Modified Gravity: Nonlinear Interactions for Massive Spin-2 Fields

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Motivation


25%
Dark Matter
70%
Dark Energy
5%
normal
matter


Λ?
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Philosophy

Approach I

1. invent a model to explain observations

   ➡ many possibilities

2. check if model is consistent, fits into larger framework, has motivations besides cosmology, etc.

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1. construct a consistent model guided by a fundamental question
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2. check if it can explain (part of) an observational phenomenon

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Consistent Field Theories

Standard Model of Particle Physics & General Relativity

Spin 0:	Higgs boson ϕ
Spin 1/2:	leptons, quarks ψ^a
Spin 1:	gluons, photon, W- & Z-boson A_μ
Spin 2:	graviton $g_{\mu\nu}$

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Well-known, works well

Maybe we could make changes here ?

Modifying Gravity

General Relativity (GR) = our theory for gravity since a century ago

Physical degrees of freedom: 2 helicity components of massless spin-2 field

Modifications of GR always introduce new degrees of freedom!
Most of the resulting theories are theoretically inconsistent.

Consistent examples: f(R), Horndeski, Galileon, ...

Ideal case: Reproduce GR in well-tested energy regimes
+ something else that solves outstanding problem(s)

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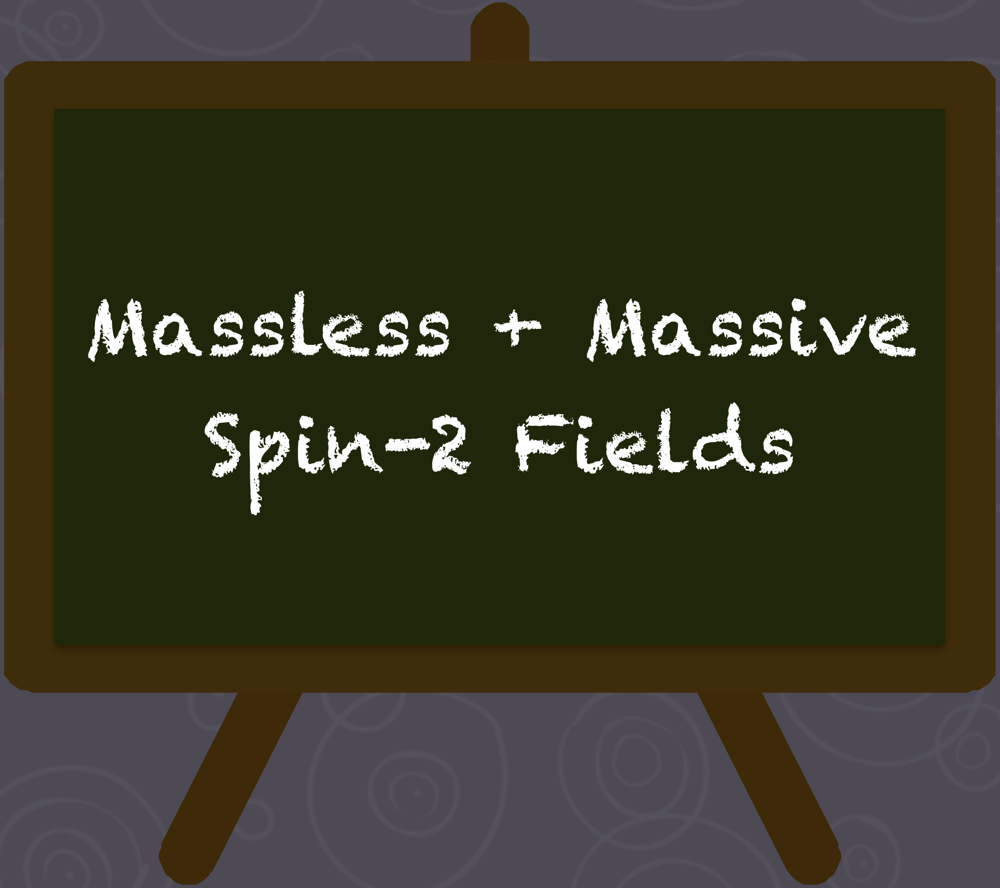
massless
& massive

MASSLESS !

How do we make a spin-2 field massive ?



