Corrections to the SM effective action and stability of EW vacuum

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with **M. Lewicki and P. Olszewski**
arXiv:1402.3826 (JHEP), arXiv:1505.05505, and to appear soon
with **T. Krajewski** arXiv:1411.6435
with **O. Czerwińska and Ł. Nakonieczny** arXiv:1508.03297

Outline:

- SM effective potential
- SM phase diagram
- BSM physics via higher-order operators
- Gauge fixing in-dependence of tunneling rate
- Modifications of the vacuum properties due to expanding background

SM Effective potential

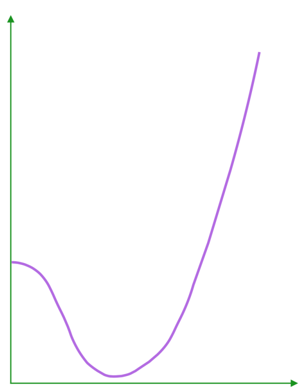
Standard Model Effective potential

$$V_{SM}(\mu) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum_i \frac{n_i}{64\pi^2} M_i^4 \left[\ln\left(\frac{M_i^2}{\mu^2}\right) - C_i \right]$$

For large field values $m^2 << \phi^2$ and $\mu = \phi$ the potential is very well approximated by

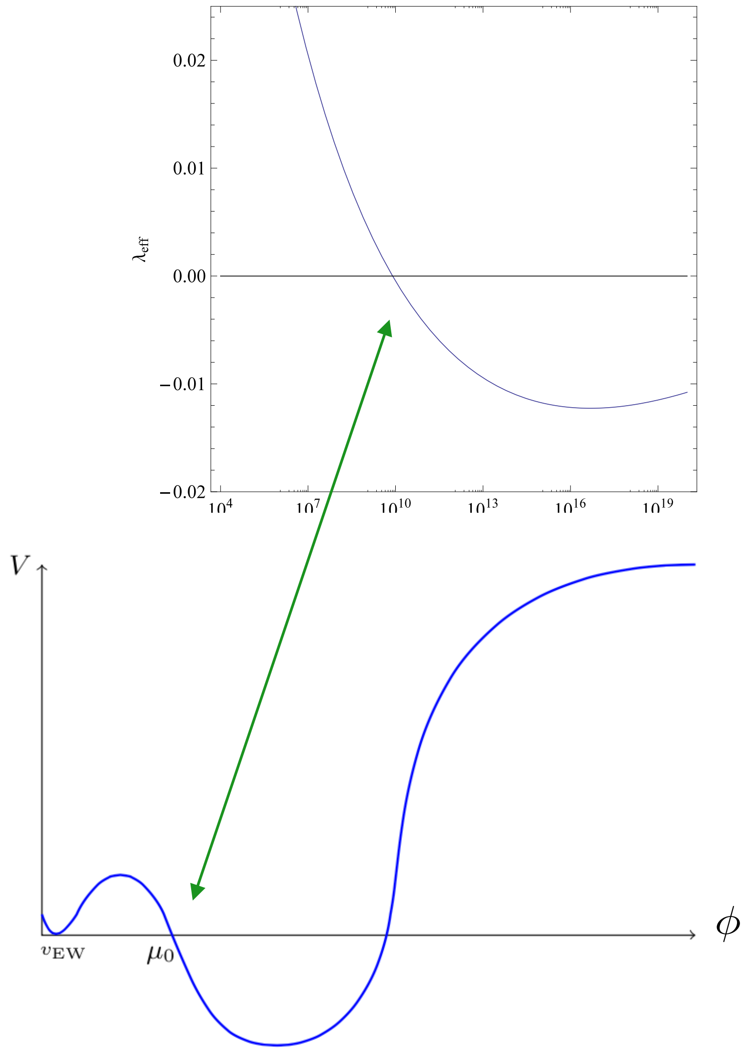
$$V_{SM}(\phi) \approx \phi^4 \left\{ \frac{\lambda}{4} + \frac{1}{64\pi^2} \left[6\left(\frac{g_2^2}{4}\right)^2 \left(\ln\left(\frac{g_2^2}{4}\right) - \frac{5}{6}\right) + 3\left(\frac{g_1^2+g_2^2}{4}\right)^2 \left(\ln\left(\frac{g_1^2+g_2^2}{4}\right) - \frac{5}{6}\right) \right.\right.$$
$$\left.\left. -12\left(\frac{y_t^2}{2}\right)^2 \left(\ln\left(\frac{y_t^2}{2}\right) - \frac{3}{2}\right) + \left(\frac{3\lambda}{2}\right)^2 \left(\ln\left(\frac{3\lambda}{2}\right) - \frac{3}{2}\right) + 3\left(\frac{\lambda}{2}\right)^2 \left(\ln\left(\frac{\lambda}{2}\right) - \frac{3}{2}\right) \right] \right\}$$

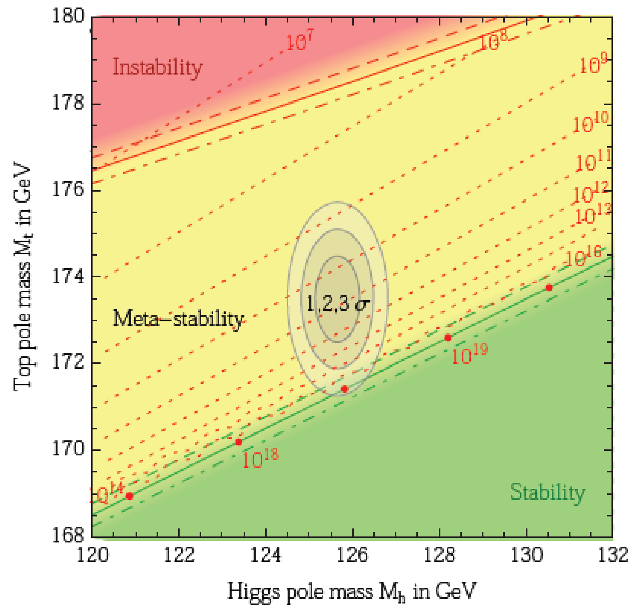
$$V_{SM}(\phi) \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4$$



← **classically** quantum corrected … →

$\lambda_{eff} < 0 \implies$ Metastability





D. Buttazzo, et al. [arXiv:1307.3536].
G. Degrassi, et al. [arXiv:1205.6497].

See lectures by G. Degrassi Corfu 2014

Tunneling

Standard semiclassical formalism

S. R. Coleman, Phys. Rev. D **15** (1977) 2929.
C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D **16** (1977) 1762.

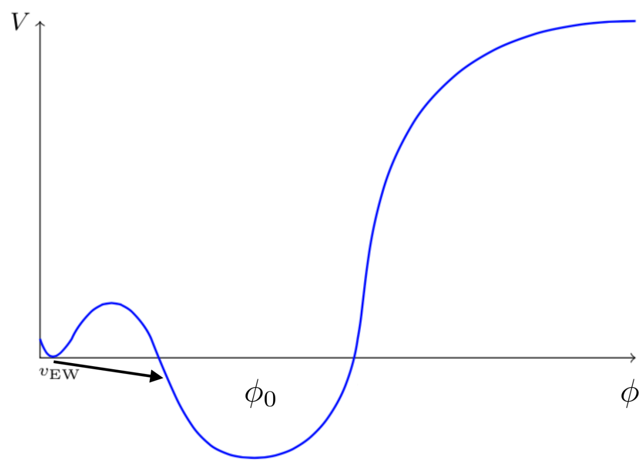
$O(4)$ symmetric solution to euclidean equation of motion

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi},$$
$$s = \sqrt{\vec{x}^2 + x_4^2}.$$

with

- $\dot{\phi}(s=0) = 0$ near the true vacuum
- $\phi(s=\infty) = \phi_{min}$ at the false vacuum

$$= v_{EW}$$



Tunneling

Action of the bounce solution

$$S_E = \int d^4x \left\{ \frac{1}{2} \sum_{\alpha=1}^{4} \left(\frac{\partial\phi(\mathbf{x})}{\partial x^\alpha} \right)^2 + V(\phi(\mathbf{x})) \right\}$$
$$= 2\pi^2 \int ds s^3 \left(\frac{1}{2}\dot{\phi}^2(s) + V(\phi(s)) \right),$$

