UV Quantum Corrections in Unimodular Gravity

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cost Action MP 1405
Quantum Structure of Spacetime

GOBIERNO DE ESPAÑA
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- What's Unimodular Gravity (UG)?

It's a gravity theory with action

$$S_{UG} \equiv -M_P^{n-2} \int d^n x \sqrt{-\hat{g}} \left(R[\hat{g}_{\mu\nu}] + L_{\text{matt}}[\psi_i, \hat{g}_{\mu\nu}] \right)$$

BUT with the space of metrics restricted by the condition

$$\hat{g} = -1$$

ie, $\det \hat{g}_{\mu\nu}$ is not a dynamical variable!!!! and the gauge symmetry is TDIFF(M), not DIFF(M)

Hence, additions of the type

$$\Lambda_0 \int d^n x \sqrt{-\hat{g}}$$

are physically irrelevant.

UG Eq. of Motion

- EM = Trace-free equations (TFE)(see book by A. Zee)

$$R_{\mu\nu} - \frac{1}{n} R \hat{g}_{\mu\nu} = M_P^{2-n} (T_{\mu\nu} - \frac{1}{n} T \hat{g}_{\mu\nu})$$

obtained by variations of S_{UG}, with momentarily unconstrained $g_{\mu\nu}$, $\hat{g}_{\mu\nu} \to g_{\mu\nu}$, under traceless variations

$$\delta g_{\mu\nu} = \text{unconstrained variation}$$
$$\delta^t g_{\mu\nu} = \delta g_{\mu\nu} - \tfrac{1}{n} \delta g^\mu_\mu \, g_{\mu\nu} \implies \delta^t g = 0.$$

- Now, the 2nd Bianchi identity $\nabla_\mu R^{\mu\nu} = \frac{1}{2} \nabla^\nu R$ and TFE imply

$$\nabla_\mu ((n-2)R + 2M_P^{2-n} T) = 0 \implies (n-2)R + 2M_P^{2-n} T = -2nC$$

- TFE and the previous consistency condition imply

$$R_{\mu\nu} - \frac{1}{2} R \hat{g}_{\mu\nu} - C \hat{g}_{\mu\nu} = M_P^{2-n} T_{\mu\nu}$$

ie, Einstein equations with a cosmological constant term but with $\hat{g}_{\mu\nu} / \hat{g} = -1$.

My motivation: Why Unimodular Gravity?

- 1) Solves in a Wilsonian way the huge disparity between the QFT "'prediction" for the vacuum energy and the experimentally observed cosmological constant: Vacuum energy is not seen by gravity. See
  i) S. Weinberg, Rev. Mod. Phys. 61 (1989) 1
  ii) G.F.R. Ellis, H. Van Elst, J. Murugan and J.P. Uzan, Class. Quantum Grav. 28(2011)22 5007
  iii) G.F.R Ellis, Gen. Relativ. Gravit. (2014) 46, 1619.

  In ii) there is a paragraph that runs thus

  "What about experiments? The experimental predictions for the two theories [General Relativity and UG] are the same, so no experiment can tell the difference between them, except for one fundamental feature: the EFE [Einstein's field equations](confirmed in the solar system and by binary pulsar measurements to high accuracy) together with QFT prediction for the vacuum energy density (confirmed by Casimir force measurements) give the wrong answer by many orders of magnitude; the TFE [UG] does not suffer this problem. In this respect, the TFE [UG] are strongly preferred by experiment."

  in i), one can read

  "In my view, the key question in deciding whether this [UG] is a plausible classical theory of gravitation is whether it can be obtained as the classical limit of any physically satisfactory quantum theory of gravitation.
- 2) When ordinary differential geometry is look at from the noncommutative geometry point of view some kind of quantisation of the volume form seems (at least to me!) to occur. See
  i) X. Calmet and A. Kobakhidze, Phys. Rev. D 72, 045010 (2005)
  ii) R. Szabo, Class.Quant.Grav. 23 (2006) R199-R242
  iii) J.M. Gracia-Bondia, "Notes on quantum gravity and noncommutative geometry", Lect.Notes Phys. 807 (2010) 3-58
  iv) A.H. Chamseddine, A. Connes, V. Mukhanov, "Quanta of Geometry: Noncommutative aspects" PRL 114(2015), 9 091302.

