UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

A microscopic approach to cosmic structure formation

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with
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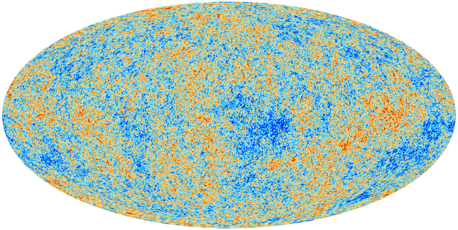
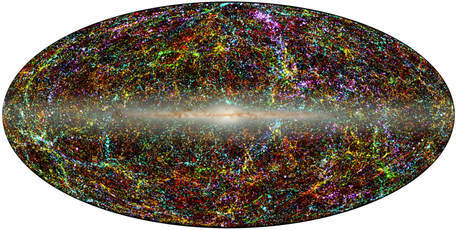
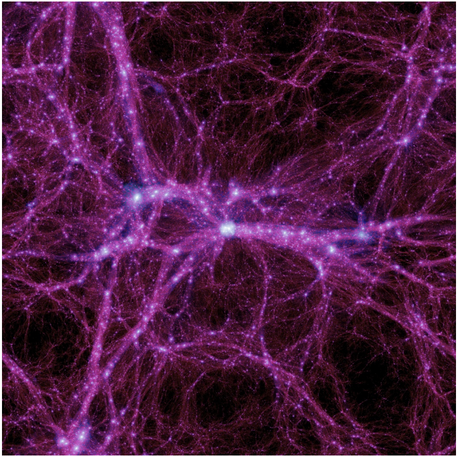
Problems in cosmic structure formation



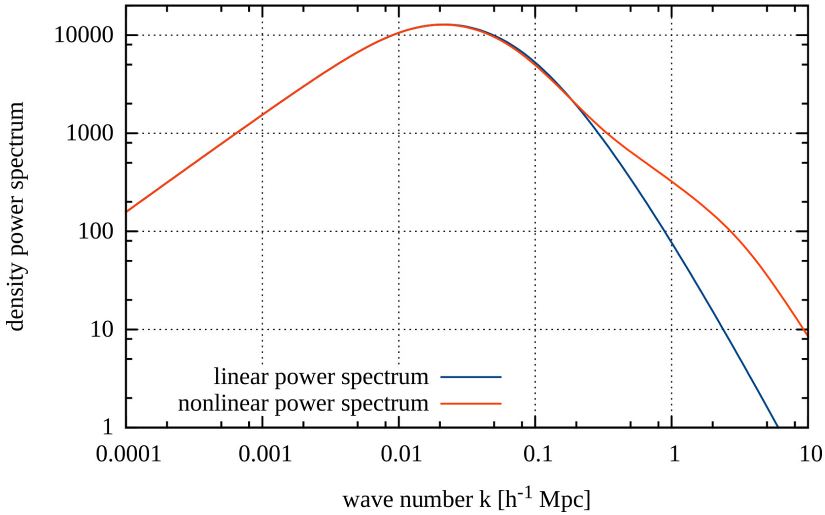


Planck; Two-micron All-Sky Survey

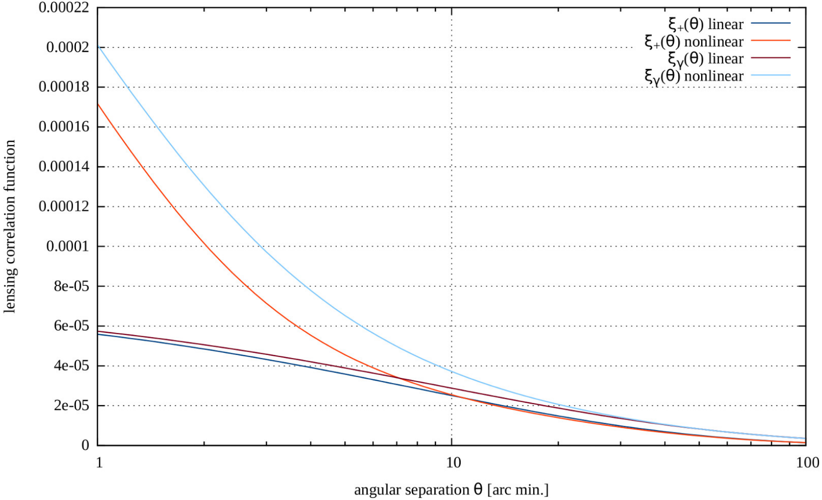


Boylan-Kolchin et al.