allows us to calculate decay probability dp of a volume d^3x

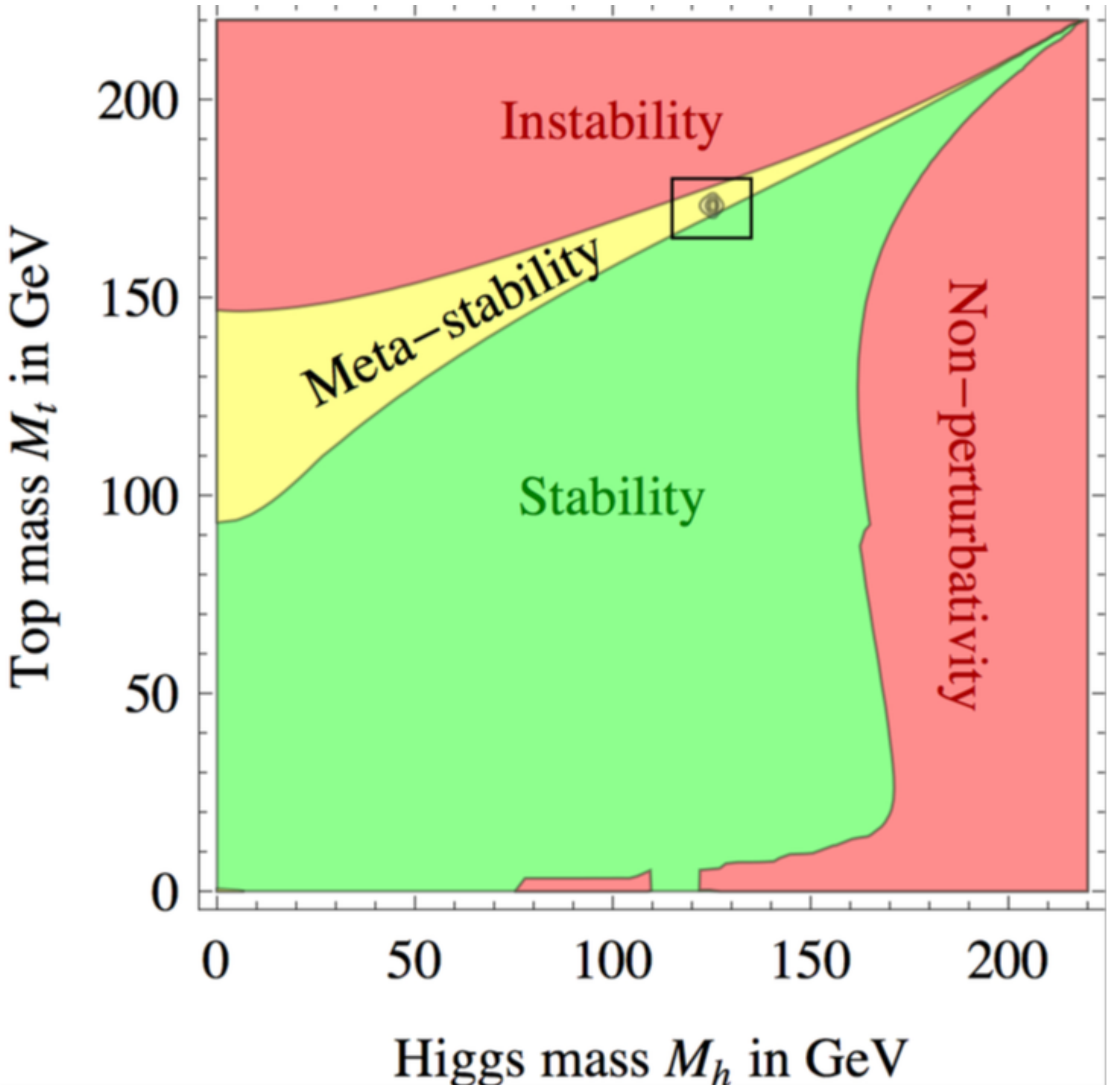
$$dp = dt\, d^3x \frac{S_E^2}{4\pi^2} \left| \frac{det'[-\partial^2 + V''(\phi)]}{det[-\partial^2 + V''(\phi_0)]} \right|^{-1/2} e^{-S_E}.$$

Simplifying

- normalisation factor replaced with width of the barrier $\propto \phi_0$
- size of the universe is $T_U = 10^{10}$yr

we can calculate the lifetime of the false vacuum ($p(\tau) = 1$)

$$\frac{\tau}{T_U} = \frac{1}{\phi_0^4 T_U^4} e^{S_E}.$$

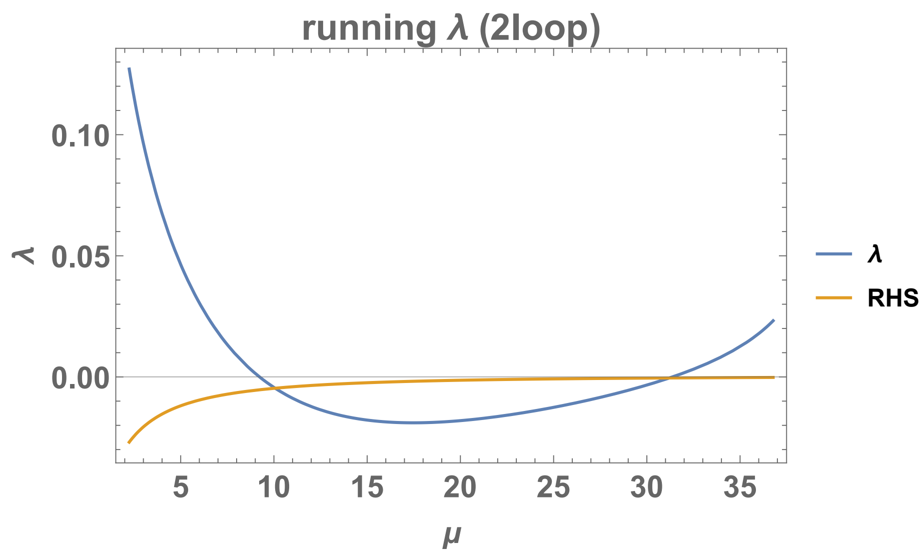


(Degrassi et al., 2012).

New extrema created by quantum corrections (Coleman-Weinberg mechanism)

condition for cancellation of corrections to the derivative of SM

$$\lambda = \frac{\hbar}{256\pi^2}\left[g_1^4 + 2g_1^2 g_2^2 + 3g_2^4 - 48h_t^4 - 3(g_1^2+g_2^2)^2 \log\frac{g_1^2+g_2^2}{4} - 6g_2^4 \log\frac{g_2^2}{4} + 48y_t^4 \log\frac{y_t^2}{2}\right]$$



Hence sensitivity to New Physics

Effective potential with nonrenormalisable interactions

We add new nonrenormalisable couplings
(similar to V. Branchina and E. Messina, [arXiv:1307.5193].)

$$V \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4 + \frac{\lambda_6}{6!}\frac{\phi^6}{M_p^2} + \frac{\lambda_8}{8!}\frac{\phi^8}{M_p^4}.$$

New Physics at Planck scale

That modify the potential around the Planck scale:

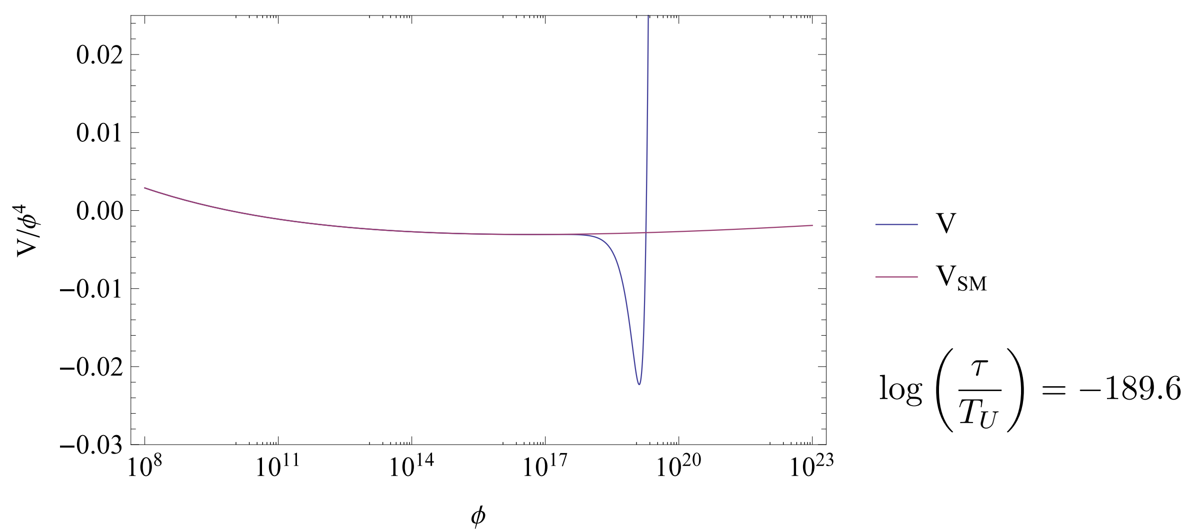


Figure: effective potential with $\lambda_6 = -1$ and $\lambda_8 = 1$.

Numerical vs Analytical again

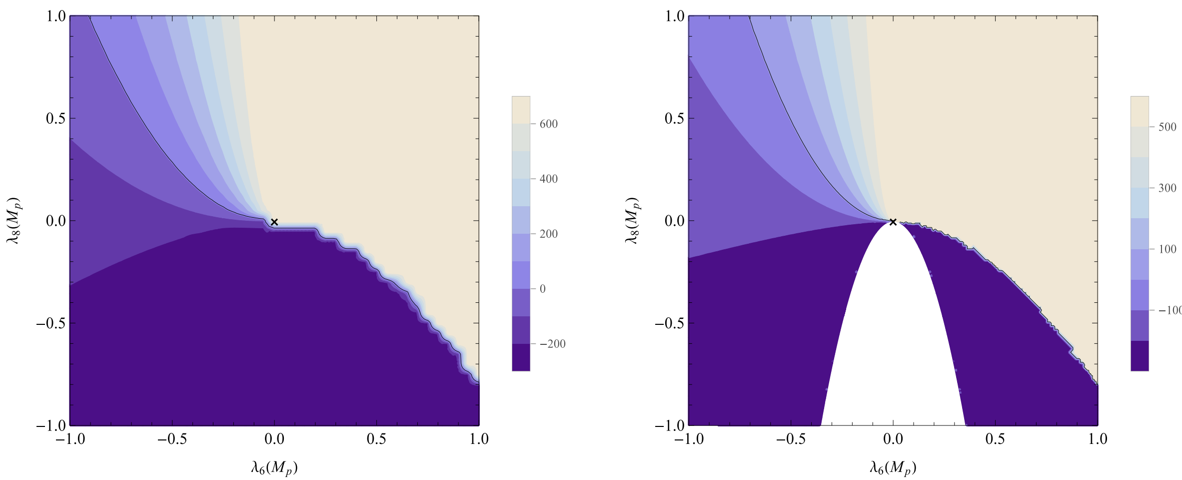


Figure: Decimal logatihm of lifetime of the universe in units of T_U as a function of the nonrenormalisable $\lambda_6(M_p)$ and $\lambda_8(M_p)$ couplings, calculated numerically (left panel) and analytically (right panel).

