Action MP 1405

Quantum Structure of Spacetime

Group Field Theories for the Atoms of Space

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Albert Einstein Institute

Humboldt Kolleg

“Open problems in Theoretical Physics: the issue of Quantum Spacetime”

Corfu, Greece, EU - 19/09/2015

Plan of the talk

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- GFT basics (from LQG point of view)
    - GFT: general definition and features
    - GFT, LQG and spin foam models
    - GFT models of 4d quantum gravity: main ingredients

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    - nature of the problem and role of renormalisation
    - perturbative GFT renormalization
    - non-perturbative GFT renormalization: recent results

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  - perturbative GFT renormalization
  - non-perturbative GFT renormalization: recent results
- Effective cosmological dynamics from GFTs
  - general perspective: universe as a condensate, cosmology as QG hydrodynamics
  - GFT condensates as homogeneous geometries
  - effective cosmological dynamics from GFT condensates

Part I:

Group Field Theory (from LQG point of view)

Group field theories

(Boulatov, Ooguri, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, Baratin, ……)

Quantum field theories over group manifold G (or corresponding Lie algebra) $\varphi : G^{\times d} \rightarrow \mathbb{C}$

QFT of spacetime, not defined on spacetime

relevant classical phase space for "GFT quanta": $(\mathcal{T}^* G)^{\times d} \simeq (\mathfrak{g} \times G)^{\times d}$

can reduce to subspaces in specific models depending on conditions on the field

d is dimension of "spacetime-to-be"

example: d=4 $\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \rightarrow \mathbb{C}$

can be defined for any (Lie) group and dimension d, any signature, ...

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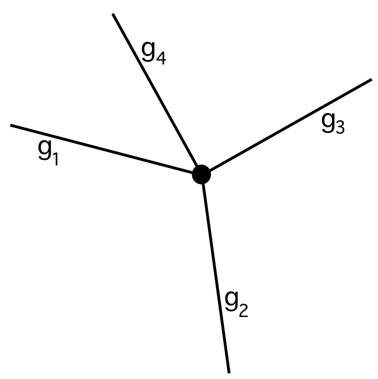

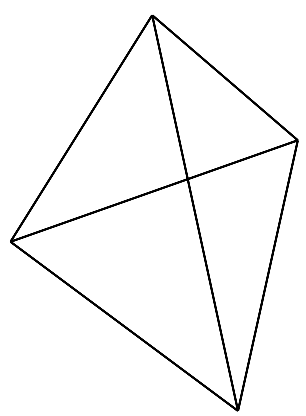

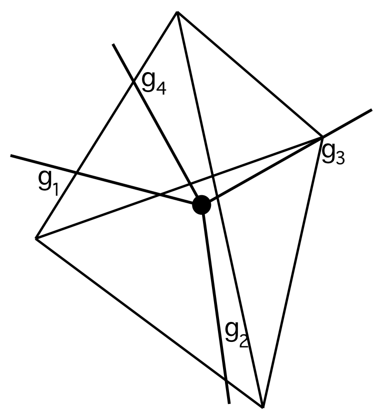

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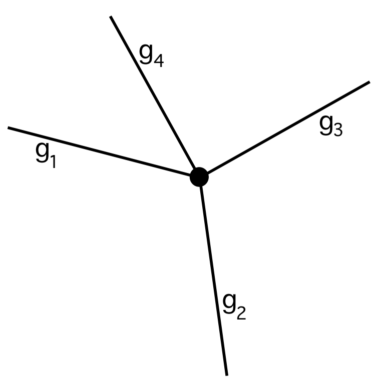

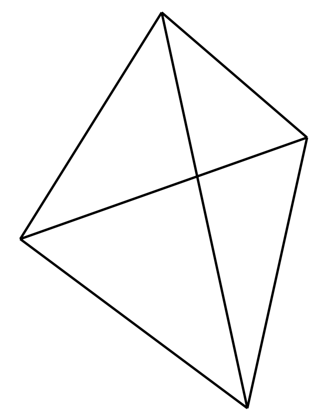

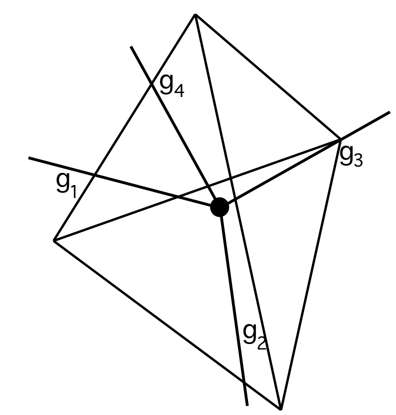

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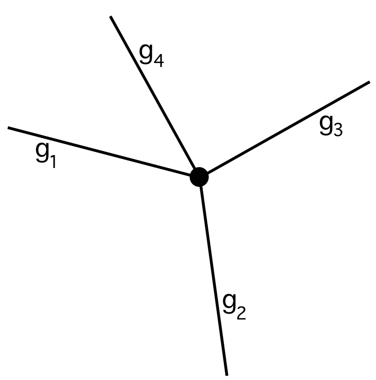

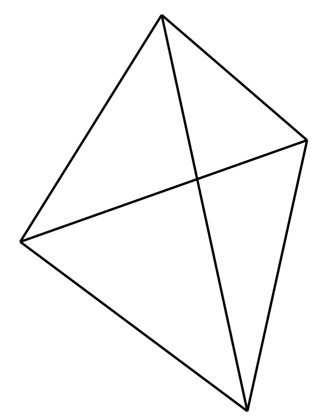
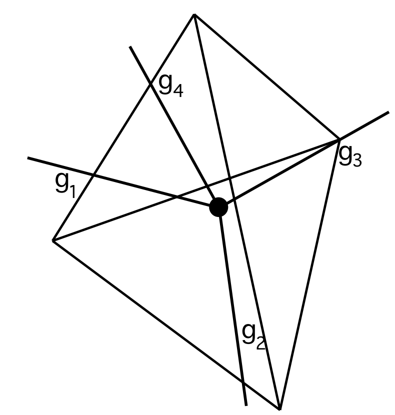

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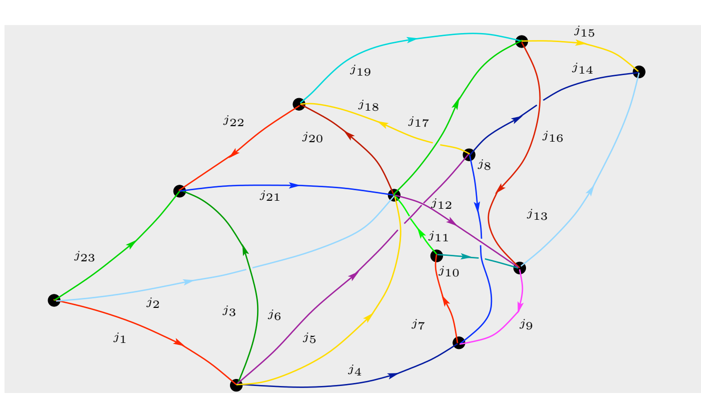
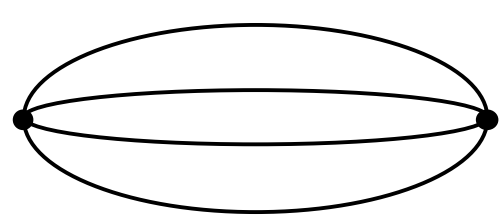
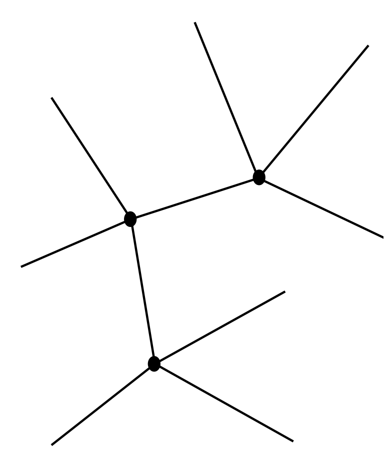
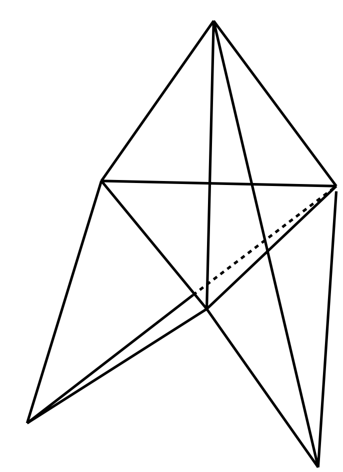

Group field theories

classical action: kinetic (quadratic) term + (higher order) interaction (convolution of GFT fields)

$$S(\varphi, \overline{\varphi}) = \frac{1}{2} \int [dg_i] \overline{\varphi(g_i)} \mathcal{K}(g_i) \varphi(g_i) + \frac{\lambda}{D!} \int [dg_{ia}] \varphi(g_{i1}) \ldots . \varphi(\bar{g}_{iD}) \mathcal{V}(g_{ia}, \bar{g}_{iD}) \quad + \quad c.c.$$

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combinatorics of field arguments in interaction: gluing of 5 tetrahedra across common triangles, to form 4-simplex ("building block of spacetime")

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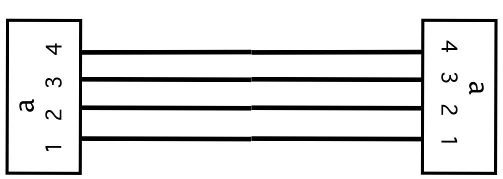

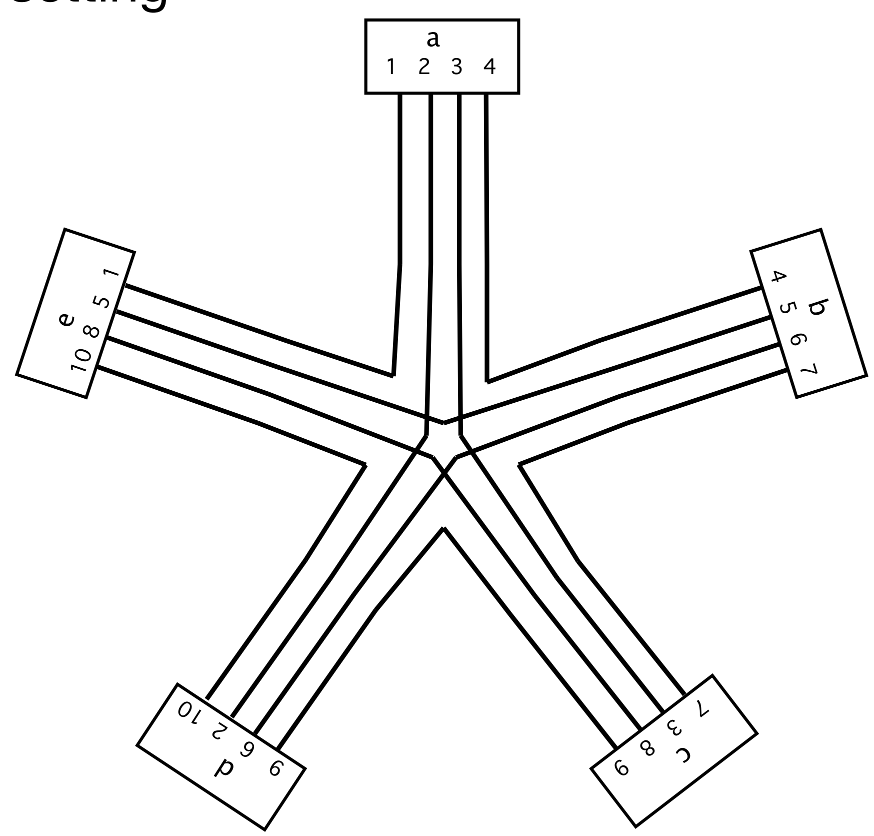

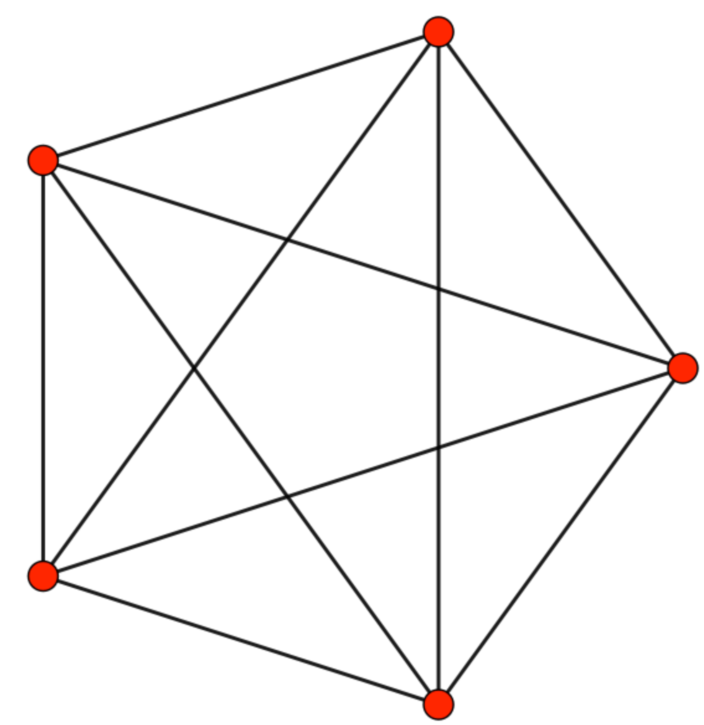

Group field theories

Feynman perturbative expansion around trivial vacuum

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Feynman diagrams (obtained by convoluting propagators with interaction kernels) =

= stranded diagrams dual to cellular complexes of arbitrary topology

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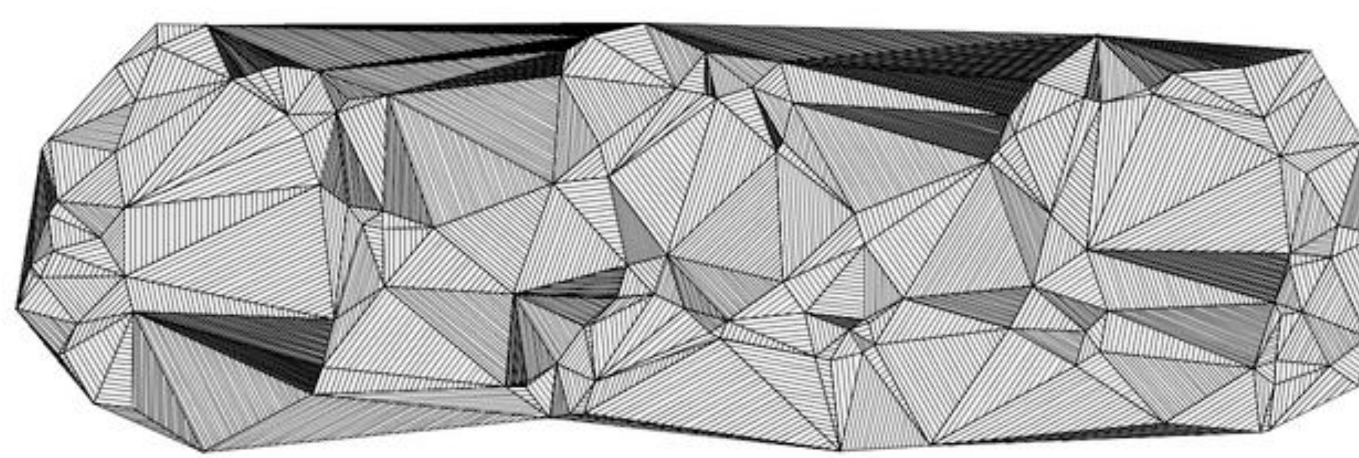

