A manifestly scale-invariant regularization and quantum effective operators

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- Outline

- Introduction: Scale invariance as a solution to the hierarchy problem
- Problem: Usual regularizations of quantum corrections break scale invariance
- Goal: Study implications of a special, scale-invariant regularization.
- Implications: New corrections to scalar potential beyond Coleman-Weinberg.
- Applications: the scalar potential in SM + dilaton.

• Introduction

- One approach to hierarchy problem: scale invariance ($x \to \rho\, x$, $\phi \to \rho^d \phi$): forbids (higgs) mass terms
- the real world is not scale invariant $\Rightarrow$ this symmetry must be broken.

  - at classical level: one can start with a scale invariant L
  - at quantum level? $\to$ the need for a subtraction/renormalization scale (μ)

Cutoff schemes: $$\ln \Lambda/m_Z = \ln \Lambda/\mu + \ln \mu/m_Z, \qquad (\Lambda \to \infty).$$

DR scheme: $$\lambda_\phi = \mu^{2\epsilon} \Big[\lambda_\phi^{(r)} + \sum_n a_n/\epsilon^n\Big], \qquad (\epsilon \to 0)$$

$\Rightarrow$ At quantum level: scale symmetry is broken explicitly by:
a dimensionful scale (cutoff, Pauli Villars) or a dimensionful coupling (DR scheme).

- Problem: in theories with scale/conformal symmetry: regularization (DR, etc...) breaks explicitly the very symmetry one wants to study at quantum level!

  - impact, particulary in non-renormalizable case, and for the hierarchy problem
  - usual (naive?) argument: "DR breaks scale symmetry more softly" $(^*)$

[Bardeen 1995]

$\Rightarrow$ Solution: replace $\mu \rightarrow f($dilaton: $\sigma)$. [Deser 1970, Englert 1976, Shaposhnikov 2009]

Evanescent power $\mu^{2\epsilon}$ in the last equation $\Rightarrow$ need $\langle\sigma\rangle \neq 0 \rightarrow$ spontaneous breaking of scale invariance!

Goal: study its implications.

$(^*)$ if the DR breaking of scale symmetry were indeed soft the result should be similar to spontaneous breaking of scale symmetry - see later

- Scale invariance at classical level

$\mathcal{L}$ of two real scalar fields:

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - V(\phi,\sigma)$$

An example:

$$V = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_m}{2}\,\phi^2\sigma^2 + \frac{\lambda_\sigma}{4}\,\sigma^4$$

Extremum: $\langle\phi\rangle\big[\,\lambda_\phi\langle\phi\rangle^2 + \lambda_m\langle\sigma\rangle^2\,\big] = 0, \quad \langle\sigma\rangle\big[\,\lambda_m\langle\phi\rangle^2 + \lambda_\sigma\langle\sigma\rangle^2\,\big] = 0,$

a) The ground state is $\langle\sigma\rangle = 0, \quad \langle\phi\rangle = 0$ and both fields are massless.

b) IF $\langle\sigma\rangle \neq 0$ a solution, then $\langle\phi\rangle \neq 0$; a non-trivial ground state exists if $\lambda_m^2 = \lambda_\phi\lambda_\sigma;\;\; \lambda_m < 0.$

$$\frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = -\frac{\lambda_m}{\lambda_\phi}, \quad\Rightarrow\quad V = \frac{\lambda_\phi}{4}\Big(\,\phi^2 + \frac{\lambda_m}{\lambda_\phi}\,\sigma^2\,\Big)^2$$

$\Rightarrow$ Spontaneous breaking of scale symmetry $\Rightarrow$ EWSB at tree-level, with a vanishing cosmo constant

• Scale invariance at classical level

$\mathcal{L}$ of two real scalar fields:

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - V(\phi,\sigma)$$

An example: [Kobakhidze et al 2007, 2014]

$$V = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_m}{2}\,\phi^2\sigma^2 + \frac{\lambda_\sigma}{4}\,\sigma^4$$

