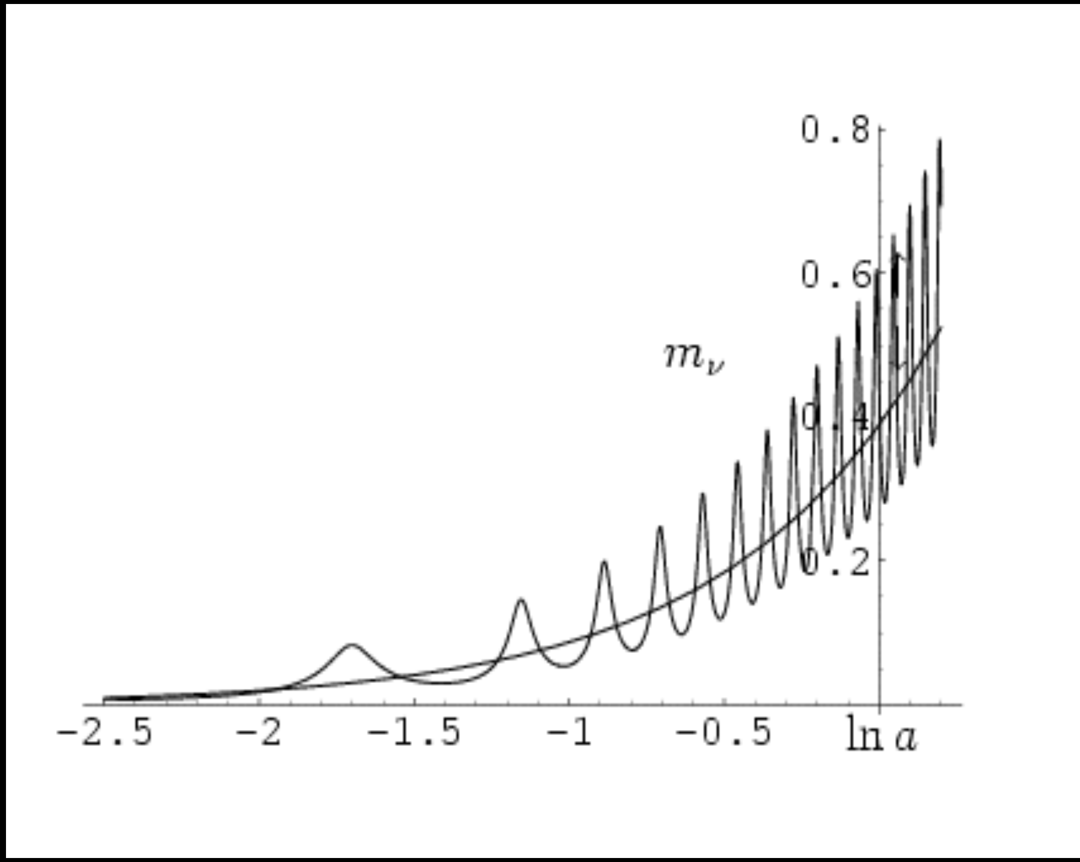
Depending on the coupling, find that the neutrino mass grows at late times and this triggers a transition to almost static dark energy.

Trigger scale set by time when neutrinos become non-rel

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\mathrm{eV}}$$





m_ν

Perturbations in Interacting Dark Energy Models [Baldi et al (2008), Tarrant et al (2010)]

Perturb everything linearly : Matter fluid example

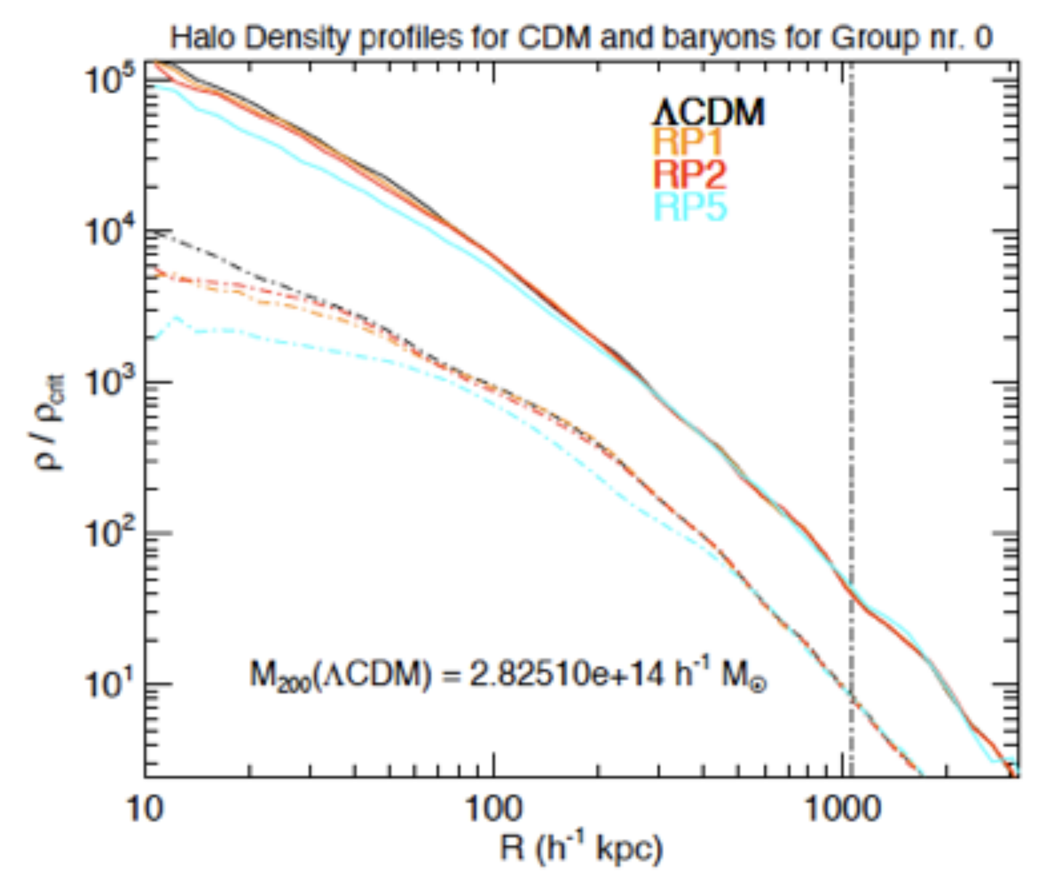
$$\ddot{\delta}_c + \left(2H - 2\beta\frac{\dot{\phi}}{M}\right)\dot{\delta}_c - \frac{3}{2}H^2[(1+2\beta^2)\Omega_c\delta_c + \Omega_b\delta_b] = 0$$

extra friction

modified grav interaction

vary DM particle mass

Include in simulations of structure formation : GADGET [Springel (2005)]



Density decreases as coupling β increases

Halo mass function modified.

Halos remain well fit by NFW profile.

Density decreases compared to ΛCDM as coupling β increases.

Scale dep bias develops from fifth force acting between CDM particles. enhanced as go from linear to smaller non-linear scales.

Still early days -- but this is where I think there should be a great deal of development (Puchwein et al 2013, Barreira et al 2014)

Dark Energy Effects

Interactions with standard model particles inevitable even if indirect.

Light scalar fields that interact with std model fields mediate fifth forces

but we dont see any long range fifth forces on earth or in the solar system.

Screening !

Dark energy changes the way photons propagate through B fields. The polarised photon can fluctuate into a DE scalar particle leading to a modification of apparent polarisation and luminosity of the sources.

Two tests [Burrage, Davis, Shaw 2008,2009]

Look for evidence of DE through changes in the scatter of luminosities of high energy sources.

Look for evidence of correlation between poln and freq of starlight .