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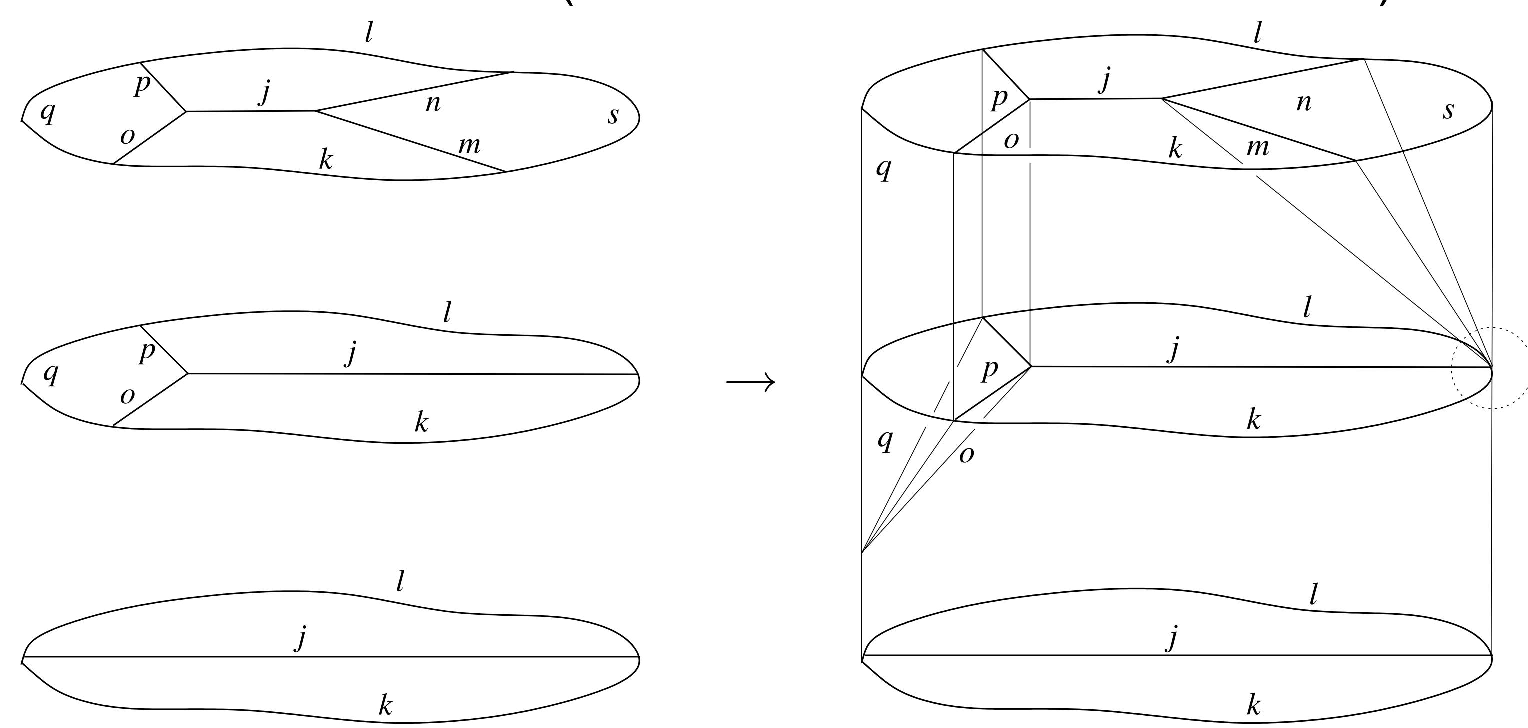
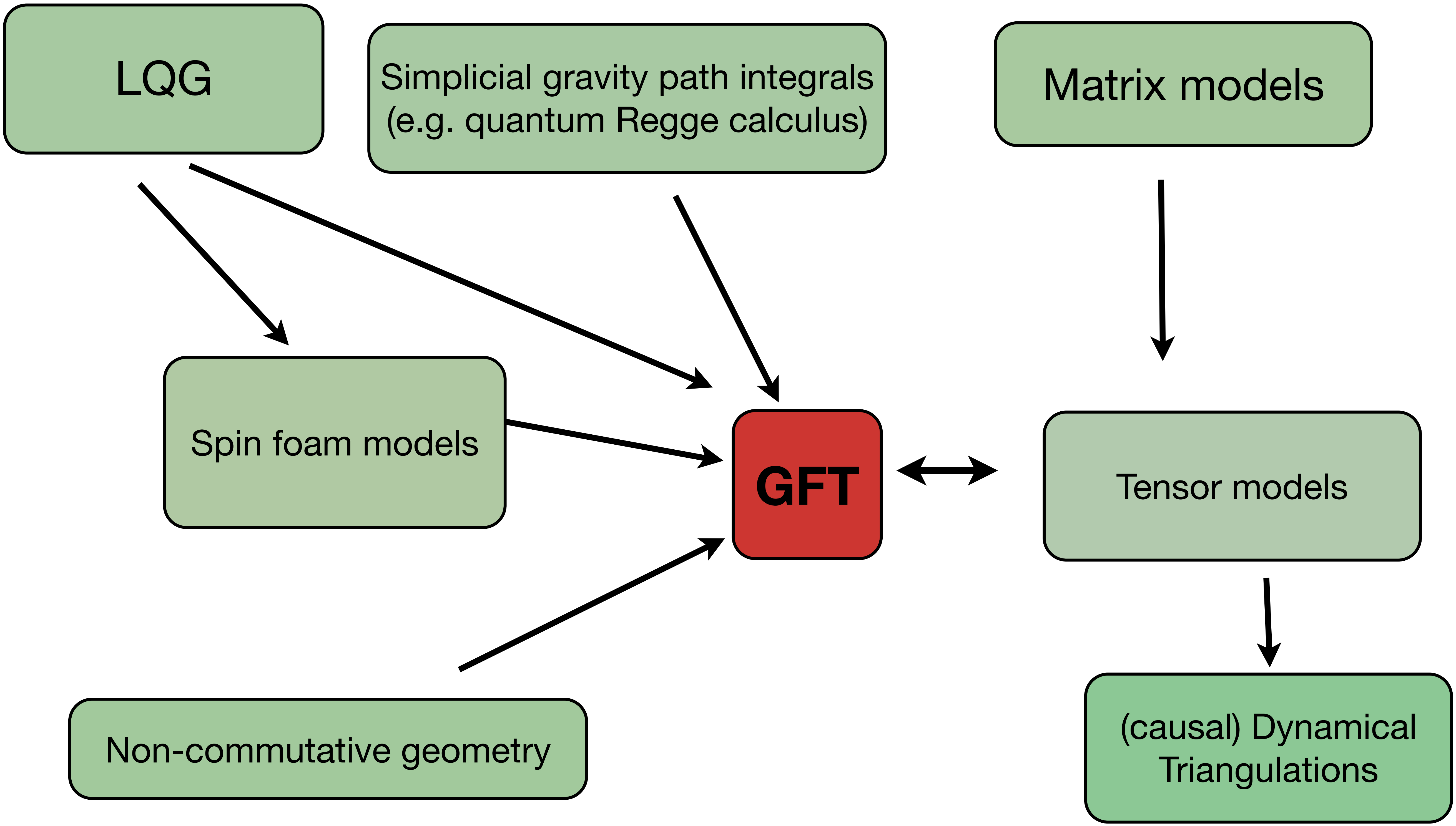


Group Field Theory: convergence of approaches



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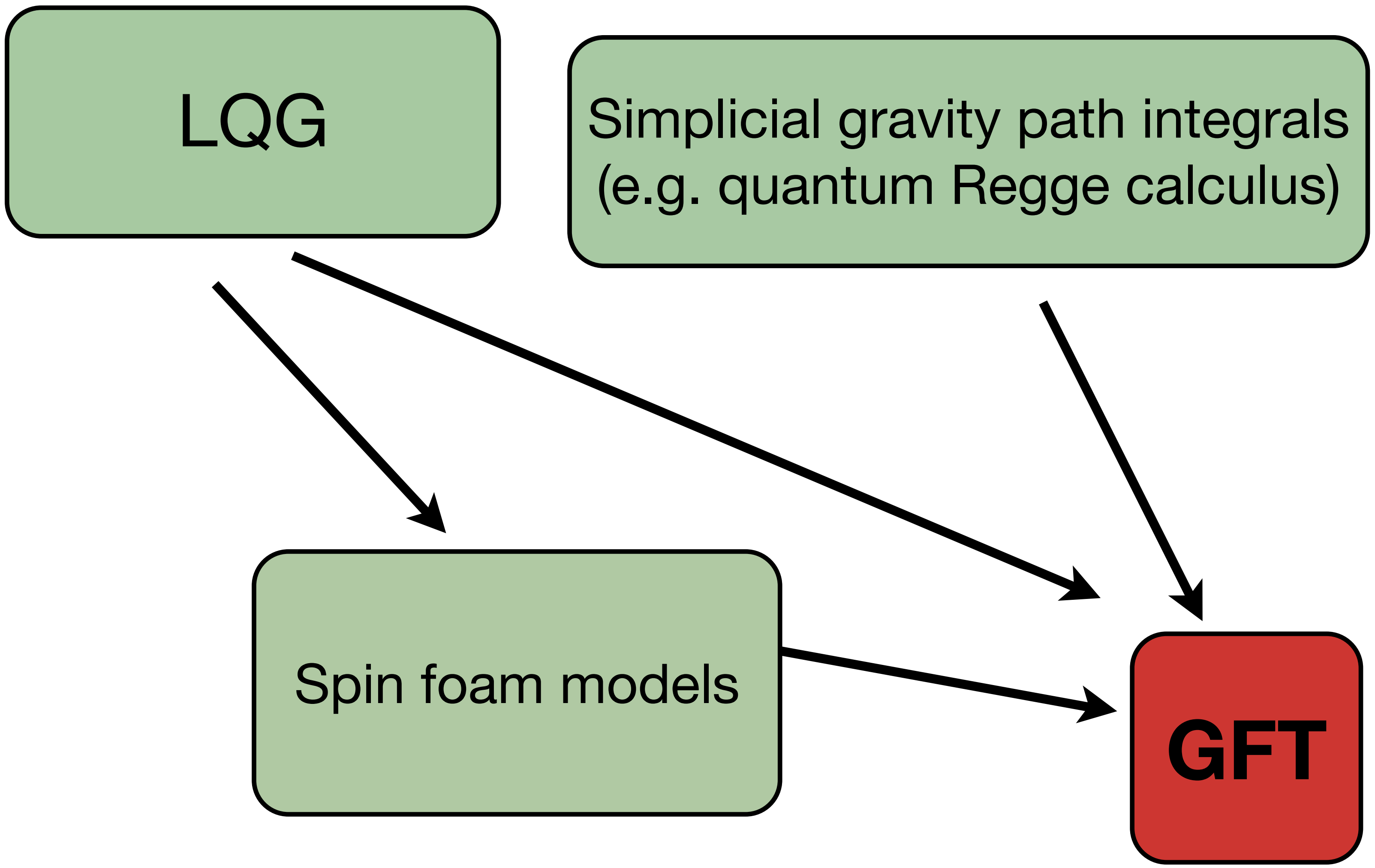

GFT as 2nd quantisation of LQG

see talk by N. Bodendorfer

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spin networks as many-body systems and 2nd quantisation —-> GFT Fock space DO, '13 ; Kittel, DO, Tomlin, to appear

(= space of "disconnected spin network vertices")

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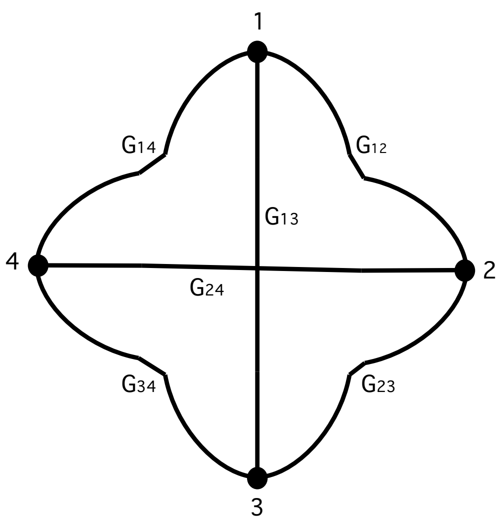

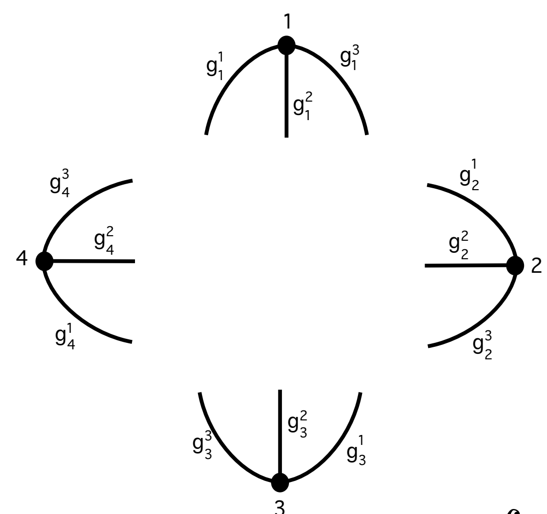

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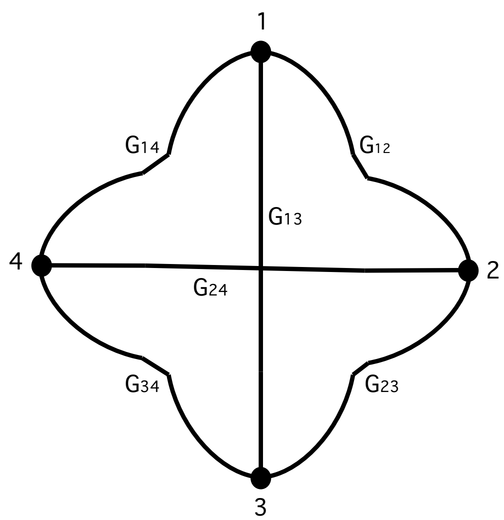

