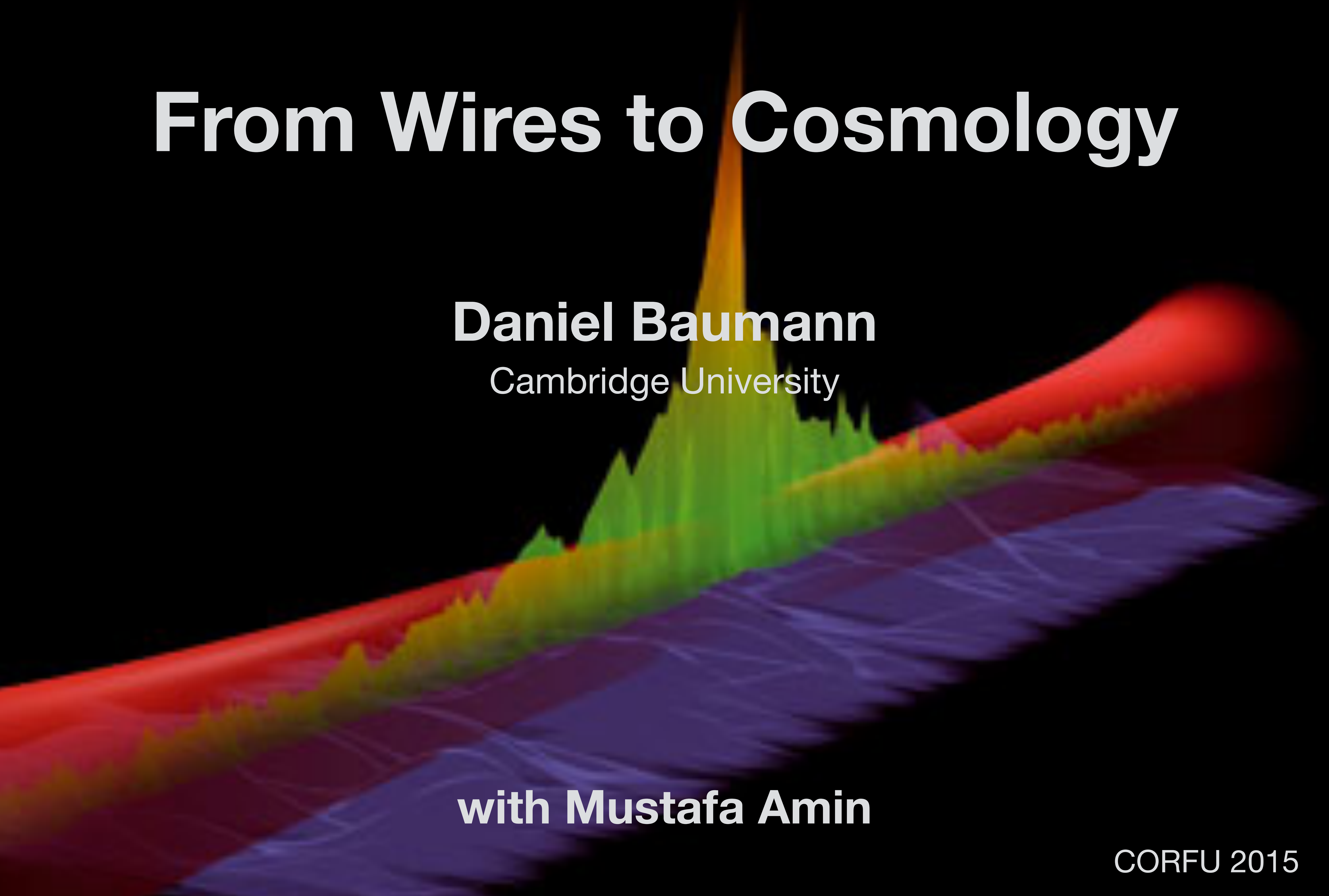
From Wires to Cosmology

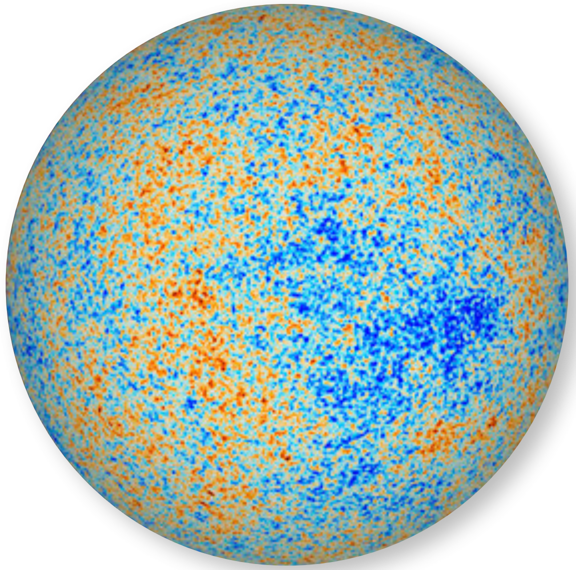


Daniel Baumann

Cambridge University

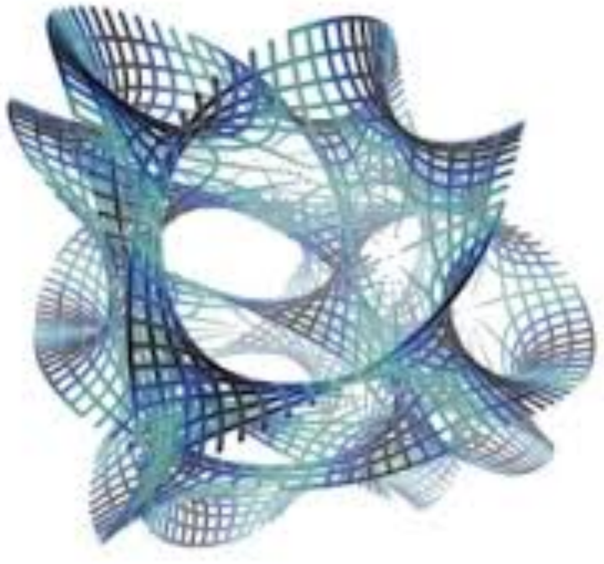
with Mustafa Amin

CORFU 2015



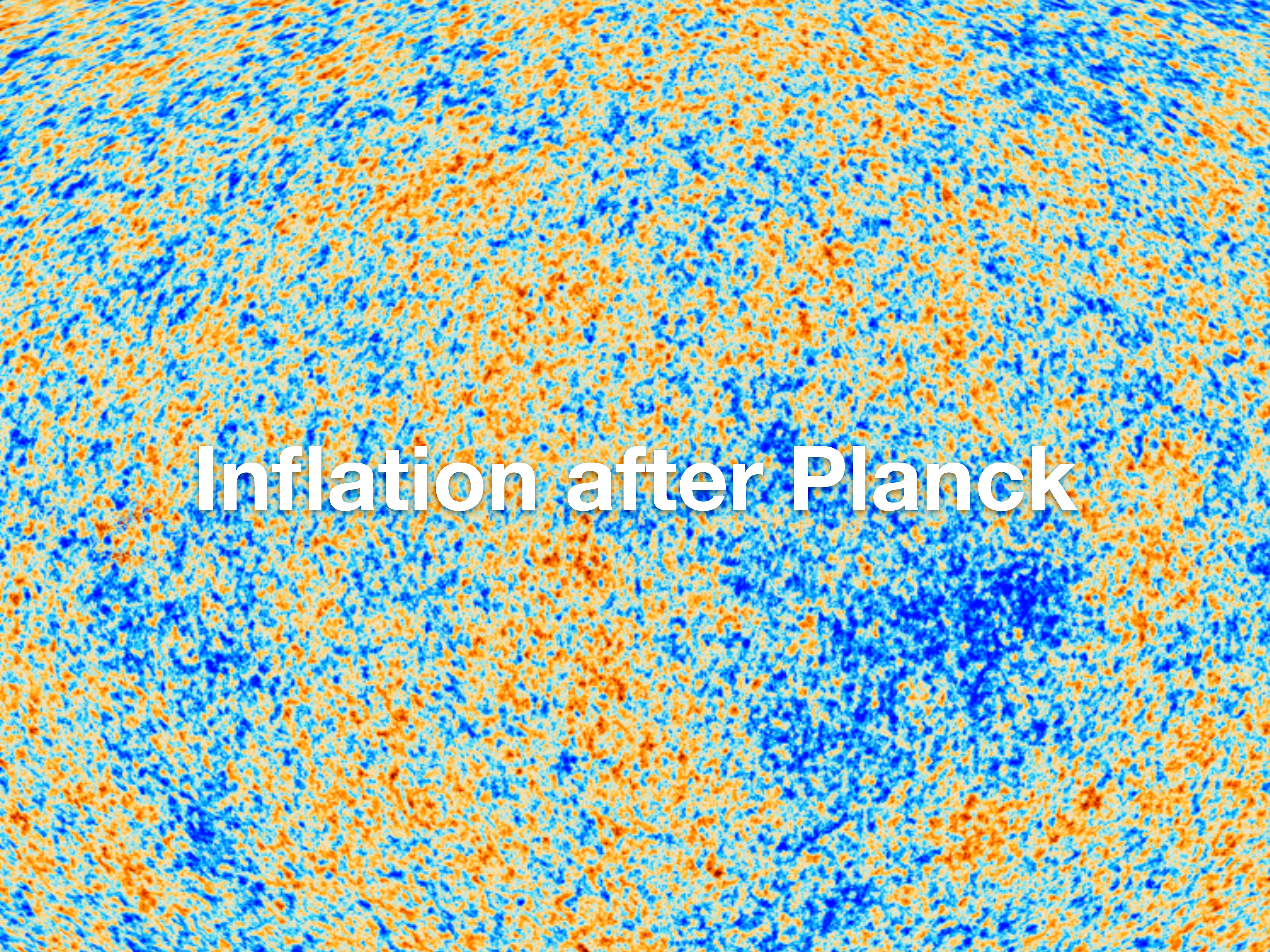
Inflation after Planck

… without the fairy tails.



Randomness during Inflation

… and its relation to conduction in wires.



Inflation after Planck

The temperature anisotropies (and polarization) of the cosmic microwave background measure ***distortions of space****:*

ζ

scalar mode

$$\mathrm{d}\ell^2 = a^2(t)\Big[1 + \underline{2\zeta(t,\boldsymbol{x})}\Big]\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

curvature perturbation

expansion of space

isotropic stretching

h_{ij}

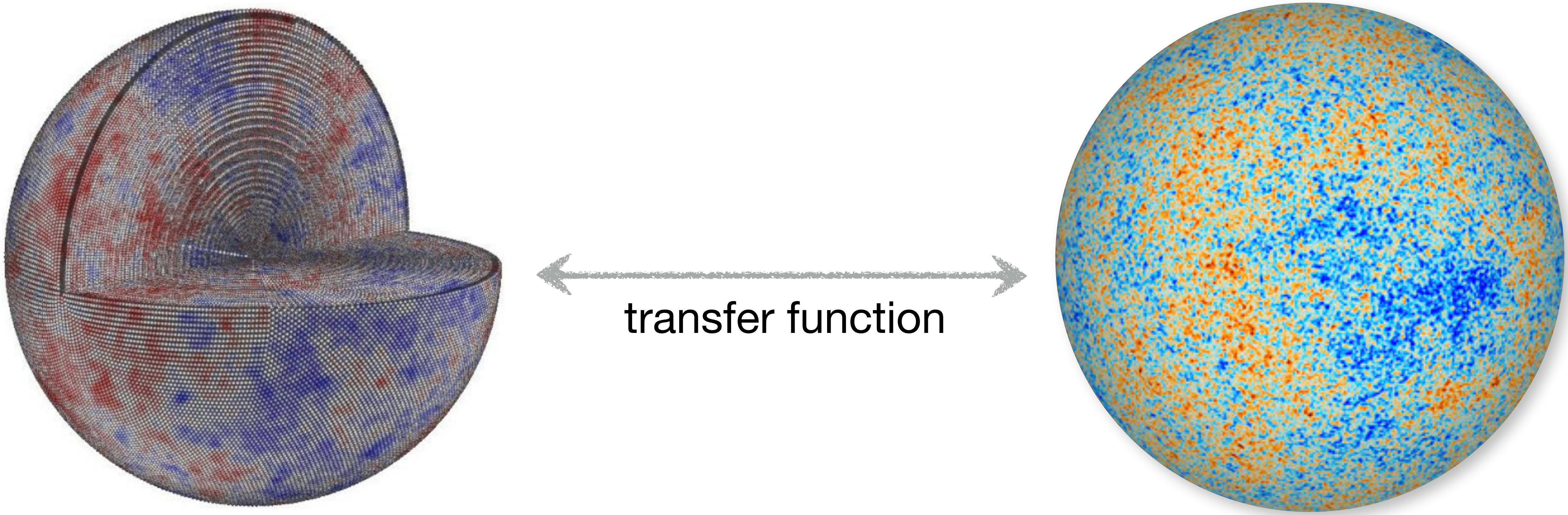
tensor mode

$$\mathrm{d}\ell^2 = a^2(t)\Big[\delta_{ij} + \underline{h_{ij}(t,\boldsymbol{x})}\Big]\mathrm{d}x^i\mathrm{d}x^j$$

gravitational waves

anisotropic stretching

These metric perturbations are small and can be traced back to their cosmic origin in perturbation theory:



primordial perturbations

transfer function

CMB anisotropies

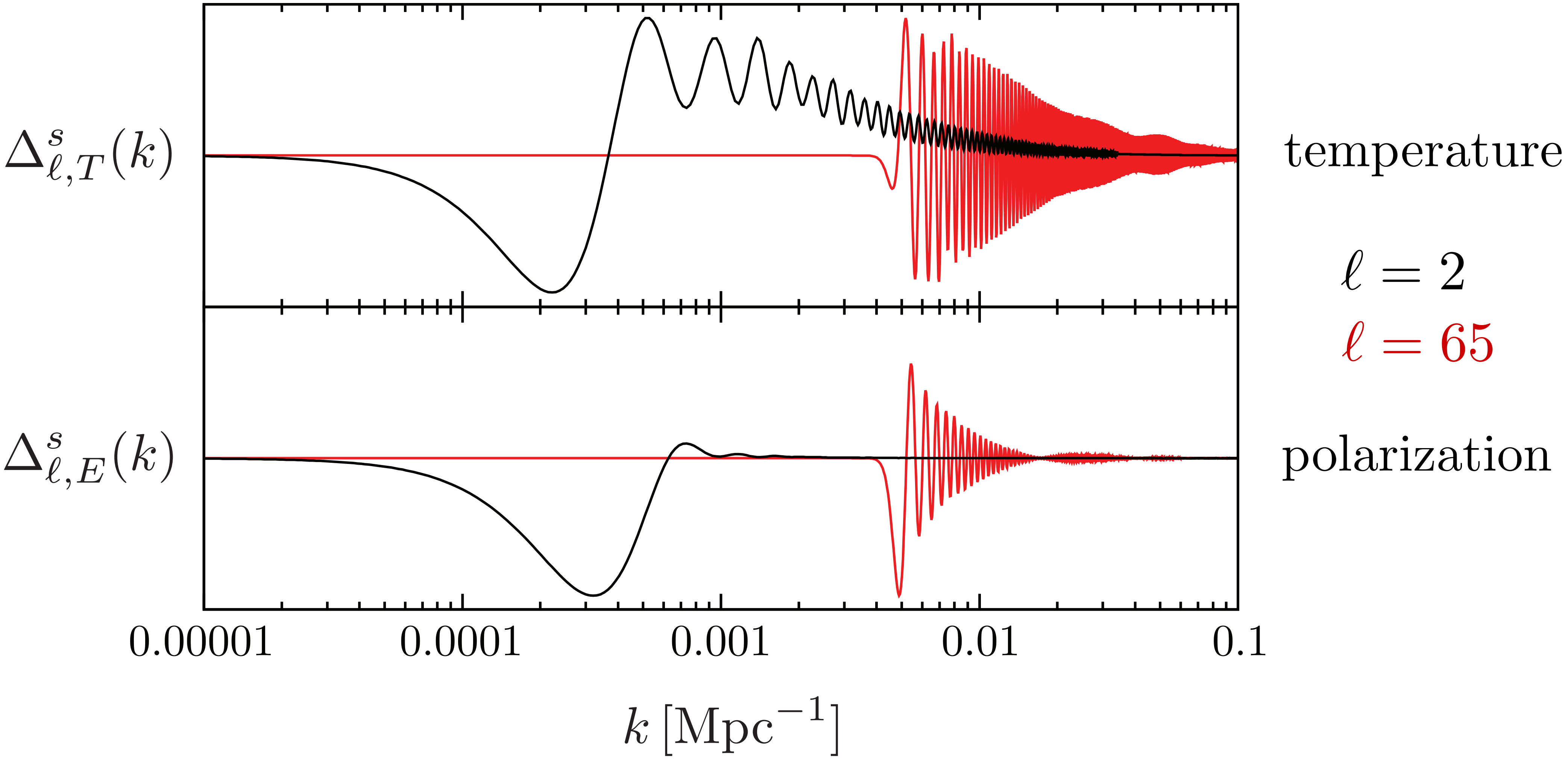
All cosmological observables are (computable) remappings of the primordial perturbations.

