IR Effect in de Sitter space, Screening of Cosmological Constant and Inflation

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Outline

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Cosmic expansion and de Sitter spacetime

- Our universe undergoes inflation at the early time and a cosmic expansion at the current time, both described by a de Sitter spacetime:

$$ds^2 = -dt^2 + e^{2Ht}dx^2,$$

- Theoretical constraints and observations:

$$H_{\rm inf} \sim 10^{14}\mathrm{GeV}, \quad H_{\rm current} \sim 10^{-42}\mathrm{GeV}.$$

- The most natural explanation of the expansion is a cosmological constant in the Einstein equation:

$$H^2 = \frac{\Lambda}{3M_P^2}$$

- Time dependence of H translates to effective cosmological constants being time dependent

Q. Did and why the expansion slow down?

Cosmological constant problem

- Classically, there is no natural value for the cosmological constant. Vacuum fluctuation suggests the it is natually of the order of manitudes:

$$\Lambda_{\rm UV} \sim (10^{18}{\rm GeV})^4$$

  This means

$$\frac{\Lambda_{\rm inf}}{\Lambda_{\rm UV}} \sim 10^{-8}, \quad \frac{\Lambda_{\rm current}}{\Lambda_{\rm UV}} \sim 10^{-120}.$$

Q. What set the Hubble scale at inflation?

Q. Why is observed $\Lambda_{\rm current}$ so tiny?

Screening of the Cosmological Constant?

IR quantum instability for massless minimally coupled scalar field in dS was first pointed out by Ford (85). It was also conjectured that quantum gravitational effects may diminish the cosmological constant. Polyakov (88) conjectured that the cosmological constant is “screened” by the infrared fluctuation of the metric.

Question: concrete mechanism? is it sufficient?

IR divergence and dS symmetry breaking

- dS space in Poincare coordinates:

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)dx_3^2, \quad a(t) = e^{Ht} \\ &= a^2(\tau)(-d\tau^2 + dx_3^2), \qquad \tau = -\frac{1}{H}e^{-Ht}, \quad -\infty < \tau < 0. \end{aligned}$$

- dS symmetry is $SO(D,1)$ and it is necessary that $\Lambda = \text{const.}$ in time.
- If one want $\Lambda = \Lambda(t)$, the desired quantum effect must be time dependent and break the de Sitter symmetry.

Quantum field in dS space

Massive scalar

- Massive scalar field in dS space admits dS invariant vacuum states: Bunch-Davies + α-vac (non-Hadamard) Allen 85, Mottola 85

Massless scalar

- Due to IR divergent effects, massless minimally coupled (mmc) scalar field does not admit dS invariant vacuum Allen, Folacci 87
- In fact, the two point function

$$\langle\phi(x)\phi(x')\rangle \sim \int_{P>H} \frac{d^3P}{(2\pi)^3}\frac{1}{2P} + \underbrace{\int_{P<H} \frac{d^3P}{(2\pi)^3}\frac{1}{2P^3}}_{\text{superhorizon modes. IR divergence}}$$

where $P = p/a$ is the physical momentum.

- If IR cutoff p_0 is introduced, the two point function is

$$\langle\phi(x)\phi(x')\rangle = \frac{H^2}{4\pi^2}\left(\frac{1}{y} - \frac{1}{2}\log y + \underline{\frac{1}{2}\log a(\tau)a(\tau')} + \frac{1}{2}\log(\frac{H^2}{p_0^2}) + 1 - \gamma\right)$$

Graviton in dS space

- Graviton in TTS gauge: $\partial_\mu h^{\mu\nu} = 0, \quad h^\mu_\mu = 0, \quad h_{\mu 0} = 0.$
- TTS graviton satisfies the same EOM as mmc scalar field and so acquires the same time-dependent dS breaking IR logarithm.

