Introduction to gauge-gravity duality and applications

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Corfu Summer School 2015

9 September 2015

Outline

I. Introduction and motivation (physics of heavy ion collisions)

II. Gauge-string (gauge-gravity) duality: origins and overview

III. Gauge-gravity duality (holography) at finite temperature and density

- holography beyond equilibrium
- holographic recipes for non-equilibrium physics
- quasinormal spectra

IV. The hydrodynamic regime

- hydrodynamics as an effective theory
- linear response
- transport properties and retarded correlation functions
--second-order hydrodynamics

Outline (continued)

V. Some applications
- transport at strong coupling
- universality of the viscosity-entropy ratio
- particle emission rates
- relation to RHIC-LHC and other experiments

Some references:

Old but useful review: O.Aharony, S.Gubser, J.Maldacena, H.Ooguri, Y.Oz, hep-th/9905111

J.Casalderrey-Solana, H.Liu, D.Mateos, K.Rajagopal, U.Wiedemann, 1101.0618 [hep-th]
Updated version is now a book published by Cambridge U.Press (2014)
"Gauge/String Duality, Hot QCD and Heavy Ion Collisions"

J.Erdmenger & M.Ammon, "Gauge/Gravity Duality: Foundations and Applications", CUP, 2015

M.Natsuume, "AdS/CFT Duality User Guide" (Lecture Notes in Physics), 2014

H.Nastase, "Introduction to the AdS/CFT correspondence", 2015

D.T.Son and A.O.S., "Viscosity, Black Holes, and Quantum Field Theory", 0704.0240 [hep-th]

P.K.Kovtun and A.O.S., "Quasinormal modes and holography", hep-th/0506184

I. Introduction and motivation (physics of heavy ion collisions)

In the last 15 years or so, holographic (gauge/gravity duality) methods were used to study strongly coupled gauge theories at finite temperature and density

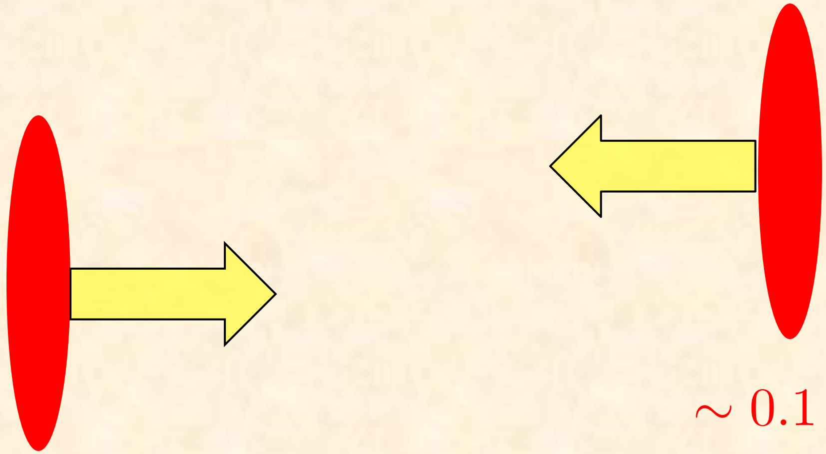
These studies were motivated by the heavy-ion collision programs at RHIC and LHC (ALICE, ATLAS) and the necessity to understand hot and dense nuclear matter in the regime of intermediate coupling $\alpha_s(T_{\mathrm{RHIC}}) \sim O(1)$

As a result, we now have a better understanding of thermodynamics and especially kinetics (transport) of strongly coupled gauge theories. Non-equlibrium behavior (thermalization) is under intense study.

Caution: these calculations are done for theoretical models such as N=4 SYM and its cousins (including non-conformal theories etc). We do not have a string dual description of QCD.

Energy density estimate

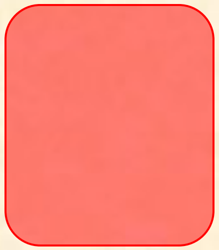
Before:



$\sim 0.1\, fm$

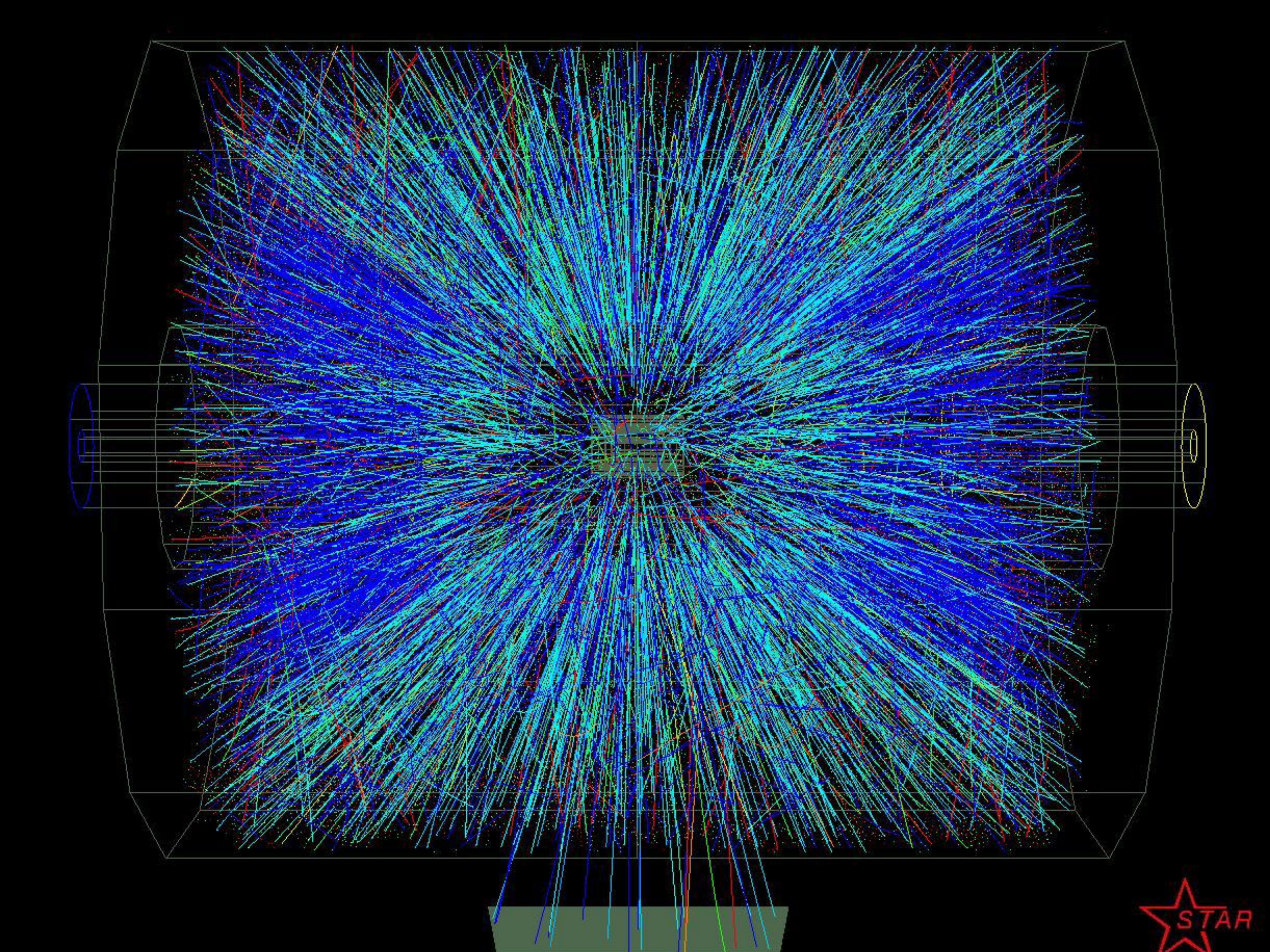
$\sim 0.1\, fm$

After:



$\sim 1\, fm$

$$\varepsilon \sim \frac{dE_T/d\eta \cdot \Delta\eta}{\tau_0\, c\, \pi R^2} \sim 5\, Gev/fm^3$$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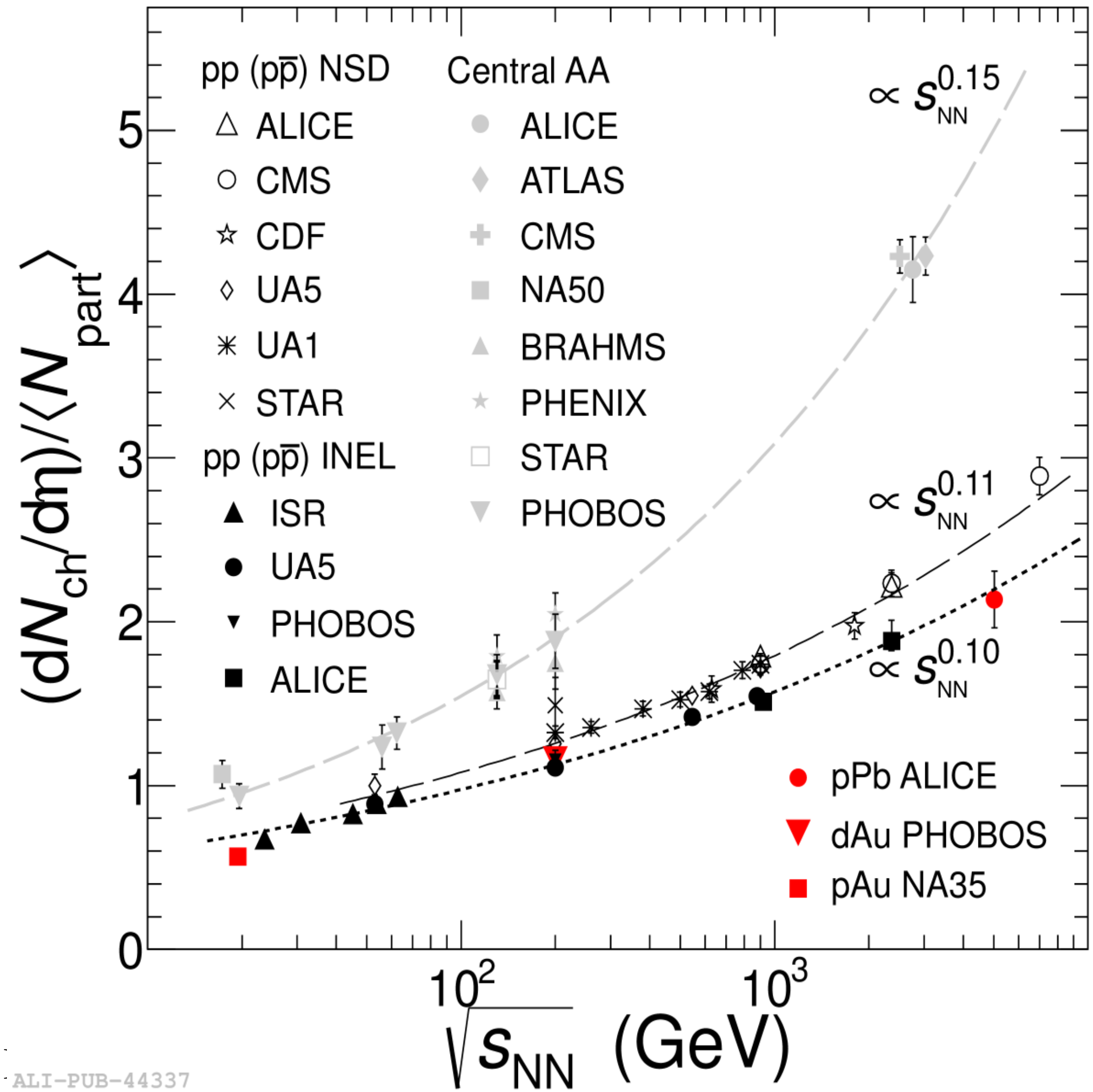
Charged partiple multiplicity in pp, pPb and central PbPb collisions



Heavy Ion Collisions

RHIC (2000-current): Au+Au $\sqrt{s_{NN}} = 0.2 \text{ TeV per nucleon}$

LHC (2010-current): Pb+Pb $\sqrt{s_{NN}} = 2.76 \text{ TeV per nucleon}$

The resulting system is:

* many-body: $N_{final} \sim 10^4$

* quantum: $\lambda_{DeBroglie} \sim l_{mfp}$

* at very high energy density $\epsilon \geq 5\ GeV/fm^3$

* in local TD equilibrium at $T \sim 10^{12}\text{ K}$

* strongly interacting: $\alpha_s(T_{\text{RHIC}}) \sim 1$

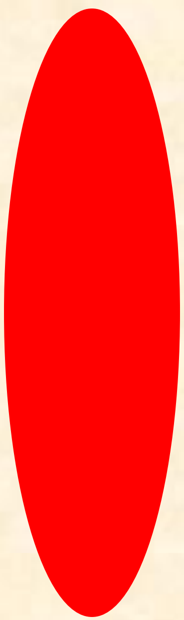
Note: LHC is only about 30% "hotter" than RHIC…

Heavy ion collision experiments at RHIC (2000-current) and LHC (2010-current) create hot and dense nuclear matter known as the “quark-gluon plasma”

(note: qualitative difference between p-p and Au-Au collisions)

Evolution of the plasma “fireball” is described by relativistic fluid dynamics (relativistic Navier-Stokes equations)

Need to know



thermodynamics (equation of state)
kinetics (first- and second-order transport coefficients)
in the regime of intermediate coupling strength:

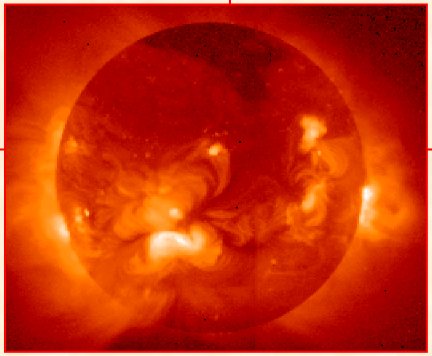
$$\alpha_s(T_{\mathrm{RHIC}}) \sim O(1)$$

initial conditions (initial energy density profile)

thermalization time (start of hydro evolution)

freeze-out conditions (end of hydro evolution)

Quantum field theories at finite temperature/density

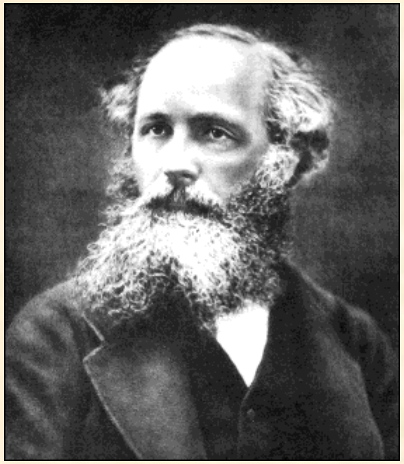


Equilibrium

entropy
equation of state
....

Near-equilibrium

transport coefficients
emission rates
....



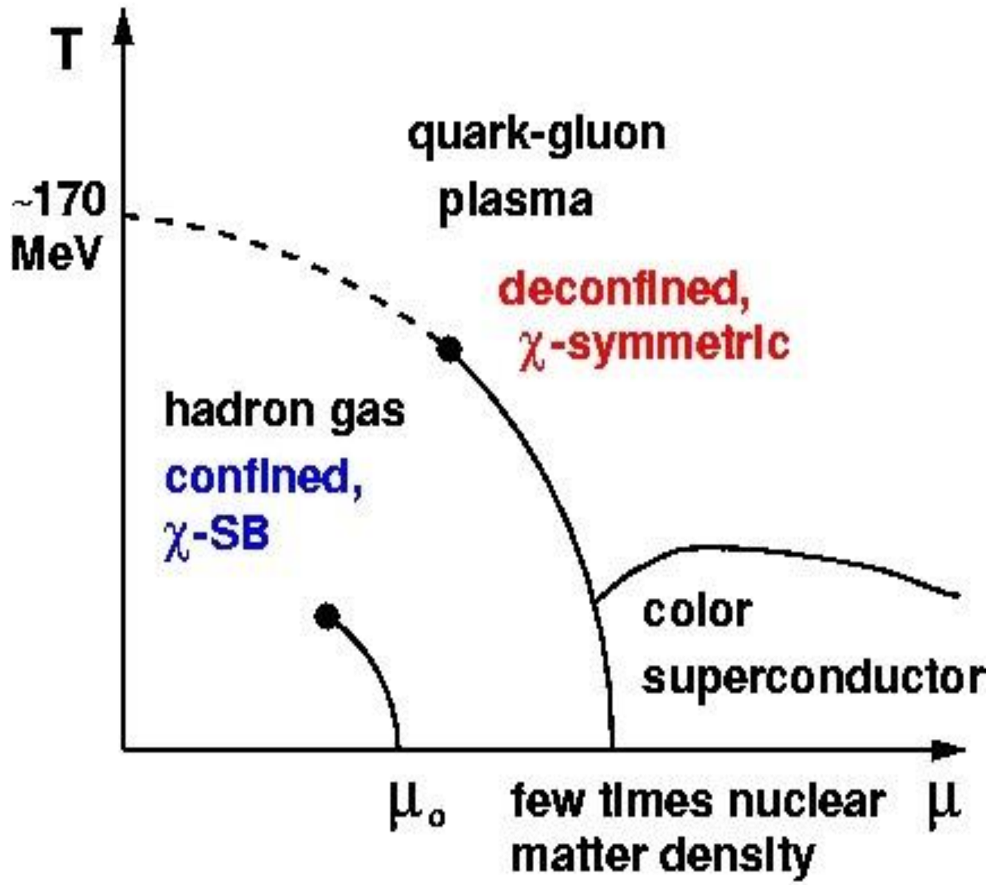
perturbative — pQCD

non-perturbative — Lattice

perturbative — kinetic theory

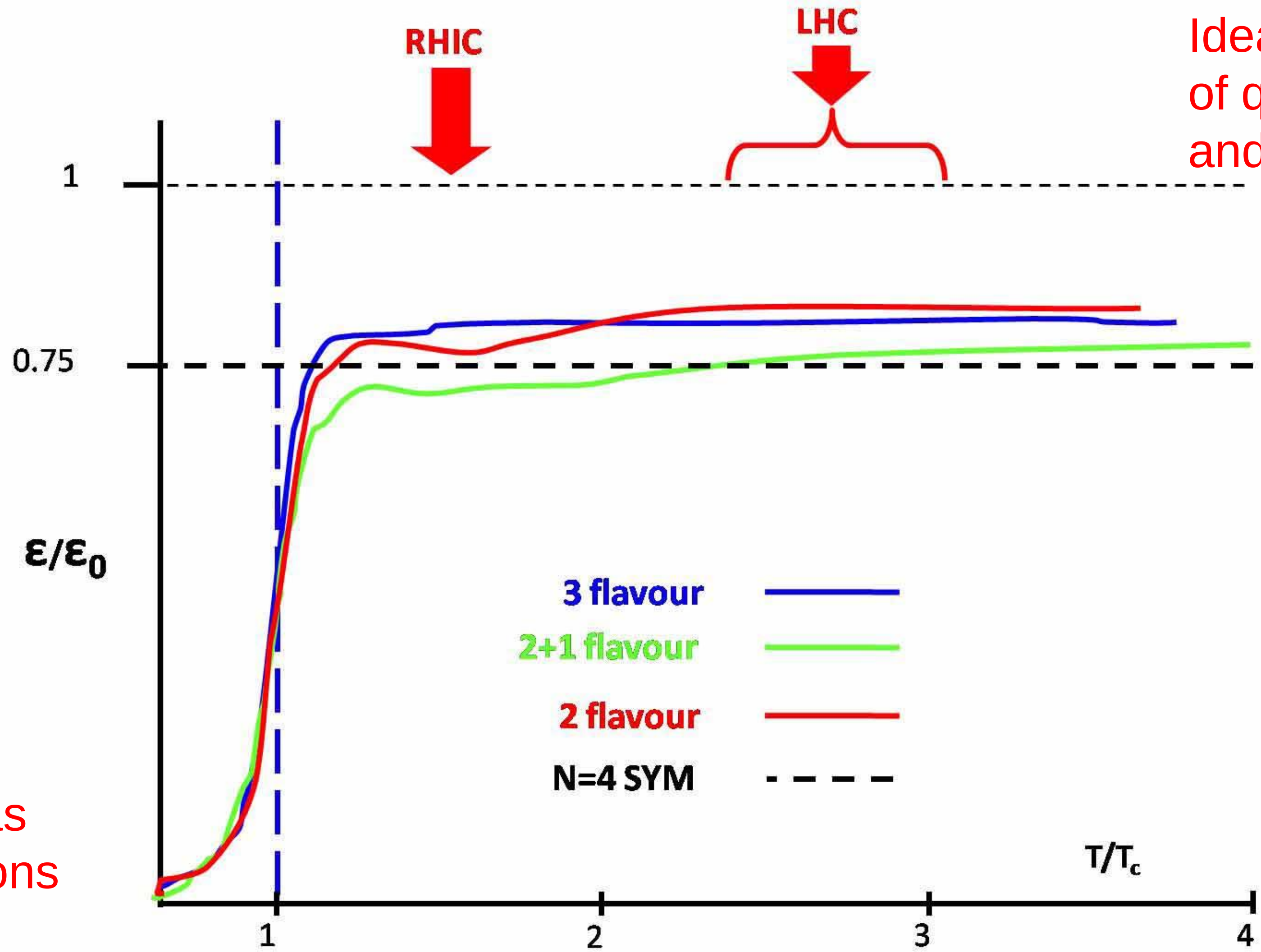
non-perturbative — ????