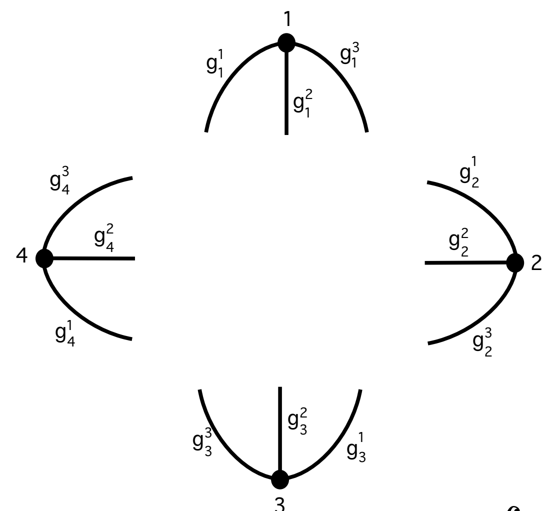

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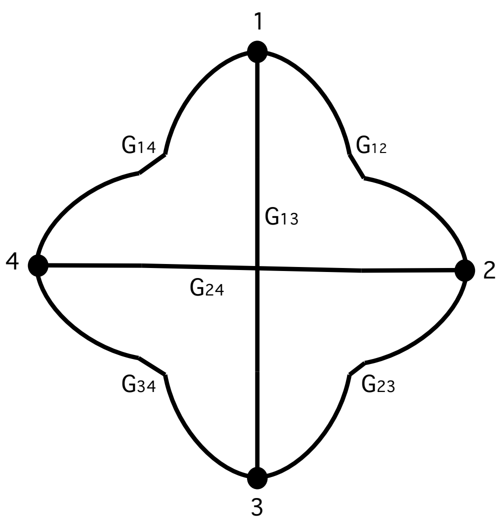

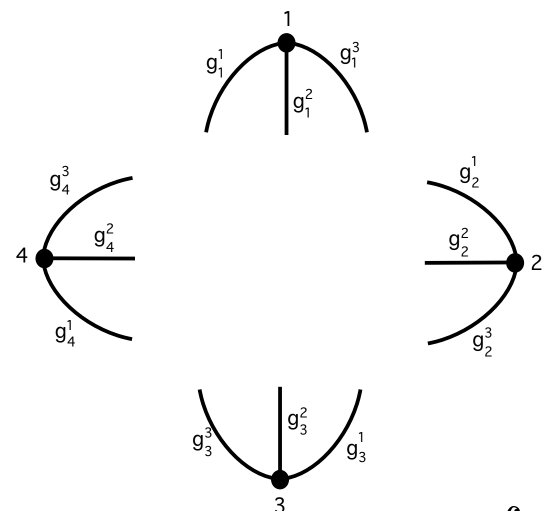

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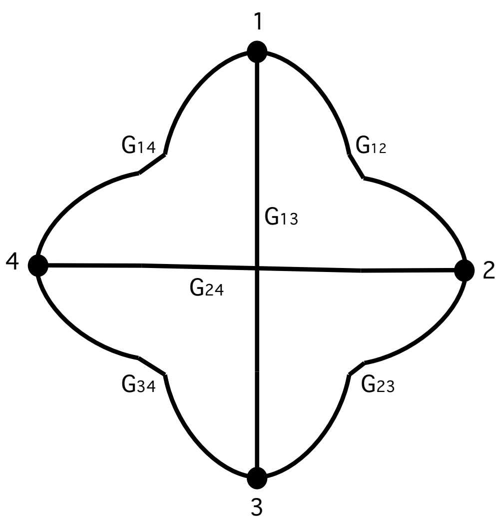

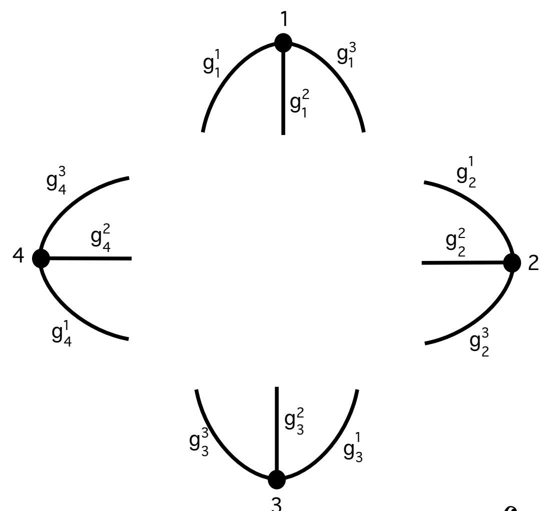

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- no continuum embedding

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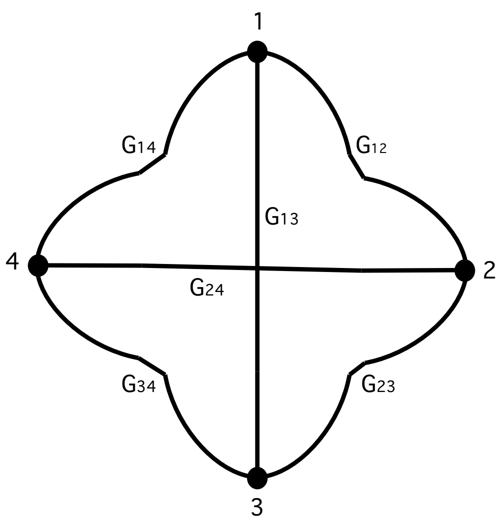

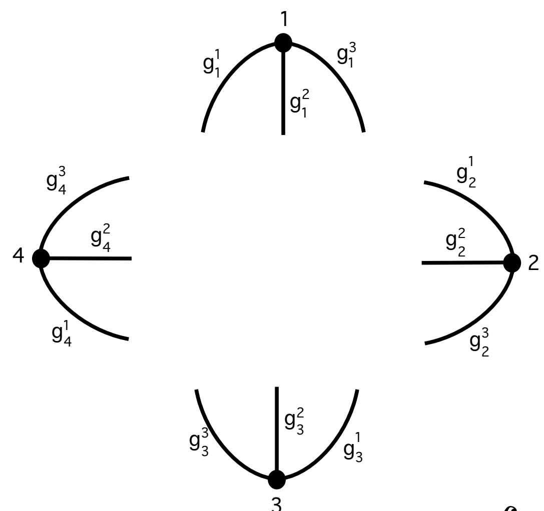

$$\mathcal{F}(\mathcal{H}_v) \;=\; \bigoplus_{V=0}^{\infty} sym\left\{\left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)}\right)\right\}$$

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$$\mathcal{H}_\gamma \subset \mathcal{H}_V \qquad \Psi_\gamma(G_{ij}^{ab}) = \prod_{[(ia),(jb)]} \int_G d\alpha_{ij}^{ab}\, \phi_V(\ldots, g_{ia}\,\alpha_{ij}^{ab}, \ldots, g_{jb}\alpha_{ij}^{ab}, \ldots) = \Psi_\gamma(g_{ia}(g_{jb})^{-1})$$

- same type of functions + same scalar product for given graph
- states for different graphs (same vertices) overlap
- no continuum embedding
- no cylindrical equivalence

GFT as 2nd quantisation of LQG

see talk by N. Bodendorfer

the GFT proposal:

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

spin networks as many-body systems and 2nd quantisation —-> GFT Fock space

DO, '13 ; Kittel, DO, Tomlin, to appear

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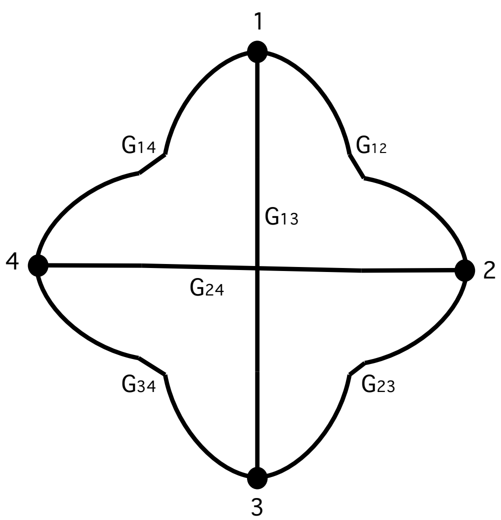

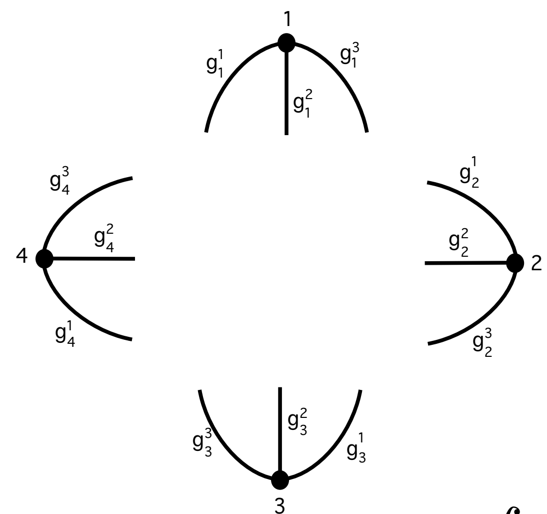

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need to accept technical differences

and change in perspective

——> fundamental discreteness

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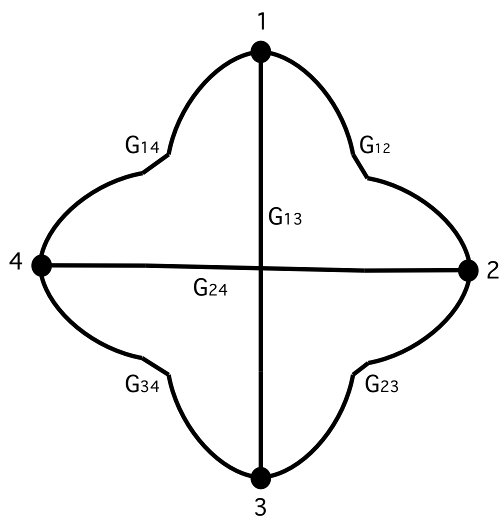

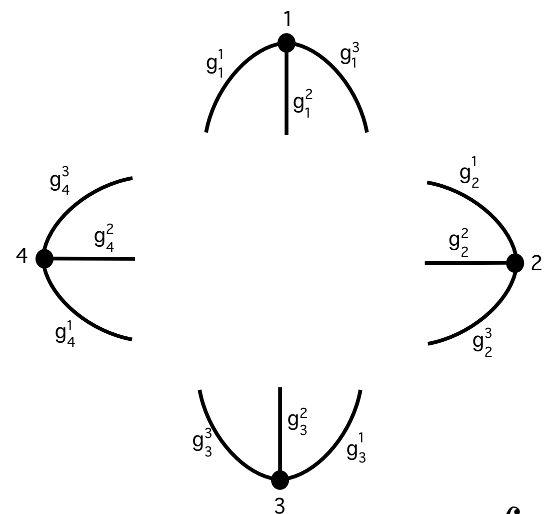

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for any canonical observable (incl. Hamiltonian constraint) -> GFT observable in 2nd quantisation

GFT as completion of spin foam models

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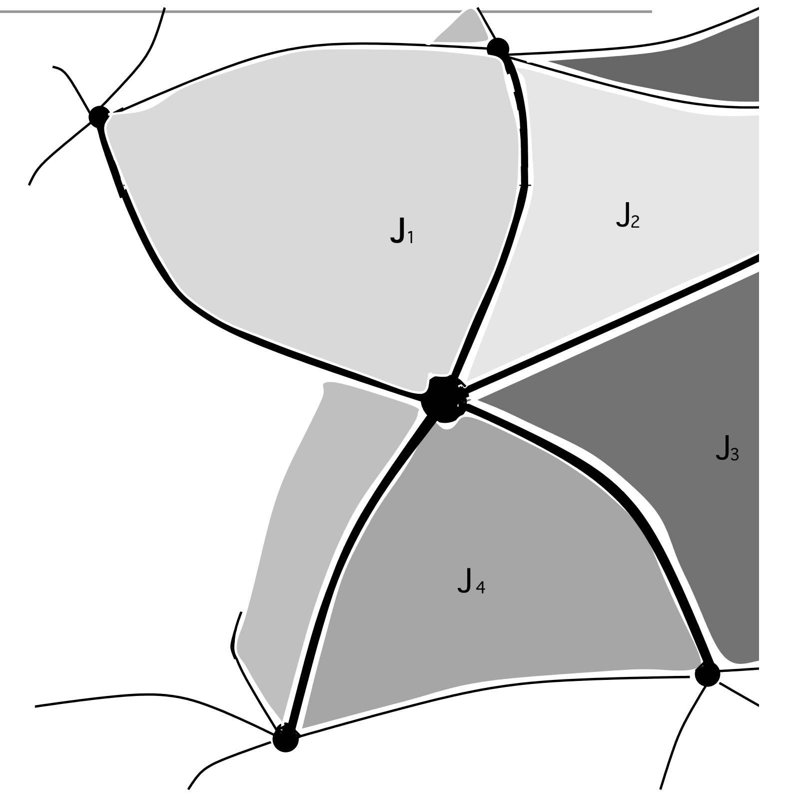
quantum spin network history = spin foam (complex with algebraic data)



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basic element of SF model: quantum amplitude for spin foam complex