$$C_\ell = \int \frac{\mathrm{d}k}{k}\, \Delta_\ell^2(k)\, P(k)$$

transfer function
= evolution × projection

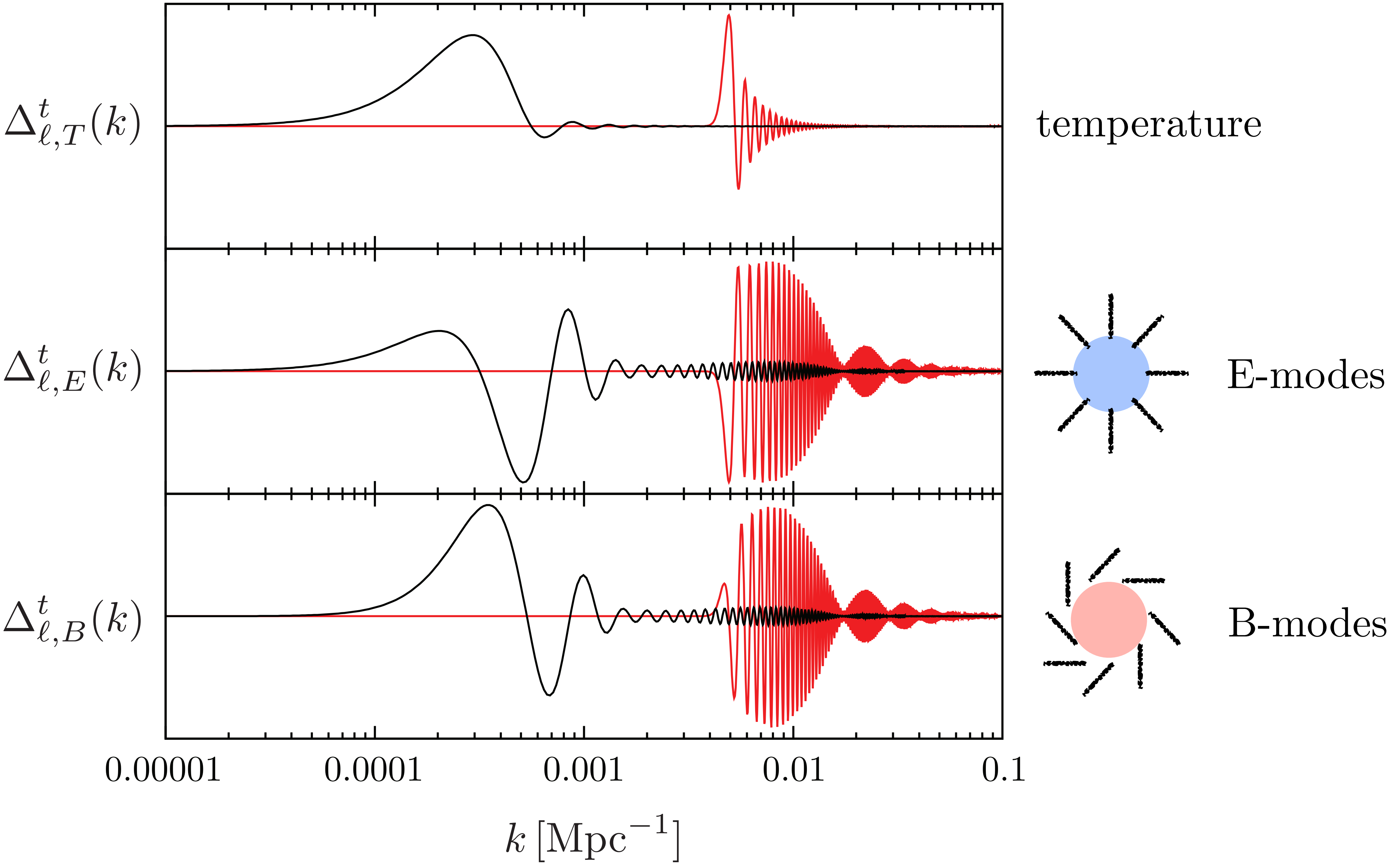
curvature perturbations

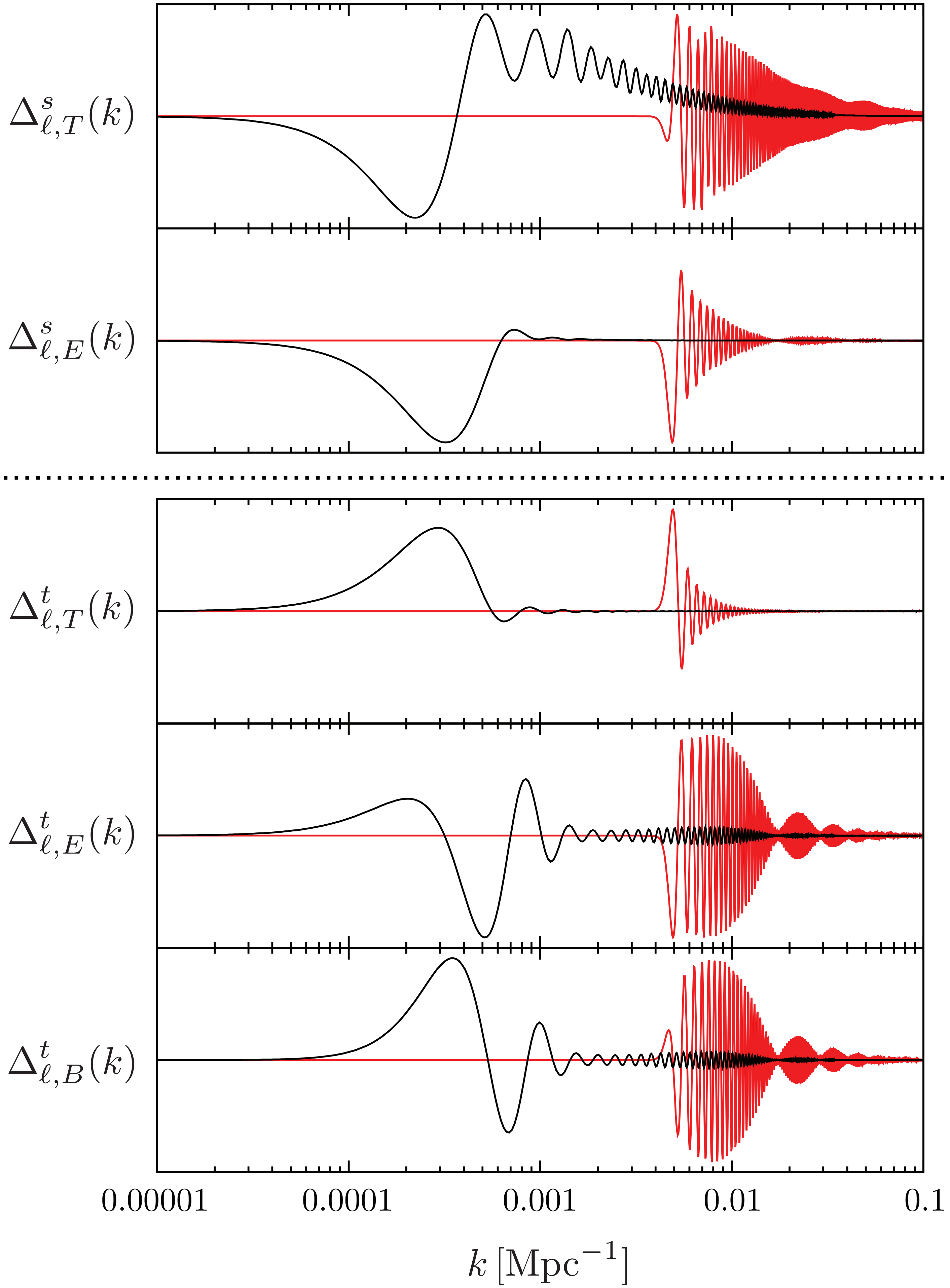
$$C_\ell^{XY} = \int \frac{\mathrm{d}k}{k}\,\Delta^s_{\ell,X}(k)\Delta^s_{\ell,Y}(k)\boxed{P_\zeta(k)}$$



gravitational waves

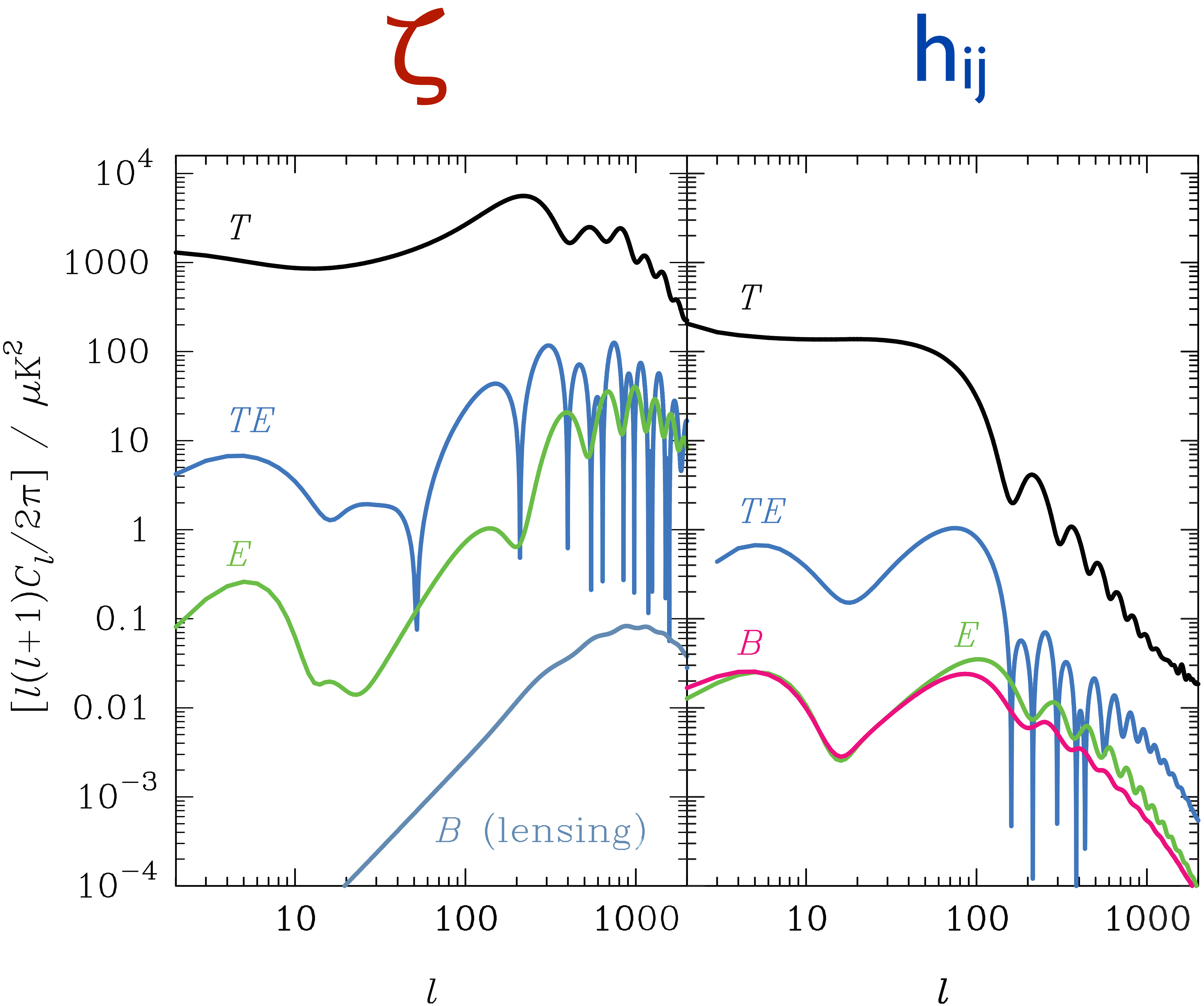
$$C_\ell^{XY} = \int \frac{\mathrm{d}k}{k}\, \Delta^t_{\ell,X}(k)\Delta^t_{\ell,Y}(k)\, \boxed{P_h(k)}$$

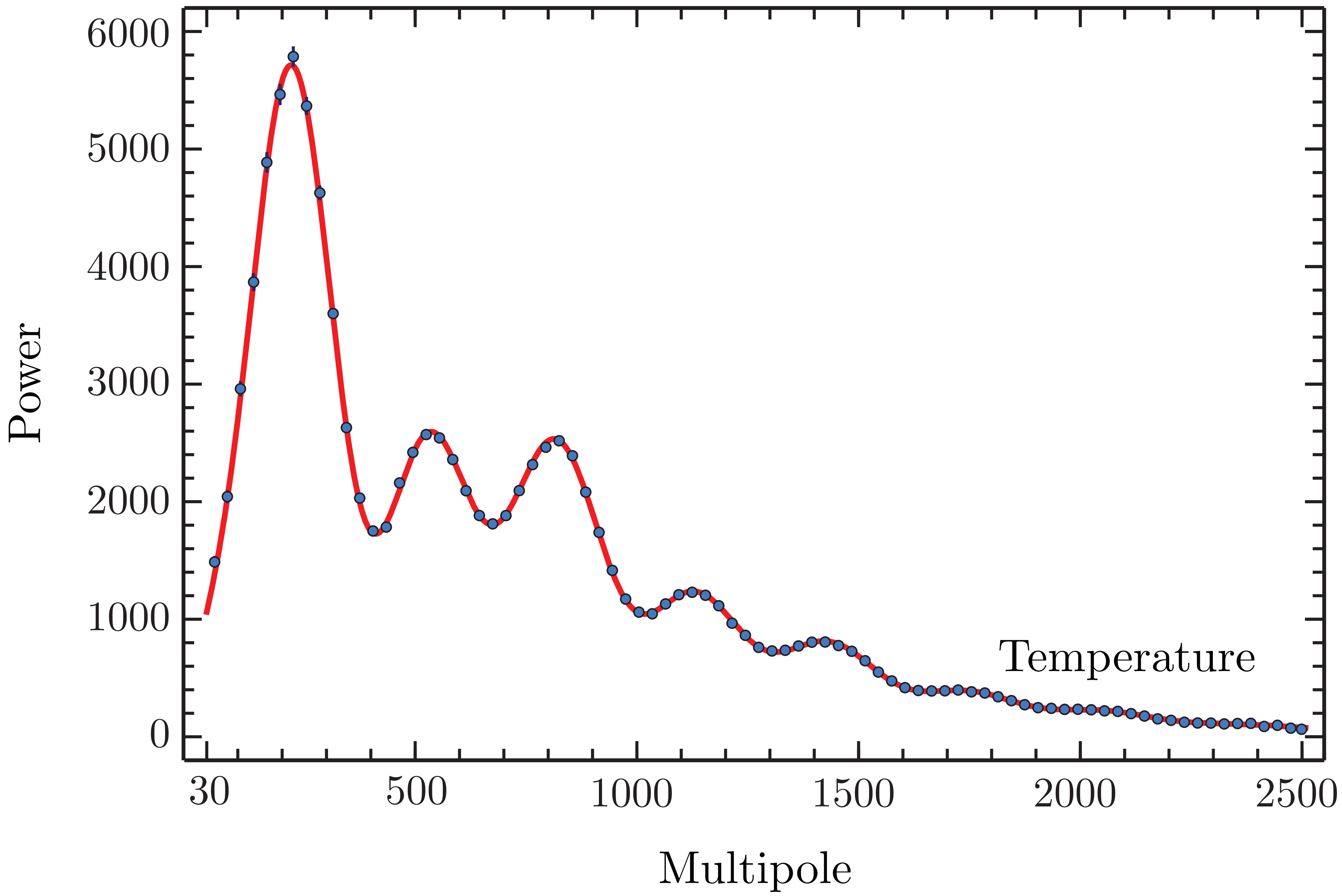


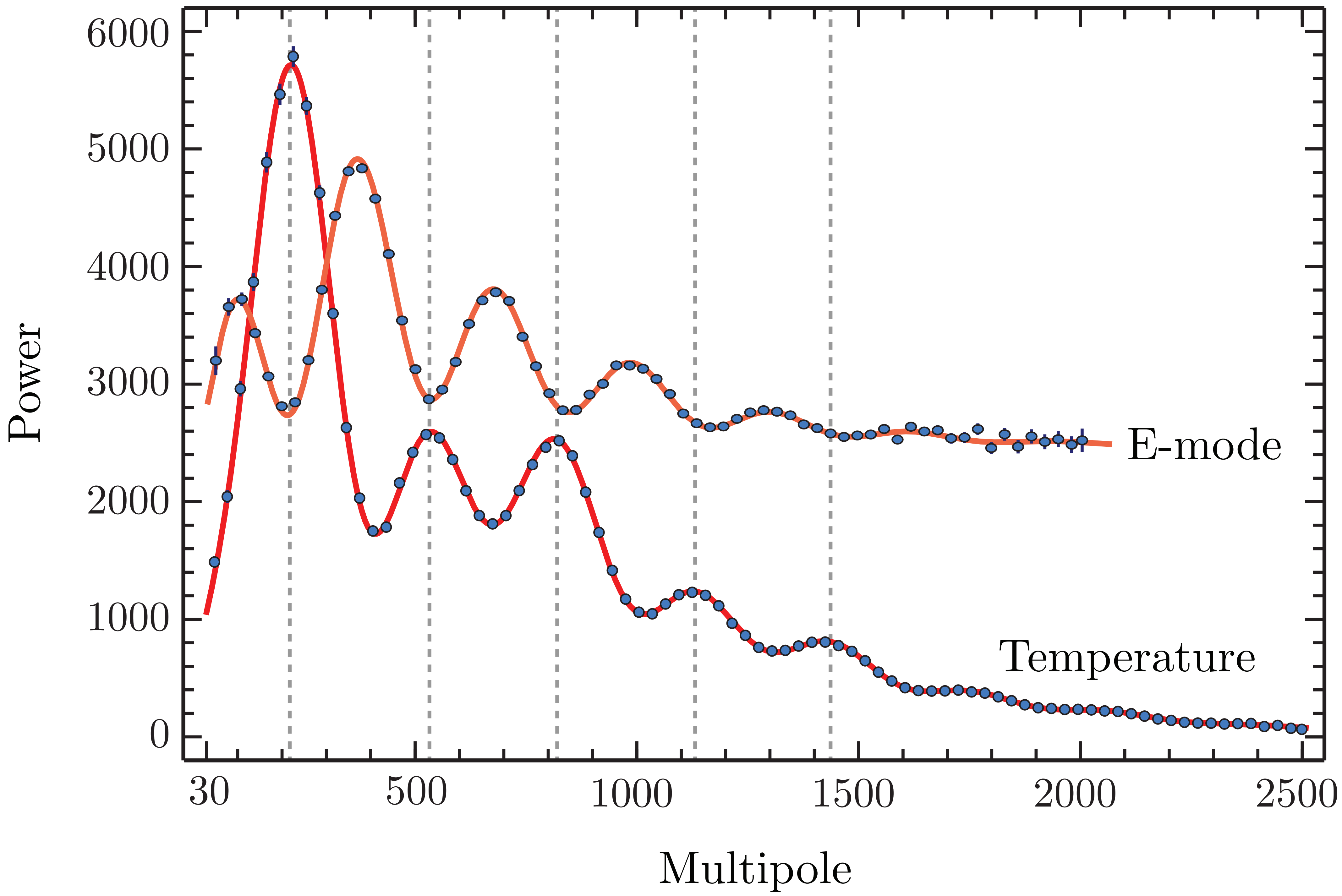


scalars

tensors


ζ
h_{ij}
10^4
1000
100
10
1
0.1
0.01
10^{-3}
10^{-4}
$[l(l+1)C_l/2\pi]\ /\ \mu K^2$
T
TE
E
B (lensing)
T
TE
B
E
10
100
1000
10
100
1000
l
l


Power
6000
5000
4000
3000
2000
1000
0
30
500
1000
1500
2000
2500
Multipole
Temperature


6000
5000
4000
3000
2000
1000
0
Power
E-mode
Temperature
30
500
1000
1500
2000
2500
Multipole

Given that we understand the evolution so well, we can use the observations to probe the ***initial conditions.***