QCD phase diagram



Energy density vs temperature for various gauge theories

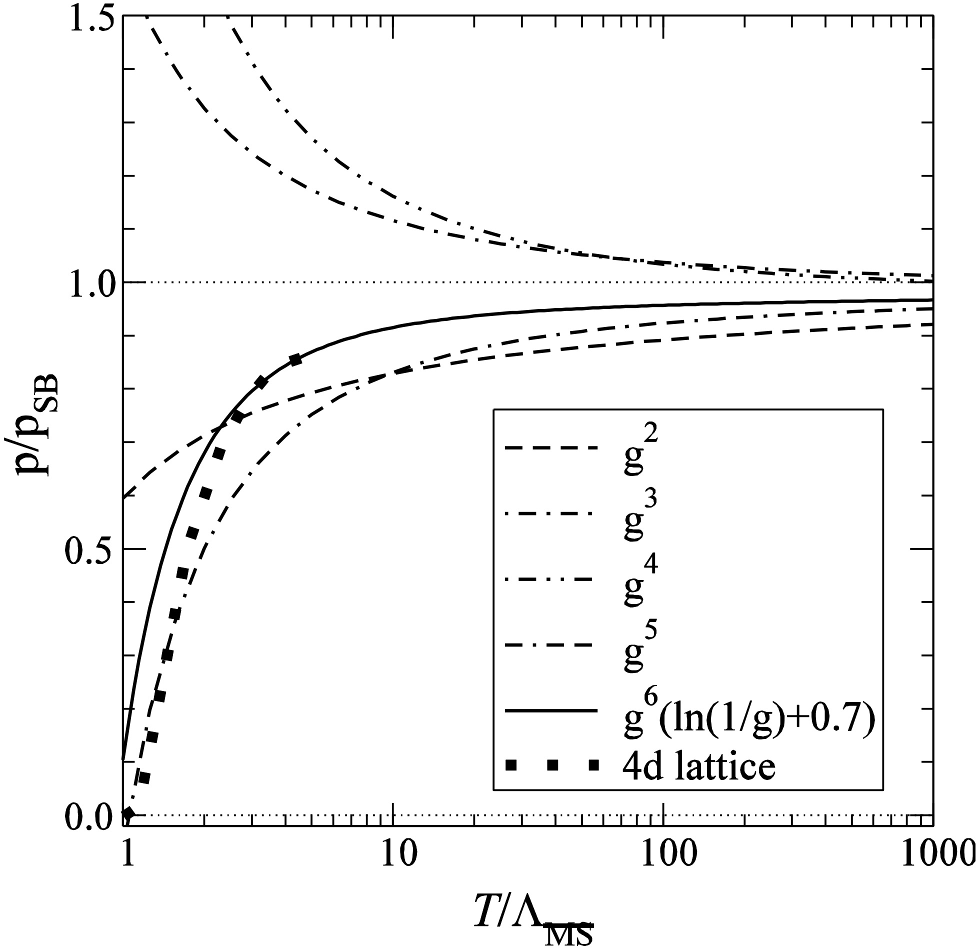
Ideal gas of quarks and gluons



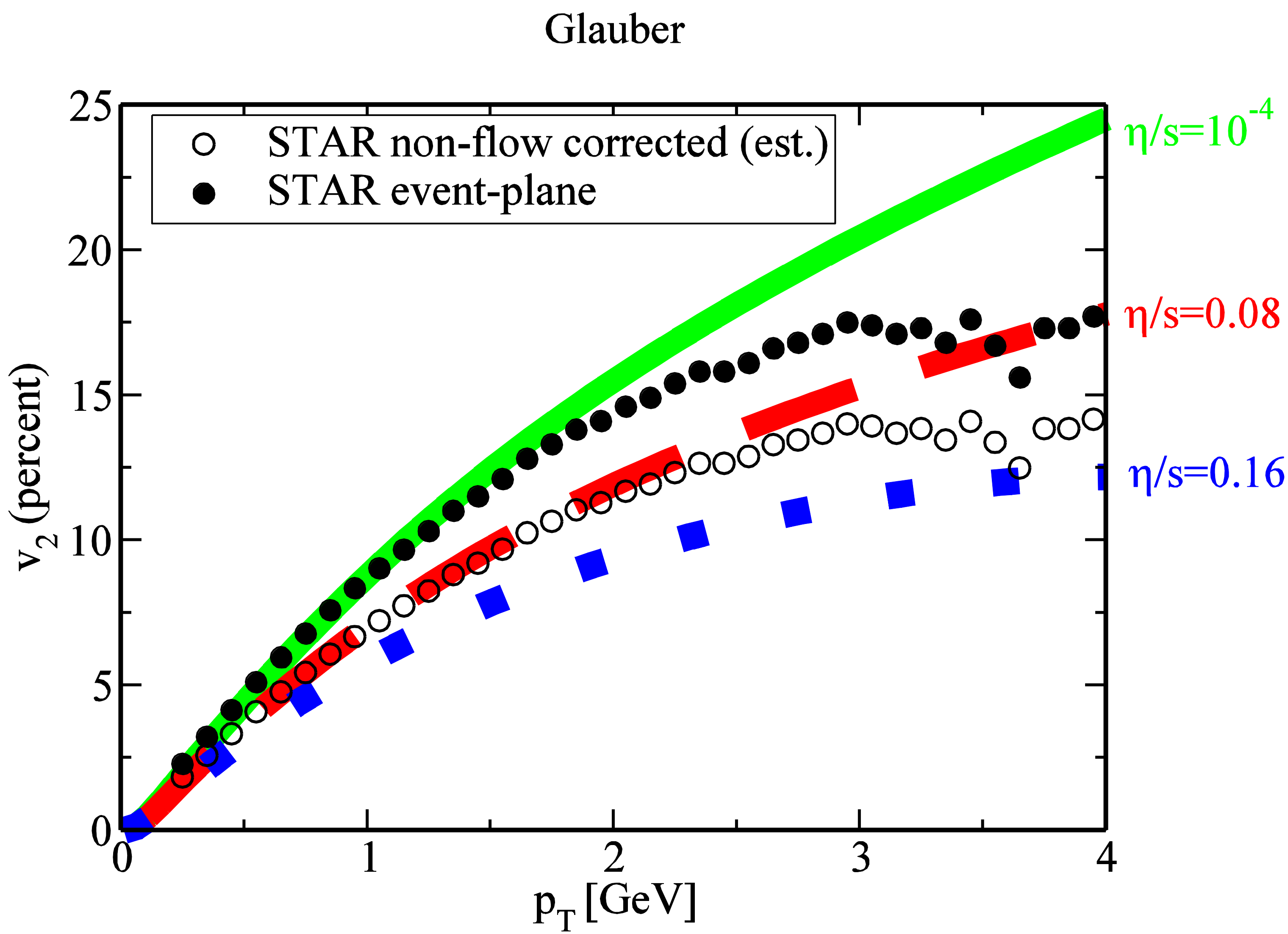
Ideal gas of hadrons

Figure: an artistic impression from Myers and Vazquez, 0804.2423 [hep-th]

Pressure in perturbative QCD



Elliptic flow with Glauber initial conditions

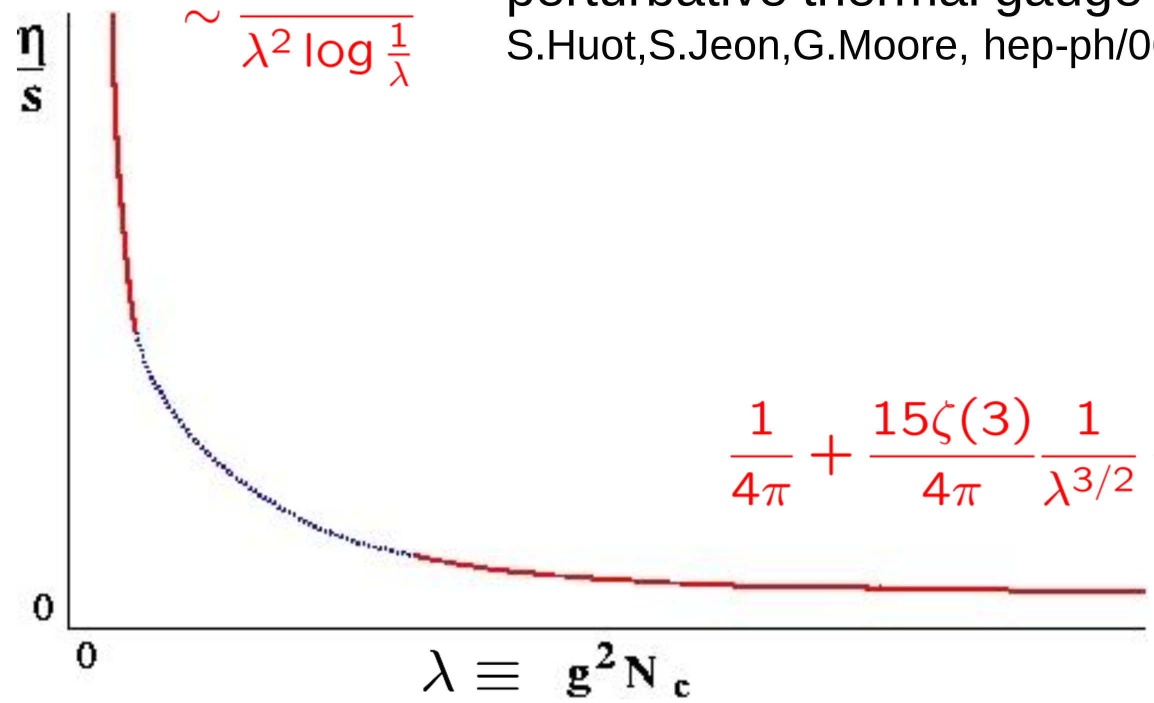


Luzum and Romatschke, 0804.4015 [nuc-th]

Shear viscosity in $\mathcal{N} = 4$ SYM

$$\sim \frac{1}{\lambda^2 \log \frac{1}{\lambda}}$$

perturbative thermal gauge theory
S.Huot,S.Jeon,G.Moore, hep-ph/0608062



$$\frac{1}{4\pi} + \frac{15\zeta(3)}{4\pi}\frac{1}{\lambda^{3/2}} + \cdots$$

Correction to $1/4\pi$: Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

II. Gauge-string (gauge-gravity) duality: Origins and overview

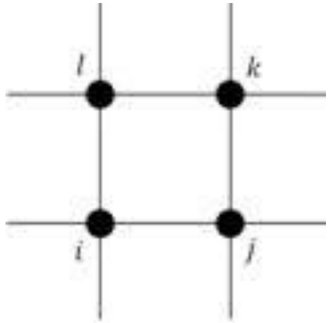
Duality can be viewed as a change of variables in the path integral

Ordinary integrals can often be simplified by a clever change of (dummy) variables

Similarly, path integrals can be simplified by choosing the "right" set of fields (d.o.f.)
(for a given corner of the parameter space)

Example: Kramers-Wannier duality (1941)

$$Z[J] = \sum_{s_j=\pm 1} \exp\left(\sum_{<ij>} J s_i s_j\right)$$



Ising model in d=2

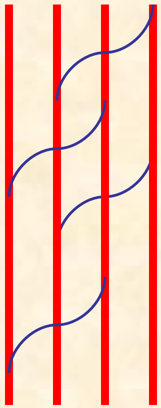
$$s_i \to \sigma_i$$

$$Z[J] = 2^N (\cosh J)^{2N}\, Z[\tilde{J}]$$

$$\sinh 2J \sinh 2\tilde{J} = 1$$

Note: can be viewed as Fourier transform on (Abelian) groups

From brane dynamics to AdS/CFT correspondence



$\Longleftrightarrow$

Open strings picture:
dynamics of N_c coincident D3 branes
at low energy is described by

Closed strings picture:
dynamics of N_c coincident D3 branes
at low energy is described by

$\mathcal{N} = 4$ supersymmetric
$SU(N_c)$ YM theory in 4 dim

$\Longleftrightarrow$

conjectured
exact equivalence

type IIB superstring theory
on $AdS_5 \times S^5$backgrond

Maldacena (1997); Gubser, Klebanov, Polyakov (1998); Witten (1998)

$\mathcal{N} = 4$ supersymmetric YM theory

Gliozzi,Scherk,Olive'77
Brink,Schwarz,Scherk'77

- Field content:

$$A_\mu \qquad \Phi_I \qquad \Psi^A_\alpha \qquad \text{all in the adjoint of } SU(N)$$

$$I = 1 \ldots 6 \qquad A = 1 \ldots 4$$

- Action:

$$S = \frac{1}{g^2_{YM}} \int d^4x \ \mathrm{tr} \left\{ \frac{1}{2} F^2_{\mu\nu} + (D_\mu \Phi_I)^2 - \frac{1}{2} [\Phi_I, \Phi_J]^2 + i\bar{\Psi}\Gamma^\mu D_\mu \Psi - \bar{\Psi}\Gamma^I [\Phi_I, \Psi] \right\}$$

(super)conformal field theory = coupling doesn't run

AdS/CFT correspondence

$\mathcal{N}=4$ supersymmetric $SU(N_c)$ YM theory in 4 dim $\Longleftrightarrow$ type IIB superstring theory on $AdS_5 \times S^5$ backgrond

conjectured
exact equivalence

Latest test: Janik'08

$$Z_{\mathrm{SYM}}[J] = \langle e^{-\int J\,\mathcal{O}\,d^4x} \rangle_{\mathrm{SYM}} = Z_{\mathrm{string}}[J]$$

Generating functional for correlation functions of gauge-invariant operators $\langle \mathcal{O}\ \mathcal{O}\ \cdots \mathcal{O} \rangle$ $\Longleftrightarrow$ String partition function

In particular

$$Z_{\mathrm{SYM}}[J] = Z_{\mathrm{string}}[J] \simeq e^{-S_{\mathrm{grav}}[J]}$$

$\lambda \equiv g_{YM}^2\, N_c \gg 1$

$N_c \gg 1$

Classical gravity action serves as a generating functional for the gauge theory correlators

AdS/CFT correspondence: the role of J

$$Z_{\mathsf{SYM}}[J] = \langle e^{-\int J\,\mathcal{O}\,d^4x} \rangle_{\mathsf{SYM}} \simeq e^{-S_{\mathsf{grav}}[J]}$$

For a given operator $\mathcal{O}$, identify the source field J, e.g. $T^{\mu\nu} \Longleftrightarrow h_{\mu\nu}$

$$e^{-S_{\mathsf{grav},M}[\phi_{\mathsf{BG}}+\delta\phi]} = Z[J = \delta\phi\big|_{\partial M}]$$

$\delta\phi$ satisfies linearized supergravity e.o.m. with b.c. $\delta\phi \to \delta\phi_0 \equiv J$

The recipe:

To compute correlators of $\mathcal{O}$, one needs to solve the bulk supergravity e.o.m. for $\delta\phi$ and compute the on-shell action as a functional of the b.c. $\delta\phi_0 \equiv J$

Warning: e.o.m. for different bulk fields may be coupled: need self-consistent solution

Then, taking functional derivatives of $e^{-S_{\mathsf{grav}}[J]}$ gives $\langle \mathcal{O}\ \mathcal{O} \rangle$

Holography at finite temperature and density

$$\langle \mathcal{O} \rangle = \frac{\mathrm{tr}\rho\mathcal{O}}{\mathrm{tr}\rho} \qquad\qquad H \to T^{00} \to T^{\mu\nu} \to h_{\mu\nu}$$

$$\rho = e^{-\beta H + \mu Q} \qquad\qquad Q \to J^{0} \to J^{\mu} \to A_{\mu}$$

Nonzero expectation values of energy and charge density translate into nontrivial background values of the metric (above extremality)=horizon and electric potential = CHARGED BLACK HOLE (with flat horizon)

$$ds^2 = -F(u)\,dt^2 + G(u)\left(dx^2 + dy^2 + dz^2\right) + H(u)\,du^2$$

$T = T_H$ temperature of the dual gauge theory

$$A_0 = P(u)$$

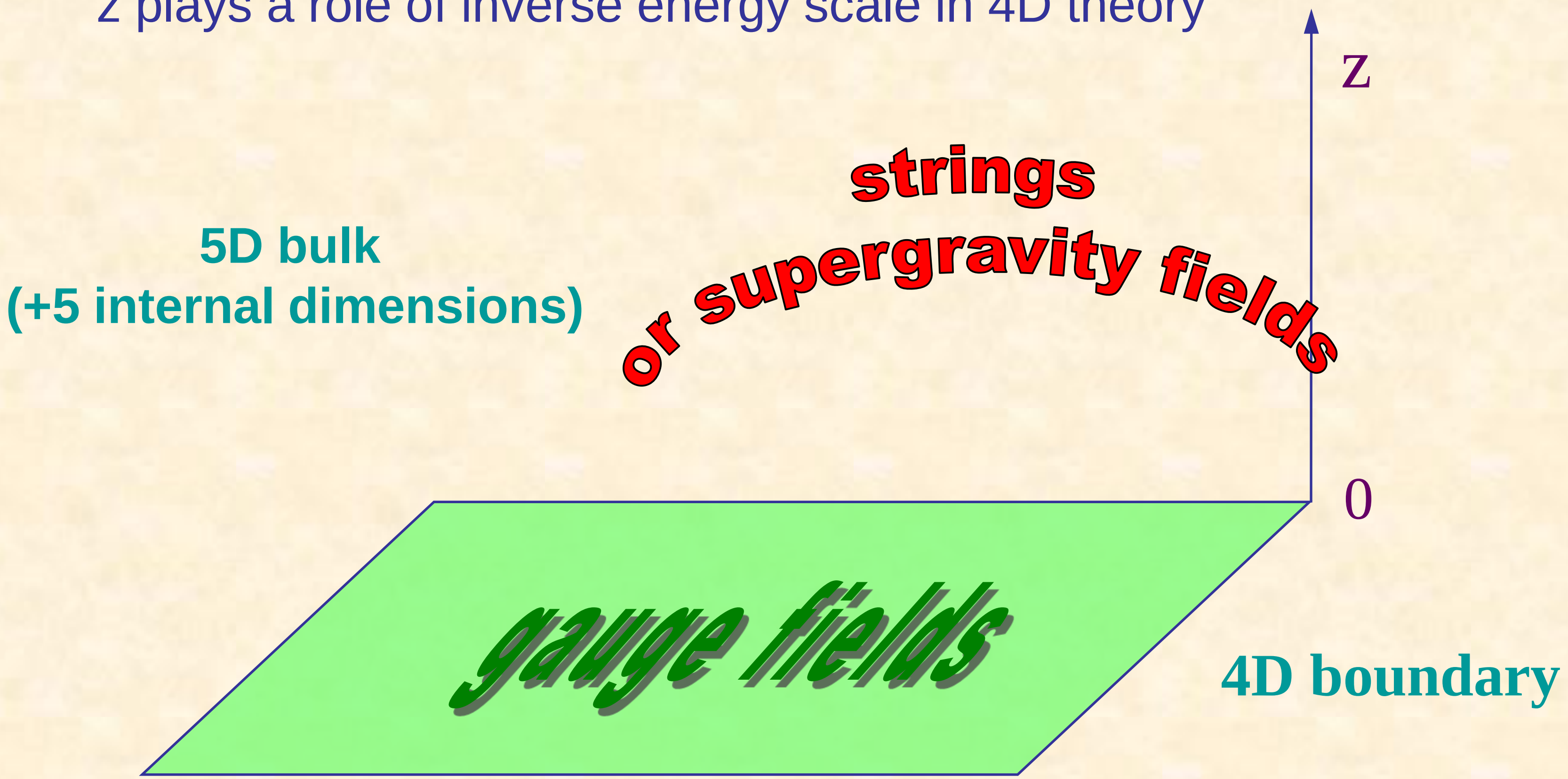
$\mu = P(\text{boundary}) - P(\text{horizon})$ chemical potential of the dual theory