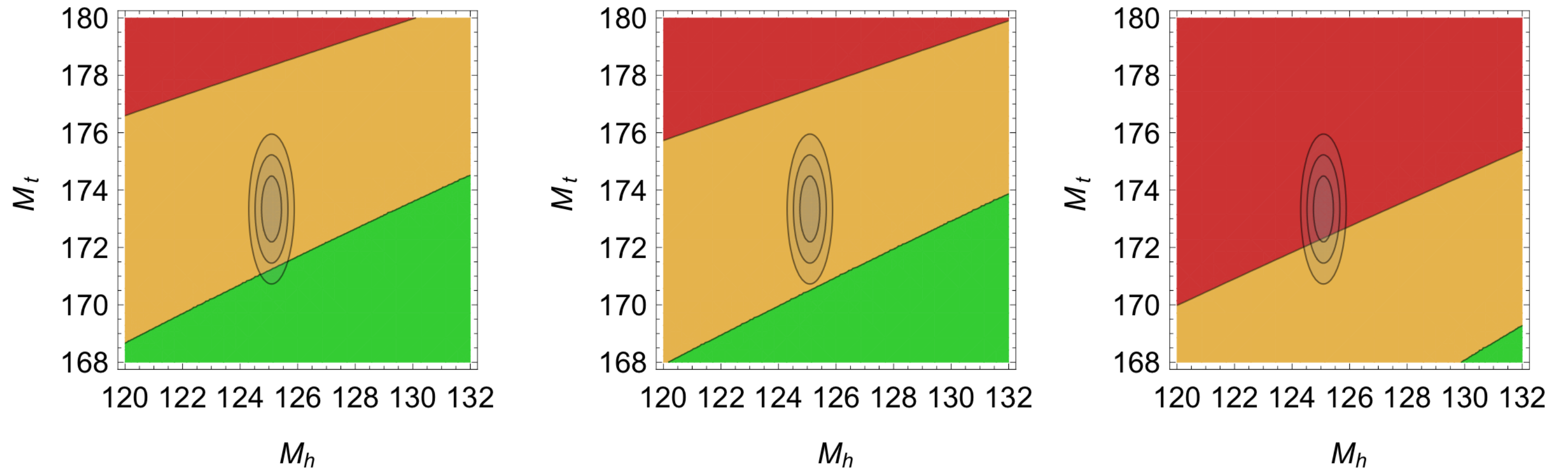


Figure 2: Standard Model phase diagram (left panel), the same diagram after including new operators $\lambda_6(M_p) = -1/2$ and $\lambda_8(M_p) = 1$ (middle panel) and $\lambda_6(M_p) = -1$ and $\lambda_8(M_p) = 1/2$ (right panel). The green region corresponds to absolute stability, the red region to instability, and the yellow region to metastability.

Magnitude of the suppression scale

Approximate lifetime:

$$\frac{\tau}{T_U} = \frac{1}{\mu^4(\lambda_{min})\,T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}}.$$

Positive λ_6 and $\lambda_8 \rightarrow$ stabilizing the potential

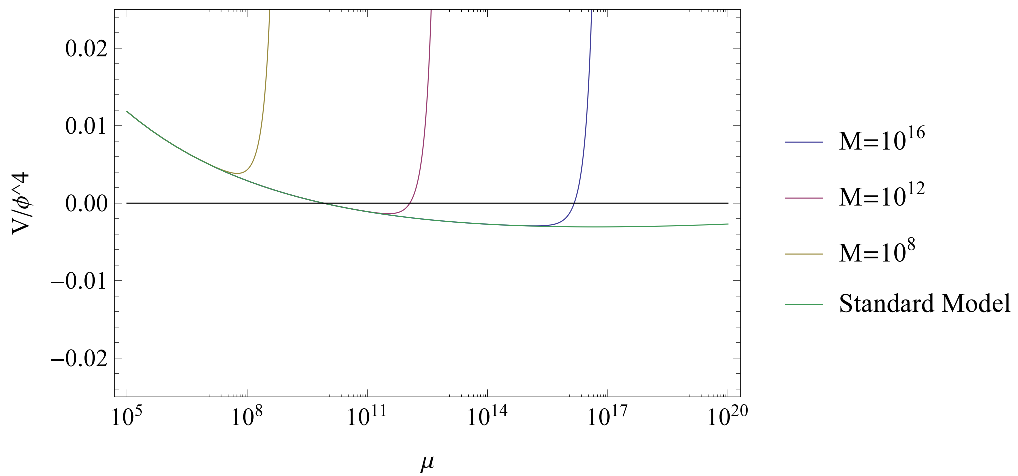


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = \lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$ are, respectively, $\log_{10}(\frac{\tau}{T_U}) = \infty, 1302, 581$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Magnitude of the suppression scale

Positive λ_8 and negative λ_6 $\rightarrow$ New Minimum

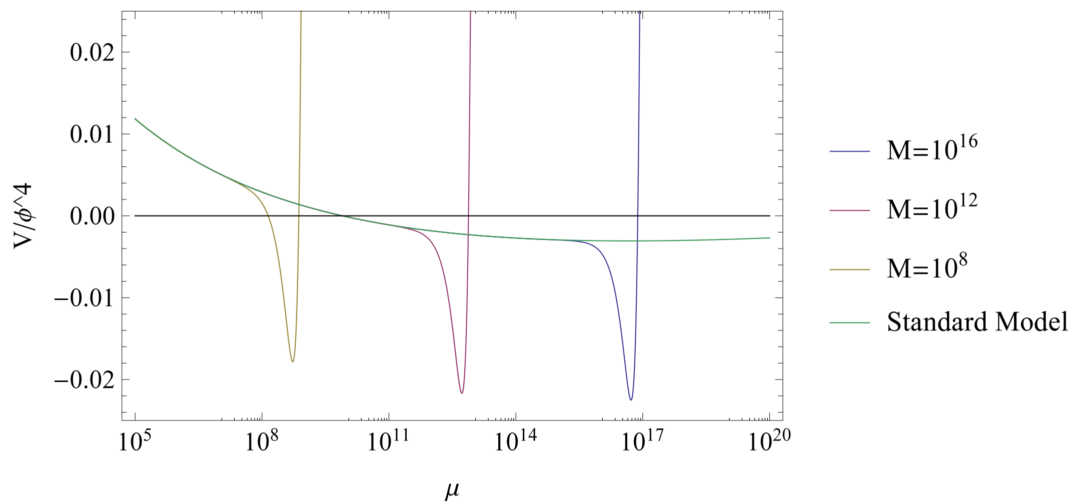


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = -1$ and $\lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$, are, respectively, $\log_{10}(\frac{\tau}{T_U}) = -45, -90, -110$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Gauge dependence of the tunneling rate

It is well known that the effective potential, and in general the effective action, are gauge-dependent objects

However, the statement about the spontaneous breaking of gauge symmetry is gauge invariant (N. K. Nielsen 1975)

The gauge invariant ”observables” are the values of the effective potential at the extrema, and the tunneling rate between different minima

When one computes the SM effective potential in a straightforward manner (say naively), nothing looks gauge independent - neither the value of the effective potential at the extrema (see L. Di Luzio and L. Mihaila 2014) nor the tunneling rate (ML,PO,ZL)

The leading gauge dependence comes from the gauge-dependent anomalous rescaling of the field

$$\mathcal{L}_{gauge\ fixing} = -\frac{1}{2\xi_W}(\partial^\mu W_\mu^{a\,2}) - \frac{1}{2\xi_B}(\partial^\mu B_\mu{}^2)$$

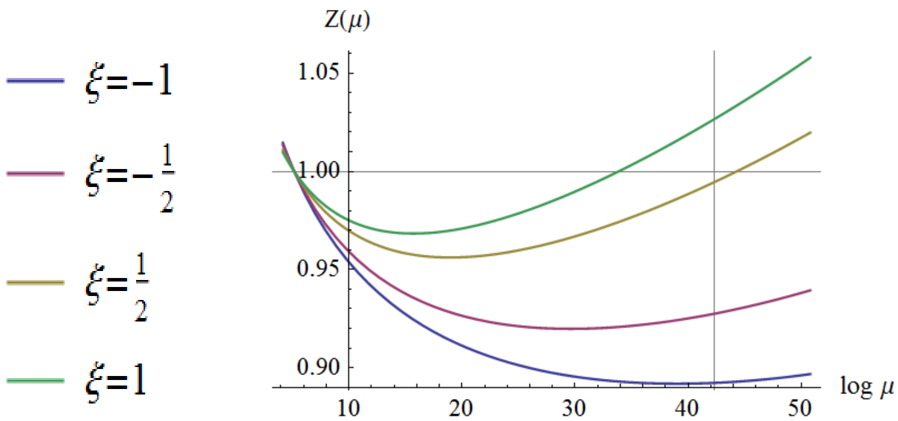
Contributes to:

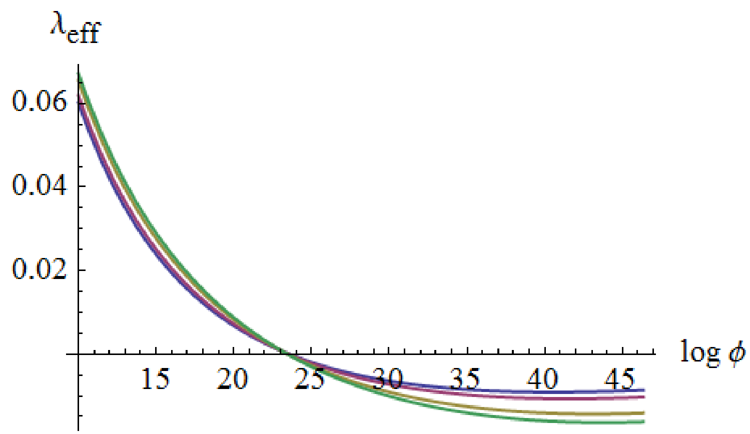
- 1-loop potential
- γ function of the scalar field