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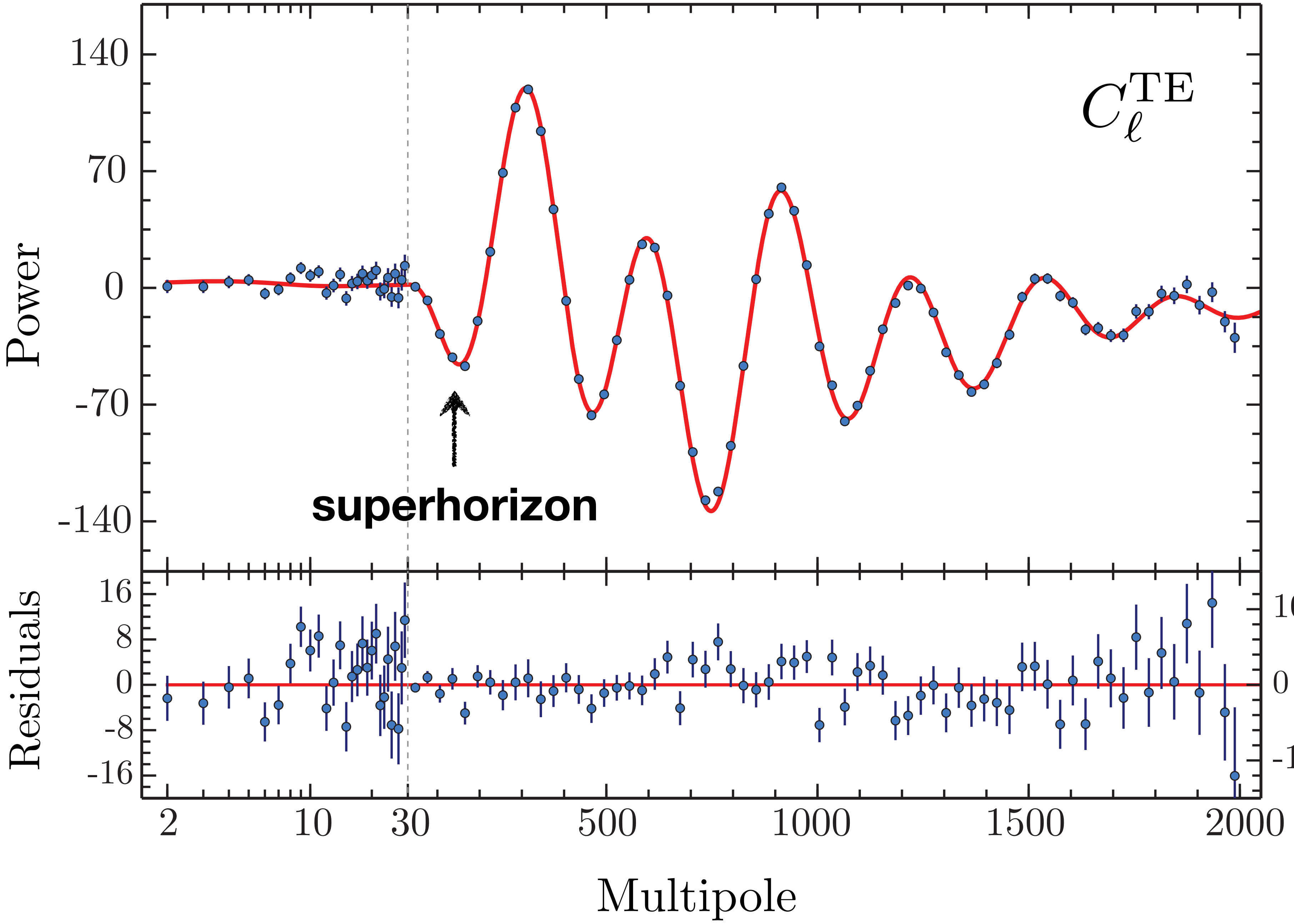
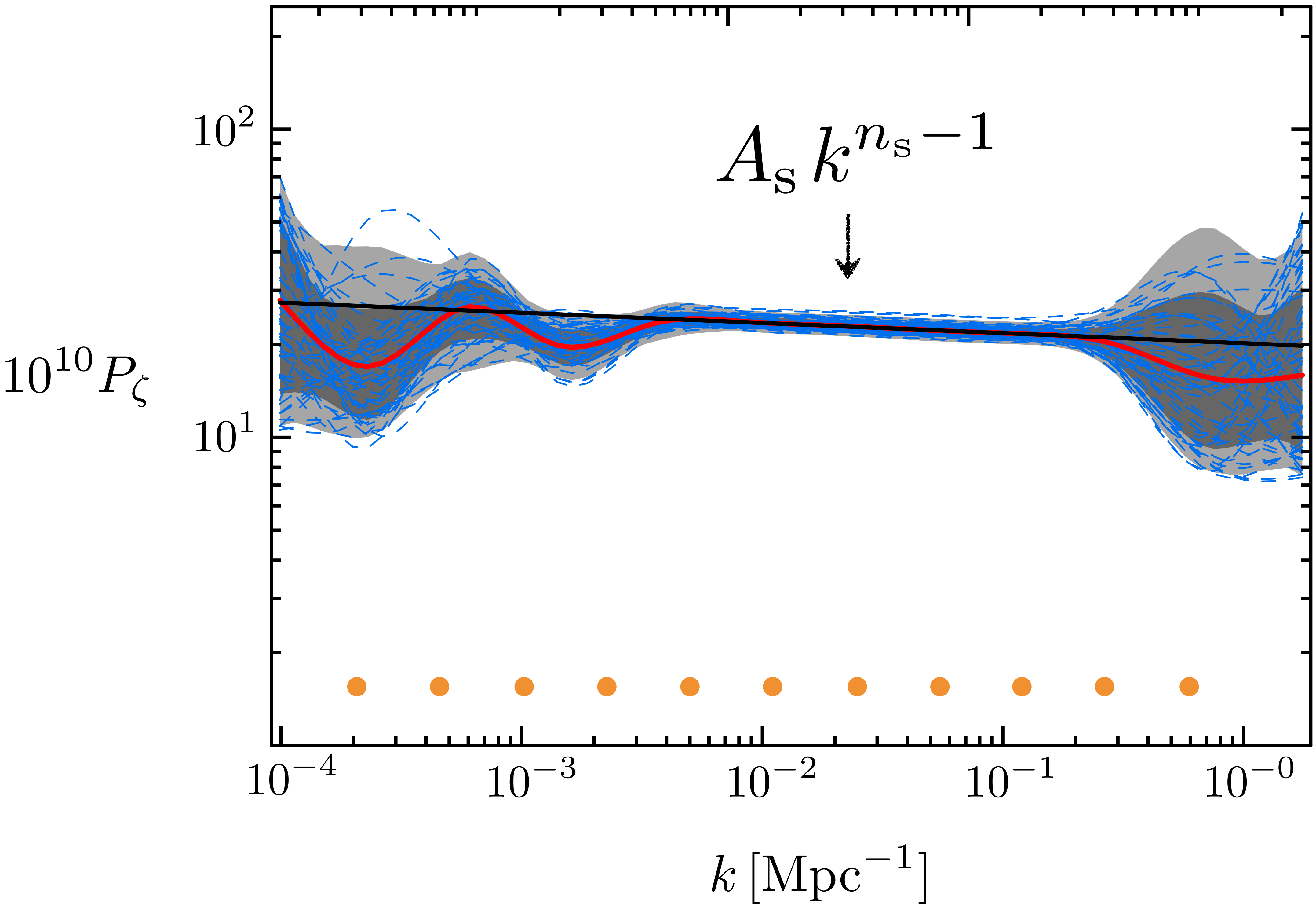
What do we really know?

1. *Perturbations existed on* ***superhorizon scales*** *at recombination:*

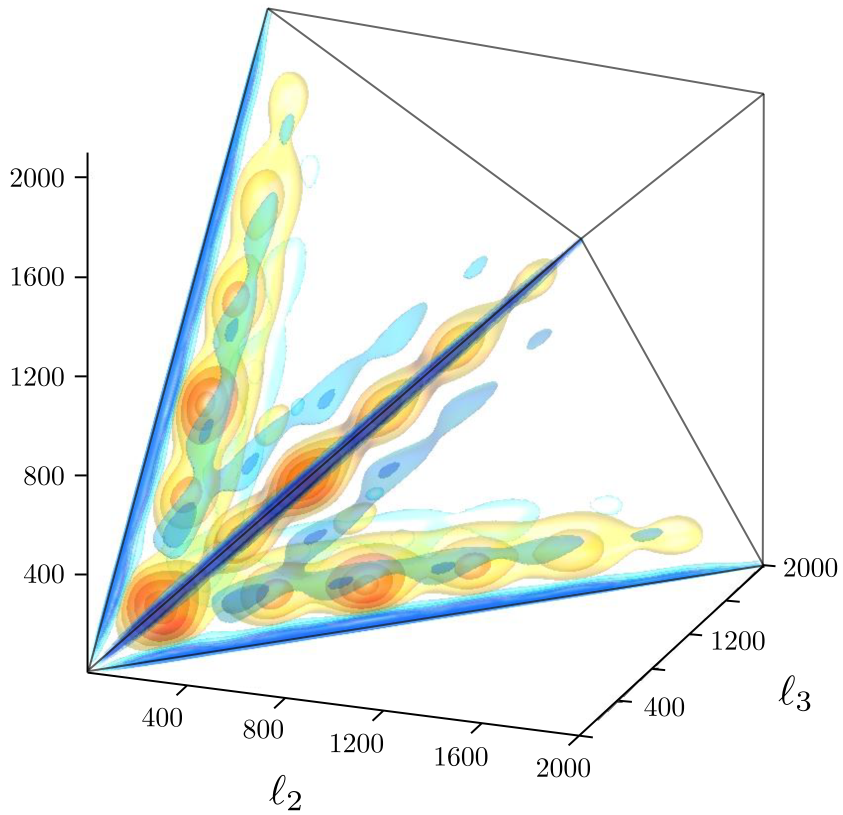


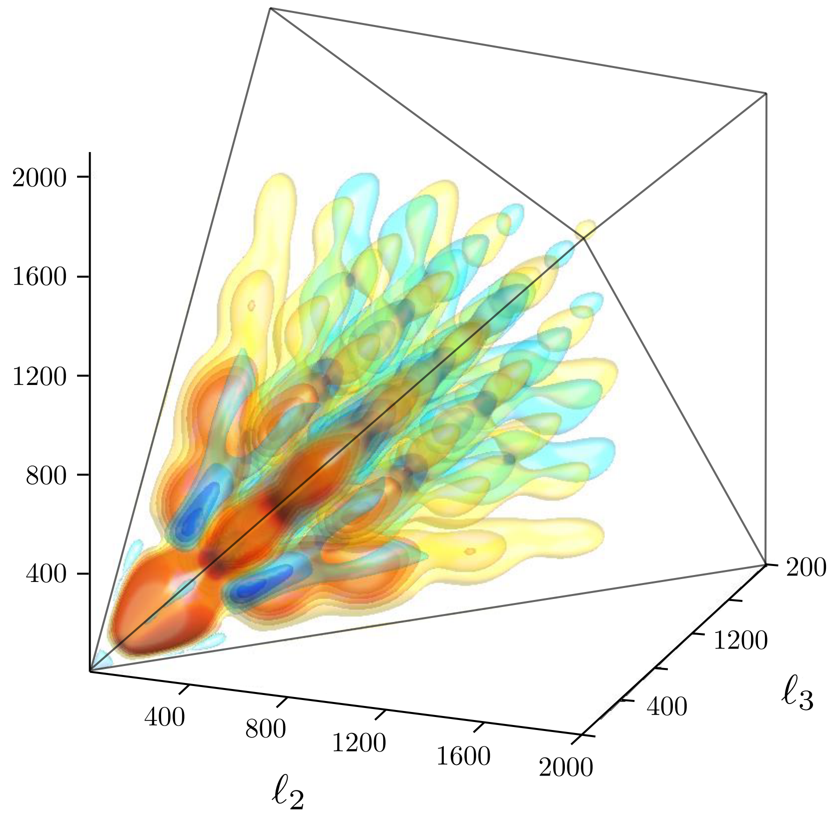
Implies that the perturbations were laid down ***before the hot big bang****.*

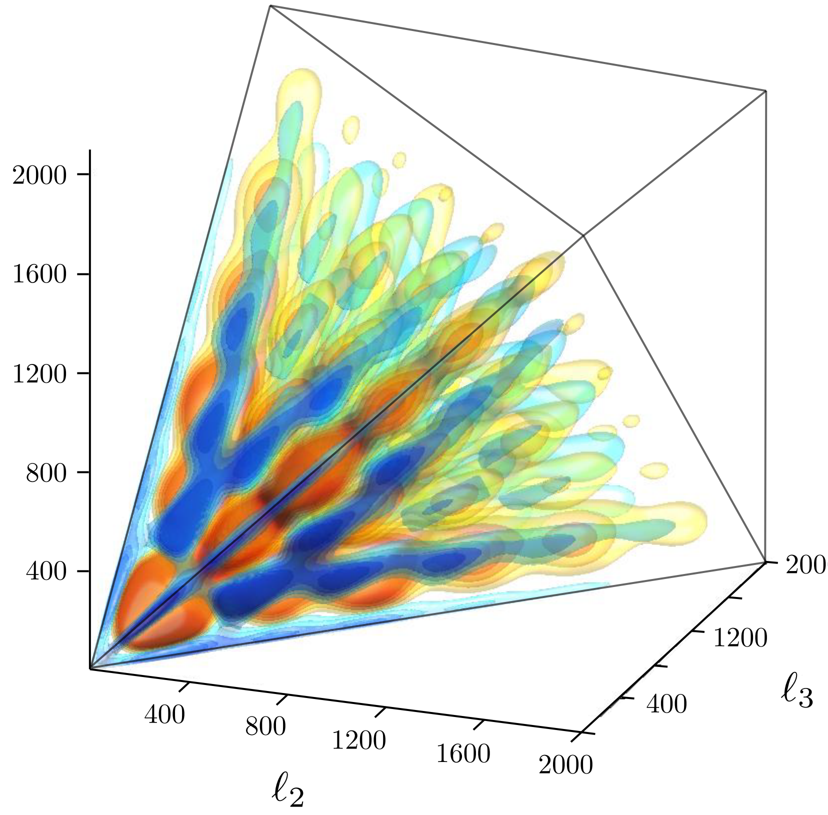
2. *Perturbations are* ***scale invariant*** *over a large range of scales:*



3. *Perturbations are **Gaussian** (to a good approximation):*


local


equilateral


orthogonal

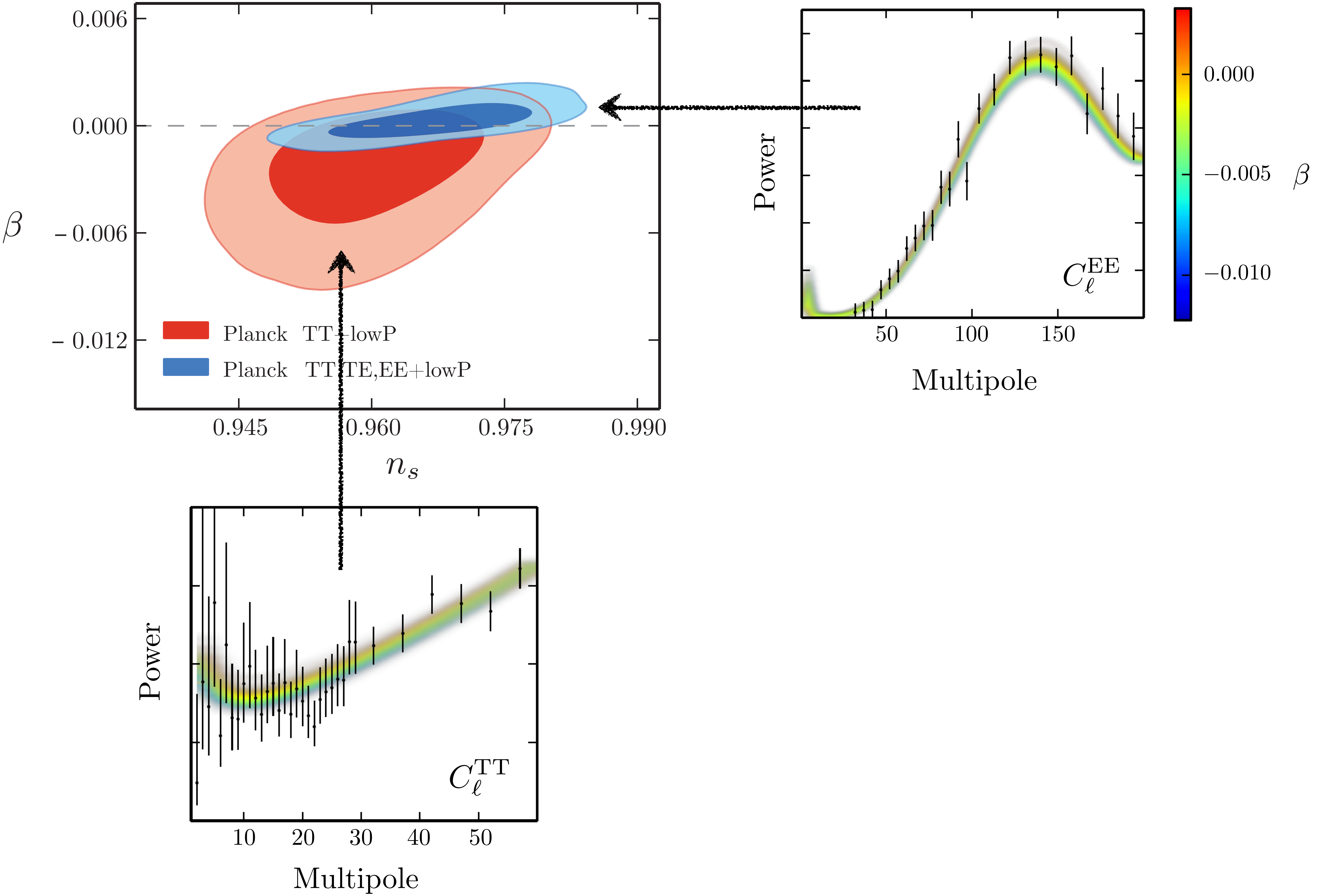
Temperature only:

$$f_{\rm NL}^{\rm local} = 1.8 \pm 5.6 \qquad f_{\rm NL}^{\rm equil} = -9.2 \pm 69 \qquad f_{\rm NL}^{\rm ortho} = -20 \pm 33$$

With polarization:

$$f_{\rm NL}^{\rm local} = 0.7 \pm 5.1 \qquad f_{\rm NL}^{\rm equil} = -9.5 \pm 44 \qquad f_{\rm NL}^{\rm ortho} = -25 \pm 22$$

4. *Perturbations are **adiabatic** (to a good approximation):*



From the CMB observations we have learned that the primordial perturbations

1. *existed before the big bang,*
2. *are nearly scale invariant,*
3. *are close to Gaussian,*
4. *are adiabatic.*

but what created these initial condition?

From the CMB observations we have learned that the primordial perturbations

1. *existed before the big bang,*
2. *are nearly scale invariant,*
3. *are close to Gaussian,*
4. *are adiabatic.*

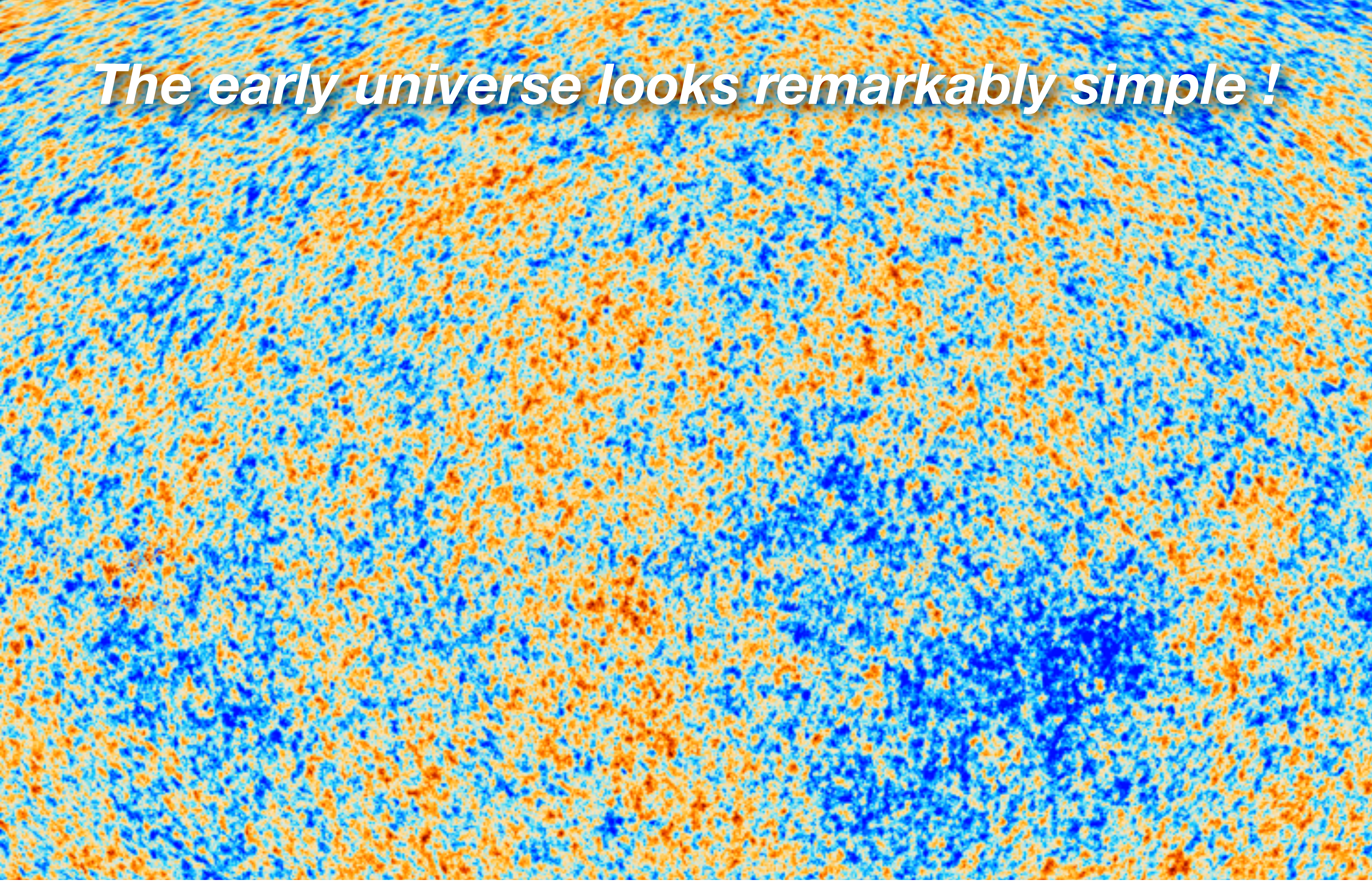
$$\left.\right\} \quad \epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1$$

but what created these initial condition?

What is the physics of inflation?

Randomness during Inflation

The early universe looks remarkably simple !

