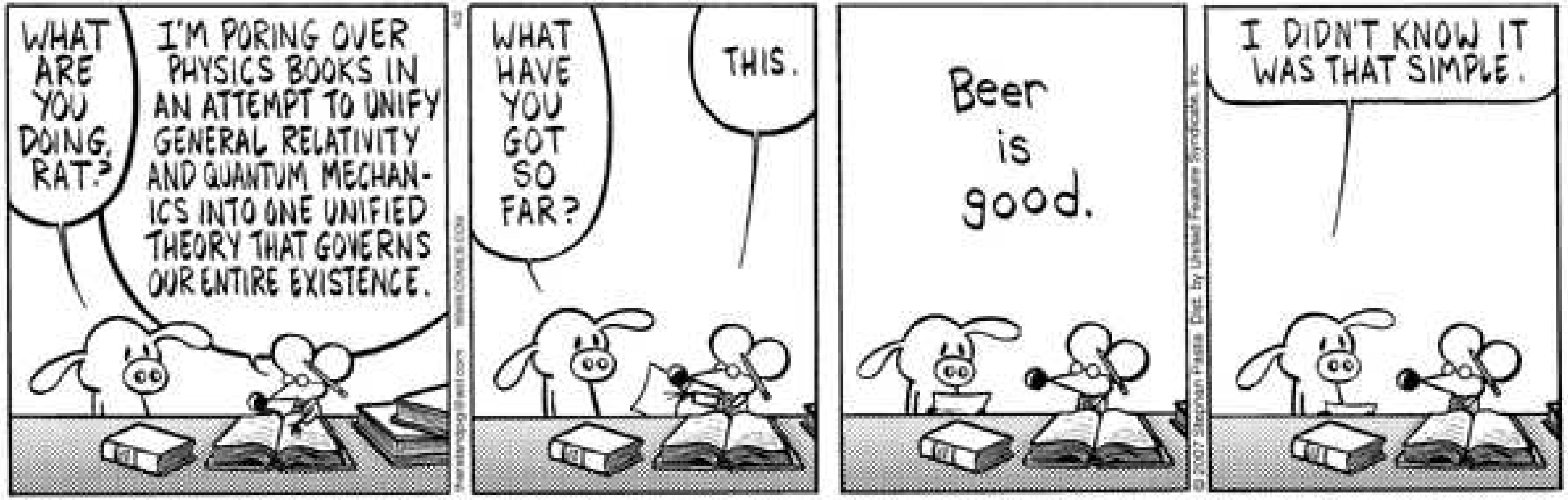

WHAT ARE YOU DOING, RAT?
I'M PORING OVER PHYSICS BOOKS IN AN ATTEMPT TO UNIFY GENERAL RELATIVITY AND QUANTUM MECHANICS INTO ONE UNIFIED THEORY THAT GOVERNS OUR ENTIRE EXISTENCE.
WHAT HAVE YOU GOT SO FAR?
THIS.
Beer is good.
I DIDN'T KNOW IT WAS THAT SIMPLE.

Higgs Physics

Sven Heinemeyer, IFCA (Santander)

Corfu, 09/2015

1. The Higgs in the Standard Model

2. The Higgs sector of the (N)MSSM

Higgs Physics

The Higgs sector in the (N)MSSM

Sven Heinemeyer, IFCA (Santander)

Corfu, 09/2015

1. Why SUSY?
2. The MSSM Higgs sector
3. Predictions for the light MSSM Higgs mass before discovery
4. The MSSM Higgs sector with complex parameters
5. The NMSSM Higgs sector

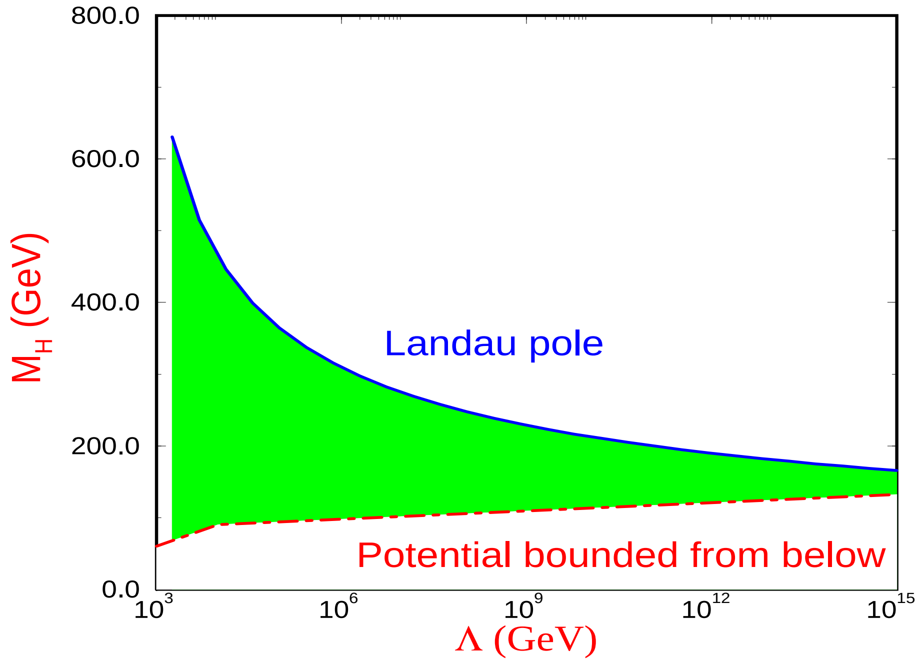
1. Why SUSY?

The Standard Model (SM) cannot be the ultimate theory

- The SM does not contain gravity
- Further problems: Hierarchy problem
- And another one: SM does not provide Cold Dark Matter candidate

Up to which energy scale Λ can the SM be valid?

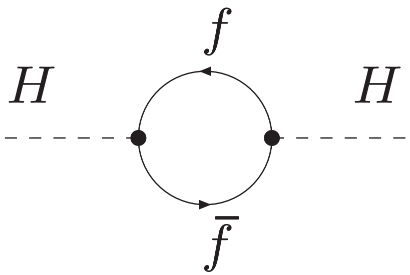
- $\Lambda < M_{\rm Pl}$: inclusion of gravity effects necessary
- stability of Higgs potential: $\Rightarrow$
- Hierarchy problem : Higgs mass unstable w.r.t. quantum corrections $\delta M_H^2 \sim \Lambda^2$



Mass is what determines the properties of the free propagation of a particle

Free propagation: 

inverse propagator: $i(p^2 - M_H^2)$

Loop corrections: 

inverse propagator: $i(p^2 - M_H^2 + \Sigma_H^f)$

QM: integration over all possible loop momenta k

dimensional analysis:

$$\Sigma_H^f \sim N_f \, \lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

$$\text{for } \Lambda \to \infty : \quad \Sigma_H^f \sim N_f \, \lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim \Lambda^2} + 2m_f^2 \underbrace{\int \frac{dk}{k}}_{\sim \ln \Lambda} \right)$$

$\Rightarrow$ quadratically divergent!

For $\Lambda = M_{\mathrm{Pl}}$:

$$\Sigma_H^f \approx \delta M_H^2 \sim M_{\mathrm{Pl}}^2 \quad \Rightarrow \quad \delta M_H^2 \approx 10^{30}\, M_H^2$$

(for $M_H \lesssim 1$ TeV)

– no additional symmetry for $M_H = 0$

– no protection against large corrections

$\Rightarrow$ Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale

E.g.: Grand Unified Theory (GUT): $\delta M_H^2 \approx M_{\mathrm{GUT}}^2$

Note however: there is another fine-tuning problem in nature, for which we have no clue so far – cosmological constant

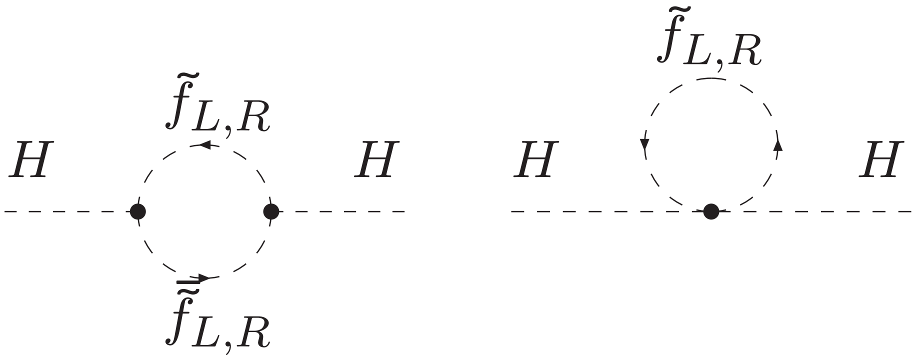
Supersymmetry:

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Effectively: SM particles have SUSY partners (e.g. $f_{L,R} \to \tilde{f}_{L,R}$)

SUSY: additional contributions from scalar fields:



$$\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2 \int d^4k \left(\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) + \text{ terms without quadratic div.}$$

for $\Lambda \to \infty$: $\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2\, \Lambda^2$

⇒ quadratic divergences cancel for

$$N_{\tilde{f}_L} = N_{\tilde{f}_R} = N_f$$
$$\lambda^2_{\tilde{f}} = \lambda^2_f$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

Soft SUSY breaking: $m^2_{\tilde{f}} = m^2_f + \Delta^2, \quad \lambda^2_{\tilde{f}} = \lambda^2_f$

$$\Rightarrow \; \Sigma_H^{f+\tilde{f}} \sim N_f \, \lambda^2_f \, \Delta^2 + \ldots$$

⇒ correction stays acceptably small if mass splitting is of weak scale

⇒ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1\,\text{TeV}$$

⇒ SUSY at TeV scale provides attractive solution of hierarchy problem

Physics beyond the SM:

Interesting (new) physics models :

- 2HDM:
  - two Higgs doublets more natural than one
- MSSM:
  - solves hierarchy problem
  - automatic electroweak symmetry breaking
  - gauge coupling unification
  - cold dark matter candidate
- Little Higgs:
  - (partially) solves the hierarchy problem
  - cold dark matter candidate
- Extra dimensions:
  - solves the hierarchy problem
  - cold dark matter candidate
- ...

$\Rightarrow$ pick your favorite model now (I pick the MSSM)

Supersymmetry:

Symmetry between

$$\text{Bosons} \leftrightarrow \text{Fermions}$$

$$Q\ |\text{Fermion}\rangle \rightarrow |\text{Boson}\rangle$$
$$Q\ |\text{Boson}\rangle \rightarrow |\text{Fermion}\rangle$$

Simplified examples:

$$Q\ |\text{top},\ t\rangle \rightarrow |\text{scalar top},\ \tilde{t}\rangle$$
$$Q\ |\text{gluon},\ g\rangle \rightarrow |\text{gluino},\ \tilde{g}\rangle$$

$\Rightarrow$ each SM multiplet is enlarged to its double size

Unbroken SUSY: All particles in a multiplet have the same mass

Reality: $m_e \neq m_{\tilde{e}}$ $\Rightarrow$ SUSY is broken . . .

. . . via soft SUSY-breaking terms in the Lagrangian (added by hand)

SUSY particles are made heavy: $\boxed{M_{\text{SUSY}} = \mathcal{O}(1\ \text{TeV})}$

Supersymmetry: Motivation

The SM is in a pretty good shape.

Why MSSM? (Is it worth to double the particle spectrum?)

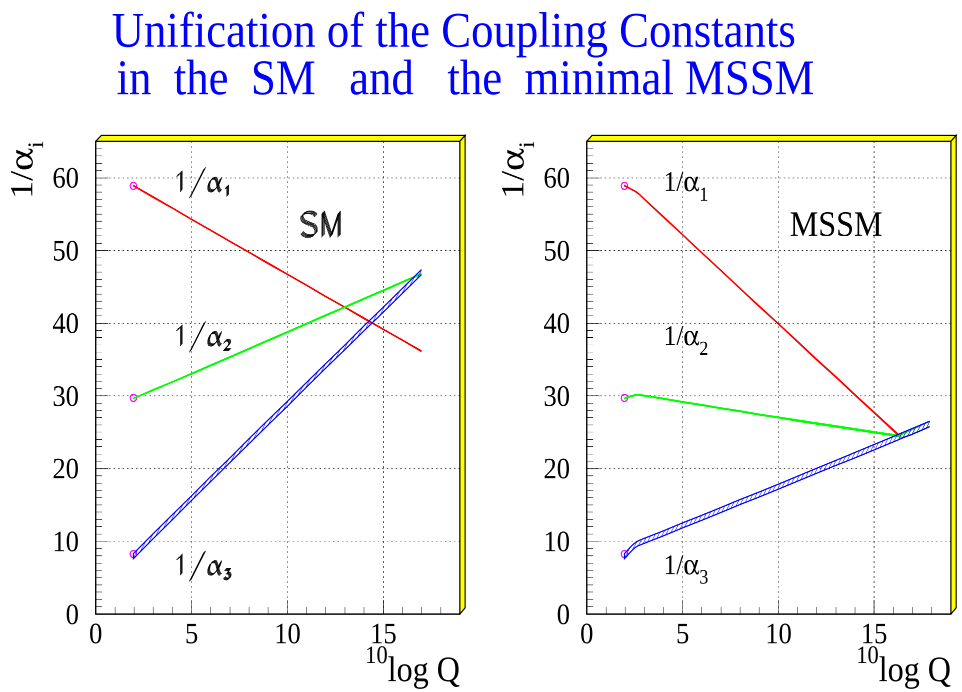
1.) Stability of the Higgs mass against higher-order corr.

2.) Unification of gauge couplings: Not possible in the SM, but in the MSSM (although it was not designed for it.)