The bulk and the boundary in AdS/CFT correspondence

$$ds^2 = \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu + dz^2}{z^2}$$

UV/IR: the AdS metric is invariant under $z \to \Lambda\, z \quad x \to \Lambda\, x$

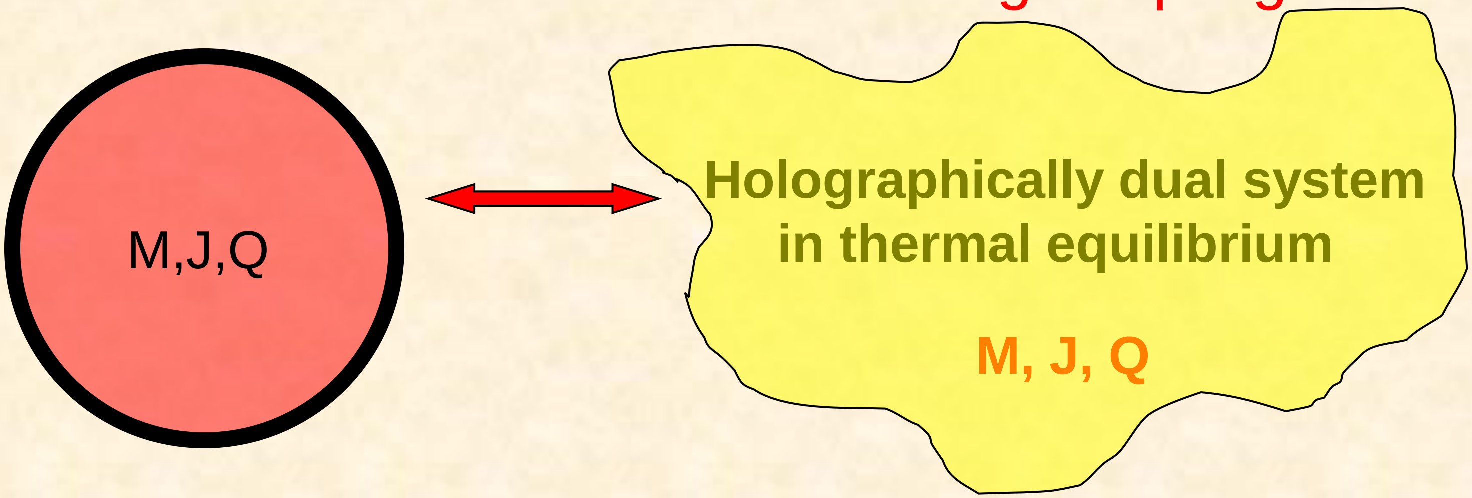
z plays a role of inverse energy scale in 4D theory



III. Gauge-gravity duality at finite temperature and density

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational+electromag fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g^{(0)}_{\mu\nu} + h_{\mu\nu}$ $A^0_\mu + a_\mu$ $\Longleftrightarrow$????

$$j_i = -D\partial_i j^0 + \cdots$$

$"\Box" \; h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$

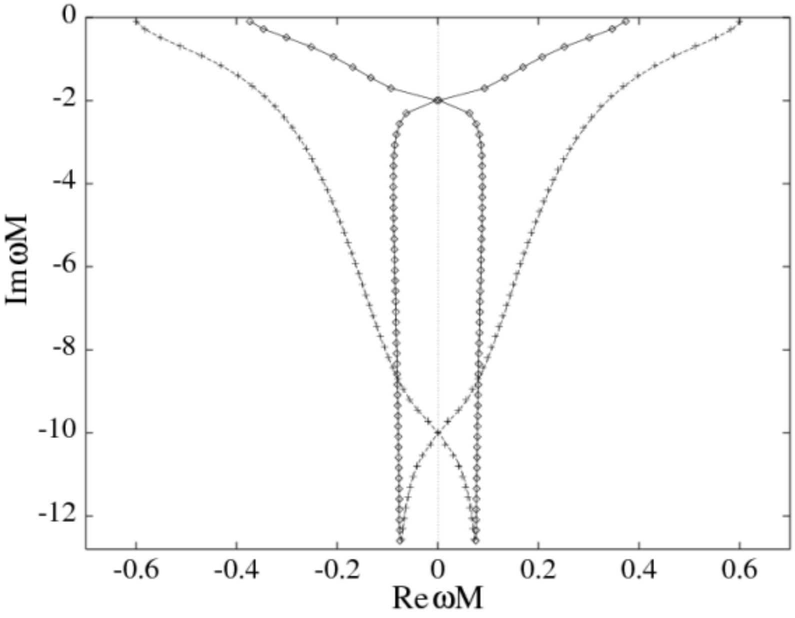
$$\partial_t j^0 + \partial_i j^i = 0$$

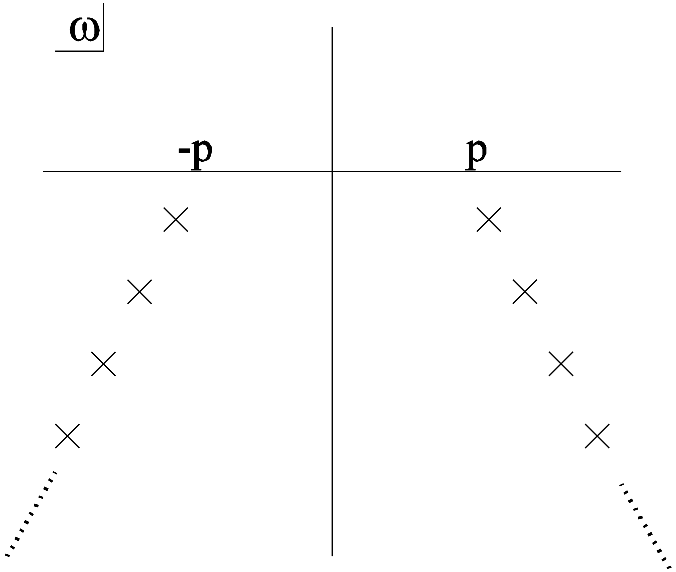
$$\partial_t j^0 = D\nabla^2 j^0$$

Quasinormal spectrum $\Longleftrightarrow$

$$\omega = -iDq^2 + \cdots$$

Quasinormal spectra of black holes/branes





Schwarzschild black hole
(asymptotically flat)

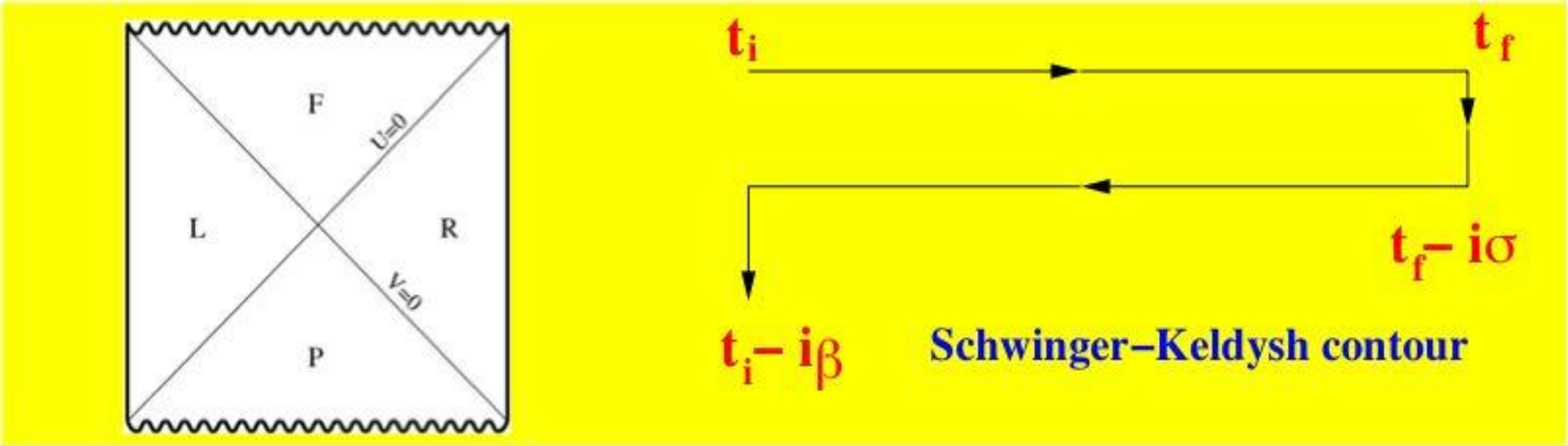
AdS-Schwarzschild black brane

Computing real-time correlation functions from gravity

To extract transport coefficients and spectral functions from dual gravity, we need a recipe for computing Minkowski space correlators in AdS/CFT

The recipe of [D.T.Son & A.S., 2001] and [C.Herzog & D.T.Son, 2002] relates real-time correlators in field theory to Penrose diagram of black hole in dual gravity

Quasinormal spectrum of dual gravity = poles of the retarded correlators in 4d theory
[D.T.Son & A.S., 2001]



The role of quasinormal modes

G.T.Horowitz and V.E.Hubeny, hep-th/9909056

D.Birmingham, I.Sachs, S.N.Solodukhin, hep-th/0112055

D.T.Son and A.O.S., hep-th/0205052; P.K.Kovtun and A.O.S., hep-th/0506184

$$S^{(2)} \sim \lim_{z=\epsilon\to 0} \int d\omega\, d^p q\, F(\omega, q)\, \Phi'(z)\Phi(z) + \text{contact terms}$$

$$\Phi(z) = \mathcal{A}\,\varphi_1(z) + \mathcal{B}\,\varphi_2(z)$$

$$\Phi(z) = \mathcal{A}\, z^{\Delta_-}\,(1+\cdots) + \mathcal{B}\, z^{\Delta_+}\,(1+\cdots) \quad \text{for} \quad z\to 0$$

I. Computing the retarded correlator: inc.wave b.c. at the horizon, normalized to 1 at the boundary

$$G^R \sim \frac{\mathcal{B}}{\mathcal{A}} + \text{contact terms}$$

II. Computing quasinormal spectrum: inc.wave b.c. at the horizon, Dirichlet at the boundary

$$\mathcal{A}(\omega, q) = 0$$

Example: R-current correlator in $4d\ \mathcal{N}=4$ SYM

in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Zero temperature: $< J_i(x)\, J_i(y) > \sim \dfrac{N_c^2}{|x-y|^6}$

$$G_E(k) = \frac{N_c^2 k_E^2}{32\pi^2} \ln k_E^2$$

$$G^{\mathrm{ret}}(k) = \frac{N_c^2 k^2}{32\pi^2} \left(\ln |k^2| - i\,\pi\theta\,(-k^2)\ \mathrm{sgn}\,\omega \right) \qquad k^2 = -\omega^2 + q^2$$

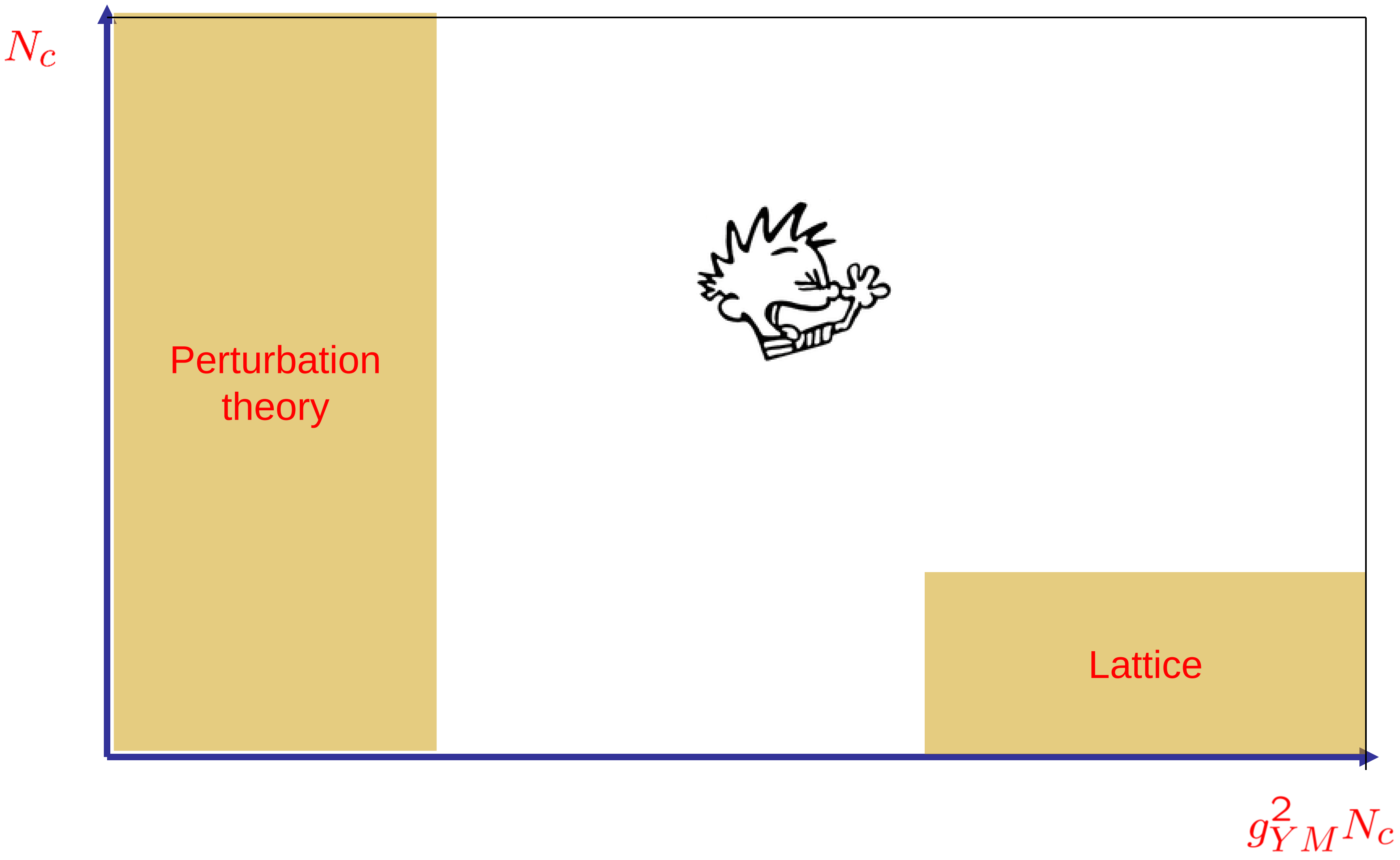
Finite temperature: $G^{\mathrm{ret}}(\omega, q)$

$$G^{\mathrm{ret}}(\omega, 0) = \frac{N_c^2 T^2}{8} \left\{ \frac{i\omega}{2\pi T} + \frac{\omega^2}{4\pi^2 T^2} \left[\psi\left(\frac{(1-i)\omega}{4\pi T} \right) + \psi\left(-\frac{(1+i)\omega}{4\pi T} \right) \right] \right\}$$

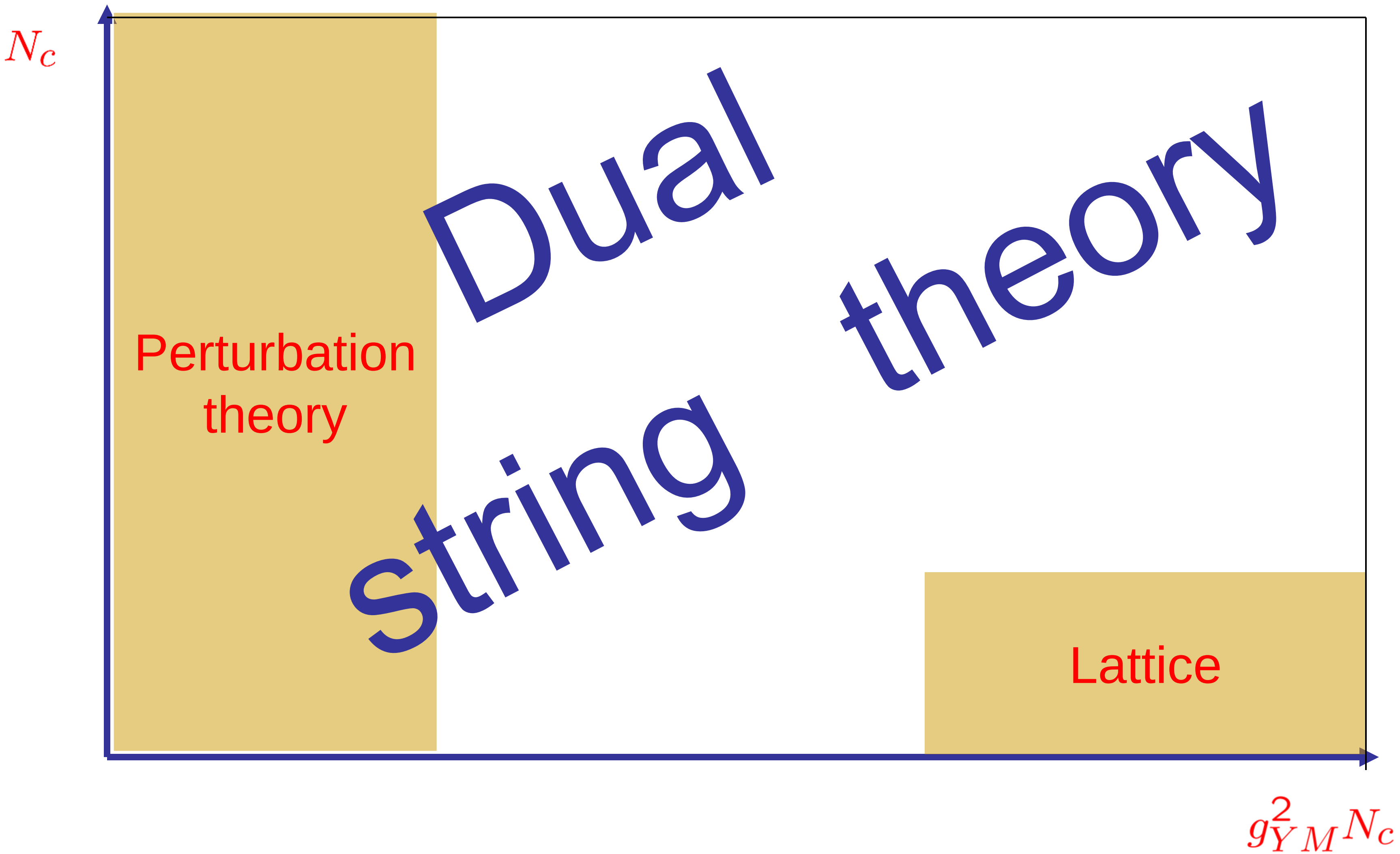
Poles of G^{ret} = quasinormal spectrum of dual gravity background

(D.Son, A.S., hep-th/0205051, P.Kovtun, A.S., hep-th/0506184)