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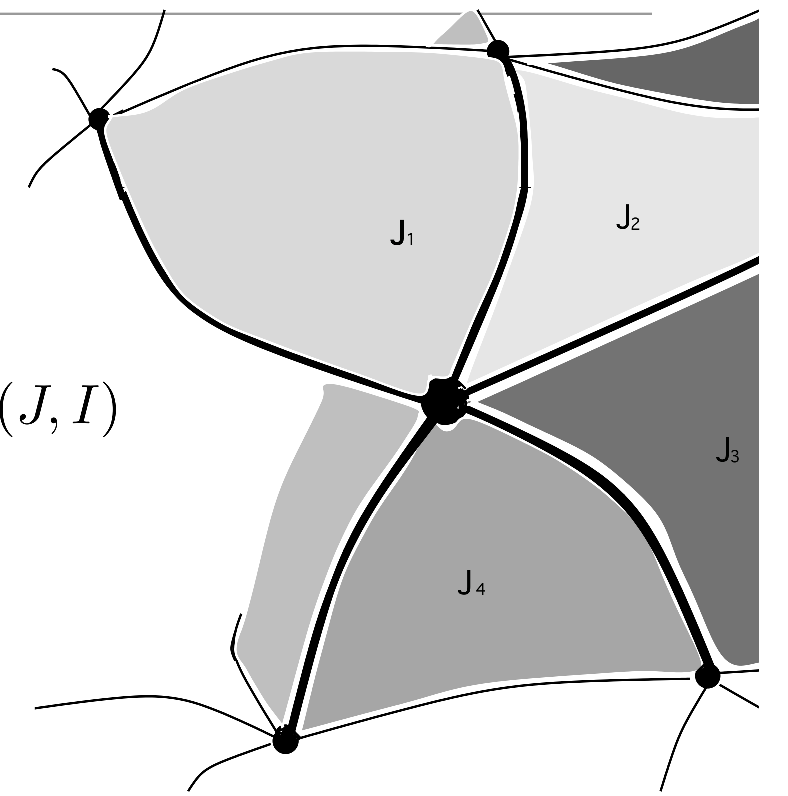

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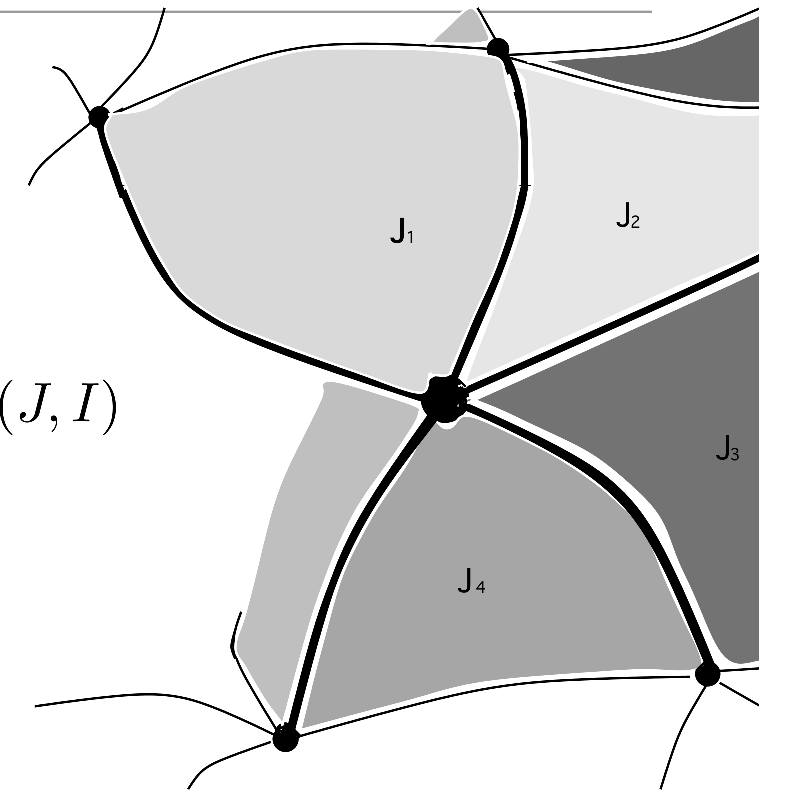

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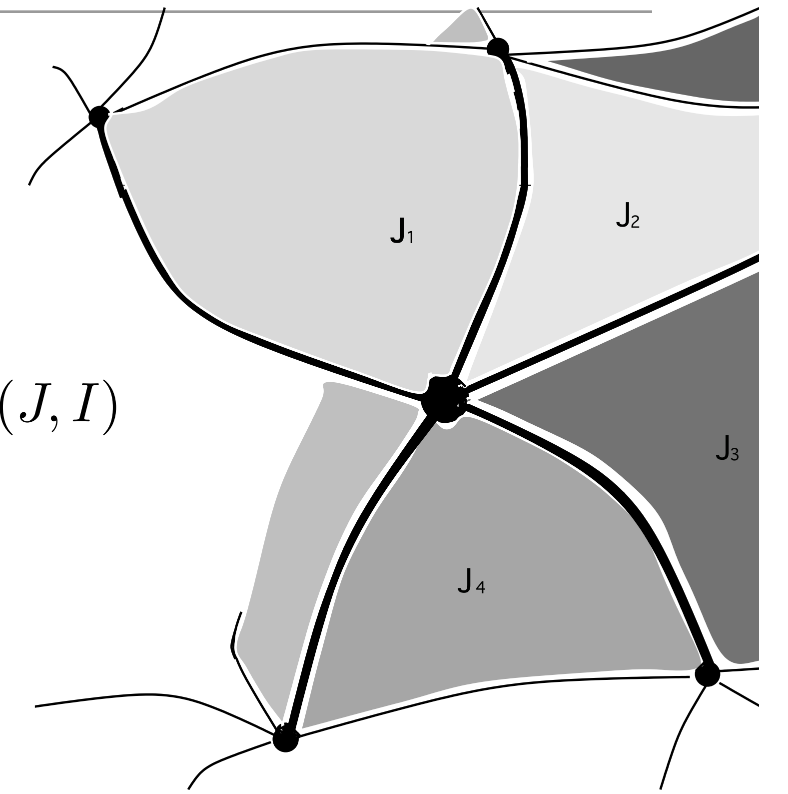

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spin foam model

with sum over complexes

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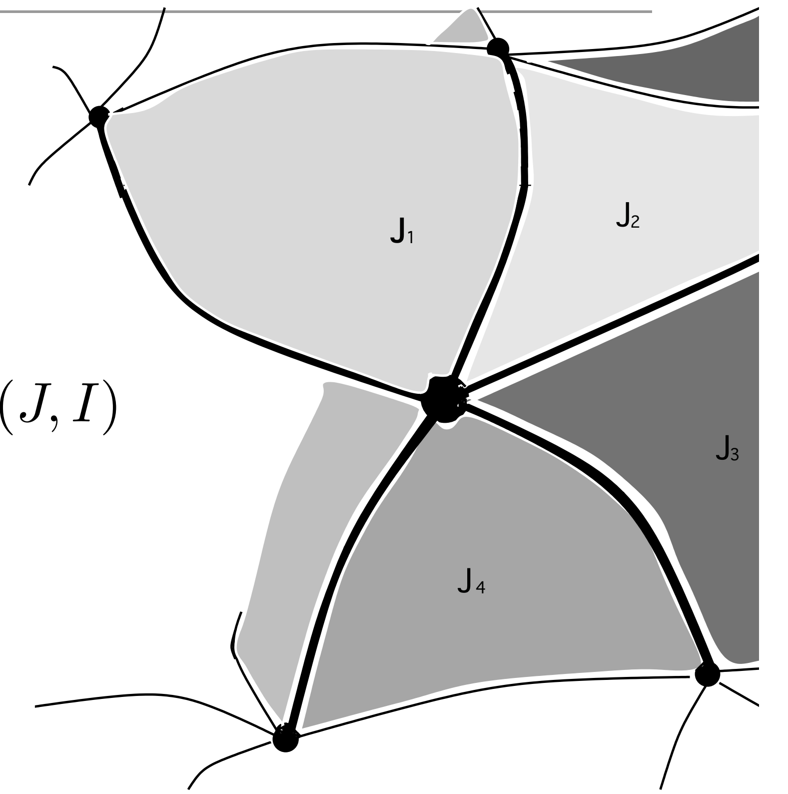

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$$Z(\Gamma) \leftrightarrow \begin{cases} A_f(J) \\ A_e(J,I) \\ A_v(J,I) \end{cases} \longleftrightarrow \begin{cases} \mathcal{K}(J,I) \sim \mathcal{K}(g) \\ \mathcal{V}(J,I) \sim \mathcal{V}(g) \end{cases} \leftrightarrow \; S(\varphi,\bar{\varphi})$$

spin foam model

with sum over complexes

as GFT perturbative expansion

(valid for any SF model)

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

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GFTs, loop quantum gravity, spin foam models

appropriate conditions on GFT fields or GFT dynamics (and choice of data) turn GFT Feynman amplitudes into lattice gauge theories/discrete gravity path integrals/spin foam models

e.g. gauge invariance of GFT fields under diagonal action of group G

example: d=3 $\varphi_\ell : SO(3)^3/SO(3) \rightarrow \mathbb{R}$

$$\forall h \in \mathrm{SO}(3), \qquad \varphi_\ell(hg_1, hg_2, hg_3) = \varphi_\ell(g_1, g_2, g_3)$$

+ simplicial interaction with only delta functions

valid for GFT definition of BF theory in any dimension

Quantum 3d simplicial geometry (Riemannian)

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classical triangle in $\mathbb{R}^3$

3 edge vectors that close $x_1, x_2, x_3 \in \mathbb{R}^3 \quad s.t. \quad \sum_i x_i = 0$

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Phase space for triangle in discrete 3d gravity

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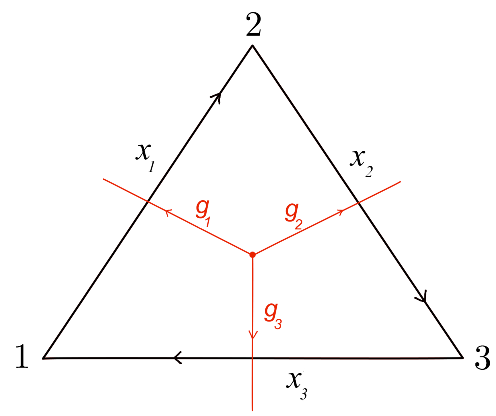

$$\varphi(g_1, g_2, g_3) \leftrightarrow \varphi(x_1, x_2, x_3)$$

GFTs, loop quantum gravity, spin foam models

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e.g. gauge invariance of GFT fields under diagonal action of group G

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$$S_{kin}[\varphi_\ell] = \int [\mathrm{d}g_i]^3 \sum_{\ell=1}^{4} \varphi_\ell(g_1, g_2, g_3)\overline{\varphi_\ell}(g_1, g_2.g_3),$$

$$\begin{aligned} S_{int}[\varphi_\ell] \quad &= \quad \lambda \int [\mathrm{d}g_i]^6 \, \varphi_1(g_1, g_2, g_3)\varphi_2(g_3, g_4, g_5)\varphi_3(g_5, g_2, g_6)\varphi_4(g_6, g_4, g_1) \\ &\quad + \lambda \int [\mathrm{d}g_i]^6 \, \overline{\varphi_4}(g_1, g_4, g_6)\overline{\varphi_3}(g_6, g_2, g_5)\overline{\varphi_2}(g_5, g_4, g_3)\overline{\varphi_1}(g_3, g_2, g_1) \end{aligned}$$

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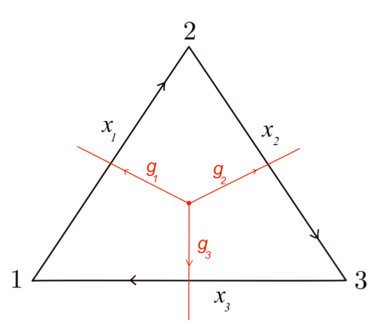

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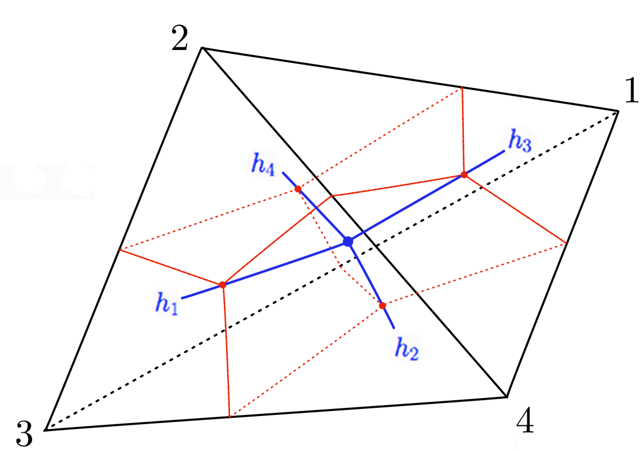

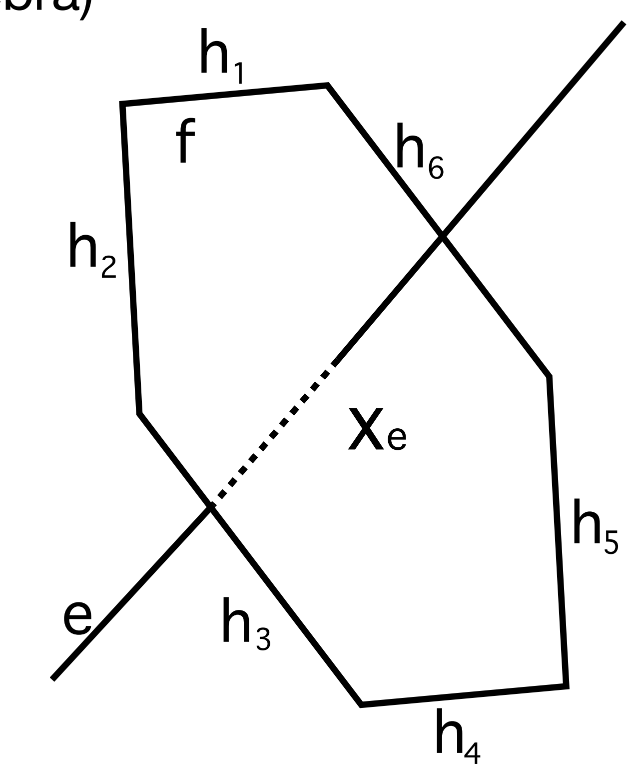

discretization of: $\quad S(e, \omega) = \int Tr(e \wedge F(\omega))$

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discrete 1st order path integral for 3d gravity/BF theory on simplicial complex dual to GFT Feynman diagram

spin foam formulation of 3d gravity/BF theory

GFTs, loop quantum gravity, spin foam models

GFT models of 4d gravity:

based on classical (Plebanski) formulation of GR as BF theory + (simplicity) constraints

start from GFT formulation of 4d BF theory
+ impose simplicity constraints (geometricity of simplicial structures)

(Barbieri, Baez, Barrett, Crane, Reisenberger, Perez, De Pietri, Engle, Pereira, Freidel, Krasnov, Rovelli, Livine, Speziale, Baratin, DO, ...)