3.) Spontaneous symmetry breaking via Higgs mechanism is automatic in SUSY GUTs

4.) SUSY provides CDM candidate

5.) ...



[*Amaldi, de Boer, Fürstenau '92*]

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \qquad \tilde{\chi}^{\pm}_{1,2} \qquad \tilde{\chi}^{0}_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: more than 100 free parameters

Nobody(?) believes that a model describing nature
has so many free parameters!

SUSY breaking:

"Hidden sector":	$\longrightarrow$	Visible sector:
SUSY breaking		MSSM

"Gravity-mediated": CMSSM/mSUGRA
"Gauge-mediated": GMSB
"Anomaly-mediated": AMSB
"Gaugino-mediated"
. . .

CMSSM/mSUGRA: mediating interactions are gravitational

GMSB: mediating interactions are ordinary electroweak and QCD gauge interactions

AMSB, Gaugino-mediation: SUSY breaking happens on a different brane in a higher-dimensional theory

$\Rightarrow$ all new low-energy parameters expressed through a few GUT scale parameters!

GUT based models: 1.) CMSSM (sometimes wrongly called mSUGRA):

⇒ Scenario characterized by

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \text{sign}\,\mu$$

m_0 : universal scalar mass parameter

$m_{1/2}$: universal gaugino mass parameter

A_0 : universal trilinear coupling

} at the GUT scale

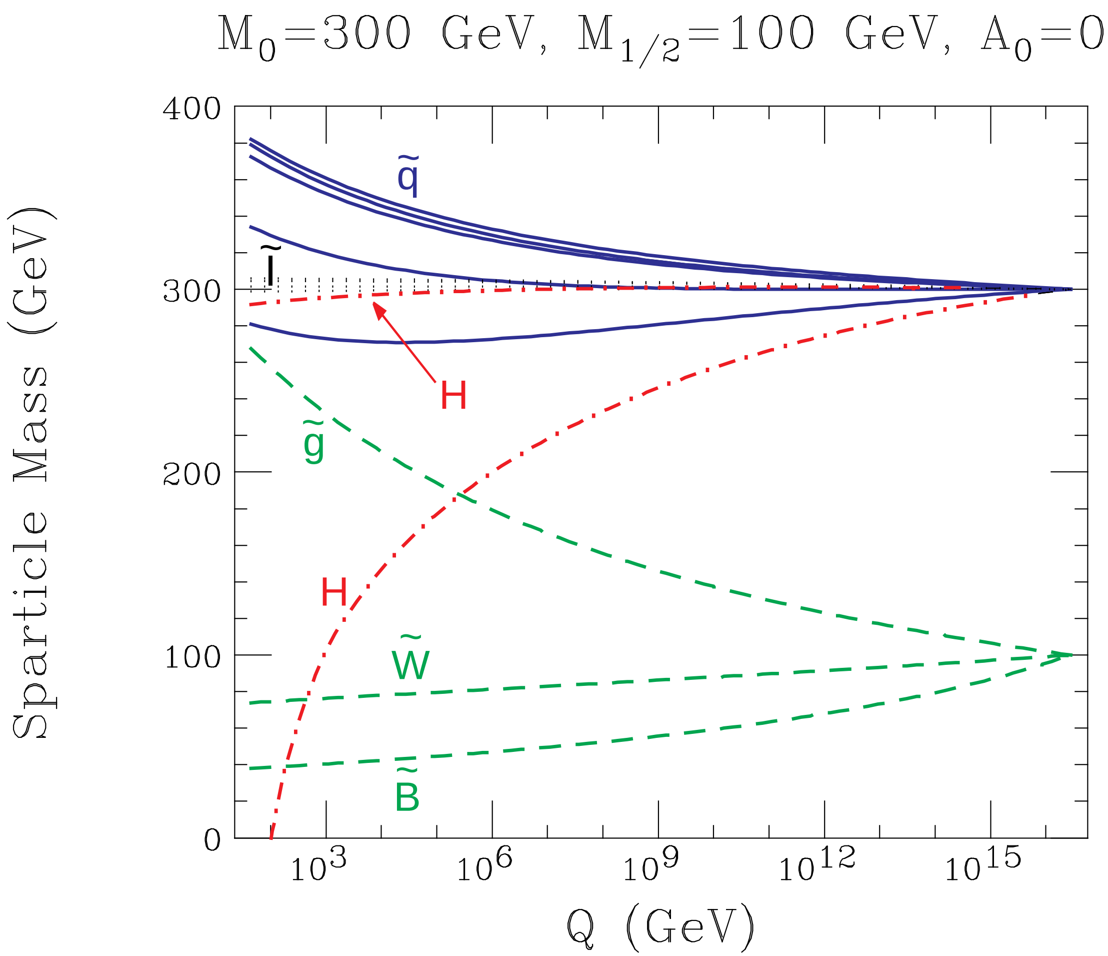
$\tan\beta$: ratio of Higgs vacuum expectation values

sign(μ) : sign of supersymmetric Higgs parameter

⇒ particle spectra from renormalization group running to weak scale

⇒ Lightest SUSY particle (LSP) is the lightest neutralino

⇒ particle spectra from renormalization group running to weak scale



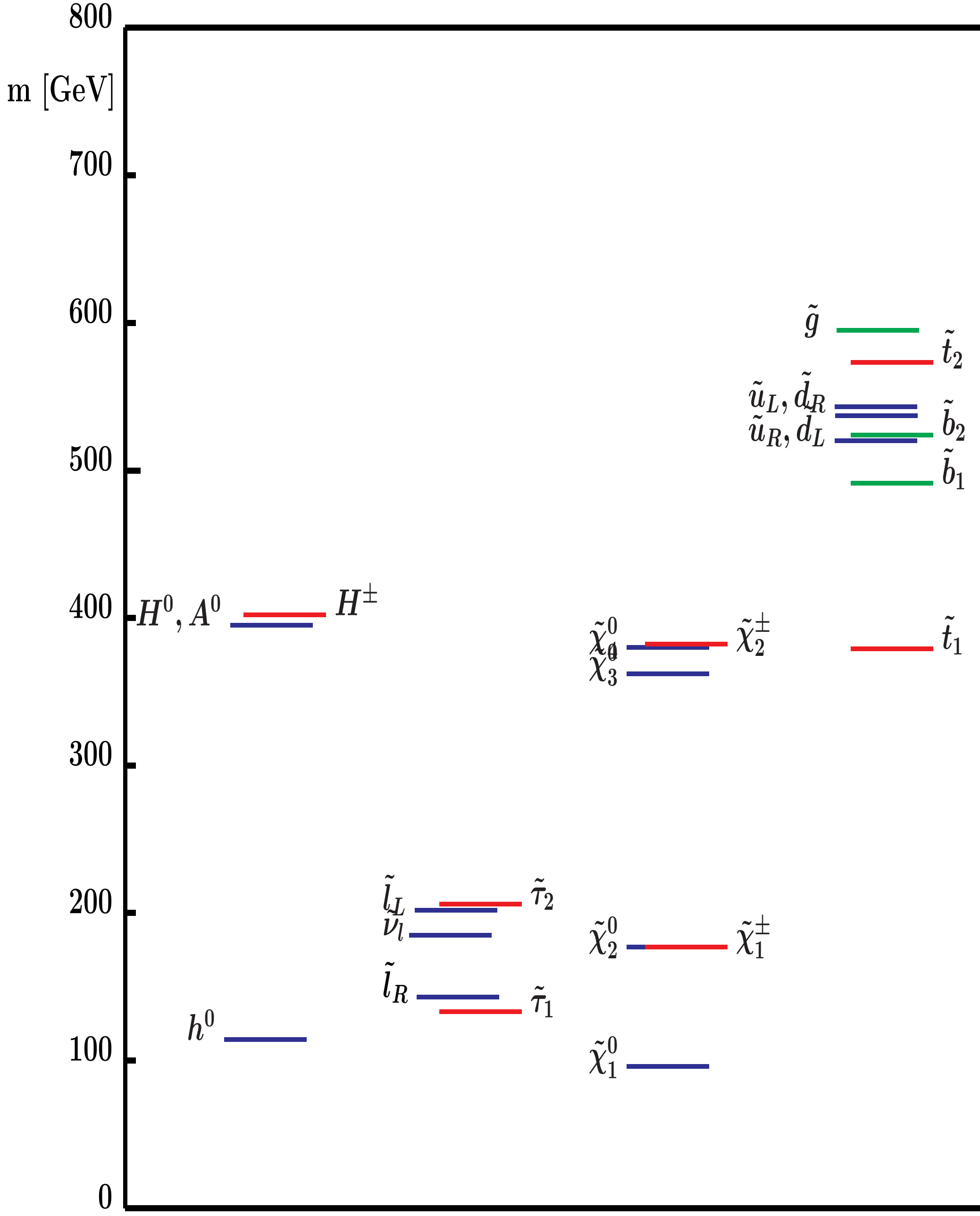
⇒ one parameter turns negative ⇒ Higgs mechanism for free

"Typical" CMSSM scenario

(SPS 1a benchmark scenario):

Strong connection between

all the sectors



GUT based models: 2.) NUHM1: (Non-universal Higgs mass model)

Assumption: no unification of scalar fermion and scalar Higgs parameter at the GUT scale

⇒ effectively M_A or μ as free parameters at the EW scale

⇒ besides the CMSSM parameters

M_A or μ

And there is more: 3.) VCMSSM
4.) mSUGRA
5.) NUHM2

. . . no time here . . .

2. The MSSM Higgs sector

Comparison with SM case:

$$\mathcal{L}_{\text{SM}} = \underbrace{m_d \bar{Q}_L \Phi d_R}_{\text{d-quark mass}} + \underbrace{m_u \bar{Q}_L \Phi_c u_R}_{\text{u-quark mass}}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \Phi_c = i\sigma_2 \Phi^*, \quad \Phi \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \Phi_c \to \begin{pmatrix} v \\ 0 \end{pmatrix}$$

In SUSY: term $\bar{Q}_L \Phi^*$ not allowed

Superpotential is holomorphic function of chiral superfields, i.e. depends only on φ_i, not on φ_i^*

No soft SUSY-breaking terms allowed for chiral fermions

$\Rightarrow$ $H_d (\equiv H_1)$ and $H_u (\equiv H_2)$ needed to give masses to down- and up-type fermions

Furthermore: two doublets also needed for cancellation of anomalies, quadratic divergences

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}(H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

Rotation to physical basis:

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \qquad \tan(2\alpha) = \tan(2\beta)\frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \qquad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

Three Goldstone bosons (as in SM): G^0, $G^\pm$

$\longrightarrow$ longitudinal components of $W^\pm$, Z

$\Rightarrow$ Five physical states: $h^0, H^0, A^0, H^\pm$

h, H: neutral, $\mathcal{CP}$-even, A^0: neutral, $\mathcal{CP}$-odd, $H^\pm$: charged

Gauge-boson masses:

$$M_W^2 = \frac{1}{2}g'^2(v_1^2 + v_2^2), \quad M_Z^2 = \frac{1}{2}(g^2 + g'^2)(v_1^2 + v_2^2), \quad M_\gamma = 0$$

Parameters in MSSM Higgs potential V (besides g, g'):

$$v_1,\ v_2,\ m_1,\ m_2,\ m_{12}$$

relation for M_W^2, M_Z^2 $\Rightarrow$ 1 condition

minimization of V w.r.t. neutral Higgs fields H_1^1, H_2^2 $\Rightarrow$ 2 conditions

$\Rightarrow$ only two free parameters remain in V, conventionally chosen as

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

$\Rightarrow$ m_h, m_H, mixing angle α, $m_{H^\pm}$: no free parameters, can be predicted

In lowest order:

$$m_{H^\pm}^2 = M_A^2 + M_W^2$$

Predictions for m_h, m_H from diagonalization of tree-level mass matrix:

$\phi_1 - \phi_2$ basis:

$$M^{2,\text{tree}}_{\text{Higgs}} = \begin{pmatrix} m^2_{\phi_1} & m^2_{\phi_1\phi_2} \\ m^2_{\phi_1\phi_2} & m^2_{\phi_2} \end{pmatrix} =$$

$$\begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2) \sin\beta \cos\beta \\ -(M_A^2 + M_Z^2) \sin\beta \cos\beta & M_A^2 \cos^2\beta + M_Z^2 \sin^2\beta \end{pmatrix}$$

$$\Big\Downarrow \quad \leftarrow \ \text{Diagonalization}, \alpha$$

$$\begin{pmatrix} m_H^{2,\text{tree}} & 0 \\ 0 & m_h^{2,\text{tree}} \end{pmatrix}$$

Tree-level result for m_h, m_H:

$$m_{H,h}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$\Rightarrow m_h \leq M_Z$ at tree level

$\Rightarrow$ Light Higgs boson h required in SUSY

Measurement of m_h, Higgs couplings

$\Rightarrow$ test of the theory (more directly than in SM)

Higgs couplings, tree level:

$$g_{hVV} = \sin(\beta-\alpha)\, g_{HVV}^{\text{SM}}, \quad V = W^\pm, Z$$

$$g_{HVV} = \cos(\beta-\alpha)\, g_{HVV}^{\text{SM}}$$

$$g_{hAZ} = \cos(\beta-\alpha)\, \frac{g'}{2\cos\theta_W}$$

$$g_{hb\bar{b}}, g_{h\tau^+\tau^-} = -\frac{\sin\alpha}{\cos\beta}\, g_{Hb\bar{b},H\tau^+\tau^-}^{\text{SM}}$$

$$g_{ht\bar{t}} = \frac{\cos\alpha}{\sin\beta}\, g_{Ht\bar{t}}^{\text{SM}}$$

$$g_{Ab\bar{b}}, g_{A\tau^+\tau^-} = \gamma_5 \tan\beta\, g_{Hb\bar{b}}^{\text{SM}}$$