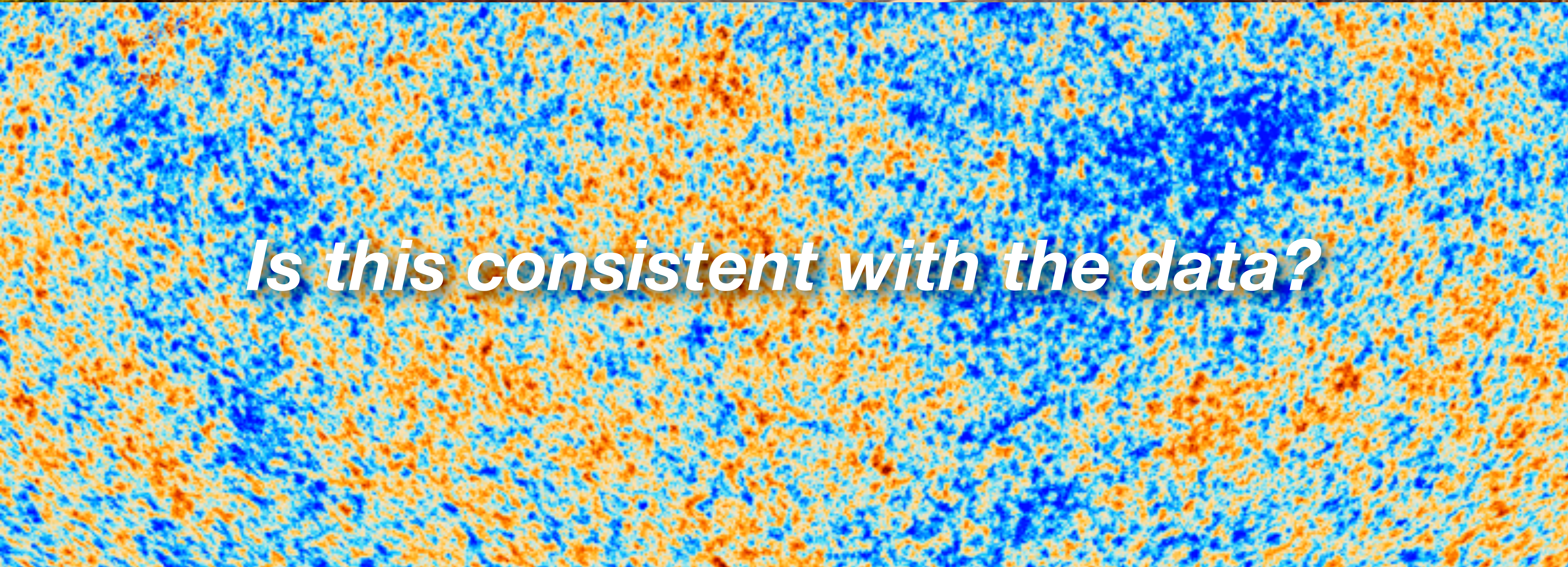


A nearly scale-invariant two-point function describes everything.

Ultraviolet theories seem remarkably complex !

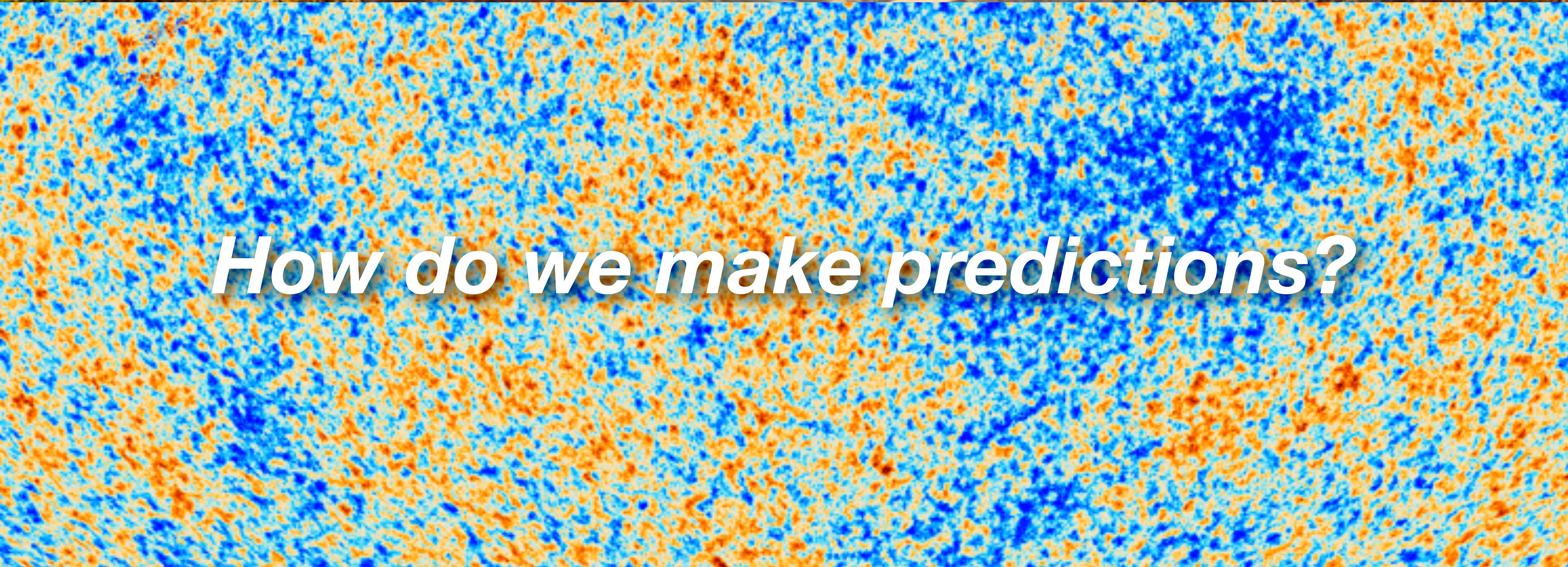
Many ingredients have to be carefully arranged to give rise to inflation.

What if some randomness survives?



Is this consistent with the data?

What if some randomness survives?

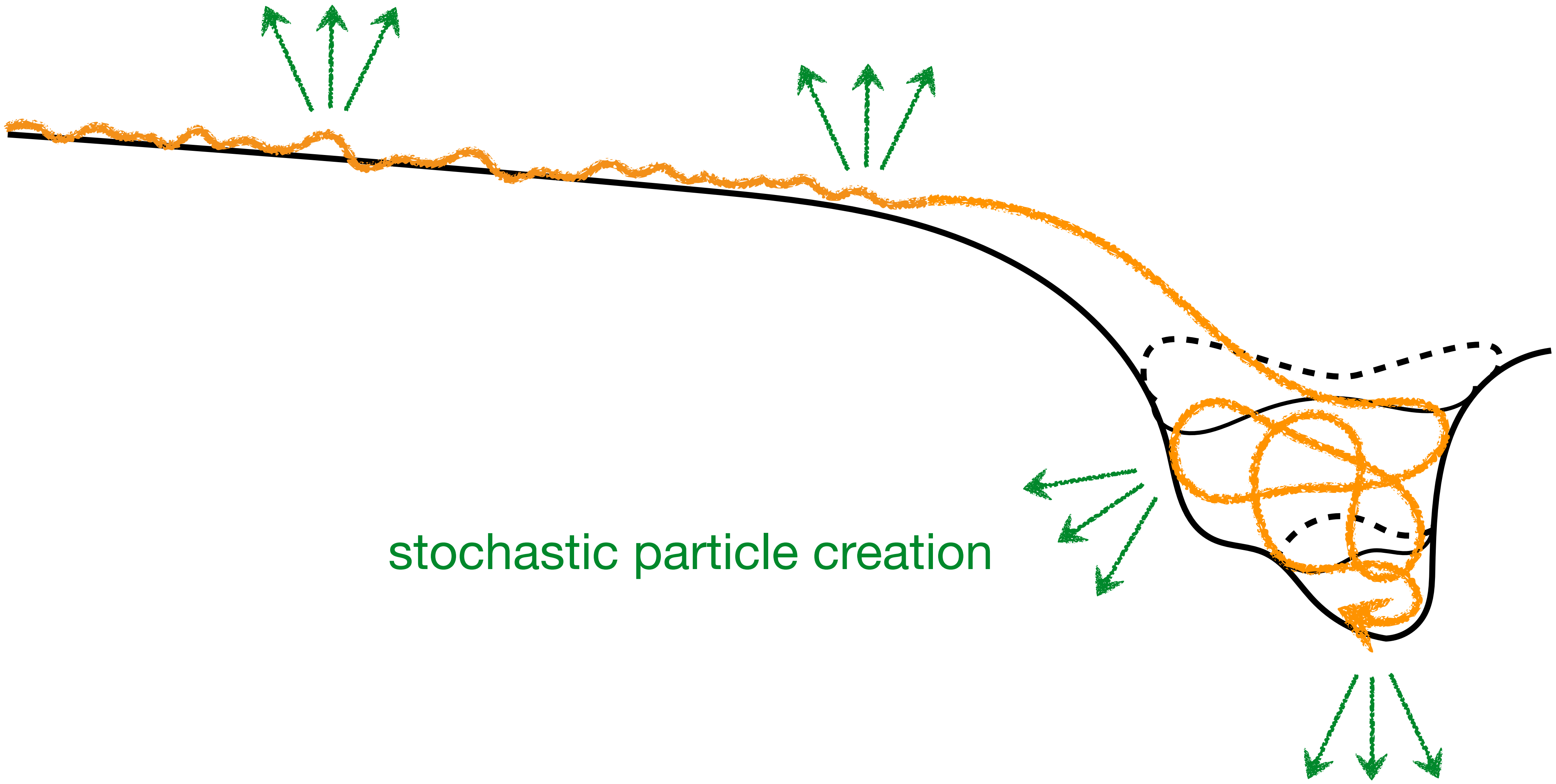


How do we make predictions?

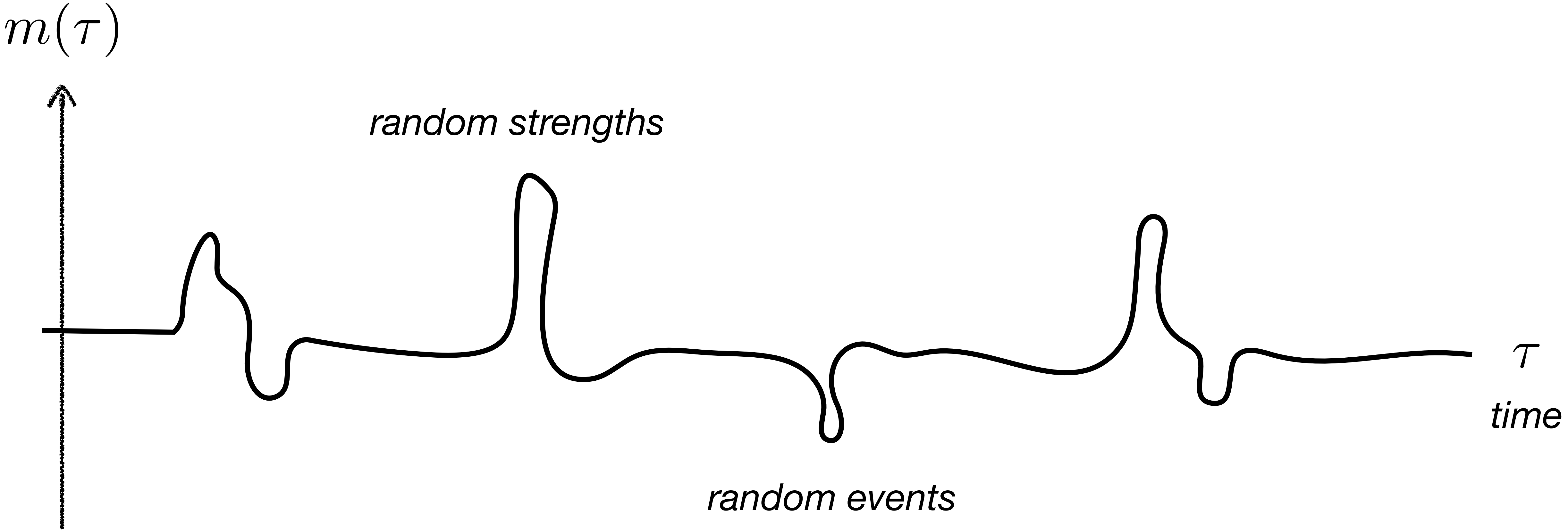
What if some randomness survives?

$$\mathcal{L}(\phi^a) = \bar{\mathcal{L}}(\phi^a) + \delta\mathcal{L}(\phi^a)$$

random



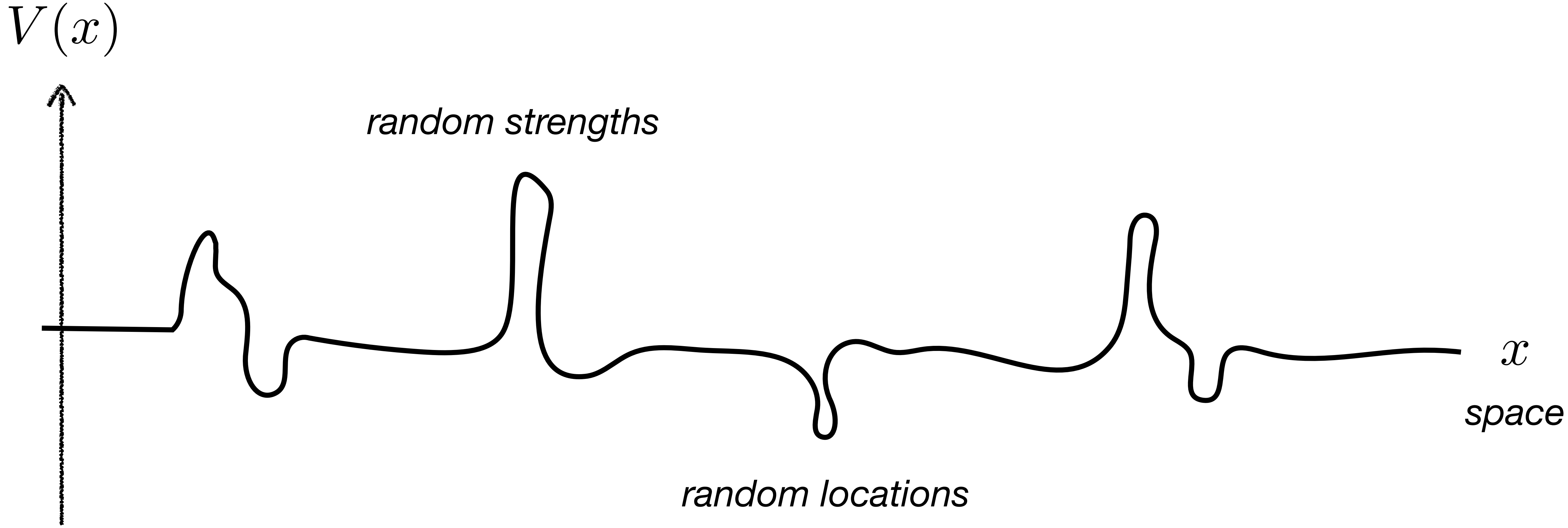
Fields may have time-dependent couplings:



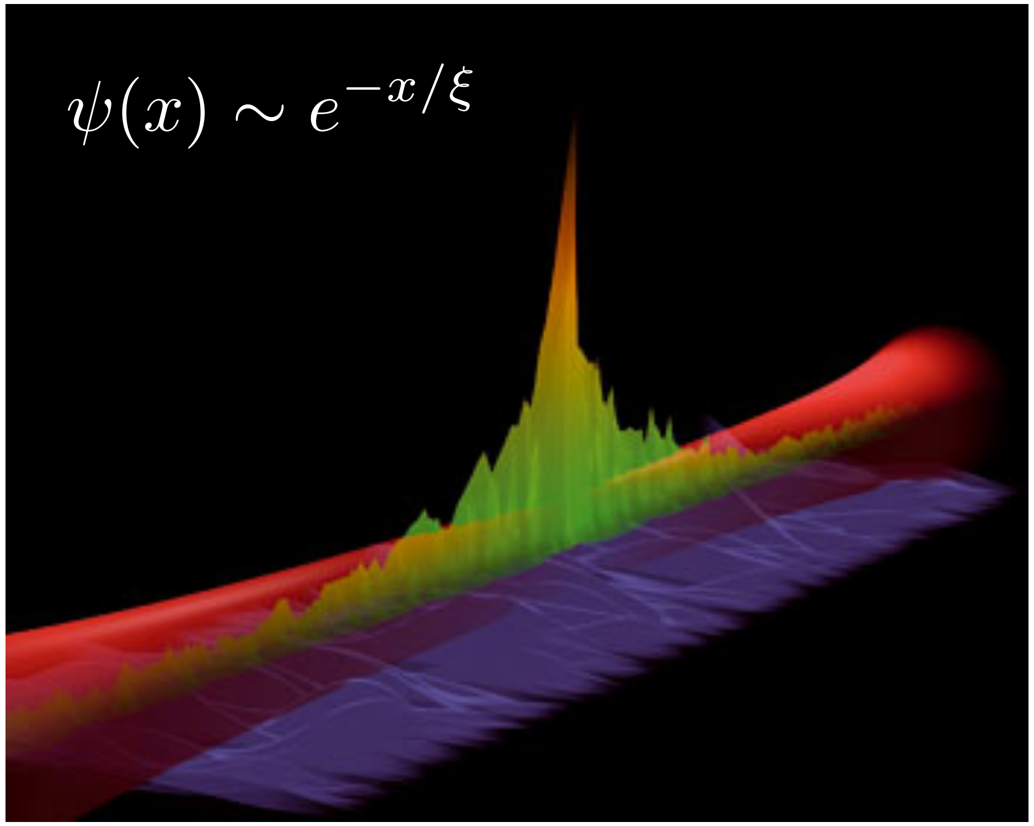
How do we compute in such scenarios?

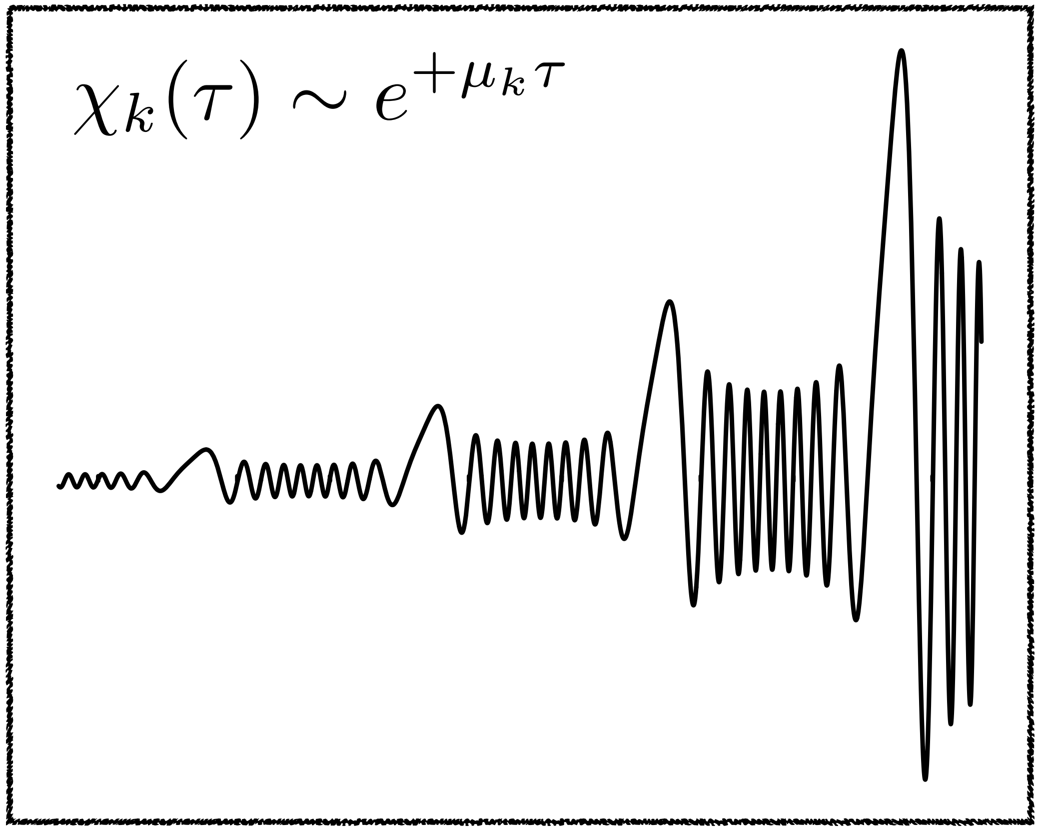
We will take inspiration from an unusual source:

current transmission in wires



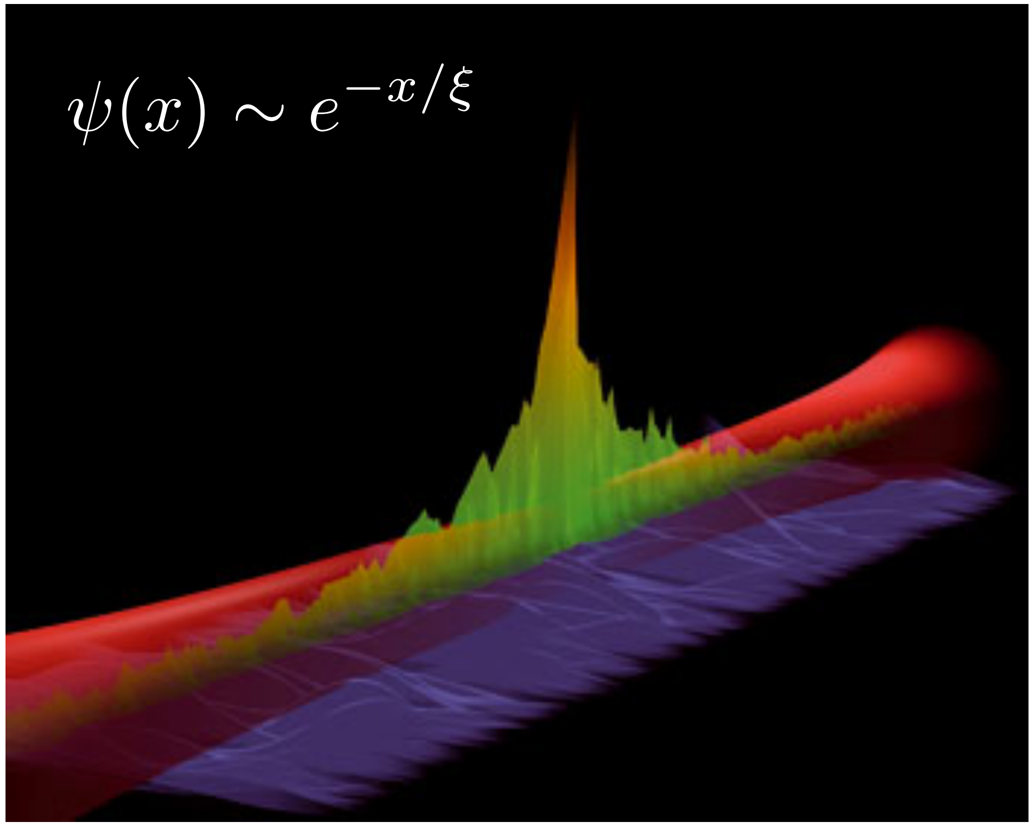
I will demonstrate a mathematical equivalence between resistance in disordered wires and stochastic particle production in cosmology.