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$$\delta\phi = 0 \;\Rightarrow\; \star B \wedge B = 0 \;\Rightarrow\; B \simeq e \wedge e$$

classically equivalent to Palatini-Holst gravity:

$$S_{Holst} = \frac{1}{G}\int_{\mathcal{M}} \left[\star e \wedge e \wedge F(\omega) + \frac{1}{\gamma}\, e \wedge e \wedge F(\omega) \right]$$

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GFTs, loop quantum gravity, spin foam models

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based on classical (Plebanski) formulation of GR as BF theory + (simplicity) constraints

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simplicity constraints =

= specific relation between SL(2,C) data and SU(2) data

decompose GFT field in SU(2) data +
geometricity conditions

$\Downarrow$

GFT dynamics to LQG quantum states

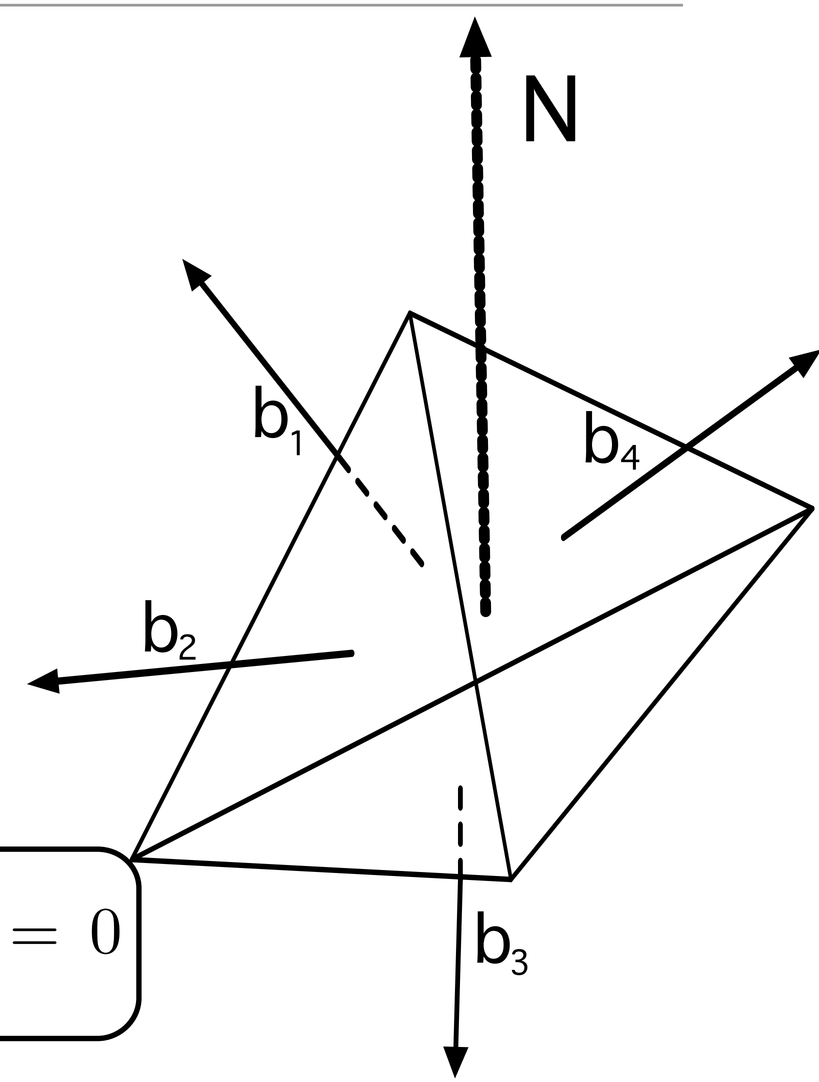
Quantum 4d simplicial geometry (riemannian)

classical tetrahedron in 4d:

$$A_i\, n_i^I = b_i^I \in \mathbb{R}^4 \qquad b_i \cdot N = 0 \qquad \sum_i b_i = 0$$

unique intrinsic geometry (up to rotations)

$$(B_i^{IJ} \in \wedge^2\mathbb{R}^4 \simeq \mathfrak{so}(4)\,,\, N^I \in \mathbb{R}^4) \qquad N_I\,(*B_i^{IJ}) = 0 \qquad \sum_i B_i^{IJ} = 0$$



phase space before constraints:

$$[\mathcal{T}^*Spin(4)]^{\times 4} \simeq [\mathcal{T}^*SU(2) \times \mathcal{T}^*SU(2)]^{\times 4}$$

how GFT help tackling open issues in LQG

open issues in LQG and spin foam models have precise GFT counterpart

QFT formalism provides powerful tools to tackle them

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(in many ways, background independent counterpart of issue of renormalizability in perturbative QG) Perez, '07

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—-> renormalizability of GFT for given spin foam amplitudes

- GFT symmetries (at both classical and quantum level) Ben Geloun, '11; Girelli, Livine, '11; Baratin, Girelli, Oriti, '11

—-> in particular, those with geometric interpretation (e.g. diffeomorphisms) Kegeles, DO, '15

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- how to define and control the continuum limit of the quantum LQG/SF dynamics?

controlling quantum dynamics of more and more interacting degrees of freedom

(large superpositions of large graphs) - inequivalent phases of LQG with different physics?

Ashtekar, Lewandowski, '94; Koslowski, '07; DO, '07; Koslowski, Sahlmann, '10, Dittrich, Geiller, '14; Gielen, DO, Sindoni, '13; DO, Tomlin, to appear

  - Non-perturbative GFT renormalization and phase diagram
  - Extraction of effective continuum dynamics in different phases

(as in condensed matter systems….)

Part II:

The problem of continuum in QG and GFT renormalisation

The problem of the continuum limit in QG

new (non-geometric, non-spatio-temporal) physical degrees of freedom (“building blocks”) for space-time

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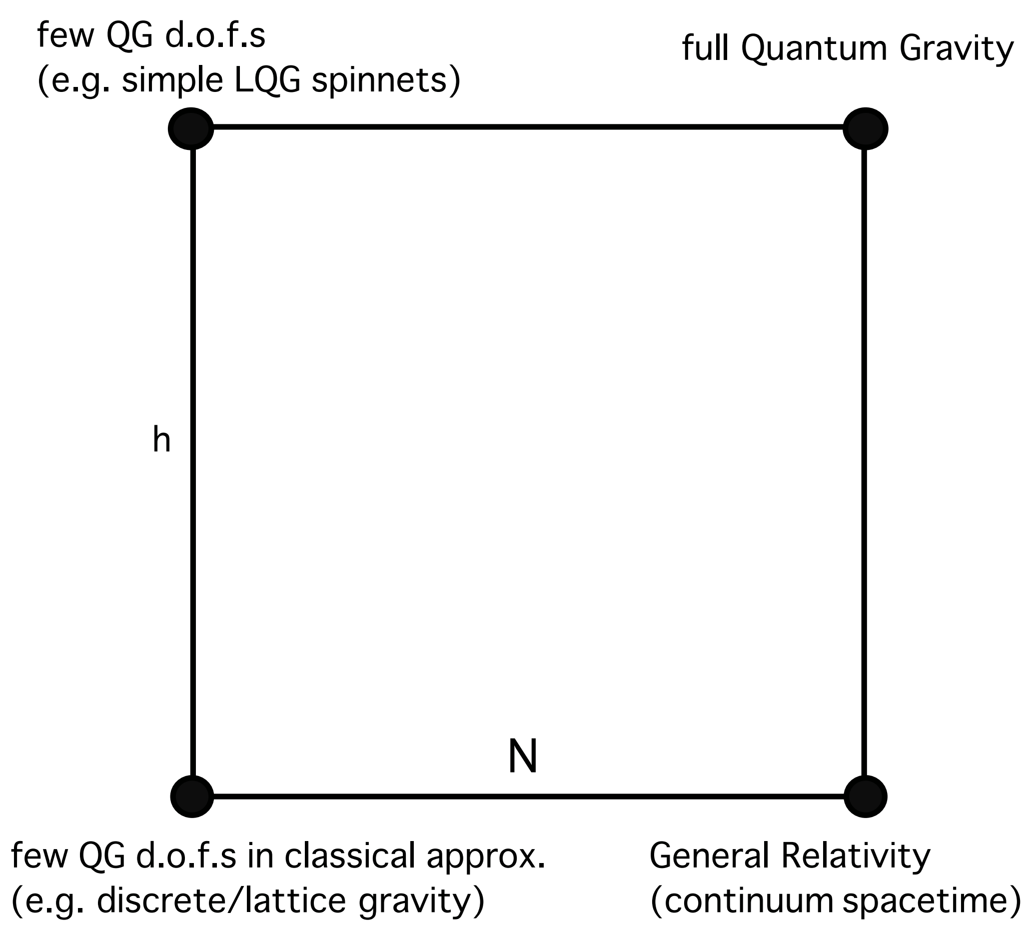

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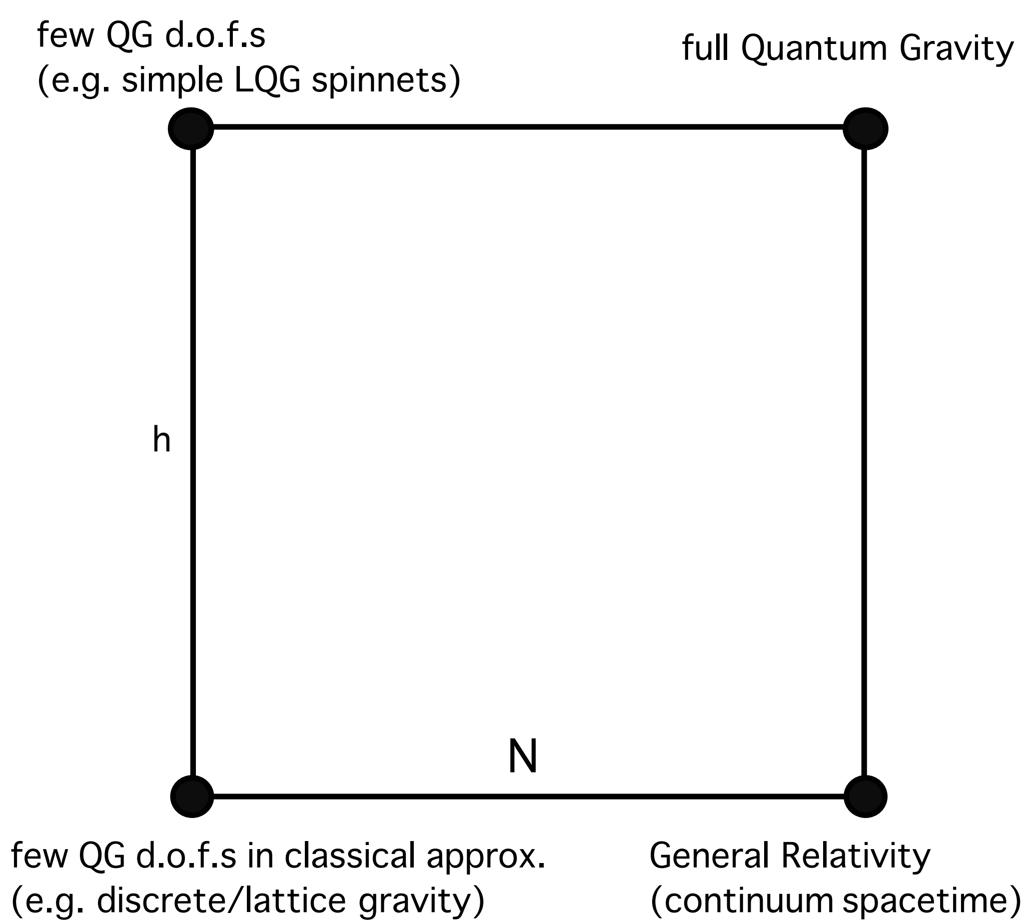
N-direction
(collective behaviour of many interacting degrees of freedom): continuum approximation

h-direction: classical approximation



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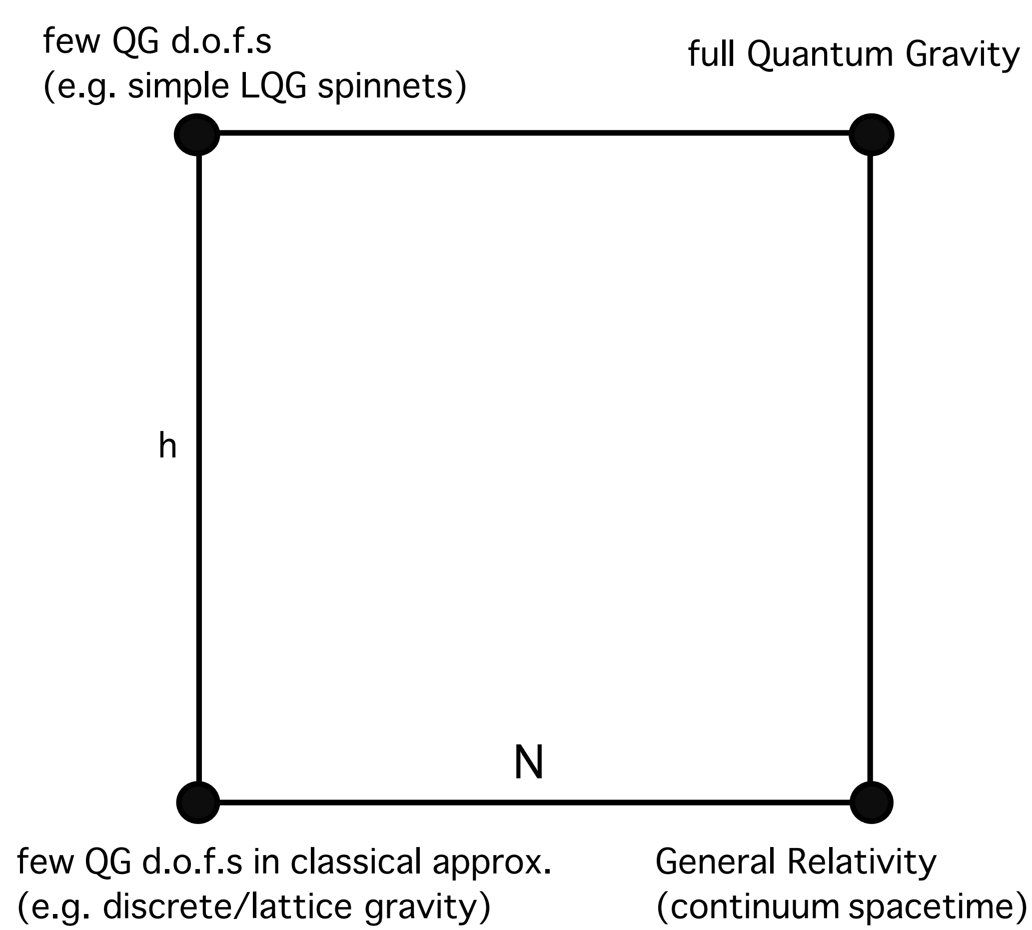
“well-understood” in spin foam models