Quantum Unimodular Gravity as a theory of gravitons: free theory

- Redundancies (ie, Gauge symmetries) of UG: Not Full Diff. but Transverse Diff, since $g = -1$:

$$\delta^{trans}\hat{g}_{\mu\nu} = \nabla_\mu \varepsilon_\nu + \nabla_\nu \varepsilon_\mu \quad \text{with} \quad \nabla_\mu \varepsilon^\mu = 0$$

- Redundancies enough to go from 9 mathematical d.o.f to 2 physical d.o.f:

$$+9 \leftarrow e_{\mu\nu}(k) \quad \text{polarizations with} \quad e_{\mu\nu}(k) = e_{\nu\mu}(k), \quad e^\mu_\mu(k) = 0$$

$$-4 \leftarrow k^\mu e_{\mu\nu}(k) = 0(\textit{transversality} \quad \textit{conditions})$$

$$\underline{-3} \leftarrow e_{\mu\nu}(k) \sim e_{\mu\nu}(k) + k_\mu \varepsilon_\nu(k) + k_\nu \varepsilon_\mu(k), \quad k_\mu \varepsilon^\mu(k) = 0$$

$$+2 \leftarrow 2 \text{ helicity states}$$

- Actually, J.J. Van der Bij, H. Van Dam and Y.J. NG (Physica 116A (1982)307 showed that the UG free propagator in Minkowski space yields the propagation of gravitons (massless helicity 2 particles) between two gauge-invariant sources and that the amplitude of this process is the same as in GR.

Quantum Unimodular Gravity as a theory of gravitons: free theory

- Actually, as shown by E. Alvarez, D. Blas, J. Garriga & E. Verdaguer NPB 756 (2006) 148, if one asks which quadratic actions on Minkowski of the general type

$$s_{quad} \equiv \sum_{i=1}^{4} C_i\ \mathscr{O}^{(i)}$$
$$\mathscr{O}^{(1)} \equiv \frac{1}{4}\partial_\mu h_{\rho\sigma}\partial^\mu h^{\rho\sigma},\ \mathscr{O}^{(2)} \equiv -\frac{1}{2}\partial^\rho h_{\rho\sigma}\partial_\mu h^{\mu\sigma}$$
$$\mathscr{O}^{(3)} \equiv \frac{1}{2}\partial_\mu h\partial_\lambda h^{\mu\lambda},\ \mathscr{O}^{(4)} \equiv -\frac{1}{4}\partial_\mu h\partial^\mu h$$

  are invariant under linear transverse diff

$$\delta^{trans} h_{\mu\nu} = \partial_\mu\varepsilon_\nu + \partial_\nu\varepsilon_\mu \quad \text{with} \quad \partial_\mu\varepsilon^\mu = 0$$

  one ends up with only two choices, namely

  - 1) Fierz-Pauli (corresponding to LDiff) and
  - 2) Linear Unimodular Gravity (corresponding to LTDiff)

- Generalizes to curved space: C. Barcelo, R. Carballo-Rubio & L.J.Garay, PRD 89 (2014) 124019.

Quantum Unimodular Gravity: Interacting theory

- Quantum Interacting theory defined by a path integral, a la BRST, over configuration space of
  - metric: $\hat{g}_{\mu\nu}$ with $\hat{g} = -1$, ghosts: c^T_μ with $\nabla^\mu c^T_\mu = 0$, etc....
- But, this is a constrained space which is not linear: Functional integration needs definition.
- Our approach: solve the constraints in terms of unconstrained fields as follows:
  - $\hat{g}_{\mu\nu} = (-g)^{-\frac{1}{n}} g_{\mu\nu}$, with $g_{\mu\nu}$ unconstrained
  - $c^T_\mu = (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu - R_{\mu\nu})c^\nu$, c_μ unconstrained
- This way 2 new redundancies (gauge symmetries) are introduced, namely
  - Weyl: $g_{\mu\nu} \to e^{2\omega} g_{\mu\nu}$ and
  - a gauge symmetry for the ghost: $c_\mu \to \nabla_\mu \phi$
- We have ended with –what is called in the Batalin-Vilkovisky formalism parlance– first-stage reducible gauge transformations:
  - a whole cascade of ghosts and antighosts are to be introduced and integrated over. SEE NEXT FRAME

Quantum Unimodular Gravity: Batalin-Vilkovisky field content

Notation: $(field)^{(n,m)}$, n = Grassmann no., m = Ghost number no.