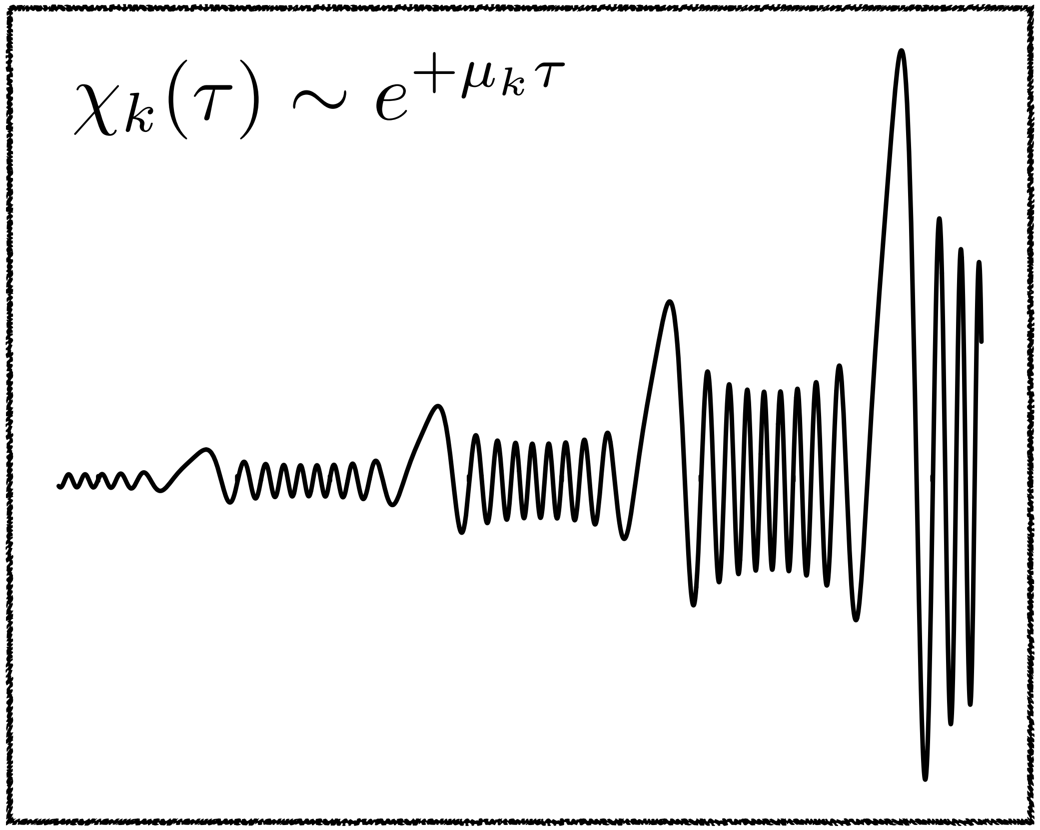




Anderson localization = **exponential particle creation**

$$\rho(L) \sim e^{L/\xi} \qquad\qquad n(T) \sim e^{+\mu T}$$





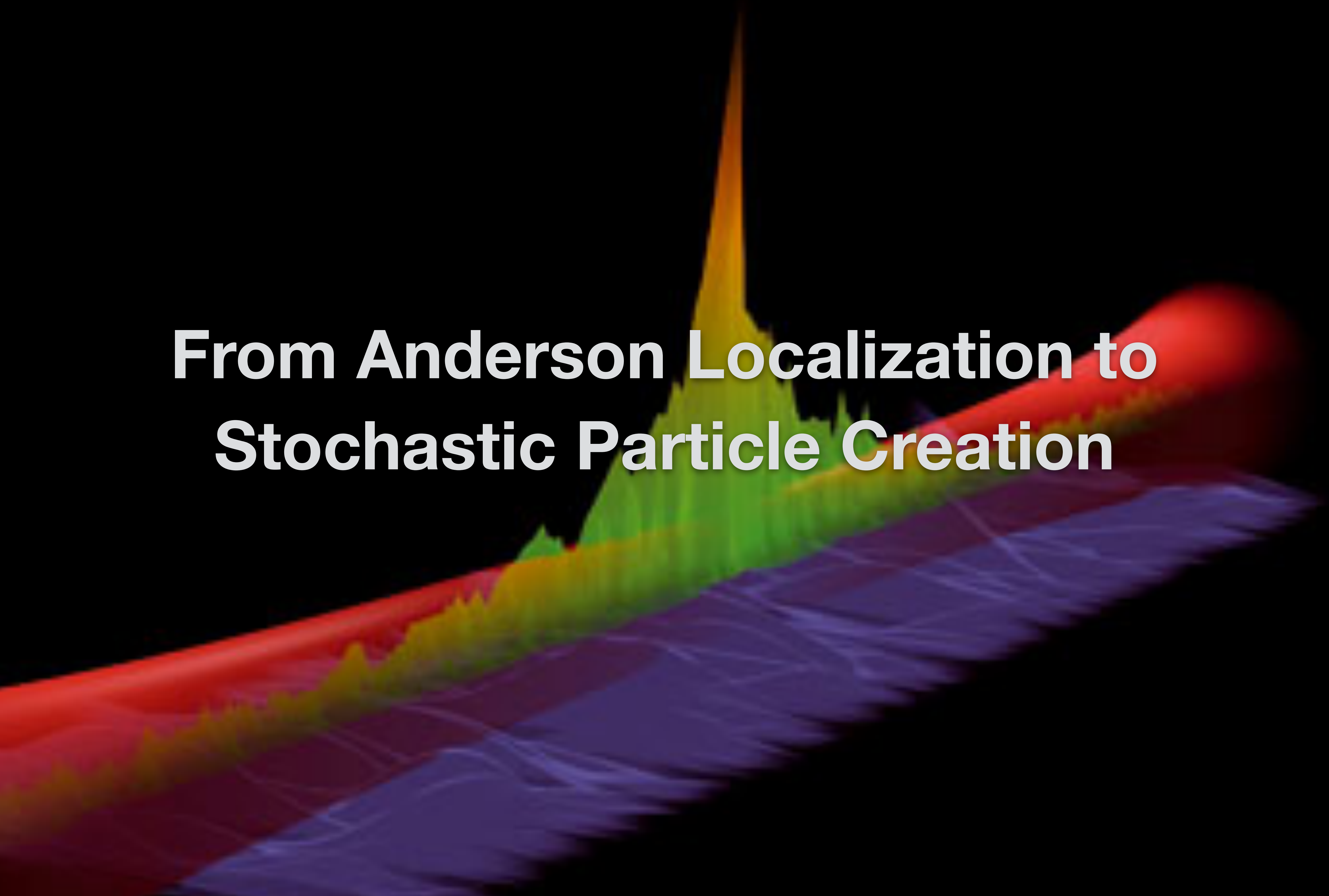
Anderson localization = **exponential particle creation**

$$\rho(L) \sim e^{L/\xi} \qquad n(T) \sim e^{+\mu T}$$

Simplicity from Complexity?

In condensed matter, emergent universal behaviour is what allows predictive power in spite of the underlying complexity of materials.

From Anderson Localization to Stochastic Particle Creation

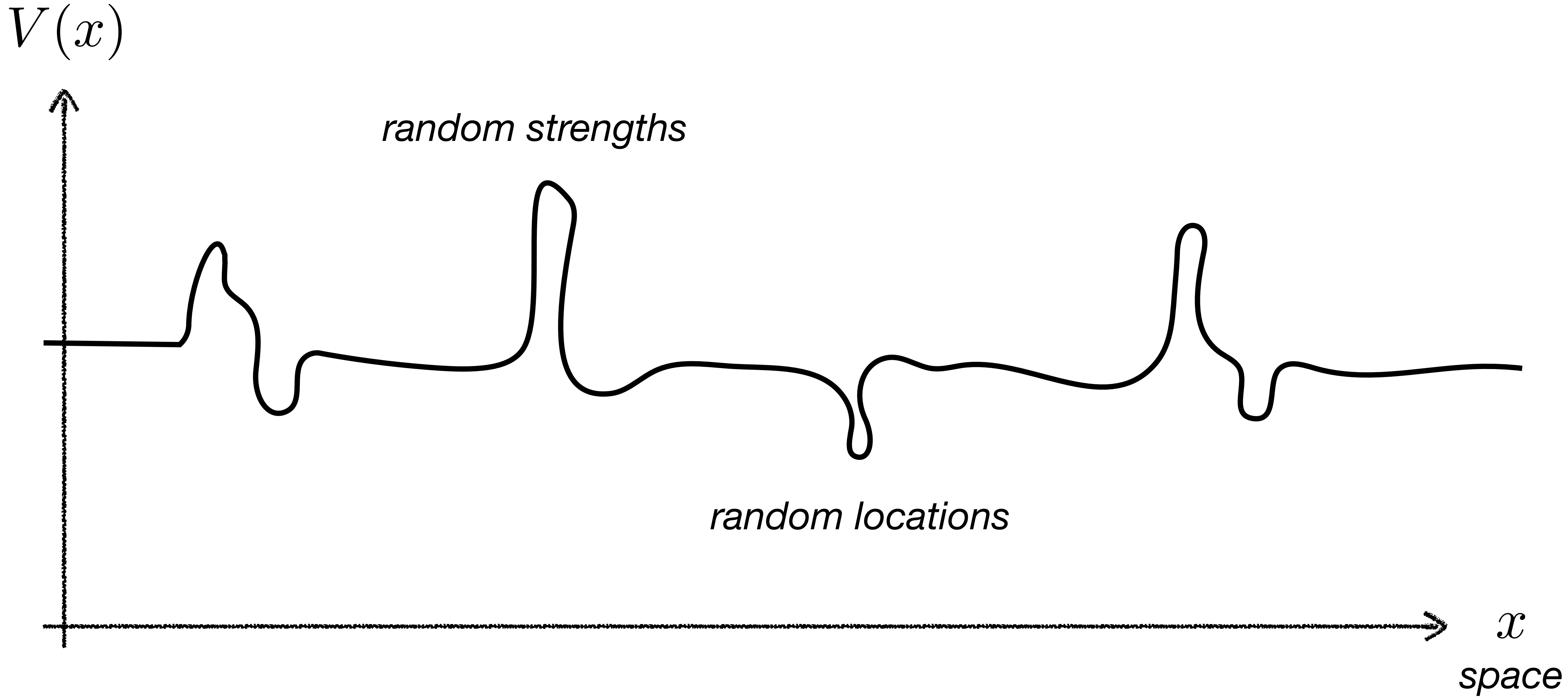


Current Transmission

Conduction in wires is described by the time-independent Schrödinger equation:

$$\boxed{\frac{d^2\psi}{dx^2} + [E - V(x)]\,\psi = 0}$$

impurities

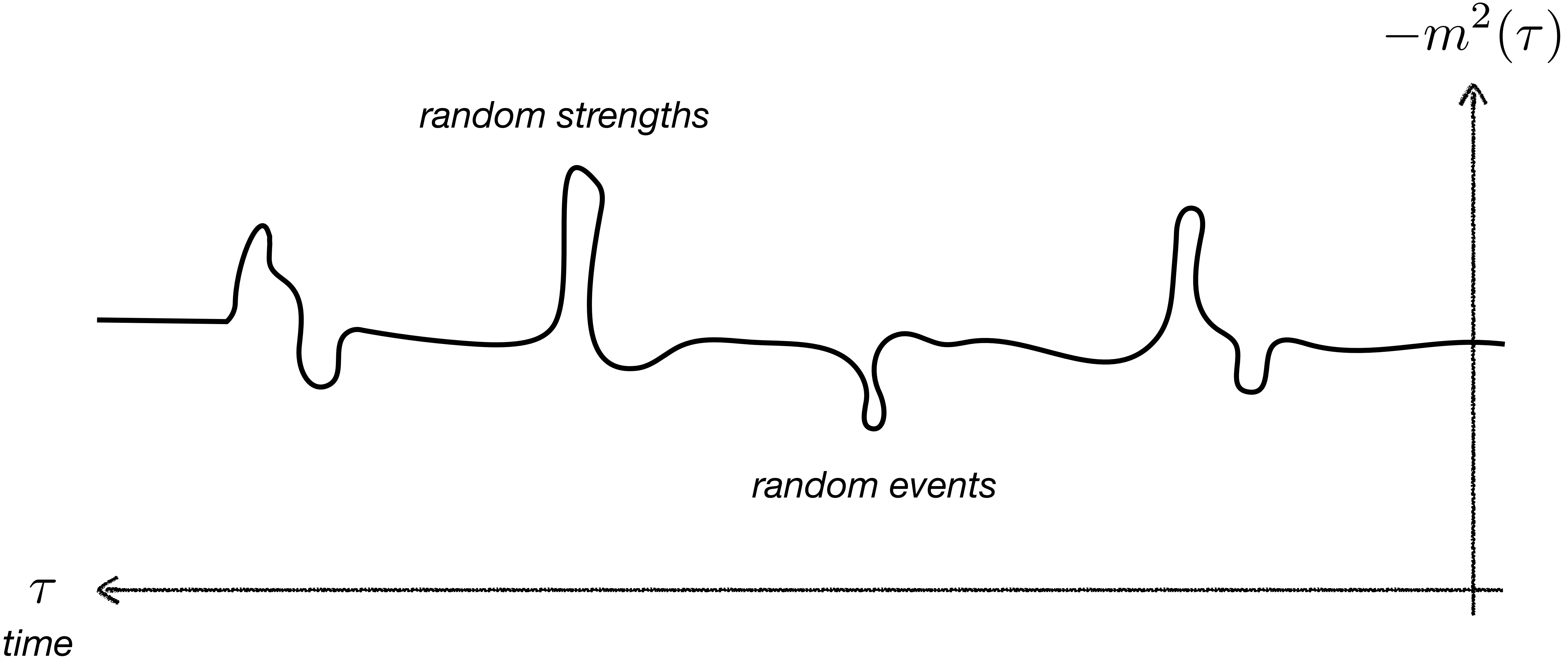


Particle Production

Particle production is describes by the time-dependent Klein-Gordon equation:

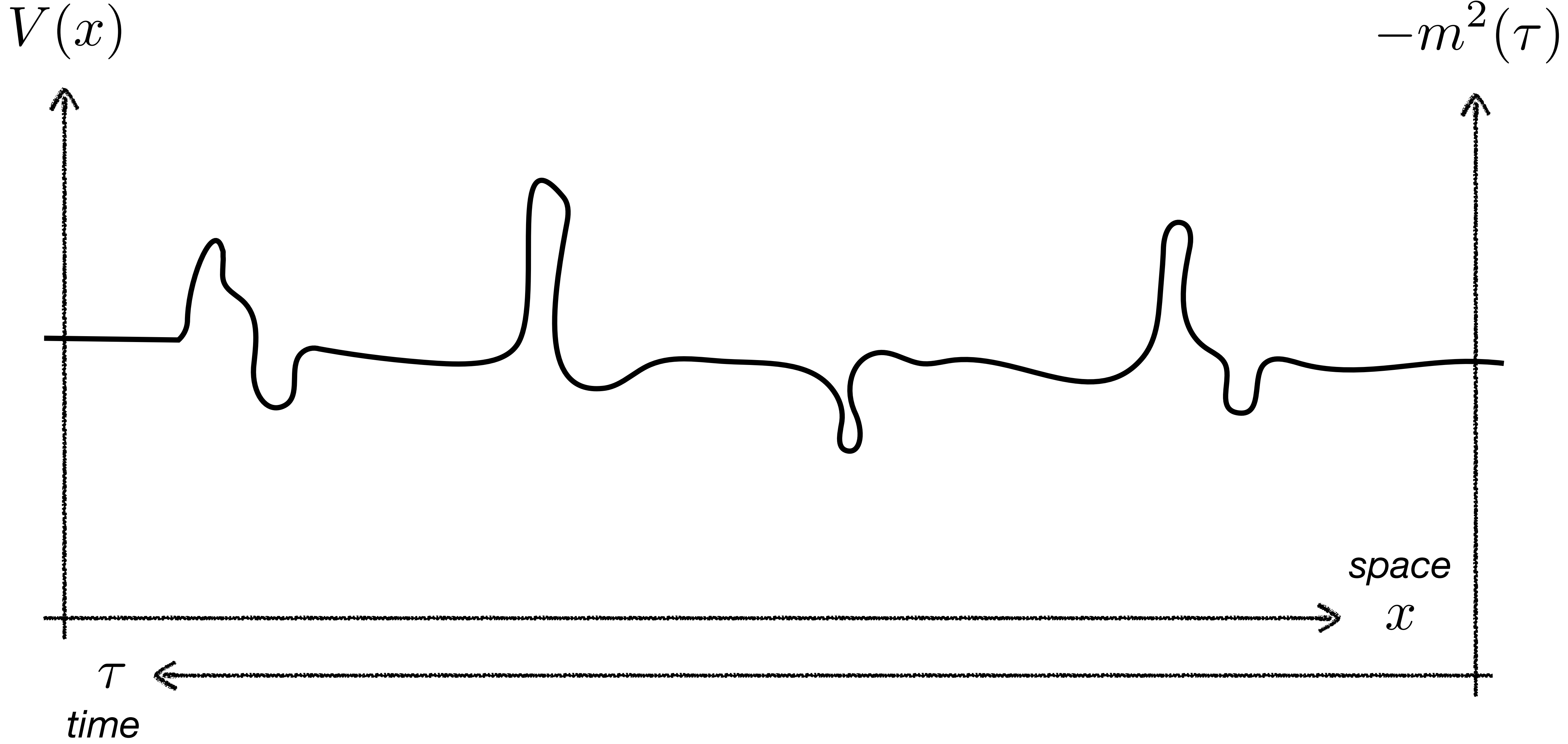
$$\frac{d^2\chi_{\vec{k}}}{d\tau^2} + \left[k^2 + m^2(\tau)\right]\chi_{\vec{k}} = 0$$