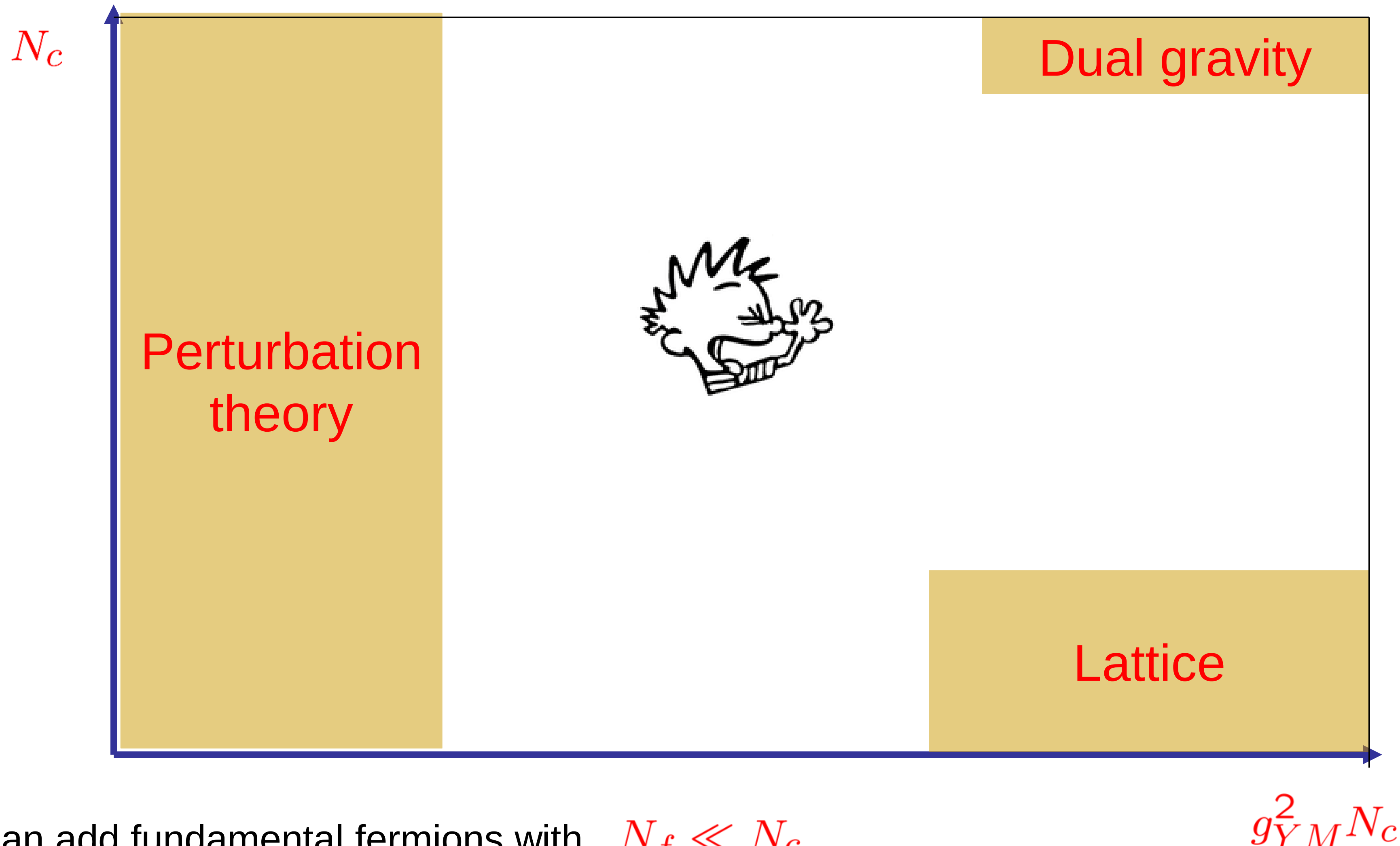
Our understanding of gauge theories is limited…



Conjecture:
specific gauge theory in 4 dim =
specific string theory in 10 dim



In practice: gravity (low energy limit of string theory) in 10 dim = 4-dim gauge theory in a region of a parameter space



Can add fundamental fermions with $N_f \ll N_c$

IV. The hydrodynamic regime

L.D.Landau and E.M.Lifshitz, *Fluid Mechanics*, Pergamon Press, Oxford, 1987

D.Forster, *Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions*, Benjamin/Cummings, New York, 1975

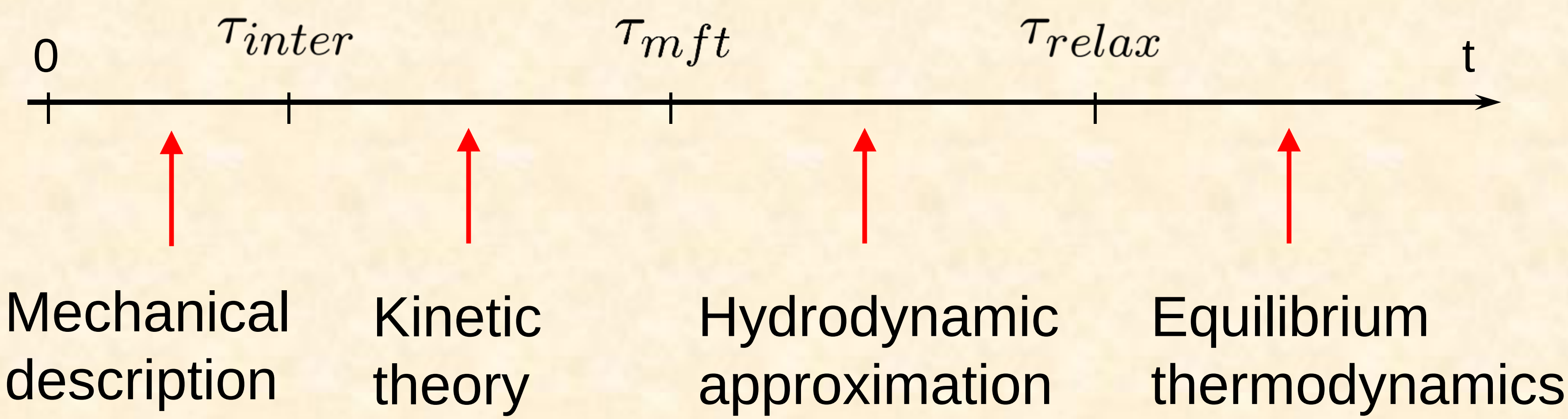
P.K. Kovtun, “Lectures on hydrodynamic fluctuations in relativistic theories”, 1205.5040 [hep-th].

P.K. Kovtun and L.G.Yaffe, “Hydrodynamic fluctuations, long-time tails, and supersymmetry”, hep-th/0303010.

The hydrodynamic regime

Hierarchy of times (e.g. in Bogolyubov's kinetic theory)

$$\tau_{mft} \ll t \ll T_{exist}$$



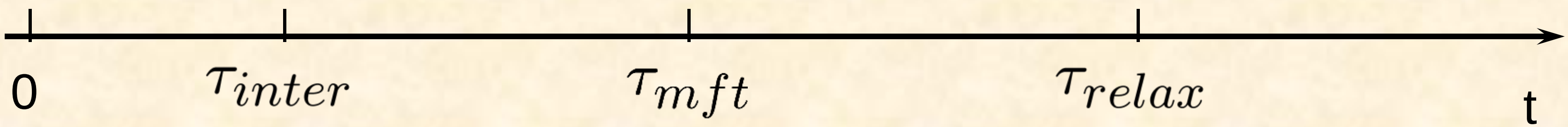
Hierarchy of scales

$$l_{mfp} \ll l \ll L$$

(L is a macroscopic size of a system)

The hydrodynamic regime (continued)

Degrees of freedom



Mechanical description

Coordinates, momenta of individual particles

Kinetic theory

Coordinate- and time-dependent distribution functions

Hydrodynamic approximation

Local densities of conserved charges

$\epsilon(x,t)$

Equilibrium thermodynamics

Globally conserved charges

E

Hydro regime: $\boxed{\tau_{\text{micro}} \ll \tau \ll t_{\text{global}}}$ $\boxed{l_{\text{micro}} \ll l \ll L_{\text{global}}}$

Hydrodynamics: fundamental d.o.f. = densities of conserved charges

Need to add constitutive relations!

Example: charge diffusion

Conservation law

$$\partial_t\, j^0 + \partial_i\, j^i = 0$$

Constitutive relation

[Fick's law (1855)]

$$j_i = -D\,\partial_i\, j^0 + \cancel{O[(\nabla j^0)^2, \nabla^2 j^0]}$$

Diffusion equation

$$\partial_t j^0 = D\nabla^2 j^0$$

Dispersion relation

$$\omega = -i\,D\,q^2 + \cdots$$

Expansion parameters: $\omega \ll T\,, \quad q \ll T$

Fluid dynamics is an effective theory valid in the long-wavelength, long-time limit

Fundamental degrees of freedom = densities of conserved charges

Equations of motion = conservation laws + constitutive relations^*

Example I

$$\left.\begin{aligned} &\partial_a J^a = 0 \\ &J^i = -D\,\nabla^i\,J^0 + \cdots \end{aligned}\right\} \quad \partial_t J^0 = D\,\nabla^2 J^0 + \cdots$$

Example II

$$\left.\begin{aligned} &\partial_a T^{ab} = 0 \\ &T^{ab} = \varepsilon u^a u^b + P(\varepsilon)\left(g^{ab} + u^a u^b\right) + \Pi^{ab} + \cdots \end{aligned}\right\}$$

Navier-Stokes eqs
Burnett eqs
…..

* Modulo assumptions e.g. analyticity

** E.o.m. universal, transport coefficients depend on underlying microscopic theory

Example: momentum diffusion and sound

Thermodynamic equilibrium: $\langle T^{00}\rangle = \epsilon\,,\ \langle T^{0i}\rangle = 0$

$$T^{ij} = P(\epsilon)\delta^{ij}$$

Near-equilibrium: $T^{00} = \epsilon + \tilde{T}^{00}$

$$T^{ij} = P\delta^{ij} + \overbrace{\frac{\partial P}{\partial \epsilon}}^{v_s^2}\tilde{T}^{00} + \tilde{T}^{ij}$$

$$\tilde{T}^{ij} = -\frac{1}{\epsilon + P}\left[\eta\left(\partial_i\tilde{T}^{0j} + \partial_j\tilde{T}^{0i} - \frac{2}{3}\delta^{ij}\partial_k\tilde{T}^{0k}\right) + \zeta\delta^{ij}\partial_k\tilde{T}^{0k}\right] + \cdots$$

Eigenmodes of the system of equations $\partial_\mu T^{\mu\nu} = 0$

Shear mode (transverse fluctuations of $\tilde{T}^{0i}$): $\omega = -\frac{i\eta}{\epsilon + P}q^2$

Sound mode: $\omega = v_s q - \frac{i}{2}\frac{1}{\epsilon + P}\left(\zeta + \frac{4}{3}\eta\right)q^2$

For CFT we have $\zeta = 0$ and $\epsilon = 3P$ $\Rightarrow$ $v_s = 1/\sqrt{3}$

Transport problems in “real life”

Thermal conductivity (transport of heat), electrical conductivity (transport of charge), diffusion (transport of charge/matter), viscosity (transport of momentum)

Transport problems in “real life”: viscosity

Viscosity of water: 1 centipoise (cP)

Viscosity of honey: 2000-10000 cP

1 Poise = 1 Pa s = 1 kg/(m s)

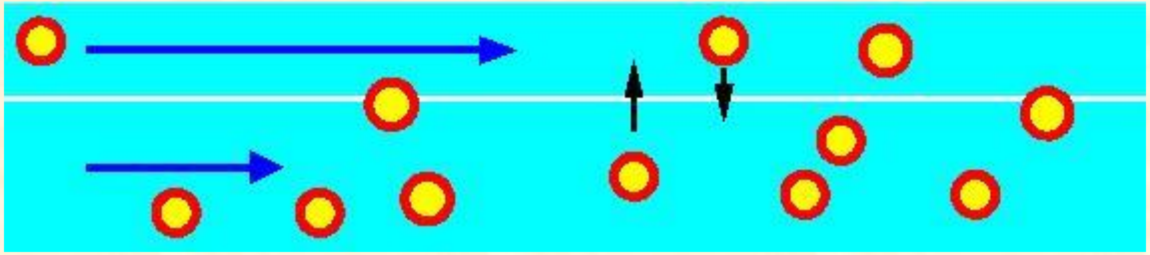
Viscosity of the quark-gluon plasma: 10^14 cP (almost glass)

What is viscosity?

Friction in Newton's equation: $$\frac{d(mv_i)}{dt} + \gamma v_i = F_i$$

Friction in Euler's equations

$$\frac{\partial(\rho v_i)}{\partial t} = -\frac{\partial}{\partial x^k}\left(P\delta_{ik} + \rho v_i v_k\right) + \frac{\partial}{\partial x^k}\sigma_{ik}^{fric}$$



$$\sigma_{ik}^{fric} \sim \partial v_i/\partial x^k \qquad\qquad \sigma_{ik}^{fric} \sim \partial v_i/\partial x^k + \partial v_k/\partial x^i$$

$$\sigma_{ik}^{fric} = \eta\left(\frac{\partial v_i}{\partial x^k} + \frac{\partial v_k}{\partial x^i} - \frac{2}{d}\delta_{ik}\frac{\partial v_l}{\partial x^l}\right) + \zeta\delta_{ik}\frac{\partial v_l}{\partial x^l} + \cdots$$

Viscosity of gases and liquids

Gases (Maxwell, 1867): $\eta \sim \rho\,\bar{v}\,l_{mfp} \sim \dfrac{m_o \bar{v}}{\sigma} \sim \dfrac{m_o^{1/2}}{\sigma}\sqrt{T}$

Viscosity of a gas is

- independent of pressure
- scales as square of temperature
- inversely proportional to cross-section

Liquids (Frenkel, 1926): $\eta \sim A(P,T)\,\exp\dfrac{W}{T}$

- W is the “activation energy”
- In practice, A and W are chosen to fit data

First-order transport (kinetic) coefficients

Shear viscosity	η
Bulk viscosity	ζ
Charge diffusion constant	D_Q
Supercharge diffusion constant	D_s
Thermal conductivity	κ_T
Electrical conductivity	σ

* Expect Einstein relations such as $\frac{\sigma}{e^2 \Xi} = D_{U(1)}$ to hold

Linear response theory

$$H(t) = H_0 + H_1(t)\theta(t - t_0)$$

$t < t_0$

$$\rho_0 = e^{-\beta H_0} = \sum_n |n >< n| e^{-\beta E_n^{(0)}}$$

$$< \mathcal{O} >= \frac{1}{Z_0} tr\, \rho_0 \mathcal{O} = \frac{1}{Z_0} \sum_n < n|\mathcal{O}|n > e^{-\beta E_n^{(0)}}$$

$t > t_0$

$$\rho(t) = \sum_n |n(t) >< n(t)| e^{-\beta E_n}$$

$$< \mathcal{O} >= \frac{1}{Z_0} tr\, \rho \mathcal{O} = \frac{1}{Z_0} \sum_n < n(t)|\mathcal{O}|n(t) > e^{-\beta E_n}$$

$$i\partial_t |n(t) >= H(t)|n(t) >$$

Linear response theory (continued)

$$|n(t)>= e^{-iH_0 t}U(t,t_0)|n>$$

$$U(t,t_0) = 1 - i\int_{t_0}^{t} dt' H_1(t') + \cdots$$

Inserting this into $\rho(t) = \sum_n |n(t)>< n(t)|e^{-\beta E_n}$ we get

$$\begin{aligned} & < \mathcal{O}(t) >= \frac{1}{Z_0} tr\, \rho\mathcal{O} = \\ = \; & < \mathcal{O} >_0 - i\int_{t_0}^{t} dt' \frac{1}{Z_0}\sum_n < n(t_0)|\mathcal{O}(t)H_1(t') - H_1(t')\mathcal{O}(t)|n(t_0) > e^{-\beta E_n} + \cdots \\ = \; & < \mathcal{O} >_0 - i\int_{t_0}^{t} dt' \; < [\mathcal{O}(t)H_1(t')] > + \cdots \end{aligned}$$

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

In the regime described by a gravity dual the correlator can be computed using the gauge theory/gravity duality

Second-order hydrodynamics

Hydrodynamics is an effective theory, valid for sufficiently small momenta

$$k\, l_{mfp} \ll 1$$

First-order hydro eqs are parabolic. They imply instant propagation of signals.

This is not a conceptual problem since hydrodynamics becomes “acausal” only outside of its validity range but it is very inconvenient for numerical work on Navier-Stokes equations where it leads to instabilities [Hiscock & Lindblom, 1985]

These problems are resolved by considering next order in derivative expansion, i.e. by adding to the hydro constitutive relations all possible second-order terms compatible with symmetries (e.g. conformal symmetry for conformal plasmas)

Second-order conformal hydrodynamics (in d dimensions)

$$T^{\mu\nu}_{\text{conformal}} = \varepsilon\, u^\mu\, u^\nu + \frac{\varepsilon}{d-1}\Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \tau_\Pi\left[{}^{<}D\sigma^{\mu\nu>} + \frac{1}{d-1}\sigma^{\mu\nu}(\nabla\cdot u)\right]$$

$$+\kappa\left[R^{<\mu\nu>} - (d-2)u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right]$$

$$+\frac{\lambda_1}{\eta^2}\sigma_\lambda^{<\mu}\sigma^{\nu>\lambda} - \frac{\lambda_2}{\eta}\sigma_\lambda^{<\mu}\Omega^{\nu>\lambda} + \lambda_3\,\Omega_\lambda^{<\mu}\Omega^{\nu>\lambda}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu\, u^\nu \qquad \sigma^{\mu\nu} = 2\nabla^{<\mu}u^{\nu>} \qquad u^\mu u_\mu = -1$$

$$D \equiv u^\mu\nabla_\mu \qquad \Omega^{\mu\nu} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta - \nabla_\beta u_\alpha\right)$$

$$A^{<\mu\nu>} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(A_{\alpha\beta} + A_{\beta\alpha}\right) - \frac{1}{d-1}\Delta^{\mu\nu}\Delta^{\alpha\beta}A_{\alpha\beta}$$

Second-order transport (kinetic) coefficients

(for theories conformal at T=0)

Relaxation time	τ_{Π}
Second order trasport coefficient	λ_1
Second order trasport coefficient	λ_2
Second order trasport coefficient	λ_3
Second order trasport coefficient	κ

In non-conformal theories such as QCD, the total number of second-order transport coefficients is quite large

Derivative expansion in hydrodynamics: first order

Hydrodynamic d.o.f. = densities of conserved charges

$\partial_\mu T^{\mu\nu} = 0$ (4 equations)

$T^{00},\ T^{0i}$ (4 d.o.f.) or $\varepsilon,\ u^\mu$

$$T^{\mu\nu} = \varepsilon\, u^\mu\, u^\nu + P(\varepsilon)\,\Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta(\varepsilon)\,\sigma^{\mu\nu} - \zeta(\varepsilon)\,\Delta^{\mu\nu}\,(\nabla\cdot u) + \cdots$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu\, u^\nu \qquad \sigma^{\mu\nu} = 2\nabla^{<\mu}u^{\nu>}$$

$$A^{<\mu\nu>} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(A_{\alpha\beta} + A_{\beta\alpha}\right) - \frac{1}{d-1}\Delta^{\mu\nu}\Delta^{\alpha\beta}A_{\alpha\beta}$$

$$u^\mu u_\mu = -1$$

Predictions of the second-order conformal hydrodynamics

Sound dispersion: $$\omega_{1,2} = \pm c_s\, q - i\Gamma\, q^2 \pm \frac{\Gamma}{c_s}\left(c_s^2 \tau_\Pi - \frac{\Gamma}{2}\right) q^3 + O(q^4)$$

$$\Gamma = \frac{d-2}{d-1}\frac{\eta}{\varepsilon + P}$$

Kubo: $$G_R^{xy,xy}(\omega, q) = P - i\,\eta\,\omega + \eta\,\tau_\Pi\,\omega^2 - \frac{\kappa}{2}\left[(d-3)\omega^2 + q^2\right]$$

In quantum field theory, the dispersion relations such as

$$\omega = \pm v_s\, q - \frac{i}{2(\epsilon + P)} \left(\frac{4}{3}\eta + \zeta\right) q^2$$

appear as poles of the retarded correlation functions, e.g.