Problem of the continuum in QG: role of RG

Renormalization Group is crucial tool (mathematical, conceptual, physical)

renormalization is not about “curing or hiding divergences”, but
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- in specific GFT case:
- fundamental formulation of QG = QFT, defined perturbatively around "no-space" (degenerate) vacuum

- need to prove consistency of the theory: perturbative GFT renormalizability

- need to understand effective dynamics at different "GFT scales": RG flow of effective actions & phase structure & phase transitions

GFT renormalisation - general scheme

see lectures by V. Rivasseau

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

general strategy:

treat GFTs as ordinary QFTs defined on Lie group manifold

use group structures (Killing form, topology, etc) to define notion of scale and to set up mode integration

subtleties of quantum gravity context at the level of interpretation

scales:

defined by propagator: spectrum of Laplacian = indexed by group representations or Lie algebra elements

- need to have control over "theory space" (e.g. via symmetries)

- main difficulty (at perturbative level):
  controlling the combinatorics of GFT Feynman diagrams to control the structure of divergences
  (more involved when gauge invariance is present)

most results for "Tensorial" GFTs

Tensorial GFTs (key insights from tensor models)

see lectures by V. Rivasseau

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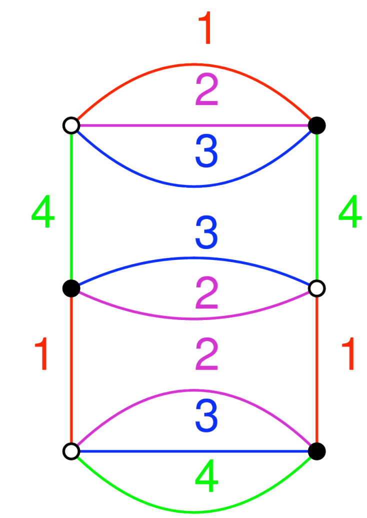
$$S(\varphi,\overline{\varphi}) = \sum_{b\in\mathcal{B}} t_b I_b(\varphi,\overline{\varphi})$$

indexed by d-colored “bubbles”



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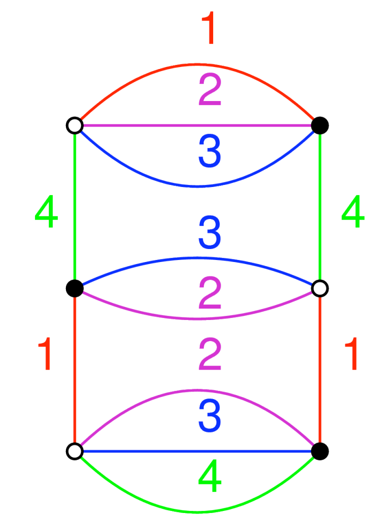

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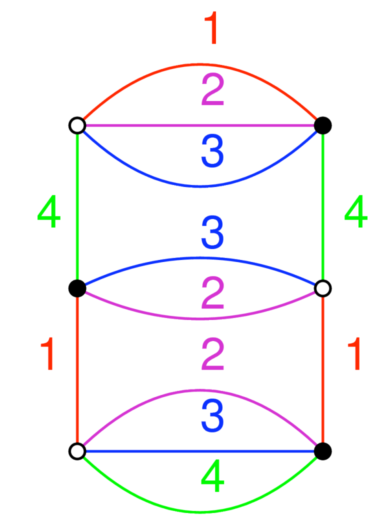

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Laplacian kinetic term

propagator $$\left(m^2 - \sum_{\ell=1}^{d} \Delta_\ell\right)^{-1}$$

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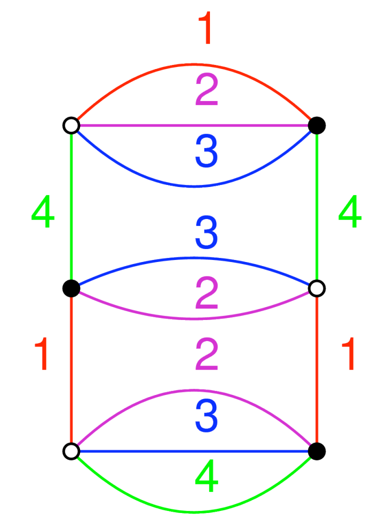

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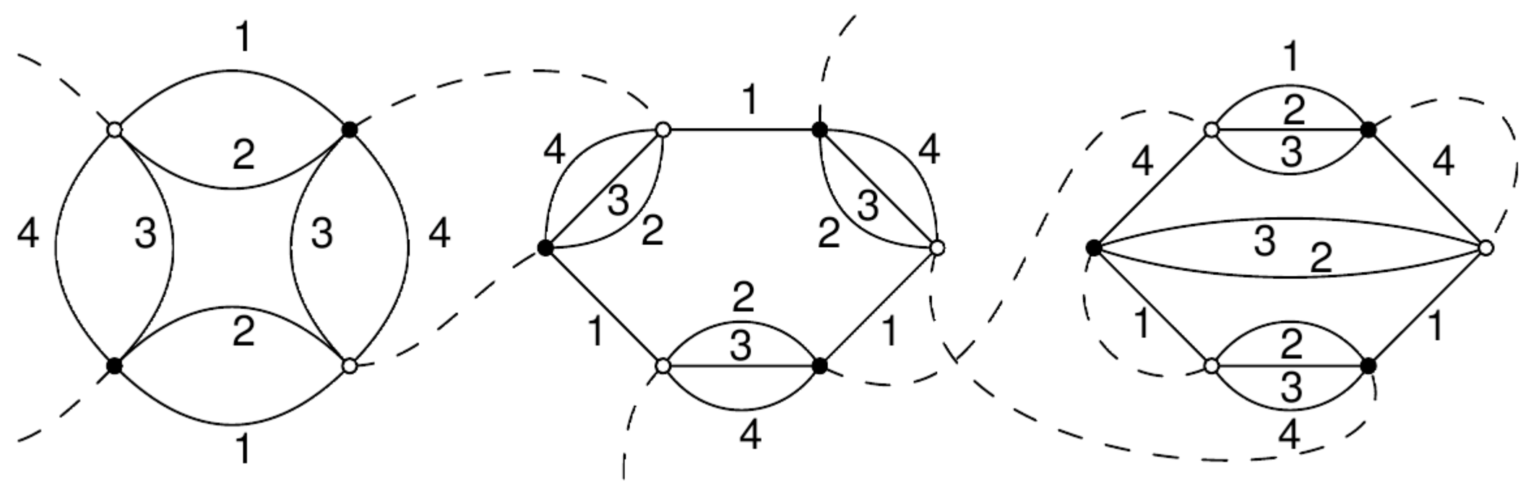

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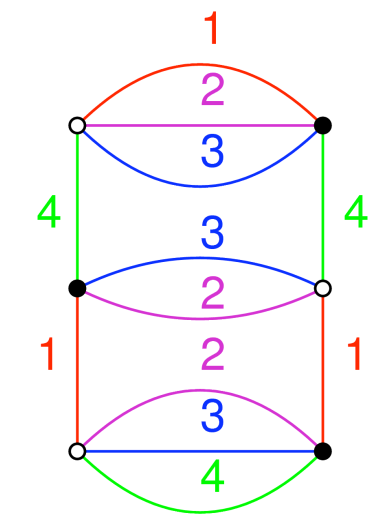

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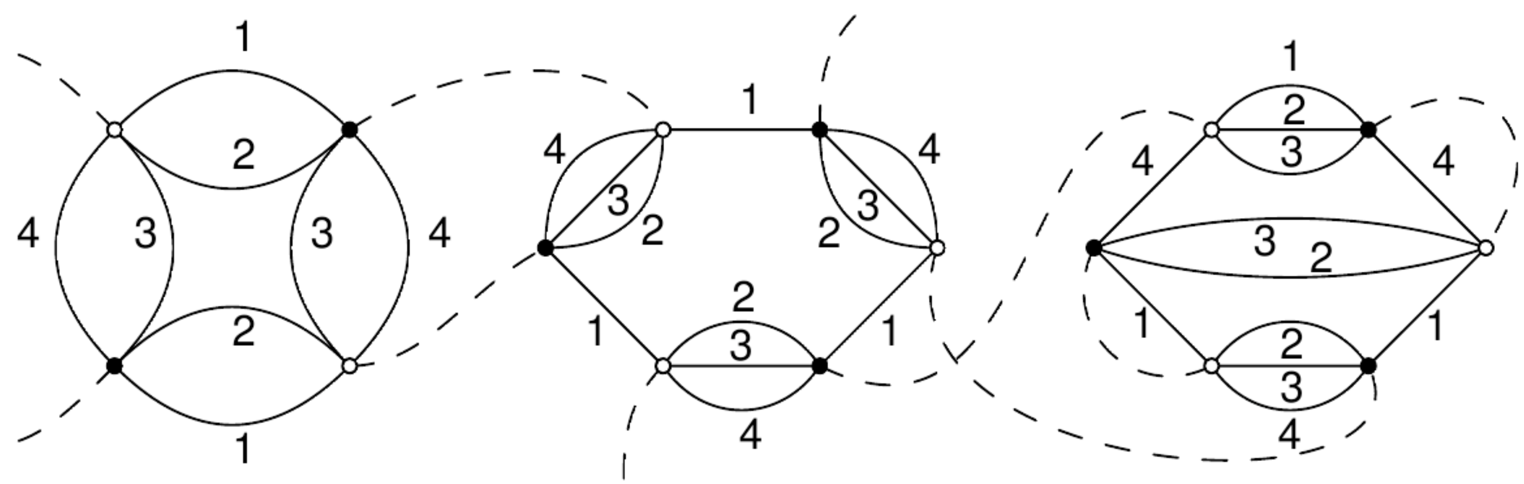
“coloring” allows control over topology of Feynman diagrams



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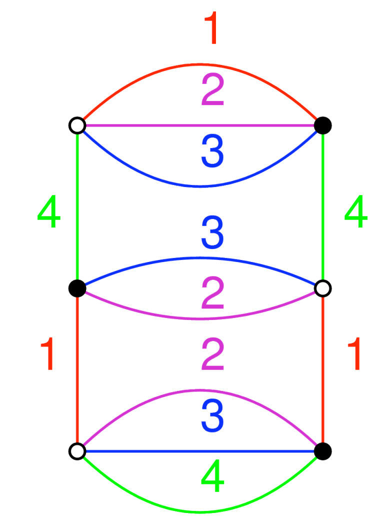

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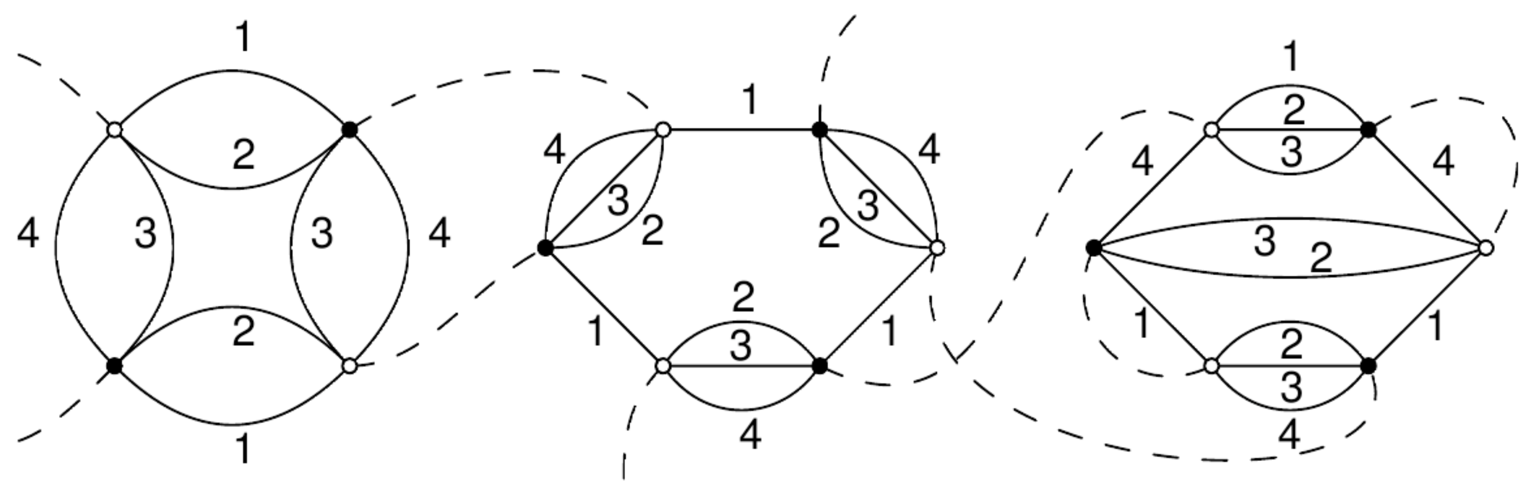

require generalization of notions of "connectedness", "contraction of high subgraphs", "locality", Wick ordering, ….

taking into account internal structure of Feynman graphs, full combinatorics of dual cellular complex, results from crystallization theory (dipole moves)

GFT perturbative renormalization

see talk by J. Ben Geloun

- systematic renormalisation group analysis of tensorial GFT models:

requires subtle analysis of combinatorics of diagrams (dual to cellular complexes)

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- several renormalizable abelian TGFT models (different groups and dimension, with/without gauge invariance)

J. Ben Geloun, V. Rivasseau, '11; J. Ben Geloun, D. Ousmane-Samary, '11 S. Carrozza, DO, V. Rivasseau, '12

- first renormalizable non-abelian TGFT model in 3d with gauge invariance (3d BF + laplacian)

S. Carrozza, DO, V. Rivasseau, '13

- first renormalizable TGFT model on homogeneous space (SU(2)/U(1))^d V. Lahoche, DO, '15
- proof of asymptotic freedom for abelian TGFT models without gauge invariance

J. Ben Geloun, D. Ousmane-Samary, '11; J. Ben Geloun, '12

- study of asymptotic freedom/safety for non-abelian TGFT models with gauge invariance

S. Carrozza, '14

Non-perturbative GFT renormalisation (continuum limit)

see talk by J. Ben Geloun

the issue: $$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

controlling quantum dynamics of more and more (up to infinity) interacting degrees of freedom

~ evaluating GFT path integral (in some non-perturbative approximation = full spin foam sum)

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IR fixed point of RG flow of GFT model

IR cutoff N —-> 0

(small J, assuming large-J integrated out)

~ definition of full GFT path integral

~ full continuum limit (all dofs of spin foam model)

$$\mathcal{Z}_N[J] = e^{W_N[J]} = \int_M d\phi\, e^{-S[\phi]-\Delta S_N[\phi]+\mathrm{Tr}_2(J\cdot\phi)}$$

$$\Gamma_N[\varphi] = \sup_J \Big(\mathrm{Tr}_2(J\cdot\varphi) - W_N(J) \Big) - \Delta S_N[\varphi]$$

$$\partial_t \Gamma_N[\varphi] = \frac{1}{2} \overline{\mathrm{Tr}}(\partial_t R_N \cdot [\Gamma_N^{(2)} + R_N]^{-1})$$

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more or less standard set-up

main difficulty: combinatorial structure of interactions

Non-perturbative GFT renormalization

see talk by J. Ben Geloun

Main results:

- Polchinski formulation based on SD equations   Krajewski, Toriumi, '14
- general set-up for Wetterich formulation based on effective action   Benedetti, Ben Geloun, DO, '14
- RG flow and phase diagram established for:
    - TGFT on compact U(1)^3 with 4th order interactions   Benedetti, Ben Geloun, DO, '14
    - TGFT on non-compact R^3 with 4th order interactions   Ben Geloun, Martini, DO, '15
    - TGFT on compact U(1)^6 with 4th order interactions and gauge invariance   Benedetti, Lahoche, '15
    - TGFT on non-compact R^d with 4th order interaction and gauge invariance   Ben Geloun, Martini, DO, to appear

Note:
get non-autonomous system of beta functions in compact case (extra scale = size of group)
non-compact case via thermodynamic limit
results in agreement with "large-N" approx of compact case

Phase diagrams qualitatively very similar (universal features?)

