The coupling of Non-linear Supersymmetry to Supergravity

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Section 1

Basic concepts of Supersymmetry

Supersymmetric Lagrangians, Kähler geometry and SSB

- Superspace and chiral superfields: $y^m = x^m + \mathrm{i}\theta\sigma^m\bar{\theta}$, $\Phi^\alpha = Z^\alpha(y) + \sqrt{2}\theta\chi^\alpha(y) + (\theta\theta)F^\alpha(y)$
- Lagrangian: $\mathscr{L} = [K(\Phi^\alpha, \bar{\Phi}^{\bar{\beta}})]_D + ([W(\Phi^\alpha)]_F + \text{ h.c.}) \to$

$$\mathscr{L} = -g_{\alpha\bar{\beta}}\partial_m Z^\alpha \partial^m \bar{Z}^{\bar{\beta}} - ig_{\alpha\bar{\beta}}\bar{\chi}^{\bar{\beta}}\bar{\sigma}^m D_m\chi^\alpha + \frac{1}{4}R_{\alpha\bar{\beta}\gamma\bar{\delta}}\chi^\alpha\chi^\gamma\bar{\chi}^{\bar{\beta}}\bar{\chi}^{\bar{\delta}}$$
$$-\frac{1}{2}D_\alpha D_\beta W\chi^\alpha\chi^\beta - \frac{1}{2}D_{\bar{\alpha}}D_{\bar{\beta}}\bar{W}\bar{\chi}^{\bar{\alpha}}\bar{\chi}^{\bar{\beta}} - g^{\alpha\bar{\beta}}D_\alpha W D_{\bar{\beta}}\bar{W}$$

- Kähler manifold - Kähler symmetry:

$$g_{\alpha\bar{\beta}} = \frac{\partial}{\partial Z^\alpha}\frac{\partial}{\partial \bar{Z}^{\bar{\beta}}}K| \quad , \quad \Gamma^\alpha_{\beta\gamma} = g^{\alpha\bar{\delta}}\partial_\beta g_{\gamma\bar{\delta}} \quad , \quad R_{\alpha\bar{\beta}\gamma\bar{\delta}} = g_{\epsilon\bar{\delta}}\partial_{\bar{\beta}}\Gamma^\epsilon_{\alpha\gamma}$$

- SSB $\Leftrightarrow \langle 0|H|0\rangle \neq 0 \to$ vacuum energy is the order parameter

1. $V = g_{\alpha\bar{\beta}}F^\alpha\bar{F}^{\bar{\beta}} = g^{\alpha\bar{\beta}}W_\alpha\bar{W}_{\bar{\beta}} = W_\alpha\bar{W}^\alpha$
2. Goldstino:

$$\delta\chi_\alpha = -\sqrt{2}F\xi_\alpha + \ldots \to -\sqrt{2}\langle F\rangle\xi_\alpha + \ldots \quad , \quad \partial_\alpha V = (D_\alpha D_\beta W)\bar{W}^\beta = 0$$

Section 2

Basic concepts of Supergravity

Simplest Lagrangians & SSB

- Geometry in terms of $\mathscr{E}$, $\mathscr{R}$
- Pure supergravity Lagrangians $\mathscr{L} = -\frac{1}{\kappa^2}\int \mathrm{d}^2\Theta\mathscr{E}\mathscr{R} + \mathrm{h.c} \rightarrow$

$$\mathscr{L} = -\frac{1}{2}eR - \frac{1}{3}eM\bar{M} + \frac{1}{3}eb^a b_a + \frac{1}{2}e\epsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\widetilde{\mathscr{D}}_c\psi_d - \psi_a\sigma_b\widetilde{\mathscr{D}}_c\bar{\psi}_d)$$

- General form, Kähler - Weyl symmetry:

$$\mathscr{L} = \frac{1}{\kappa^2}\int \mathrm{d}^2\Theta 2\mathscr{E}\{\frac{3}{8}(\bar{\mathscr{D}}\bar{\mathscr{D}} - \frac{8}{6}\mathscr{R})\exp(-\frac{\kappa^2 K}{3}) + \kappa^2 W\} + \mathrm{h.c.}$$
$$= -\frac{1}{\kappa^2}\int \mathrm{d}^2\Theta\mathscr{E}\mathscr{R} + \int \mathrm{d}^2\Theta 2\mathscr{E}\{-\frac{1}{8}(\bar{\mathscr{D}}\bar{\mathscr{D}} - \frac{8}{6}\mathscr{R})K + W\} + \ldots + \mathrm{h.c.}$$