- More important.
- One needs to remember that kinetic contribution to the action is mulipied by Z.

$$\gamma = \frac{1}{16\pi^2}\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2 - 3y_b^2 - y_\tau^2 + \frac{3}{20}\zeta_B g_1^2 + \frac{3}{4}\zeta_W g_2^2\right)$$

L. Di Luzio, L. Mihaila 1404.7450





$$\xi = \xi_W(m_t) = \xi_B(m_t)$$

At one loop effective potential contains gauge-dependent terms

$$V^{1,\xi} = -\frac{1}{256\pi^2}\lambda h^4 \left[\xi_B g_1^2 \left(\log \frac{\lambda h^4(\xi_B g_1^2+\xi_W g_2^2)}{4\mu^4} - 3\right) + \xi_W g_2^2 \left(\log \frac{\lambda^3 h^{12}\xi_W^2 g_2^4(\xi_B^2 g_1^2+\xi_W g_2^2)}{64\mu^{12}} - 9\right)\right]$$

As pointed out by A. Andreassen, W. Frost and M. Schwartz 2014, who followed E. Weinberg and D. Metaxas 1996 and S. Coleman and E. Weinberg 1973, the key to save in the calculations the gauge independence of the potantial at the extrema is to realize, that to create extrema radiatively, loop corrections have to cancel between themselves or the tree-level contributions

In CW model $$\lambda \sim \frac{\hbar e^4}{16\pi^2}$$

In the SM the equivalent condition is

$$\lambda = \frac{\hbar}{256\pi^2}\left[g_1^4 + 2g_1^2 g_2^2 + 3g_2^4 - 48h_t^4 - 3(g_1^2+g_2^2)^2 \log\frac{g_1^2+g_2^2}{4} - 6g_2^4 \log\frac{g_2^2}{4} + 48y_t^4 \log\frac{y_t^2}{2}\right]$$

which holds at the extrema $h = \mu$

Hence λ is of the order $\hbar\, g^4$ and gives a higher order contribution

It has been shown that that taking this relation into account in counting radiative contributions in the SM makes the value of the potential at the extrema gauge independent at LO $(\hbar\, g^4)$ and NLO $(\hbar\, g^6)$

In general, the tunneling rate has the form

$$\Gamma = Ae^{-B}$$

Weinberg and Mataxas argued that if the reordering of the radiative corrections used above holds everywhere, not only at the extrema, then the exponent B shall be gauge independent at the NLO.

Leading Order

Gauge-fixing dependent terms in the effective potential are order g^6 and corrections to the kinetic term are g^2.

Observation, which allows one to ease the problem, is that once one includes in the euclidean action which is used to compute the bounce the renormalization factor in the 2-derivative term, and treats it consistently as a field dependent quantity, then one can go over to the new field variable $h \to \sqrt{Z(h)}h$ in terms of which the whole action becomes gauge independent at the modified leading order (that is assuming $\lambda \sim \hbar$), and only mildly gauge dependent in the more standard expression, through small logarithmic terms.

The LO procedure leading to gauge independent estimate the tunneling rate can easily be extended to the analysis of the role of the effective nonrenormalisable operators, and the results shown correspond to such a case.

Gauge fixing in-dependence

$$\text{order } g^6$$

Gauge fixing independence in abelian Higgs model in t'Hooft gauge

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm int} + \mathcal{L}_{\rm gf} + \mathcal{L}_\psi$$

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\varphi_i\,\partial^\mu\varphi_i - \frac{1}{2}m^2\varphi_i\varphi_i - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_{\rm int} = -Z_\varphi\, g[\epsilon_{ij}(\partial_\mu\varphi_i)\varphi_j]A^\mu + Z_\varphi\,\frac{g^2}{2}\varphi_i\varphi_i A_\mu A^\mu - Z_\lambda\,\frac{\lambda}{4!}(\varphi_i\varphi_i)^2 + \mathcal{L}_{\rm ct}$$

$$\mathcal{L}_{\rm ct} = \frac{1}{2}(Z_\varphi - 1)\partial_\mu\varphi_i\,\partial^\mu\varphi_i - \frac{1}{2}(Z_{m^2} - 1)m^2\varphi_i\varphi_i - \frac{1}{4}(Z_A - 1)F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_{\rm gf} = -\frac{1}{2\xi}(\partial_\mu A^\mu + g\,v\varphi_2)^2$$

$$\mathcal{L}_\psi = \partial_\mu\psi^*\,\partial^\mu\psi + g\,v\varphi_1\,\psi^*\psi$$

$$\varphi_1 \to w + \varphi_1$$

Running and scaling

$$\beta_g = \frac{g^3}{48\pi^2}$$

$$\gamma_\varphi = -\frac{g^2(3-\xi)}{16\pi^2}$$

$$\beta_\lambda = \frac{3g^2}{4\pi^2}\left(3g^2-\lambda\right)$$

$$\beta_{m^2} = -\frac{3g^2}{8\pi^2}m^2$$

$$\beta_\xi = -\xi\,\frac{g^2}{48\pi^2}$$

$$\beta_v = -v\,\frac{g^2}{16\pi^2}\left(\frac{2}{3}+(3-\xi)\right)$$

$$\beta_w = \frac{g^2}{16\pi^2}[w(3-\xi)-2v]$$

$$\gamma_\varphi = \frac{1}{2}\frac{\mathrm{d}\,Z_\varphi}{\mathrm{d}\log\mu}\ , \qquad \Gamma(\mu) = \mathrm{e}^{\,-\int^{\log\mu}\gamma(\mu')\,\mathrm{d}\log(\mu')}\ , \qquad \varphi_1(\mu) = \Gamma(\mu)\,\varphi_1$$

Explicit running

$$g(\mu) = g_0 + \frac{g_0^3}{48\pi^2} \log \frac{\mu}{\mu_0} + \mathcal{O}(g^5)$$

$$\lambda(\mu) = \lambda_0 + \frac{3}{4\pi^2} \left[\left(3g_0^4 - \lambda_0 g_0^2\right) \log \frac{\mu}{\mu_0} - \frac{g_0^6}{\pi^2} \log^2 \frac{\mu}{\mu_0} \right] + \mathcal{O}(g^8)$$

$$m^2(\mu) = m_0^2 \left(1 - \frac{3\, g_0^2}{8\pi^2} \log \frac{\mu}{\mu_0}\right) + \mathcal{O}(g^8)$$

$$\Gamma(\mu) = \Gamma_0 \left(1 + \frac{g_0^2}{16\pi^2} (3 - \xi) \log \frac{\mu}{\mu_0}\right) + \mathcal{O}(g^8)$$

$$\xi(\mu) = \xi_0 + \mathcal{O}(g^2)$$

$$v(\mu) = v_0 + \mathcal{O}(g^2)$$

$$w(\mu) = w_0 + \frac{g_0^2}{16\pi^2} \left(w_0(3 - \xi_0) - 3v_0\right) \log \frac{\mu}{\mu_0} + \mathcal{O}(g^4)$$

where $g_0 = g(\mu_0)$, etc.

Renormalized effective action

$$\mathcal{L} = \frac{1}{2}\left[1 + K_{g^2}(\varphi_1,\, \mu)\right](\partial\varphi_1)^2 \; - \; V_{g^4}(\varphi_1,\, \mu) - V_{g^6}(\varphi_1,\, \mu)$$

$$g \to g(\mu)\,,\ \ldots$$

$$\varphi^\circ \to \Gamma(\mu)\varphi_1 + w(\mu)$$

$$\hat{\varphi}(x) = \Gamma_0\, \varphi_1(x) + w_0\ , \quad \mathcal{L} = \frac{1}{2}\left[1 + K_{g^2}(\hat{\varphi})\right]\partial_\mu\hat{\varphi}\partial^\mu\hat{\varphi} - (V_{g^4} + V_{g^6})(\hat{\varphi})$$

$$K_{g^2} = 3\frac{g_0^2}{(4\pi)^2}\log\frac{g_0^2\hat{\varphi}^2}{\bar{\mu}_0^2} - \xi_0\frac{g_0^2}{(4\pi)^2}\left(\log\frac{-g_0^2 v_0\hat{\varphi}}{\bar{\mu}_0^2} + 1\right) - 2\frac{g_0^2}{(4\pi)^2}\frac{v_0}{\hat{\varphi}}$$

$$V_{g^4} = \frac{m_0^2}{2}\hat{\varphi}^2 + \frac{\lambda_0}{4!}\hat{\varphi}^4 + \frac{3\, g_0^4\hat{\varphi}^4}{64\pi^2}\left(\log\frac{g_0^2\hat{\varphi}^2}{\bar{\mu}_0^2} - \frac{5}{6}\right)$$

$$V_{g^6} = \frac{g_0^2}{32\pi^2}\left[v_0 - (2v_0 + \xi_0\hat{\varphi})\log\frac{-g_0^2 v_0\hat{\varphi}}{\bar{\mu}_0^2}\right]\frac{\partial V_{g^4}(\hat{\varphi})}{\partial\hat{\varphi}}$$

Action is explicitly μ-independent

Gauge fixing independence

$$\text{EOM:} \quad \frac{\delta\Gamma[\phi]}{\delta\phi}\bigg|_{\phi=\phi_{sol}} = 0\,,$$

$$\text{gauge fixing independence:} \quad \xi\frac{\partial}{\partial\xi}\Gamma[\phi_{sol}] = v\frac{\partial}{\partial v}\Gamma[\phi_{sol}] = 0$$

$$S_B = \Gamma_E[\varphi_B]\,, \qquad \frac{\delta\Gamma_E[\phi]}{\delta\phi}\bigg|_{\phi=\varphi_B} = 0 \quad (+\text{ specific boundary conditions for } \phi_B)$$

desired property: $\xi\frac{\partial}{\partial\xi}S_B = v\frac{\partial}{\partial v}S_B = 0$

