A stable vacuum from vector dark matter

Bohdan GRZADKOWSKI
University of Warsaw

- The Vector Dark Matter (VDM) model
- Vacuum stability
- Landau poles
- Experimental constraints
- Direct detection of dark matter
- Summary

The Vector Dark Matter (VDM) model


- ...
The model:

• extra $U(1)_X$ gauge symmetry ($A^\mu_X$),

• a complex scalar field $S$, whose vev generates a mass for the $U(1)$’s vector field, $S = (0, 1, 1, 1)$ under $U(1)_Y \times SU(2)_L \times SU(3)_c \times U(1)_X$.

• SM fields neutral under $U(1)_X$,

• in order to ensure stability of the new vector boson, a $\mathbb{Z}_2$ symmetry is assumed to forbid $U(1)$-kinetic mixing between $U(1)_X$ and $U(1)_Y$. The extra gauge boson $A^\mu_X$ and the scalar $S$ field transform under $\mathbb{Z}_2$ as follows

$$A^\mu_X \rightarrow -A^\mu_X, \quad S \rightarrow S^*, \text{ where } S = \phi e^{i\sigma}, \text{ so } \phi \rightarrow \phi, \quad \sigma \rightarrow -\sigma.$$
The scalar potential

\[ V = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2. \]

The vector bosons masses:

\[ M_W = \frac{1}{2} g v, \quad M_Z = \frac{1}{2} \sqrt{g^2 + g'^2} v \quad \text{and} \quad M_{Z'} = g_x v_x, \]

where

\[ \langle H \rangle = \begin{pmatrix} 0 \\ v \sqrt{2} \end{pmatrix} \quad \text{and} \quad \langle S \rangle = \frac{v_x}{\sqrt{2}}. \]

Positivity of the potential implies

\[ \lambda_H > 0, \quad \lambda_S > 0, \quad \kappa > -2 \sqrt{\lambda_H \lambda_S}. \]

The minimization conditions for scalar fields

\[ (2\lambda_H v^2 + \kappa v_x^2 - 2\mu_H^2) v = 0 \quad \text{and} \quad (\kappa v_x^2 + 2\lambda_S v_x^2 - 2\mu_S^2) v_x = 0. \]
For $\kappa^2 < 4\lambda_H\lambda_S$ the global minima are

$$v^2 = \frac{4\lambda_S\mu_H^2 - 2\kappa\mu_S^2}{4\lambda_H\lambda_S - \kappa^2} \quad \text{and} \quad v_x^2 = \frac{4\lambda_H\mu_S^2 - 2\kappa\mu_H^2}{4\lambda_H\lambda_S - \kappa^2}$$

Both scalar fields can be expanded around corresponding vev’s as follows

$$S = \frac{1}{\sqrt{2}}(v_x + \phi_S + i\sigma_S), \quad H^0 = \frac{1}{\sqrt{2}}(v + \phi_H + i\sigma_H) \quad \text{where} \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}. $$

The mass squared matrix $\mathcal{M}^2$ for the fluctuations $(\phi_H, \phi_S)$ and their eigenvalues read

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & \kappa vv_x \\ \kappa vv_x & 2\lambda_S v_x^2 \end{pmatrix}$$

$$M_{\pm}^2 = \lambda_H v^2 + \lambda_S v_x^2 \pm \sqrt{\lambda_S^2 v_x^4 - 2\lambda_H\lambda_S v^2 v_x^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_x^4}$$

$$\mathcal{M}^2_{\text{diag}} = \begin{pmatrix} M_{h_1}^2 & 0 \\ 0 & M_{h_2}^2 \end{pmatrix}, \quad R = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}, \quad \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R^{-1} \begin{pmatrix} \phi_H \\ \phi_S \end{pmatrix},$$
where \( M_{h_1} = 125.7 \text{ GeV} \) is the mass of the observed Higgs particle. Then we obtain

\[
\sin 2\alpha = \frac{\text{sign}(\lambda_{SM} - \lambda_H) 2M_{12}^2}{\sqrt{(M_{11}^2 - M_{22}^2)^2 + 4(M_{12}^2)^2}}, \quad \cos 2\alpha = \frac{\text{sign}(\lambda_{SM} - \lambda_H)(M_{11}^2 - M_{22}^2)}{\sqrt{(M_{11}^2 - M_{22}^2)^2 + 4(M_{12}^2)^2}}.
\]