$$\langle T_{00}(k)\, T_{00}(-k)\rangle \sim \frac{q^2\, T^4}{\omega^2 - q^2/3 + i\omega q^2/3\pi T}$$

- in the hydro approximation - $\omega/T \ll 1\,, \quad q/T \ll 1$

Supersymmetric sound mode ("phonino") in $4d \; \mathcal{N}=4$ SYM

Conserved charge	$\Longrightarrow$	Hydrodynamic mode (infinitely slowly relaxing fluctuation of the charge density)	$\Longrightarrow$	Hydro pole in the retarded correlator of the charge density

$$\partial_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu}_{equib} + \delta T^{\mu\nu} \qquad \langle T_{\mu\nu}(-k)\, T_{\rho\sigma}(k)\rangle$$

Sound wave pole: $\omega = \pm v_s\, q - i\dfrac{2}{3sT}\left(\eta + \dfrac{3}{4}\zeta\right) q^2 + \cdots \qquad v_s = \sqrt{\dfrac{\partial P}{\partial \epsilon}}$

$$\partial_\mu S^\mu_\alpha = 0 \qquad S^\mu_\alpha + \delta S^\mu_\alpha \qquad \langle \bar{S}^\mu_{\dot\alpha}(-k) S^\nu_\beta(k)\rangle$$

Supersound wave pole: $\omega = \pm v_{ss}\, q - i D_s q^2 + \cdots \qquad v_{SS} = \dfrac{P}{\epsilon}$

Lebedev & Smilga, 1988 (see also Kovtun & Yaffe, 2003)

Sound and supersymmetric sound in $4d\ \mathcal{N}=4$ SYM

In 4d CFT

$$\epsilon = 3\,P$$

$$\zeta = 0$$

$$\Longrightarrow$$

$$v_s = \sqrt{\frac{\partial P}{\partial \epsilon}} = \frac{1}{\sqrt{3}}$$

$$v_{SS} = \frac{P}{\epsilon} = \frac{1}{3}$$

Sound mode: $\omega = \pm \frac{q}{\sqrt{3}} - i\frac{2\,\eta}{3sT}q^2 + \cdots$

Supersound mode: $\omega = \pm \frac{q}{3} - iD_s q^2 + \cdots$

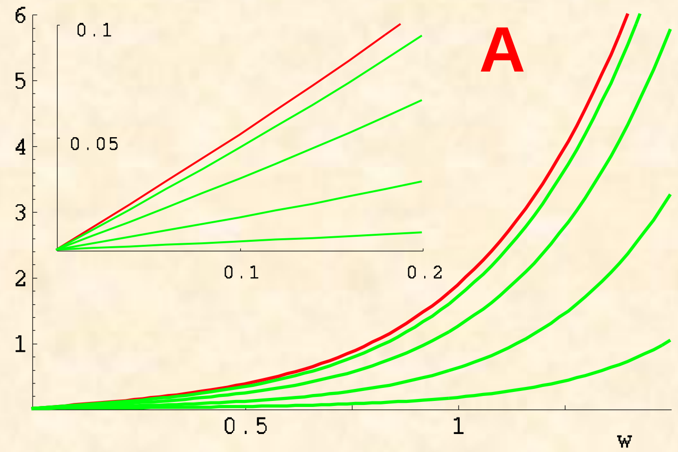
Quasinormal modes in dual gravity

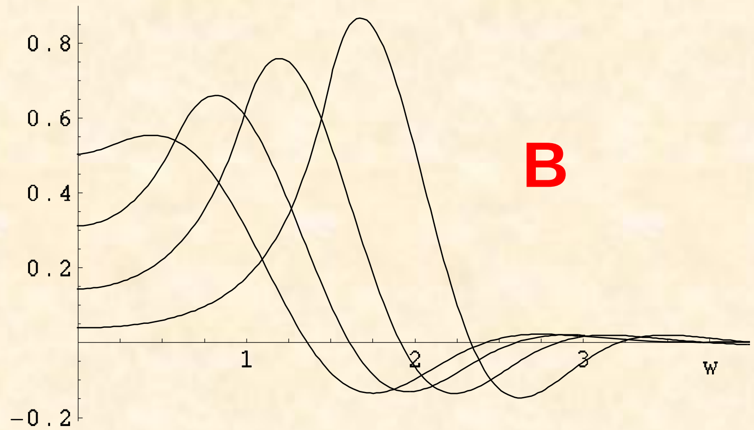
Graviton: $\omega = \pm \frac{q}{\sqrt{3}} - i\frac{1}{6\pi T}q^2 + \cdots \Longrightarrow \frac{\eta}{s} = \frac{1}{4\pi}$

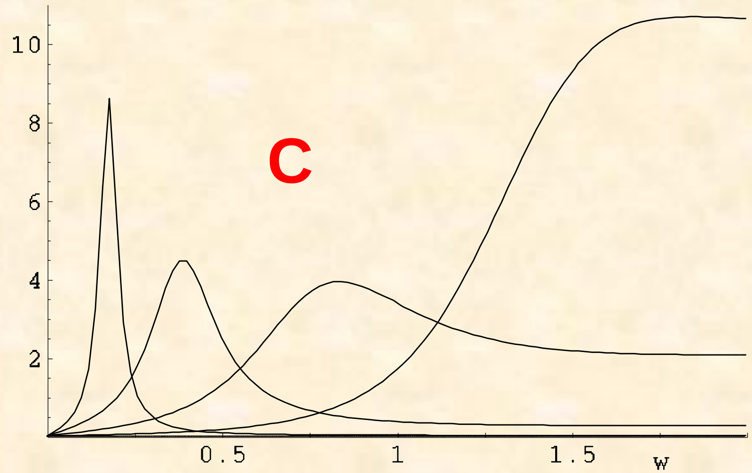
Gravitino: $\omega = \pm \frac{q}{3} - i\frac{2\sqrt{2}}{9\pi T}q^2 + \cdots \Longrightarrow D_s = \frac{2\sqrt{2}}{9\pi T}$

Spectral function and quasiparticles

$$\chi_{\mu\nu,\alpha\beta}(k) = \int d^4x\, e^{-ikx} \left\langle \left[T_{\mu\nu}(x) T_{\alpha\beta}(0) \right] \right\rangle = -2\, \mathrm{Im}\, G^R_{\mu\nu,\alpha\beta}(\omega, q)$$







A: scalar channel

B: scalar channel - thermal part

C: sound channel

V. Some applications

First-order transport coefficients in N = 4 SYM

in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Shear viscosity $\eta = \frac{\pi}{8} N_c^2 T^3 \left[1 + O\left(\frac{1}{(g^2 N_c)^{3/2}}, \frac{1}{N_c^2} \right) \right]$

Bulk viscosity $\zeta = 0$

for non-conformal theories see
Buchel et al; G.D.Moore et al
Gubser et al.

Charge diffusion constant $D_R = \frac{1}{2\pi T} + \cdots$

Supercharge diffusion constant $D_s = \frac{2\sqrt{2}}{9\pi T}$ (G.Policastro, 2008)

Thermal conductivity $\frac{\kappa_T\ \mu^2}{\eta\ T} = 8\pi^2 + \cdots$

Electrical conductivity $\sigma = e^2 \frac{N_c^2\, T}{16\, \pi} + \cdots$

Universality of shear viscosity in the regime described by gravity duals

$$ds^2 = f(w)\left(dx^2 + dy^2\right) + g_{\mu\nu}(w) dw^\mu dw^\nu$$

$$\left.\begin{aligned}
\eta &= \lim_{\omega\to 0} \frac{1}{2\omega}\int dt\, dx e^{i\omega t}\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle \\
\sigma_{abs} &= -\frac{16\pi G}{\omega}\,\mathrm{Im}\, G^R(\omega) \\
&= \frac{8\pi G}{\omega}\int dt\, dx e^{i\omega t}\,\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle
\end{aligned}\right\} \quad \eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

Graviton's component h_y^x obeys equation for a minimally coupled massless scalar. But then $\sigma_{abs}(0) = A_H$.

Since the entropy (density) is $s = A_H/4G$ we get

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

First and second order transport coefficients of *conformal* holographic fluids *to leading order* in supergravity approximation

$$\eta = s/4\pi\,,$$

$$\tau_\Pi = \frac{d}{4\pi T}\left(1+\frac{1}{d}\left[\gamma_E+\psi\left(\frac{2}{d}\right)\right]\right)\,,$$

$$\kappa = \frac{d}{d-2}\,\frac{\eta}{2\pi T}\,,$$

$$\lambda_1 = \frac{d\eta}{8\pi T}\,,$$

$$\lambda_2 = \left[\gamma_E+\psi\left(\frac{2}{d}\right)\right]\,\frac{\eta}{2\pi T}\,,$$

$$\lambda_3 = 0$$

Bhattacharyya et al, 2008

Note: $2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0$

Classification of fluctuations and universality

$$ds^2 = \frac{r^2}{R^2}\left(-f(r)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{R^2}{r^2 f}dR^2$$

$\delta g_{\mu\nu} \sim e^{-i\omega t + iqz} h_{\mu\nu}(r)$ O(2) symmetry in x-y plane

Shear channel: h_{tx} h_{zx} h_{ty} h_{zy} $\Rightarrow$ Z_1

Sound channel: h_{tt} h_{tz} h_{zz} $h_{xx} + h_{yy}$ $\Rightarrow$ Z_2

Scalar channel: h_{xy} $h_{xx} - h_{yy}$ $\Rightarrow$ Z_3

Other fluctuations (e.g. $\delta\varphi_1, \ldots \delta\varphi_n$) may affect sound channel

But not the shear channel $\Rightarrow$ universality of η/s

Two-point correlation function of stress-energy tensor

Field theory

Zero temperature: $\langle T_{\mu\nu} T_{\alpha\beta} \rangle_{T=0} = \Pi_{\mu\nu,\alpha\beta}\, F(k^2) + Q_{\mu\nu,\alpha\beta}\, G(k^2)$

Finite temperature: $\langle T_{\mu\nu} T_{\alpha\beta} \rangle_{T} = S^{(1)}_{\mu\nu,\alpha\beta}\, G_1(\omega,q) + S^{(2)}_{\mu\nu,\alpha\beta}\, G_2(\omega,q)$

$$+ \quad S^{(3)}_{\mu\nu,\alpha\beta}\, G_3(\omega,q) + S^{(4)}_{\mu\nu,\alpha\beta}\, G_4 + S^{(5)}_{\mu\nu,\alpha\beta}\, G_5$$

Dual gravity

- Five gauge-invariant combinations Z_1, Z_2, Z_3, Z_4, Z_5 of $h_{\mu\nu}$ and other fields determine G_1, G_2, G_3, G_4, G_5
- Z_1, Z_2, Z_3, Z_4, Z_5 obey a system of coupled ODEs
- Their (quasinormal) spectrum determines singularities of the correlator

Example: stress-energy tensor correlator in $4d\ \mathcal{N}=4$ SYM in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Zero temperature, Euclid: $G_E(k) = \frac{N_c^2 k_E^4}{32\pi^2} \ln k_E^2$

Finite temperature, Mink:

$$< T_{tt}(-\omega, -q), T_{tt}(\omega, q) >^{\text{ret}} = \frac{3 N_c^2 \pi^2 T^4 q^2}{2(\omega^2 - q^2/3 + i\omega q^2/3\pi T)} + \cdots$$

(in the limit $\omega/T \ll 1\,, \quad q/T \ll 1$)

The pole
(or the lowest quasinormal freq.)

$$\omega = \pm \frac{1}{\sqrt{3}} q - \frac{i}{6\pi T} q^2 + \frac{3 - 2\ln 2}{24\pi^2 \sqrt{3} T^2} q^3 + \cdots$$

Compare with hydro:

$$\omega = \pm v_s q - \frac{i}{2sT} \left(\zeta + \frac{4}{3}\eta \right) q^2 + \cdots$$

In CFT: $v_s = \frac{1}{\sqrt{3}}\,, \quad \zeta = 0$

$$\Rightarrow \quad \eta = \pi N_c^2 T^3 / 8$$

Also, $s = \pi^2 N_c^2 T^3 / 2$ (Gubser, Klebanov, Peet, 1996)

Example 2 (continued): stress-energy tensor correlator in

$4d\ \mathcal{N}=4$ SYM in the limit $N_c \to \infty\,, \quad g_{YM}^2 N_c \to \infty$

Zero temperature, Euclid: $$G_E(k) = \frac{N_c^2 k_E^4}{32\pi^2} \ln k_E^2$$

Finite temperature, Mink:

$$< T_{tx}(-\omega,-q), T_{tx}(\omega,q) >^{\text{ret}} \sim \frac{N_c^2 T^4 \omega^2}{\omega - iq^2/4\pi T} + \cdots$$

(in the limit $\omega/T \ll 1\,, \quad q/T \ll 1$)

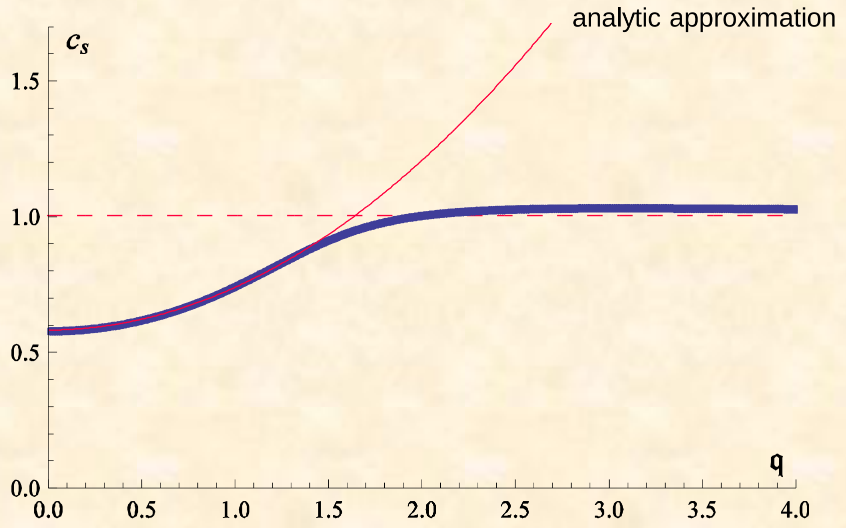
The pole
(or the lowest quasinormal freq.) $$\omega = -\frac{i}{4\pi T} q^2 - \frac{i(1-\ln 2)}{32\pi^3 T^3} q^4 + \cdots$$

Compare with hydro: $$\omega = -\frac{i\,\eta}{sT} q^2 + \cdots$$

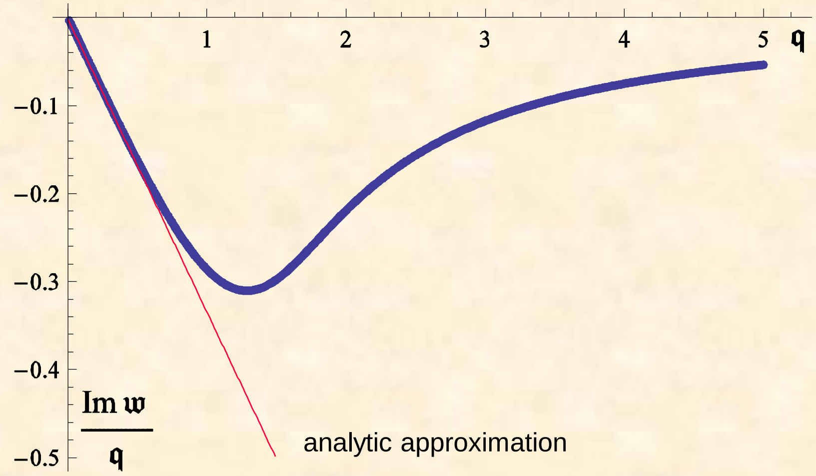
$$s = \pi^2 N_c^2 T^3/2 \qquad \Rightarrow \quad \eta = \pi N_c^2 T^3/8$$

Sound dispersion in $4d\ \mathcal{N}=4$ SYM

$\text{Re}\,(\omega)/q$

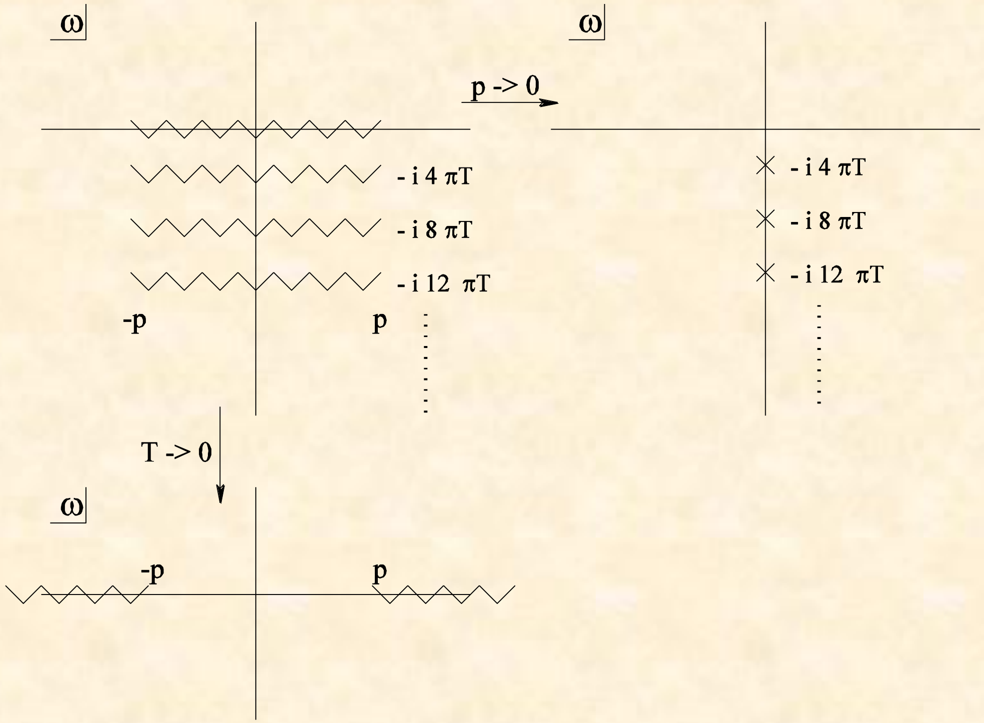


q

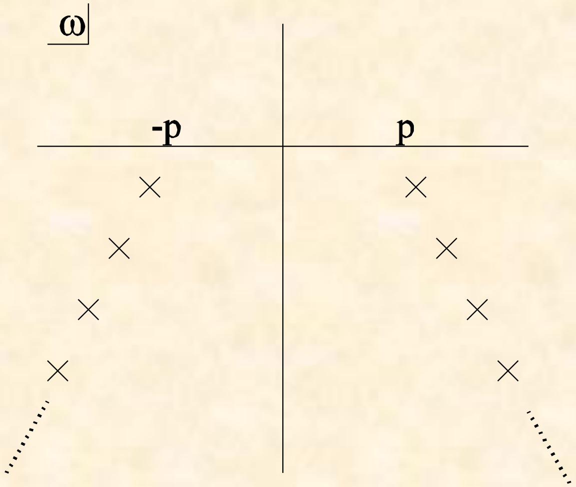


Analytic structure of the correlators

$g^2 N = 0$



$g^2 N = \infty$



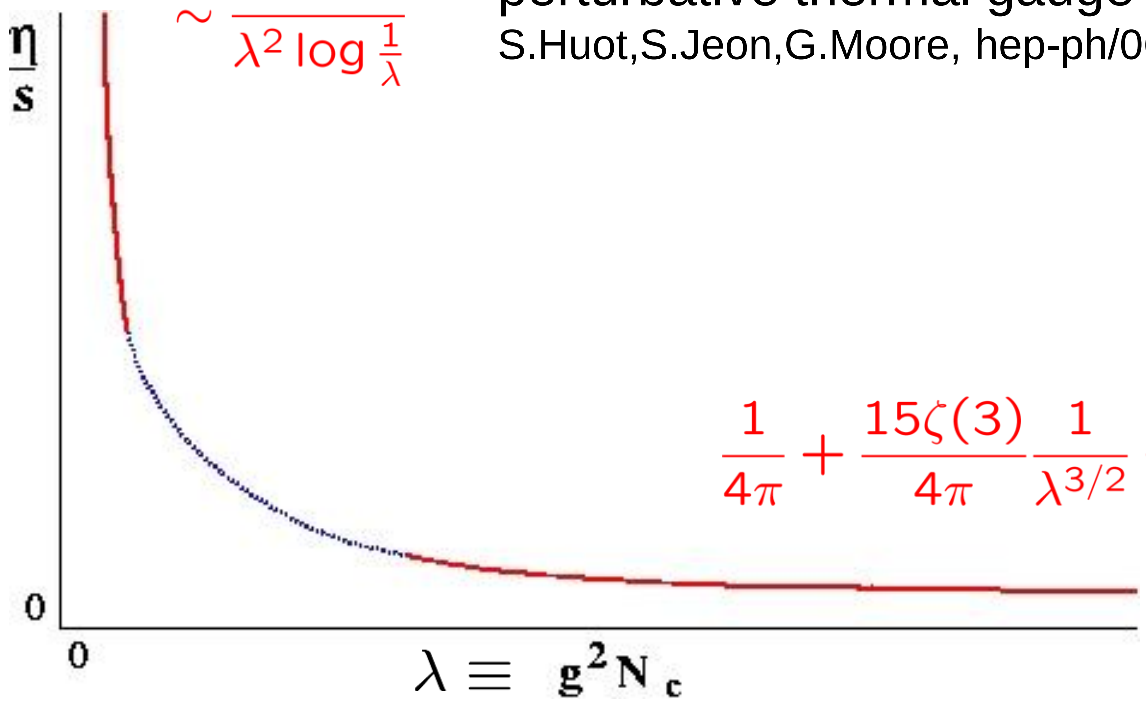
Strong coupling: A.S., hep-th/0207133

Weak coupling: S. Hartnoll and P. Kumar, hep-th/0508092

Shear viscosity in $\mathcal{N} = 4$ SYM

$\sim \frac{1}{\lambda^2 \log \frac{1}{\lambda}}$

perturbative thermal gauge theory
S.Huot,S.Jeon,G.Moore, hep-ph/0608062



$$\frac{1}{4\pi} + \frac{15\zeta(3)}{4\pi}\frac{1}{\lambda^{3/2}} + \cdots$$

Correction to $1/4\pi$: Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

Electrical conductivity in $\mathcal{N} = 4$ SYM

Weak coupling: $\sigma = 1.28349 \, \dfrac{e^2 (N_c^2 - 1) T}{\lambda^2 \left[\ln \lambda^{-1/2} + O(1) \right]}$