$\Rightarrow$ $g_{hVV} \le g_{HVV}^{\text{SM}}$, g_{hVV}, g_{HVV}, g_{hAZ} cannot all be small

$g_{hb\bar{b}}, g_{h\tau^+\tau^-}$: significant suppression or enhancement w.r.t. SM coupling possible

The decoupling limit:

For $M_A \gtrsim 150$ GeV:

The lightest MSSM Higgs
is SM-like

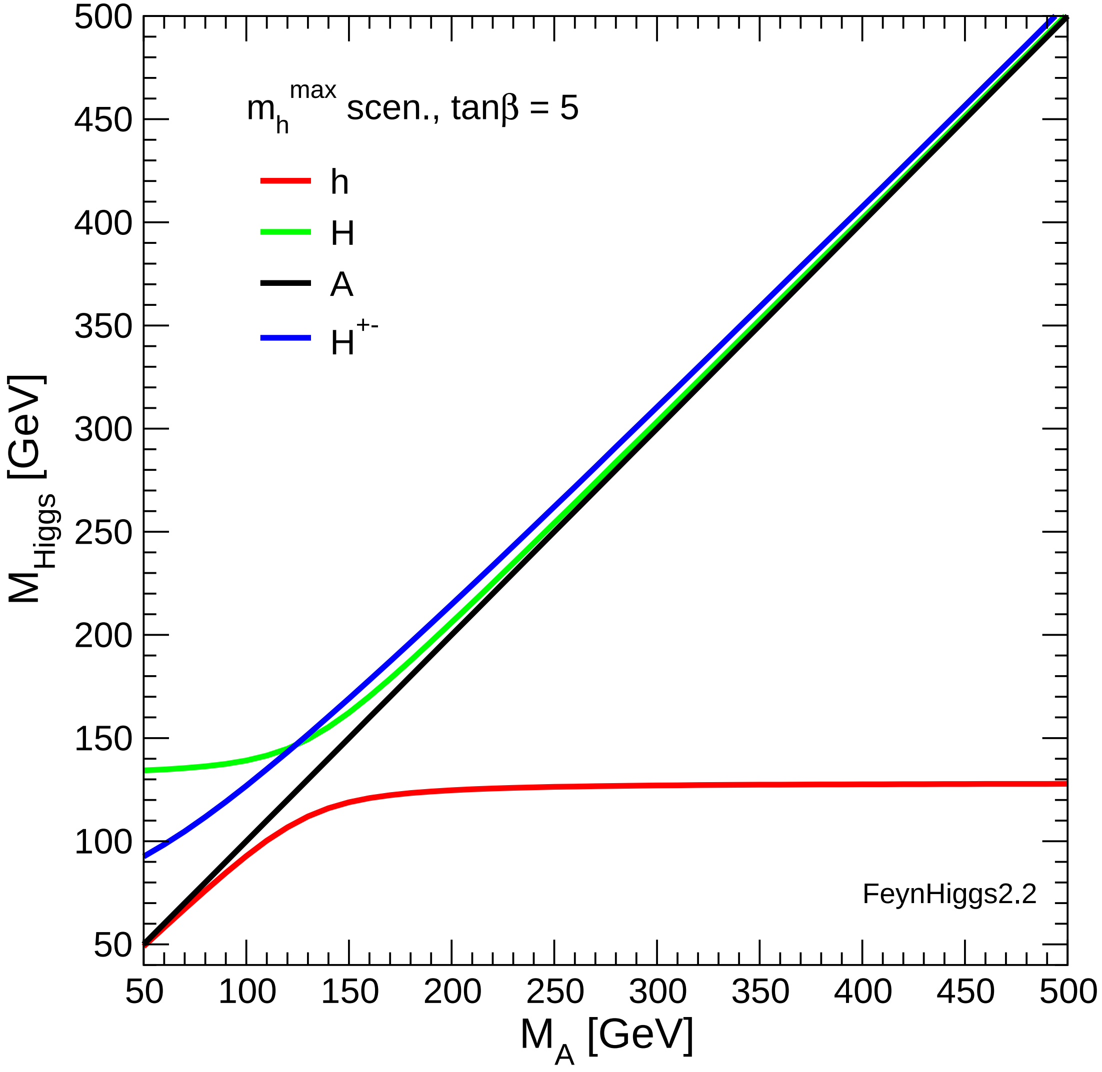
$\Rightarrow$ SM analysis applies!

The heavy MSSM Higgses:

$M_A \approx M_H \approx M_{H^\pm}$

$\to$ coupling to gauge bosons ~ 0

$\Rightarrow$ no decay $H \to WW^{(*)}, \ldots$



The lightest MSSM Higgs boson

MSSM predicts upper bound on M_h:

tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches!

Large radiative corrections: → excursion

Yukawa couplings: $\frac{\mathrm{e}\, m_t}{2 M_W s_\mathrm{W}}, \frac{\mathrm{e}\, m_t^2}{M_W s_\mathrm{W}}, \ldots$

⇒ Dominant one-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Present status of M_h prediction in the MSSM:

Complete 1L, 'almost complete' 2L available, also very leading 3L . . .

What is a mass

Definition: The mass of a particle is the pole of the propagator

Example: scalar particle

Propagator:

$$\frac{i}{q^2 - m^2}$$

q^2 : four-momentum squared
m^2: constant in the Lagrangian

If one chooses $q^2 = m^2$ then the propagator has a pole.
This q^2 is then the mass of the particle.

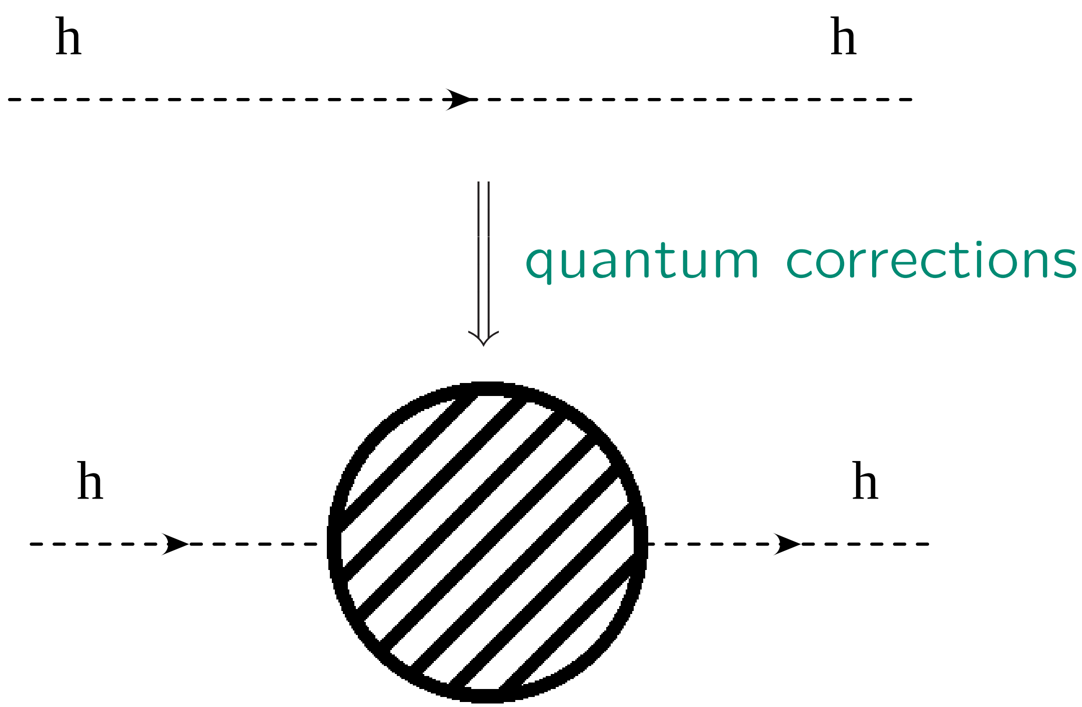
$\Rightarrow$ Pole of the propagator corresponds to zeroth of the inverse propagator.

Inverse propagator:

$$-i(q^2 - m^2)$$

Problem: quantum corrections

Higgs propagator:

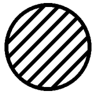


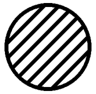
Inverse propagator:

$$-i(q^2 - m^2) \quad \longrightarrow \quad -i\left(q^2 - m^2 + \hat{\Sigma}_h(q^2)\right)$$

$\hat{\Sigma}_h(q^2)$: renormalized Higgs self-energy

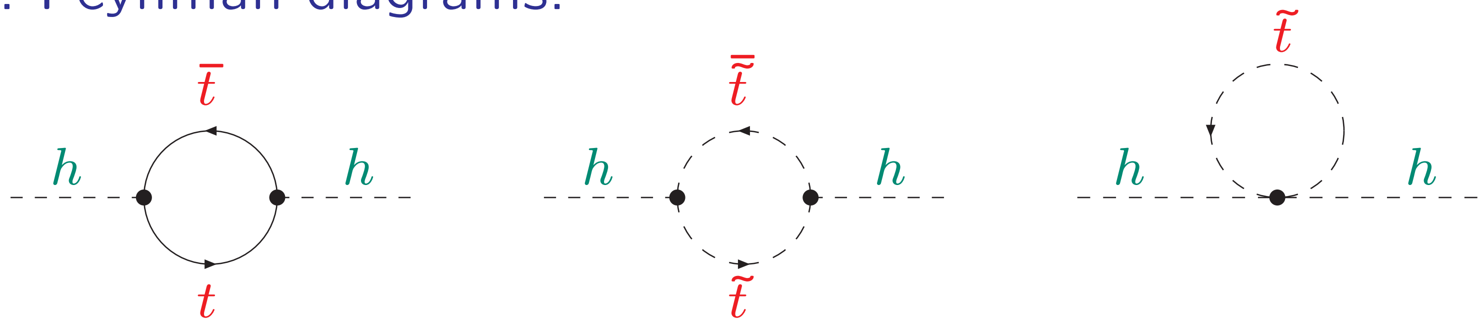
Calculation of the blob:

 $= \hat{\Sigma}(q^2) = \hat{\Sigma}^{(1)}(q^2) + \hat{\Sigma}^{(2)}(q^2) + \ldots$

 : all MSSM particles contribute

main contribution: $t/\tilde{t}$ sector ($\tilde{t}$: scalar top, SUSY partner of the t)

1-Loop: Feynman diagrams:



Dominant 1-loop corrections: $\Delta m_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

size of the corrections: $\mathcal{O}$(50 GeV)

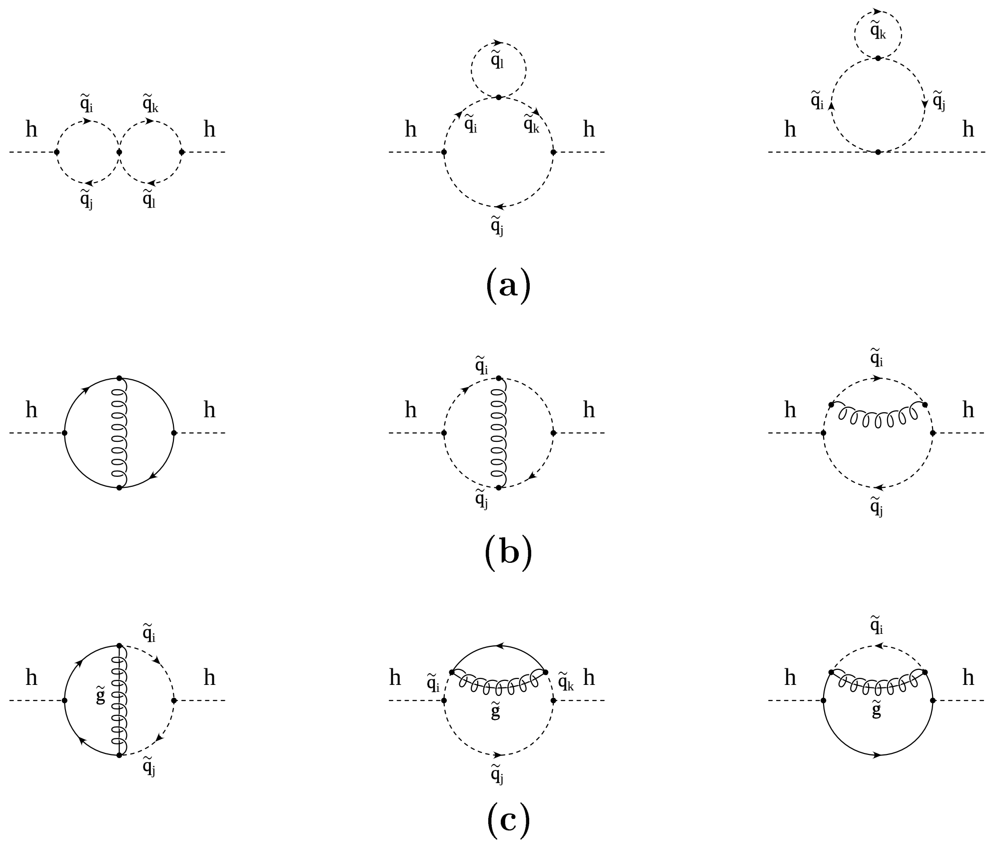
$\Rightarrow$ 2-Loop calculation necessary!

2-loop: $\hat{\Sigma}^{(2)}(0)$

[*S. H., W. Hollik, G. Weiglein '98*]

dominant contributions of $\mathcal{O}(\alpha_t\alpha_s)$:

(a) pure scalar diagrams
(b) diagrams with gluonexchange
(c) diagrams with gluinoexchange



Quite complicated calculation . . .
⇒ Need for computer algebra programms

['*98 - '13:*] ⇒ many more corrections calculated!