Woodard, Tsamis 94

Effects of the IR div

The time dependence of the propagator induces time dependence to physical quantities of the theory

- E.g.1. graviton loop slows expansion:
  Two loops tadpole diagrams modify the expansion rate $H_{\rm eff}$ to

$$H_{\rm eff} = H\left(1 - \frac{H^2}{4\pi^2 M_P^2} \log^2 a(\tau) + \cdots\right).$$

Woddard, Tsamis 95

- E.g.2. graviton loop screens matter couplings
  Gravition loop corrections to matter and gauge field theory has been studied by different groups of people: Woodard etal, Kitamoto, Kitazawa, Giddings, ...
- e.g.

$$\lambda_{\rm eff} = \lambda(1 - \frac{21H^2}{16\pi^2 M_P^2} \log a)$$

Kitamoto, Kitazawa 12

How large is the screening effects?

- The IR logarithmic effects from graviton and mmc scalar to the CC

$$\left(\frac{H}{M_P}\log a\right)^n$$

are suppressed by the Planck scale.

- It will take an incredibly long time to obtain large effect from the IR logarithms:

$$\log a > \frac{M_P}{H}.$$

Besides perturbation theory breaks down in such regime.

- It seems the effect of the IR logarithm are so tiny to resolve the CC problem ?

It is necessary to introduce some new mechanism in order for these IR effect be *magnified* and seen.

Outline

Main features of our model

We consider a dilaton-gravity theory with a non-minimal coupling of a Brans-Dicke scalar

- $$S = \int \sqrt{-g} d^4x \left[\frac{M^2}{2} e^{-2\phi/\eta} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right],$$
  where M is a fundamental mass scale, η is a mass scale which determines the strength of the non-minimal coupling of ϕ to R. $V(\phi)$ is a potential
- Allowing classical vev of scalar $\phi = v$ and incoorperates the IR quantum effects of graviton loop, then we may have
  $$v_{\rm eff} = v + \delta v(\tau)$$
- In Einstein frame, this time dependent vev modifies the cosmological constant,
  $$S_E \sim \int d^4x \sqrt{-g_E}\; V(v) e^{4v_{\rm eff}/\eta}.$$
  This brings in a time dependence (screening) in the cosmological constant. Note that the time dependent IR effect is exponentiated and can become much more sizable.

Outline

Our model

- Consider the action

$$S = \int \sqrt{-g} d^4x \left[\frac{M^2}{2} e^{-2\phi/\eta} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right],$$

- The potential admits a vacuum at $\phi = v$ and defines a dS bkgd:

$$g_{\mu\nu} = a_0^2(t) \eta_{\mu\nu}, \quad a_0 = e^{H_0 t}$$

with

$$H_0^2 = \frac{e^{2v/\eta} V(v)}{3M^2}$$

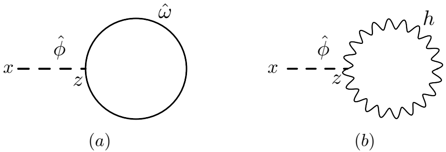
- With some mild conditions on V, the background is classically stable

Effective vev from one-loop graviton effect

- We consider the one-loop correction to the v.e.v of the dilaton scalar

$$\delta v = \langle \Omega | \phi(x) | \Omega \rangle .$$

- We are interested in the time dependent part of the correction. For this we need only to consider the propagation of mmc mode of the graviton in the loop, which generates time dependent IR logarithm. This is given by the one loop tadpole diagram



- Note: Graviton perturbations can be categorized into massless minimally coupled (mmc) modes and conformally coupled (cc) modes. Only mmc graviton mode give rises to time dependent IR logaritm.

