Asymptotically safe inflation from quadratic gravity

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Quantum generation of initial spectrum

Quantum fluctuations

$$(a\,\delta\phi_k)'' + \left(k^2 + \frac{z''}{z}\right)(a\,\delta\phi_k) = 0 \qquad z = \frac{a\,(\partial_t\phi)}{H}$$

Two-point correlations function:

$$\langle 0|\mathcal{R}_k\mathcal{R}_p|0\rangle = \frac{(2\pi)^3}{2\,k^3}\,\mathcal{P}_{\mathcal{R}}(k)\,\delta^3(k+p) \qquad \mathcal{R} = -\,H\,\frac{\delta\phi}{\partial_t\phi}$$

Spectral index and tensor-to-scalar-ratio

$$\mathcal{P}_{\mathcal{R}}(k) = A_s\left(\frac{k}{k_0}\right)^{n_s-1+\frac{\log(k/k_0)}{2}\frac{\mathrm{d}n_s}{\mathrm{d}\log(k)}}$$

$$\mathcal{P}_t(k) = A_t\left(\frac{k}{k_0}\right)^{n_t} \qquad r \equiv \frac{A_t}{A_s} \qquad k_0 \sim 0.05\,\mathrm{Mpc}^{-1}$$ **pivot scale**

Slow-roll inflation and power spectrum

Scalar field - **inflaton** - us a perfect fluid with:

$$\rho_\phi \sim \frac{1}{2}(\partial_t \phi)^2 + V(\phi) \qquad p_\phi \sim \frac{1}{2}(\partial_t \phi)^2 - V(\phi)$$

Definitions of **slow-roll parameters**:

$$\epsilon(\phi) = \frac{1}{2\,\kappa^2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \qquad \eta(\phi) = \frac{1}{\kappa^2}\left(\frac{V''(\phi)}{V(\phi)}\right)$$

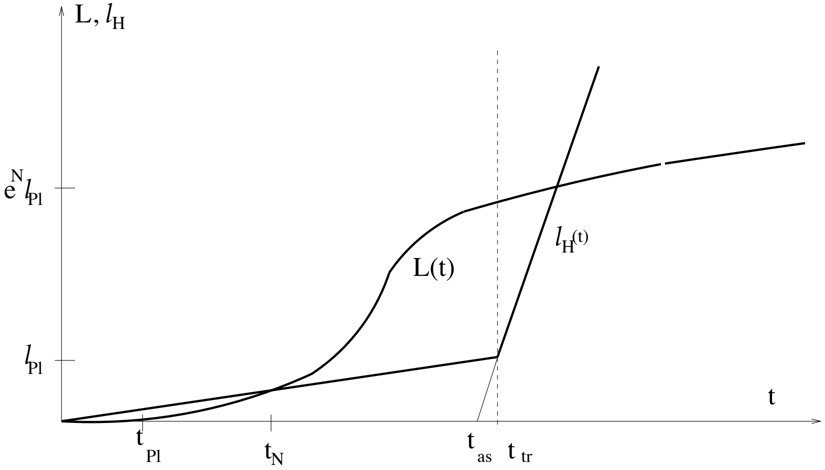
$$\text{Slow-roll inflation} \quad \Leftrightarrow \quad \epsilon(\phi) \ll 1 \quad \wedge \quad \eta(\phi) \ll 1$$

The values of the slow-roll parameters identify:

Spectral index n_s, tensor-to-scalar-ratio r

$$n_s = 1 + 2\,\eta(\phi_i) - 6\,\epsilon(\phi_i) \qquad r = 16\,\epsilon(\phi_i)$$

Problem: Super-Planckian initial conditions



$H(t) \sim a/t \quad a > 1$ A. Bonanno, M. Reuter, JCAP 08 (2007) 024

Asymptotic safety

Problem

Quantum Einstein gravity isn't a (perturbative) renormalizable theory.