End of excursion: Higgs mass calculations

The lightest MSSM Higgs boson

MSSM predicts upper bound on M_h:

tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches!

Large radiative corrections: $\rightarrow$ excursion

Yukawa couplings: $\frac{\mathrm{e}\, m_t}{2 M_W s_\mathrm{W}}, \frac{\mathrm{e}\, m_t^2}{M_W s_\mathrm{W}}, \ldots$

$\Rightarrow$ Dominant one-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

> The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Present status of M_h prediction in the MSSM:

Complete 1L, ‘almost complete’ 2L available, also very leading 3L ...

Mixing of the $\mathcal{CP}$-even Higgs bosons:

Propagator/Mass matrix at tree-level:

$$\begin{pmatrix} q^2 - m_H^2 & 0 \\ 0 & q^2 - m_h^2 \end{pmatrix}$$

Propagator / mass matrix with higher-order corrections
($\to$ Feynman-diagrammatic approach):

$$M_{hH}^2(q^2) = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i, j = h, H)$: renormalized Higgs self-energies

$\mathcal{CP}$-even fields can mix

$\Rightarrow$ complex roots of $\det(M_{hH}^2(q^2))$: $\mathcal{M}_{h_i}^2 (i = 1, 2)$: $\mathcal{M}^2 = M^2 - iM\Gamma$

Upper bound on M_h in the MSSM:

"Unconstrained MSSM" :

M_A, $\tan\beta$, 5 parameters in $\tilde{t}$–$\tilde{b}$ sector, μ, $m_{\tilde{g}}$, M_2

$$M_h \lesssim 135 \text{ GeV}$$

for $m_t = 173.2 \pm 0.9\,\text{GeV}$ and $m_{\tilde{t}} \lesssim 2$ TeV

(including theoretical uncertainties from unknown higher orders)

$\Rightarrow$ observable at the LHC

Obtained with:

FeynHiggs

`www.feynhiggs.de`

[*T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '98 – '15*]

$\rightarrow$ all Higgs masses, couplings, BRs, XSs (easy to link, easy to use :-)

Upper bound on M_h in the MSSM:

"Unconstrained MSSM" :

M_A, $\tan\beta$, 5 parameters in $\tilde{t}$–$\tilde{b}$ sector, μ, $m_{\tilde{g}}$, M_2

$$\boxed{M_h \lesssim 135 \text{ GeV}} \qquad \text{Note}: 125 < 135!$$

for $m_t = 173.2 \pm 0.9\,\text{GeV}$ and $m_{\tilde{t}} \lesssim 2$ TeV

(including theoretical uncertainties from unknown higher orders)

$\Rightarrow$ observable at the LHC

Obtained with:

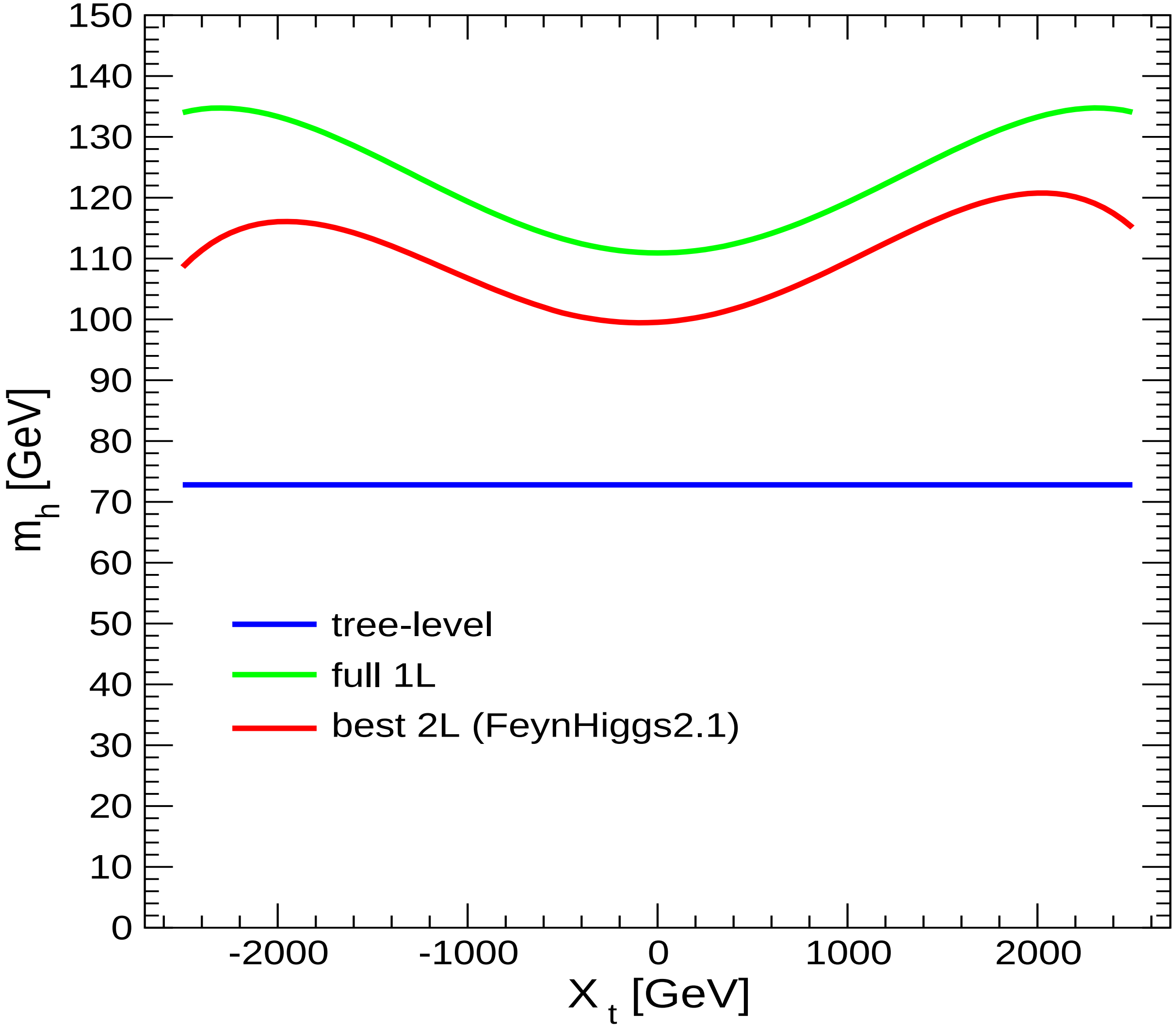
FeynHiggs

`www.feynhiggs.de`

[*T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '98 – '15*]

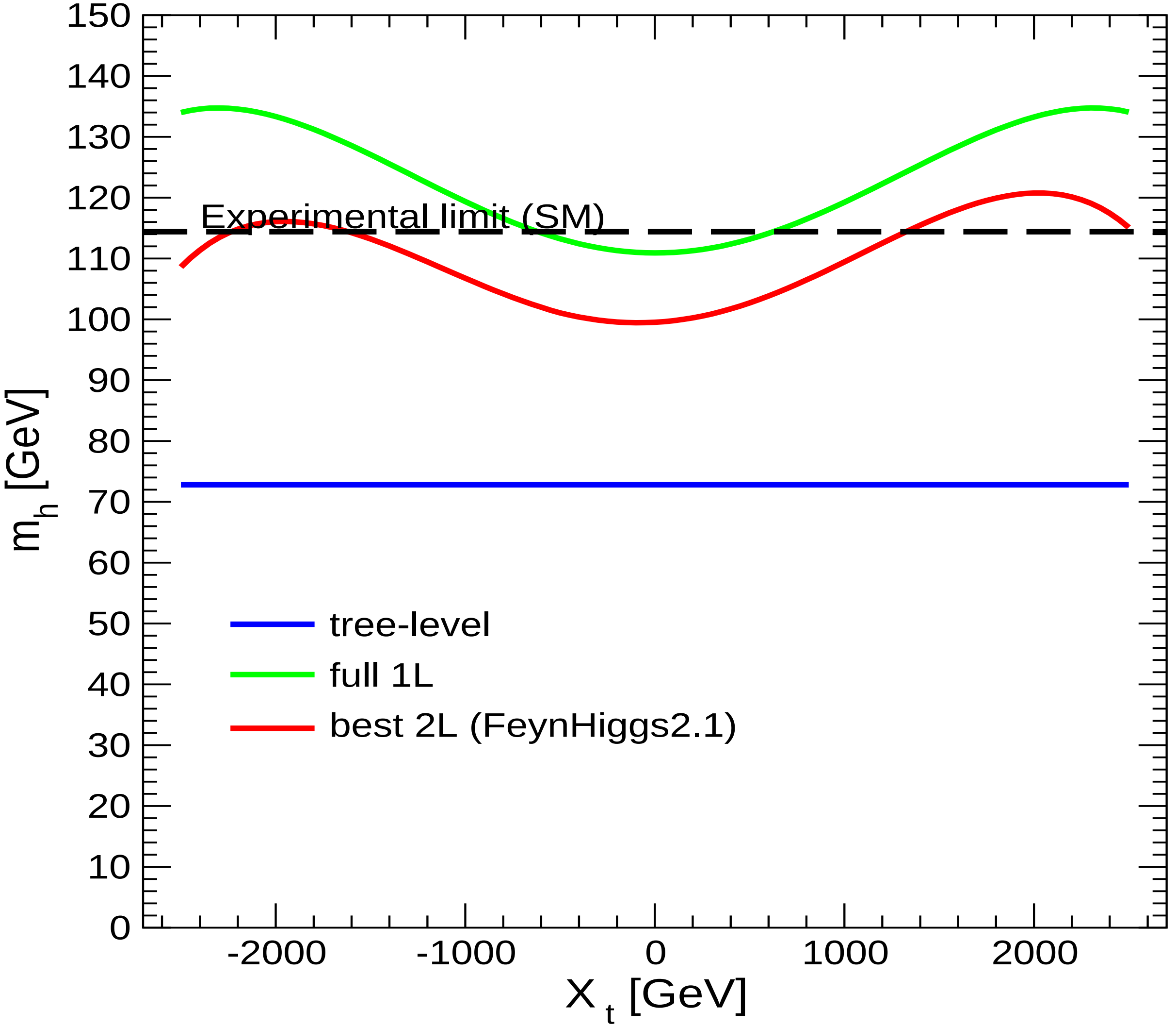
$\to$ all Higgs masses, couplings, BRs, XSs (easy to link, easy to use :-)

Example for one set of MSSM parameters



Effects of the two-loop corrections to the lightest Higgs mass:

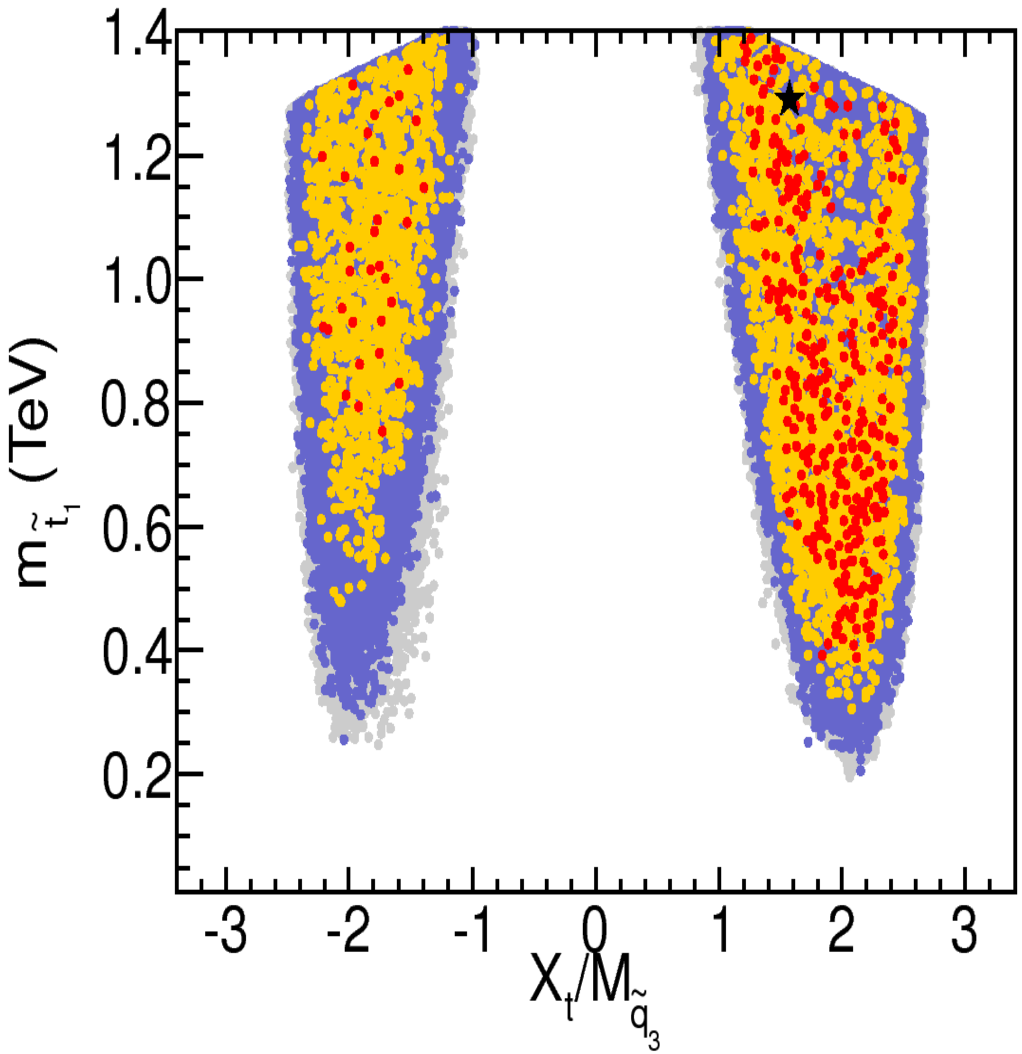
Example for one set of MSSM parameters



Comparison with experimental limits

$\Rightarrow$ strong impact on bound on SUSY parameters

Stop masses: [P. Bechtle, S.H., O. Stål, T. Stefaniak, G. Weiglein, L. Zeune '15 – PRELIMINARY]