- Scalar potential: $V = \exp(K)[g^{\alpha\bar{\beta}}(D_\alpha W)(\bar{D}_{\bar{\beta}}\bar{W}) - 3\bar{W}W] = g_{\alpha\bar{\beta}}F^\alpha\bar{F}^{\bar{\beta}} - \frac{1}{3}M\bar{M} = \exp(\mathscr{G})[g^{\alpha\bar{\beta}}\mathscr{G}_\alpha\mathscr{G}_{\bar{\beta}} - 3]$
- Gravitino mass term: $m_\psi = \langle exp(\mathscr{G}/2)\rangle$
- Goldstino: $\delta n = \mathrm{const.} + \ldots$ only if $\langle\mathscr{G}_\alpha\rangle \neq 0$ or $\langle\frac{D_\alpha W}{W}\rangle \sim \langle F_\alpha\rangle \neq 0$

Superconformal formalism

1. Extend Poincare supersymmetry $\rightarrow$ superconformal symmetry
2. Use chiral compensator superfield $\rightarrow$ $n+1$ chiral superfields with components X^I, Ω^I, F^I
   - General Lagrangian:

$$\mathscr{L} = [\mathscr{N}(X,\bar{X})]_D + [\mathscr{W}(X)]_F + [f_{AB}(X)\bar{\lambda}^A P_L \lambda^B]_F$$

   - Kähler manifold: $G_{I\bar{J}} = \mathscr{N}_{I\bar{J}} = \frac{\partial^2 \mathscr{N}}{\partial X^I \partial \bar{X}^J}$ but Kähler symmetry?
3. Break superfluous symmetries
   - $D-$gauge: $\mathscr{N} = -\frac{3}{\kappa^2} = -3M_P^2$
   - Parametrization with $U(1)$ symmetry: $X^I = Yx^I(\bar{Z}^{\bar{\alpha}}), X_I = Yx_I(Z_\alpha) \rightarrow$ projective Kähler manifold, Kähler symmetry restored

$$\mathscr{K} = -3\ln[-\frac{1}{3}\frac{\mathscr{N}}{Y\bar{Y}}]$$

   - After the complete gauge-fixing: conventional supergravity Lagrangians

Superconformal tensor calculus

- Distinguish chiral compensator superfield S_0, define $\mathscr{R} = S_0^{-1}\Sigma(S_0)$
- Useful identities:

$$[\Lambda\mathscr{R}]_F + \text{h.c.} = [\Lambda\frac{\bar{S}_0}{S_0} + \bar{\Lambda}\frac{S_0}{\bar{S}_0}]_D + (\text{surface term})$$

$$[\Lambda \cdot Z \cdot \Sigma(Z)]_F + \text{h.c.} = [Z\bar{Z}(\Lambda + \bar{\Lambda})]_D + (\text{surface term})$$

- Example: $[\Lambda \cdot \mathscr{R} \cdot S_0^2]_F + \text{h.c.} = [S_0\bar{S}_0(\Lambda + \bar{\Lambda})]_D + (\text{surface term})$
- How to write Lagrangians: e.g. $R + R^2$ models

$$\mathscr{L} = -[S_0 S_0]_D + [h(\frac{\mathscr{R}}{S_0}, \frac{\bar{\mathscr{R}}}{\bar{S}_0})S_0\bar{S}_0]_D + ([W(\frac{\mathscr{R}}{S_0})S_0^3]_F + \text{h.c.})$$