$\lambda \ll 1$

Strong coupling: $\sigma = \dfrac{e^2 N_c^2 T}{16 \pi} + O\left(\dfrac{1}{\lambda^{3/2}}\right)$

$\lambda \gg 1$

* Charge susceptibility can be computed independently: $\Xi = \dfrac{N_c^2 T^2}{8}$

D.T.Son, A.S., hep-th/0601157

Einstein relation holds: $\dfrac{\sigma}{e^2 \Xi} = D_{U(1)} = \dfrac{1}{2\pi T}$

Universality of η/s

Theorem:

For a thermal gauge theory, the ratio of shear viscosity to entropy density is equal to $1/4\pi$
in the regime described by a dual gravity theory

$(\text{e.g. at} \quad g^2_{YM} N_c = \infty \, , N_c = \infty \quad \text{in} \quad \mathcal{N} = 4 \;\; \text{SYM})$

Remarks:

- Extended to non-zero chemical potential:

  Benincasa, Buchel, Naryshkin, hep-th/0610145

- Extended to models with fundamental fermions in the limit $N_f/N_c \ll 1$

  Mateos, Myers, Thomson, hep-th/0610184

- *String/Gravity dual to QCD is currently unknown*

Universality of shear viscosity in the regime described by gravity duals

$$ds^2 = f(w)\left(dx^2 + dy^2\right) + g_{\mu\nu}(w)dw^\mu dw^\nu$$

$$\left.\begin{aligned}
\eta &= \lim_{\omega\to 0}\frac{1}{2\omega}\int dt\,dx e^{i\omega t}\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle \\
\sigma_{abs} &= -\frac{16\pi G}{\omega}\,\mathrm{Im}\,G^R(\omega) \\
&= \frac{8\pi G}{\omega}\int dt\,dx e^{i\omega t}\,\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle
\end{aligned}\right\}\quad \eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

Graviton's component h_y^x obeys equation for a minimally coupled massless scalar. But then $\sigma_{abs}(0) = A_H$.

Since the entropy (density) is $s = A_H/4G$ we get

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Three roads to universality of η/s

- **The absorption argument**

  D. Son, P. Kovtun, A.S., hep-th/0405231

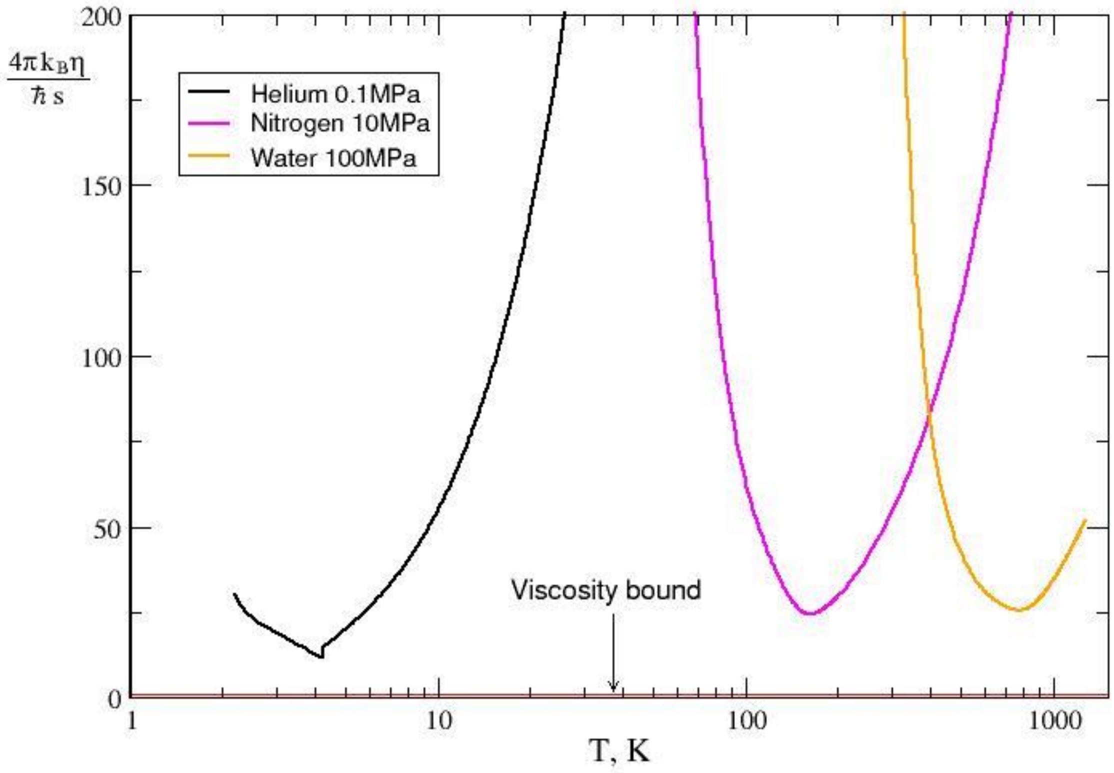
- **Direct computation of the correlator in Kubo formula from AdS/CFT** A.Buchel, hep-th/0408095

- **"Membrane paradigm" general formula for diffusion coefficient + interpretation as lowest quasinormal frequency = pole of the shear mode correlator + Buchel-Liu theorem**

  P. Kovtun, D.Son, A.S., hep-th/0309213, A.S., 0806.3797 [hep-th], P.Kovtun, A.S., hep-th/0506184, A.Buchel, J.Liu, hep-th/0311175

A viscosity bound conjecture

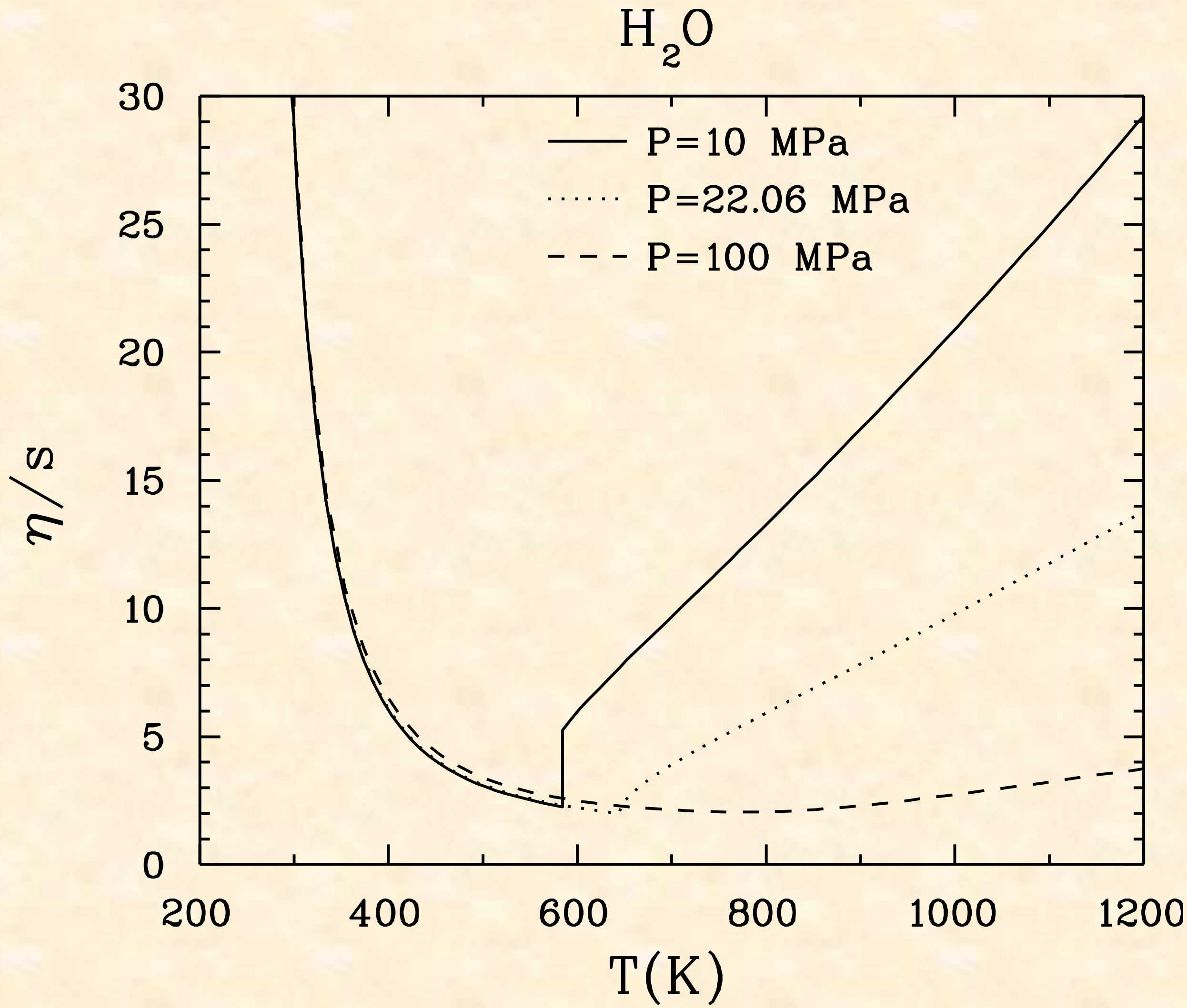
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13}\, K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

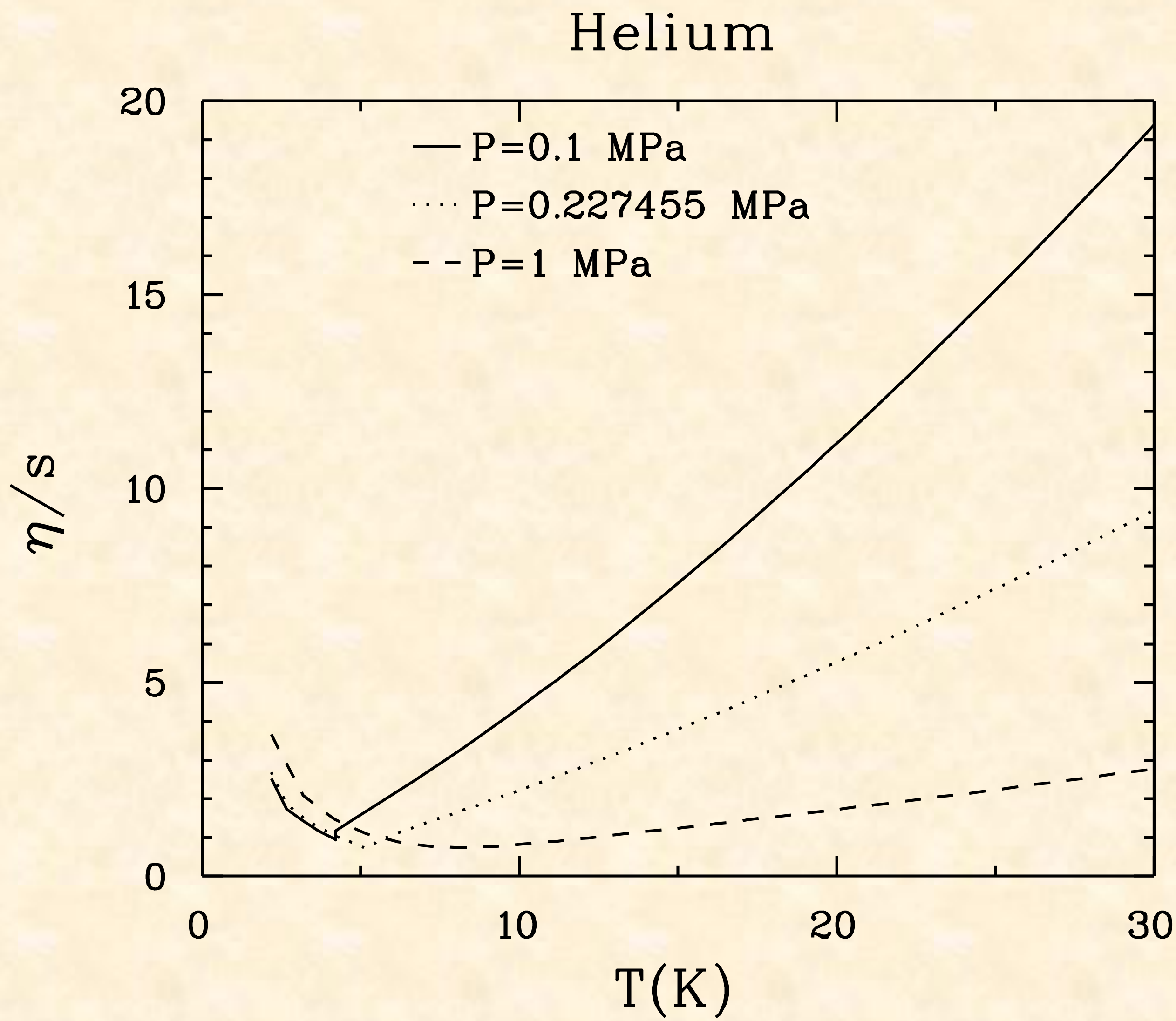
Xe	84
Kr	57
CO_2	32
H_2O	25
C_2H_5OH	22
Ne	17
He	8.8

P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231



$(\eta/s)_{\text{min}} \sim 25$ in units of $\dfrac{\hbar}{4\pi k_B}$

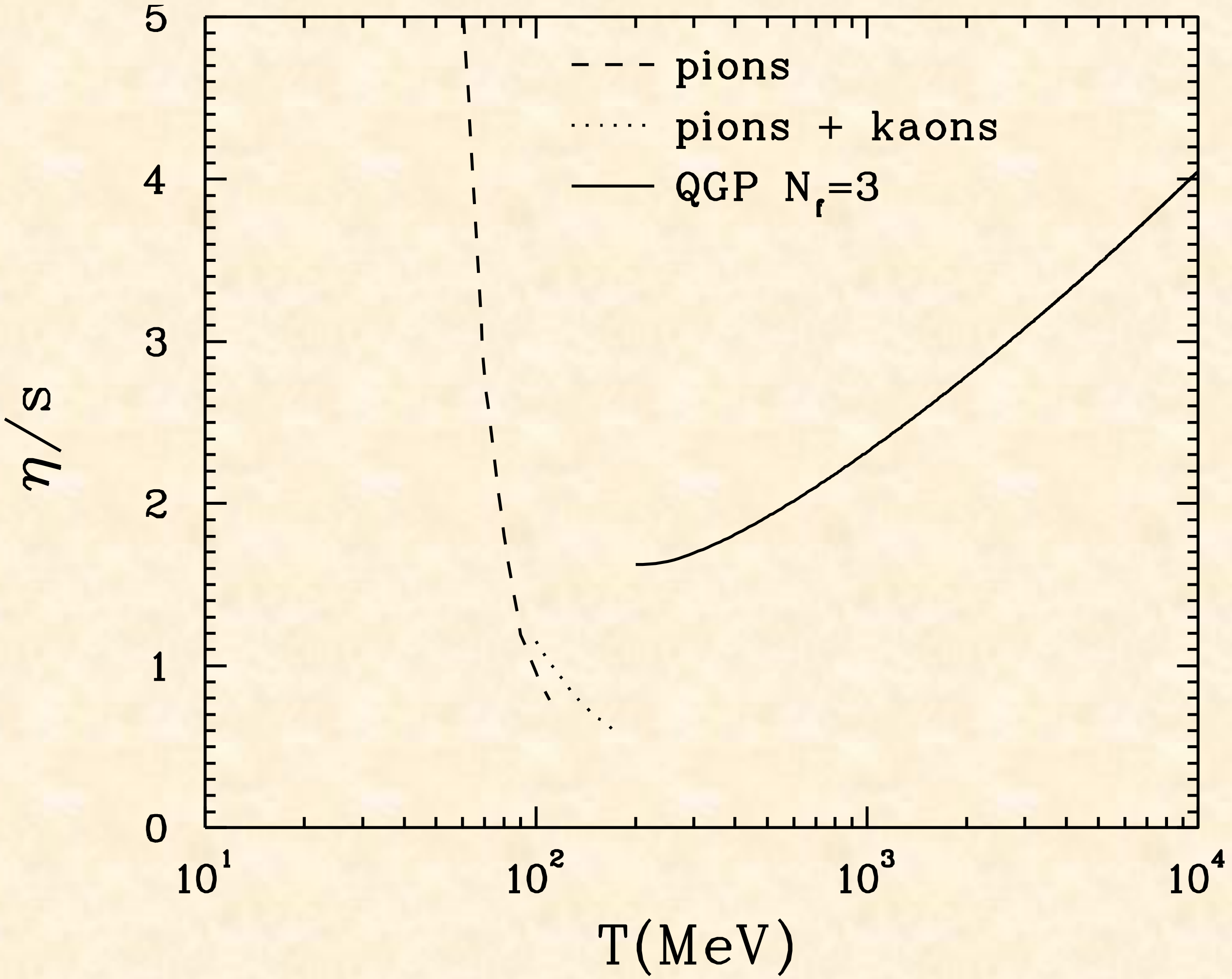
Chernai, Kapusta, McLerran, nucl-th/0604032



$$(\eta/s)_{\text{min}} \sim 8.8 \quad \text{in units of } \frac{\hbar}{4\pi k_B}$$

Chernai, Kapusta, McLerran, nucl-th/0604032

QCD



Chernai, Kapusta, McLerran, nucl-th/0604032

A hand-waving argument

$$\eta \sim \rho\, v\, l \sim \rho\, v^2 \tau \sim n\, m\, v^2 \tau \sim n\, \epsilon \tau$$

$$s \sim n$$

Thus $$\frac{\eta}{s} \sim \epsilon \tau \geq \hbar$$

Gravity duals fix the coefficient: $$\frac{\eta}{s} \geq \hbar / 4\pi$$

Shear viscosity - (volume) entropy density ratio from gauge-string duality

In ALL theories (in the limit where dual gravity valid) : $\frac{1}{4\pi} + \text{corrections}$

In particular, in N=4 SYM: $\frac{1}{4\pi} + \frac{15\zeta(3)}{4\pi}\frac{1}{\lambda^{3/2}} + \cdots$

Other higher-derivative gravity actions

$$S = \int d^D x \sqrt{-g}\left(R - 2\Lambda + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \cdots\right)$$

Y.Kats and P.Petrov: 0712.0743 [hep-th]

M.Brigante, H.Liu, R.C.Myers, S.Shenker and S.Yaida: 0802.3318 [hep-th], 0712.0805 [hep-th].