Note that since vev of \( H \) is fixed at 246.22 GeV, with \( \kappa = 0 \) (no mass mixing) and \( \lambda_H \neq \lambda_{SM} \) it is only \( \phi_S \) which can have the observed Higgs mass of 125.7 GeV. Even though the mass matrix is diagonal in this case, however in order to satisfy our convention that \( M_{h_1} = 125.7 \text{ GeV} \) a rotation by \( \alpha = \pm \pi/2 \) is required in such a case.

There are 5 real parameters in the potential: \( \mu_H, \mu_S, \lambda_H, \lambda_S \) and \( \kappa \). Adopting the minimization conditions \( \mu_H, \mu_S \) could be replaced by \( v \) and \( v_x \). The SM vev is fixed at \( v = 246.22 \text{ GeV} \). Using the condition \( M_{h_1} = 125.7 \text{ GeV} \), \( v_x^2 \) could be eliminated in terms of \( v^2, \lambda_H, \kappa, \lambda_S, \lambda_{SM} = M_{h_1}^2/(2v^2) \):

\[
v_x^2 = v^2 \frac{4\lambda_{SM}(\lambda_H - \lambda_{SM})}{4\lambda_S(\lambda_H - \lambda_{SM}) - \kappa^2}
\]

Eventually there are 4 independent parameters:

\[ (\lambda_H, \kappa, \lambda_S, g_x), \]

where \( g_x \) is the \( U(1)_X \) coupling constant.
Figure 1: Contour plots for masses of the non-standard \((h_2)\) Higgs particle in the plane \((\lambda_H, \kappa)\). In the bottom part of the plot \((\lambda_H < \lambda_{SM} = M_{h_1}^2 / (2v^2) = 0.13)\) the heavier Higgs is the currently observed one, while in the upper part \((\lambda_H > \lambda_{SM})\) the lighter state is the observed one. White regions in the upper and lower parts are disallowed by the positivity conditions for \(v_x^2\) and \(M_{h_2}^2\) respectively.

- Positivity of \(v_x^2\) implies for \(\lambda_H > \lambda_{SM}\) that \(\lambda_H > \frac{\kappa^2}{4\lambda_S} + \lambda_{SM}\)

- Positivity of \(M_{h_2}^2\) implies for \(\lambda_H < \lambda_{SM}\) that \(\lambda_H > \frac{\kappa^2}{4\lambda_S}\)
Figure 2: Contour plots for the vacuum expectation value of the extra scalar $v_x \equiv \sqrt{2}\langle S \rangle$ (left panel) and of the mixing angle $\alpha$ (right panel) in the plane $(\lambda_H, \kappa)$. 
Vacuum stability

\[ V = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2 \]

2-loop running of parameters adopted

\[ \lambda_H(Q) > 0, \quad \lambda_S(Q) > 0, \quad \kappa(Q) + 2\sqrt{\lambda_H(Q)\lambda_S(Q)} > 0 \]

**Figure 3:** Running of various parameters at 1- and 2-loop, in solid and dashed lines respectively. For this choice of parameters \( \lambda_H(Q) > 0 \) at 2-loop (right panel blue) but not at 1-loop. \( \lambda_S(Q) \) is always positive (right panel red), running of \( \kappa(Q) \) is very limited, however the third positivity condition \( \kappa(Q) + 2\sqrt{\lambda_H(Q)\lambda_S(Q)} > 0 \) is violated at higher scales even at 2-loops (right panel green).
The mass of the Higgs boson is known experimentally therefore within the SM the initial condition for running of $\lambda_H(Q)$ is fixed

$$\lambda_H(m_t) = M^2_{h_1}/(2v^2) = \lambda_{SM} = 0.13$$

For VDM this is not necessarily the case:

$$M^2_{h_1} = \lambda_H v^2 + \lambda_S v^2_x \pm \sqrt{\lambda^2_S v^4_x - 2\lambda_H \lambda_S v^2 v^2_x + \lambda^2_H v^4 + \kappa^2 v^2 v^4_x}.$$

VDM:

- Larger initial values of $\lambda_H$ such that $\lambda_H(m_t) > \lambda_{SM}$ are allowed delaying the instability (by shifting up the scale at which $\lambda_H(Q) < 0$).