Massless + Massive Spin-2 Fields

General Relativity

= classical nonlinear field theory
for metric tensor $g_{\mu\nu}$

- Einstein-Hilbert action: $S_{\mathrm{EH}}[g] = M_{\mathrm{P}}^2 \int \mathrm{d}^4x \sqrt{g} \left(R(g) - 2\Lambda \right)$

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0$

- Maximally symmetric solutions: $\bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu}$

➡ correspond to Einstein - de Sitter spacetimes for which notion of mass and spin exists (label representation of isometry group; for instance flat space with $\Lambda = 0$ and Poincaré isometry)

Massless Gravity

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Linear perturbations of Einstein's equations, $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$:

$$\bar{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma} \delta g_{\rho\sigma} = 0 \qquad \bar{\mathcal{E}} \sim \nabla\nabla + \Lambda$$

➡ equation for a massless spin-2 field with 2 degrees of freedom,

tensor analogue of $\Box \phi = 0$

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Hamiltonian analysis ➡ 2 d.o.f. also at the nonlinear level

General Relativity

=

nonlinear theory
of massless spin-2

Linear Massive Gravity

Fierz & Pauli (1939)

Equation for a massive spin-2 field:

$$\bar{\mathcal{E}}_{\mu\nu}^{\ \ \rho\sigma}\delta g_{\rho\sigma} + \frac{m_{\rm FP}^2}{2}\left(\delta g_{\mu\nu} - \mathbf{a}\,\bar{g}_{\mu\nu}\delta g\right) = 0$$

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- propagates 5 degrees of freedom for $\mathbf{a} = \mathbf{1}$
- for $\mathbf{a} \neq \mathbf{1}$ there is an additional scalar mode which gives rise to a ghost instability

Ghosts

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maybe you think of Faddeev-Popov ghosts,
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Modifications of General Relativity tend to be haunted by ghosts.
Modifying gravity is EXTREMELY difficult!

Dangerous ghosts

Ghost = field with negative kinetic energy

$$\mathcal{L} = (\partial_t\phi)^2 \cdots \quad \text{healthy}$$

$$\mathcal{L} = -(\partial_t\phi)^2 \cdots \quad \text{ghost}$$

- consequences: classical instability, negative probabilities at quantum level
  - ➡ must be avoided!
- explicit check for ghosts by computing the Hamiltonian

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- massive spin-2: six instead of five propagating degrees of freedom
  - ➡ need extra constraint to remove the ghost
- Fierz-Pauli theory: constraint arises only for $\mathbf{a} = 1$ in mass term

Fierz-Pauli theory is linear.
General Relativity is nonlinear.

Can we write down a
nonlinear mass term ?

General Structure

What would the nonlinear theory look like ?

Compare e.g. to massive vectors:

$$\mathcal{L}_{\text{Proca}} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\swarrow\ \text{kinetic term}} - \underbrace{\frac{m^2}{2}A^{\mu}A_{\mu}}_{\searrow\ \text{mass term}}$$

➡ $m^2 = 0$ corresponds to massless theory

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$\swarrow$ $\searrow$

kinetic term mass term

➡ $m^2 = 0$ corresponds to massless theory

➡ Nonlinear massive gravity
= General Relativity + non-derivative interactions for metric tensor

Nonlinear Mass Term

... should not contain derivatives nor loose indices.

But if we try to contract the indices of the metric, we get: $g^{\mu\nu} g_{\mu\nu} = 4$

This is not a mass term.

Simplest way out: Introduce second "metric" to contract indices:

$$g^{\mu\nu} f_{\mu\nu} = \mathrm{Tr}\left(g^{-1} f\right) \qquad f^{\mu\nu} g_{\mu\nu} = \mathrm{Tr}\left(f^{-1} g\right)$$

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➡ Massive gravity action is of the form

$$S_{\mathrm{MG}}[g] \;=\; S_{\mathrm{EH}}[g] - \int \mathrm{d}^4 x \, V(g, f)$$

kinetic term   mass term

Nonlinear Mass Term

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g^{μ}

What determines $f_{\mu\nu}$?

➡ Massive gravity

$$S_{\mathrm{MG}}[g] = \underbrace{S_{\mathrm{EH}}[g]}_{\text{kinetic term}} - \underbrace{\int \mathrm{d}^4x\, V(g,f)}_{\text{mass term}}$$

Fixed Reference Metric

Nonlinear massive gravity action:

$$S_{\mathrm{MG}}[g] \;=\; m_g^2 \int \mathrm{d}^4x\sqrt{g}\,\Big(R(g) - 2\Lambda\Big) \;-\; \int \mathrm{d}^4x\; V(g, f)$$

- $g_{\mu\nu}$ is the only dynamical variable
- $f_{\mu\nu}$ is fixed reference metric that needs to be put in by hand

Fixed Reference Metric

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- $g_{\mu\nu}$ is the only dynamical variable
- $f_{\mu\nu}$ is fixed reference metric that needs to be put in by hand

What if we let $f_{\mu\nu}$ be determined dynamically ?

