The Standard Model in Noncommutative Geometry: fermions as internal Dirac spinors

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(based on JNCG in print with F. D'Andrea)

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Establish if in the noncommutative geometry approach to the Standard Model (νSM) the fundamental fermions can be regarded as 'quantum' Dirac spinors of the internal space.

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2. *Formulate the concept of quantum spin space and of Dirac spinors in terms of Morita equivalence involving the underlying algebra A and certain quantum analogue of Clifford bundle algebra*
3. *See what happens in νSM.*

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Proviso: *quantum* $=$ *noncommutative (NC)*

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Framework: NC or *spectral* geometry à la Connes et. al.

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ordinary (spin) manifold $\times$ a finite quantum space,

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The gauge fields are encoded by the usual Dirac operator $\not{D}$ on M plus certain Hermitian matrix D operating on H_F.

Intro 2

This "almost commutative" geometry is mathematically a product of two *spectral triples* (S.T.):
The first one is the *canonical* S.T. on M

$$\big(\, C^\infty(M), L^2(\Sigma), \not{D} \,\big), \tag{1}$$

a prototype of commutative S.T. Under some assumptions one can reconstruct the data $M, g, \Sigma \,\&\, \not{D}$.

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Answer: ’Yes’, if two extra fields are added (& a different grading).

Other modifications

In fact to get the correct experimental value of the Higgs mass, various modifications of the CC model have been proposed:
- enlarge H thus introducing new fermions [Ste09]
- turn one of the elements in D into a field by hand [CC12] rather than getting it as a fluctuation of the metric;
- relax the 1st order condition [CCvS13] & allow new terms in D;
- enlarge A [DLM13] and use the *twisted* spectral triple [DM13]

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NCG gives a possibility to employ also quantum groups [BDDD13], and actually just algebras instead of groups.

Dirac spinors: classical

An oriented Riemannian manifold M is spin^c iff ...

- $SO(n)$ frame bundle lifts to $Spin_c(n)$
- $w_2(M)$ is a $\mathbb{Z}_2$-reduction of a class in $H^2(M, \mathbb{Z})$
- $\boxed{\text{there is a Morita equivalence } \mathcal{C}\ell(M) - C(M) \text{ bimodule } \Sigma}$

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Automatically $\Sigma = \Gamma(S)$, where S is the complex vector bundle of Dirac spinors on M in conventional diff. geom., on which $C(M)$ acts by pointwise multiplication, $\mathcal{C}\ell(M)$ by Clifford multiplication $\cdot$, and one constructs canonical $\not{D}$. Now, for $f \in C^\infty(M)$,

$$i[\not{D}, f] = \mathrm{d}f\cdot$$

and such operators generate $\mathcal{C}\ell(M)$.

If dim$M$ is even the S.T. (1) is $\mathbb{Z}_2$-graded; $\exists$ a grading $\gamma \in \mathcal{C}\ell(M)$.

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There is an algebraic characterization for spin manifolds as well: a spin^c manifold is spin iff $\exists$ a real structure (charge conjugation).

Another class of N.C. S.T.

Def

A <u>finite-dimensional</u> spectral triple (A, H, D) consists of:
- (real or complex) $*$ subalgebra A of matrices acting on
- fin. dim. Hilbert space H, and
- Hermitian matrix D on H.
(A, H, D) is even if H is $\mathbb{Z}_2$-graded, A is even and D is odd; we denote by γ the grading operator.
(A, H, D) is real if $\exists$ an antilinear isometry J on H, s.t.

$$J^2 = \epsilon \, \mathrm{id}_H \ , \qquad JD = \epsilon' DJ \ , \qquad J\gamma = \epsilon'' \gamma J$$

for some $\epsilon, \epsilon', \epsilon'' \in \{\pm 1\}$, plus the 0th order condition:

$$[a, JbJ^{-1}] = 0 \qquad \forall \, a, b \in A,$$

and the 1st order condition:

$$[[D, a], JbJ^{-1}] = 0 \qquad \forall \, a, b \in A. \tag{2}$$

Dirac spinors: quantum

Quite as in [Lord,Rennie,Varilly12]:

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We call Clifford algebra the complex $*$*-algebra* $\mathcal{C}\ell(A)$ *generated by* A, $[D, A]$ *and by* γ *(if any).*

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Now, can't have a $\mathcal{C}\ell(A)$-A bimodule (for noncommutative A), but the 0th and 1st order $\Rightarrow$ H is a $\mathcal{C}\ell(A)$-A° bimodule, where

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Def (FD'A, LD)

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Examples:

- Classical case
- $H = A$, $J(a) = a^*$ and $D = 0$.