$M_h = 125 \pm 3$ GeV

$\star$: best-fit point

red: $\Delta\chi^2 < 2.3$

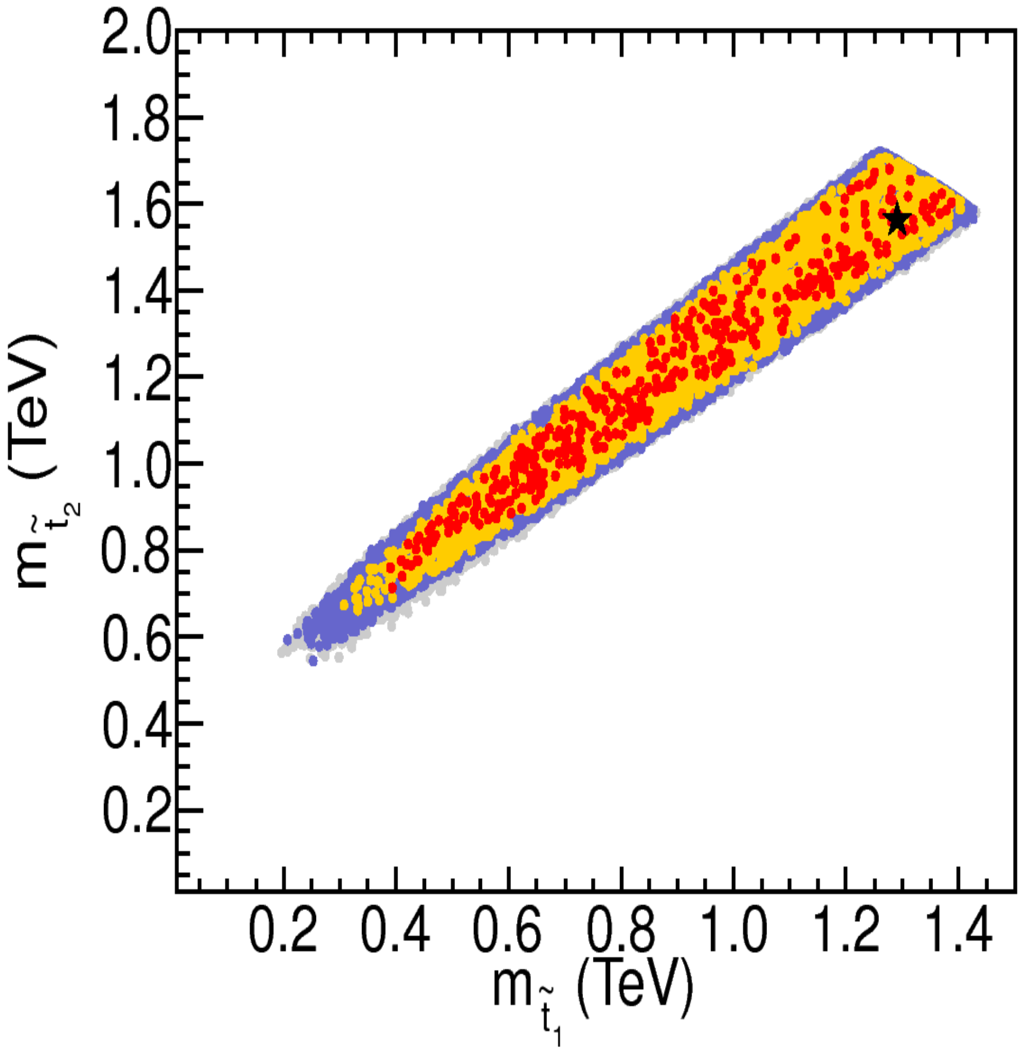
orange: $\Delta\chi^2 < 5.99$

blue: all points HiggsBounds allowed

gray: all scan points

$\Rightarrow M_h \sim 125$ GeV requires large X_t and/or large M_{SUSY}

Stop masses: [P. Bechtle, S.H., O. Stål, T. Stefaniak, G. Weiglein, L. Zeune '15 – PRELIMINARY]



$M_h = 125 \pm 3$ GeV

$\star$: best-fit point

red: $\Delta\chi^2 < 2.3$

orange: $\Delta\chi^2 < 5.99$

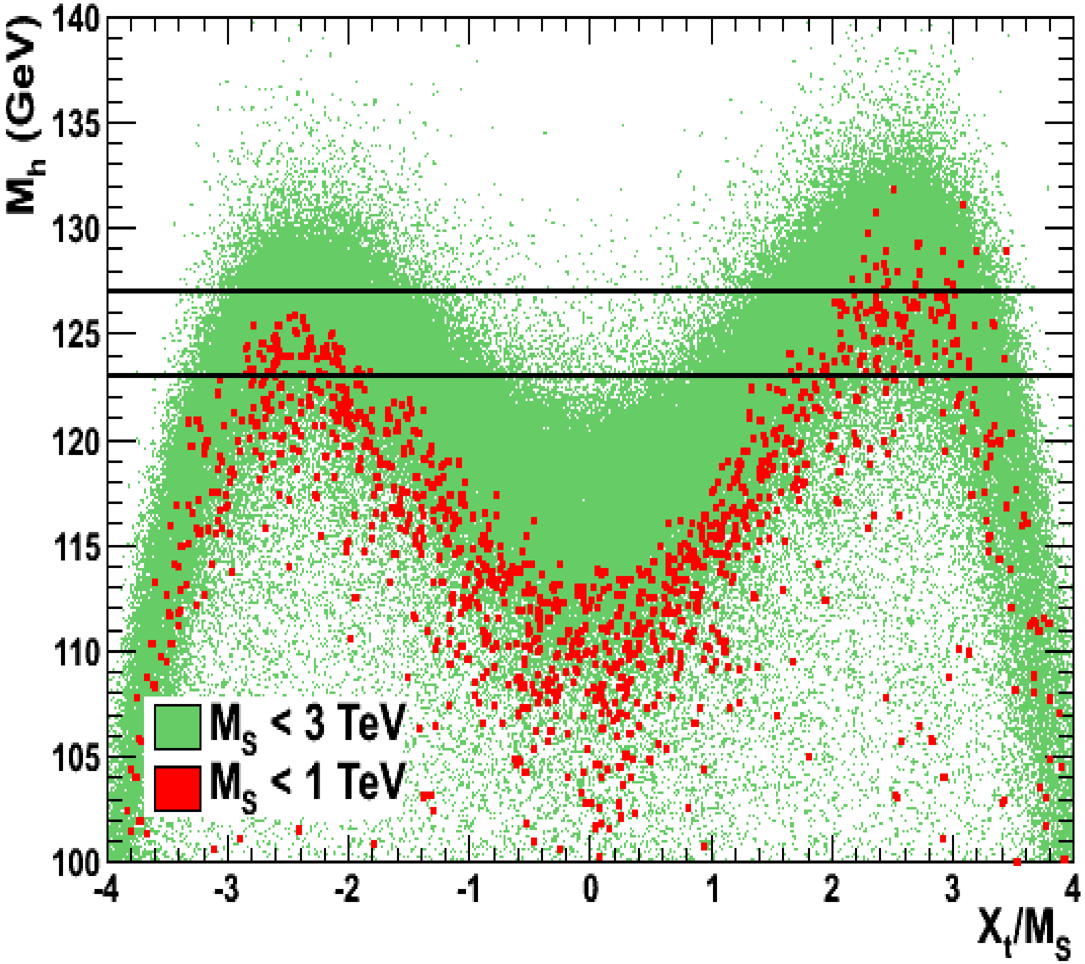
blue: all points HiggsBounds allowed

gray: all scan points

$\Rightarrow$ light and heavy stops compatible with $M_h \simeq 125$ GeV

Stop masses for $M_h = 125$ GeV

[*A. Arbey et al., '11*]



⇒ $M_h \sim 125$ GeV requires large X_t and/or large M_{SUSY}

⇒ no clear prediction for the LHC!

Remaining theoretical uncertainties in prediction for M_h in the MSSM:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

– From unknown higher-order corrections:
  $\Rightarrow \Delta M_h \approx 3$ GeV

– From uncertainties in input parameters

  $m_t, \ldots, M_A, \tan\beta, m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{g}}, \ldots$

  $\Delta m_t \approx 1$ GeV $\Rightarrow \Delta M_h \approx 1$ GeV

$\Rightarrow$ Recent improvement published!

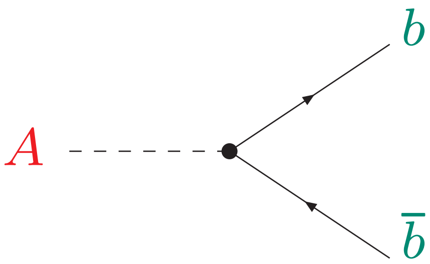
[*arXiv:1312.4937 [hep-ph]*]

$\Rightarrow$ ask me for details over coffee :-)

The heavy MSSM Higgs bosons

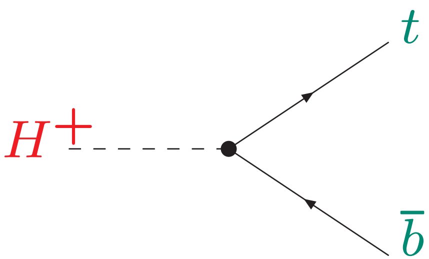
Differences compared to the SM Higgs:

Additional enhancement factors compared to the SM case:



$$y_b \quad \to \quad y_b \, \frac{\tan\beta}{1+\Delta_b}$$

At large $\tan\beta$: either $H \approx A$ or $h \approx A$



$$y_b \, \frac{\tan\beta}{1+\Delta_b}$$

$$\Delta_b = \frac{2\alpha_s}{3\,\pi} m_{\tilde{g}}\, \mu \, \tan\beta \times I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}})$$
$$+ \frac{\alpha_t}{4\,\pi} A_t\, \mu \, \tan\beta \times I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \mu)$$

$\Rightarrow$ other parameters enter $\Rightarrow$ strong μ dependence

Most powerful LHC search modes for heavy MSSM Higgs bosons:

$$b\bar{b} \to H/A \to \tau^+\tau^- + X$$
$$gb \to tH^\pm + X,\ H^\pm \to \tau\nu_\tau$$
$$pp \to t\bar{t} \to H^\pm + X,\ H^\pm \to \tau\nu_\tau$$

Enhancement factors compared to the SM case:

$$H/A \ : \ \frac{\tan^2\beta}{(1+\Delta_b)^2} \times \frac{\mathrm{BR}(H \to \tau^+\tau^-) + \mathrm{BR}(A \to \tau^+\tau^-)}{\mathrm{BR}(H \to \tau^+\tau^-)_{\mathrm{SM}}}$$

$$H^\pm \ : \ \frac{\tan^2\beta}{(1+\Delta_b)^2} \times \mathrm{BR}(H^\pm \to \tau\nu_\tau)$$

$\Rightarrow$ Δ_b effects (often neglected by ATLAS/CMS analyses)