Possible solution

Wilsonian (non-perturbative) renormalization group theory.

Wilsonian RG generates a flow in energy k for infinitely many couplings:

$$\lambda_i(k) = \lambda_i^* + \sum_n c_n V_i^n k^{-\theta_n} \qquad \theta_n \text{ critical exponents}$$

The microscopic theory is identified by UV fixed points of β functions:

$$\beta_{\lambda_i}(\lambda_1^*, \lambda_2^*, \dots) = 0$$

In the Einstein-Hilbert truncation:

$$\Gamma(k) = \frac{1}{16\pi G(k)} \int \sqrt{-g}\ \{-R + 2\Lambda(k)\} + S_{\rm gf} + S_{\rm gh}$$

It can be found that there are the following fixed points:

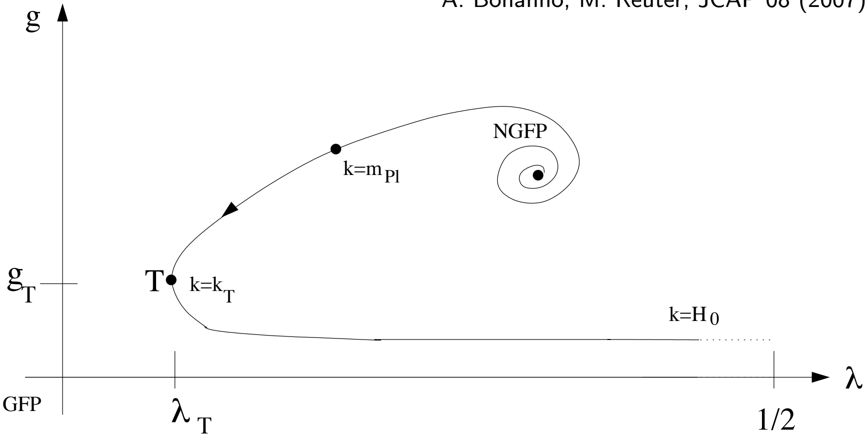
- **Gaussian fixed point** $g^* = 0$ and $\lambda^* = 0$ (free theory, saddle point);
- **Non-Gaussian fixed point** $g^* > 0$ and $\lambda^* > 0$ (UV attractive);

Asymptotic Safety

From a non-perturbative (Wilsonian) point of view, Einstein gravity is a perfectly renormalizable theory; **NGFP** is the **UV completion for gravity**.

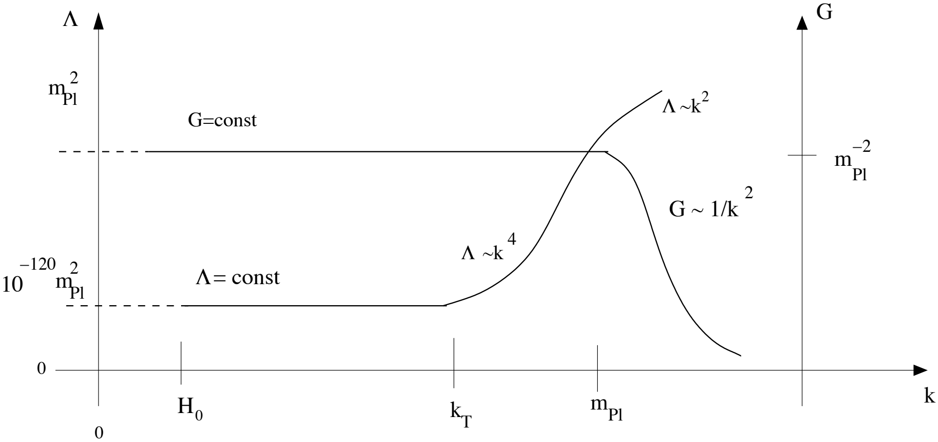
RG trajectories in nature

M. Reuter, H. Weyer, JCAP 0412 (2004) 001
A. Bonanno, M. Reuter, JCAP 08 (2007) 024



- Originated at NGFP (quantum regime);
- Passing extremely close to the GFP;
- Long classical regime (Einstein gravity).