Dark Energy Direct Detection Experiment [Burrage, EC, Hinds 2015,Hamilton et al 2015]

Atom Interferometry

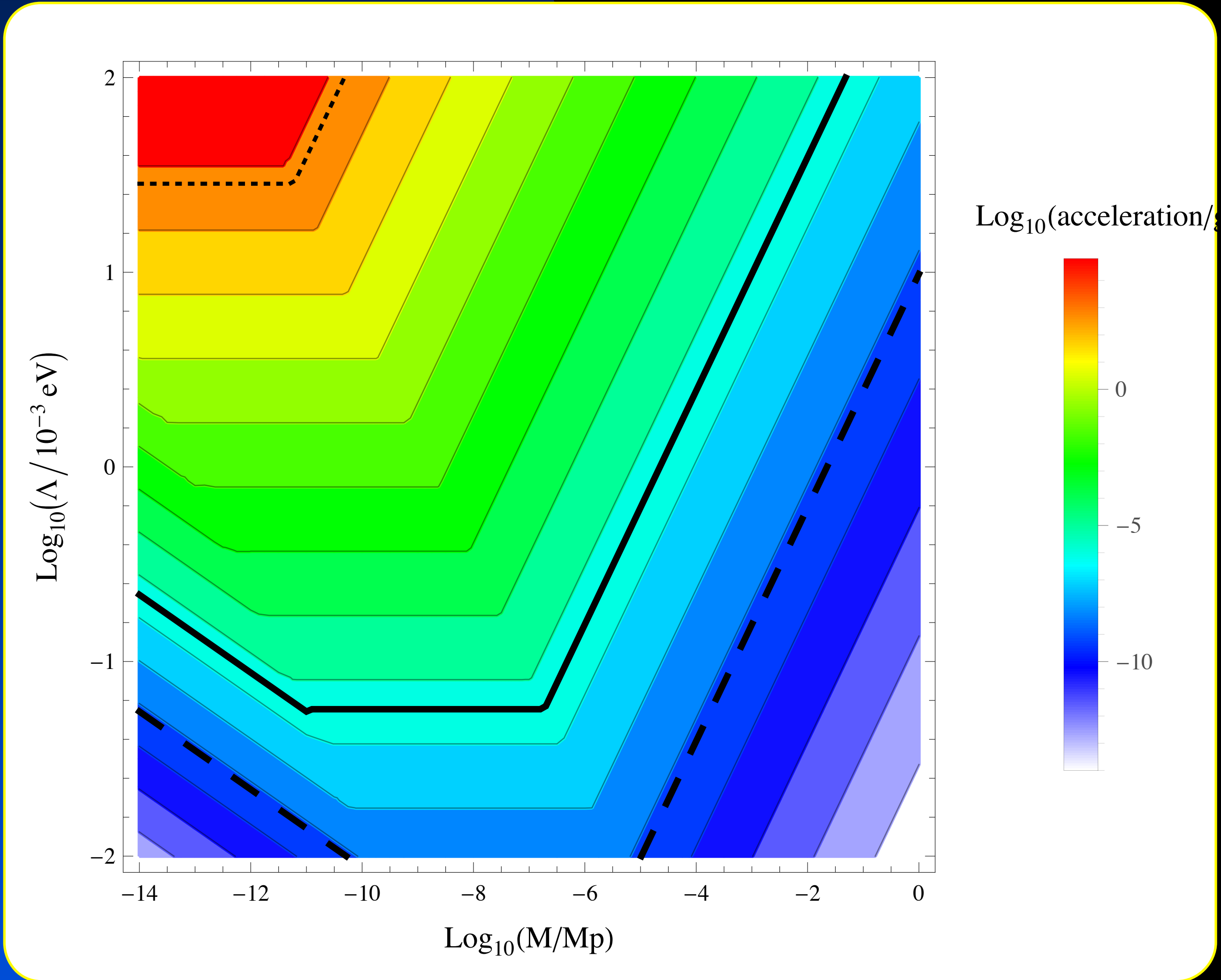
Idea: Individual atoms in a high vacuum chamber are too small to screen the chameleon field and so are very sensitive to it - can detect it with high sensitivity. Can use atom interferometry to measure the chameleon force - or more likely constrain the parameters !

$$\nabla^2\phi = -\frac{\Lambda^2}{\phi^2} + \frac{\rho}{M}$$

$$F_r = \frac{GM_AM_B}{r^2}\left[1 + 2\lambda_A\lambda_B\left(\frac{M_P}{M}\right)^2\right]$$

$$\lambda_i = 1 \text{ for } \rho_i R_i^2 < 3M\phi_{bg}$$

$$\lambda_i = \frac{3M\phi_{bg}}{\rho_i R_i^2} \text{ for } \rho_i R_i^2 > 3M\phi_{bg}$$



Sph source A and test object B near middle of chamber experience force between them - usually ƛ<<1 in cosmology but for atom ƛ=1 - reduced suppression

Modifying Gravity rather than looking for Dark Energy - non trivial

Any theory deviating from GR must do so at late times yet remain consistent with Solar System tests. Potential examples include:

•f(R), f(G) gravity -- coupled to higher curv terms, changes the dynamical eqns for the spacetime metric. Need chameleon mechanism [Starobinski 1980, Carroll et al 2003, ...]

- Modified source gravity -- gravity depends on nonlinear function of the energy.
- Gravity based on the existence of extra dimensions -- DGP gravity

We live on a brane in an infinite extra dimension. Gravity is stronger in the bulk, and therefore wants to stick close to the brane -- looks locally four-dimensional.

Tightly constrained -- both from theory [ghosts] and observations

- Scalar-tensor theories including higher order scalar-tensor lagrangians -- recent examples being Galileon models
- Massive gravity - single massive graviton bounds m>O(1meV) from demand perturbative down to O(1)mm - too large to conform with GR at large distances

[Burrage et al 2013]

Designer f (R) or f(G) models [Hu and Sawicki (2007), ...]

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{2\kappa^2} + \mathcal{L}_m \right]$$