Nielsen identities:

$$\alpha\frac{\partial\Gamma[\phi]}{\partial\alpha} = \int C^\alpha[\phi]\,\frac{\delta\Gamma[\phi]}{\delta\phi}$$

$$\xi\frac{\partial K_{g^2}}{\partial\xi} = 2\frac{\partial C^\xi_{g^2}}{\partial\varphi_1}\,, \quad \xi\frac{\partial V_{g^6}}{\partial\xi} = C^\xi_{g^2}\frac{\partial V_{g^4}}{\partial\varphi_1} \qquad v\frac{\partial K_{g^2}}{\partial v} = 2\frac{\partial C^v_{g^2}}{\partial\varphi_1}\,, \quad v\frac{\partial V_{g^6}}{\partial v} = C^v_{g^2}\frac{\partial V_{g^4}}{\partial\varphi_1}$$

Nielsen functions

$$C^{\xi}_{g^2} = -\frac{g_0^2\,\hat{\varphi}}{32\pi^2}\,\log\frac{-g_0^2\,v_0\hat{\varphi}}{\bar{\mu}_0^2}\,,$$

$$C^{v}_{g^2} = -\frac{g_0^2}{32\pi^2}\left[v_0 + (2v_0 + \xi_0\hat{\varphi})\log\frac{-g_0^2\,v_0\hat{\varphi}}{\bar{\mu}_0^2}\right]$$

$$\mathcal{L} = \mathcal{L}^0 + \mathcal{L}^1 + \ldots \quad , \qquad \mathcal{L}^0(\varphi) = \frac{1}{2}(\partial_\mu\varphi)^2 + V_{g^4}(\varphi)$$

$$\varphi_B = \varphi_B^0 + \varphi_B^1 + \ldots \qquad \text{where by definition,}$$

$$0 = \left.\frac{\delta\mathcal{L}^0(\varphi)}{\delta\varphi}\right|_{\varphi=\varphi_B^0} \Leftrightarrow \partial_\mu^2\varphi_B^0 = V'_{g^4}(\varphi_B^0)\,.$$

bounce is derived from the lowest nontrivial order, $o(g^4)$, Lagrangian

with these ingredients S_B gauge fixing independent to the order g^6

Back to higher-order operators

$$\delta \mathcal{L}_{g^4} = \frac{\lambda_6}{6} \frac{(\varphi_i \varphi_i)^3}{\Lambda^2} + \frac{\lambda_8}{8} \frac{(\varphi_i \varphi_i)^4}{\Lambda^4} + \ldots .$$

but Nielsen identity:

$$v \frac{\partial K_{g^2}}{\partial v} = 2 \frac{\partial C^v_{g^2}}{\partial \varphi_1} , \qquad v \frac{\partial V_{g^6}}{\partial v} = C^v_{g^2} \frac{\partial V_{g^4}}{\partial \varphi_1}$$

$$\delta^{\rm nonren} V = \frac{\lambda_{60}}{6} \frac{\hat{\varphi}^6}{\Lambda^2} + \frac{\lambda_{80}}{8} \frac{\hat{\varphi}^8}{\Lambda^4} + \ldots + \frac{g_0^2}{32\pi^2} \left[v_0 - (2v_0 + \xi_0 \hat{\varphi}) \log \frac{-g_0^2 v_0 \hat{\varphi}}{\bar{\mu}_0^2} \right] \cdot \left(\lambda_{60} \frac{\hat{\varphi}^4}{\Lambda^2} + \lambda_{80} \frac{\hat{\varphi}^6}{\Lambda^4} + \ldots \right)$$

Action supplemented this way is explicitly scale invariant and gauge fixing invariant

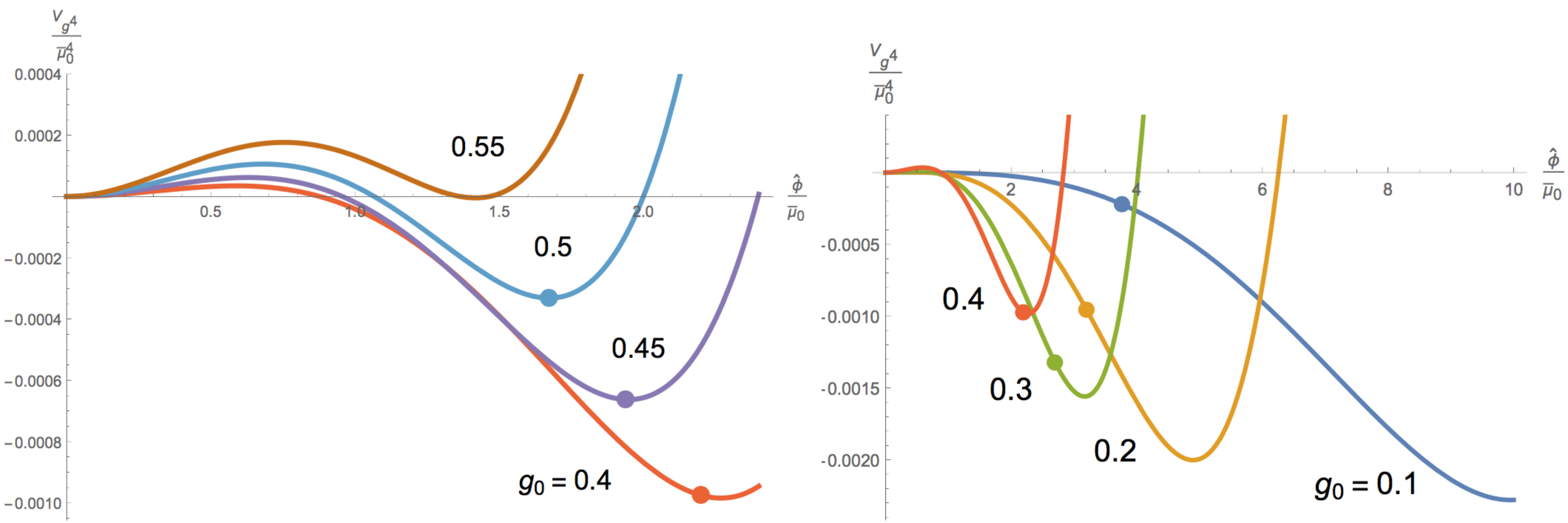
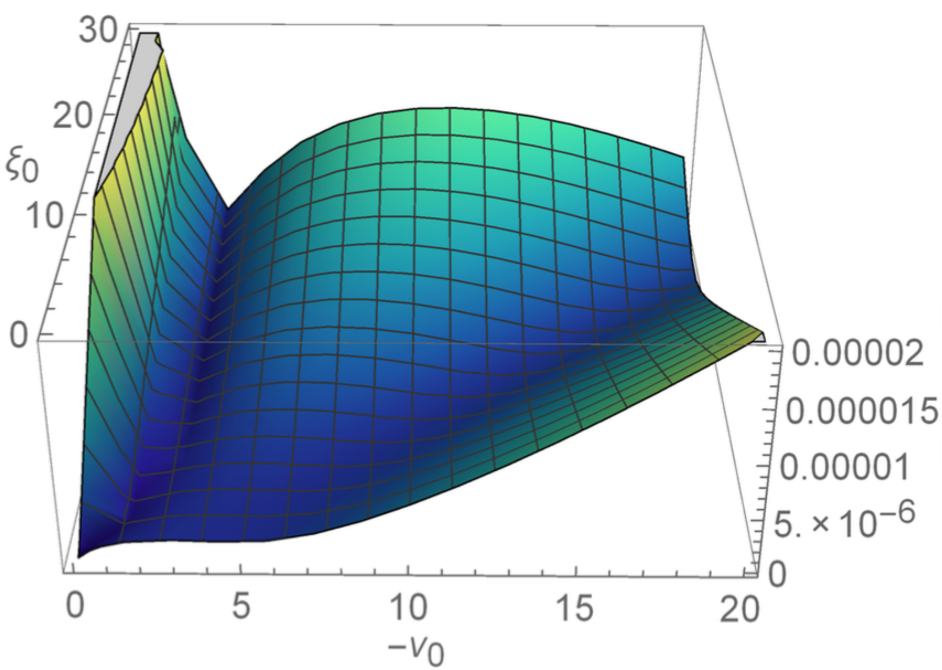
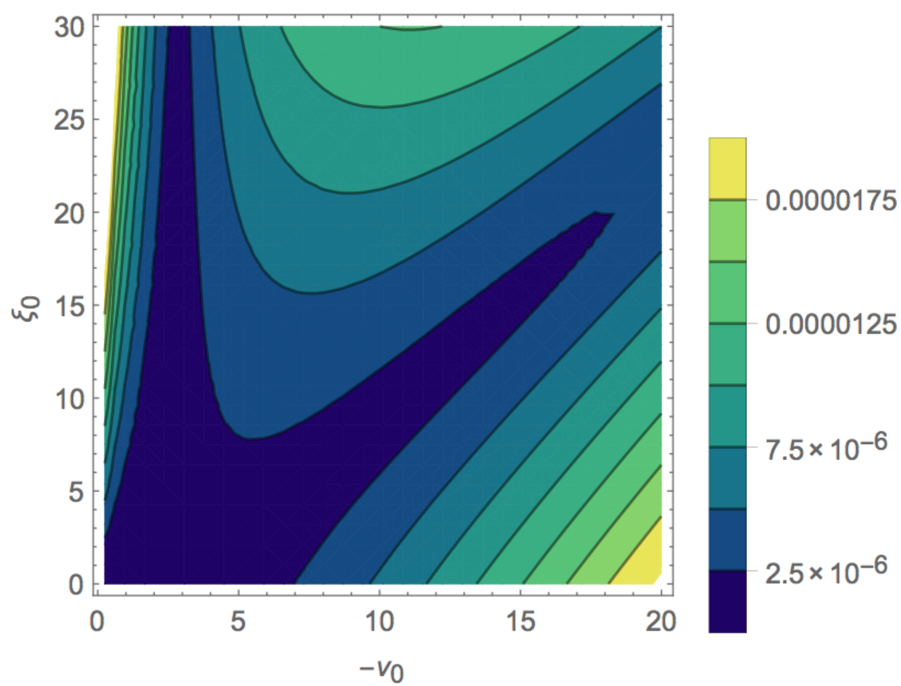


Figure 3: Plots of the potential at the lowest order, V_{g^4}, for a specific choice of couplings (see text). The renormalisation scale μ_0 is used as a unit of energy.