non-adiabatic events



From Wires to Cosmology

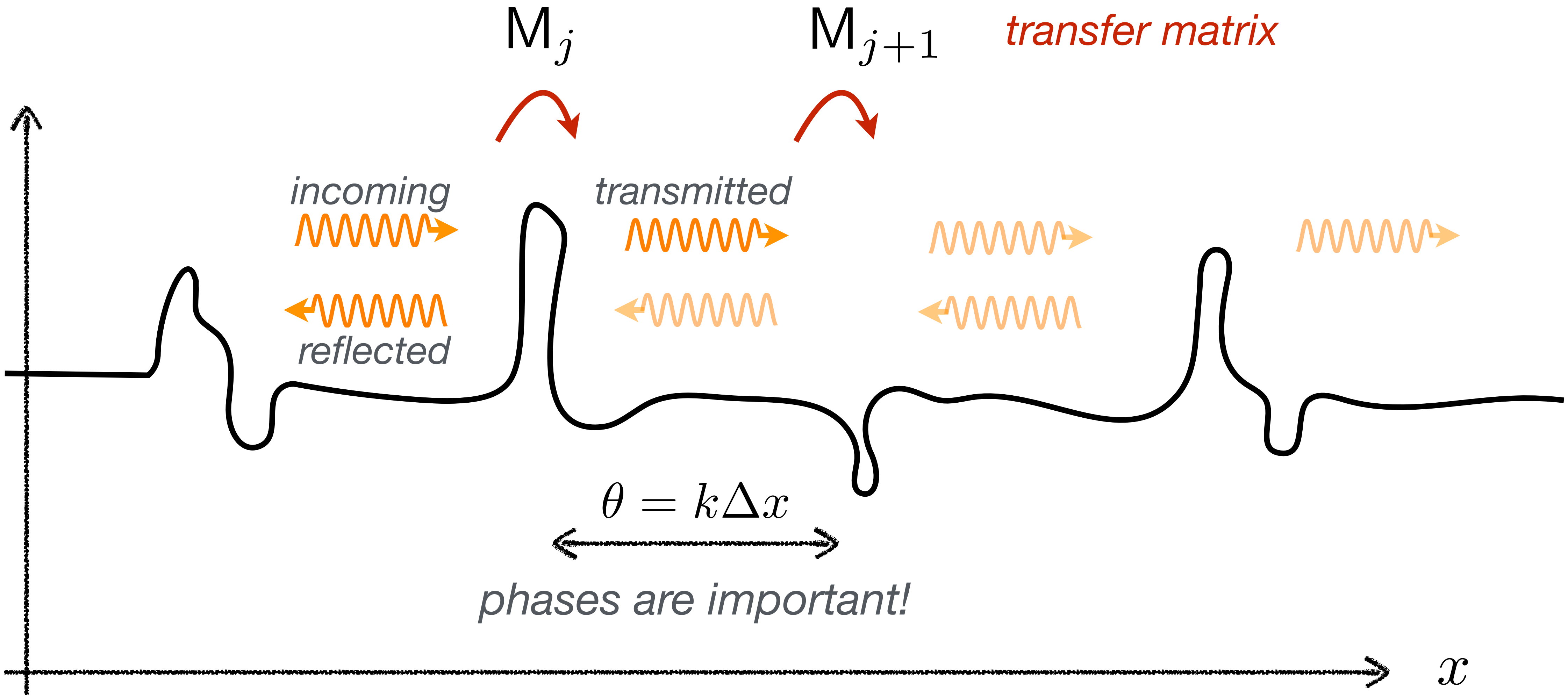
$$\frac{d^2\psi}{dx^2} + [E - V(x)]\,\psi = 0$$



$$\frac{d^2\chi_{\vec{k}}}{d\tau^2} + \left[k^2 + m^2(\tau)\right]\chi_{\vec{k}} = 0$$

Anderson Localization

$$|\psi(L)\rangle = \cdots \mathsf{M}_{j+1}\mathsf{M}_j \cdots |\psi(0)\rangle$$



M_j

M_{j+1}

transfer matrix

incoming

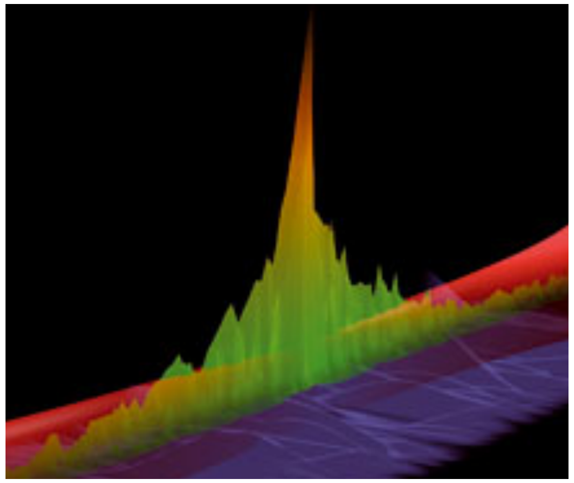
transmitted

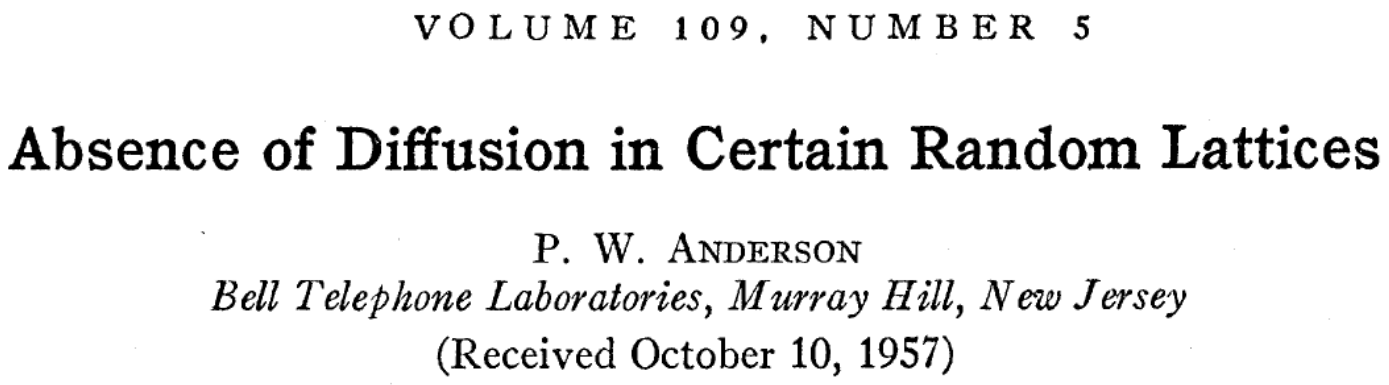
reflected

$\theta = k\Delta x$

phases are important!

x





VOLUME 109, NUMBER 5

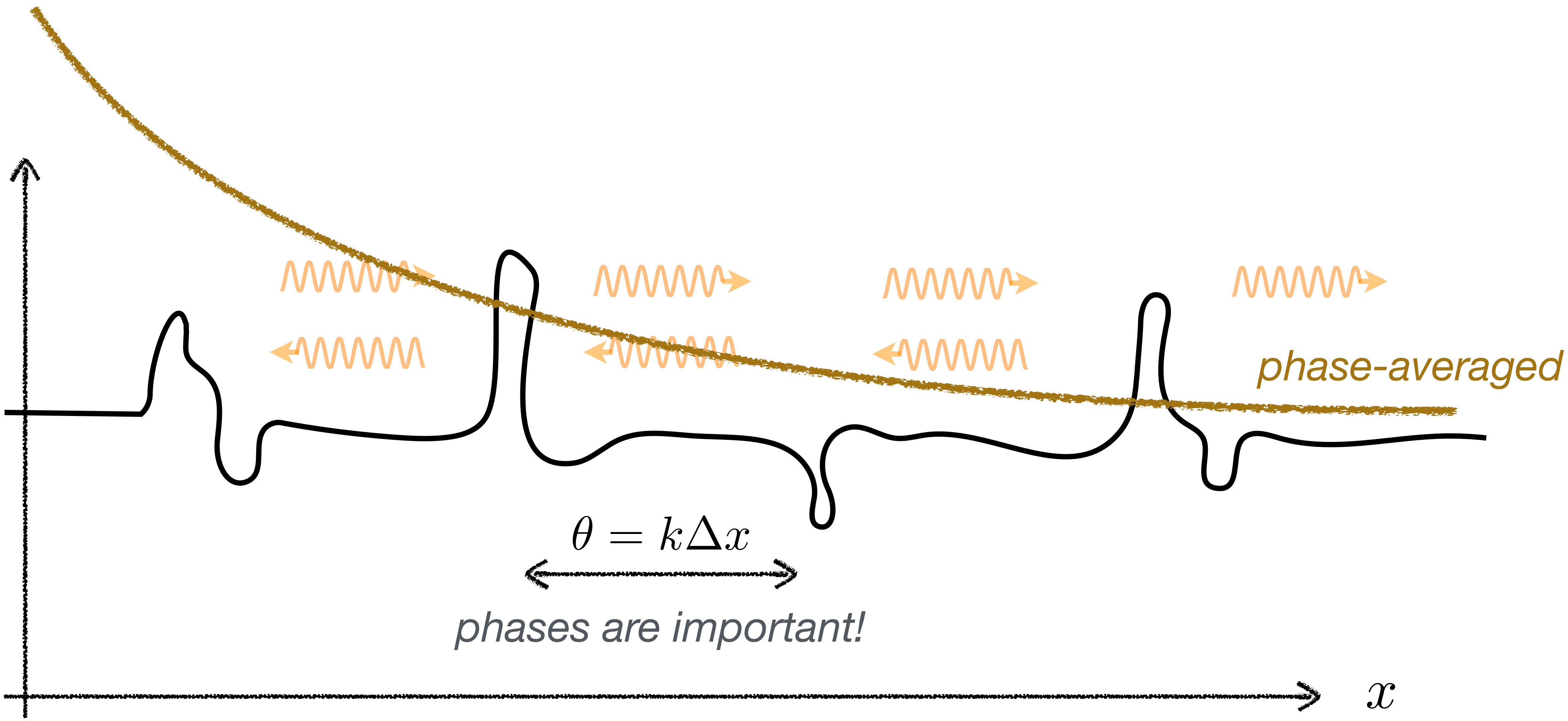
Absence of Diffusion in Certain Random Lattices

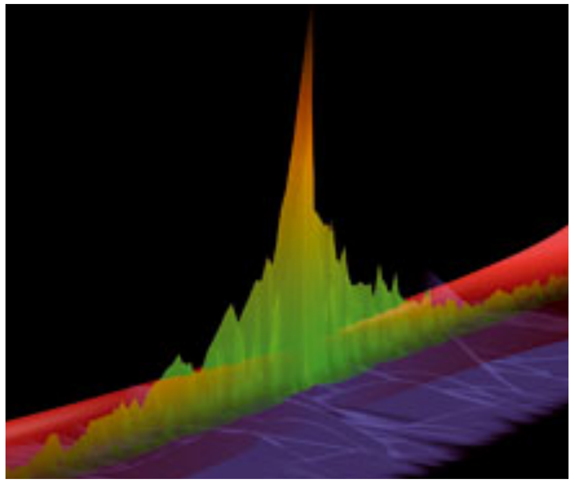
P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey

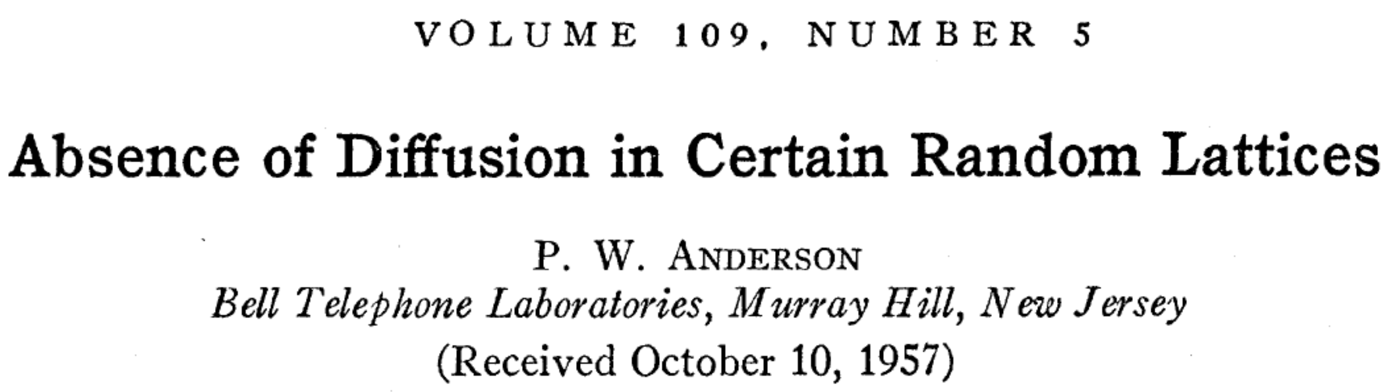
(Received October 10, 1957)

Anderson Localization

$$\psi(x) \sim e^{-x/\xi}$$







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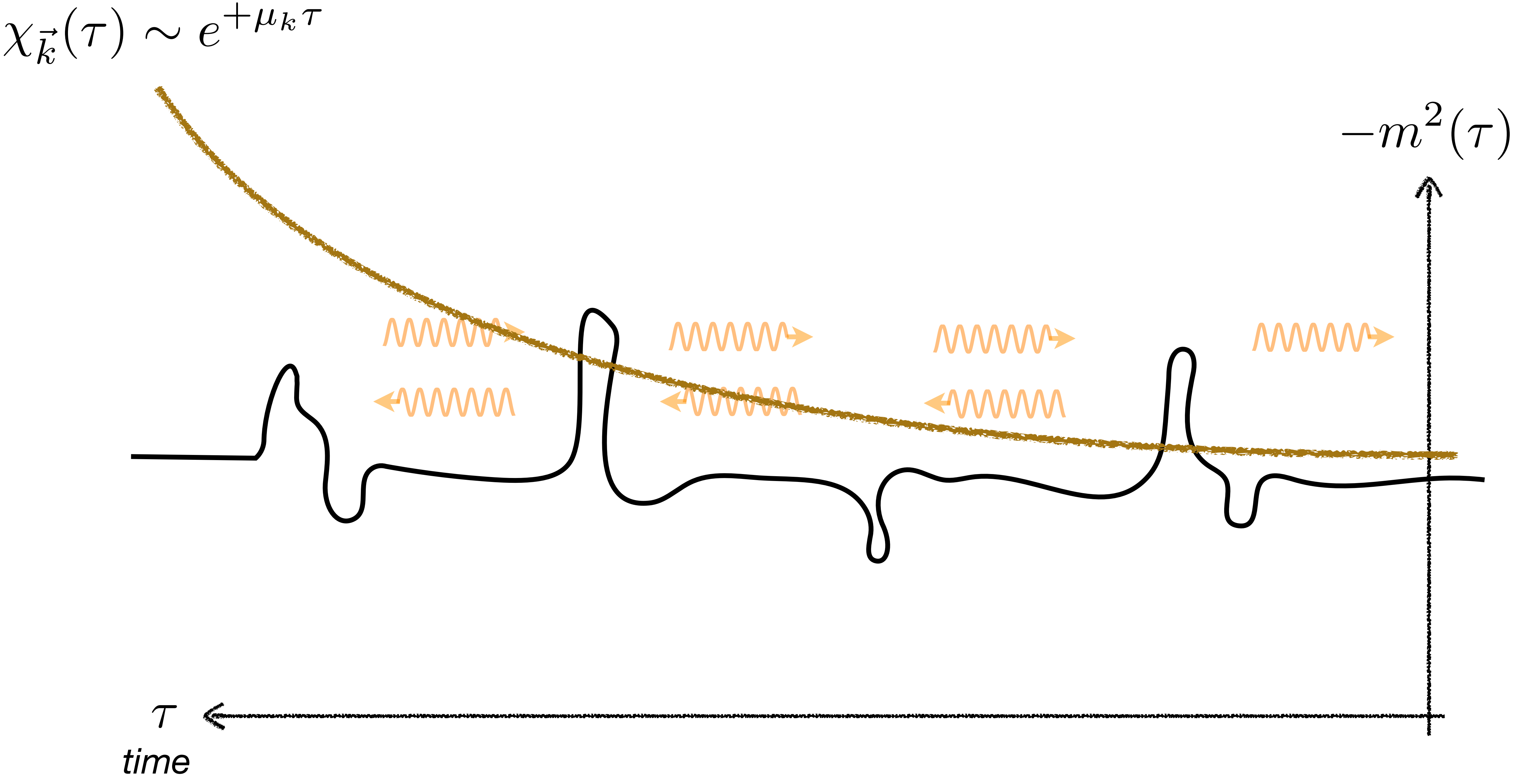
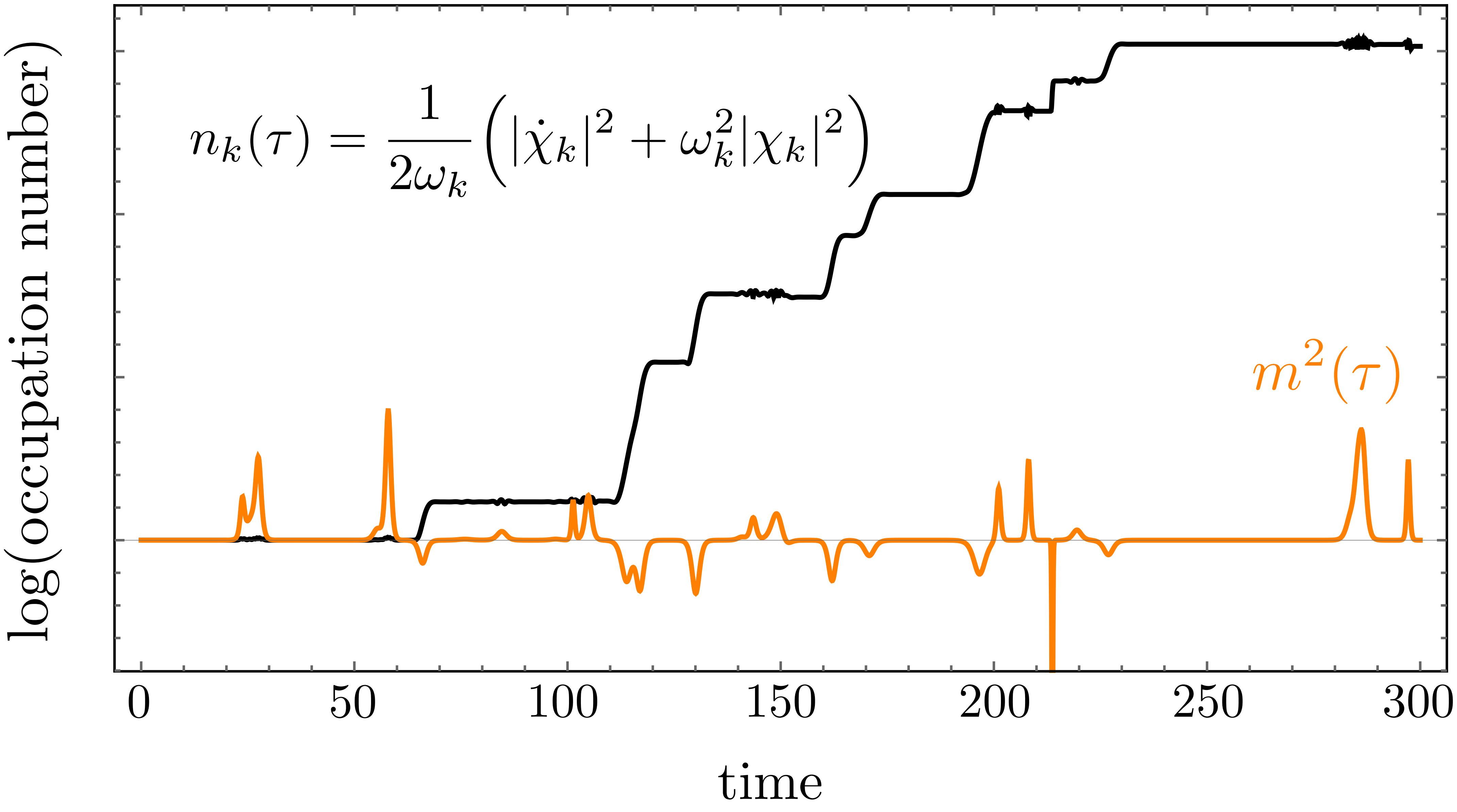
Stochastic Particle Production



We expect exponential particle production to be the analog of Anderson localization.