The mass eigenstates:

$$\begin{aligned} m^2_{\tilde\phi} &= 2\,\lambda_\phi\big(1-\lambda_m/\lambda_\phi\big)\,\langle\phi\rangle^2 = -2\lambda_m\,(1-\lambda_m/\lambda_\phi)\,\langle\sigma\rangle^2 \\ m_\sigma &= 0 \end{aligned}$$

$\Rightarrow$ σ: Goldstone mode of scale invariance (dilaton). [Shaposhnikov et al 2009, Ross et al 2014]

Expect: $\langle\sigma\rangle \sim M_{\text{Planck}}$ $\Rightarrow$ To ensure a hierachy $m_{\tilde\phi} \sim \langle\phi\rangle \sim \mathcal{O}(100)$ GeV, one tunes classically λ_m :

$$\langle\phi\rangle \ll \langle\sigma\rangle \quad \text{if} \quad \lambda_\phi \gg |\lambda_m| \gg \lambda_\sigma, \qquad \lambda_m^2 = \lambda_\phi\lambda_\sigma$$

$\lambda_m \sim 1/\langle\sigma\rangle^2$, $\lambda_\sigma \sim 1/\langle\sigma\rangle^4$. At quantum level: is extra tuning needed?

• Scale invariance at quantum level

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - V(\phi,\sigma)$$

- DR: $d = 4 - 2\epsilon$: $[\mathcal{L}] = d$, $[\lambda] = [V^{(4)}] = d - 4(d-2)/2 = 4 - d \Rightarrow \lambda \to \mu^{4-d}\,\lambda$.

A scale invariant regularization: $\mu \to \mu(\sigma,\phi)$. Then $V \to \tilde{V} \equiv \mu(\phi,\sigma)^{4-d}\,V$

$$\begin{aligned} U &= \tilde{V} - \frac{i}{2}\int \frac{d^d p}{(2\pi)^d}\,\mathrm{Tr}\ln\left[p^2 - \tilde{M}^2(\phi,\sigma) + i\varepsilon\right], \\ &= \tilde{V} - \frac{1}{64\pi^2}\sum_{s=\phi,\sigma}\tilde{M}_s^4\left[\frac{2}{4-d} - \ln \tilde{M}_s^2/\kappa\right], \end{aligned}$$

$$(\tilde{M}^2)_{\alpha\beta} = \frac{\partial^2\tilde{V}}{\partial\alpha\partial\beta} \equiv \tilde{V}_{\alpha\beta}, \qquad (M^2)_{\alpha\beta} = V_{\alpha\beta}; \quad \alpha,\beta = \phi,\sigma.$$

$$\begin{aligned} (\tilde{M}^2)_{\alpha\beta} &= \mu^{4-d}\left[(M^2)_{\alpha\beta} + (4-d)\,\mu^{-2}\,N_{\alpha\beta}\right], \\ \sum_{s=\phi,\sigma}\tilde{M}_s^4 &= \mu^{2\,(4-d)}\left[\mathrm{Tr}\,M^4 + 2\,(4-d)\,\mu^{-2}\,\mathrm{Tr}\,(M^2N)\right], \\ N_{\alpha\beta} &\equiv \mu\,(\mu_\alpha\,V_\beta + \mu_\beta\,V_\alpha) + (\mu\,\mu_{\alpha\beta} - \mu_\alpha\,\mu_\beta)\,V, \end{aligned}$$

$\Rightarrow$ "Evanescent" corrections to $(M^2)_{\alpha\beta}$ bring finite quantum corrections to U, due to $(4-d)\times\frac{2}{(4-d)}$.