Bimetric Theory

Nonlinear bimetric action:

$$\begin{aligned} S_{\mathrm{b}}[g,f] &= m_g^2 \int \mathrm{d}^4x \sqrt{g}\left(R(g) - 2\Lambda\right) \\ &+ m_f^2 \int \mathrm{d}^4x \sqrt{f}\left(R(f) - 2\tilde{\Lambda}\right) - \int \mathrm{d}^4x \, V(g,f) \end{aligned}$$

- both metrics are dynamical and treated on equal footing
- no need to put in anything by hand
- more natural from field theoretical point of view
- should describe massive & massless spin-2 field (5+2 d.o.f.)

This looks nice…

…but what about  ?

The Nonlinear Ghost

Can we extend the Fierz-Pauli mass term
by nonlinear interactions ?

$$\frac{m_{\mathrm{FP}}^2}{2}\left(\delta g_{\mu\nu} - \bar{g}_{\mu\nu}\delta g\right) + \mathbf{c_1}\delta g_\mu{}^\rho \delta g_{\rho\nu} + \mathbf{c_2}\delta g \delta g_{\mu\nu} + \ldots$$

Can we choose coefficients $\mathbf{c_i}$ such that the ghost remains absent ?

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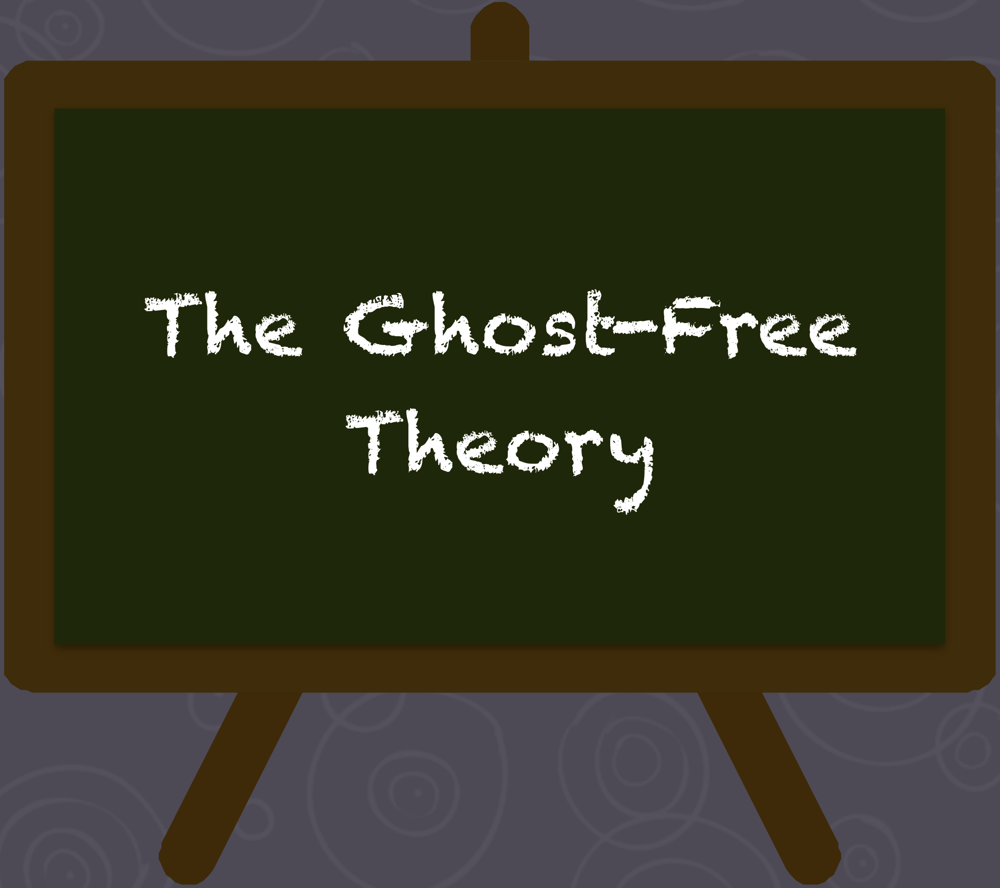
Can we choose coefficients $\mathbf{c_i}$ such that the ghost remains absent ?