Problems in cosmic structure formation

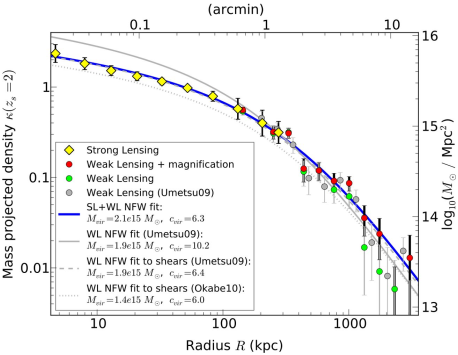


Problems in cosmic structure formation



Problems in cosmic structure formation

Abell 2261, Clash project



Density profile, Coe et al.

Cosmological perturbation theory

Conventional:

- Hydrodynamical equations:

$$\dot{\delta} + \vec{\nabla} \cdot \vec{u} = 0$$
$$\dot{\vec{u}} + 2H\vec{u} = -\vec{\nabla}\phi$$
$$\vec{\nabla}^2\phi = 4\pi G\bar{\rho}\delta$$

- But: *dark matter is no fluid*
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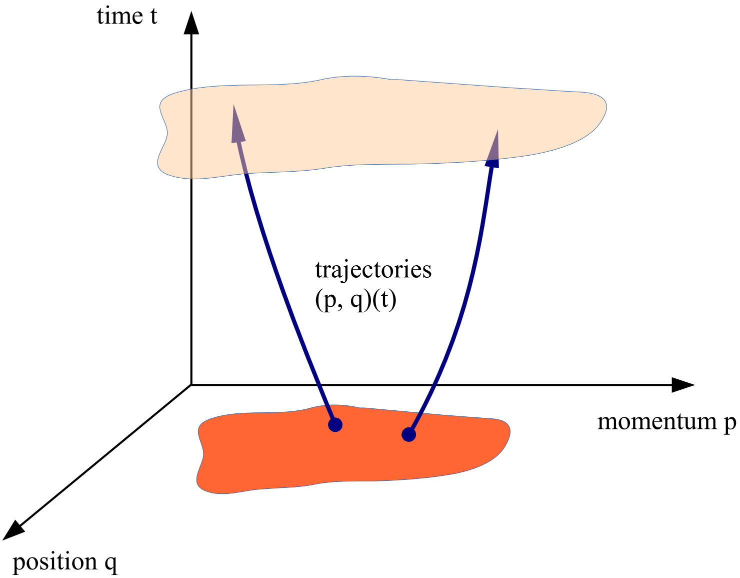
New approach:

- Non-equilibrium statistics of N classical particle trajectories
- Describe particle ensemble by partition sum (generating functional) Z
- Derive statistical properties by functional derivatives

Phase-space trajectories



Phase-space trajectories

- Classical particles follow Hamiltonian equations of motion,

$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p}\,, \quad \dot{p} = -\frac{\partial \mathcal{H}}{\partial q}\,, \quad x = (q, p)$$

- Trajectories are described by (retarded) Green's function $G_{\mathrm{R}}(t, t')$,

$$\bar{x}(t) = \underbrace{G_{\mathrm{R}}(t, 0)x^{(\mathrm{i})}}_{\text{free motion}} - \underbrace{\int_0^t G_{\mathrm{R}}(t, t')K(t')\mathrm{d}t'}_{\text{interaction}}$$

- In static space, with potential v,

$$G_{\mathrm{R}}(t, t') = \begin{pmatrix} 1 & (t - t')/m \\ 0 & 1 \end{pmatrix}, \quad K = \begin{pmatrix} 0 \\ \nabla v \end{pmatrix}$$