Gauge dependence of the potential

$$\sqrt{\int_0^{\varphi_{\min}} \left(V_{g^6}\right)^2 \, \mathrm{d}\varphi}$$





Gravity Corrections in Curved Space

Effective action in curved background: gauge-less Higgs model

$$
\begin{aligned}
\Gamma = & -\frac{1}{16\pi G}\int\sqrt{-g}d^4x(R+2\Lambda)+\int\sqrt{-g}d^4x\Bigg\{\bar{\chi}\left[i\gamma^\mu\nabla_\mu-\frac{1}{\sqrt{2}}yh\right]\chi+\\
& +\frac{1}{2}\nabla_\mu h\nabla^\mu h-\frac{1}{2}\left(m_h^2-\xi_h R\right)h^2-\frac{\lambda_h}{4}h^4-\frac{\lambda_{hX}}{4}h^2X^2+\\
& +\frac{1}{2}\nabla_\mu X\nabla^\mu X-\frac{1}{2}\left(m_X^2-\xi_X R\right)X^2-\frac{\lambda_X}{4}X^4+\\
& +\frac{\hbar}{64\pi^2}\Bigg[\frac{1}{2}y_t^2\bar{\chi}\left(i\gamma^\mu\nabla_\mu+2\frac{1}{\sqrt{2}}y_t h\right)\chi-\frac{3}{2}y_t^2\nabla_\mu h\nabla^\mu h-2y_t^2\ln\left(\frac{b}{\mu^2}\right)\nabla_\nu h\nabla^\nu h+\\
& -\frac{1}{3}tr\left(\Box a\ln\left(\frac{a}{\mu^2}\right)\right)+\frac{8}{3}\Box b\ln\left(\frac{b}{\mu^2}\right)-tr\left(a^2\ln\left(\frac{a}{\mu^2}\right)\right)+\frac{3}{2}tr a^2+8b^2\ln\left(\frac{b}{\mu^2}\right)-12b^2+\\
& +\frac{1}{3}y_t^2h^2\ln\left(\frac{b}{\mu^2}\right)R-y_t^4h^4\ln\left(\frac{b}{\mu^2}\right)+\\
& -\frac{4}{180}\left(-R_{\alpha\beta}R^{\alpha\beta}+R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\right)\left(\ln\left(\frac{a_+}{\mu^2}\right)+\ln\left(\frac{a_-}{\mu^2}\right)-2\ln\left(\frac{b}{\mu^2}\right)\right)+\\
& -\frac{4}{3}R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\ln\left(\frac{b}{\mu^2}\right)\Bigg]\Bigg\},
\end{aligned}
\tag{2.32}
$$

where:

$$b = \frac{1}{2} y_t^2 h^2 - \frac{1}{12} R,$$

$$a = \begin{bmatrix} m_X^2 - (\xi_X - \frac{1}{6})R + 3\lambda_x X^2 + \frac{\lambda_{hX}}{2} h^2 & \lambda_{hX} hX \\ \lambda_{hX} hX & m_h^2 - (\xi_h - \frac{1}{6})R + 3\lambda_h h^2 + \frac{\lambda_{hX}}{2} X^2 \end{bmatrix}$$

The eigenvalues of the matrix a are

$$a_{\pm} = \frac{1}{2} \Bigg\{ \left[m_X^2 + m_h^2 - \left(\xi_X + \xi_h - \frac{2}{6} \right) R + \left(3\lambda_h + \frac{1}{2} \lambda_{hX} \right) h^2 + \left(3\lambda_X + \frac{1}{2} \lambda_{hX} \right) X^2 \right] + \\ \pm \sqrt{ \left[m_X^2 - m_h^2 - \left(\xi_X - \xi_h \right) R + \left(\frac{1}{2} \lambda_{hX} - 3\lambda_h \right) h^2 + \left(3\lambda_X - \frac{1}{2} \lambda_{hX} \right) X^2 \right]^2 + 4 \left(\lambda_{hX} hX \right)^2 } \Bigg\}$$

$$\beta_{y_t} = \frac{\hbar}{(4\pi)^2}\frac{5}{2}y_t^3,$$

$$\beta_{\lambda_h} = \frac{\hbar}{(4\pi)^2}\left[18\lambda_h^2 - 2y_t^4 + 4y_t^2\lambda_h + \frac{1}{2}\lambda_{hX}^2\right],$$

$$\beta_{\lambda_X} = \frac{\hbar}{(4\pi)^2}\left[18\lambda_X^2 + \frac{1}{2}\lambda_{hX}^2\right],$$

$$\beta_{\lambda_{hX}} = \frac{\hbar}{(4\pi)^2}\left[4\lambda_{hX}^2 + 6\lambda_{hX}(\lambda_h + \lambda_X) + 2\lambda_{hX}y^2\right]$$

$$\beta_{m_h^2} = \frac{\hbar}{(4\pi)^2}\left[6\lambda_h m_h^2 + 2y_t^2 m_h^2 + \lambda_{hX}m_X^2\right],$$

$$\beta_{m_X^2} = \frac{\hbar}{(4\pi)^2}\left[6\lambda_x m_X^2 + \lambda_{hX}m_h^2\right],$$

$$\beta_{\xi_h} = \frac{\hbar}{(4\pi)^2}\left[6\lambda_h(\xi_h - \frac{1}{6}) + \lambda_{hX}(\xi_x - \frac{1}{6}) + 2y_t^2(\xi_h - \frac{1}{6})\right],$$

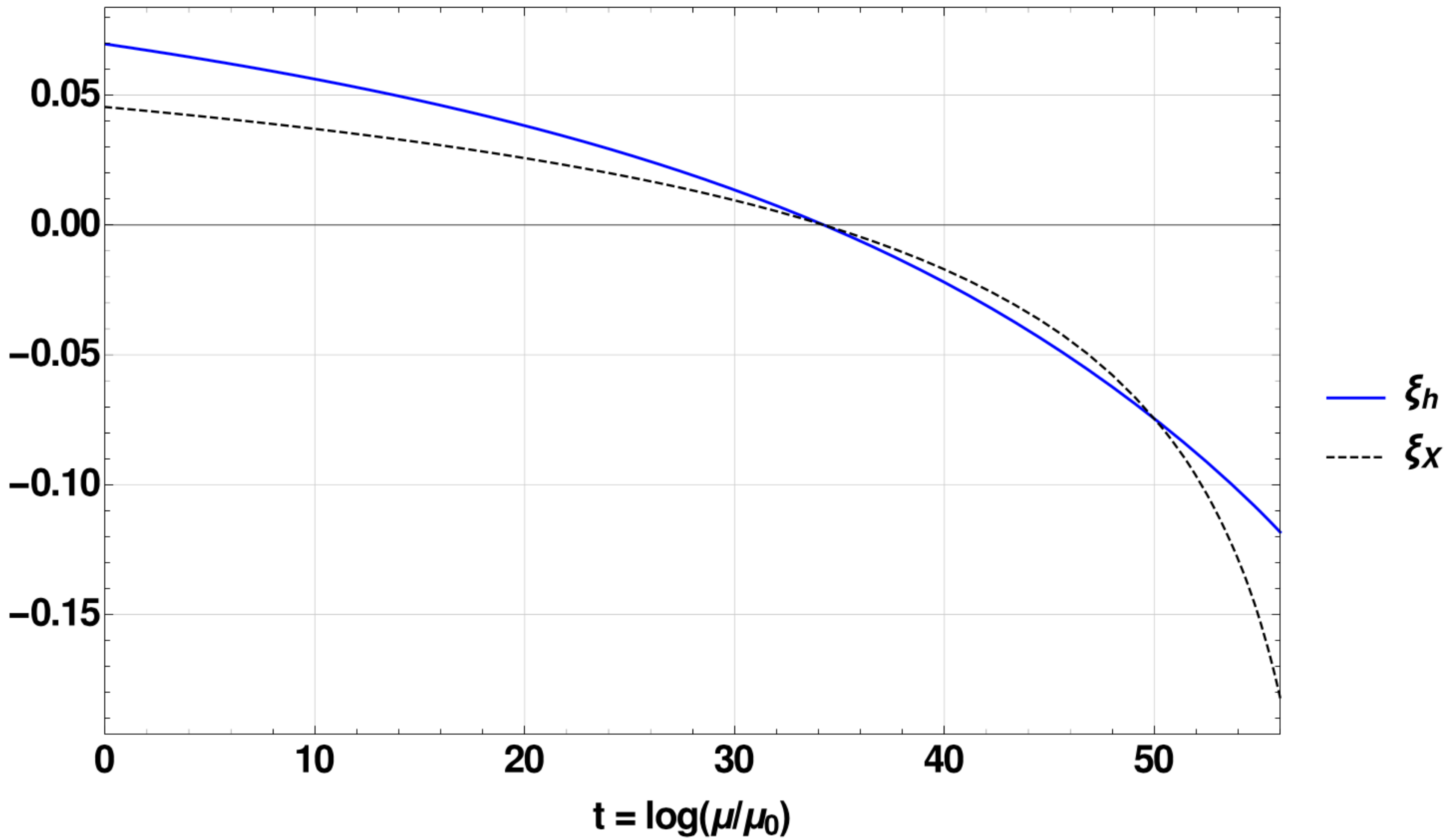
$$\beta_{\xi_X} = \frac{\hbar}{(4\pi)^2}\left[6\lambda_X(\xi_X - \frac{1}{6}) + \lambda_{hX}(\xi_h - \frac{1}{6})\right].$$