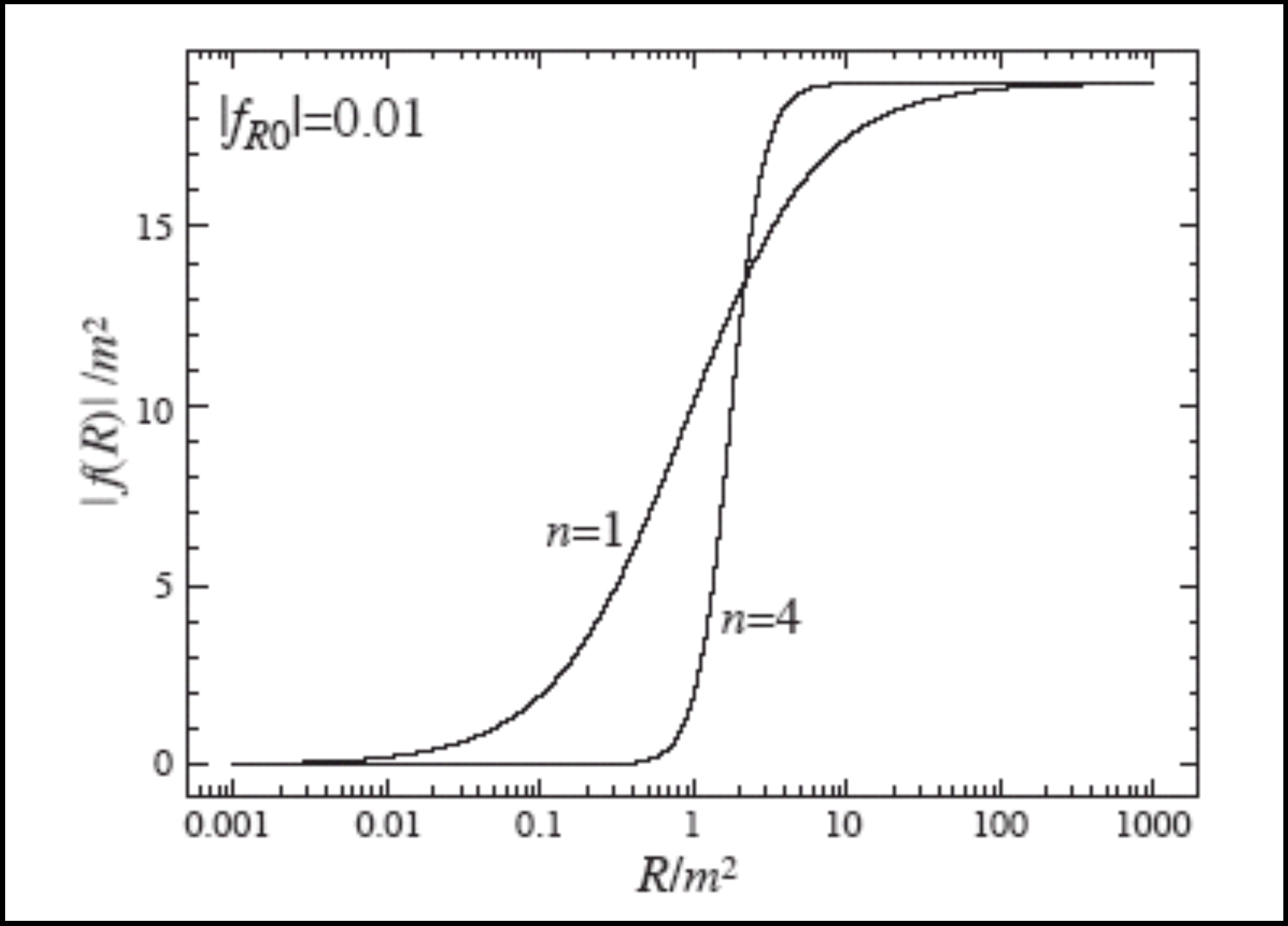
Construct a model to satisfy observational requirements:

1. Mimic LCDM at high z as suggested by CMB
2. Accelerate univ at low z
3. Include enough dof to allow for variety of low z phenomena
4. Include phenom of LCDM as limiting case.

$$\lim_{R\to\infty} f(R) = \text{const.}\,, \qquad \lim_{R\to 0} f(R) = 0\,,$$

$$f(R) = -m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1}\,,$$

$$f_{RR} \equiv \frac{d^2 f(R)}{dR^2} > 0$$



Effective chameleon mechanism

What should we do to help determine the nature of DE ?

1. We need to define properly theoretically predicted observables, or determine optimum ways to parameterise consistency tests (i.e. how should we parameterise w(z)?)

2. Need to start including dynamical dark energy, interacting dark matter-dark energy and modified gravity models in large scale simulations - [Wyman et al 2013, Li et al 2013 Puchwein et al 2013, Jennings et al 2012, Barreira et al 2012, Brax et al 2013].

3. Include the gastrophysics + star formation especially when considering baryonic effects in the non-linear regimes - `mud wrestling'.

4. On the theoretical side, develop models that go beyond illustrative toy models. Extend Quintessential Axion models. Are there examples of actual Landscape predictions? De Sitter vaccua in string theory is non trivial.

5. Recently massive gravity and galileon models have been developed which have been shown to be free of ghosts. What are their self-acceleration and consistency properties?

6. Will we be able to reconstruct the underlying Quintessence potential from observation?

7. Will we ever be able to determine whether $w \neq -1$?

8. Look for alternatives, perhaps we can shield the CC from affecting the dynamics through self tuning-- The Fab Four

9. Given the complexity (baroque nature ?) of some of the models compared to that of say Λ, should we be using Bayesian model selection criterion to help determine the relevance of any one model.

Things are getting very exciting with DES beginning to take data and future Euclid missions, LSST, as well as proposed giant telescopes, GMT, ELT, SKA - travelling in new directions !

What's the best way to parameterise the DE eqn of state ?

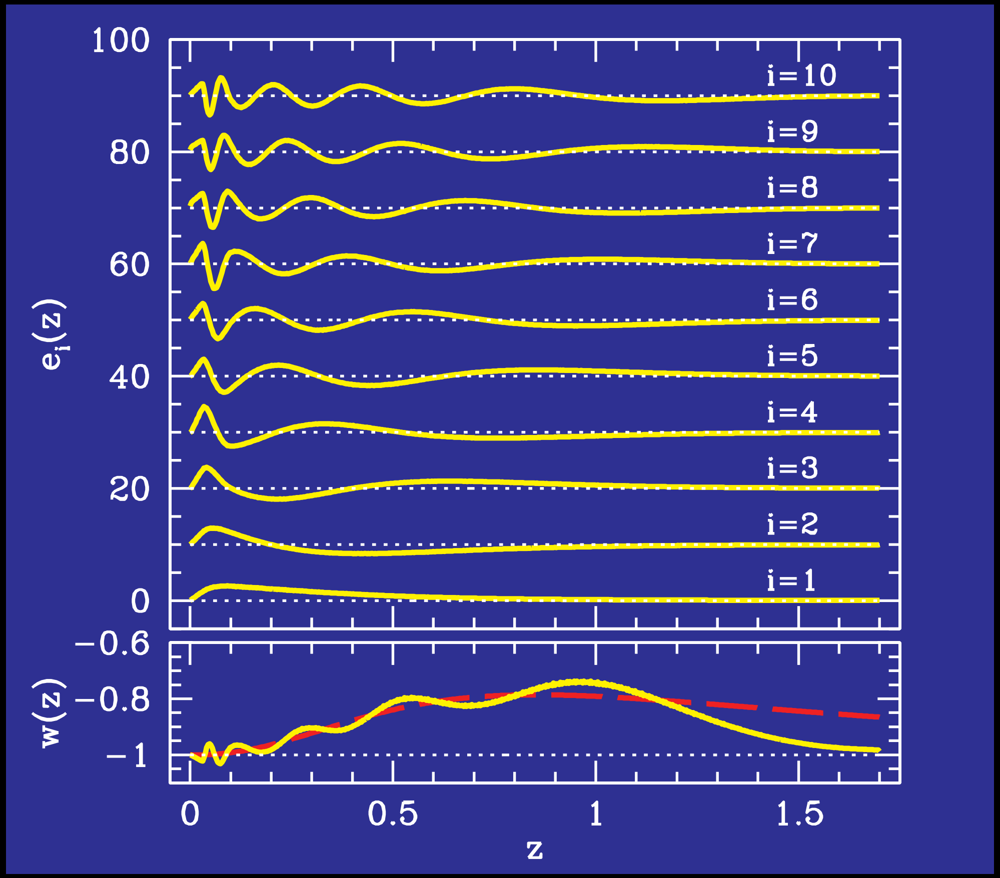
Important for surveys like DES, EUCLID, LSST

1. Principal components -

$$w(z) - w_b(z) = \sum_i \alpha_i e_i(z)$$

w_b – baseline eos

e_i – Fischer matrix eig modes



[Mortonson,Hu and Huterer 2011]

2. w(z)

$$w(z) = w_0 + w_1 \left[\frac{z}{1+z}\right] \quad \text{Chevallier – Polarski – Linder}$$