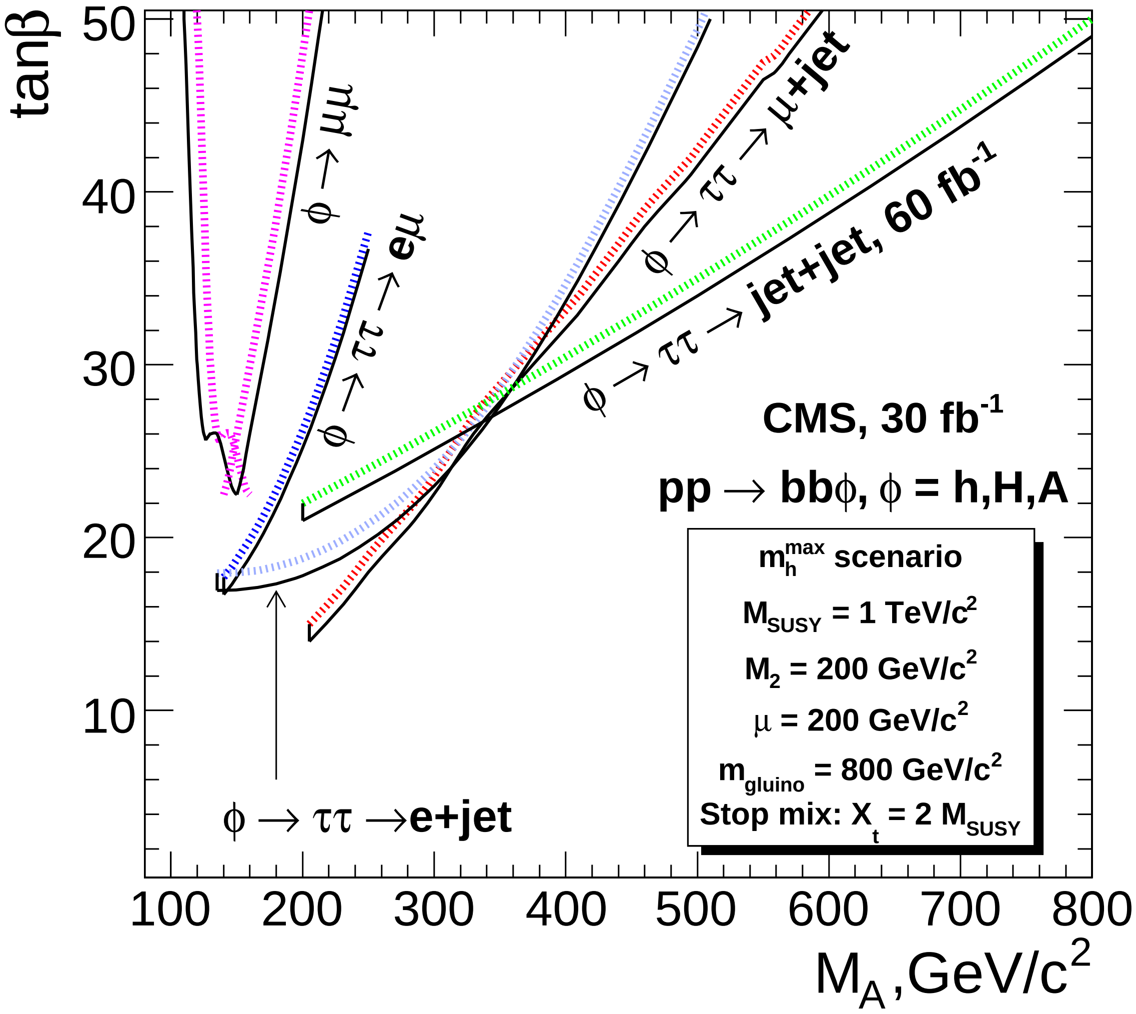
also relevant for $\mathrm{BR}(H/A \to \tau^+\tau^-)$, $\mathrm{BR}(H^\pm \to \tau\nu_\tau)$

also relevant: correct evaluation of $\Gamma(H/A/H^\pm \to \mathrm{SUSY})$

$\Rightarrow$ additional effects on $\mathrm{BR}(H/A \to \tau^+\tau^-)$, $\mathrm{BR}(H^\pm \to \tau\nu_\tau)$

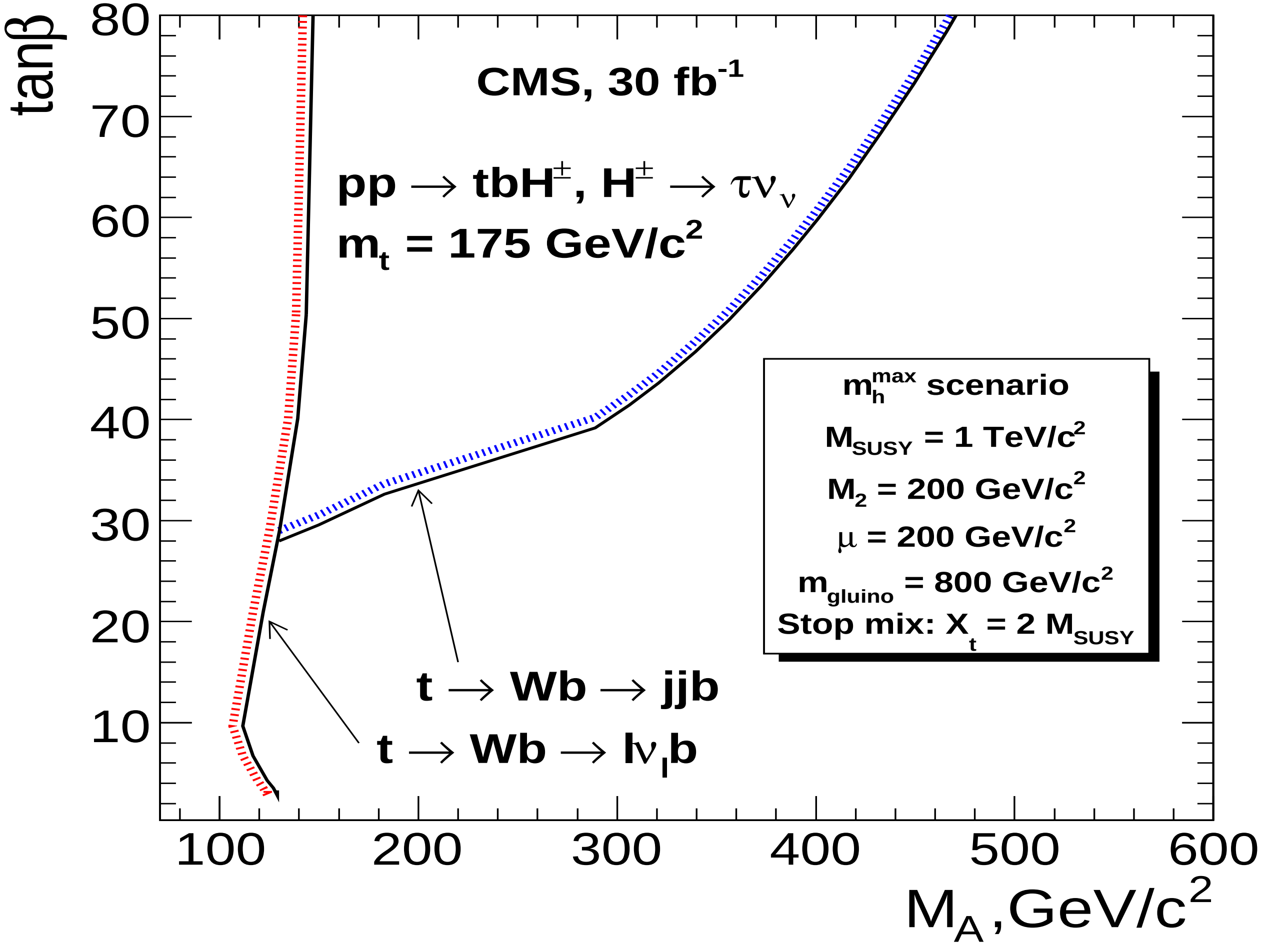
Pre-LHC predictions for neutral heavy Higgs bosons:

MSSM Higgs discovery contours in M_A–$\tan\beta$ plane $(\Phi = H, A)$
($m_h^{\rm max}$ benchmark scenario): [*CMS PTDR '06*]



Pre-LHC predictions for charged Higgs boson searches:

MSSM Higgs discovery contours in M_A–$\tan\beta$ plane ($m_h^{\rm max}$ benchmark scenario): [*CMS PTDR '06*]



light charged Higgs:

$$M_{H^\pm} < m_t$$

heavy charged Higgs:

$$M_{H^\pm} > m_t$$

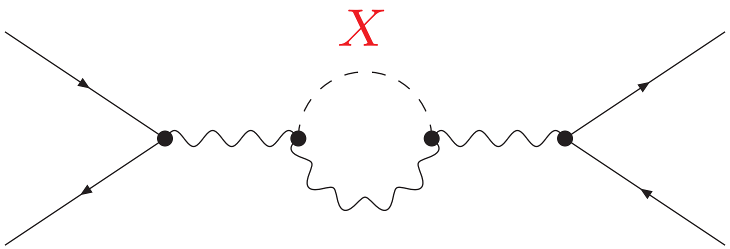
3. Predictions for the light MSSM Higgs mass before discovery

Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$M_W, \sin^2\theta_{\rm eff}, a_\mu,$ ...	$\leftrightarrow$	SM, MSSM , ...

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections, e.g. M_X



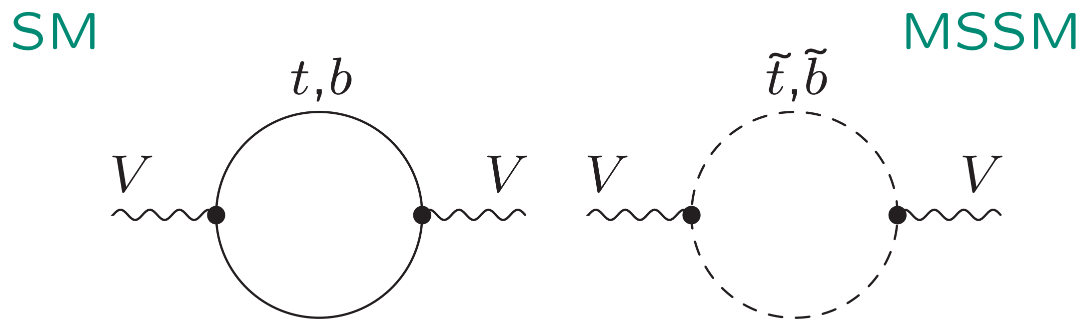
$\Downarrow$

MSSM: limits on M_X

Very high accuracy of measurements and theoretical predictions needed

Differences between the MSSM and the SM:

1.) New contributions from SUSY particles:



2.) CPV effects via new complex phases

3.) large Yukawa corrections: $\sim m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

4.) large corrections from the $b/\tilde{b}$ sector for large $\tan\beta$

5.) non-decoupling SUSY effects: $\sim \log \frac{M_{\text{SUSY}}}{M_W}$

Corrections to M_W, $\sin^2\theta_{\text{eff}}$ $\rightarrow$ approximation via the ρ-parameter:

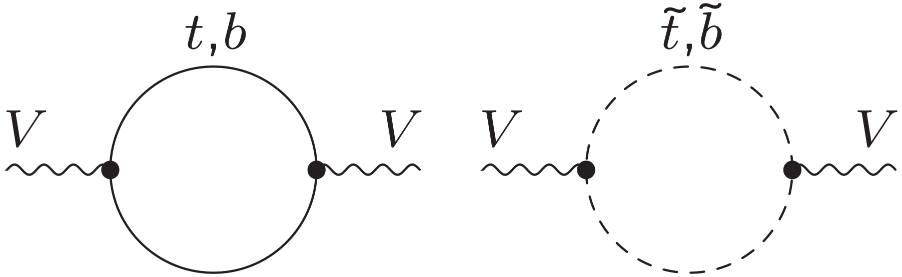
ρ measures the relative strength between
neutral current interaction and charged current interaction

$$\rho = \frac{1}{1 - \Delta\rho} \qquad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

(leading, process independent terms)

$\Delta\rho$ gives the main contribution to EW observables:

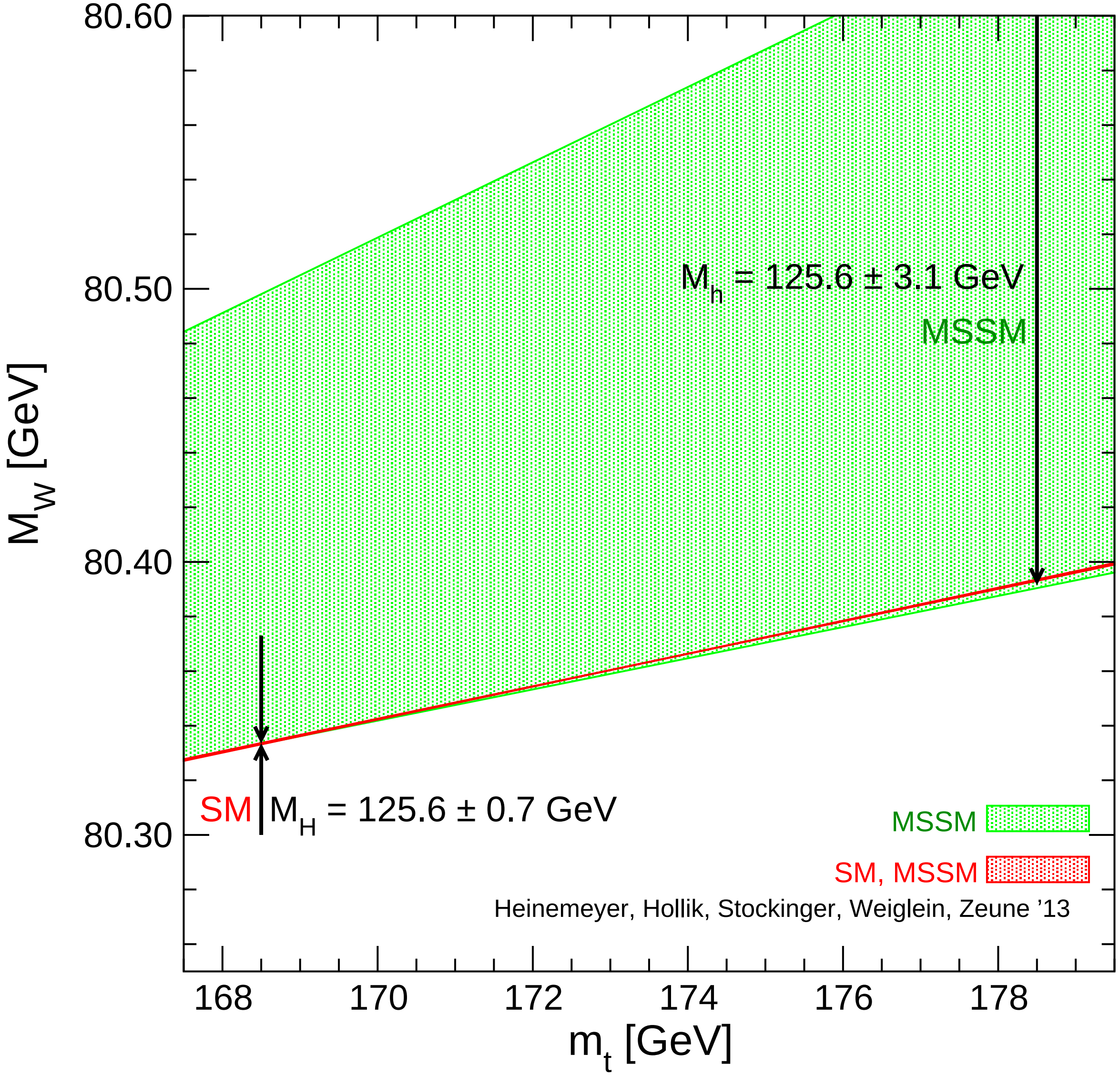
$$\Delta M_W \approx \frac{M_W}{2}\frac{c_W^2}{c_W^2 - s_W^2}\Delta\rho, \qquad \Delta\sin^2\theta_W^{\text{eff}} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2}\Delta\rho$$