• The scale-invariant one-loop potential

$$U(\phi,\sigma) = V(\phi,\sigma) + \frac{1}{64\pi^2}\Big\{\sum_{s=\phi,\sigma} M_s^4(\phi,\sigma)\Big[\ln\frac{M_s^2(\phi,\sigma)}{\mu^2(\phi,\sigma)} - \frac{3}{2}\Big] + \Delta U(\phi,\sigma)\Big\}$$

$$\Delta U = \frac{-4}{\mu^2}\Big\{V\,\big[(\mu\mu_{\phi\phi} - \mu_\phi^2)\,V_{\phi\phi} + 2\,(\mu\mu_{\phi\sigma} - \mu_\phi\mu_\sigma)V_{\phi\sigma} + (\mu\mu_{\sigma\sigma} - \mu_\sigma^2)\,V_{\sigma\sigma}\big]$$

$$+\ 2\mu\,(\mu_\phi\,V_{\phi\phi} + \mu_\sigma\,V_{\phi\sigma})\,V_\phi + 2\mu\,(\mu_\phi\,V_{\phi\sigma} + \mu_\sigma\,V_{\sigma\sigma})\,V_\sigma\Big\}, \qquad \mu_\alpha = \frac{\partial\mu}{\partial\alpha},\ \ \mu_{\alpha\beta} = \frac{\partial^2\mu}{\partial\alpha\partial\beta},$$

with $\alpha,\beta=\phi,\sigma$. If $\mu=\mu(\sigma)$ only:

$$\Delta U = \frac{-4}{\mu(\sigma)^2}\Big\{2\sigma\,\big(V_\sigma\,V_{\sigma\sigma} + V_\phi\,V_{\phi\sigma}\big) - V\,V_{\sigma\sigma}\Big\}$$

If μ=constant, $\Delta U = 0$. On tree-level ground state: $\Delta U = 0$.

$\Rightarrow$ ΔU: new, one-loop finite correction, beyond the Coleman-Weinberg term.

- The scale-invariant one-loop potential. Minimal case: $\mu = z\,\sigma$, z: constant. $\left[\mu = z\,\sigma^{2/(d-2)}\right]$

$$\Delta U = \frac{\lambda_\phi \lambda_m \phi^6}{\sigma^2} - \left(16\lambda_\phi\lambda_m + 6\lambda_m^2 - 3\lambda_\phi\lambda_\sigma\right)\phi^4 - \left(16\lambda_m + 25\lambda_\sigma\right)\lambda_m\,\phi^2\sigma^2 - 21\lambda_\sigma^2\sigma^4$$

- ΔU contains higher dimensional operators. It is independent of the subtraction parameter z!

- total U is unstable if $\lambda_m < 0$, due to $\lambda_m \phi^6/\sigma^2 < 0$! Higher orders can stabilize it.

- if $\lambda_m^2 = \lambda_\phi\,\lambda_\sigma$, $\lambda_m < 0$ for tree-level EWSB, then:

$$\Delta U = \frac{\lambda_m}{\lambda_\phi}\left(\frac{\phi^2}{\sigma^2} + \frac{\lambda_m}{\lambda_\phi}\right)\left(\lambda_\phi^2\,\phi^4 - 4\,\lambda_\phi\,(4\,\lambda_\phi + \lambda_m)\,\phi^2\,\sigma^2 - 21\lambda_m^2\,\sigma^4\right)$$

- U can be Taylor expanded: $\sigma = \langle\sigma\rangle + \delta\sigma$, $\delta\sigma =$ quantum fluctuation

- spectrum at quantum level: massive ϕ and a massless dilaton σ (Goldstone) - flat direction

- can only predict the ratio $\langle\phi\rangle/\langle\sigma\rangle$.

$\Rightarrow$ Potential unstable under quantum fluctuations. Higher orders may stabilize it.

$\Rightarrow$ Quantum effective operators present, with known, finite coefficient, independent of z.