Non-perturbative GFT renormalization

see talk by J. Ben Geloun

Example of phase diagram:

U(1)^3 model in large-N approximation

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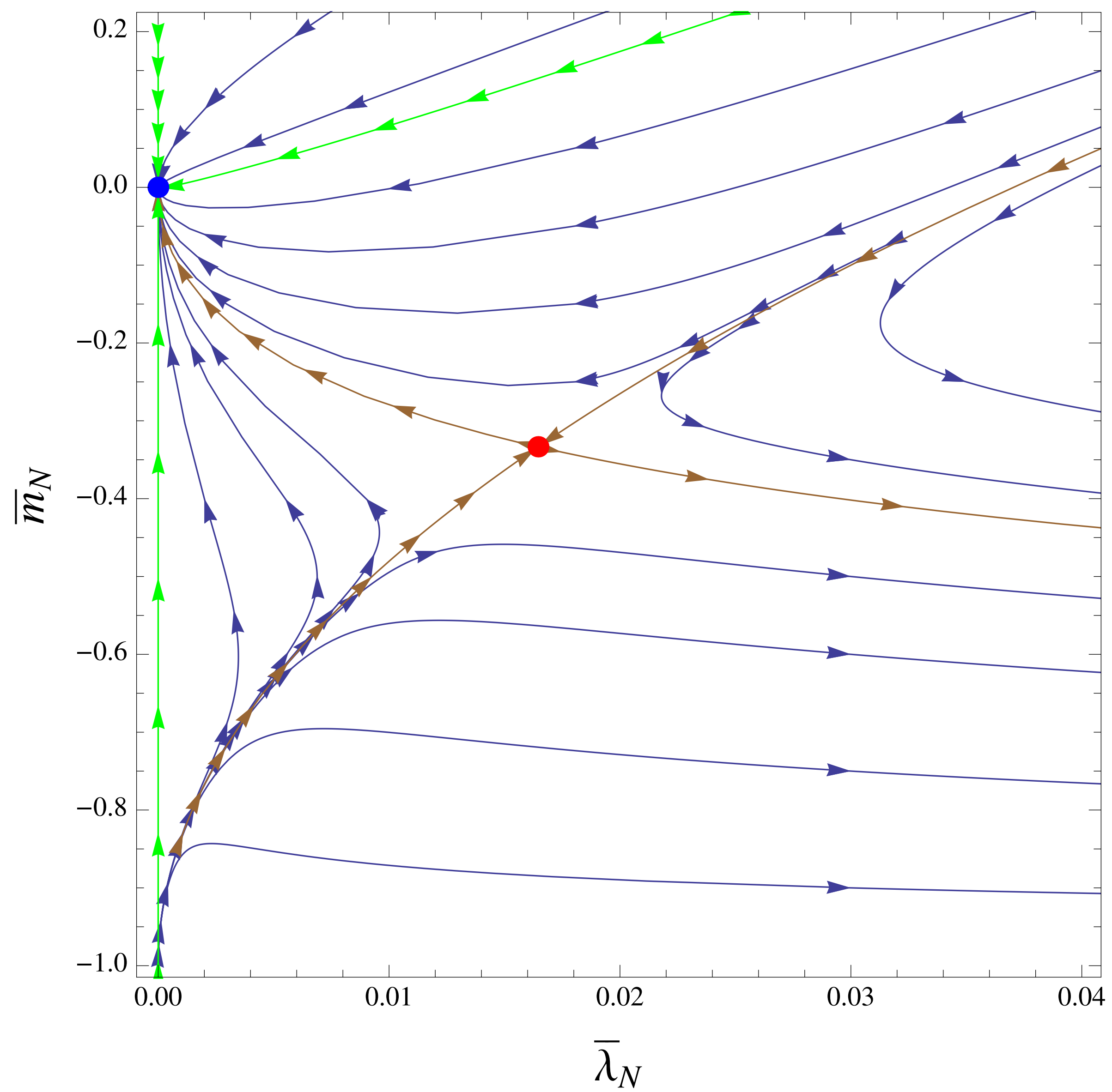
$$\Gamma_N(\varphi) = \frac{Z_N}{2}\mathrm{Tr}_2(\varphi\cdot K\cdot\varphi) + \frac{m_N}{2}\mathrm{Tr}_2(\varphi^2) + S^{\mathrm{int}}$$

$$S'^{\mathrm{int}} = \frac{\lambda_N}{4}\Big(\mathrm{Tr}_{4;1}(\varphi^4) + \mathrm{Sym}(1\to 2\to 3)\Big)$$

$$\mathrm{Tr}_2(\varphi\cdot K\cdot\varphi) = \sum_{p_i\in\mathbb{N}} \varphi_{123}\left(\tfrac{1}{3}\sum_i p_i\right)\varphi_{123}$$

$$\mathrm{Tr}_2(\varphi^2) = \sum_{p_i\in\mathbb{N}} \varphi_{123}^2$$

$$\mathrm{Tr}_{4;1}(\varphi^4) = \sum_{p_i,p_i'\in\mathbb{N}} \varphi_{123}\,\varphi_{1'23}\,\varphi_{1'2'3'}\,\varphi_{12'3'}$$



one Gaussian UV FPs, one non-Gaussian IR fixed point of Wilson-Fischer type

one symmetric phase

one broken or condensate phase - order parameter is expectation value of field operator

Part III:

Emergent cosmology from GFT condensation

The “geometrogenesis” hypothesis

in canonical LQG context:
T. Koslowski, 0709.3465 [gr-qc]

in covariant SF/GFT context:
DO, 0710.3276 [gr-qc]

also in tensor models
V. Rivasseau, ‘13

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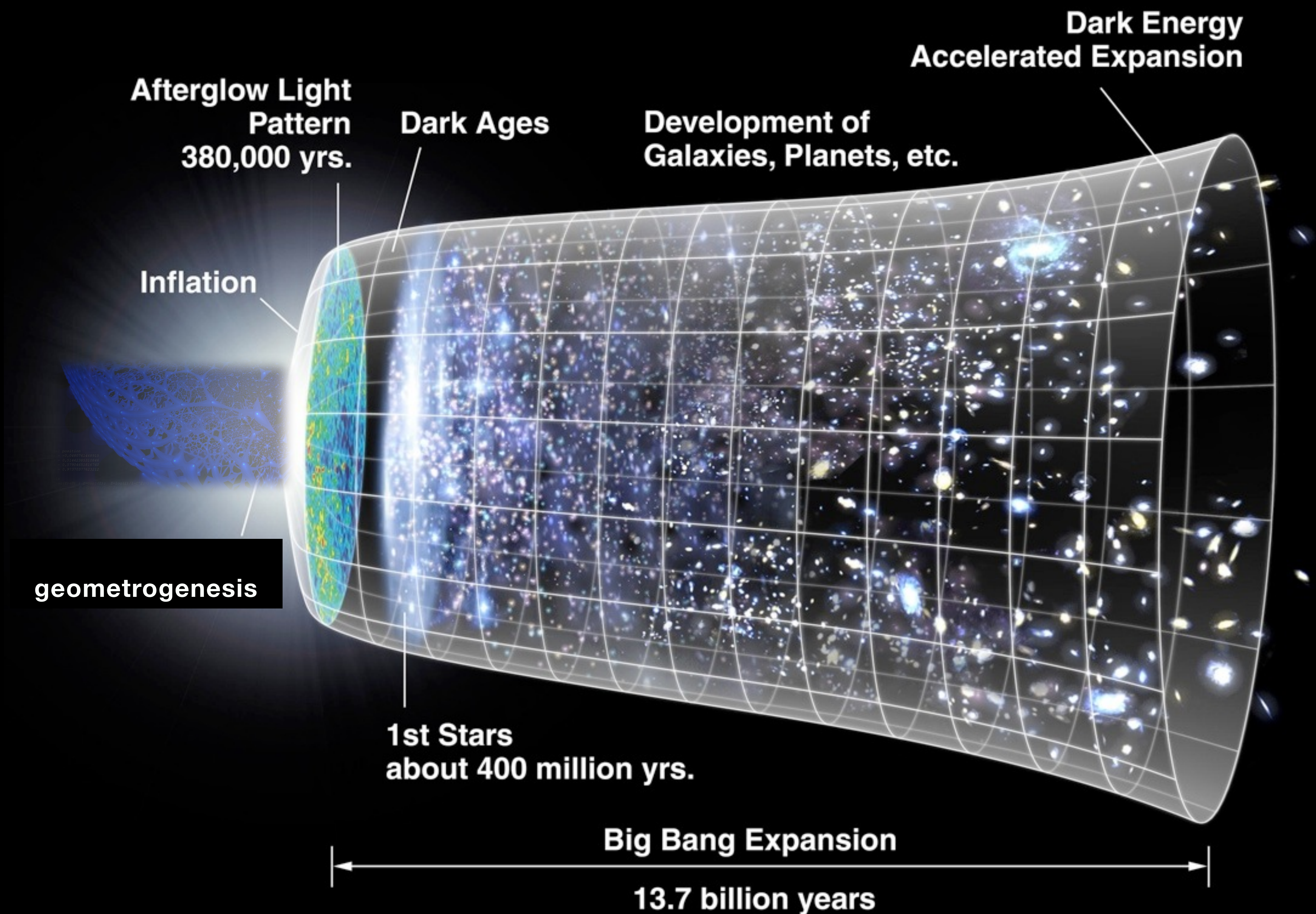
in this perspective, what is the role of (quantum) cosmology?



Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
geometrogenesis
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Two points of view on quantum gravity and quantum spacetime

two views:

1. quantum gravity = quantum theory of gravitational field ~ quantum General Relativity

2. quantum gravity = microscopic theory of pre-geometric quantum degrees of freedom
("quantum (field) theory of atoms of space")

→ gravitational field result of collective dynamics
spacetime and geometry are emergent entities

in case 2.

(quantum) cosmological degrees of freedom governed by statistical distribution
not quantum theory of homogeneous geometries (quantum cosmology)

cosmological dynamics is the hydrodynamic approximation of full quantum gravity
(most macroscopic, coarse grained, global description of the microscopic pre-geometric system)

Quantum cosmology or cosmological hydrodynamics?

….. option 2 suggests a picture in which a "quantum cosmology wavefunction" describes a homogeneous patch/region of space, with many such regions to be patched together to form an arbitrary spatial configuration

Bojowald, '14

candidate single-patch dynamics: (quantum) Friedmann-like eqn

expect full dynamics for "cosmological wave-function" to be non-linear - no superposition, no Hilbert space

if probability interpretation (on minisuperspace), only in statistical sense

multi-patch cosmology more naturally understood as coarse grained "hydrodynamic description"

cosmology as hydrodynamics

advantages:
perspective and tools from condensed matter theory

"easier", at least conceptually: inhomogeneities are always present, no real truncation of dofs

Cosmology as hydrodynamics of (quantum) spacetime

re-thinking the "Cosmological Principle": "every point is equivalent to any other" ~ homogeneity of space

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really means: a certain approximation is assumed valid:

universe is in state where inhomogeneities can be neglected, in relation to dynamics of homogeneous modes

~ universe is in state where effects on largest wavelengths of shorter wavelengths is negligible

~ can neglect wavelengths (much) shorter than scale factor

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dynamics of microscopic degrees of freedom can be neglected + effects of small wavelengths can be neglected

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implies:

degrees of freedom of local region can describe whole of system (in a coarse grained, statistical sense)

i.e. whole universe (dynamics) well-approximated by local patch (dynamics)

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recall: standard hydrodynamics from classical many-particle system

e.g. N particles in R

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from knowledge of microstate, get reduced 1-particle density by coarse graining:

$$\rho(x,p) = \int [dx_i][dp_i]\, D_N\left(x,p;x_2,p_2;....;x_N,p_N\right)$$

density (probability measure)
in phase space

which particle is chosen is irrelevant because of permutation symmetry

such that:

$$\int dxdp\, \rho(x,p) \;=\; 1$$

$$\rho(x,p)dxdp = \left[\int [dx_i][dp_i]\left(\sum_i \delta(x-x_i)\,\delta(p-p_i)\right)\right] dxdp$$

probability to find a particle in
the phase space region dxdp

$$\rho(x) \;=\; \int dp\, \rho(x,p)$$

more apt for QM systems

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- a “point” in the fluid corresponds to a region containing a large number of microscopic constituents
- valid at long wavelengths (not sensitive to small-scale dynamics)

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how to define the basic cosmological hydrodynamic variable?