Principle

Equilibrium thermodynamics:

$$Z_{\mathrm{gc}} = \int \mathrm{d}\Gamma \, \mathrm{e}^{-\beta(H(x)-\mu N)}$$

$$\langle N \rangle = \frac{1}{\beta} \frac{\partial \ln Z_{\mathrm{gc}}}{\partial \mu}$$

Principle

Non-equilibrium, statistical quantum field theory:

$$Z_0[J] = \mathrm{Tr} \int \mathcal{D}\varphi \, \mathrm{e}^{\mathrm{i} \int (\mathcal{L}_0 + J\varphi) \mathrm{d}^4 x}$$

$$Z[J] = \mathrm{e}^{\mathrm{i}\hat{S}_\mathrm{I}} Z_0[J]$$

$$\langle \varphi(1)\varphi(2) \rangle = \frac{\delta}{\mathrm{i}\delta J(1)} \frac{\delta}{\mathrm{i}\delta J(2)} Z$$

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$$\langle \rho(1)\rho(2) \rangle = \frac{\delta}{\mathrm{i}\delta H_\rho(1)} \frac{\delta}{\mathrm{i}\delta H_\rho(2)} Z$$

Some detail

- Collective fields:

$$H = (\rho, B, \ldots)^\top$$

- Generating functional:

$$Z[H, J, K] = \mathrm{e}^{\mathrm{i}\hat{S}} \mathrm{e}^{\mathrm{i}H\hat{\Phi}} \int \mathrm{d}\Gamma^{(\mathrm{i})} \mathrm{e}^{\mathrm{i}\int \langle J, \bar{x} \rangle \mathrm{d}t}$$

- Interaction operator:

$$\hat{S}_\mathrm{I} = -\int \mathrm{d}1 \int \mathrm{d}2 \frac{\delta}{\delta H_B(1)} v(12) \frac{\delta}{\delta H_\rho(2)}$$

- Perturbation theory: $\mathrm{e}^{\mathrm{i}\hat{S}} = 1 + \mathrm{i}\hat{S} - \frac{1}{2}\hat{S}^2 + \ldots$

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Specialisation to cosmology

1. Choose initial phase-space measure

$$\mathrm{d}\Gamma^{(\mathrm{i})} = P(q,p)\,\mathrm{d}^{3N}q\,\mathrm{d}^{3N}p$$

Specialisation to cosmology

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fully specified by initial power spectrum

$$P(q,p) = \frac{V^{-N}}{\left((2\pi)^{3N}\det C_{pp}\right)^{1/2}}\exp\left(-\frac{1}{2}p^\top C_{pp}p\right)$$

Specialisation to cosmology

1. Choose initial phase-space measure

$$\mathrm{d}\Gamma^{(\mathrm{i})} = P(q,p)\,\mathrm{d}^{3N}q\,\mathrm{d}^{3N}p$$

2. Change time coordinate

$$t \to \tau = D_+(t) - D_+(t_\mathrm{i})$$

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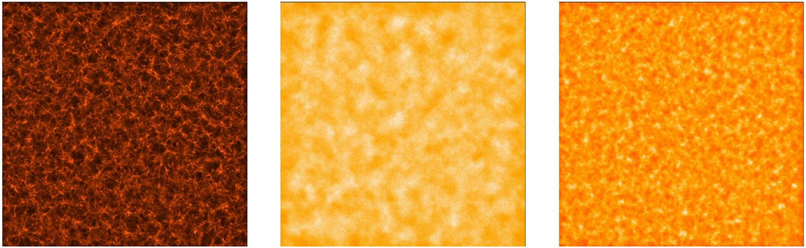
3. Adapt Green's function to expanding universe