$\Delta\rho^{\text{SUSY}}$ from $\tilde{t}/\tilde{b}$ loops > 0 $\Rightarrow M_W^{\text{SUSY}} \gtrsim M_W^{\text{SM}}$

Prediction for M_W in the SM and the MSSM :

[S.H., W. Hollik, D. Stockinger, G. Weiglein, L. Zeune '12]



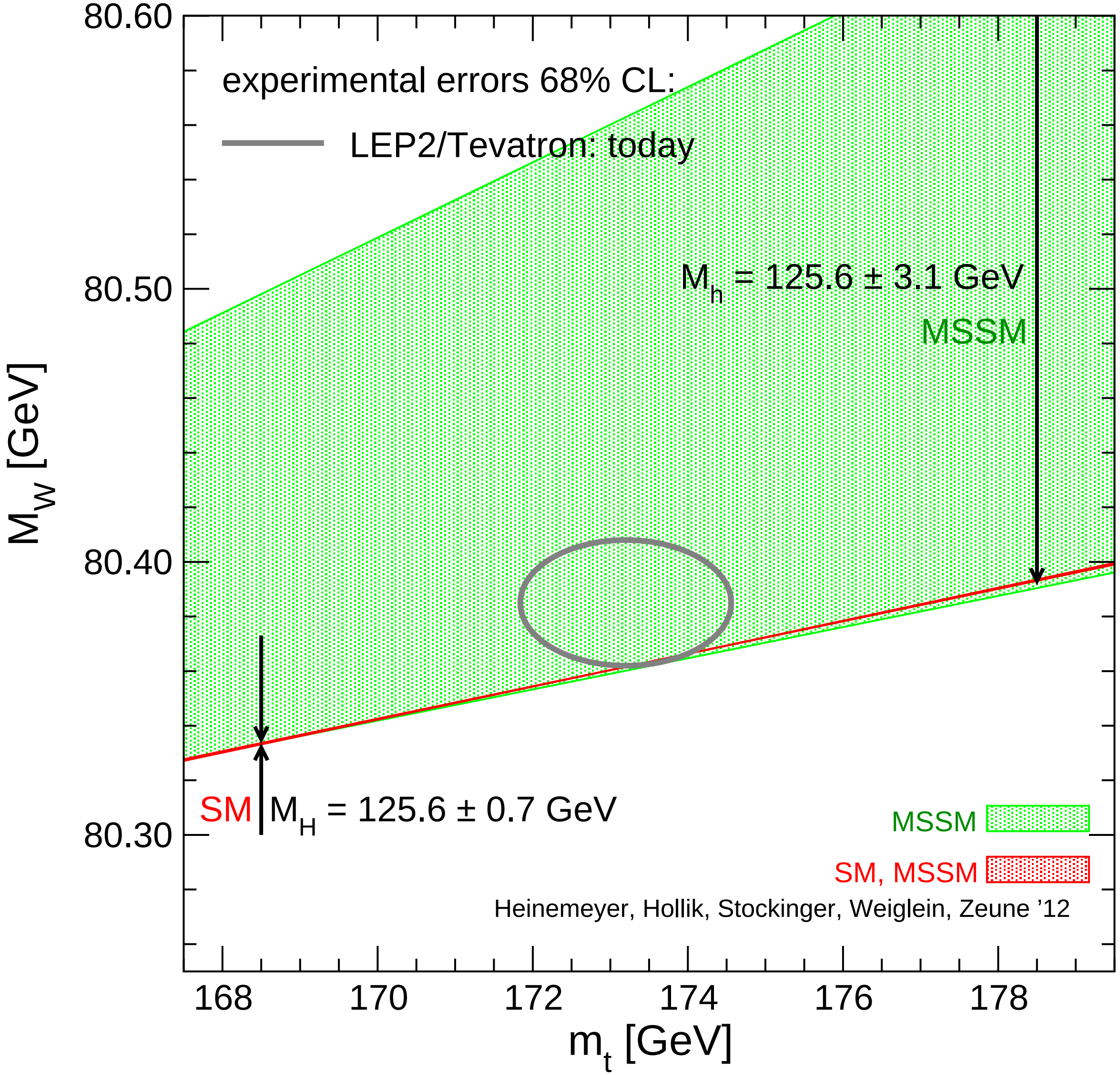
MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of $M_H^{\rm SM}$

Prediction for M_W in the SM and the MSSM :

[S.H., W. Hollik, D. Stockinger, G. Weiglein, L. Zeune '12]



MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of M_H^{SM}

MSSM: indirect constraints on M_h from existing data?

- Electroweak precision observables (EWPO) ?
- B physics observables (BPO) ?
- Cold dark matter (CDM) ?
- SUSY/Higgs data ?

  $\Rightarrow$ combination of EWPO, BPO, CDM, SUSY/Higgs ?

MSSM: indirect constraints on M_h from existing data?

- Electroweak precision observables (EWPO) ?
- B physics observables (BPO) ?
- Cold dark matter (CDM) ?
- SUSY/Higgs data ?

  $\Rightarrow$ combination of EWPO, BPO, CDM, SUSY/Higgs ?

SUSY limits: information on $m_{\tilde{q}_{1,2}}$, $m_{\tilde{g}}$

Heavy Higgs results: informatin on $m_{\tilde{t}}, m_{\tilde{b}}, \ldots$

EWPO $(g-2)_\mu$: information on $\tan\beta$ and/or $m_{\tilde{\chi}^0}$, $m_{\tilde{\chi}^\pm}$ and/or $m_{\tilde{\mu}}$, $m_{\tilde{\nu}_\mu}$

BPO BR$(b \to s\gamma)$: information on $\tan\beta$ and/or $M_{H^\pm}$ and/or $m_{\tilde{t}}$, $m_{\tilde{\chi}^\pm}$

CDM (LSP gives CDM) : information on $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\tau}}$ or M_A or ...

MSSM: indirect constraints on M_h from existing data?

- Electroweak precision observables (EWPO) ?
- B physics observables (BPO) ?
- Cold dark matter (CDM) ?
- SUSY/Higgs data ?

  $\Rightarrow$ combination of EWPO, BPO, CDM, SUSY/Higgs ?

SUSY limits: information on $m_{\tilde{q}_{1,2}}$, $m_{\tilde{g}}$

Heavy Higgs results: informatin on $m_{\tilde{t}}, m_{\tilde{b}}, \ldots$

EWPO $(g-2)_\mu$: information on $\tan\beta$ and/or $m_{\tilde{\chi}^0}$, $m_{\tilde{\chi}^\pm}$ and/or $m_{\tilde{\mu}}$, $m_{\tilde{\nu}_\mu}$

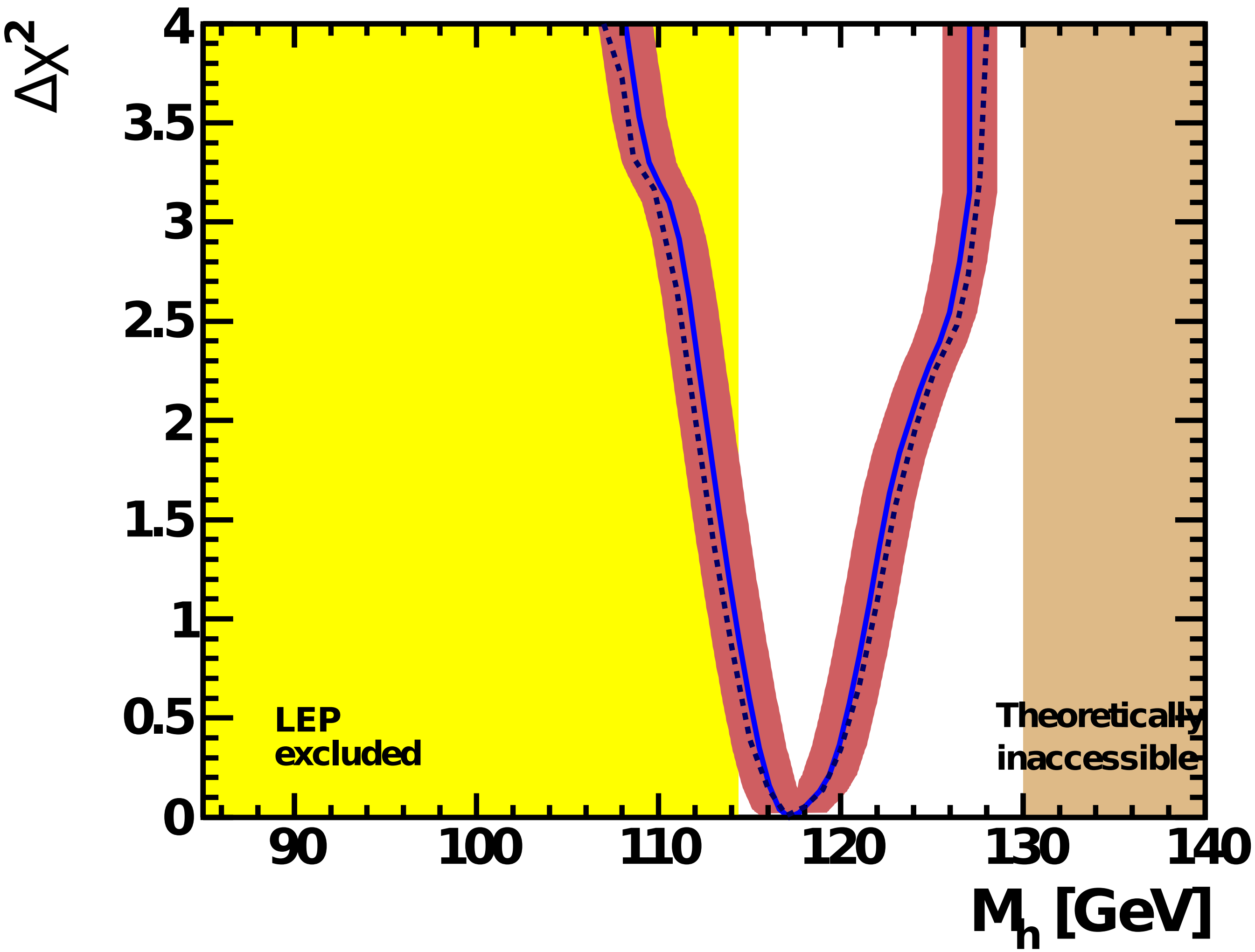
BPO BR$(b \to s\gamma)$: information on $\tan\beta$ and/or $M_{H^\pm}$ and/or $m_{\tilde{t}}$, $m_{\tilde{\chi}^\pm}$

CDM (LSP gives CDM) : information on $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\tau}}$ or M_A or ...

$\Rightarrow$ combination (so far) makes only sense if all parameters are connected!
$\Rightarrow$ GUT based models, ... $\Rightarrow$ M_h in the CMSSM, NUHM1, ...

CMSSM: post-LHC (1 fb^{-1}) red band plot:

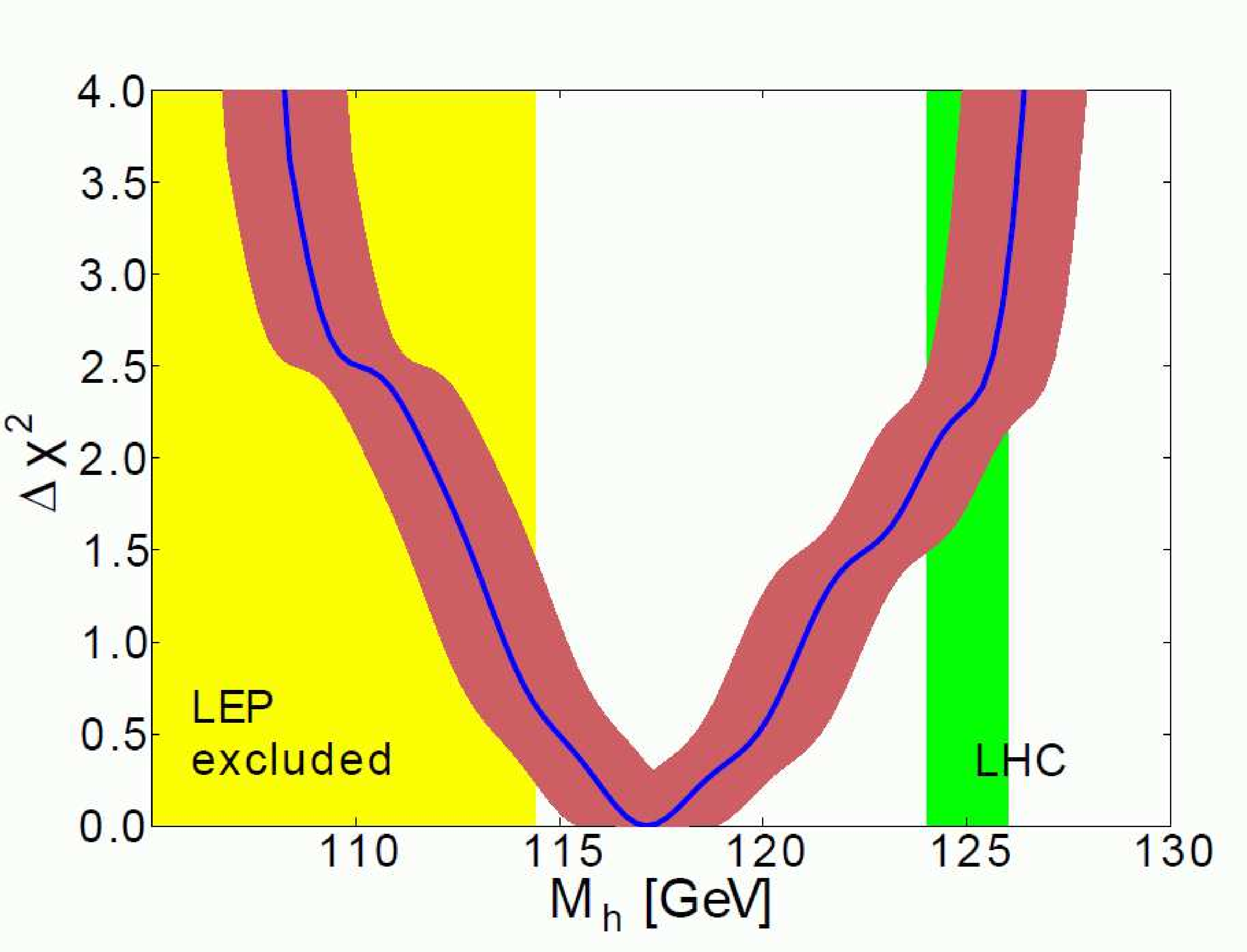
MasterCode [2011]



$M_h = 118 \pm 3\,(\text{exp}) \pm 1.5(\text{theo})$ GeV $\Rightarrow$ substantially higher than in SM!