- Gauge-fixing: fix S_0 suitably, most common choice: $S_0 = 1$

Section 3

Non-linear Supersymmetry

Motivation and the Volkov-Akulov Lagrangian

- Fermionic and bosonic d. o. f. do not match
  1. Brane dynamics: String Theory
  2. Low-energy phenomenology of supersymmetry breaking
     - Supersymmetry breaking scale $\langle |F|^2 \rangle = M^4_{\rm SUSY}$, $\kappa^2 = 8\pi G_N = \frac{1}{M_P^2}$
     - Low-energy breaking: Goldstino component of the gravitino dominates with $m_G \ll M_{\rm SUSY}$,, sGoldstino becomes superheavy
  3. Inflationary Cosmology
- Supersymmetry transformations: $\theta' = \theta + \xi \to$ can set $\theta = \kappa\lambda$

$$\delta_\xi \lambda^\alpha = \frac{1}{\kappa}\xi^\alpha - i\kappa(\lambda\sigma^m\bar{\xi} - \xi\sigma^m\bar{\lambda})\partial_m\lambda^\alpha$$

- Using $A^a_m = \delta^a_m - i\kappa^2\partial_m\lambda\sigma^a\bar{\lambda} + i\kappa^2\lambda\sigma^a\partial_m\bar{\lambda}$, construct Lagrangian (Volkov - Akulov 1973)

$$\mathscr{L} = -\frac{1}{2\kappa^2}\det A = -\frac{1}{2\kappa^2} - \frac{i}{2}(\lambda\sigma^m\partial_m\bar{\lambda} - \partial_m\lambda\sigma^m\bar{\lambda}) + \mathcal{O}(\kappa^2)$$

Constrained Superfields

- Chiral superfield X, $X^2 = 0$ (Komargodski-Seiberg 2009) $\rightarrow$

$$X(y,\theta) = \frac{GG}{2F}(y) + \sqrt{2}\theta G(y) + (\theta\theta)F(y)$$

- General Lagrangian (without derivatives):

$$\mathscr{L} = [X\bar{X}]_D + ([fX]_F + h.c.)$$
$$= fF + f\bar{F} + \frac{\bar{G}^2}{2\bar{F}}\partial^\mu\partial_\mu(\frac{G^2}{2F}) + i\partial_\mu\bar{G}\bar{\sigma}^\mu G + F\bar{F} \rightarrow$$

$$\mathscr{L} = -f^2 + i\partial_\mu\bar{G}\bar{\sigma}^\mu G + \frac{1}{4f^2}\bar{G}^2\partial^2 G^2 - \frac{1}{16f^6}G^2\bar{G}^2\partial^2 G^2\partial^2\bar{G}^2$$

$\rightarrow$ equivalent to Volkov-Akulov Lagrangian for $f^2 = \frac{1}{2\kappa^2}$

Section 4

The coupling of Non-linear Supersymmetry to Supergravity

The geometrical description of the coupling

- The coupling:

$$\mathscr{L} = -[(1 - X\bar{X})S_0\bar{S}_0]_D + ([(fX + W_0 + \frac{1}{2}TX^2)S_0^3]_F + \mathrm{h.c.})$$

with Kähler potential: $K = -3\ln(1 - X\bar{X}) = 3X\bar{X}$

- Kähler symmetry:

$$K \to K' = K - 3(X + \bar{X}), W \to W' = e^{3X}W$$

- The geometrical formulation, $\lambda = f + 3W_0$:

$$\mathscr{L}' = -[(1 + X + \bar{X})S_0\bar{S}_0]_D + ([(fX + W_0 + \frac{1}{2}TX^2)e^{3X}S_0^3]_F + \mathrm{h.c.})$$
$$= [(-\frac{1}{2}\frac{\mathscr{R}}{S_0} + W_0 - \frac{1}{2T}(\frac{\mathscr{R}}{S_0} - \lambda)^2)S_0^3]_F + \mathrm{h.c.}$$

In terms of component fields

1. Using the constrained chiral superfield X, $X^2 = 0$:

$$\mathscr{L} = \int \mathrm{d}^2\Theta 2\mathscr{E}\{-\frac{3}{8}(\bar{\mathscr{D}}\bar{\mathscr{D}} - \frac{8}{6}\mathscr{R})(X\bar{X} - 1) + fX + W_0\} + \mathrm{h.c.}$$

2. Using the constrained supergravity multiplet $\mathscr{R}$, $(\mathscr{R} - \lambda)^2 = 0$:

$$\mathscr{L}' = -\int d^2\Theta \mathscr{E}\mathscr{R} + \int d^2\Theta 2\mathscr{E} W_0 + \mathrm{h.c.}$$