- Unimodular Background metric $\tilde{g}^{(0,0)}_{\mu\nu} = (-\bar{g})^{-\frac{1}{n}}\,\bar{g}_{\mu\nu}$ and quantum gravitational field $h^{(0,0)}_{\mu\nu}$: $g_{\mu\nu} = \bar{g}_{\mu\nu} + (-\bar{g})^{\frac{1}{n}}\,h_{\mu\nu}$
- Transverse Diffeomorphisms
  - Ghost fields: $c^{(1,1)}_{\mu}$ (from TDiff of $h^{(0,0)}_{\mu\nu}$), $\phi^{(0,2)}$ (from transversality of c^{T}_{μ})
  - Antighosts and auxiliary fields couples
    - $(b^{(1,-1)}_{\mu}, f^{(0,0)}_{\mu})$ to gauge-fix the symmetry parametrized by $c^{(1,1)}_{\mu}$
    - $(\bar{c}^{(0,2)}, \pi^{(1,-1)})$ and $(c'^{(0,0)}, \pi'^{(1,1)})$ to gauge-fix the symmetry parametrized by $\phi^{(0,2)}$
- Weyl transformations
  - Ghost fields: $c^{(1,1)}$ (from Weyl of $h^{(0,0)}_{\mu\nu}$)
  - Antighosts and auxiliary fields couple
    - $(b^{(1,-1)}, f^{(0,0)})$ to gauge-fix Weyl

Background BRST transformations

- The FULL BRST operator

$$s = s_D + s_W$$

  acting of the fields is introduced, so that
  - $s_D^2 = 0$, $s_W^2 = 0$ and $\{s_D, s_W\} = 0$
  - s, s_D and s_W all have Grassmann no. = 1 and ghost no = 1.
- s_D, coming form the TDiff, and s_W coming from Weyl transformations, act on the fields as follows:

Background BRST transformations definition

field	s_D	s_W
$\tilde{g}_{\mu\nu}$	0	0
$h_{\mu\nu}$	$\nabla_\mu c^T_\nu + \nabla_\nu c^T_\mu + c^{\rho\,T}\nabla_\rho h_{\mu\nu} + \nabla_\mu c^{\rho\,T} h_{\rho\nu} + \nabla_\nu c^{\rho\,T} h_{\rho\mu}$	$2c^{(1,1)}\left(\tilde{g}_{\mu\nu} + h_{\mu\nu}\right)$
$c^{(1,1)\mu}$	$\left(Q^{-1}\right)^\mu_\nu \left(c^{\rho\,T}\nabla_\rho c^{T\nu}\right) + \nabla^\mu \phi^{(0,2)}$	0
$\phi^{(0,2)}$	0	0
$b^{(1,-1)}_\mu$	$f^{(0,0)}_\mu$	0
$f^{(0,0)}_\mu$	0	0
$\bar{c}^{(0,-2)}$	$\pi^{(1,-1)}$	0
$\pi^{(1,-1)}$	0	0
$c'^{\,(0,0)}$	$\pi'^{\,(1,1)}$	0
$\pi'^{\,(1,1)}$	0	0
$c^{(1,1)}$	$c^{T\rho}\nabla_\rho c^{(1,1)}$	0
$b^{(1,-1)}$	$c^{T\rho}\nabla_\rho b^{(1,-1)}$	$f^{(0,0)}$
$f^{(0,0)}$	$c^{T\rho}\nabla_\rho f^{(0,0)}$	0

where $\left(Q^{-1}\right)^\mu_\nu$ denotes the inverse of the operator $Q_{\mu\nu} = \tilde{g}_{\mu\nu}\Box - R_{\mu\nu}$,
∇_μ and $R_{\mu\nu}$ defined with respect to $\tilde{g}_{\mu\nu}$

Quantum UG: The DeWitt effective action

- The DeWitt effective action $W[\tilde{g}_{\mu\nu}]$

$$e^{iW[\tilde{g}_{\mu\nu}]} = \int \mathscr{D}h_{\mu\nu}\mathscr{D}c_\mu\mathscr{D}b_\mu\mathscr{D}f_\mu\mathscr{D}\bar{c}\mathscr{D}\pi\mathscr{D}c'\mathscr{D}\pi'\mathscr{D}c\mathscr{D}b\mathscr{D}f \quad e^{iS_{UG}[\tilde{g}_{\mu\nu}+h_{\mu\nu}]+iS_{gf}}$$

$$S_{UG}[g_{\mu\nu}] = -M_P^{n-2}\int d^nx\, R[(-g)^{-\frac{1}{n}}g_{\mu\nu}]$$
$$S_{gf} = \int d^nx\, s(X_{TD}+X_W)\,,$$