Boulware & Deser (1972): Beyond linear order this is impossible!

No consistent nonlinear massive gravity / bimetric theory ?

> Massive gravity stinks.
> If you want to modify gravity,
> try something else...

Quote from lecture notes by Kurt Hinterbichler, 2010
(now turned into a very nice review!)



The Ghost-Free Theory

Development

Creminelli, Nicolis, Papucci, Trincherini (2005):
attempt to construct ghost-free candidate theory; fails only because of unfortunate sign mistake

de Rham, Gabadadze, Tolley (2010):
construction of candidate theory for massive gravity in flat reference frame; ghost-free in "decoupling limit"

Hassan, Rosen, ASM, von Strauss (2011/12):
proof of absence of ghost in fully nonlinear theory

Hassan & Rosen (2011):
generalisation to ghost-free bimetric theory

Interaction Potential

de Rham, Gabadadze, Tolley (2010);
Hassan & Rosen (2011)

$$V\left(g,f\right) = m^4\sqrt{g}\sum_{n=1}^{3}\beta_n\, e_n\left(\sqrt{g^{-1}f}\right)$$

- arbitrary spin-2 mass scale m
- 3 interaction parameters β_n
- elementary symmetric polynomials $e_n(S)$
- square-root matrix S defined through $S^2 = g^{-1}f$

Hassan & Rosen (2011);
Hassan, ASM, Rosen (2011);
Hassan, ASM, von Strauss (2012)

- free bimetric theory

Hassan & Rosen (2011)

$$S_\mathrm{b}[g,f] = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f)$$

$$\boxed{V(g,f) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n\left(\sqrt{g^{-1}f}\right) = m^4 \sqrt{f} \sum_{n=0}^{4} \beta_{4-n}\, e_n\left(\sqrt{f^{-1}g}\right)}$$

$$e_1(S) = \mathrm{Tr}[S] \qquad e_2(S) = \tfrac{1}{2}\left((\mathrm{Tr}[S])^2 - \mathrm{Tr}[S^2]\right)$$

$$e_3(S) = \tfrac{1}{6}\left((\mathrm{Tr}[S])^3 - 3\,\mathrm{Tr}[S^2]\mathrm{Tr}[S] + 2\,\mathrm{Tr}[S^3]\right)$$

What is the physical content of ghost-free bimetric theory ?

Proportional solutions

Hassan, ASM, von Strauss (2012)

$$\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu} \qquad \text{with} \quad c = \text{const.}$$

- gives two copies of Einstein's equations $(\alpha \equiv m_f/m_g)$:

$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_g(\alpha, \beta_n, c)\bar{g}_{\mu\nu} = 0$$
$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_f(\alpha, \beta_n, c)\bar{g}_{\mu\nu} = 0$$

- consistency condition: $\Lambda_g(\alpha, \beta_n, c) = \Lambda_f(\alpha, \beta_n, c)$ determines c

➡ Maximally symmetric backgrounds with $R_{\mu\nu}(\bar{g}) = \Lambda_g \bar{g}_{\mu\nu}$

Mass spectrum

Hassan, ASM, von Strauss (2012)

Perturbations around proportional backgrounds:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad f_{\mu\nu} = c^2 \bar{g}_{\mu\nu} + \delta f_{\mu\nu}$$

Can be diagonalised into mass eigenstates:

$\delta G_{\mu\nu} \propto \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}$ massless (2 d.o.f.)

$\delta M_{\mu\nu} \propto \delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}$ massive (5 d.o.f.)

Linearised equations:

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} \delta G_{\rho\sigma} = 0$$

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} \delta M_{\rho\sigma} + \frac{m_{\mathrm{FP}}^2}{2} \left(\delta M_{\mu\nu} - \bar{g}_{\mu\nu} \delta M \right) = 0$$

with Fierz-Pauli mass $m_{\mathrm{FP}} = m_{\mathrm{FP}}(\alpha, \beta_n, c)$

Ghost-free bimetric theory

=

nonlinear theory
of massless & massive spin-2

What is the physical metric ?

How does matter couple to the tensor fields ?