$\rightarrow$ both have the same physical content: (we use the unitary gauge)

- Gravitino mass: $m_{3/2} = |W_0|$
- Cosmological constant: 0 if $\lambda = 6W_0, \lambda = 0 \text{ or } f = \pm 3W_0$
- Final form:

$$\mathscr{L} = \mathscr{L}' = -\frac{1}{2}eR + \frac{1}{2}e\epsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\widetilde{\mathscr{D}}_c\psi_d - \psi_a\sigma_b\widetilde{\mathscr{D}}_c\bar{\psi}_d)$$
$$-eW_0\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b - e\bar{W}_0\psi_a\sigma^{ab}\psi_b$$

Without imposing constraints (I)

- Alternatively, consider the Lagrangian

$$\bar{\mathscr{L}} = \left[\left(-\frac{1}{2}\frac{\mathscr{R}}{S_0} + W_0 + \frac{1}{2}\rho(\frac{\mathscr{R}}{S_0} - \lambda)^2\right) S_0^3\right]_F + \text{h.c. at } \rho \to \infty$$

$$\Rightarrow \bar{\mathscr{L}} = -[b - S - \bar{S}]_D + \left([a - \frac{1}{2\rho}S^2]_F + \text{h.c.}\right)$$

- Scalar potential: $V = \frac{1}{\rho^2(b-2A_R)^2}\left\{\frac{1}{3}(A_R^2 + A_I^2)(b + A_R) - 2a\rho A_R\right\}$
- $-b \leqslant A_R < \frac{b}{2} \quad , \quad b > 0$

1. $\langle V\rangle = \langle\frac{\partial V}{\partial A_R}\rangle = \langle\frac{\partial V}{\partial A_I}\rangle = 0 \Rightarrow$ no minimum & sgoldstino does not decouple
2. $\langle\frac{\partial V}{\partial A_R}\rangle = \langle\frac{\partial V}{\partial A_I}\rangle = 0$ and $\langle V\rangle = 0$ only at $\rho \to \infty \Rightarrow$ sgoldstino does not decouple

Without imposing constraints (II)

- Idea: $f(\mathscr{R})$ supergravity

$$\mathscr{L}'' = \left[\left(-\frac{1}{2}\frac{\mathscr{R}}{S_0} + W_0 + \frac{1}{2}\rho(\frac{\mathscr{R}}{S_0} - \lambda)^2 + \frac{1}{\rho}\left(S\frac{\mathscr{R}}{S_0} - F(S)\right)\right) S_0^3\right]_F + \text{h.c.} \Rightarrow$$

$$\mathscr{L}'' = -\left[(1 - \frac{1}{\rho}(S + \bar{S}))S_0\bar{S}_0\right]_D + \left\{[(W_0 + \frac{1}{2}\rho(F' - \lambda)^2 - \frac{1}{\rho}F)S_0^3]_F + \text{h.c.}\right\}$$

- Leading behaviour of the scalar potential V for $\rho \to \infty$:
  $V = \frac{\rho^4}{3}|F''(F' - \lambda)|^2 \to$ consider minimum for $F' = \lambda$:
  1. Scalar mass: $m_\phi = \frac{\rho^3}{3}(F'')^2$
  2. Cosmological constant: $\lambda = 6W_0 \quad \text{or} \quad \lambda = 0$
  3. Solution:
  $$F(\phi) = \phi_0 W_0 + \lambda(\phi - \phi_0) \pm \frac{3W_0}{2c}(\phi - \phi_0)^2 \pm \frac{1}{3!}\frac{2W_0}{c^2}(\phi - \phi_0)^3 + \dots$$
  4. SSB: $\langle|\mathcal{F}^\phi|\rangle = \left\langle\left|e^{K/2}\sqrt{g^{\phi\bar{\phi}}}\bar{D}_{\bar{\phi}}\bar{W}\right|\right\rangle \overset{\rho\to\infty}{\longrightarrow} \sqrt{3}|W_0| \neq 0$,
  $m_{3/2} = \langle|W|e^{K/2}\rangle \overset{\rho\to\infty}{\longrightarrow} |W_0|$