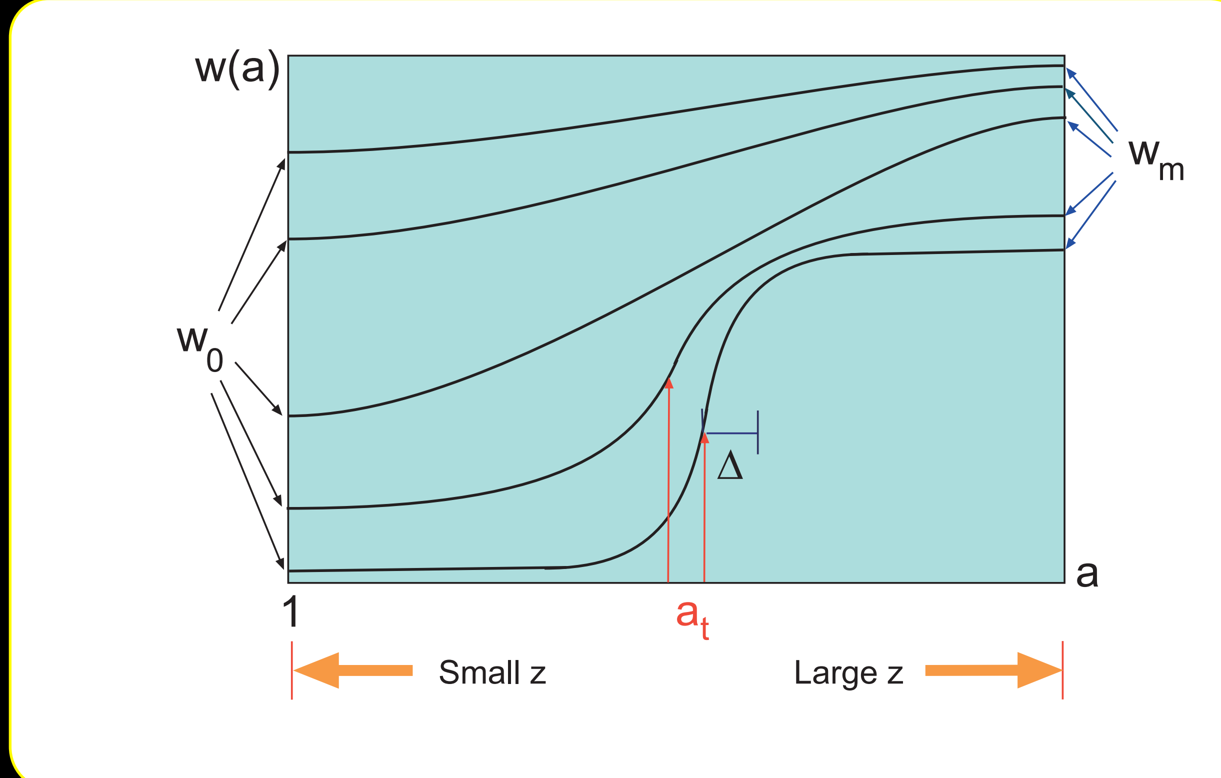
$$w(z) = w_0 + w_1 \left[\ln\left(\frac{1}{1+z}\right)\right] \quad \text{Gerke Efstathiou}$$

$$w(z) = w_0 + (w_m - w_0)\Gamma(a, a_t, \Delta)$$

Corasaniti et al

allows for tracker like behaviour although very tight bounds emerging from Planck on allowed density of early dark energy

$$\Omega_e < 0.009$$



3. w(Ω_{DE})

$$w(\Omega_e) = w_0 + w_1\Omega_e + w_2\Omega_e^2$$

Tarrant et al 2013

good for dynamical DE such as Quintessence as long as monotonic evolution.

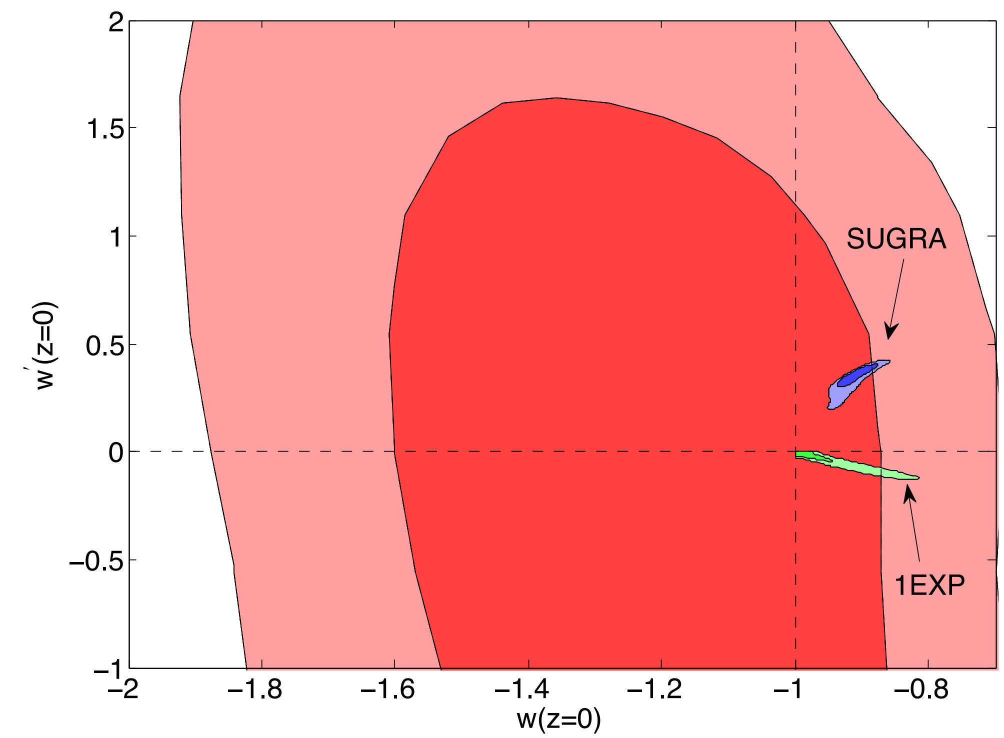


FIG. 5. The 2D 68% (dark shading) and 95% (light shading) marginalised contours in the $w_e|_{z=0}$–$w'_e|_{z=0}$ plane for the 1EXP and SUGRA quintessence models superimposed upon the corresponding contours of the dark energy clock

How early is early dark energy? [Pettorino,Amendola and Wetterich 2013]

$$\Omega_{de}(a) = \begin{cases} \Omega_e & a < a_c \ , \\ \frac{\Omega_{de0}}{\Omega_{de0}+\Omega_{m0}a^{-3}+\Omega_{r0}a^{-4}} & a \geq a_c \ . \end{cases}$$

$$a_c = \left[\frac{\Omega_e \Omega_{m_0}}{\Omega_{de0}(1-\Omega_e)}\right]^{1/3} \ .$$

$$w(a) = -\frac{1}{3[1-\Omega_{de}(a)]}\frac{d\ln\Omega_{de}}{d\ln a} + \frac{a_{eq}}{3(a+a_{eq})},$$

EDE2 model

Intro nice parameterisation of EDE which shows how CMB constraints depend on epoch when DE was non-negligible - the later it occurs the weaker the bounds.

Ω_e<0.05 if occurs for z<100

Ω_e<0.01 if present at least scattering.

EDE in the CMB

Not really - heat map from yesterdays Champions League game, Chelsea v Maccabi Tel Aviv

Testing models - consider coupled dark energy-dark matter.

Have seen provides a nice way to explain coincidence problem.

What is most general phenomenological model we can construct?