$$\gamma_h = \frac{\hbar}{(4\pi)^2}y_t^2,$$

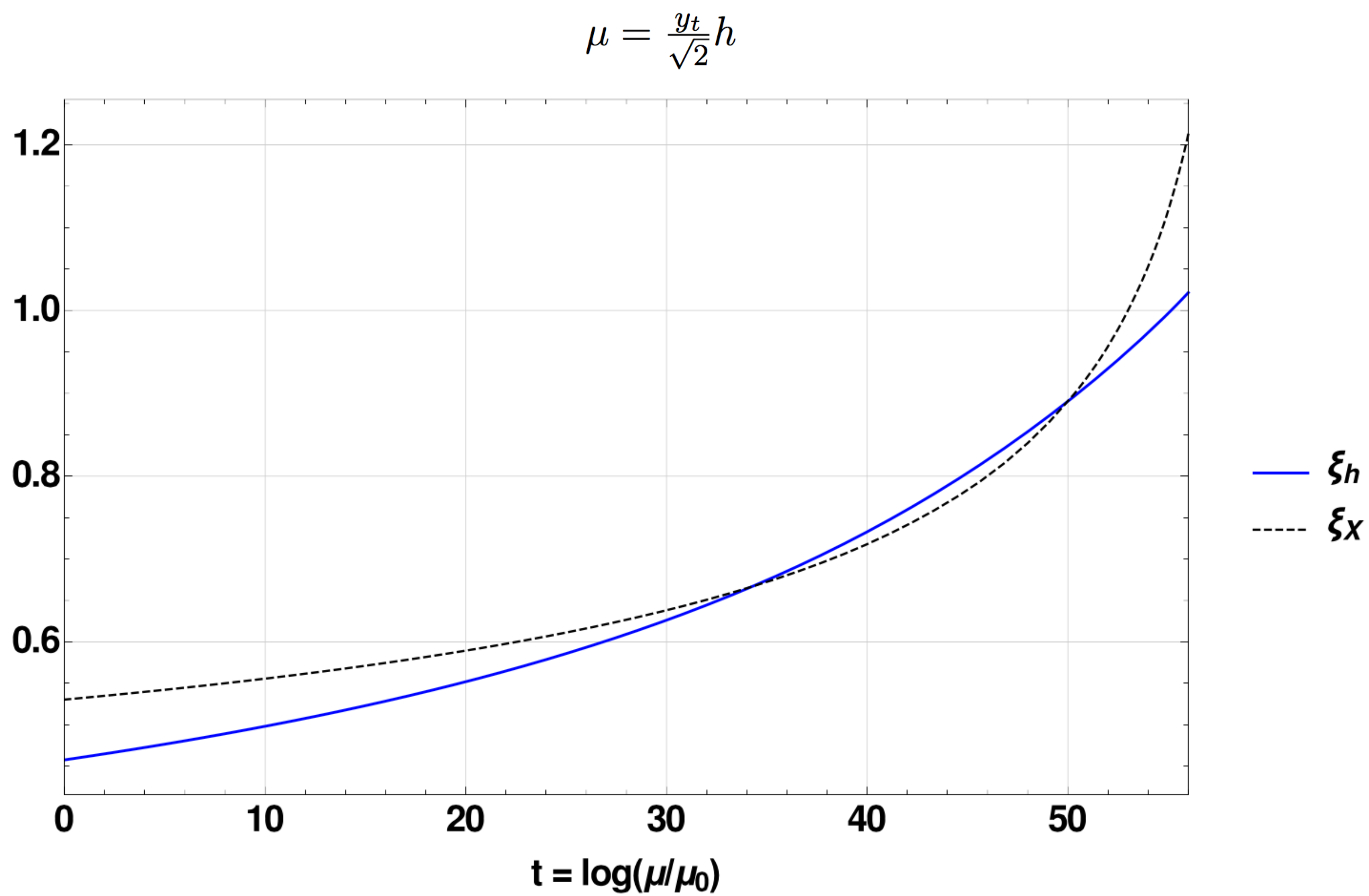
$$\gamma_X = 0,$$

$$\gamma_\chi = \frac{\hbar}{(4\pi)^2}\frac{1}{4}y_t^2.$$

$\mu = \frac{y_t}{\sqrt{2}} h$



(b) The running of the nonminimal couplings to the gravity for the scalar fields, the initial conditions were $\xi_h = \xi_X = 0$ at the $\mu = m_t$. The energy range is $\mu_0 = 2.7K - \mu_{max} = 10^{11}$ GeV.



(c) The running of the nonminimal couplings to the gravity for the scalar fields, the initial conditions were $\xi_h = \xi_X = \frac{1}{3}$ at the $\mu = m_t$. The energy range is $\mu_0 = 2.7K - \mu_{max} = 10^{11}$ GeV.

In Robertson-Walker background one may express curvature invariant through energy density and preassure

$$R = -3\bar{M}_P^{-2}\left[-p + \frac{1}{3}\rho\right]$$

RD

$$-R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = -12H^2\frac{\ddot{a}}{a} = 2M_P^{-4}\rho\left(\frac{1}{3}\rho + p\right) = \frac{4}{3}\left(\bar{M}_P^{-2}\rho\right)^2$$

$$R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = 12\left[H^4 + \left(\frac{\ddot{a}}{a}\right)^2\right] = 12M_P^{-4}\left[\frac{1}{9}\rho^2 + \frac{1}{4}\left(\frac{1}{3}\rho + p\right)^2\right] = \frac{8}{3}\left(\bar{M}_P^{-2}\rho\right)^2$$

$$\rho = \sigma\nu^4 + (\tfrac{y_t}{\sqrt{2}}h)^4$$

$$\mu = \tfrac{y_t}{\sqrt{2}}h$$

Quadratic part of the potential

in RD

$$V(h^2) = \frac{1}{2}m_h^2 h^2 + \frac{1}{64\pi^2}\frac{4}{180}\left(-R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\right)\tilde{b} =$$

$$= \left[\frac{1}{2}m_h^2 + \frac{1}{64\pi^2}\frac{4}{180}\left(-R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\right)\frac{\tilde{b}}{h^2}\right] h^2 =$$

$$= \left[\frac{1}{2}m_h^2 + \frac{1}{64\pi^2}\frac{4}{180}\frac{4}{3}\left(\bar{M}_P^{-2}\rho\right)^2\frac{\tilde{b}}{h^2}\right]_{|h=v_h} h^2 = m_{eff}^2 h^2,$$

in dS or MD

$$V(h^2) = \left[\frac{1}{2}m_h^2 + \tilde{\beta}T^2 - \frac{1}{2}\xi_h R\right] h^2 = \left[-\frac{1}{2}|m_h^2| + \tilde{\beta}\nu^2 + 2\xi_h \bar{M}_P^{-2}\sigma\nu^4\right] h^2$$

Critical Temperature in RD

$$\frac{1}{2}m_h^2 + \frac{1}{64\pi^2}\frac{4}{180}\left(-R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\right)\frac{\tilde{b}}{v_h^2} + \tilde{\beta}T^2 = 0$$

$$T_c = \sqrt{\frac{|m_h^2|}{2\tilde{\beta}} - \frac{|m_h^2|^4}{16\tilde{\beta}^5}\frac{1}{64\pi^2}\frac{16\tilde{b}}{640v_h^2}\bar{M}_P^{-4}}$$

Critical Temperature in dS

$$V(h^2) = \left[\frac{1}{2}m_h^2 + \tilde{\beta}T^2 - \frac{1}{2}\xi_h R\right]h^2$$

vs

$$T^{dS} = \frac{H}{2\pi}$$

$$T_c^{dS} = \sqrt{\frac{1}{2}|m_h^2|\frac{1}{1 + 24\pi^2|\xi_h|}}$$

Large field region

Stability in RD

$$V(h^4) = \frac{\lambda_{eff}(h)}{4} h^4 + V^{(1)}_{grav}$$

$$V(h^4) = \frac{\lambda_{eff}(h)}{4} h^4 + \frac{1}{64\pi^2}\frac{4}{3} R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} \ln\left(\frac{b}{\mu^2}\right) =$$

$$= \frac{1}{4}\left[\lambda_{eff}(h) + \frac{4}{64\pi^2}\frac{4}{3}\frac{8}{3}\left(\bar{M}_P^{-2}\rho\right)^2 \frac{\tilde{c}}{h^4}\right]_{|h=h_0} h^4 = \frac{1}{4}\bar{\lambda}_{eff}(h) h^4,$$

$$\rho = 4\pi h_0^2 \bar{M}_P^2 \sqrt{\frac{9\tilde{d}}{32\tilde{c}}}$$

➡ For $\tilde{d} = |\lambda_{eff}| \sim 0.02$ we obtain the energy scale $\nu \sim 10^{14}\,\mathrm{GeV}$

Stability in dS

$$V(h^4) = \frac{1}{4}\left[\lambda_{eff}(h) - 2\xi_h \frac{R}{h^2}\right]_{|h=h_0} h^4$$

$$\rho = \frac{1}{8\xi_h}\bar{M}_P^2 h_0^2 \tilde{d}$$ ➡ $$\nu \sim 7\cdot 10^{13}\,\mathrm{GeV}$$