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- use lattice to replace smooth manifold: points replaced by fundamental lattice cells

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cosmology is (non-linear) dynamics for such density and for geometric (global) observables computed from it

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key strategy:

coarse graining of kinematical QG configurations

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one special case:

quantum condensates (BEC)

effective hydrodynamics directly read out of microscopic quantum dynamics (in simplest approximation)

(Quantum) Cosmology from GFT

S. Gielen, DO, L. Sindoni, PRL, arXiv:1303.3576 [gr-qc]; JHEP, arXiv:1311.1238 [gr-qc]

problem 1:
identify quantum states in fundamental theory with continuum spacetime interpretation

problem 2:
extract from fundamental theory an effective macroscopic dynamics for such states

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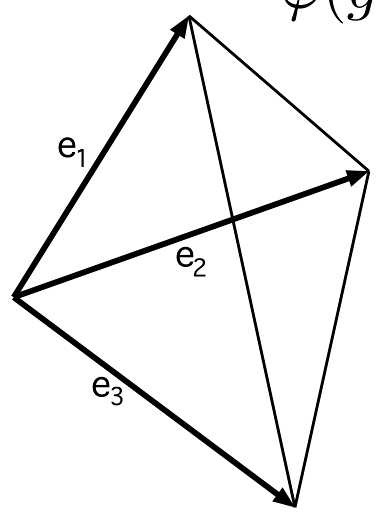
S. Gielen, ’14; G. Calcagni, ’14; L. Sindoni, ’14; S. Gielen, DO, ’14; S. Gielen, ’14; S. Gielen, ’15; DO, L. Sindoni, E. Wilson-Ewing, to appear

GFT states and approximate continuum geometries

- work with GFT with simplicial geometric interpretation (Riemannian SO(4) case for simplicity)

$$\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \to \mathbb{C}$$ describes geometric tetrahedron



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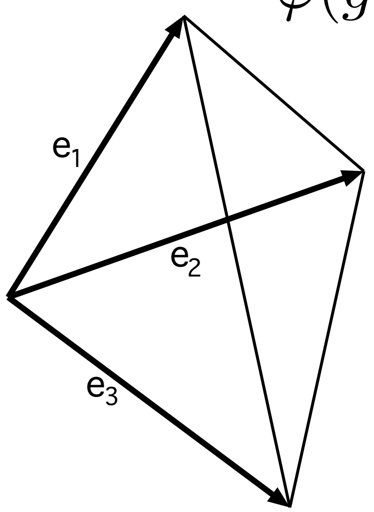

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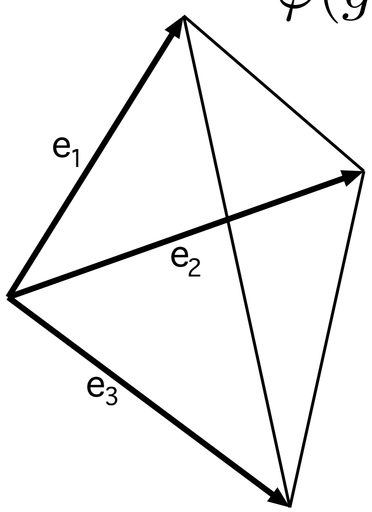

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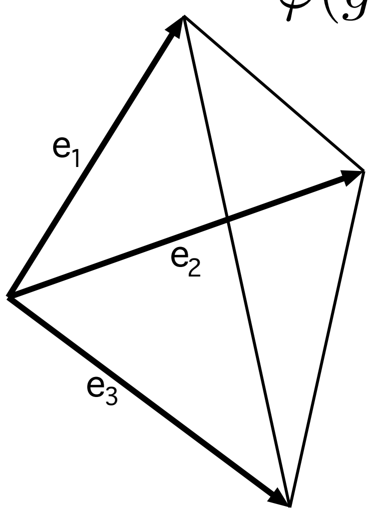

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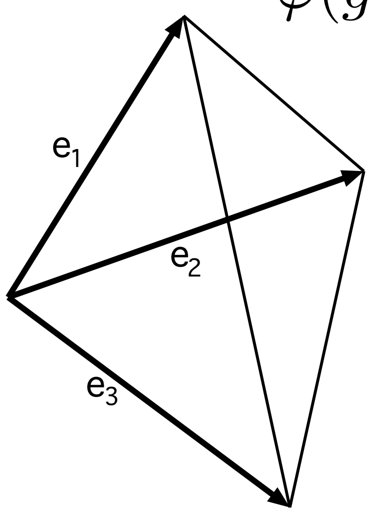

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- classical criterion for homogeneity (for GFT data):

$$g_{ij(m)} = g_{ij(k)} \quad \forall k, m = 1, \ldots, N$$

i.e. all GFT quanta are labelled by the same (gauge invariant) data

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→ quantum GFT condensates are continuum homogeneous (quantum) spaces

similar constructions in LQG (Alesci, Cianfrani) and LQC (Bojowald, Wilson-Ewing, ...)

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a simple choice of quantum GFT condensate
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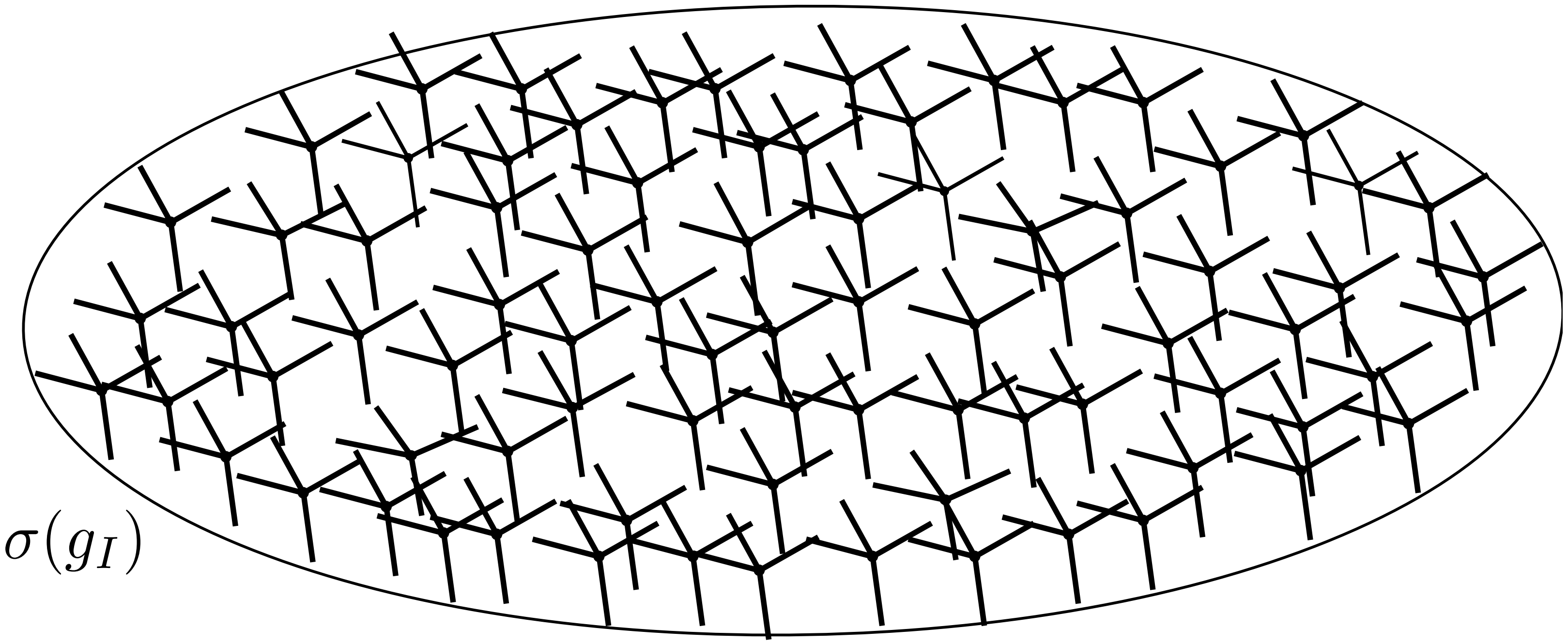

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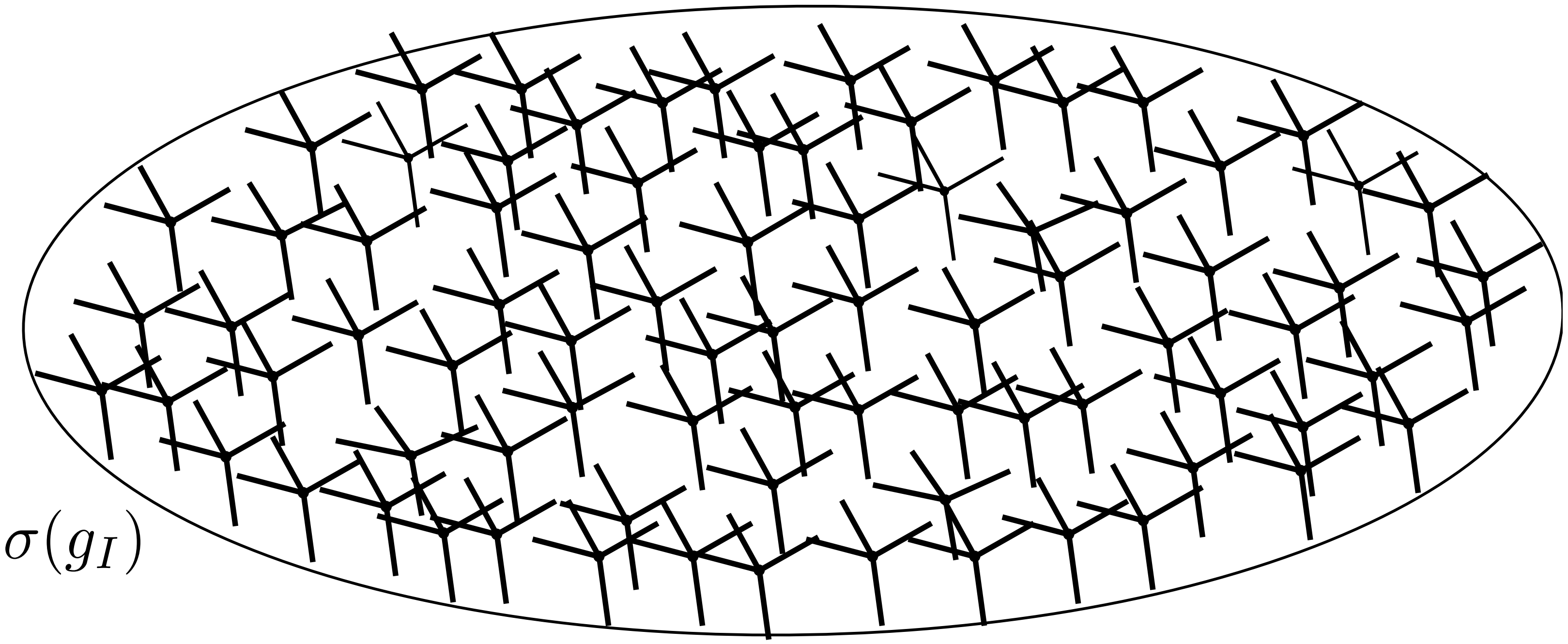

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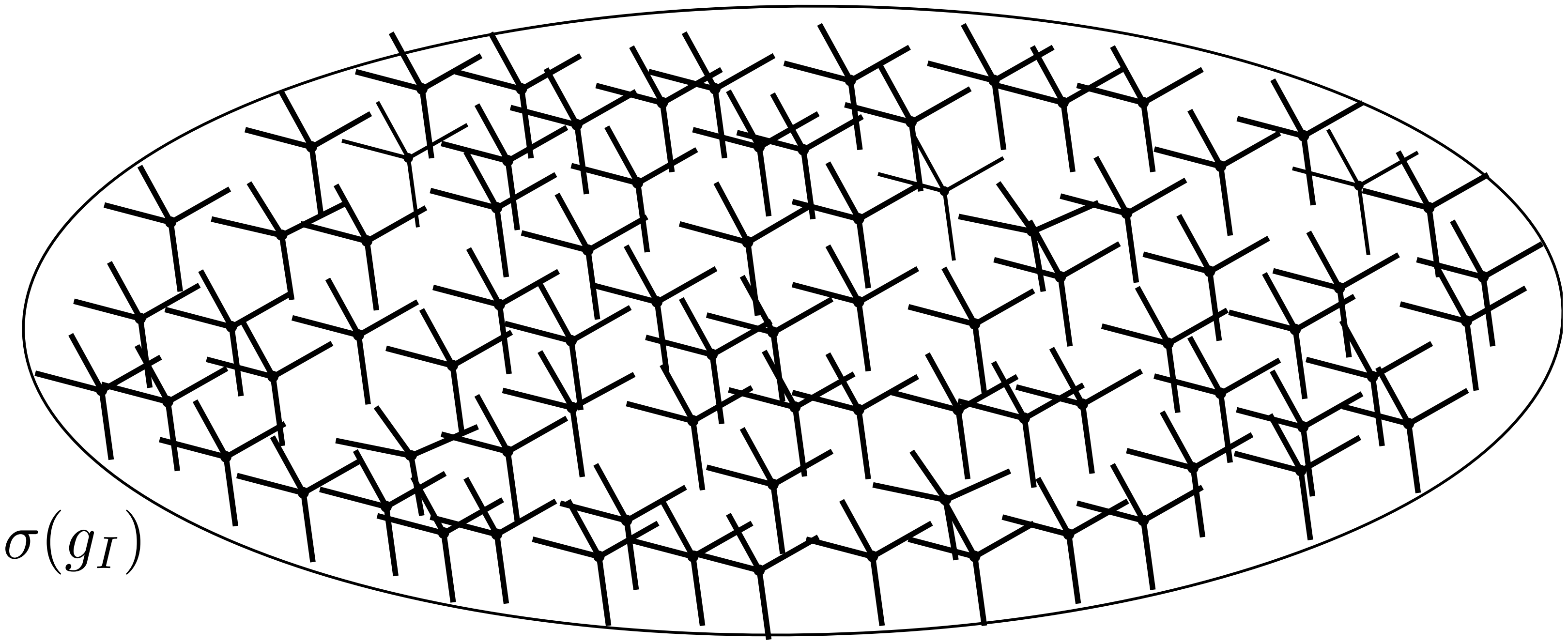

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- data for homogeneous anisotropic geometries
- truly non-perturbative quantum states (infinite QG dofs, superposition of graphs)
- support perturbations at any sampling scale N
- 2nd quantized coherent states $\hat{\varphi}(g_I)|\sigma\rangle = \sigma(g_I)\,|\sigma\rangle$
- can be studied using BEC techniques

Effective cosmological dynamics from GFT

S. Gielen, DO, L. Sindoni,

PRL, arXiv:1303.3576 [gr-qc];

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similar equations to M. Bojowald et al., arXiv:1210.8138 [gr-qc]

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in some simple models and with various simplifying assumptions, effective dynamics take form of (modified) Friedman equation (also with scalar field);
work recently extended to fundamental GFT/spin foam models (DO, L. Sindoni, E. Wilson-Ewing, to appear)

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→ Cosmology as Quantum Gravity (condensate) hydrodynamics!

Thank you for your attention!