Three distinct classes of mixed models with couplings intro at the level of the action [Pourtsidou, Skordis , EC 2013]

- Consider Dark Energy (DE) coupled to Cold Dark Matter (c) [e.g. Kodama & Sasaki '84, Ma & Bertschinger '95]
- $T^{(c)}$ and $T^{(DE)}$ are not separately conserved:

$$\nabla_\mu T^{(c)\mu}{}_\nu = -\nabla_\mu T^{(DE)\mu}{}_\nu = J_\nu \neq 0$$

- Various forms of coupling have been considered. Examples:

$$J_\nu \propto \rho_c \nabla_\nu \phi \quad \text{[Amendola '00]}$$
$$J_\nu \propto \rho_c\, u_\nu^{(c)} \quad \text{[Valiviita et al '08]}$$

- FRW background with $\bar{J}_\nu = (\bar{J}_0, \bar{J}_i)$ and linear perturbations $(\delta J_0, \delta J_i)$. Note that $\bar{J}_i = 0$ because of isotropy. The CDM energy density equation becomes

$$\dot{\bar{\rho}}_c + 3\mathcal{H}\bar{\rho}_c = -\bar{J}_0$$

With thanks to my collaborator Alkistis Pourtsidou for lending me some of the following slides.

Using the fluid pull back formalism we consider the fluid/particle number density n.

- The action for GR and a fluid is

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}f(n)$$

- $f(n)$ is (in principle) an arbitrary function, whose form determines the equation of state and speed of sound of the fluid

- For pressureless matter (CDM) $f(n) \propto n$

- Stress-energy tensor is given by

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}$$

- Can match ρ, P to the fluid function $f(n)$ as

$$\Rightarrow \rho = f, \quad P = n\frac{df}{dn} - f$$

- We want to construct a model where the fluid with number density n (e.g. CDM) is explicitly coupled to a DE field ϕ

- Invariants: $Y = \frac{1}{2}(\nabla_\mu\phi)^2$, $Z = u^\mu\nabla_\mu\phi$

- Our general Lagrangian has the form

$$L = L(n, Y, Z, \phi)$$

- Example: Usual quintessence has

$$L = Y + V(\phi) + f(n)$$

Type 1 models. $L(n, Y, Z, \phi) = F(Y, \phi) + f(n, \phi)$ ex: $f(n) = g(n) e^{\alpha(\phi)}$

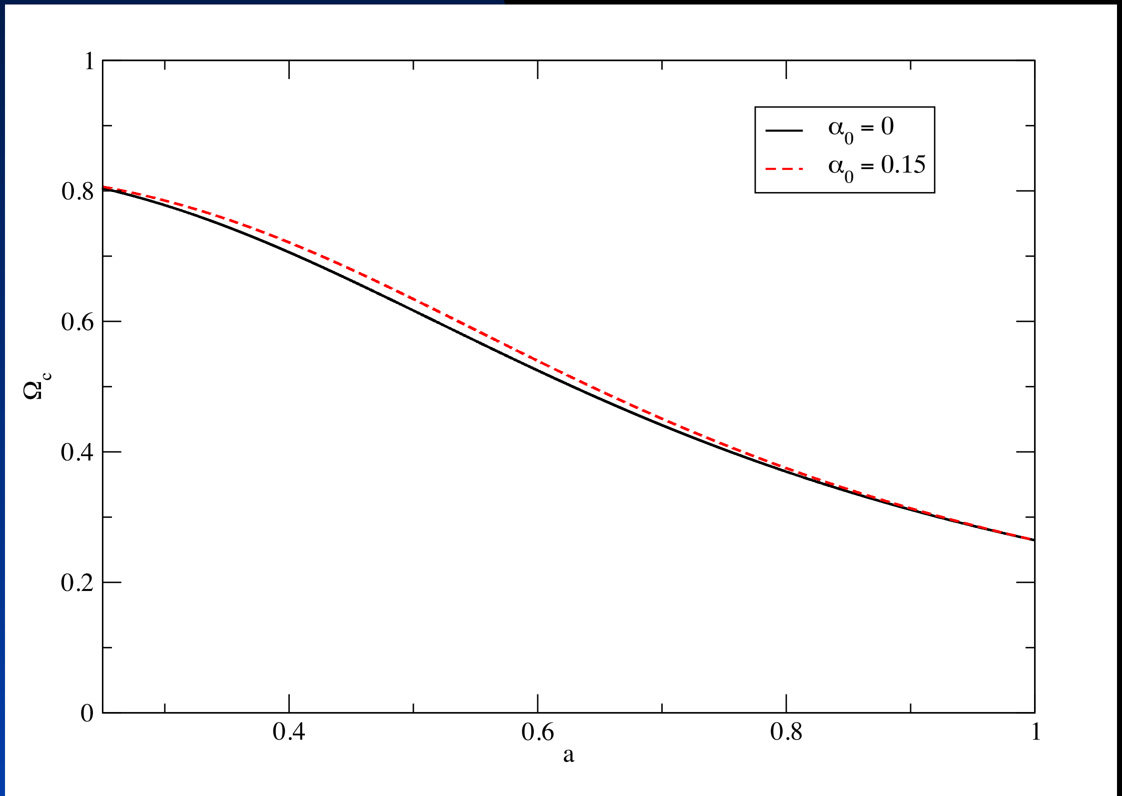
Could be K-essence scalar field coupled to matter, or Quintessence if F=Y+V(ϕ)

Coupling current $J_\mu = -\rho \frac{d\alpha(\phi)}{d\phi} \nabla_\mu \phi$ [generalized Amendola model]

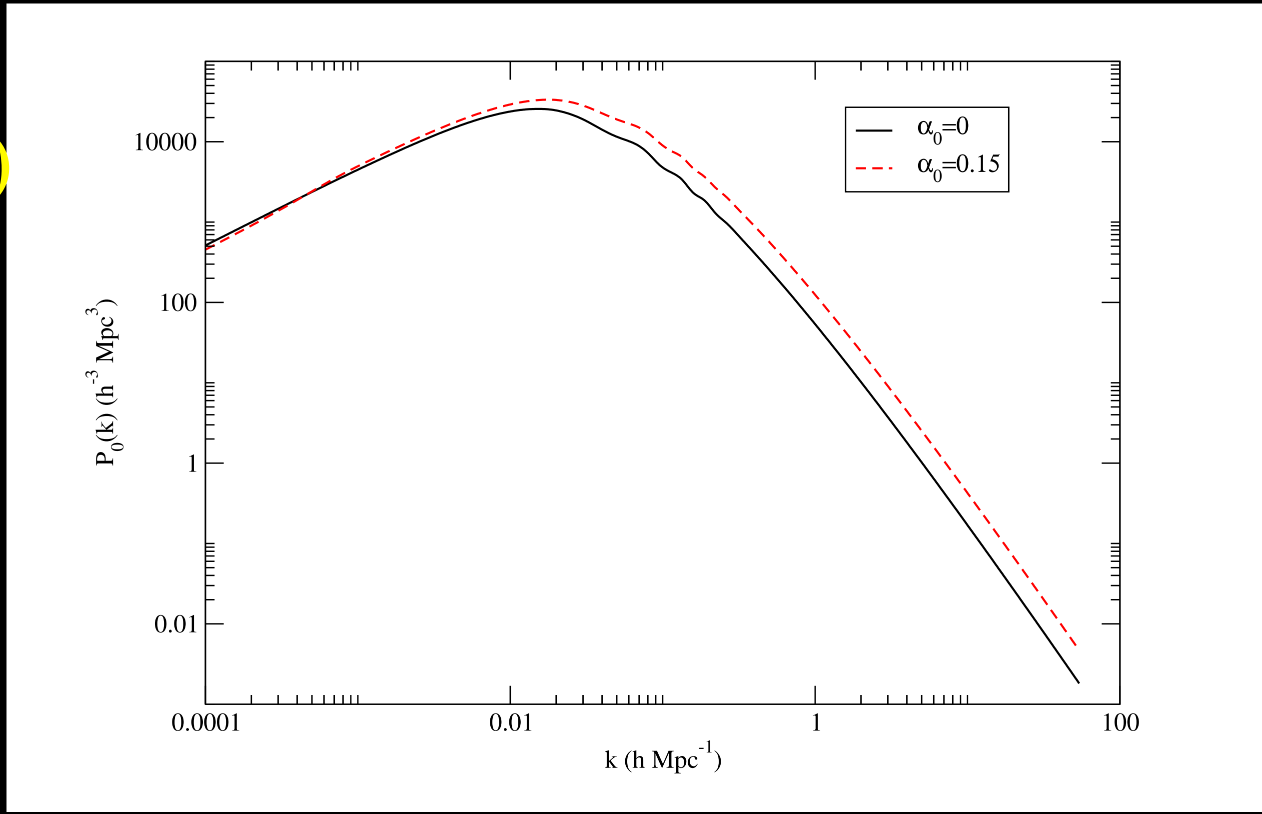
Choose $\alpha(\phi) = \alpha_0 \phi$ with α_0 const and study observational signatures in CMB and matter power spectra (modified CAMB code).