R.Myers,M.Paulos, A.Sinha: 0903.2834 [hep-th] (and ref. therein – many other papers)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 8c_1 + \ldots\right) \qquad \frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{1}{2N}\right)$$

for superconformal Sp(N) gauge theory in d=4

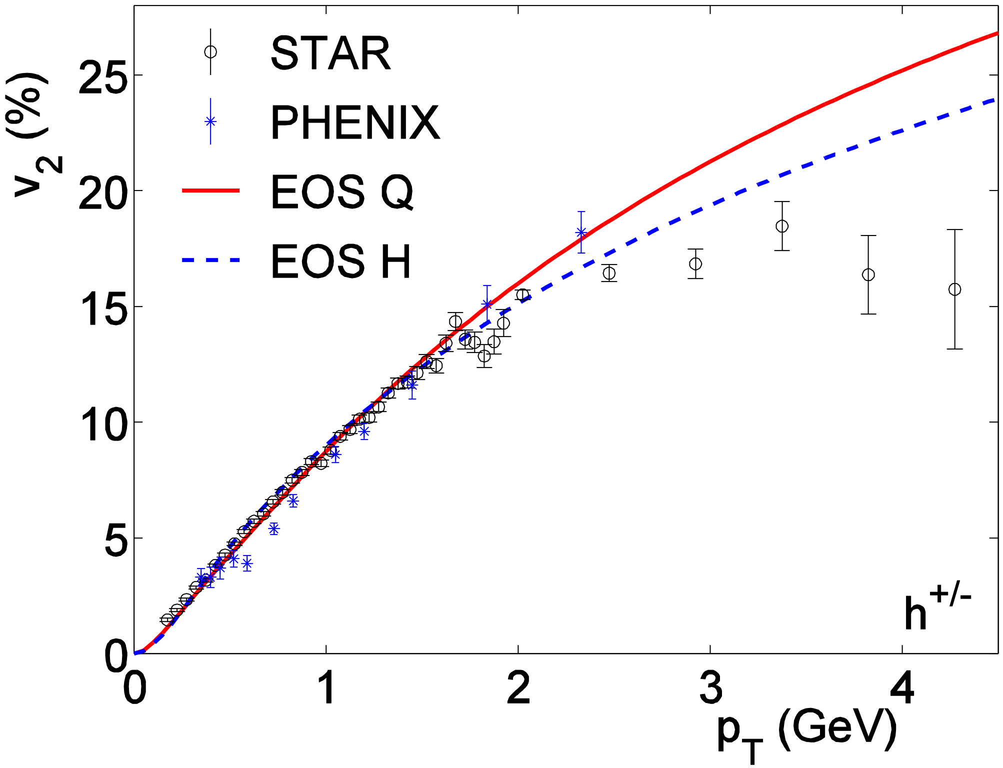
Also: The species problem: T.Cohen, hep-th/0702136; A. Dolbado, F.Llanes-Estrada: hep-th/0703132

Viscosity "measurements" at RHIC

Viscosity is ONE of the parameters used in the hydro models describing the azimuthal anisotropy of particle distribution

$$\frac{d^2N^i}{dp_T d\phi} = N_0^i \left[1 + 2v_2^i(p_T)\, \cos 2\phi + \cdots\right]$$

$v_2^i(p_T)$ -elliptic flow for particle species "i"



Elliptic flow reproduced for

$$0 < \eta/s \leq 0.3$$

e.g. Baier, Romatschke, nucl-th/0610108

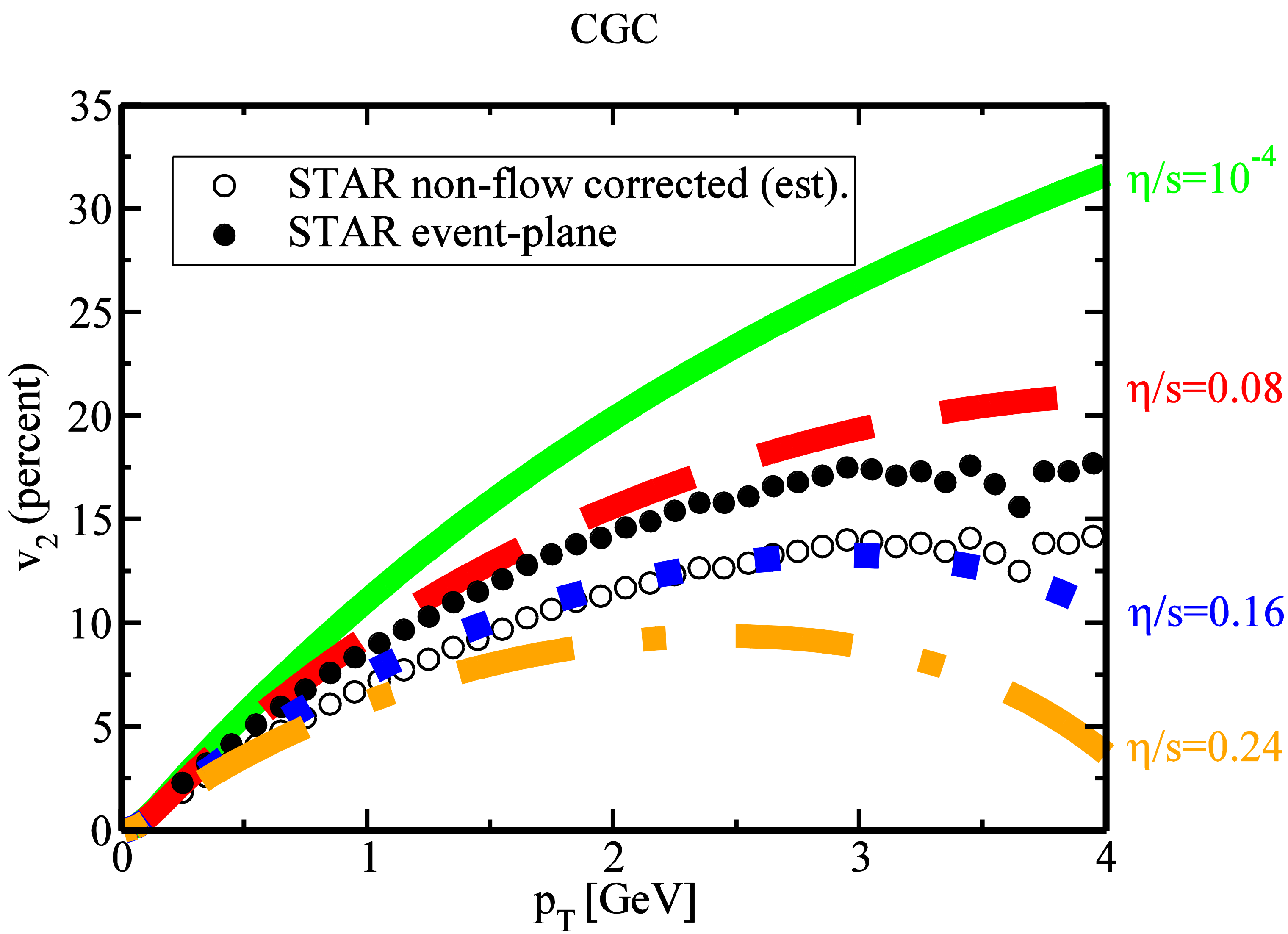
Perturbative QCD:

$$\eta/s\,(T_{\mathrm{RHIC}}) \approx 1.6 \sim 1.8$$

Chernai, Kapusta, McLerran, nucl-th/0604032

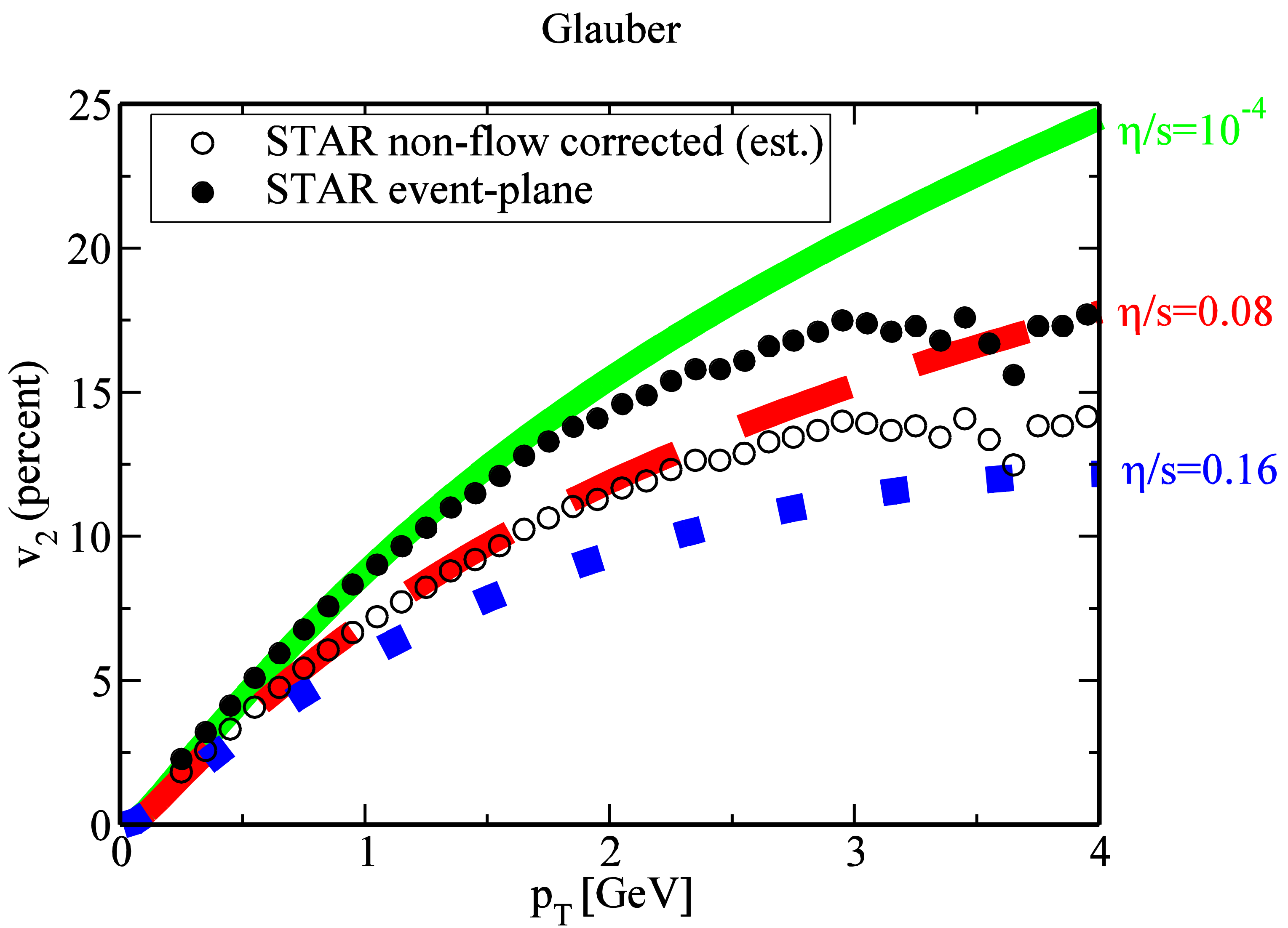
SYM: $\eta/s \approx 0.09 \sim 0.28$

Elliptic flow with color glass condensate initial conditions



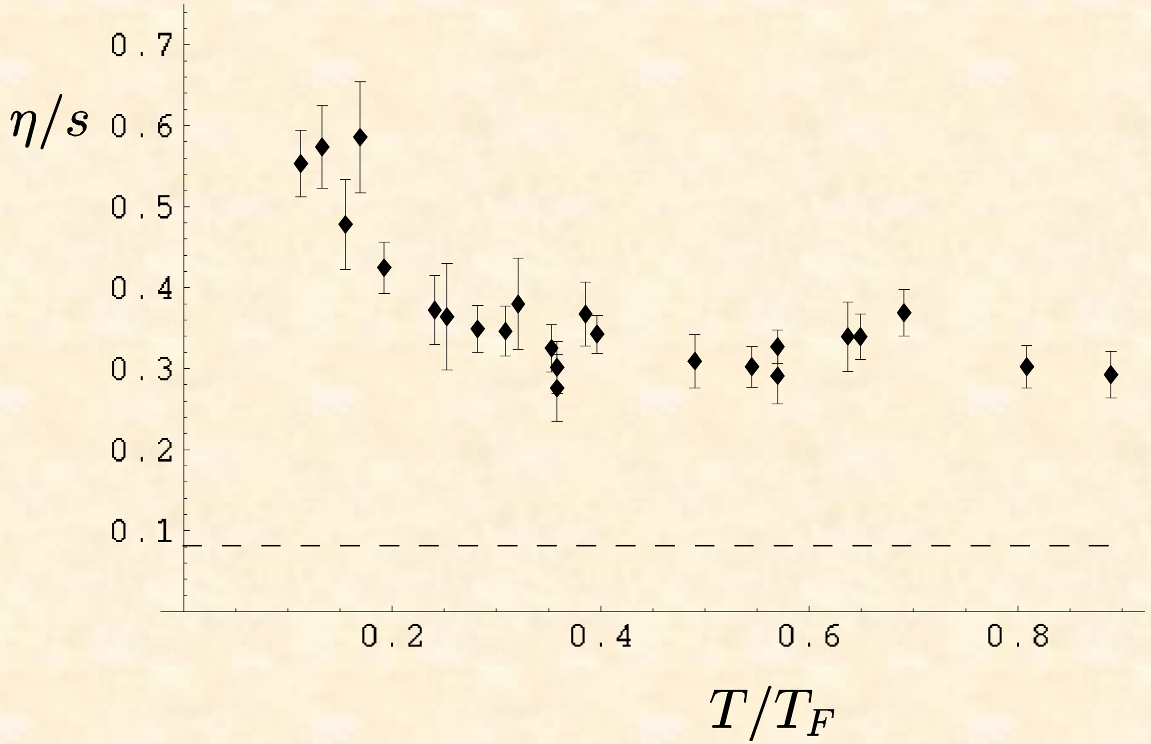
Luzum and Romatschke, 0804.4015 [nuc-th]

Elliptic flow with Glauber initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Viscosity-entropy ratio of a trapped Fermi gas



$\eta/s \sim 4.2$ in units of $\dfrac{\hbar}{4\pi k_B}$

T.Schafer, cond-mat/0701251

(based on experimental results by Duke U. group, J.E.Thomas et al., 2005-06)

Viscosity/entropy ratio in QCD: current status

Theories with gravity duals in the regime where the dual gravity description is valid

$$\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$$

(universal limit)

Kovtun, Son & A.S; Buchel; Buchel & Liu, A.S

QCD: RHIC elliptic flow analysis suggests

$$0 < \frac{\eta}{s} < 0.2$$

QCD: (Indirect) LQCD simulations

$$0.08 < \frac{\eta}{s} < 0.16$$

$$1.2\, T_c < T < 1.7\, T_c$$

H.Meyer, 0805.4567 [hep-th]

Trapped strongly correlated cold alkali atoms

$$\left(\frac{\eta}{s}\right)_{\text{min}} \approx 0.5$$

T.Schafer, 0808.0734 [nucl-th]

Liquid Helium-3

$$\left(\frac{\eta}{s}\right)_{\text{min}} \approx 0.7$$

Shear viscosity at non-zero chemical potential

$\mathcal{N} = 4$ SYM

$q_i \in U(1)^3 \subset SO(6)_R$ $\Longleftrightarrow$ Reissner-Nordstrom-AdS black hole

$Z = \mathrm{tr}\, e^{-\beta H + \mu_i q_i}$ with three R charges

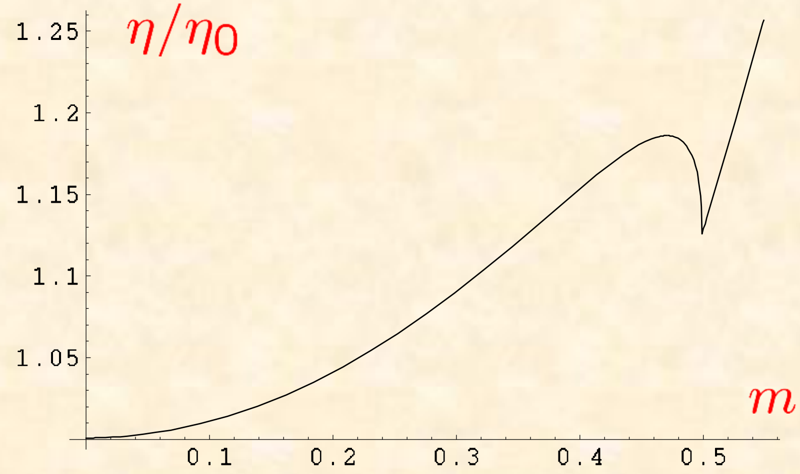
(see e.g. Yaffe, Yamada, hep-th/0602074) (Behrnd, Cvetic, Sabra, 1998)

We still have $\dfrac{\eta}{s} = \dfrac{1}{4\pi}$

J.Mas
D.Son, A.S.
O.Saremi
K.Maeda, M.Natsuume, T.Okamura

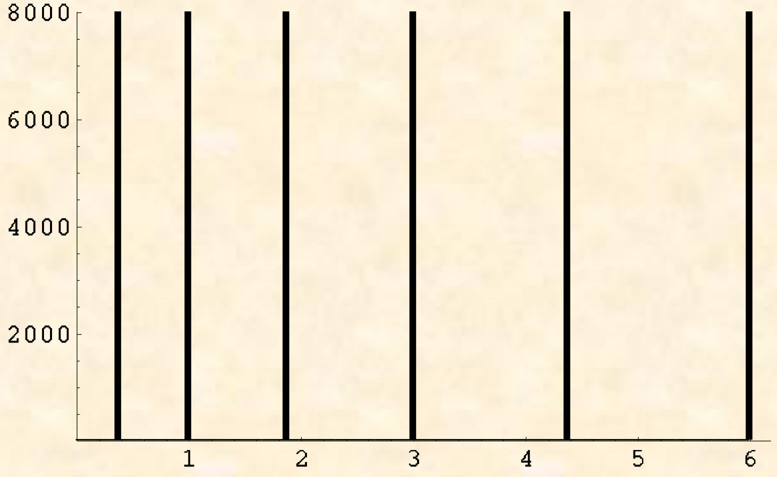
$$\eta = \pi N^2 T^3 \frac{m^2(1-\sqrt{1-4m^2}-m^2)^2}{(1-\sqrt{1-4m^2})^3}$$

$m \equiv \mu/2\pi T$

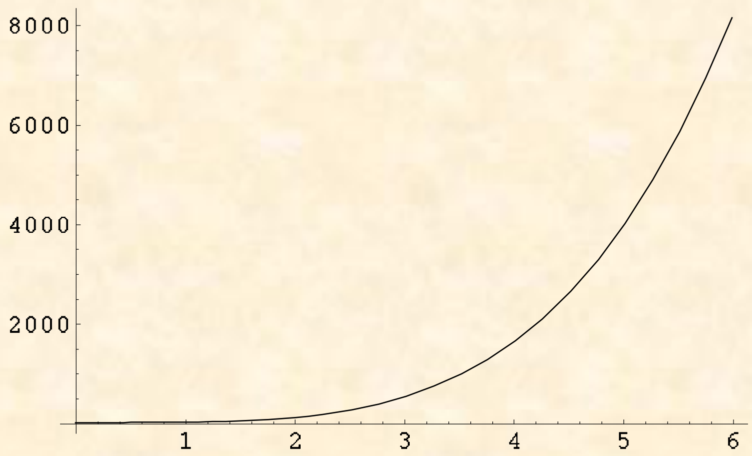


Spectral function and quasiparticles in finite-temperature "AdS + IR cutoff" model