- We work with the in-in formalism

$$\langle\Omega|\phi(x)|\Omega\rangle = \langle 0|\,\tilde{T}\{e^{i\int_{\tau_i}^{\tau} H_{\text{int}-} d\tau'}\}\phi(x)\,T\{e^{-i\int_{\tau_i}^{\tau} H_{\text{int}+} d\tau''}\}|0\rangle$$

- One loop result gives the leading IR logarithm

$$\langle\Omega|\phi(x)|\Omega\rangle = -\frac{\zeta}{\eta}\frac{3H^2}{4\pi^2}\log^2 a(\tau) + \text{sub-leading}$$

and the effective vev is obtained as

$$v_{\text{eff}}(\tau) = v - \frac{\zeta^2}{\eta}\frac{3H^2}{4\pi^2}\log^2 a(\tau) + \text{sub-leading},$$

where

$$\zeta(v) = \frac{1}{\sqrt{1+\frac{6M^2}{\eta^2}e^{-2v/\eta}}}$$

- Subleading terms include $\log a, 1/a, \cdots$ etc.

- Perturbation breaks down at late time when $\log a$ becomes large. However DRG (Dynamical RG) method allows to resum the leading IR logarithm effectively

  Boyanovsky, Vega 03; Burgess, Leblond, Holmanm, Shandera 10

  The idea is similar to that in QFT where resummation of large logarithm can be obtained by the RG equation.
- We obtain

$$v_{\text{eff}}(\tau) = r + \frac{1}{q} W(pqe^{-qr})$$

where

$$p := \frac{3M^2}{\eta}, \qquad q := \frac{2}{\eta}, \qquad r := v - \frac{3M^2}{\eta} e^{-2v/\eta} - \frac{3H_0^2 \log^2 a_0(\tau)}{4\pi^2 \eta}$$

and $W(z)$ is the Lambert-W function

$$z = W(z) e^{W(z)}.$$

The result is valid for all time as long as $\epsilon \ll 1$.

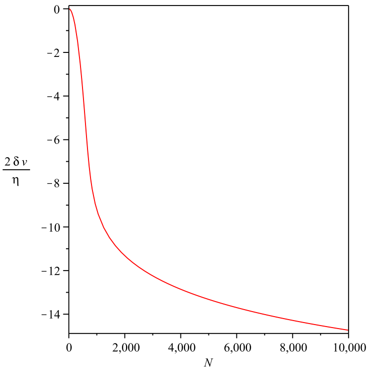


Figure: Plot of $\frac{2\delta v}{\eta}$ against N for $x = 1, y = 0.01$

- $$x := \frac{Me^{-v/\eta}}{H_0}, \quad y := \frac{Me^{-v/\eta}}{\eta}$$

 parameterize the model

Validity of approximation

- At the tree level, the Hubble parameter is given by

$$H_0^2 = \frac{1}{3M^2} e^{\frac{2v}{\eta}} V(v).$$

- The one loop Hubble constant in the string frame is given by

$$H^2 = \frac{1}{3M^2} e^{2v_{\mathrm{eff}}/\eta} V(v) = H_0^2 e^{2\delta v/\eta}$$

- We can compute the "slow rolll" parameters

$$\varepsilon = \frac{d \ln H}{d\mathcal{N}}, \quad \eta := \frac{d \ln \varepsilon}{d\mathcal{N}},$$

to measure the size of the backreaction. Need

$$\varepsilon, \eta \ll 1$$

in order to trust the quantum field theory computation.

- Given x and y, ε and η stay very small for a large range of N from the initial time, and then increase rapidly at around N of the order of $N \sim xy^{-1} \ln y^{-1}$.