Cosmic history



A. Bonanno, M. Reuter, JCAP 08 (2007) 024

A. Bonanno, M. Reuter, 2008 J. Phys.: Conf.Ser. 140 012008

Asymptotically safe inflation from quadratic gravity

AS inflation Weinberg Phys. Rev. D 81, 083535 (2010)

Consider a general truncation to obtain a de Sitter solution which is unstable but lasts $N > 60$ e-folds.

Neglecting the irrilevant operators, we consider the **UV Lagrangian**:

$$\mathcal{L}_k = \frac{1}{16\pi g(k)}[R - 2\lambda(k)] - \beta(k)R^2$$

Where g_k, λ_k, β_k are dimensionless running coupling constants, such that:

$$\lim_{k\to\infty}\{g_k, \lambda_k, \beta_k\} = \{g_*, \lambda_*, \beta_*\}$$

Solving the linearized flow equations for $g(k)$, $\lambda(k)$ and $\beta(k)$, we obtain:

$$g_k = \frac{6\pi g(\mu)\, k^2}{6\pi\mu^2 + 23 g(\mu)\,(k^2 - \mu^2)} \tag{1}$$

$$\beta_k = \beta_* + b_0 \left(\frac{k^2}{\mu^2}\right)^{-\frac{\theta_3}{2}} \qquad \lambda_k \sim c_0\, k^{-2} \tag{2}$$

Where μ is an IR renormalization scale, and $\theta_3 \sim 1$ is a critical exponent. By substituing (1) and (2) into the action and doing the **scale identification**:

$$k^2 \to \xi R$$

We obtain the **effective action near the inflationary era**:

$$S = \frac{1}{2\kappa^2}\int \sqrt{-g}\, \left\{R + \frac{1}{6m^2}\, R^2 + \frac{\alpha}{3\sqrt{3}\, m}\, R^{\frac{3}{2}} - m^2\Lambda\right\} \mathrm{d}^4 x$$

Inflation in f(R) gravity model

Let us consider the following general action:

$$S[g_{\mu\nu}] = \frac{1}{2\kappa^2} \int \sqrt{-g} \, \{R + F(R)\} \, \mathrm{d}^4 x$$

If $F''(R) \neq 0$, we can do a **conformal transformation**:

$$g_{\mu\nu} \quad \longrightarrow \quad g^{\mathrm{E}}_{\mu\nu} = \varphi g_{\mu\nu}$$

So that:

$$S[g^{\mathrm{E}}_{\mu\nu}] \equiv \int \sqrt{-g_{\mathrm{E}}} \left\{ \frac{\varphi R_{\mathrm{E}}}{2\kappa^2} - \frac{1}{2} g^{\mu\nu}_{\mathrm{E}} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right\} \mathrm{d}^4 x$$

$$V(\phi) = \frac{1}{2\kappa^2 \varphi^2} \{(\varphi - 1) \cdot \chi(\varphi) - F(\chi(\varphi))\} \qquad \varphi = \mathrm{e}^{\sqrt{\frac{2}{3}}\,\kappa\phi}$$

We can study **inflation scenario** coming from the scalar potential $V(\phi)$.

We obtain the following **scalar inflationary potential**:

$$V_{\pm}(\phi) = -\frac{m^2\,\mathrm{e}^{-2\sqrt{\frac{2}{3}}\kappa\phi}}{256\,\kappa^2}\Bigg\{-192\,\left(\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi}-1\right)^2+3\,\alpha^4-128\,\Lambda+$$