- X_{TD} and X_W are to be chosen so that the term quadratic in the quantum fields is the closest to a minimal –the large energy behaviour is of Laplacian (to some power) type– differential operator: See JHEP 1508 (2015) 078.
- Recall that $W[\tilde{g}_{\mu\nu}]$ is gauge-invariant when $\tilde{g}_{\mu\nu}$ satisfies the classical equations of motion (ie, it's on-shell)

Quantum UG: Nonminimal Operator

The operator involving $h_{\mu\nu}$, f and c' is non-minimal. We need to use the Barvinsky & Vilkovisky technique (A. O. Barvinsky and G. A. Vilkovisky, Phys. Rept. **119**, 1 (1985)) to compute it.
The non minimal piece can be written

$$S = \int d^n x \, \Psi^A F_{AB} \Psi^B$$

$$\Psi^A = \begin{pmatrix} h^{\mu\nu} \\ f \\ c' \end{pmatrix}$$

Barvinsky & Vilkovisky method

The main idea is to introduce a parameter λ in the non-minimal part of the operator

$$F_{AB}(\nabla|\lambda) = \gamma_{AB}\Box + \lambda J^{\alpha\beta}_{AB}\nabla_\alpha\nabla_\beta + M_{AB} = D_{AB}(\nabla|\lambda) + M_{AB} \qquad 0 \leq \lambda \leq 1$$

so the effective action can be defined as

$$W(1) = W(0) - \frac{1}{2}\int_0^1 d\lambda' Tr\left[\frac{d\hat{F}(\lambda)}{d\lambda'}\hat{G}(\lambda')\right]$$

And if we find the inverse of $\hat{F}$ in the sense

$$\hat{F}(\nabla)\hat{K}(\nabla) = \Box^m + \hat{M}(\nabla)$$

we can expand the Green function as a power series in $\hat{M}$

$$\hat{G} = -\hat{K}\sum_{p=0}^{4}(-1)^p\,\hat{M}_p\frac{\mathbb{I}}{\Box^{m(p+1)}} + \ldots$$

so the trace can be computed with some effort and help from Mathematica's xAct. Indeed,$\rightarrow$

