Holographic Entanglement Entropy from Numerical Relativity

Based on work with Daniel Grumiller, Stefan Stricker 1506.02658 (JHEP); & Wilke van der Schee (15XX.XXXXX)

Christian Ecker

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Motivation

Central question:
How evolves a strongly coupled quantum system from a far-from equilibrium state to equilibrium?

Challenges:
- Due to strong coupling perturbative methods are not applicable.
- Far-from equilibrium we are outside the regime of linear response theory.
Quark-gluon plasma in heavy ion collisions

Quark gluon plasma (QGP) is a deconfined phase of quarks and gluons which is produced in heavy ion collision (HIC) experiments at RHIC and LHC.

Why AdS/CFT?
- QGP produced in HIC's behaves like a strongly coupled liquid rather than a weakly coupled gas.
- Initial fireball thermalizes on a very short timescale (~1fm/c).
- Fast thermalization is a robust feature of holographic HIC models.
AdS/CFT correspondence: [Maldacena 97]

Type IIB string theory on AdS$_5 \times S^5$ is equivalent to $\mathcal{N}=4$ super symmetric SU($N_c$) Yang-Mills theory in 4D.

Supergravity limit:

Strongly coupled large $N_c$ $\mathcal{N}=4$ SU($N_c$) SYM theory is equivalent to classical (super)gravity on AdS$_5$.

Strategy:

- Use $\mathcal{N}=4$ SYM as toymodel for QCD.
- Build a gravity model dual to HICs, like colliding gravitational shock waves.
- Switch on the computer and solve the 5-dim. gravity problem numerically.
- Use the holographic dictionary to compute observables in the 4 dim. field theory form the gravity result.
Holographic thermalization

Thermalization = Black hole formation

- Holographic dictionary translates thermodynamic properties (S,T,M) of black holes to the corresponding properties of the gauge theory.
- Computing black hole formation on AdS in general requires methods from numerical relativity.
- The observable we use to study thermalization is entanglement entropy.
Divide the system into two parts A, B. The total Hilbert space factorizes:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

The reduced density matrix of A is obtained by the trace over $\mathcal{H}_B$

$$\rho_A = \text{Tr}_B \rho$$

Entanglement entropy is defined as the von Neumann entropy of $\rho_A$:

$$S_A = -\text{Tr}_A \rho_A \log \rho_A$$
Entanglement entropy in a two quantum bit system

Consider a quantum system of two spin 1/2 dof's. Observer Alice has only access to one spin and Bob to the other spin.

A product state (not entangled) in a two spin 1/2 system:

$$|\psi\rangle = \frac{1}{2} (|\uparrow_A\rangle + |\downarrow_A\rangle) \otimes (|\uparrow_B\rangle + |\downarrow_B\rangle)$$

A (maximally) entangled state in a two spin 1/2 system:

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow_A\rangle \otimes |\downarrow_B\rangle - |\downarrow_A\rangle \otimes |\uparrow_B\rangle)$$

Entanglement entropy is a measure for how much a given quantum state is entangled.
Entanglement entropy in quantum field theories

The Basic Method to compute entanglement entropy in quantum field theories is the **replica method**.

Involves path integrals over n-sheeted Riemann surfaces ~ it's **complicated**!

With the **replica method** one gets **analytic results** for 1+1 dim. CFTs. [Holzhey-Larsen-Wilczek 94]

One finds **universal scaling** with interval size:

\[ S_A = \frac{c}{3} \log \frac{L}{\alpha} + \text{finite} \]

**central charge of the CFT**

**UV cut off**

**Message:** Computing entanglement entropy in interacting QFTs is complicated and analytically only possible in 1+1 dim. CFTs.

The **AdS/CFT** provides a **simpler method** that works also in **higher dimensions**.

Christian Ecker (TU-Wien)  September 11, 2015  Corfu
Holographic entanglement entropy

Within AdS/CFT entanglement entropy can be computed from the area of minimal (extremal) surfaces in the gravity theory.

\[ S_A = \frac{\text{Area}(\Sigma)}{4G_N} \]

[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07]
Holographic entanglement entropy

- In practice computing extremal co-dim. 2 hyper-surfaces is numerically involved.
- Can we somehow simplify our lives?

Yes we can!
Entanglement entropy from geodesics

For infinitely extended stripe regions respecting the symmetries of the geometry, computing entanglement entropy essentially reduces to computing the geodesics length.

$$S_A = \text{const.} \frac{\text{Length}(\Gamma)}{4G_N}$$
There are two standard numerical methods for solving two point boundary value problems.

- **Shooting:** Very sensitive to initialization on asymptotic AdS spacetimes.

- **Relaxation:** Converges very fast if good initial geodesic is provided.

Geodesic equation as two point boundary value problem.

\[ \ddot{X}^\mu(\tau) + \Gamma^\mu_{\alpha\beta} \dot{X}^\alpha(\tau) \dot{X}^\beta(\tau) = 0 \]

**BCs:** \((V(\pm 1), Z(\pm 1), X(\pm 1)) = (t_0, 0, L/2)\)
Shock wave collisions

- HIC is modeled by two colliding sheets of energy with infinite extend in transverse direction and Gaussian profile in beam direction.
Entanglement entropy for shock wave collisions

- Entanglement entropy and effective horizon entropy **grow linearly.**
- The **fall off** behavior is however **different!**
Strong subadditivity

- A fundamental property of entanglement entropy is strong subadditivity.
- Hard to prove within QFT, intuitive in the dual gravity picture.

\[ S_A + S_B \geq S_{A \cup B} + S_{A \cap B} \]
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$$S_A + S_B \geq S_{A \cup B} + S_{A \cap B}$$
Numerical check of strong subadditivity

$S_A + S_B \neq S_{A \cup B} + S_{A \cap B}$

Preliminary
Summary

- I have shown you the **first entanglement entropy simulations** for holographic HIC models. (15XX.XXXXX)
- In the shock wave geometry the **entanglement entropy** and horizon entropy grow in the same way but show different fall off behavior.
- Successfully checked the **strong subadditivity** condition.
- **Take home message**: Complicated stuff in QFT often has a very intuitive geometric interpretation on the gravity side.

Outlook

- Effective horizon entropy is gauge dependent – is **entanglement entropy**, which is gauge independent, an alternative **measure for entropy production in HIC's**?
- Go **beyond** the **supergravity** approximation: study the influence of **string corrections** on thermalization patterns.