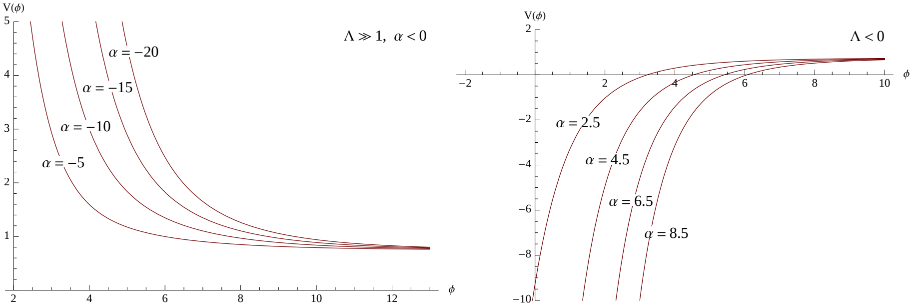
$$+3\,\alpha^2\left(\alpha^2+16\,\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi}-16\right)\pm 6\,\alpha^2\sqrt{\alpha^2\left(\alpha^2+16\,\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi}-16\right)}+$$

$$+\sqrt{32}\alpha\left(\left(\alpha^2+8\,\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi}-8\right)\mp\sqrt{\alpha^2\left(\alpha^2+16\,\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi}-16\right)}\right)^{\frac{3}{2}}\Bigg\}$$

We use this potential to describe **slow-roll inflation**.

The shape of the potential depends on the values (α, Λ).

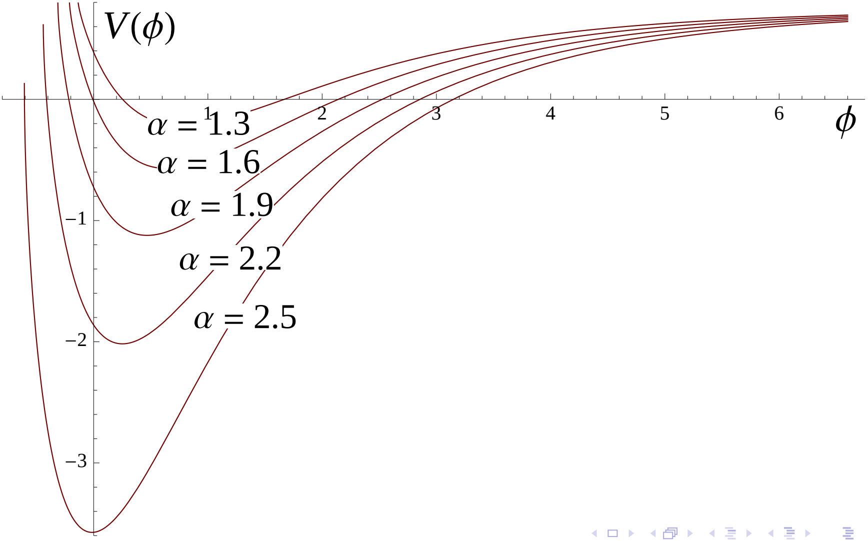
Examples:



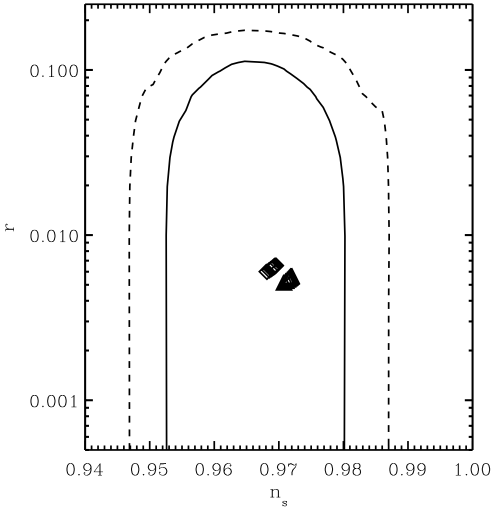
We choose ranges for α and Λ so that:

- $V(\phi)$ has a minimum (**oscillatory** phase);
- We can have a "graceful" **exit** from inflation $\Leftrightarrow V(\phi_{\text{min}}) \leq 0$.