Quantum UG: An involved trace

$$
\begin{aligned}
\nabla_\mu \nabla_\nu \nabla_\alpha \nabla_\beta \frac{\mathbb{I}}{\Box^2} = {} & \frac{\sqrt{g}}{8(n-4)\pi^2} \Big\{ \Big[\frac{1}{36} \left(R_{\mu\nu} R_{\alpha\beta} + R_{\mu\alpha} R_{\nu\beta} + R_{\mu\beta} R_{\nu\alpha} \right) + \frac{1}{180} \left(R^\lambda_\mu \left(11 R_{\nu\alpha\beta\lambda} - R_{\beta\alpha\nu\lambda} \right) + R^\lambda_\nu \left(11 R_{\mu\alpha\beta\lambda} - R_{\beta\alpha\mu\lambda} \right) + R^\lambda_\alpha \left(11 R_{\mu\nu\beta\lambda} - R_{\beta\nu\mu\lambda} \right) + R^\lambda_\beta \left(11 R_{\mu\nu\alpha\lambda} - R_{\alpha\nu\mu\lambda} \right) \right) \\
& + \frac{1}{90} \left(R_\mu{}^\lambda{}_\nu{}^\sigma \left(R_{\lambda\alpha\sigma\beta} + R_{\lambda\beta\sigma\alpha} \right) + R_\mu{}^\lambda{}_\alpha{}^\sigma \left(R_{\lambda\nu\sigma\beta} + R_{\lambda\beta\sigma\nu} \right) + R_\mu{}^\lambda{}_\beta{}^\sigma \left(R_{\lambda\nu\sigma\alpha} + R_{\lambda\alpha\sigma\nu} \right) \right) + \\
& + \frac{1}{20} \left(\nabla_\mu \nabla_\nu R_{\alpha\beta} + \nabla_\mu \nabla_\alpha R_{\nu\beta} + \nabla_\mu \nabla_\beta R_{\nu\alpha} + \nabla_n \nabla_a R_{\mu\beta} + \nabla_\nu \nabla_\beta R_{\mu\alpha} + \nabla_a \nabla_\beta R_{\mu\nu} \right) \Big] \mathbb{I} + \\
& + \frac{1}{12} \left[R_{\mu\nu} \hat{\mathscr{R}}_{\alpha\beta} + R_{\mu\alpha} \hat{\mathscr{R}}_{\nu\beta} + R_{\mu\beta} \hat{\mathscr{R}}_{\nu\alpha} + R_{\nu\alpha} \hat{\mathscr{R}}_{\mu\beta} + R_{\nu\beta} \hat{\mathscr{R}}_{\mu\alpha} + R_{\alpha\beta} \hat{\mathscr{R}}_{\mu\nu} \right] + \\
& + \frac{1}{2} \left[\nabla_\mu \nabla_\nu \hat{\mathscr{R}}_{\alpha\beta} + \nabla_\mu \nabla_\alpha \hat{\mathscr{R}}_{\nu,b} + \nabla_\nu \nabla_\alpha \hat{\mathscr{R}}_{\mu\beta} \right] + \frac{1}{8} \Big[\hat{\mathscr{R}}_{\mu\nu} \hat{\mathscr{R}}_{\alpha\beta} + \hat{\mathscr{R}}_{\alpha\beta} \hat{\mathscr{R}}_{\mu\nu} + \hat{\mathscr{R}}_{\mu\alpha} \hat{\mathscr{R}}_{\nu\beta} + \\
& + \hat{\mathscr{R}}_{\nu\beta} \hat{\mathscr{R}}_{\mu\alpha} + \hat{\mathscr{R}}_{\mu\beta} \hat{\mathscr{R}}_{\nu\alpha} + \hat{\mathscr{R}}_{\nu\alpha} \hat{\mathscr{R}}_{\mu\beta} \Big] - \frac{1}{12} \Big[\hat{\mathscr{R}}_{\mu\lambda} \left(R^\lambda_{\alpha\nu\beta} + R^\lambda_{\beta\nu\alpha} \right) + \hat{\mathscr{R}}_{\nu\lambda} \left(R^\lambda_{\alpha\mu\beta} + R^\lambda_{\beta\mu\alpha} \right) + \\
& + \hat{\mathscr{R}}_{\alpha\lambda} \left(R^\lambda_{\nu\mu\beta} + R^\lambda_{\beta\mu\nu} \right) + \hat{\mathscr{R}}_{\beta\lambda} \left(R^\lambda_{\mu\nu\alpha} + R^\lambda_{\alpha\nu\mu} \right) \Big] - \frac{1}{2} \Big[- \frac{1}{9} \left(R_{\alpha\mu\beta\nu} + R_{\beta\mu\alpha\nu} \right) R \mathbb{I} + \\
& + g_{\mu\nu} \Big[\Big(\frac{1}{36} R_{\alpha\beta} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\alpha\sigma\beta} + \frac{1}{90} R_{\rho\sigma\lambda\alpha} R^{\rho\sigma\lambda}{}_\beta - \frac{1}{45} R_{\alpha\lambda} R^\lambda_\beta + \frac{1}{60} \Box R_{\alpha\beta} + \frac{1}{20} \nabla_\alpha \nabla_\beta R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\alpha\lambda} \hat{\mathscr{R}}^\lambda_\beta + \hat{\mathscr{R}}_{\beta\lambda} \hat{\mathscr{R}}^\lambda_\alpha \right) - \frac{1}{12} \left(\nabla_\alpha \nabla^\lambda \hat{\mathscr{R}}_{\lambda\beta} + \nabla_\beta \nabla^\lambda \hat{\mathscr{R}}_{\lambda\alpha} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\alpha\beta} \Big] + \\