$T < T_c$



$T > T_c$



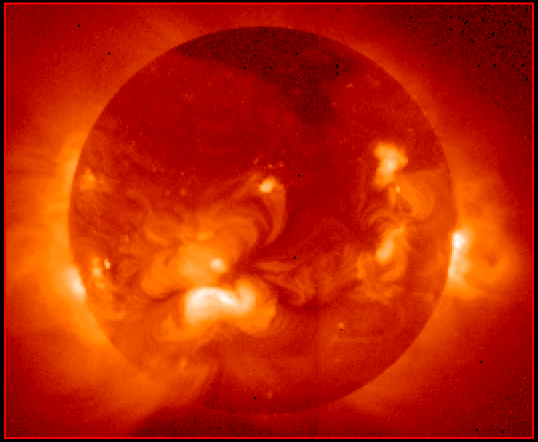
$$\chi(\omega) \sim N_c^2 \sum_{n=0}^{\infty} \omega_n^2 \rho(\omega_n)\, \delta(\omega - \omega_n)$$

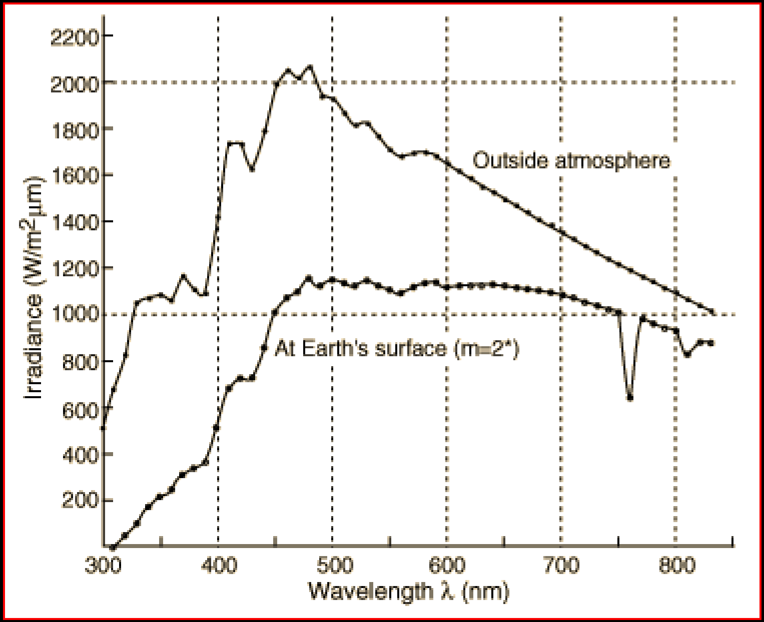
$$\chi(\omega) = \frac{N_c^2}{16\pi} \frac{\omega^2 \sinh(\omega/2T)}{\cosh(\omega/2T) - \cos(\omega/2T)}$$

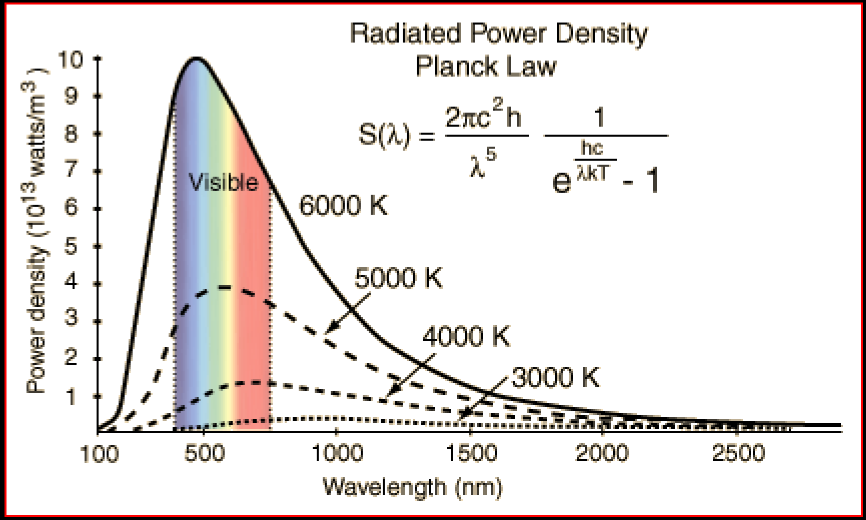
$\mathcal{N} = 4$ SYM

Photon and dilepton emission from supersymmetric Yang-Mills plasma

S. Caron-Huot, P. Kovtun, G. Moore, A.S., L.G. Yaffe, hep-th/0607237







Photon emission from SYM plasma

Photons interacting with matter: $e\, J_\mu^{\text{EM}}\, A^\mu$

To leading order in e $$d\Gamma_\gamma = \frac{d^3k}{(2\pi)^3}\,\frac{e^2}{2|k|}\,\eta^{\mu\nu} C^<_{\mu\nu}(k^0 = |k|)$$

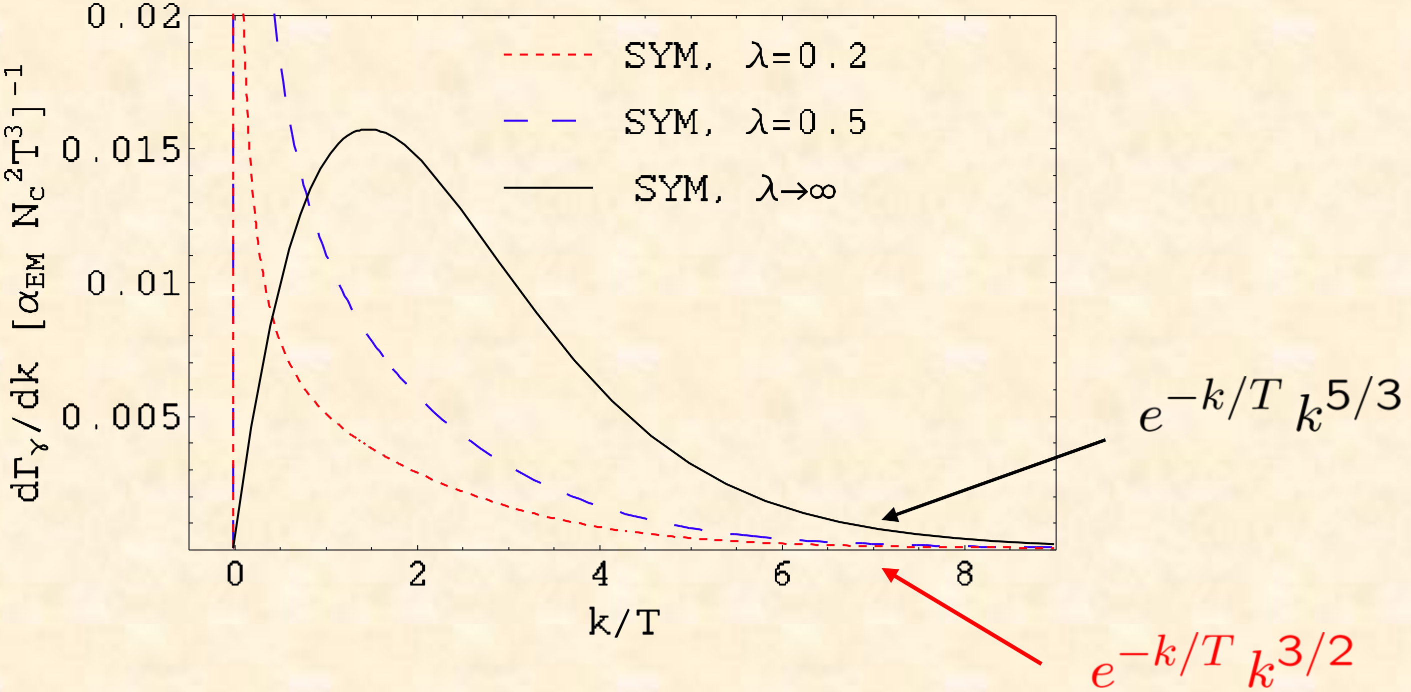
$$C^<_{\mu\nu} = \int d^4X e^{-iKX} \langle J_\mu^{\text{EM}}(0) J_\nu^{\text{EM}}(X)\rangle$$

Mimic J_μ^{EM} by gauging global R-symmetry $U(1) \subset SU(4)$

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=4\,\text{SYM}} + e\, J_\mu^3\, A^\mu - \frac{1}{4} F_{\mu\nu}^2$$

Need only to compute correlators of the R-currents J_μ^3

Photoproduction rate in SYM



(Normalized) photon production rate in SYM for various values of ‘t Hooft coupling

$$\frac{d\Gamma_\gamma}{dk\,\alpha_{em}N_c^2T^3} = n_B(k)\left(\frac{k}{4\pi T}\right)^2 \left|\, {}_2F_1\left(1-\frac{(1+i)k}{4\pi T}, 1+\frac{(1-i)k}{4\pi T}; 1-\frac{ik}{2\pi T}; -1\right)\right|^{-2}$$

Transport coefficients are non-trivial functions of the parameters of the underlying microscopic theory, even in the simplest case of conformal liquids

$$\frac{\eta}{s} = F\left(N_c, N_f, \frac{m}{T}, \frac{\Lambda}{T}, \ldots\right)$$

Some information can be obtained from kinetic theory at weak coupling and from gauge-string duality at strong coupling (for theories with string or gravity duals). In the latter case, it is natural to look for "universal" (independent of the specific string construction) results.

In the limit of infinite N (gauge group rank) and infinite coupling (i.e. in the supergravity approximation from dual string theory point of view)

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Policastro, Kovtun, Son, AOS, 2001-2008
Buchel, J.Liu, 2003; Buchel, 2004
Iqbal, H.Liu, 2008

$$\frac{\sigma}{\chi} = \frac{1}{4\pi T}\frac{d}{d-2}$$

[for conformal relativistic fluids]: Kovtun, Ritz, 2008

$2\eta\tau_{\Pi} - 4\lambda_1 - \lambda_2 = 0$ [for conformal relativistic fluids]: Haack, Yarom, 2008

More speculative statements inspired by holography

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

Kovtun, Son, AOS, 2004 (violated by some models with Higher derivative gravity, Kats and Petrov, 2007; Brigante, Myers, H.Liu, Myers, Shenker, Yaida, 2008)

$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{d-1} - c_s^2\right)$$

Buchel, 2008 (counterexample: Buchel, 2012)

See also Kanitscheider and Skenderis, 2009

$$c_s^2 \leq c_{s,conf}^2 = \frac{1}{d-1}$$

Hohler and Stephanov, 2009;
Cherman, Cohen, Nellore, 2009

$$\frac{\sigma}{\chi} \geq \frac{\hbar v^2}{4\pi T}\frac{d}{d-2}$$

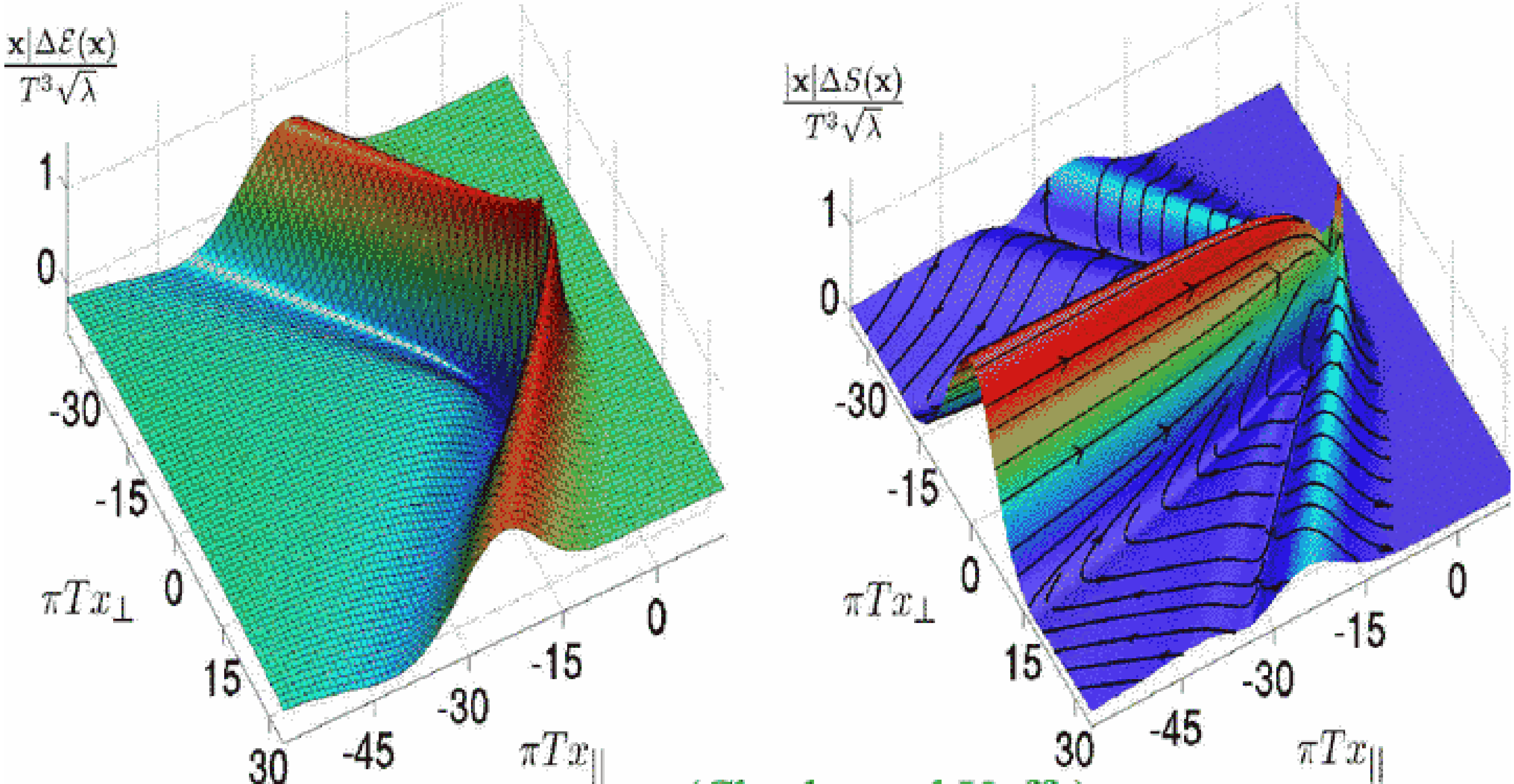
Kovtun and Ritz, 2008

$$D \gtrsim \frac{\hbar v_F}{k_B T}$$

Hartnoll, 2014

See also “Fluctuation bounds…” by Kovtun 1407.0690 [hep-th]

Energy and Momentum Density



(Chesler and Yaffe)

Now consider strongly interacting systems at finite density and LOW temperature

AdS/CFT and condensed matter physics

S. Hartnoll
"Lectures on holographic methods for condensed matter physics",
0903.3246 [hep-th]

C. Herzog
"Lectures on holographic superfluidity and superconductivity",
0904.1975 [hep-th]

M. Rangamani
"Gravity and hydrodynamics: Lectures on the fluid-gravity correspondence",
0905.4352 [hep-th]

S.Sachdev
"Condensed matter and AdS/CFT",
1002.2947 [hep-th]

AdS/CFT and condensed matter physics

* Studies of systems at low temperature, finite density

* Quantum critical points, holographic superconductors

* Top-down and bottom up approaches

* Searching for a Fermi surface

$$G_F^{-1}(\omega, q) = -i\omega + v_F(q - q_F) - c_1\omega^\theta$$

Faulkner, Liu, McGreevy, Vegh, 0907.2694 [hep-th]
Cubrovic, Zaanen, Schalm, Science, **325**, 439 (2009)

Probing quantum liquids with holography

Quantum liquid in p+1 dim	Low-energy elementary excitations	Specific heat at low T
Quantum Bose liquid	phonons	$\sim \; T^p$
Quantum Fermi liquid (Landau FLT)	fermionic quasiparticles + bosonic branch (zero sound)	$\sim \; T$

Departures from normal Fermi liquid occur in

- 3+1 and 2+1 –dimensional systems with strongly correlated electrons

- In 1+1 –dimensional systems for any strength of interaction (Luttinger liquid)

One can apply holography to study strongly coupled Fermi systems at low T

L.D.Landau (1908-1968)

The simplest candidate with a known holographic description is

$SU(N_c)$ $\mathcal{N}=4$ SYM coupled to N_f $\mathcal{N}=2$ fundamental hypermultiplets

at finite temperature T and nonzero chemical potential associated with the "baryon number" density of the charge $U(1)_B \subset U(N_f)$

There are two dimensionless parameters: $\dfrac{n_q^{1/3}}{T}$ $\dfrac{M}{T}$

n_q is the baryon number density

M is the hypermultiplet mass

The holographic dual description in the limit $N_c \gg 1\,,\ g_{YM}^2 N_c \gg 1\,,\ N_f$ finite is given by the D3/D7 system, with D3 branes replaced by the AdS-Schwarzschild geometry and D7 branes embedded in it as probes.

Karch & Katz, hep-th/0205236

AdS-Schwarzschild black hole (brane) background

$$ds^2 = \frac{r^2}{R^2}\left[-\left(1-\frac{r_H^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \left(1-\frac{r_H^4}{r^4}\right)^{-1}\frac{R^2}{r^2}dr^2$$

D7 probe branes

$$S_{DBI} = -N_f T_{D7}\int d^8\xi\sqrt{-\det(g_{ab}+2\pi\alpha' F_{ab})}$$

The worldvolume U(1) field A_μ couples to the flavor current J^μ at the boundary

Nontrivial background value of A_0 corresponds to nontrivial expectation value of J^0

We would like to compute

- the specific heat at low $(T n_q^{-1/3} \ll 1)$ temperature
- the charge density correlator $G^R \sim \langle J^0(k)\, J^0(-k)\rangle$

★ The specific heat (in p+1 dimensions):

$$c_V = \mathcal{N}_q\, p \left(\frac{4\pi}{p+1}\right)^{2p+1} \frac{T^{2p}}{n_q} \left[1 + O(T n_q^{-\frac{1}{p}})\right]$$

(note the difference with Fermi $c_V \sim T$ and Bose $c_V \sim T^p$ systems)

★ The (retarded) charge density correlator $G^R \sim \langle J^0(k)\, J^0(-k)\rangle$
has a pole corresponding to a propagating mode (zero sound)
- even at zero temperature

$$\omega = \pm \frac{q}{\sqrt{p}} - \frac{i\,\Gamma(\frac{1}{2})\, q^2}{n_q^{\frac{1}{p}} \Gamma(\frac{1}{2} - \frac{1}{2p}) \Gamma(\frac{1}{2p})} + O(q^3)$$

(note that this is NOT a superfluid phonon whose attenuation scales as q^{p+1})

New type of quantum liquid?

Other avenues of (related) research

Bulk viscosity for non-conformal theories (Buchel, Gubser,…)

Non-relativistic gravity duals (Son, McGreevy,…)

Gravity duals of theories with SSB (Kovtun, Herzog,…)

Bulk from the boundary (Janik,…)

Navier-Stokes equations and their generalization from gravity (Minwalla,…)

Quarks moving through plasma (Chesler, Yaffe, Gubser,…)

Epilogue

- On the level of theoretical models, there exists a connection between near-equilibrium regime of certain strongly coupled thermal field theories and fluctuations of black holes
- This connection allows us to compute transport coefficients for these theories
- At the moment, this method is the only theoretical tool available to study the near-equilibrium regime of strongly coupled thermal field theories
- The result for the shear viscosity turns out to be universal for all such theories in the limit of infinitely strong coupling
- Influences other fields (heavy ion physics, condmat)