- Even if the initial $\lambda_H$ is smaller than its SM value, $\lambda_H(m_t) < \lambda_{SM}$, still there is a chance to lift the instability scale if appropriate initial value of the portal coupling $\kappa(m_t)$ is chosen.

$$\beta^{(1)}_{\lambda_H} = \beta^{SM}_{\lambda_H} (1) + \kappa^2$$
Figure 4: The stability frontier for the $H$ direction: these plots identify the renormalisation scale $t^* = \log_{10}(Q^*)$ at which $\lambda_H(Q^*) = 0$ and the vacuum becomes unstable, as a function of $(\lambda(m_t), \kappa(m_t))$. The horizontal solid black line corresponds to $\lambda_H(m_t) = \lambda_{SM} \simeq 0.13$. 
Figure 5: The “in between” stability frontier: these plots identify the scale $t^* = \log_{10}(Q^*)$ at which the positivity condition $\kappa(Q) + 2\sqrt{\lambda_H(Q)\lambda_S(Q)} > 0$ fails and the vacuum becomes unstable, as a function of $(\lambda(m_t), \kappa(m_t))$ for fixed choices of $(g_x(m_t), \lambda_S(m_t))$ specified above each panel. The horizontal solid black line corresponds to $\lambda_H(m_t) = \lambda_{SM} \simeq 0.13$. The gray area is excluded by the requirement that there is no Landau poles up to the Planck mass.
Figure 6: Contour plots of $\lambda_H(M_{Pl})$ in the plane of $(\lambda(m_t), \kappa(m_t))$ for fixed $g_x(m_t)$ and $\lambda_S(m_t)$ specified above each panel. The horizontal solid black line corresponds to $\lambda_H(m_t) = \lambda_{SM} \simeq 0.13$. The plots allow one to identify regions (white) in which the $\lambda_H(Q)$ Landau pole is below the Planck scale.
Experimental constraints

• no invisible $h_1$ decays: $h_1 \rightarrow Z'Z'$, $h_1 \rightarrow h_2 h_2$,

• LEP constraints for $e^+ e^- \rightarrow Z h_2$ satisfied,

• LHC constraints on

$$\kappa_V \equiv \frac{g_{h_1 VV}}{g_{h_1 VV}^{SM}} \text{ with } 0.85 < \kappa_V < 1$$

• limits from electroweak precision data (S,T) satisfied at 95% CL

$$S = \frac{16\pi \cos^2 \theta_W}{g^2} \delta \Pi_{ZZ}'(0), \quad T = \frac{4\pi}{e^2} \left( \frac{\delta \Pi_{WW}(0)}{M_W^2} - \frac{\delta \Pi_{ZZ}(0)}{M_Z^2} \right),$$

• DM abundance ($\Omega_{DM} h^2$) remains within the $5\sigma$ limit (micrOMEGAs and explicite calculation)
Figure 7: Combined plots of allowed and disallowed parameter space in the plane \((\lambda_H(m_t), \kappa(m_t))\) for \(g_x(m_t) = g_1(m_t)\) and \(\lambda_S(m_t) = \lambda_{SM}(m_t) = 0.13\). The thin red line denotes the frontier above which a Landau pole appears below \(\lambda_H(M_{Pl})\). The thin blue line denotes the absolute stability frontier. Below the thin green line the positivity condition fails at some renormalisation scale (its wavy shape is a numerical artifact). The green area denotes LEP exclusions on Higgs-like scalars. In the outer red area positivity fails at the low scale, while in the orange area no physical solution of the vev \(v_x\) exists. The blue area denotes an excess of the \(h_1\) Higgs couplings to vector bosons \((\kappa_V)\). The remaining allowed region is in white. The green points are those for which also \(\Omega_{DM}h^2\) constraint is fulfilled.
Figure 8: Same as in fig. 7 however colouring of allowed points is here with respect to $\sin \alpha$. 
\( g_x(m_t) = 0.25, \lambda_S(m_t) = 0.05 \)

\[ \kappa \left( m_t \right) \]

\[ \lambda_H \left( m_t \right) \]

\[ g_x(m_t) = 0.6, \lambda_S(m_t) = 0.2 \]

\[ \kappa \left( m_t \right) \]

\[ \lambda_H \left( m_t \right) \]