What about the internal S.T. of νS.M. ?

H_F

We identify (for any of 3 generations)

$$H_F = \mathbb{C}^{32} \simeq M_{8\times 4}(\mathbb{C})$$

with basis labelled by particles and antiparticles arranged as

$$v_1 = \begin{bmatrix} \nu_R & u_R^1 & u_R^2 & u_R^3 \\ e_R & d_R^1 & d_R^2 & d_R^3 \\ \nu_L & u_L^1 & u_L^2 & u_L^3 \\ e_L & d_L^1 & d_L^2 & d_L^3 \\ \bar{\nu}_R & \bar{e}_R & \bar{\nu}_L & \bar{e}_L \\ \bar{u}_R^1 & \bar{d}_R^1 & \bar{u}_L^1 & \bar{d}_L^1 \\ \bar{u}_R^2 & \bar{d}_R^2 & \bar{u}_L^2 & \bar{d}_L^2 \\ \bar{u}_R^3 & \bar{d}_R^3 & \bar{u}_L^3 & \bar{d}_L^3 \end{bmatrix}$$

(1,2,3=colors).

A general linear operators on H_F is a finite sum $L = \sum_i a_i \otimes b_i$, with $a_i \in M_8(\mathbb{C})$ acting on the left and $b_i \in M_4(\mathbb{C})$ on the right.

In particular $A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \ni (\lambda, q, m)$ is represented by

$$a = \begin{bmatrix} \left[\begin{array}{cc|cc} \lambda & 0 & 0 & 0 \\ 0 & \bar{\lambda} & 0 & 0 \\ \hline 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{q}} \\ 0 & 0 & & \end{array}\right] & 0_4 \\ 0_4 & \left[\begin{array}{c|ccc} \lambda & 0 & 0 & 0 \\ \hline 0 & & & \\ 0 & & m & \\ 0 & & & \end{array}\right] \end{bmatrix} \otimes 1_4 \,. \qquad (3)$$

The real conjugation J_F is the operator

$$J_F \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_2^* \\ v_1^* \end{bmatrix} . \tag{4}$$

$A_F^\circ = J_F A_F J_F \subset \mathrm{End}_{\mathbb{C}}(H_F)$ consists of elements of the form:

$$a^\circ = \begin{bmatrix} 1_4 & 0_4 \\ 0_4 & 0_4 \end{bmatrix} \otimes \left[\begin{array}{c|ccc} \lambda & 0 & 0 & 0 \\ \hline 0 & & & \\ 0 & & m & \\ 0 & & & \end{array}\right] + \begin{bmatrix} 0_4 & 0_4 \\ 0_4 & 1_4 \end{bmatrix} \otimes \left[\begin{array}{cc|cc} \lambda & 0 & 0 & 0 \\ 0 & \bar{\lambda} & 0 & 0 \\ \hline 0 & 0 & & \\ 0 & 0 & & q \end{array}\right] .$$

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Note that if $A \subset \mathrm{End}_{\mathbb{C}}(H_F)$ is a real $*$-subalgebra, its complex linear span $A_{\mathbb{C}}$ has the same commutant in $\mathrm{End}_{\mathbb{C}}(H_F)$.
The map $a \mapsto a^\circ = J_F \bar{a} J_F$ (here $\bar{a} = (a^*)^t$) gives two isomorphisms $A_F \to A_F^\circ$ and $(A_F)_{\mathbb{C}} \to (A_F^\circ)_{\mathbb{C}}$.

Lemma (By direct computation)

The commutant of A_F *in* $M_8(\mathbb{C})$ *is the algebra* C_F *with elements*

$$\begin{bmatrix} q_{11} & & & q_{12} & \\ & \alpha & & & \\ & & \beta 1_2 & & \\ q_{21} & & & q_{22} & \\ & & & & \delta 1_3 \end{bmatrix}, \tag{5}$$

where $\alpha, \beta, \delta \in \mathbb{C}$, $q = (q_{ij}) \in M_2(\mathbb{C})$.
The commutant of A_F *in* $\mathrm{End}_{\mathbb{C}}(H)$ *is*
$A'_F = C_F \otimes M_4(\mathbb{C}) \simeq M_4(\mathbb{C})^{\oplus 3} \oplus M_8(\mathbb{C})$, *of* $\dim_{\mathbb{C}} = 112$.