Note the evolution of CDM density: $\bar{\rho}_c = \bar{\rho}_{c,0} a^{-3} e^{\alpha(\phi)}$

Ω_C



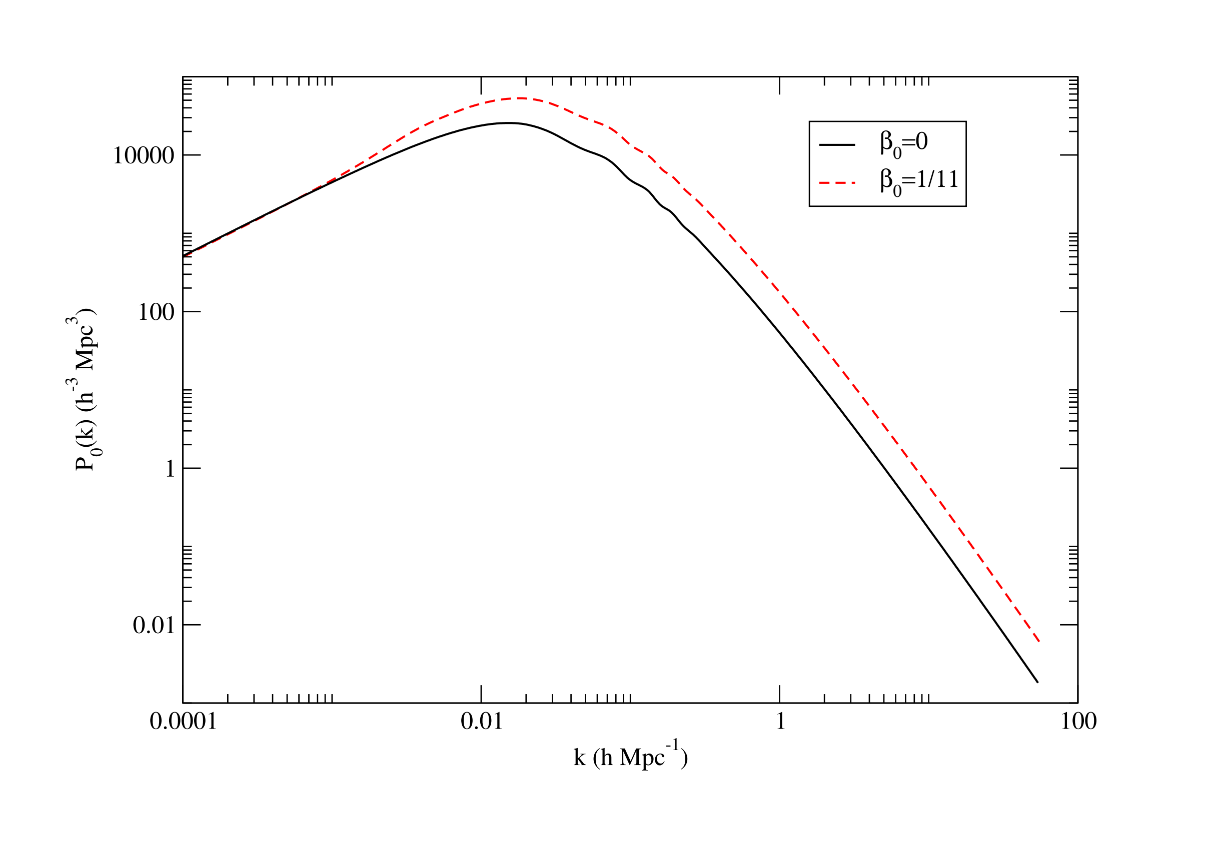
P(k)

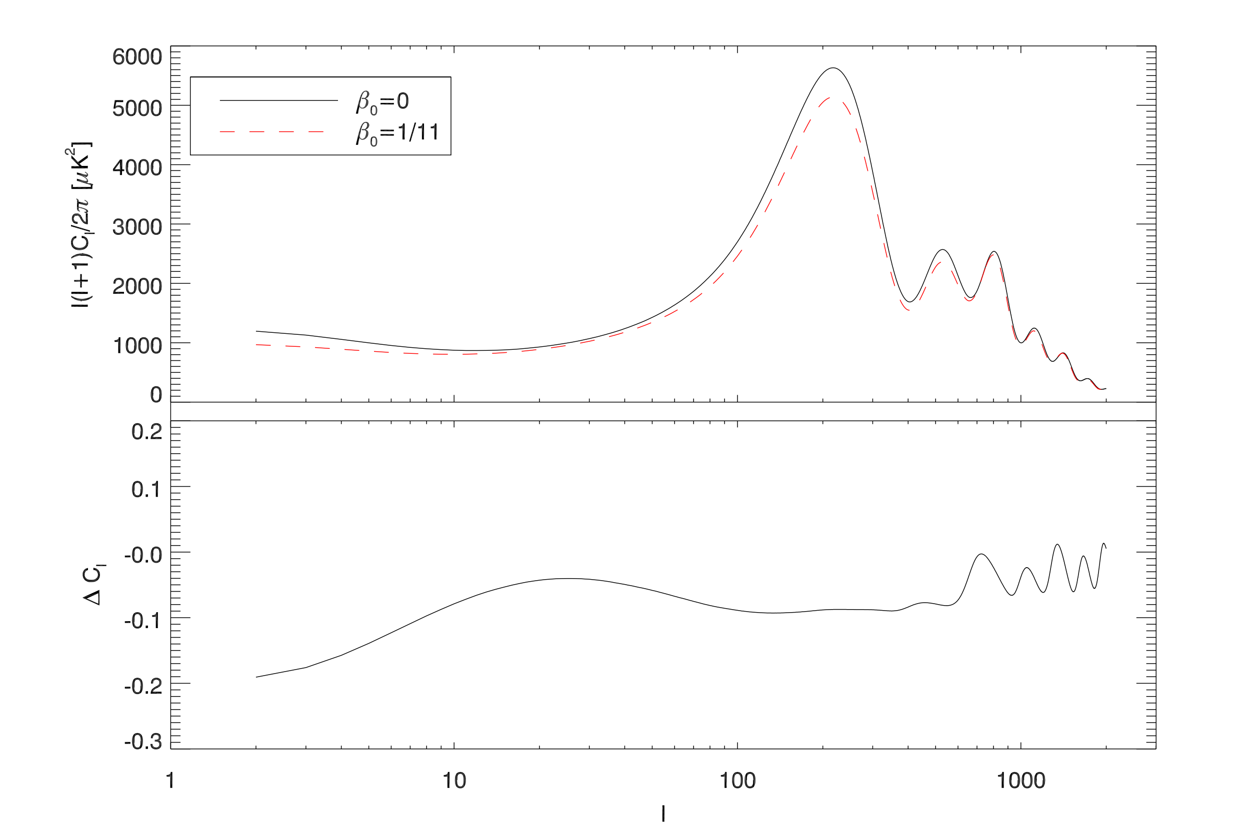


More DM at early times, equality earlier - only small scale pertns have time to enter horizon and grow during radiation dom - growth enhanced, small scale power increases, larger sigma8

Type 2 models. $L(n, Y, Z, \phi) = F(Y, \phi) + f(n, Z)$ ex: $\bar{\rho}_c = \bar{\rho}_{c,0} a^{-3} \bar{Z}^{\frac{\beta_0}{1-\beta_0}}$

Since $\bar{Z} = -\dot{\bar{\phi}}/a$, $\bar{\rho}_c$ depends on the time derivative $\dot{\bar{\phi}}$ instead of $\bar{\phi}$ itself which is a notable difference from the Type-1 case.