Improved Zel'dovich trajectories



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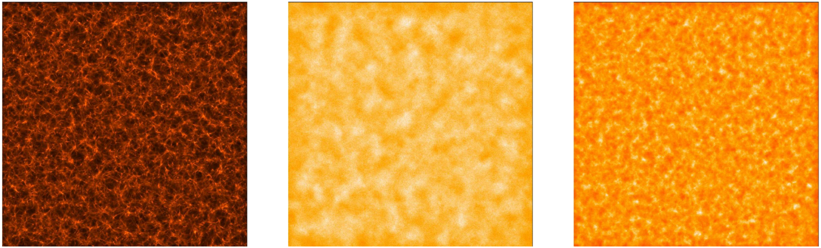

$$\vec{q}(\tau) = \vec{q}^{\,(\mathrm{i})} + \vec{p}^{\,(\mathrm{i})} \int_0^{\tau} \exp\left(h(\tau')\right) \mathrm{d}\tau'$$

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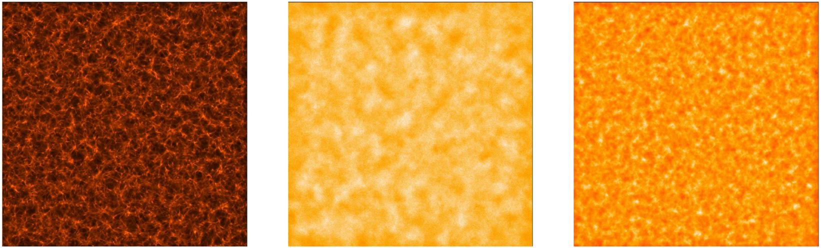

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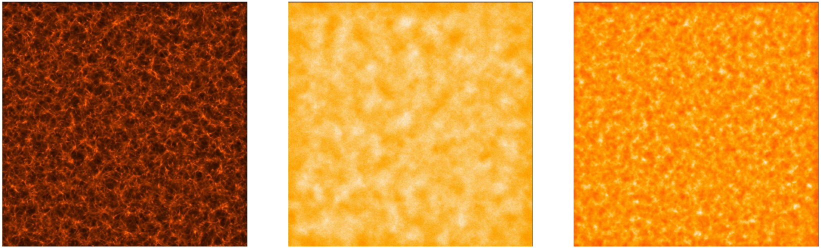

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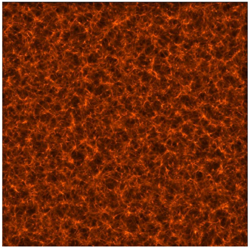

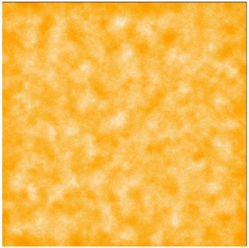

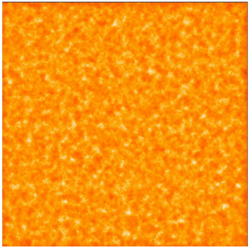

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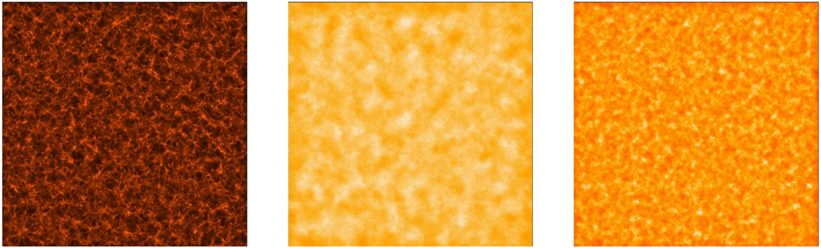