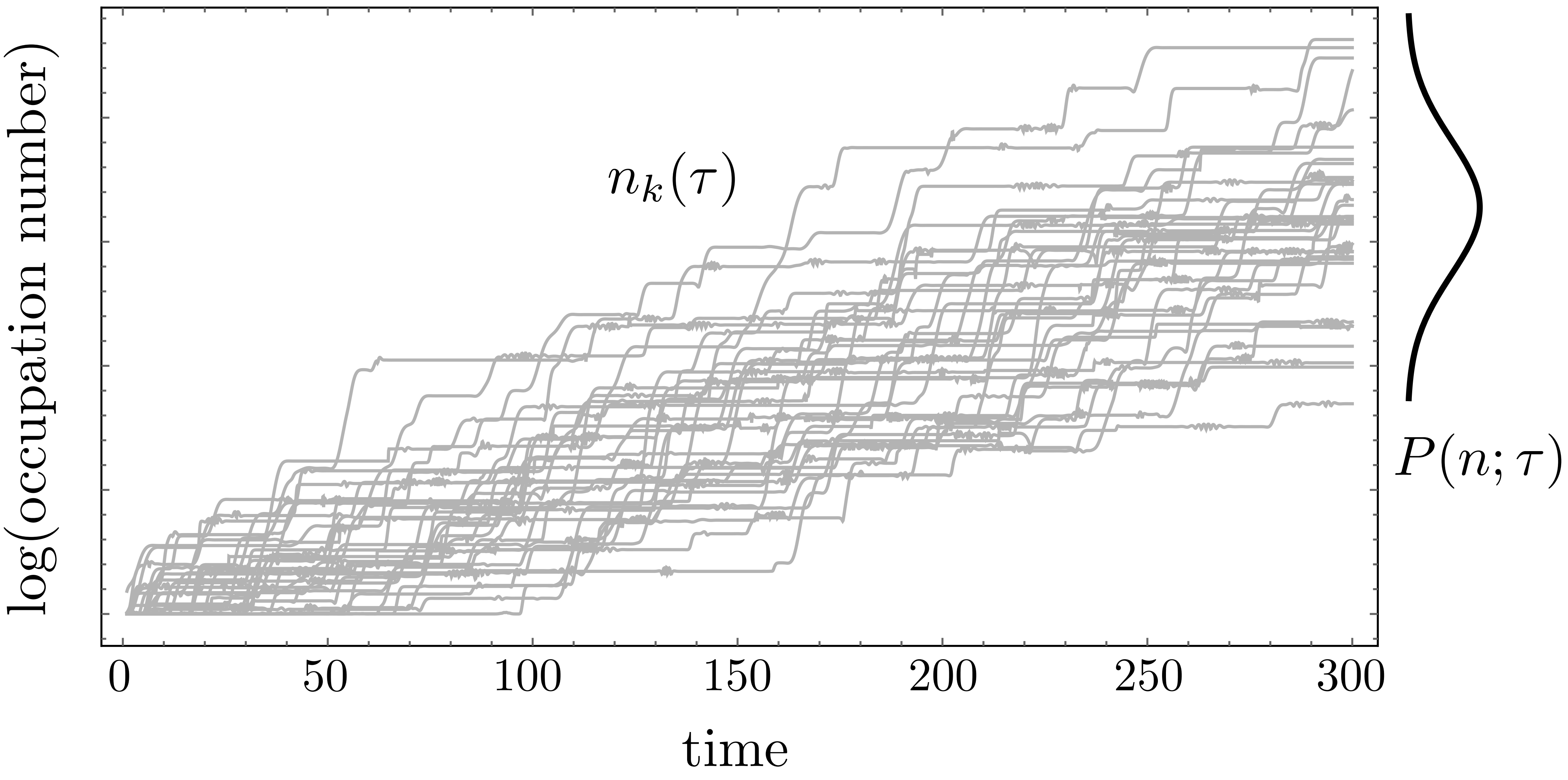
Stochastic Particle Production

Indeed, this is what we find in simulations:



Stochastic Particle Production

The evolution is stochastic:



probability distribution

Brownian Motion

The probability distribution satisfies a **Fokker-Planck equation**:

$$\boxed{\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n;\tau) = \underbrace{(1+2n)\frac{\partial P}{\partial n}}_{\text{drift}} + \underbrace{n(1+n)\frac{\partial^2 P}{\partial n^2}}_{\text{diffusion}}}$$

Amin and DB

mean particle production rate

(analog: mean free path)

(computable from the microscopic properties of the scattering events)

At late times, the solution approaches a log-normal distribution.

Moments of the Distribution

mean

$$\langle n \rangle = \frac{1}{2}\left(e^{2\mu_k \tau} - 1\right)$$

$$\langle \ln n \rangle = \mu_k \tau$$

variance

$$\frac{\mathrm{Var}(n)}{\langle n \rangle^2} \xrightarrow{\mu_k \tau \gg 1} e^{2\mu_k \tau}$$

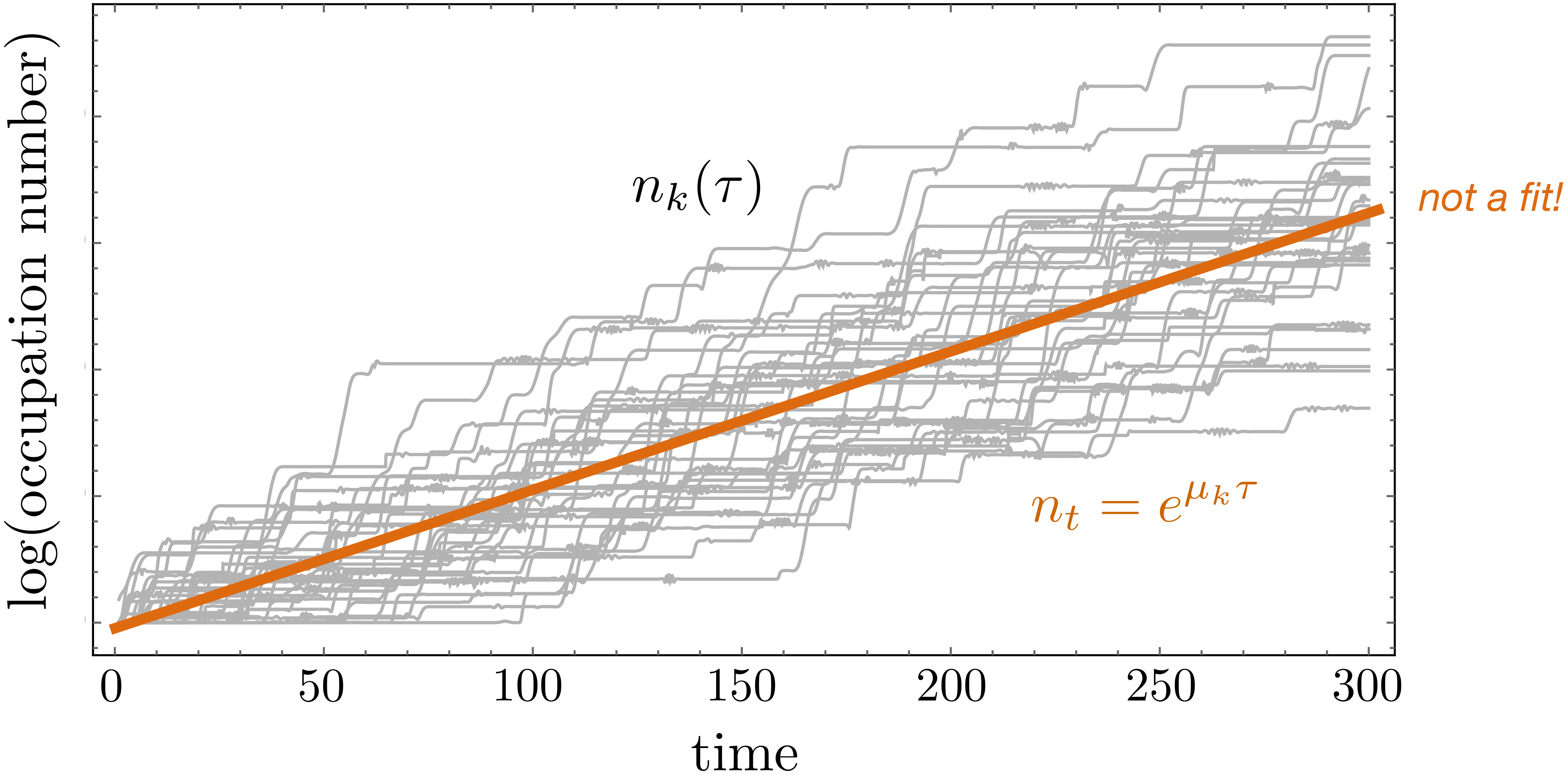
$$\frac{\mathrm{Var}(\ln n)}{\langle \ln n \rangle^2} \xrightarrow{\mu_k \tau \gg 1} \frac{2}{\mu_k \tau}$$

The most probable value of the number density is

$$n_t \equiv \exp(\langle \ln n \rangle) = e^{\mu_k \tau}$$

Numerical Test

The statistics of the produced particles and their evolution is predicted by the Fokker-Planck equation:

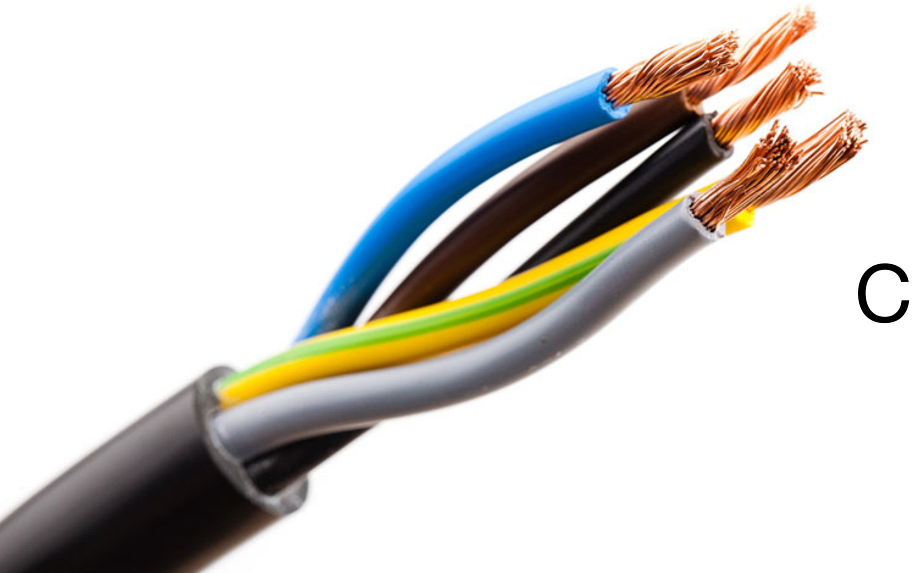


Multi-Field Generalization

The dynamics of the early universe may involve **multiple fields**:

$$\underbrace{\left[\mathsf{1}\,\partial_\tau^2 + \mathsf{P}(k,\tau)\partial_\tau + \mathsf{F}(k,\tau)\right]}_{\equiv\, \mathsf{U}(k,\tau)} \cdot \vec{\chi}_k \ = \ 0$$

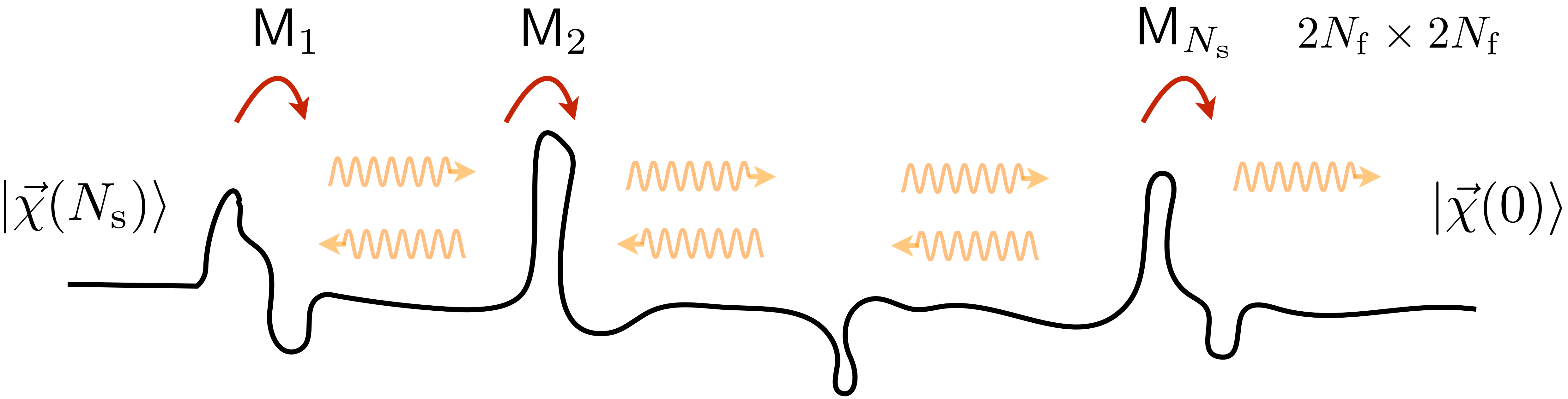
↕



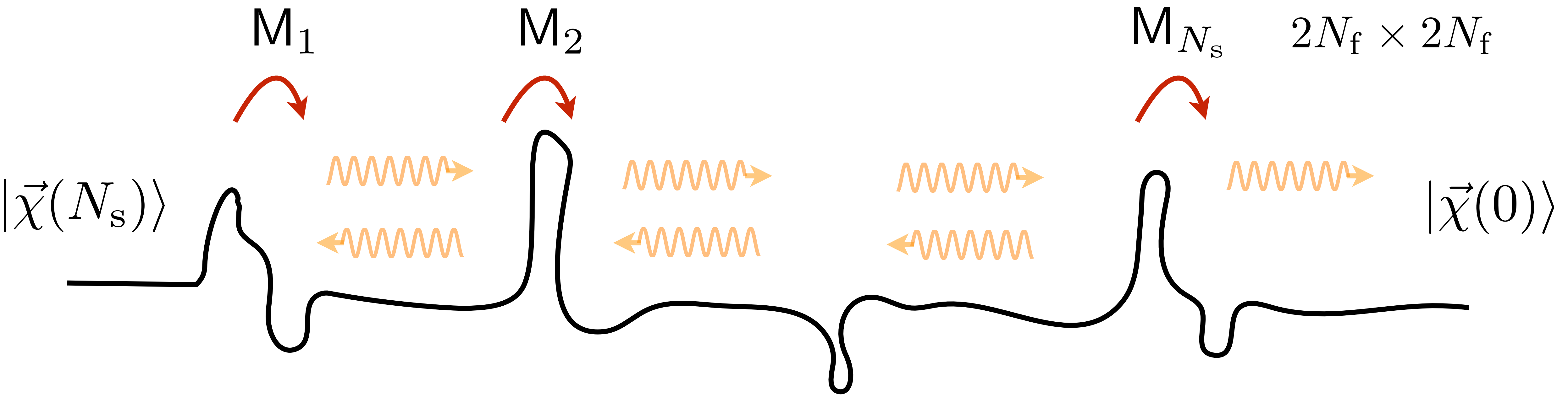
Real wires are not one-dimensional.

Current conduction occurs in **multiple channels**.

Multi-field particle production can also be formulated as a scattering problem:



Multi-field particle production can also be formulated as a scattering problem:



The state after many scatterings is

$$|\vec{\chi}(N_\mathrm{s})\rangle \;=\; \mathsf{M}\,|\vec{\chi}(0)\rangle \quad \text{where} \quad \mathsf{M} \equiv \mathsf{M}_{N_\mathrm{s}} \cdots \mathsf{M}_2\mathsf{M}_1 \longleftarrow$$

product of random matrices

The total number of particles is

$$n = \mathrm{Tr}(\mathsf{n}) = \sum_{a=1}^{N_\mathrm{f}} n_a \quad \text{where} \quad \mathsf{n} \sim \mathsf{M}\mathsf{M}^\dagger\,.$$

particles in each “channel”

Fokker-Planck Equation

Dorokhov, Mello, Pereura, and Kumar

The joint probability for the number densities satisfies the following Fokker-Planck equation:

$$\boxed{\begin{aligned}\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n_a;\tau) \;=\; & \sum_{a=1}^{N_{\mathrm{f}}}\left[(1+2n_a)+\frac{1}{N_{\mathrm{f}}+1}\sum_{b\neq a}\frac{n_a+n_b+2n_an_b}{n_a-n_b}\right]\frac{\partial P}{\partial n_a}\\ & +\frac{2}{N_{\mathrm{f}}+1}\sum_{a=1}^{N_{\mathrm{f}}}n_a(1+n_a)\frac{\partial^2 P}{\partial n_a^2}\end{aligned}}$$

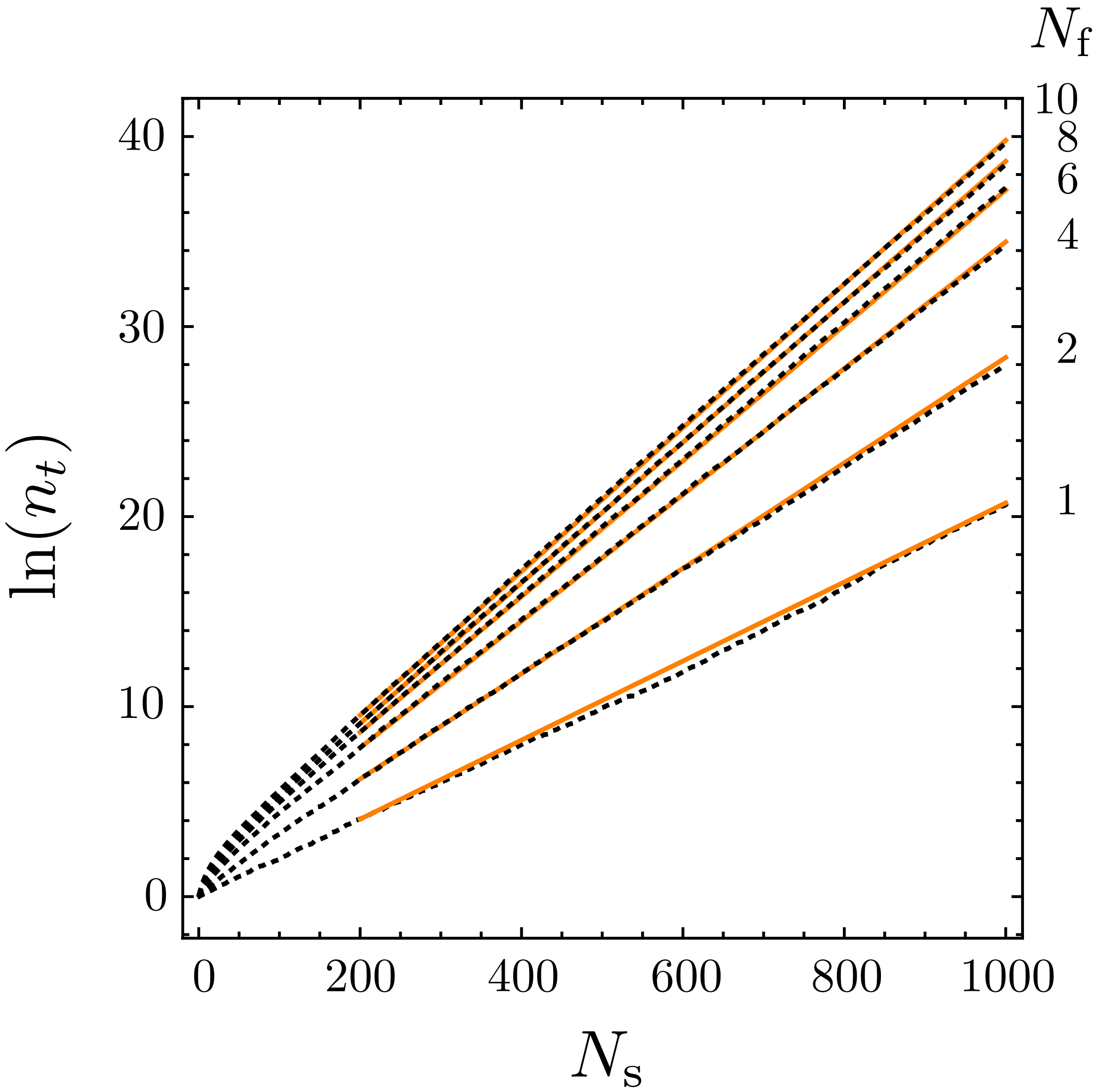
As before, we use this to predict the statistics of the particle production.