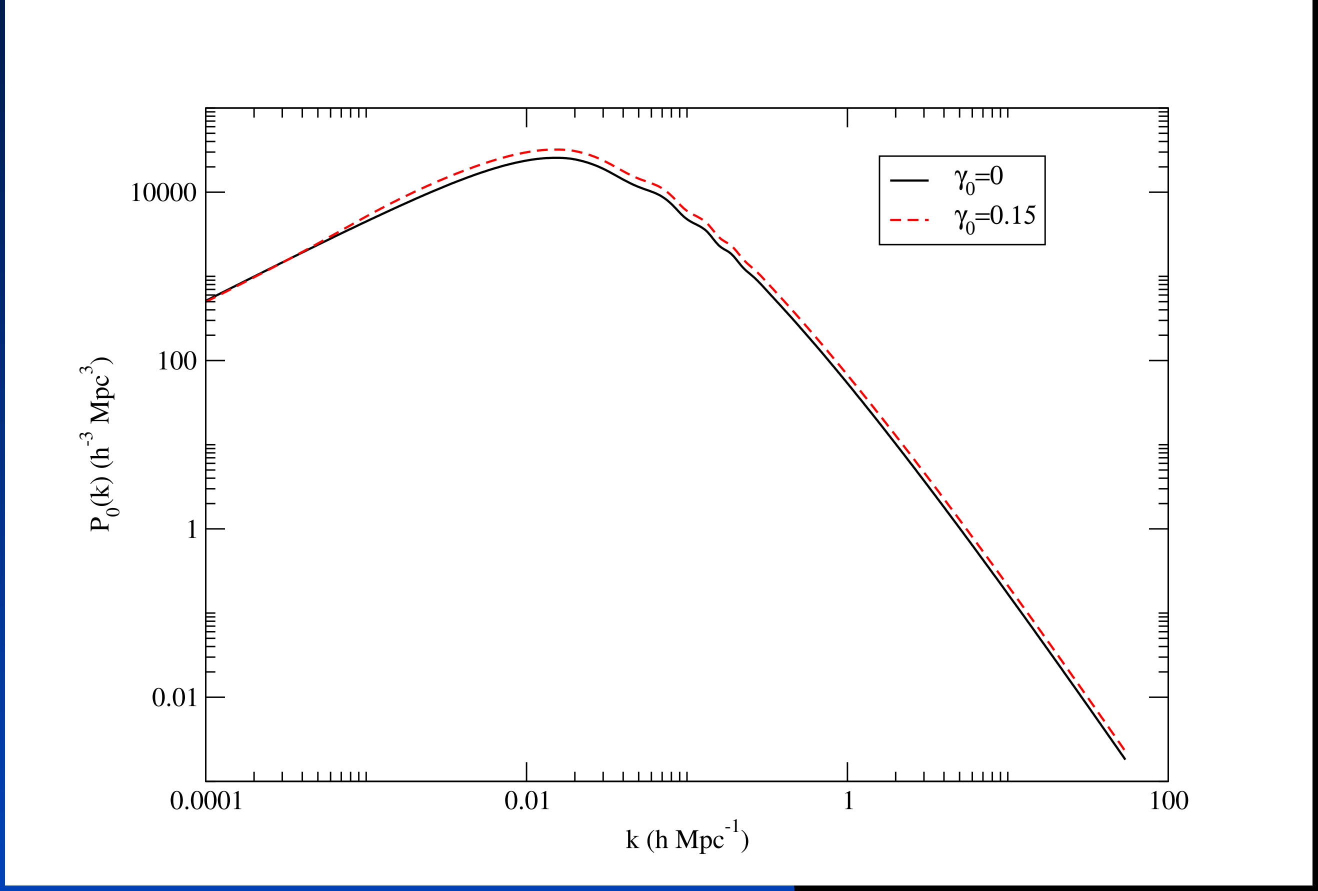
Type 3 models. $L(n, Y, Z, \phi) = F(Y, Z, \phi) + f(n)$ ex: $F = Y + V(\phi) + \gamma(Z)$ $\gamma(Z) = \gamma_0 Z^2$

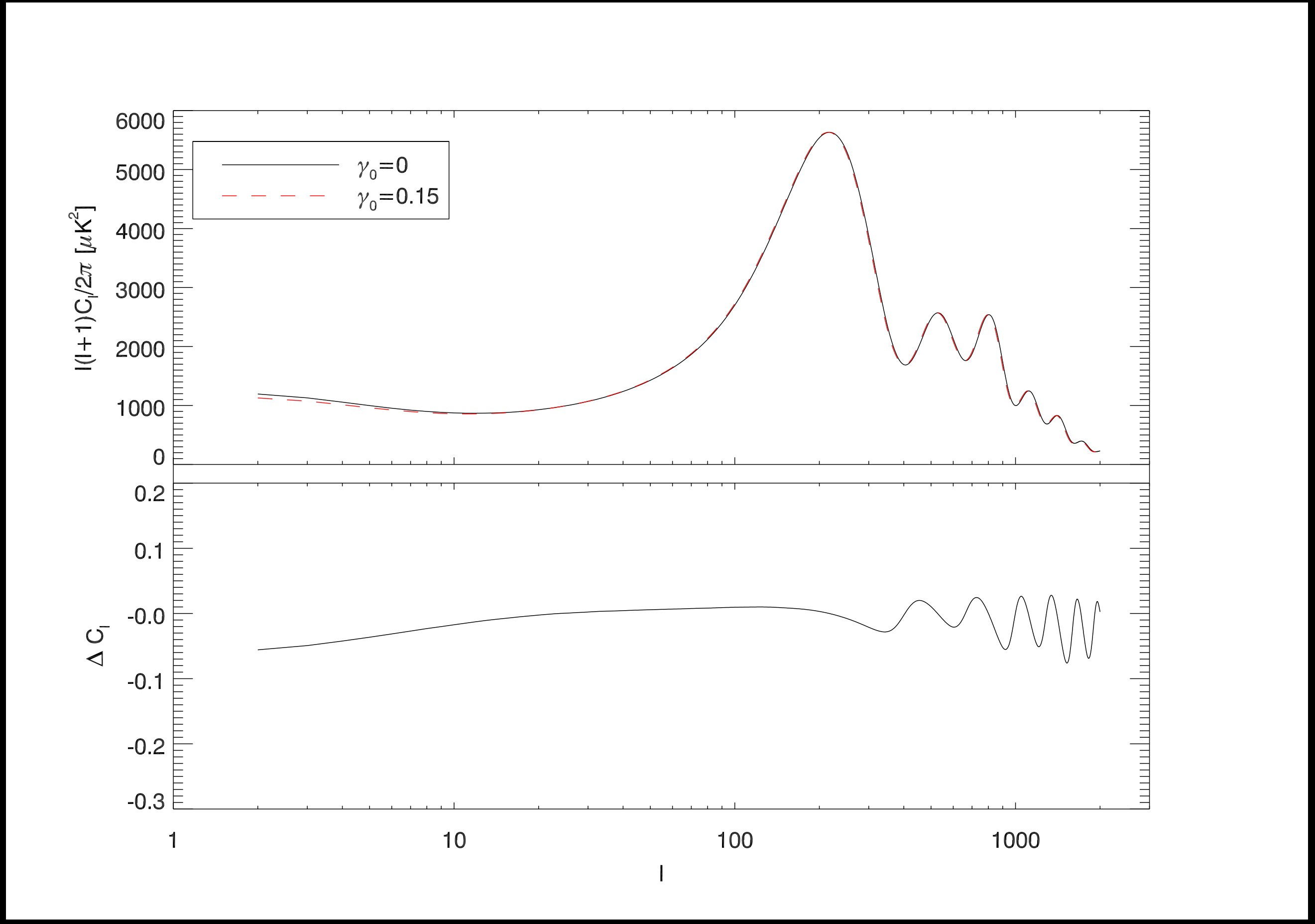
Type 3 models have $\bar{J}_0 = 0$ They involve pure momentum transfer

no coupling at the background field equations!

$$\dot{\bar{\rho}}_c + 3\mathcal{H}\bar{\rho}_c = 0$$

Furthermore, the energy-conservation equation remains uncoupled even at the linear level, i.e. $\delta \equiv \delta\rho/\bar{\rho}$ obeys uncoupled equation.