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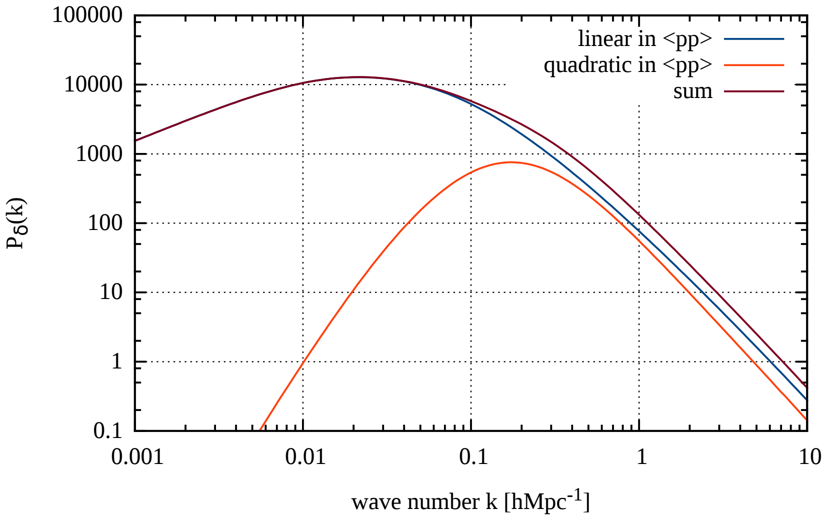
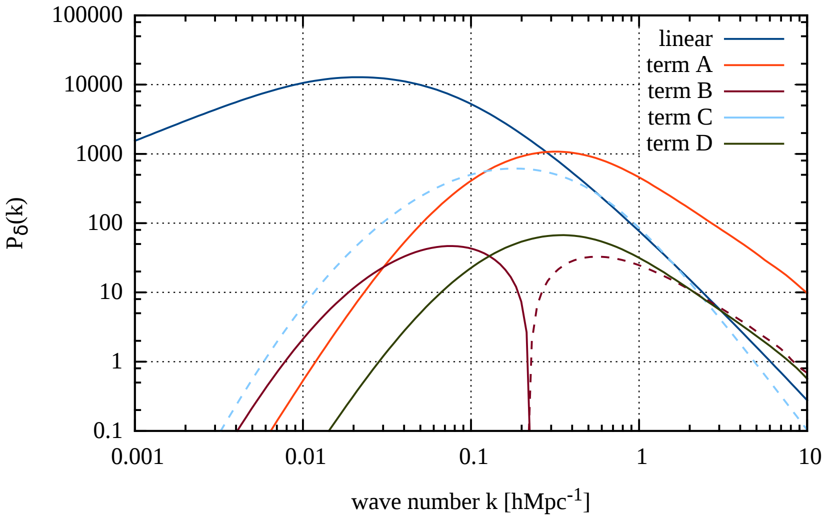
$$\hat{v}(k) = -\frac{3}{2k^2}\frac{a}{g^2} \propto a^{-2} \text{ (EdS)}$$

Density power spectrum



Improved Zel'dovich trajectories, no interaction

Density power spectrum



Improved Zel'dovich trajectories, first-order interaction, modified potential

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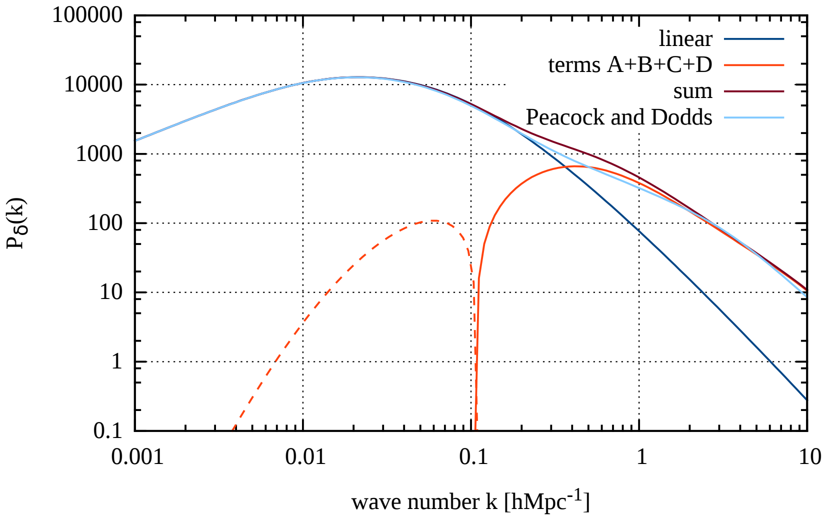

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Summary

1. Non-equilibrium statistical field theory for dark-matter particles set up
2. Hamiltonian equations of motion, simple Green's function
3. Expansion parameter is deviation from unperturbed (improved Zel'dovich) trajectories
4. *n*-point statistics for collective fields obtained by functional derivatives
5. First-order perturbation theory reproduces numerical results already quite well

Possible applications