$(A_F^\circ)'$

The map $x \mapsto J_F \bar{x} J_F$ is an isomorphism between A_F' and $(A_F^\circ)'$. From this,

Lemma

The commutant $(A_F^\circ)'$ of A_F° has elements

$$a \otimes e_{11} + \begin{bmatrix} b & \\ & c \end{bmatrix} \otimes e_{22} + \begin{bmatrix} b & \\ & d \end{bmatrix} \otimes (e_{33} + e_{44}) \qquad (6)$$

with $a \in M_8(\mathbb{C})$, $b, c, d \in M_4(\mathbb{C})$.

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with $a \in M_8(\mathbb{C})$, $b, c, d \in M_4(\mathbb{C})$.

Lemma

$A'_F \cap (A_F^\circ)' \simeq \mathbb{C}^{\oplus 10} \oplus M_2(\mathbb{C})$.

It follows that $\dim_{\mathbb{C}}(A'_F + (A_F^\circ)') = 210 \quad (= 2 \cdot 112 - 14)$.
The (real) subspace of hermitian matrices has $\dim_{\mathbb{R}} = 210$.

D_F: the 1st order condition

Prop (Krajewski)

$D_F \in \mathrm{End}_{\mathbb{C}}(H_F)$ satisfies the 1st order condition (2) *iff*

$$D_F = D_0 + D_1$$

where $D_0 \in (A_F^\circ)'$ and $D_1 \in A_F'$.
Furthermore $[D_F, J_F] = 0$ iff

$$D_1 = J_F D_0 J_F.$$

If $D_F = D_F^*$, we have 112 free real parameters.

D_F: reformulation of the 1st order

$\mathcal{C}\ell(A_F)$ and the property *spin* constrains only D_0, and it is useful to reformulate the Props above as follows. Let

$$D_R = (\Upsilon_R e_{51} + \bar{\Upsilon}_R e_{15}) \otimes e_{11} \,, \tag{7}$$

with $\Upsilon_R \in \mathbb{C}$. Note that $D_R \in A'_F \cap (A^\circ_F)'$ and $J_F D_R = D_R J_F$.

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Prop

Most general $D_F = D_F^*$ satisfying the 1st order condition is

$$D_F = D_0 + D_1 + D_R \tag{8}$$

where $D_0 = D_0^* \in (A^\circ_F)'$ and $D_1 = D_1^* \in A'_F$ have null coefficeint of $e_{15} \otimes e_{11}$ and $e_{51} \otimes e_{11}$, and $D_1 = J_F D_0 J_F$ if D_F and J_F commute.

Grading

Consider now a grading operator γ_F anticommuting with J_F, which means KO-dimension 6 (or every except 0 and 4 if $J_F \rightsquigarrow J_F\gamma_F$, $D_F \rightsquigarrow D_F\gamma_F$, and/or forgetting γ_F).

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Lemma

Any γ_F-odd Dirac operator satisfying the 1st order condition can be written in the form

$$D_F = D_0 + D_1 + \epsilon D_R$$

as in (8), with both D_0 and D_1 γ_F-odd operators and $\epsilon = 0$ or 1 depending on the parity of D_R.
If moreover γ_F either commutes or anticommutes with J_F, then

$$D_1 = J_F D_0 J_F.$$

Some natural choices of γ_F, and the corresponding form of odd D_0 (D_1 is spurious) are:

The standard grading

$$\gamma_F = +1 \text{ on } p_R,\ \bar{p}_L,\ \text{and } -1 \text{ on } p_L,\ \bar{p}_R \qquad (9)$$

(KO-dim $=6$).
Any γ_F-odd D_0 has the form:

$$\begin{bmatrix} & & * & * & & \circledast & \circledast & \circledast \\ & & * & * & * & \circledast & \circledast & \circledast \\ * & * & & & & & & \\ * & * & & & & & & \\ & * & & & & & & \\ \circledast & \circledast & & & & & & \\ \circledast & \circledast & & & & & & \\ \circledast & \circledast & & & & & & \end{bmatrix} \otimes e_{11} + \begin{bmatrix} & & * & * & & & & \\ & & * & * & & & & \\ * & * & & & & & & \\ * & * & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \end{bmatrix} \otimes (1 - e_{11}) \ ,$$