& + g_{\mu\alpha} \Big[\Big(\frac{1}{36} R_{\nu\beta} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\nu\sigma\beta} + \frac{1}{90} R_{\rho\sigma\lambda\nu} R^{\rho\sigma\lambda}{}_\beta - \frac{1}{45} R_{\nu\lambda} R^\lambda_\beta + \frac{1}{60} \Box R_{\nu\beta} + \frac{1}{20} \nabla_\nu \nabla_\beta R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\nu\lambda} \hat{\mathscr{R}}^\lambda_\beta + \hat{\mathscr{R}}_{\beta\lambda} \hat{\mathscr{R}}^\lambda_\nu \right) - \frac{1}{12} \left(\nabla_\nu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\beta} + \nabla_\beta \nabla^\lambda \hat{\mathscr{R}}_{\lambda\nu} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\nu\beta} \Big] + \\
& + g_{\mu\beta} \Big[\Big(\frac{1}{36} R_{\nu\alpha} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\nu\sigma\alpha} + \frac{1}{90} R_{\rho\sigma\lambda\nu} R^{\rho\sigma\lambda}{}_\alpha - \frac{1}{45} R_{\nu\lambda} R^\lambda_\alpha + \frac{1}{60} \Box R_{\nu\alpha} + \frac{1}{20} \nabla_\nu \nabla_\alpha R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\nu\lambda} \hat{\mathscr{R}}^\lambda_\alpha + \hat{\mathscr{R}}_{\alpha\lambda} \hat{\mathscr{R}}^\lambda_\nu \right) - \frac{1}{12} \left(\nabla_\nu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\alpha} + \nabla_\alpha \nabla^\lambda \hat{\mathscr{R}}_{\lambda\nu} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\nu\alpha} \Big] + \\
& + g_{\nu\alpha} \Big[\Big(\frac{1}{36} R_{\mu\beta} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\mu\sigma\beta} + \frac{1}{90} R_{\rho\sigma\lambda\mu} R^{\rho\sigma\lambda}{}_\beta - \frac{1}{45} R_{\mu\lambda} R^\lambda_\beta + \frac{1}{60} \Box R_{\mu\beta} + \frac{1}{20} \nabla_\mu \nabla_\beta R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\mu\lambda} \hat{\mathscr{R}}^\lambda_\beta + \hat{\mathscr{R}}_{\beta\lambda} \hat{\mathscr{R}}^\lambda_\mu \right) - \frac{1}{12} \left(\nabla_\mu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\beta} + \nabla_\beta \nabla^\lambda \hat{\mathscr{R}}_{\lambda\mu} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\mu\beta} \Big] + \\
& + g_{\nu\beta} \Big[\Big(\frac{1}{36} R_{\mu\alpha} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\mu\sigma\alpha} + \frac{1}{90} R_{\rho\sigma\lambda\mu} R^{\rho\sigma\lambda}{}_\alpha - \frac{1}{45} R_{\mu\lambda} R^\lambda_\alpha + \frac{1}{60} \Box R_{\mu\alpha} + \frac{1}{20} \nabla_\mu \nabla_\alpha R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\mu\lambda} \hat{\mathscr{R}}^\lambda_\alpha + \hat{\mathscr{R}}_{\alpha\lambda} \hat{\mathscr{R}}^\lambda_\mu \right) - \frac{1}{12} \left(\nabla_\mu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\alpha} + \nabla_\alpha \nabla^\lambda \hat{\mathscr{R}}_{\lambda\mu} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\mu\alpha} \Big] + \\
& + g_{\alpha\beta} \Big[\Big(\frac{1}{36} R_{\mu\nu} R + \frac{1}{90} R^{\lambda\sigma} R_{\lambda\mu\sigma\nu} + \frac{1}{90} R_{\rho\sigma\lambda\mu} R^{\rho\sigma\lambda}{}_\nu - \frac{1}{45} R_{\mu\lambda} R^\lambda_\nu + \frac{1}{60} \Box R_{\mu\nu} + \frac{1}{20} \nabla_\mu \nabla_\nu R \Big) \mathbb{I} + \\
& + \frac{1}{12} \left(\hat{\mathscr{R}}_{\mu\lambda} \hat{\mathscr{R}}^\lambda_\nu + \hat{\mathscr{R}}_{\nu\lambda} \hat{\mathscr{R}}^\lambda_\mu \right) - \frac{1}{12} \left(\nabla_\mu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\nu} + \nabla_\nu \nabla^\lambda \hat{\mathscr{R}}_{\lambda\mu} \right) + \frac{1}{12} R \hat{\mathscr{R}}_{\mu\nu} \Big] \Big] + \\
& + \frac{1}{4} \left(g_{\mu\nu} g_{\alpha\beta} + g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} \right) \Big[\Big[\frac{1}{180} \left(R_{\lambda\sigma\rho\gamma} R^{\lambda\sigma\rho\gamma} - R_{\lambda\delta} R^{\lambda\sigma} \right) + \frac{1}{30} \Box R - \frac{1}{72} R^2 \Big] \mathbb{I} + \frac{1}{12} \hat{\mathscr{R}}_{\lambda\sigma} \hat{\mathscr{R}}^{\lambda\sigma} \Big] \Big\}
\end{aligned} \tag{1}
$$