Matter coupling

Yamashita, de Felice, Tanaka;
de Rham, Heisenberg, Ribeiro (2015)

Only one metric can couple to matter!

$$
\begin{aligned}
S_{gf} = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \quad & + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) \\
& - m^4 \int \mathrm{d}^4x \sqrt{g} \sum_{n=0}^{4} \beta_n e_n \left(\sqrt{g^{-1} f}\right) \\
& + \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\mathrm{matter}}(g, \phi)
\end{aligned}
$$

- ➜ only coupling that does not re-introduce the ghost
- ➜ $g_{\mu\nu}$ is gravitational metric

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- $g_{\mu\nu}$ is gravitational metric

The gravitational metric is not massless !!

Physical Interpretation

Baccetti, Martin-Moruno, Visser (2012);
Hassan, ASM, von Strauss (2012/14);
Akrami, Hassan, Koennig, ASM, Solomon (2015)

Bimetric theory = General Relativity + corrections

Recall: $\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu}$

Assume that $\alpha = m_f / m_g$ is small (i.e. weak gravity!)

- the gravitational metric is almost massless
- the massive spin-2 field interacts only weakly with matter

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$\alpha \to 0$ is the General Relativity limit of bimetric theory

Ghost-free bimetric theory

=

General Relativity +
additional tensor field

Applications


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“Screening” does not work. But maybe extra symmetries?

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See Yashar’s talk this afternoon!

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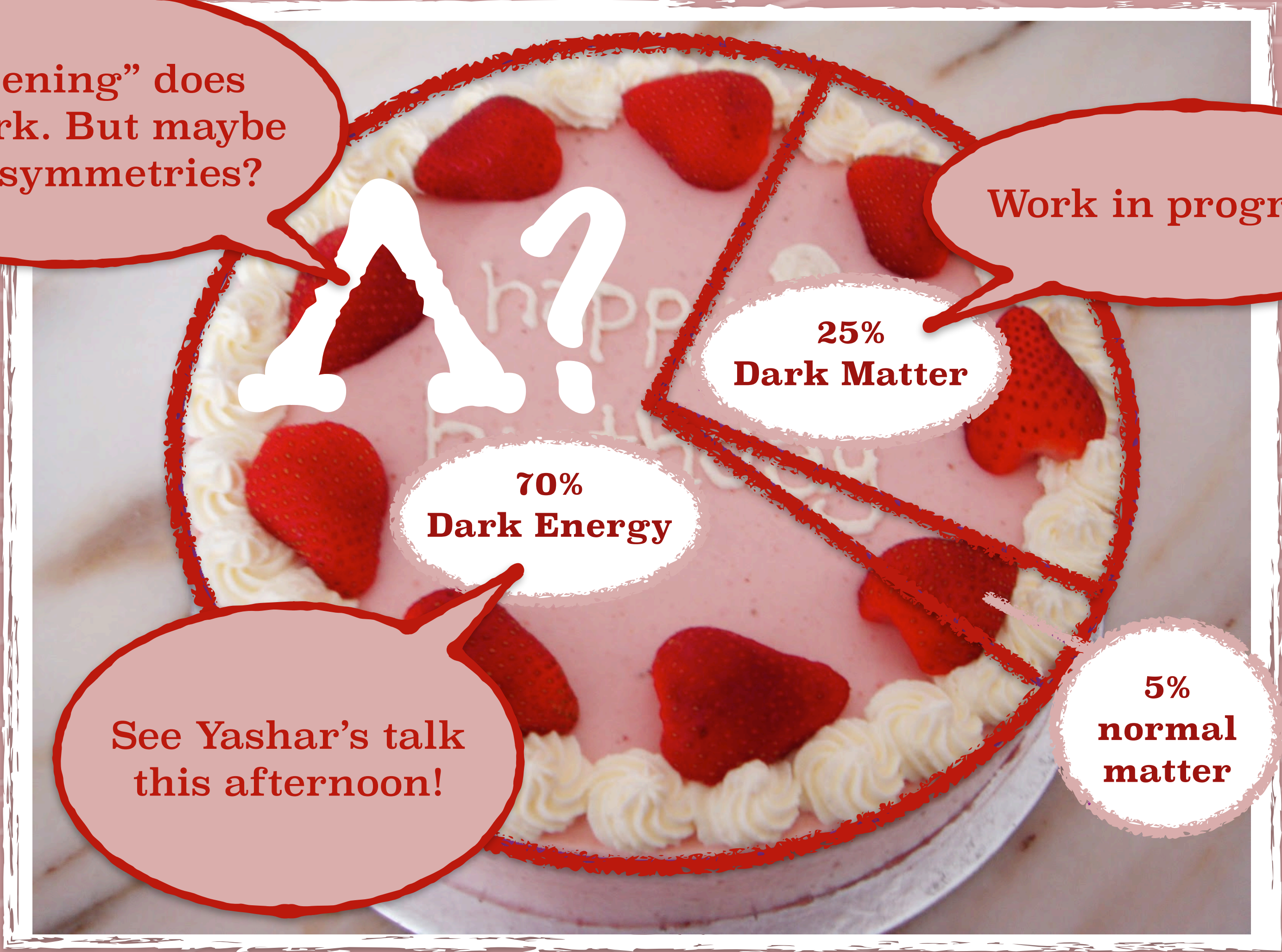