• Minimizing the one-loop U: $\lambda_\phi \gg |\lambda_m| \gg \lambda_\sigma$, and $\mu = z\,\sigma$. (*)

$$U = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_m}{2}\phi^2\sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4 + \frac{1}{64\pi^2}\Big\{\sum_{s=1,2} M_s^4\Big[\ln\frac{M_s^2}{z^2\sigma^2} - \frac{3}{2}\Big]$$
$$+ \lambda_\phi\lambda_m\frac{\phi^6}{\sigma^2} - \big(16\,\lambda_\phi\lambda_m + 6\lambda_m^2 - 3\lambda_\phi\lambda_\sigma\big)\phi^4 - 16\,\lambda_m^2\,\phi^2\sigma^2\Big\} + \mathcal{O}(\lambda_m^3)$$

$$\text{min:}\quad \rho \equiv \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = -\frac{\lambda_m}{\lambda_\phi}\Big[1 - \frac{6\lambda_\phi}{64\pi^2}\big(4\ln 3\lambda_\phi - 17/3\big)\Big] + \mathcal{O}(\lambda_m^2)$$

$$m_{\tilde\phi}^2 = (U_{\phi\phi} + U_{\sigma\sigma})_{\text{min}}; \qquad \delta m_{\tilde\phi}^2 = \frac{1}{64\pi^2}\big(\Delta U_{\phi\phi} + \Delta U_{\sigma\sigma}\big)_{\text{min}}$$

$$\delta m_{\tilde\phi}^2 = \frac{-\langle\sigma\rangle^2}{32\pi^2}\Big[4\lambda_m^2(4+13\rho) + 18\lambda_\sigma(7\lambda_\sigma - \lambda_\phi\rho) + \lambda_m\big[25\lambda_\sigma(1+\rho) - 3\lambda_\phi\rho(-32+5\rho+\rho^2)\big]\Big] \sim \lambda_m^2\,\langle\sigma\rangle^2$$

- fixing the dimensionless subtraction parameter: take $z = \langle\phi\rangle/\langle\sigma\rangle \Rightarrow \mu = \langle\phi\rangle$, as usual.

$\Rightarrow$ No tuning needed beyond (*) to keep $\delta m_{\tilde\phi}^2 \ll \langle\sigma\rangle^2$. No dangereous $\lambda_\phi\langle\sigma\rangle^2$. may hold to all orders

$\Rightarrow$ Callan-Symanzik: $z\,dU/dz = 0$. [see related work of C. Tamarit 2014]

• Restrictions on other expressions for $\mu(\sigma,\phi)$:

Adding a term: $\Delta\mathcal{L}_G = -\frac{1}{2}\,(\xi_\phi\,\phi^2+\xi_\sigma\,\sigma^2)\,R$, needed in some models to generate the Planck scale

[Shaposhnikov et al 2009]

$$\mu = z\left(\xi_\phi\,\phi^2+\xi_\sigma\,\sigma^2\right)^{1/2}$$

Then: $$\Delta U = -(\xi_\phi\phi^2+\xi_\sigma\sigma^2)^{-2}\Big[(21\,\lambda_\phi\,\xi_\phi+\lambda_m\,\xi_\sigma)\,\xi_\phi\lambda_\phi\,\phi^8+(21\lambda_\sigma\,\xi_\sigma+\lambda_m\,\xi_\phi)\,\xi_\sigma\lambda_\sigma\,\sigma^8+\cdots\Big]$$

$\Rightarrow$ negative coefficients of ϕ^8, σ^8 for $\lambda_m^2=\lambda_\phi\lambda_\sigma$. U unstable at large fields.

$$\Delta U\Big|_{\lambda_m=0} = -3\,\lambda_\phi^2\,\xi_\phi\,\phi^6\Big[9\,\xi_\sigma\,\sigma^2+7\,\xi_\phi\,\phi^2\Big]\,(\xi_\phi\phi^2+\xi_\sigma\sigma^2)^{-2}$$

$\Rightarrow$ in the classical decoupling limit: non-decoupling quantum effects, unless $\langle\sigma\rangle\rightarrow\infty$

More general case of: $\mu(\phi,\sigma)=z\,\sigma\,\exp\big[h(\phi/\sigma)\big]$ - similar conclusion.