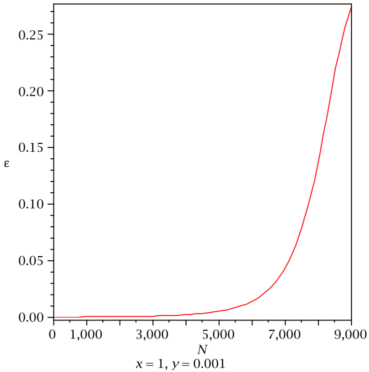


Figure: Plot of ε against N

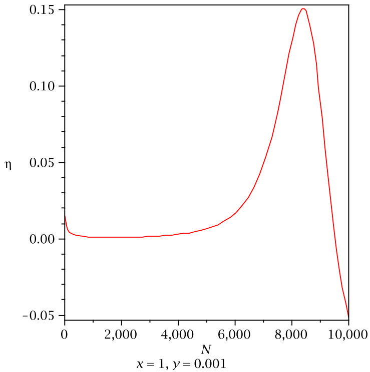


Figure: Plot of η against N

Outline

Hubble constant in Einstein frame

- To examine the physical effects of the time dependent vev, we need to go back to the Einstein frame by performing a Weyl scaling of the string frame metric
$$g_{\mu\nu} = g^E_{\mu\nu} e^{2(\frac{\varphi}{\eta}+\beta)},$$
where φ denote the dynamical part of ϕ above the vev,
$$\phi = v_{\text{eff}} + \varphi$$
and $\beta = \delta v/\eta$.
- We obtain the Planck mass
$$M_P = Me^{-\frac{v}{\eta}}.$$
and the Hubble parameter
$$H_E = e^{\frac{2\delta v}{\eta}} H_0.$$
- As δv is always negative, we get a screening of the Hubble constant and the CC.

Slow roll inflation

- The slow roll parameters,

$$\varepsilon_E := \frac{d}{dt_E} H_E^{-1}, \quad \eta_E := \frac{1}{H_E \varepsilon_E} \frac{d\varepsilon_E}{dt_E}$$

are given by

$$\varepsilon_E = 2e^{\frac{\delta v}{\eta}} \varepsilon, \quad \eta_E = e^{\frac{\delta v}{\eta}} (\eta - \varepsilon)$$

- It is clear that as long as we can trust the quantum field theory computations, the corresponding cosmology in the Einstein frame describes a slow roll inflation with

$$\varepsilon_E, \eta_E \ll 1.$$

Remarks

1. Slow roll inflation is achieved without a slow roll potential: In contrast to the simplest slow roll inflation, where expansion is driven by the slow rolling of an inflaton field down an almost flat potential, here inflation is driven by the IR effects of the gravitons themselves.
2. No eta problem in our model: Slow roll inflation has

$$\eta_E = m_\phi^2/3H_E^2$$

and it is difficult to maintain $m_\phi^2 \ll H_E^2$ due to large quantum corrections on m_ϕ^2: Like Higgs hierarchy problem, generically

$$m_\phi^2 \sim \Lambda_{\mathrm{UV}}^2 \gg H_E^2.$$

SUSY improves it a little but it must be SSB during inflation, leading to to

$$m_\phi^2 \sim H_E^2.$$

In our model, inflation is driven by time dependent IR effect and this is unaffected by the UV corrections.

Screening of the cosmological constant

- Universe is described by a de Sitter metric during inflation at the early universe and the current cosmic expansion
- Current value of the Hubble constant is tiny

$$H_{E,\mathrm{now}} \simeq 10^{-60} M_P.$$

  We have analyzed and concluded that the screening effect of graviton loop is not sufficient to bring the Hubble constant down to this present tiny value from its value during inflation. Probably it is the reheating process right after the inflation which is responsible for the tiny CC.
- On the other hand, we apply our analysis to the inflationary phase and find that the screening effect on H_E can give the inflationary scale

$$H_{E,\mathrm{inf}} = 10^{-4} M_P$$

  from the natural initial codition

$$H_E(0) = M_P.$$

Outline

In this talk, IR effect of graviton is studied. We found that:

1. IR divergence of graviton in the loop induces a time dependence on the vev of the dilaton field
2. This effect is exponentiated when transformed back to the Einstein frame, and can produce a significant amount of screening for the cosmological constant.

As applications,

1. We provide an alternative mechanism to achieve inflation, but without the need of an adhoc inflationary potential as in slow roll inflation.
2. Moreover, starting with a natural initial value for the CC, our model can explain why it has the inflation scale $H_{E,\mathrm{inf}}$ during the last 60-efolding.

Further studies:

- No free parameters in our model. Other predictions?
  e.g before the last 60-efolding.
- Time dependent effects of reheating is important