Parameterising these mixed models - extend the PPF formalism [Hu 08, Skordis 08]

$$G_{\mu\nu} = T^{(\mathrm{known})}_{\mu\nu} + U_{\mu\nu}$$

Tensor $U_{\mu\nu}$ contains the unknown fields/modifications, i.e. effective dark energy. Can depend on additional fields, metric etc. Example $f(R)$ gravity with $f_R = \frac{df}{dR}$:

$$U_{\mu\nu} = \nabla_\mu \nabla_\nu f_R - f_R R_{\mu\nu} + \left(\frac{1}{2} f - \nabla^2 f_R\right) g_{\mu\nu}$$

Assuming that there are no interactions between the two sectors, use $\nabla_\mu G^\mu{}_\nu = 0$ and $\nabla_\mu T^\mu{}_\nu = 0$ to get

$$\nabla_\mu U^\mu{}_\nu = 0$$

$\rightarrow$ Field equations for the modifications.

[Skordis, Pourtsidou, EC 2015]

- Consider Dark Energy (DE) coupled to Cold Dark Matter (c).
- $\nabla_\mu G^\mu{}_\nu = 0$ still true, but $T = T^{(\mathrm{c})}$ and $U = T^{(\mathrm{DE})}$ not separately conserved

$$\nabla_\mu T^{(\mathrm{c})\mu}{}_\nu = -\nabla_\mu T^{(\mathrm{DE})\mu}{}_\nu = J_\nu$$

- Split $J_\nu = \bar{J}_\nu + \delta J_\nu$ (note $\delta J_i = \nabla_i S$).
- FRW background with $\bar{J}_0$, $\bar{J}_i = 0$
- We want to parameterise δJ_0 and S in terms of metric and fluid variables.

- δJ_0 and S are written in terms of the DM and DE fluid variables, the metric variables and their derivatives.
- Notation: $\delta = \delta\rho/\bar{\rho}$

$$\delta J_0 = -6A_1\hat{\Phi} - 6A_2(\dot{\hat{\Phi}} + \mathcal{H}\hat{\Psi}) + A_3\delta_{\rm DE} + A_4\delta_c + A_5\theta_{\rm DE} + A_6\theta_c + \bar{J}_0\Psi,$$

$$S = -6B_1\hat{\Phi} - 6B_2(\dot{\hat{\Phi}} + \mathcal{H}\hat{\Psi}) + B_3\delta_{\rm DE} + B_4\delta_c + B_5\theta_{\rm DE} + B_6\theta_c,$$

- We have 12 free functions. Different models have different sets of non-zero (A_i, B_i).

- $\bar{J}_0 = \Gamma\bar{\rho}_c$ [Valiviita et al]. This model has

$$\delta J_0 = \bar{J}_0(\delta_c + \Psi); \quad S = \bar{J}_0\theta_c$$

$\Rightarrow$ The only non-zero coefficients are:

$$A_4 = B_6 = \bar{J}_0$$

- $\bar{J}_0 = -\beta\bar{\rho}_c\dot{\bar{\phi}}$ [Amendola]

$\Rightarrow$ The non-zero coefficients are:

$$A_3 = \frac{\bar{J}_0}{1 + w_\phi}, \quad A_4 = \bar{J}_0, \quad A_5 = \beta\bar{\rho}_c a^2 \frac{dV}{d\phi}$$

$$B_5 = \bar{J}_0$$

Model/Coefficients	Q	A_1	A_2	A_3	A_4	A_5	A_6	B_1	B_2	B_3	B_4	B_5	B_6
Coupled Quintessence	$-\beta_A\bar{\rho}_c\dot{\bar{\phi}}$	-	-	$\frac{Q}{1+w}$	Q	$\beta_A\bar{\rho}_c a^2 V_\phi$	-	-	-	-	-	Q	-
$J_\mu \propto u_\mu$	$a\Gamma_{int}\bar{\rho}_c$	-	-	-	Q	-	-	-	-	-	-	-	Q
elastic scattering	-	-	-	-	-	-	-	-	-	-	-	$-\rho_{DE}(1+w)an_D\sigma_D$	$-B_5$
Type-1	$-\bar{\rho}_c\alpha_\phi\dot{\bar{\phi}}$	-	-	$\frac{Qc_s^2}{1+w}$	Q	$Q\left[\frac{\alpha_{\phi\phi}}{\alpha_\phi} - \frac{c_s^2\dot{\bar{\phi}}\bar{K}_\phi}{(1+w)\bar{K}}\right]$	-	-	-	-	-	Q	-
Type-2	$\frac{\bar{Z}\beta_Z\bar{\rho}_c}{1+\bar{Z}\beta}\dot{\bar{Z}}$	-	A_2	A_3	A_4	A_5	-	-	-	-	-	Q	-
Type-3	-	-	-	-	-	-	-	-	-	B_3	-	B_5	$-B_5 + \frac{3\mathcal{H}\bar{Z}F_Z c_s^2}{1-\frac{\bar{Z}\bar{F}_Z}{\bar{\rho}_c}}$

TABLE II: Specific models and their PPF coefficients. The coupled Quintessence model is a subcase of Type 1 with $\alpha_\phi = \beta_A$. The elastic scattering model is in fact distinct from Type-3 (see text at the end of section III D). For the coefficients A_2, A_3, A_4 and A_5 in the case of Type-2 see (70). For the coefficients B_3 and A_5 in the case of Type-3 see (86). For the remaining functions the reader is referred to each specific example in the text.

Basic assumptions: Bgd cosmology is FRW soln, field eqns are at most 2nd order in time derivatives and are gauge invariant.

Once you know the field eqns PPF parameterisation is useful tool for phenomenological model building.

Interesting that in all the models we looked at A_1, A_6, B_1, B_2, B_4 are all zero. What models are there where they are non-zero?

See also very nice related work in Amendola, Barreiro and Nunes 2014 [Assisted coupled quintessence]; Amendola et al 2013 [Observables and unobservables in DE cosmologies]

Summary

1.Depending on your faith in string landscape approach we have a solution to CC problem. If not, its solution remains to be determined.

2. Sequestering cancels CC at all orders - impact of gravity loops ?

3. Fab Four - provides a way of living with a large changing cosmological constant ! Realistic models ?

4. Quintessence type approaches require light scalars which bring with them fifth force constraints that need satisfying.

5. Need to screen this which leads to models such as axions, chameleons,non-canonical kinetic terms etc.. -- these have their own issues.

6. Alternatively could consider modified gravity such as massive gravity but this brings with it constraints.

7. Increased interest in coupled DE-DM models which can be analysed by PPF formalism and can include new couplings such as scalar field to velocity components.