A non-standard grading

Let

$$\gamma'_F = \gamma_F(L - B). \tag{10}$$

(opposite parity of chiral leptons w.r.t. quarks, still KO-dim =6).
Any γ'_F-odd D'_0 has the form:

$$\left[\begin{array}{cccc|cccc} & & * & * & & & & \\ & & * & * & * & & & \\ * & * & & & & \circledast & \circledast & \circledast \\ * & * & & & & \circledast & \circledast & \circledast \\ \cline{5-8} & * & & & & \circledast & \circledast & \circledast \\ & & \circledast & \circledast & \circledast & & & \\ & & \circledast & \circledast & \circledast & & & \\ & & \circledast & \circledast & \circledast & & & \end{array}\right] \otimes e_{11} + \left[\begin{array}{cccccccc} & & * & * & & & & \\ & & * & * & & & & \\ * & * & & & & & & \\ * & * & & & & & & \\ & & & & & \bigstar & \bigstar & \bigstar \\ & & & & \bigstar & & & \\ & & & & \bigstar & & & \\ & & & & \bigstar & & & \end{array}\right] \otimes e_{11}^{\perp}$$

where $\bigstar$ can differ for $\otimes e_{22}$ and $\otimes(e_{33} + e_{44})$.

Special case:

(Modified) Chamseddine-Connes's D_0'

$$\left[\begin{array}{cccc|cccc}
 & & \bar{\Upsilon}_\nu & & & & & \\
 & & & \bar{\Upsilon}_e & \bar{\Omega} & & & \\
\Upsilon_\nu & & & & & & & \\
 & \Upsilon_e & & & & & & \\
\hline
 & \Omega & & & & \color{red}{\Delta} & & \\
 & & & & \color{red}{\Delta} & & & \\
 & & & & & & & \\
 & & & & & & &
\end{array}\right] \otimes e_{11} + \left[\begin{array}{cccc|cccc}
 & & \bar{\Upsilon}_u & & & & & \\
 & & & \bar{\Upsilon}_d & & & & \\
\Upsilon_u & & & & & & & \\
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 & & & & & \color{red}{\Delta} & & \\
 & & & & \color{red}{\Delta} & & & \\
 & & & & & & & \\
 & & & & & & &
\end{array}\right] \otimes e_{11}^{\perp} \,,$$

where Υ's, $\Omega \in \mathbb{C}$, and $\Delta \in \mathbb{R}$ (term mixing leptons and quarks).

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\hline
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 & & & & & \Delta & & \\
 & & & & \Delta & & & \\
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where Υ's, $\Omega \in \mathbb{C}$, and $\Delta \in \mathbb{R}$ (term mixing leptons and quarks).

The Chamseddine-Connes is a further special case with $\Omega = \Delta = 0$ (which is odd w.r.t. the both gradings γ_F and γ_F').

The spin property

By straightforward (though meticulous and tedious) considerations:

Prop

A spectral triple with γ_F-odd operator D_F is not spin.
For a γ'_F-odd Modified Chamseddine-Connes's operator D'_F with all coefficients different from zero, if at least one of

$$1. \quad \Upsilon_\nu \neq \pm\Upsilon_u \, , \qquad 2. \quad \Upsilon_e \neq \pm\Upsilon_d \, ,$$

holds, then:

- i) *the odd spectral triple (A_F, H_F, D'_F, J_F) is not spin;*
- ii) *the even spectral triple $(A_F, H_F, D'_F, \gamma'_F, J_F)$ is spin;*

Conclusions

Corollary

i) *the spectral triple with D_F of Connes-Chamseddine ($\Delta = 0$ or $\Omega = 0$), is not spin.*

ii) *our D'_F is a minimal modifiation of Connes-Chamseddine D_F, for which the even spectral triple is spin.*

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Corollary

i) *the spectral triple with D_F of Connes-Chamseddine ($\Delta = 0$ or $\Omega = 0$), is not spin.*

ii) *our D'_F is a minimal modifiation of Connes-Chamseddine D_F, for which the even spectral triple is spin.*

Final remarks

i) *γ'_F does not belong to the algebra generated by A_F and $[D, A_F]$, so the modified even spectral triple is not orientable. Thus it is pin rather than spin, and the elements of H are "quantum Dirac pinors".*

ii) *It is irreducible in the sense that $\{0\}$ and H are the only subspaces stable under A, D'_F, J and γ'_F.*

iii) *So far Math. Phys. implications, e.g. "small" Ω, Δ; lepto-quark interactions (cf. [PSS97]); how the Higgs mass is modified ...: under scrutiny.*