Work in progress…

See Yashar’s talk this afternoon!

Dark Matter

Hassan, ASM, von Strauss (2012/14)

Consider again the General Relativity limit of bimetric theory: $\alpha \to 0$

➡ massive spin-2 field decouples from matter, interacts only with gravity

At the same time, the gravitational interactions are weak and resemble General Relativity!

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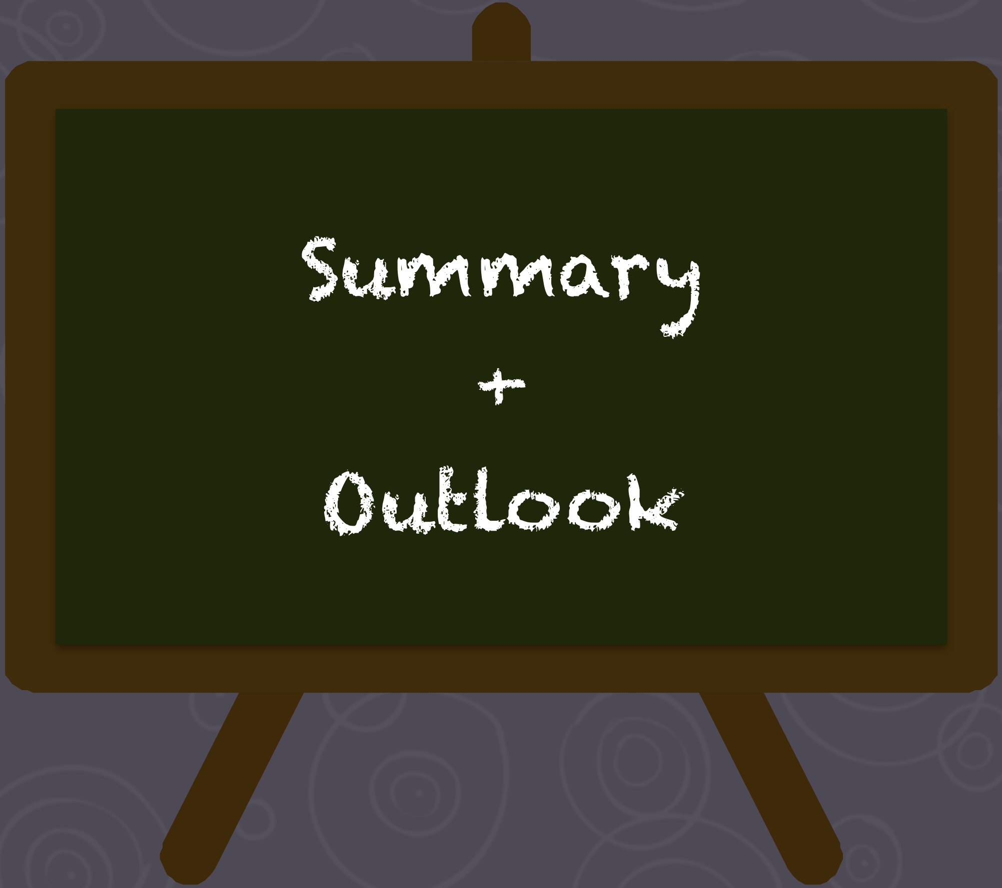
Other attempts (extra matter in $f_{\mu\nu}$ - sector): Blanchet & Heisenberg (2015)



Summary + Outlook

Ghost- free bimetric theory…

- is one of the few known consistent modifications of General Relativity
- describes nonlinear interactions of massless and massive spin-2 fields
- can be interpreted as gravity in the presence of an extra spin-2 field
- gives rise to a viable cosmology with self-accelerating backgrounds

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Open questions:

- dark matter ?   inflation ?
- extra symmetries ("partial masslessness") ?
- further extensions ?

Thank you for you attention!