$\Rightarrow$ The form of $\mu(\phi,\sigma)$ is restricted to avoid such non-decoupling effects $\Rightarrow$ Minimal $\mu=\mu(\sigma)$ only!

• Summary

- scale invariance often used to address the hierarchy problem but all regularizations break explicitly the symmetry one wants to study at quantum level.

⇒ we studied a scale-invariant regularization, with spontaneous breaking of this symmetry.

Implications:

⇒ One-loop scale invariant scalar potential U.

⇒ ΔU: new correction to U, beyond Coleman-Weinberg term ("evanescent" origin).

⇒ ΔU: independent of subtraction parameter;

⇒ $\Delta U \sim \phi^6/\sigma^2$ finite, effective operator(s), destabilize U at large ϕ.

⇒ mass correction to ϕ under control at one-loop (no extra tuning needed).

⇒ next: study the scalar potential for scale invariant SM (+ dilaton). Non-renormalizability?

⇒ applications to theories in which preserving scale invariance at loop level needed (CFT's, ...)

- Scale invariant Standard Model one-loop potential:

$$\tilde{V} = \mu^{4-d} V, \quad V = \lambda_\phi |H|^4 + \lambda_m |H|^2 \sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4; \qquad H = (0,\phi)/\sqrt{2}.$$

$$\begin{aligned} M_G^2 &= \lambda_\phi \phi^2 + \lambda_m \sigma^2, \qquad M_\phi^2, M_\sigma^2 \\ M_W^2 &= \frac{1}{4} g^2 \phi^2, \qquad M_Z^2 = \frac{1}{4}(g^2 + g'^2)\phi^2, \qquad M_t^2 = \frac{1}{2} y_t^2 \phi^2. \end{aligned}$$

$$\begin{aligned} U &= \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_m}{2}\phi^2\sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4 + \frac{1}{64\pi^2}\Big\{ \frac{3}{2}(\lambda_\phi\phi^2 + \lambda_m\sigma^2)^2 \Big[\ln\frac{\lambda_\sigma\phi^2 + \lambda_m\sigma^2}{z^2\sigma^2} - \frac{3}{2} \Big] \\ &+ \lambda_\phi\lambda_m \frac{\phi^6}{\sigma^2} - \big(16\lambda_\phi\lambda_m + 6\lambda_m^2 - 3\lambda_\phi\lambda_\sigma\big)\phi^4 - \big(16\lambda_m + 25\lambda_\sigma\big)\,\lambda_m\,\phi^2\sigma^2 - 21\lambda_\sigma^2\sigma^4 \\ &+ \sum_{s=\phi,\sigma} M_s^4 \ln\frac{M_s^2}{z^2\sigma^2} - \frac{3}{2}\Big[(9\lambda_\phi^2 + \lambda_m^2)\phi^4 + 2\lambda_m\,(3\lambda_\phi + 4\lambda_m + 3\lambda_\sigma)\,\phi^2\sigma^2 + (\lambda_m^2 + 9\lambda_\sigma^2)\,\sigma^4 \Big] \\ &+ \frac{3}{8} g^4 \phi^4 \Big[\ln\frac{g_2^2\phi^2}{4z^2\sigma^2} - \frac{5}{6} \Big] + \frac{3}{16}\,(g^2 + g'^2)^2 \phi^4 \Big[\ln\frac{g^2\phi^2}{4\,z^2\,\sigma^2} - \frac{5}{6} \Big] - 3\phi^4 y_t^4 \Big[\ln\frac{\phi^2 y_t^2}{2z^2\sigma^2} - \frac{3}{2} \Big] \Big\} \end{aligned}$$

- Phenomenology?