Quantum UG: logarithmic UV divergencies

By doing this we find (the UV divergent part of) the off-shell effective action

$$W_\infty = \frac{1}{16\pi^2}\frac{1}{n-4}\int d^4x \left(\frac{119}{90}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + \left(\frac{1}{6\alpha^2} - \frac{359}{90}\right)R_{\mu\nu}R^{\mu\nu} + \frac{1}{72}\left(22 - \frac{3}{\alpha^2}\right)R^2\right)$$

We can get the on-shell result using the equations of motion of the background field

$$R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} = 0$$
$$R_{\mu\nu}R^{\mu\nu} = \frac{1}{4}R^2$$
$$R = \text{constant}$$

and

$$E_4 \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2 = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

Quantum UG: logaritmic On-shell UV divergencies

The one loop on-shell DeWitt effective action reads

$$W_\infty^{\text{on-shell}} = \frac{1}{16\pi^2}\frac{1}{n-4}\int d^4x\left(\frac{119}{90}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{83}{120}R^2\right) = $$
$$= \frac{1}{16\pi^2}\frac{1}{n-4}\int d^4x\left(\frac{119}{90}E_4 - \frac{83}{120}R^2\right)$$

This is physically irrelevant, in contrast with GR with a Cosmological Constant term (Christensen-Duff, NPB 170[FSI] (1980) 480)

$$W_\infty^{GR} \equiv \frac{1}{16\pi^2(n-4)}\int \sqrt{|g|}d^4x\,\left(\frac{53}{45}\,E_4 - \frac{522}{45}\Lambda^2\right)$$

Quantum UG: Quadratic UV divergencies

- Dimensional Regularization applied to the Proper time expression for the one-loop effective action yields the Logarithmic UV divergences.
- To work out the UV Quadratic divergences a cut-off of the small proper-times is to be introduced:
- $W_{reg} = -\frac{1}{2}\int_{\Lambda_{UV}^{-2}}^{\infty} \frac{dt}{t} K(t,D)$, D differential operator, $K(t,D)$ Heat Kernel
- $K(t,F) = \frac{1}{(4\pi t)^2}\sum_{i=0} t^{i/2} a_i(F)$ as $t \to 0^+$
- $W_\infty = \frac{1}{2} a_0 \Lambda_{UV}^4 + a_2 \Lambda_{UV}^2 + a_4 \ln(\Lambda_{UV}/\mu) +$ UV finite terms for large Λ_{UV}
- There does not exist general expressions for a_2 for general nonminimal operators. Fortunately, as shown by Toms in PRD 26 (1982) 2713, a_2 is given by the residue at the pole in $n = 4$ of the Dimensionally regularized Green function of D; This can be computed by using the Barvinsky & Vilkovisky technique for our nonminimal operator. We have obtained

$$a_2 = \frac{3+\sqrt{\pi}}{6}\int d^4x\, R \quad \text{(off-shell)} \Longrightarrow a_2^{(on-shell)} = C\frac{3+\sqrt{\pi}}{6}\int d^4x \quad \text{(on-shell)}$$

- The quadratic UV divergencies are also physically irrelevant

Summary and outlook

- We've set up a formalism that can be used to compute in a systematic way the DeWitt effective action of Unimodular Gravity
- We have checked by explicit computation that Unimodular Gravity shares with General Relativity without Cosmological Constant the ancient ('t Hooft and Veltman, 1974) beautiful property of one-loop UV finiteness.

Some open problems (personal taste)

- Do GR without C.C. and UG have the same S-matrix? (My educated guess: Yes)
- All good ideas (eg, Noncommutative Geometry) have found accommodation in the String framework, will this framework let us down this time and not admit Unimodular Gravity?