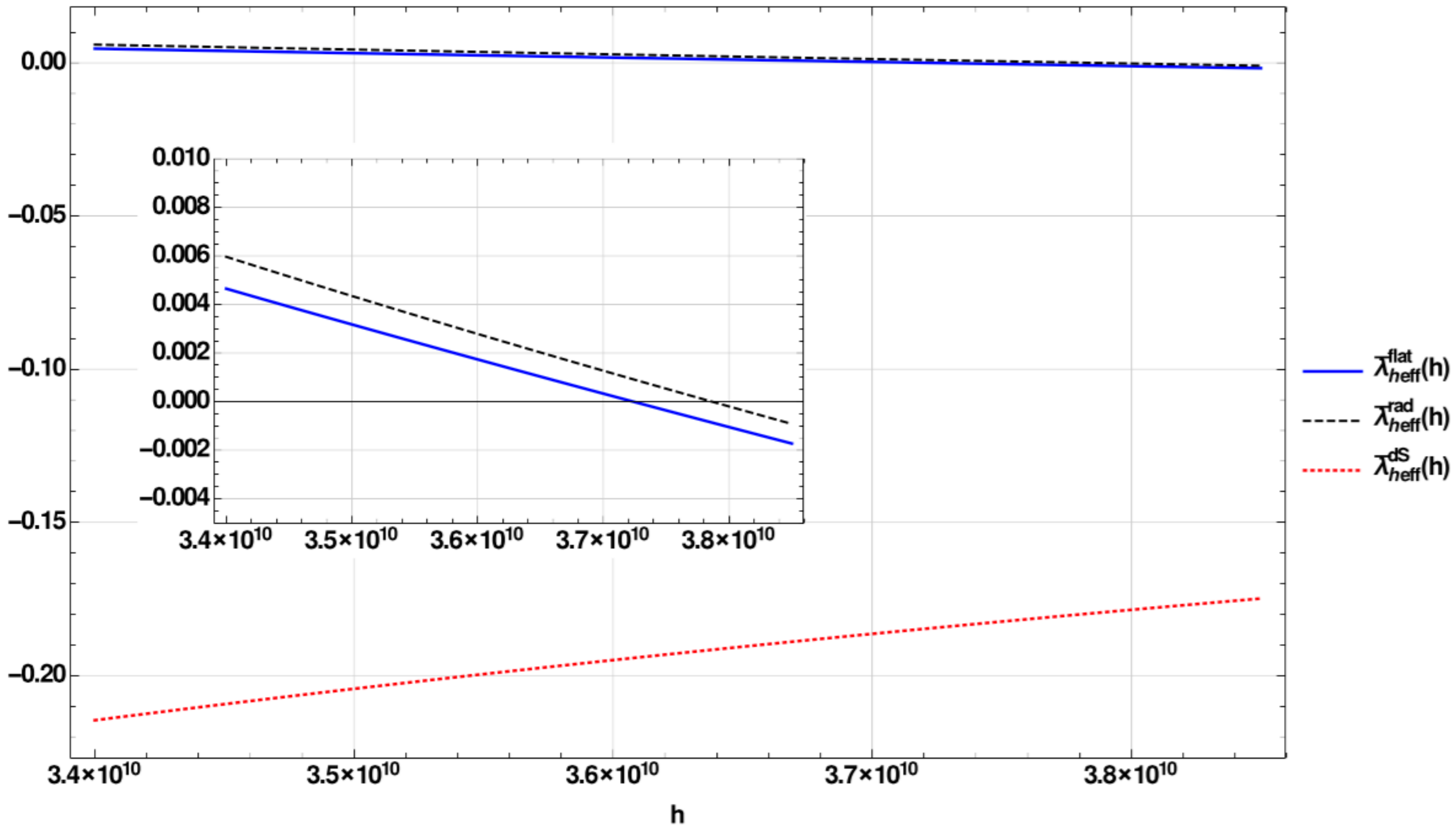


Figure 10: The effective quartic Higgs coupling, as defined by the relation $\bar{\lambda}_{heff}(h) \equiv \frac{4V^{(1)}(h)}{h^4}$, for various equations of state: $flat$ – flat spacetime result, rad – radiation dominance ($p = \frac{1}{3}\rho$), dS – de Sitter like ($p = -\rho$). The energy density was given by $\rho = \rho_{hc} + (\frac{y_t h}{\sqrt{2}})^4$, where ρ_{hc} was specified by the relation (4.36) and equal to $\rho_{hc} = (2.04 \cdot 10^{14} GeV)^4$. The X field was constant and set as equal to $X = v_X$. The non-minimal couplings were $\xi_h = \xi_X = 0$ at the $\mu = m_t$. The insert shows a close up of the difference for the flat spacetime and the radiation dominated era.

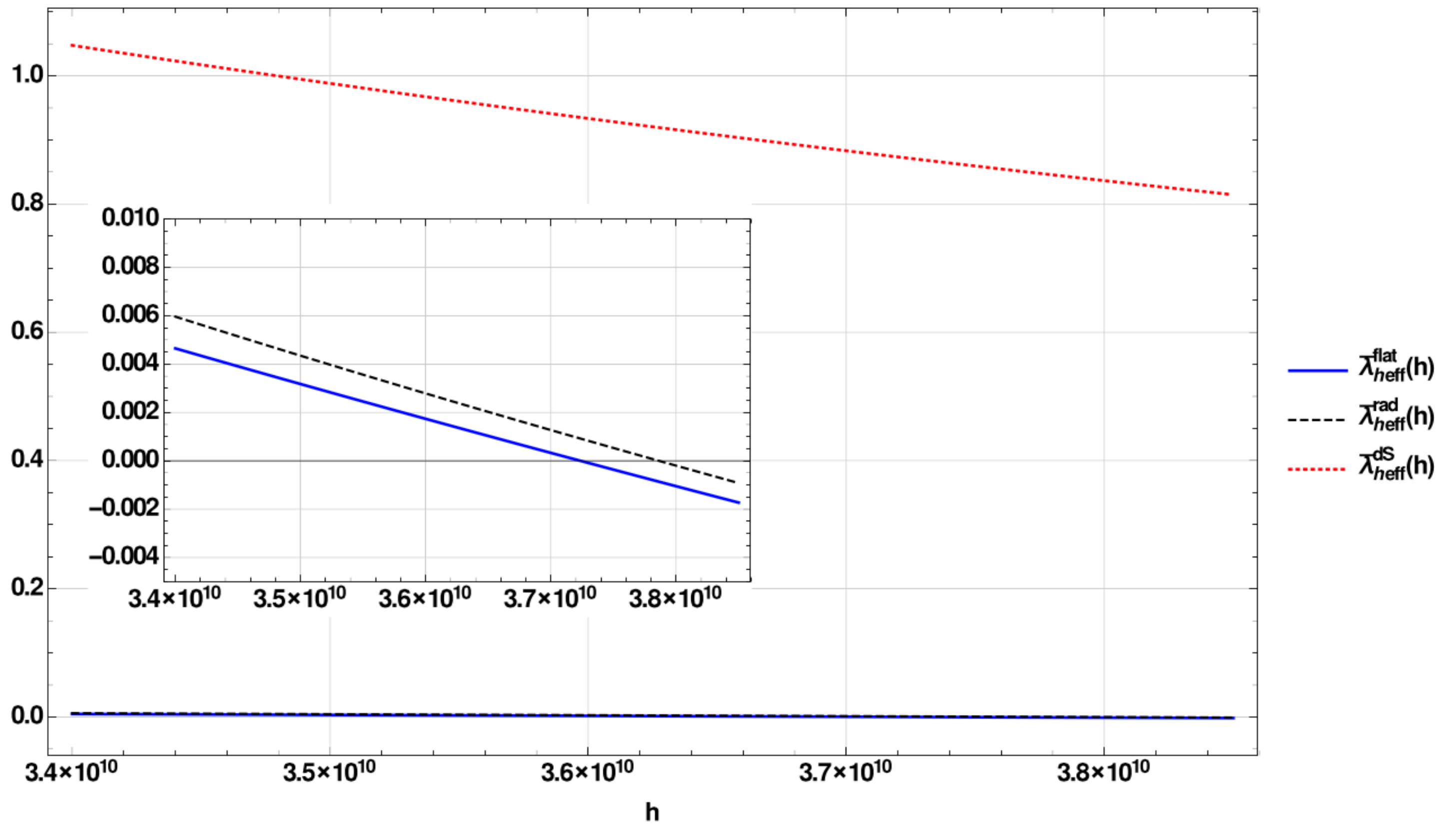


Figure 11: The effective quartic Higgs coupling, as defined by the relation $\bar{\lambda}_{heff}(h) \equiv \frac{4V^{(1)}(h)}{h^4}$, for various equations of state: *flat* – flat spacetime result, *rad* – radiation dominance ($p = \frac{1}{3}\rho$), *dS* – de Sitter like ($p = -\rho$). The energy density was given by $\rho = \rho_{hc} + (\frac{y_t h}{\sqrt{2}})^4$, where ρ_{hc} was specified by the relation (4.36) and equal to $\rho_{hc} = (2.04 \cdot 10^{14} GeV)^4$. The X field was constant and set as equal to $X = v_X$. The non-minimal couplings were $\xi_h = \xi_X = \frac{1}{3}$ at the $\mu = m_t$. The insert shows a close up of the difference for the flat spacetime and the radiation dominated era.

Quantum gravity effects:

In Loop Quantum Cosmology holonomy corrections can be summarized as

$$\rho \to \rho \left(1 - \frac{\rho}{\rho_{cr}} \right)$$

Hence, for given ρ the correction becomes smaller.

Summary

- SM vacuum can be stabilized by higher order operators if they appear at suffciently low energy scale $10^{10} - 10^{11}\,\mathrm{GeV}$
- SM vacuum lifetime can be dramatically shortened by higher order operators for any suppression scale
- Beyond the leading order one needs to define proper expansion of the action to demonstrate perturbatively the cancellation of gauge-dependent contributions to the lifetime of the EW vacuum
- In the abelian Higgs model such a procedure can be carried out at the level of the renormalized effective action
- Peoperties of the electroweak vacuum - critical temperature and lifetime - can be modified by a fast expansion of the gravitational background