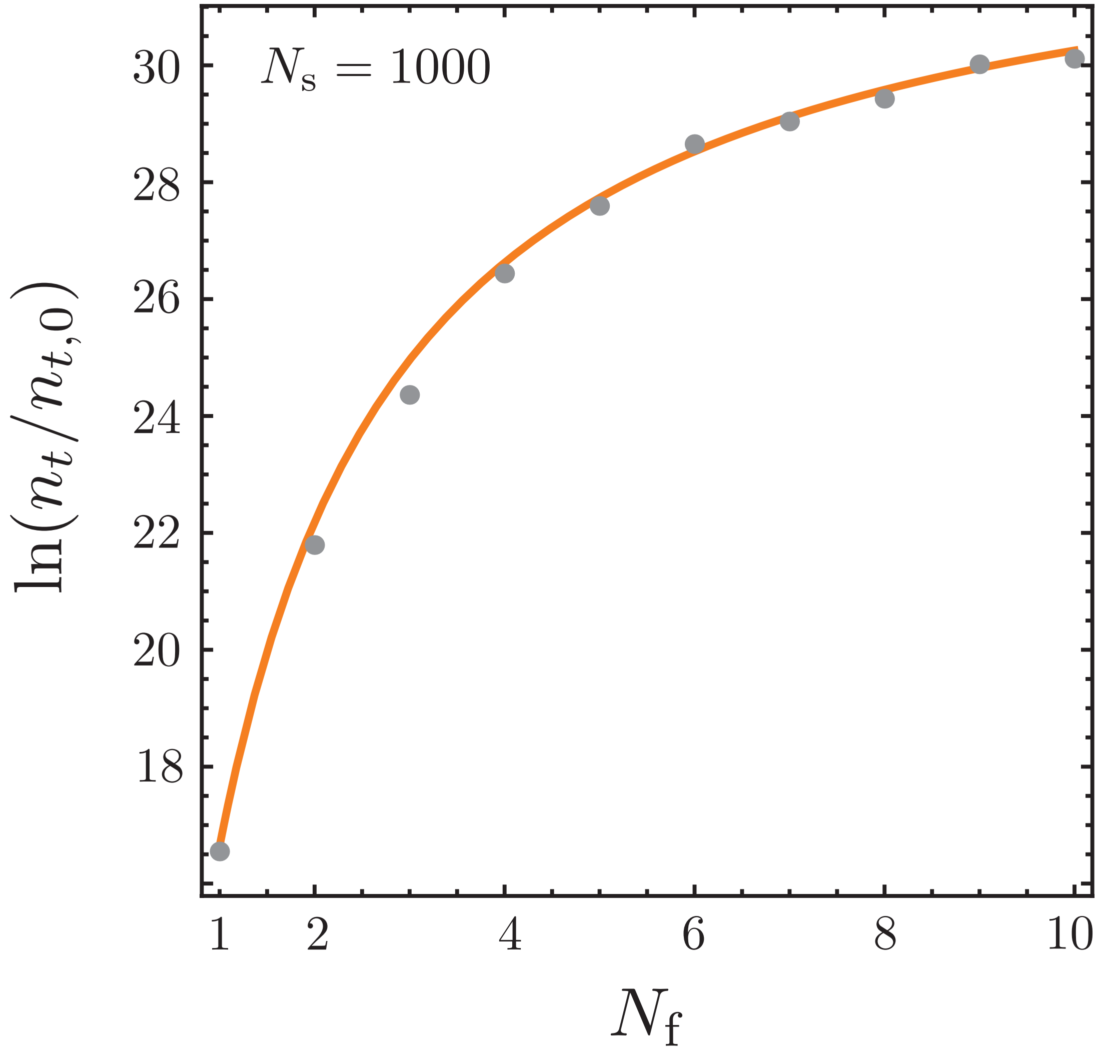
We find $\boxed{\ln(n_t) \propto \frac{2N_{\mathrm{f}}}{N_{\mathrm{f}}+1}N_{\mathrm{s}}}$ where $n = \sum_{a=1}^{N_{\mathrm{f}}} n_a$.

Results

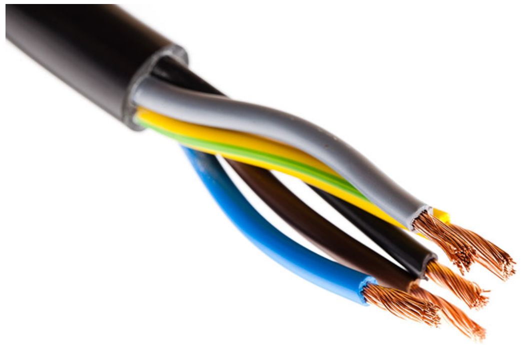
Analytics and numerics agree remarkably well:

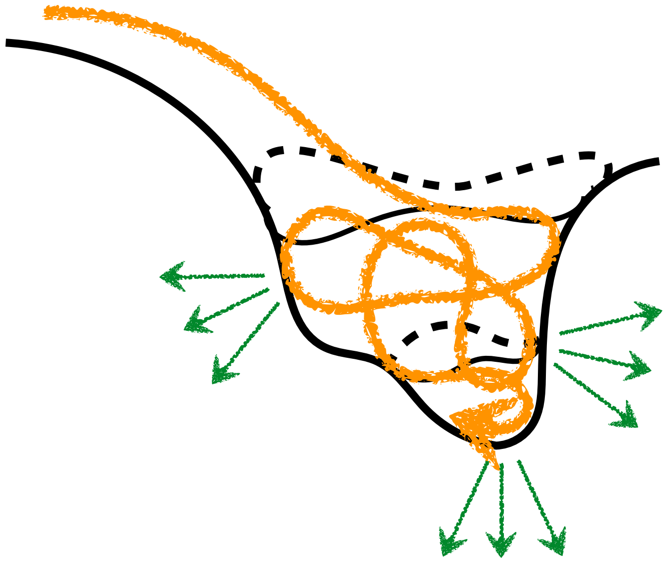
$$\boxed{\ln(n_t) \propto \frac{2N_{\mathrm{f}}}{N_{\mathrm{f}}+1}N_{\mathrm{s}}}$$





Simplicity/Universality





ℓ : mean free path

$N_{\rm c}$: number of channels

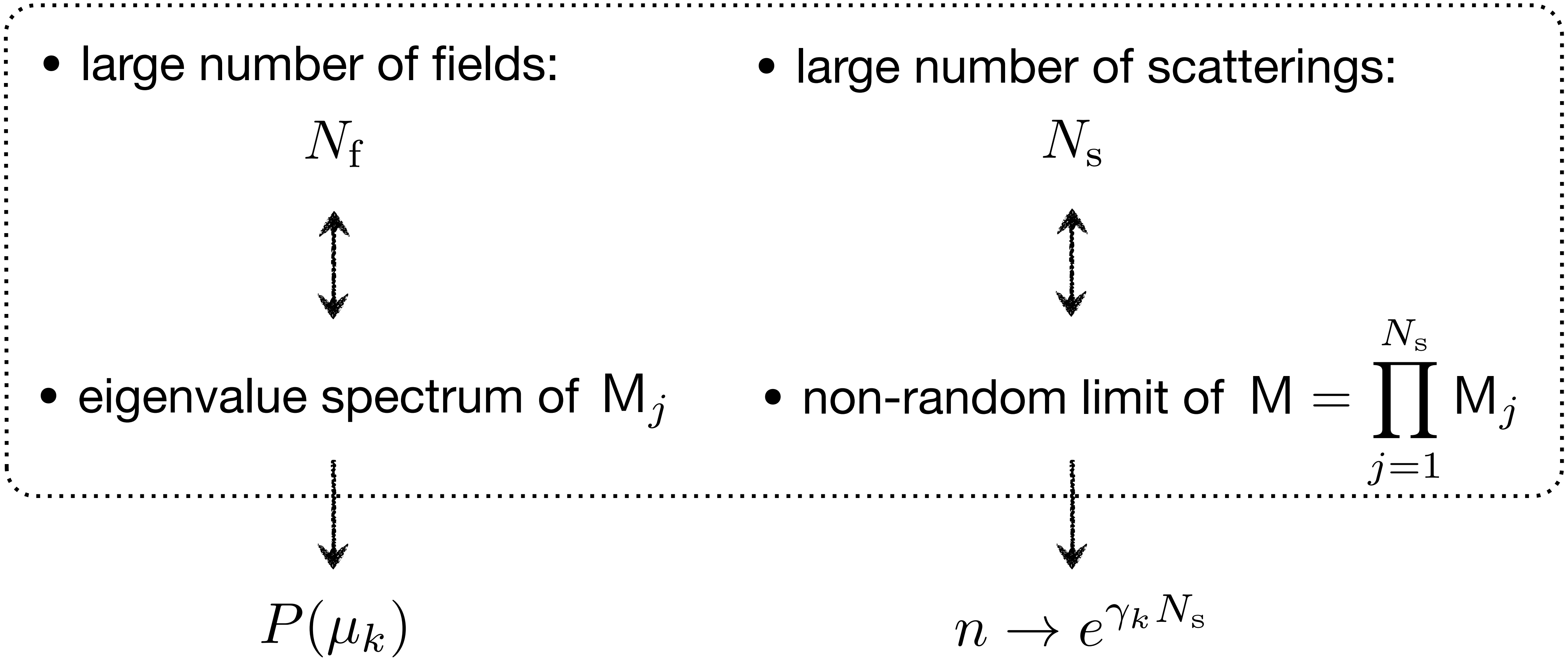
μ_k : mean particle production rate

$N_{\rm f}$: number of fields

universality: regimes exist where the dependence on parameters vanishes.

Universality from Random Matrix Theory

We can exploit two large N’s:



RMT predicts:

- large number of fields:

$$N_\mathrm{f}$$

$$\updownarrow$$

- eigenvalue spectrum of M_j

$$\downarrow$$

$$P(\mu_k)$$

- large number of scatterings:

$$N_\mathrm{s}$$

$$\updownarrow$$

- non-random limit of $\mathsf{M} = \prod_{j=1}^{N_\mathrm{s}} \mathsf{M}_j$

$$\downarrow$$

$$n \rightarrow e^{\gamma_k N_\mathrm{s}}$$

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↕

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RMT predicts:

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Oseledec

For $N_{\mathrm{s}} \to \infty$, the $2N_{\mathrm{f}}$ random eigenvalues $e^{\pm\nu_a}$ of $\mathsf{MM}^\dagger \sim \mathsf{n}$ tend to the non-random values $e^{\pm\gamma_a N_{\mathrm{s}}}$, with γ_a independent of N_{s}.

↑ *Lyapunov exponent*

For finite N_{s}, the ν_a's have small Gaussian fluctuations around their asymptotic limit $\gamma_a N_{\mathrm{s}}$.

Outlook

Emergent Simplicity from Complexity?

We have seen hints of **universality** emerging in the evolution of the particle number density:

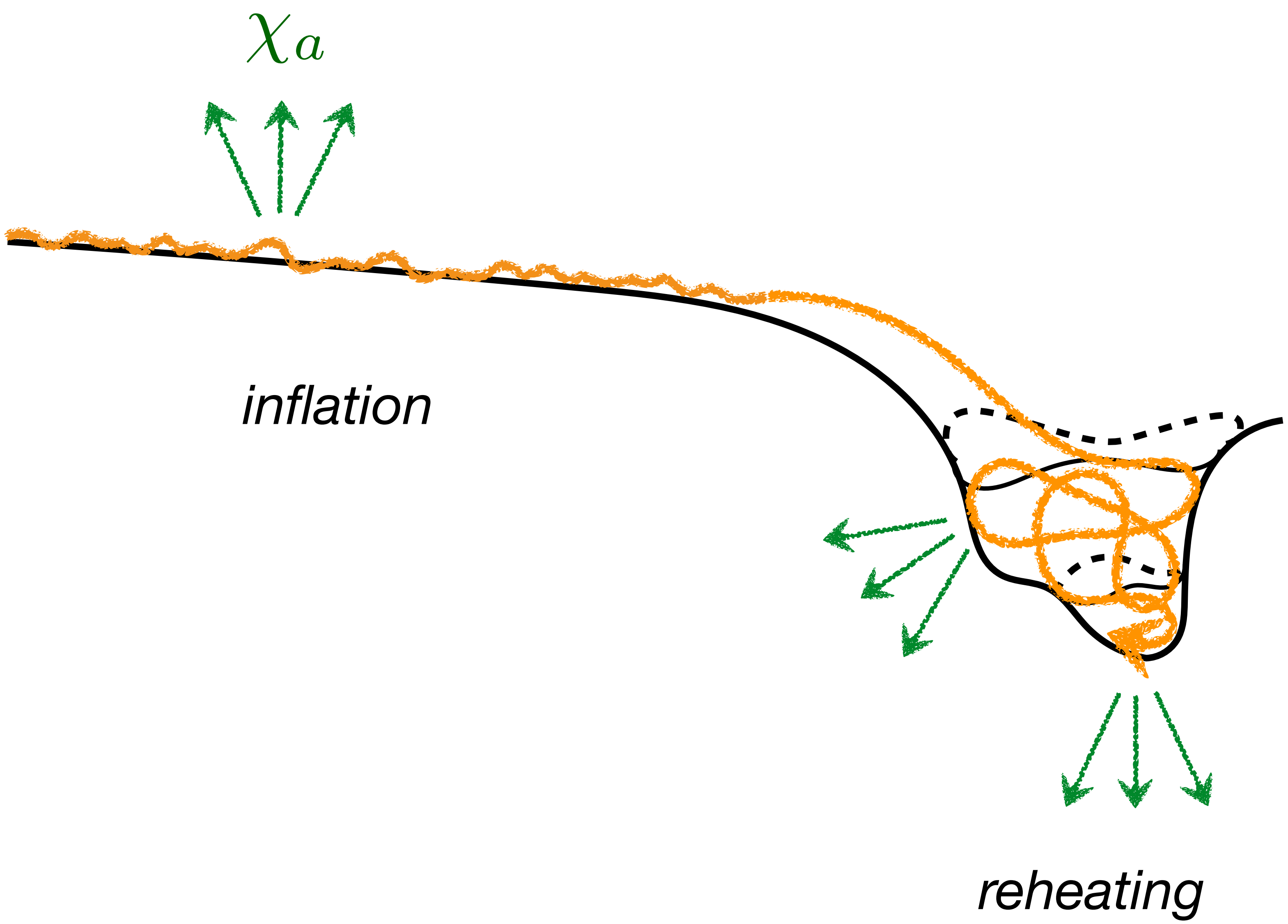
$$n_k \sim e^{\mu_k \tau}$$

↑

microscopic details have collapsed
into the Lyapunov exponent

Statistics are characterized by μ_k and N_{f} .

It remains to be seen how this is reflected in **cosmological observables**:



background dynamics

↓

particle production

$\langle n_{\vec{k}_1} n_{\vec{k}_2} \ldots \rangle$

↕

curvature fluctuations

$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \ldots \rangle$

Application: Inflation

Nacir, Porto, Senatore, and Zaldarriaga

The produced particles can backreact on the evolution of curvature perturbations during inflation:

$$\left(\partial_t^2 + 3H\partial_t - \frac{\nabla^2}{a^2}\right)\zeta \quad \propto \quad \underbrace{n}_{\text{source}}{}^{*}$$

stochastic noise

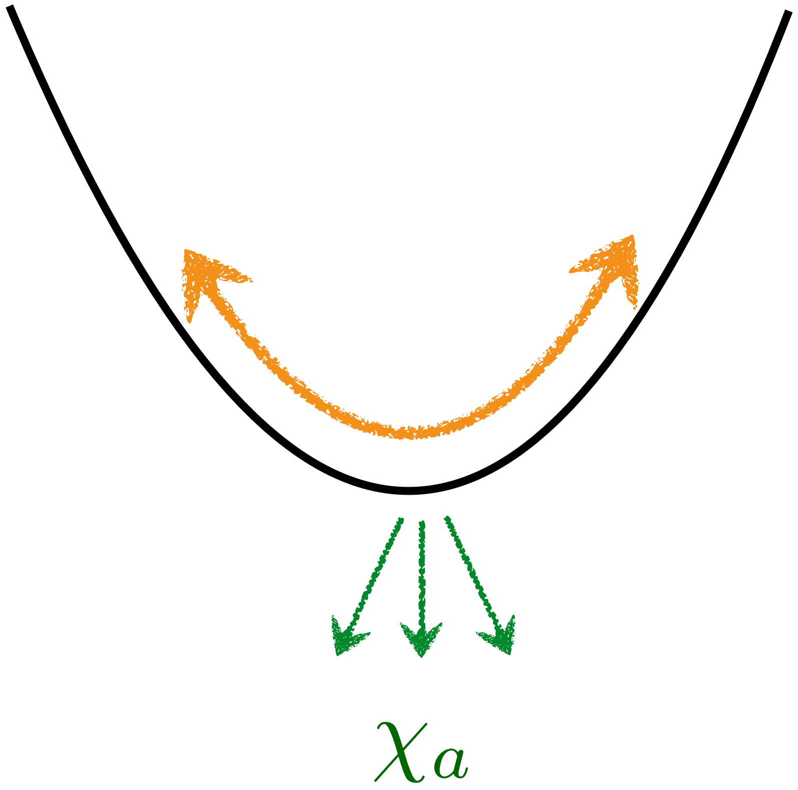
$$(n)_S \equiv \langle n \rangle_{\zeta=0}$$

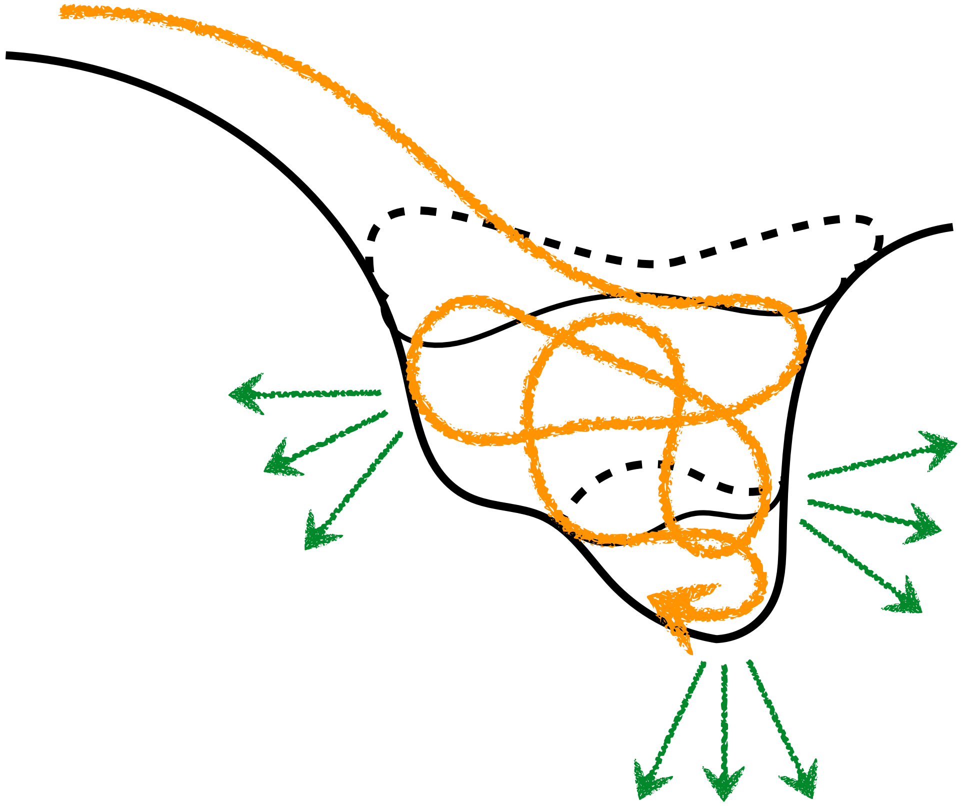
linear response

$$(n)_R \equiv \int^t \mathrm{d}t' \, G_{\mathrm{ret}}^{\langle n \rangle}(t,t')\,\zeta(t')$$

* Outside the horizon: $n \to \chi^2$

Application: Reheating





Model-insensitive description of a complicated reheating process.

Kofman, Linde, Starobinsky

Open Questions

- Do universal conductance fluctuations have a counterpart in inflation?
- Does the large variance of the produced particles leave an imprint?

Green

- How natural is scale-invariance?
- How do our results compare to explicit examples?

discussions with Bachlechner, Dias, Frazer, Marsh and McAllister.

Thank you for you attention.