These features are verified for $V(\phi) = V_+(\phi)$ if $\alpha \in [1, 3]$ and $\Lambda \in [0, 1.5]$



- **Planck 2015**: $n_s = 0.968 \pm 0.006 \qquad r < 0.11$
- **AS inflation**: $n_s \in [0.968, 0.970] \qquad r \in [0.005, 0.006]$



Oscillatory phase after inflation

After the end of inflation, the inflaton field ϕ begins to oscillate around the minimum ϕ_{min} of $V(\phi)$.

To study this phase, we can approximate:

$$V(\phi) \sim \frac{a}{2}\left[(\phi - \phi_{\text{min}})^2 - b\right]$$

Where:

- $\phi_{\text{min}} = \phi_{\text{min}}(\alpha, \Lambda)$
- $a(\alpha, \Lambda) = V''(\phi_{\text{min}})$
- $b(\alpha, \Lambda) = -2\,\frac{V(\phi_{\text{min}})}{V''(\phi_{\text{min}})}$

The time evolution of the field $\phi(t)$ is given by

$$\ddot{\phi}(t) + 3\,H(t)\,\dot{\phi}(t) + V'(\phi(t)) = 0$$

Where:

$$H(t) = \left[\frac{1}{3}\left(\frac{1}{2}\dot{\phi}(t)^2 + V(\phi(t))\right)\right]^{1/2}$$

Putting:

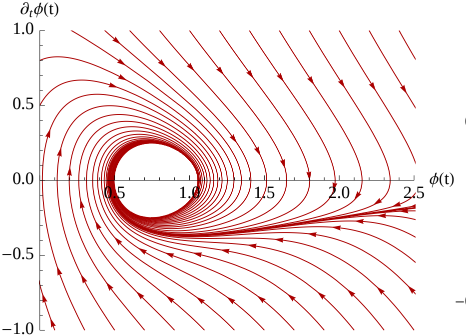
- $x(t) = \sqrt{a}\,(\phi(t) - \phi_{\text{min}})$
- $y(t) = \dot{\phi}(t)$

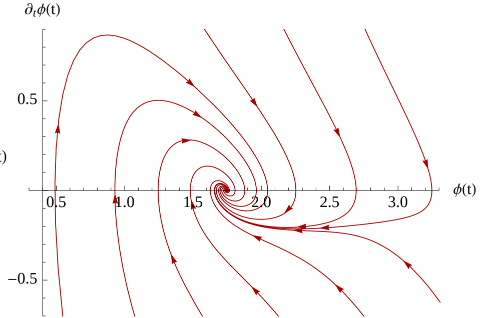
The initial equation is equivalent to the following **dynamical system**:

$$\begin{cases} \dot{y} = -\left[\frac{3}{2}\left(y^2 + x^2 - \text{ab}\right)\right]^{\frac{1}{2}} y - \sqrt{a}\,x \\ \dot{x} = \sqrt{a}\,y \end{cases}$$

The **long time behavior** is determined by the sign of $ab = -2\,V(\phi_{\min})$

- $V(\phi_{\min}) < 0 \Rightarrow$ **Limit cycle behavior** (our case)
- $V(\phi_{\min}) > 0 \Rightarrow (\phi_{\min}, V(\phi_{\min}))$ is an **attractive node**
- $V(\phi_{\min}) = 0$ is an **Hopf bifurcation point**





Conclusions

- AS inflation emerges naturally from the structure of the UV critical surface;
- Our model is significatly different from the Starobinsky model because it predicts a tensor-to-scalar ratio which is significantly higher, and a dynamics characterized by a limit-cycle behavior at the inflation exit;
- It is in agreement with Planck 2015 data;
- Present CMB data can put important constraints on the structure of the effective Lagrangian at the Planck scale;
- Limitation: simple tensorial structure of the effective Lagrangian which assumes a functional dependence of the $f(R)$ type.

Thanks for your attention