\( \text{LEP excluded} \)

\( \sin \alpha \)
Direct detection of dark matter

\[ \sigma_{Z'N} = \frac{\mu^2}{4\pi} g_x^2 g_{hNN}^2 \sin^2 2\alpha \left( \frac{1}{M_{h_1}^2} - \frac{1}{M_{h_2}^2} \right)^2 \]

- scan range: \(0.1 < g_x < 1, 0 < \lambda_H < 0.25\) and \(-0.5 < \kappa < 0.5\)

- \(\lambda_H > \lambda_{SM}\) (heavy dark matter): \(63 \text{ GeV} \lesssim M_{Z'} \lesssim 1000 \text{ GeV}\).

- \(\lambda_H < \lambda_{SM}\) (light dark matter): \(60 \text{ GeV} \lesssim M_{Z'} \lesssim 120 \text{ GeV}\),

**Figure 9:** The figure shows the DM-nucleon cross section, \(\sigma_{Z'N}\), as a function of the DM mass \(M_{Z'}\) for points which satisfy all other constraints for \(\lambda_H > \lambda_{SM}\). The singlet quartic coupling is fixed at \(\lambda_S = 0.2\). Colouring corresponds to the strength of the gauge coupling \(g_x\). The solid lines are the experimental limits for \(\sigma_{Z'N}\) from XENON100, LUX (2013) and anticipated results for XENON 1T.
Figure 10: The left panel illustrates correlation between $M_{h_2}$ and $M_{Z'}$, while the right one shows predictions for $\Omega_{DM} h^2$ as a function of $M_{Z'}$. The colouring corresponds to the cross section $g_x$. Above the right box resonances and channels which open as $M_{Z'}$ increases are shown. Coordinates in the parameter space $(\lambda_H, \kappa, \lambda_S)$ and corresponding $M_{h_2}$ and $v_x$ are shown above the right panel.
Figure 11: The figure shows the DM-nucleon cross section, $\sigma_{Z'N}$, as a function of the DM mass $M_{Z'}$ for points which satisfy all other constraints for $\lambda_H < \lambda_{SM}$. The singlet quartic coupling is fixed at $\lambda_S = 0.2$. Colouring corresponds to the strength of the gauge coupling $g_x$. The nearly horizontal lines are the experimental limits for $\sigma_{Z'N}$ from XENON100, LUX (2013) and anticipated results for XENON 1T.
$\sigma_{Z'N} \text{ [cm}^2\text{]}$

$M_{h_2} \text{ [GeV]}$

$\sin^2(2\alpha)$

$M_{Z'} \text{ [GeV]}$

- Diagram showing results from XENON100 (2012), LUX (2013), and XENON 1T.

- Data points and lines indicating suppression of double-scalar production with incoming $Z'$.
Figure 12: The left panel illustrates correlation between between $M_{h_2}$ and $M_{Z'}$, while the right one shows predictions for $\Omega_{DM} h^2$ as a function of $M_{Z'}$. The colouring corresponds to the cross section $\sigma_{Z'N}$. Above the right box resonances and channels which open as $M_{Z'}$ increases are shown. Coordinates in the parameter space $(\lambda_H, \kappa, \lambda_S)$ and corresponding $M_{h_2}$ and $v_x$ are shown above the right panel.
Summary

- VDM model has been presented: $Z'$ (DM), $h_2$ (extra Higgs)
- Vacuum stability was addressed: absolute stability
- Cosmological consequences were discussed, VDM easily consistent with $\Omega_{DM} h^2$ and $\sigma_{Z'N}$
- Collider phenomenology: in progress