CMSSM: post-LHC (5+5 fb^{-1}) red band plot:

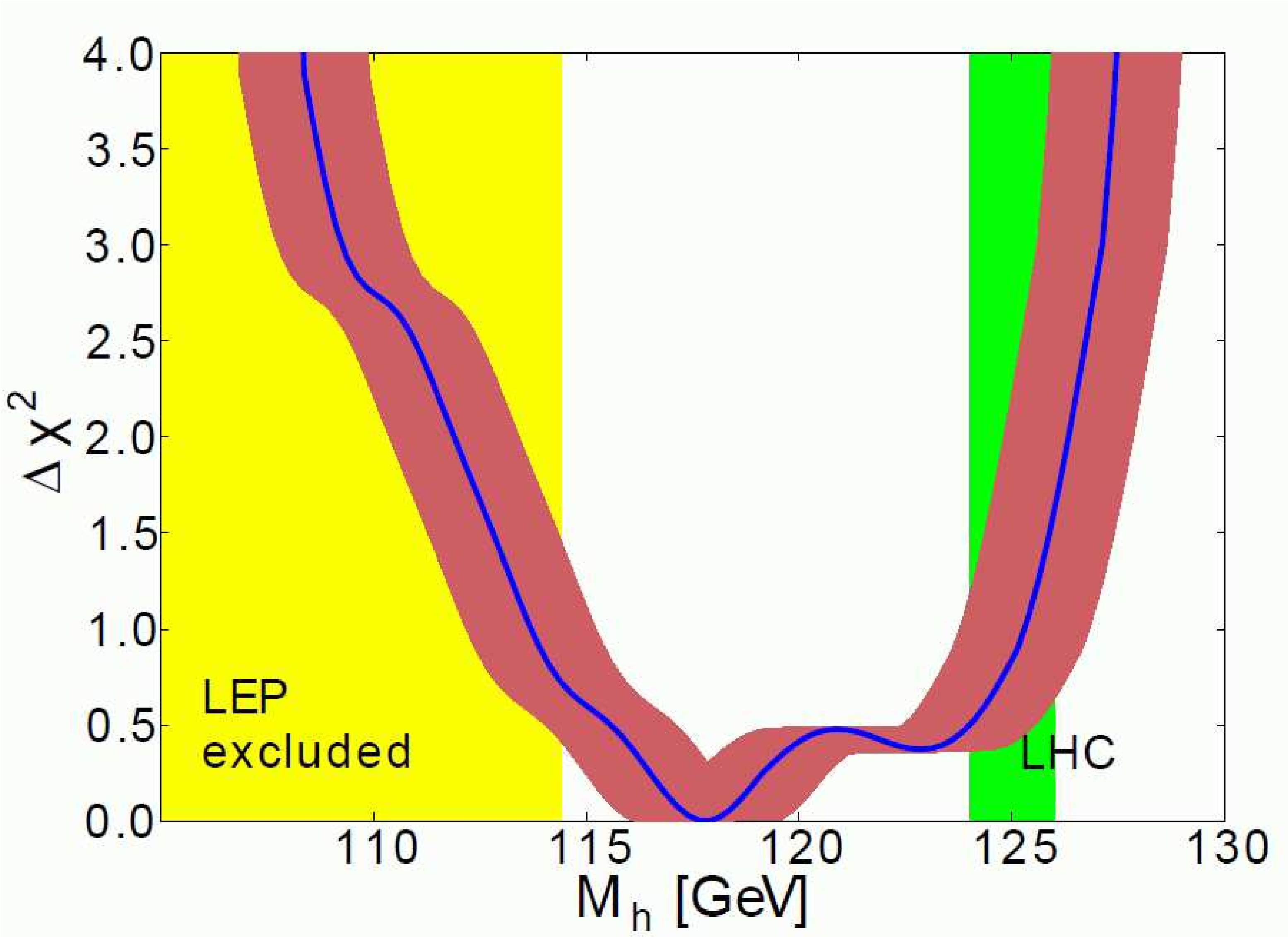
[2012]



$M_h = 117 \pm 4\,(\text{exp}) \pm 1.5(\text{theo})$ GeV $\quad \Delta\chi^2(M_h = 125 \text{ GeV}) \lesssim 2$

NUHM1: post-LHC (5+5 fb^{-1}) red band plot:

[2012]



$M_h \approx 118^{+7}_{-4}$ (exp) $\pm$ 1.5(theo) GeV $\qquad \Delta\chi^2(M_h = 125 \text{ GeV}) \approx 0.5$

4. The NMSSM Higgs sector (Z_3 invariant NMSSM)

MSSM Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = (\tilde{m}_1^2 + |\mu \quad |^2) H_1 \bar{H}_1 + (\tilde{m}_2^2 + |\mu \quad |^2) H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$
$$+ \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2$$

4. The NMSSM Higgs sector (Z_3 invariant NMSSM)

NMSSM Higgs sector: Two Higgs doublets + one Higgs singlet

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$S = v_s + S_R + IS_I$$

$$V = (\tilde{m}_1^2 + |\not{\mu}\lambda S|^2)H_1\bar{H}_1 + (\tilde{m}_2^2 + |\not{\mu}\lambda S|^2)H_2\bar{H}_2 - m_{12}^2(\epsilon_{ab}H_1^a H_2^b + \text{h.c.})$$
$$+ \frac{g'^2 + g^2}{8}(H_1\bar{H}_1 - H_2\bar{H}_2)^2 + \frac{g^2}{2}|H_1\bar{H}_2|^2$$
$$+ |\lambda(\epsilon_{ab}H_1^a H_2^b) + \kappa S^2|^2 + m_S^2|S|^2 + (\lambda A_\lambda(\epsilon_{ab}H_1^a H_2^b)S + \frac{\kappa}{3}A_\kappa S^3 + \text{h.c.})$$

Free parameters:

$$\lambda,\ \kappa,\ A_\kappa,\ M_{H^\pm},\ \tan\beta,\ \mu_{\text{eff}} = \lambda v_s$$

Higgs spectrum:

$$\mathcal{CP}\text{−even} : h_1, h_2, h_3$$
$$\mathcal{CP}\text{−odd} : a_1, a_2$$
$$\text{charged} : H^+, H^-$$
$$\text{Goldstones} : G^0, G^+, G^-$$

Neutralinos:

$$\mu \to \mu_{\text{eff}}$$

compared to the MSSM: one singlino more

$$\to \tilde{\chi}^0_1, \tilde{\chi}^0_2, \tilde{\chi}^0_3, \tilde{\chi}^0_4, \tilde{\chi}^0_5$$

Mass of the lightest $\mathcal{CP}$-even Higgs:

$$m^2_{h,\text{tree,NMSSM}} = m^2_{h,\text{tree,MSSM}} + M_Z^2 \frac{\lambda^2}{g^2} \sin^2 2\beta$$

Mass of the $\mathcal{CP}$-odd Higgs:

$$\text{MSSM}: M_A^2 = -m_{12}^2(\tan\beta + \cot\beta) = \mu B(\tan\beta + \cot\beta)$$

$$\text{NMSSM}: \text{"} M_A^2 \text{"} = \mu_{\text{eff}} B_{\text{eff}}(\tan\beta + \cot\beta)$$

with $B_{\text{eff}} = A_\lambda + \kappa\, s,\ \mu_{\text{eff}} = \lambda\, s$ $\quad\Rightarrow$ one very light a_1

Mass of the charged Higgs:

$$\text{MSSM}: M^2_{H^\pm} = M_A^2 + M_W^2 = M_A^2 + \frac{1}{2} v^2 g^2$$

$$\text{NMSSM}: M^2_{H^\pm} = M_A^2 + v^2 \left(\frac{g^2}{2} - \lambda^2\right)$$

Mass of the lightest $\mathcal{CP}$-even Higgs:

$$m^2_{h,\text{tree,NMSSM}} = m^2_{h,\text{tree,MSSM}} + M_Z^2 \frac{\lambda^2}{g^2} \sin^2 2\beta$$

Mass of the $\mathcal{CP}$-odd Higgs:

$$\text{MSSM}: M_A^2 = -m_{12}^2(\tan\beta + \cot\beta) = \mu B(\tan\beta + \cot\beta)$$

$$\text{NMSSM}: \text{"}M_A^2\text{"} = \mu_{\text{eff}} B_{\text{eff}}(\tan\beta + \cot\beta)$$

$$\text{with } B_{\text{eff}} = A_\lambda + \kappa\, s,\ \mu_{\text{eff}} = \lambda\, s$$

$\Rightarrow$ one very light a_1

Mass of the charged Higgs:

$$\text{MSSM}: M_{H^\pm}^2 = M_A^2 + M_W^2 = M_A^2 + \frac{1}{2} v^2 g^2$$

$$\text{NMSSM}: M_{H^\pm}^2 = M_A^2 + v^2 \left(\frac{g^2}{2} - \lambda^2 \right)$$

$\Rightarrow M_{h_1}^{\text{MSSM,tree}} \le M_{h_1}^{\text{NMSSM,tree}}$, one light a_1, $M_{H^\pm}^{\text{MSSM,tree}} \ge M_{H^\pm}^{\text{NMSSM,tree}}$

5. The MSSM Higgs sector with $\mathcal{CP}$ violation

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} e^{i\xi}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$
$$+ \underbrace{\frac{g'^2 + g^2}{8}}(H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

2 $\mathcal{CP}$-violating phases: ξ, arg(m_{12}) $\Rightarrow$ can be set/rotated to zero

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_{H^\pm}^2$$

The Higgs sector of the cMSSM at tree-level:

- phase of m_{12} :

  $m_{12} = 0$ and $\mu = 0 \Rightarrow$ additional $U(1)$ (PQ) symmetry

  reality: $m_{12} \neq 0$, $\mu \neq 0$

  $\Rightarrow$ perform PQ transformation with ϕ_{PQ}

$$\begin{aligned} m_{12}^{2\prime} &= |m_{12}^2| e^{i(\phi_{m_{12}} - \phi_{\text{PQ}})} \\ \mu' &= |\mu| e^{i(\phi_\mu - \phi_{\text{PQ}})} \end{aligned}$$

  $\Rightarrow$ m_{12} can always be chosen real

- phase of H_2: ξ :

  mixing between $\mathcal{CP}$-even and $\mathcal{CP}$-odd states:

$$\mathcal{M}_{\mathcal{CP}-\text{even},\mathcal{CP}-\text{odd}} = \begin{pmatrix} 0 & m_{12}^2 \sin\xi \\ -m_{12}^2 \sin\xi & 0 \end{pmatrix}$$

  Tadpoles have to vanish: $T_A^{\text{tree}} \propto \sin\xi \, m_{12}^2 \stackrel{!}{=} 0$
  $\Rightarrow \xi = 0 \Rightarrow$ no $\mathcal{CP}$V at tree-level

The Higgs sector of the cMSSM at the loop-level:

Complex parameters enter via loop corrections:

– μ : Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings $\Rightarrow$ $X_{t,b,\tau} = A_{t,b,\tau} - \mu^* \{\cot\beta\,, \tan\beta\}$ complex

– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– M_3 : gluino mass parameter

$\Rightarrow$ can induce $\mathcal{CP}$-violating effects

Result:

$$(A, H, h) \to (h_3, h_2, h_1)$$

with

$$m_{h_3} > m_{h_2} > m_{h_1}$$

$\Rightarrow$ strong changes in Higgs couplings to SM gauge bosons and fermions

$\tilde{t}/\tilde{b}$ sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ($X_t = A_t - \mu^*/\tan\beta$, $X_b = A_b - \mu^* \tan\beta$):

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

soft SUSY-breaking parameters A_t, A_b also appear in ϕ-$\tilde{t}/\tilde{b}$ couplings

$$m^2_{\tilde{t}_{1,2}} = m_t^2 + \frac{1}{2}\left(M^2_{\tilde{t}_L} + M^2_{\tilde{t}_R} \mp \sqrt{(M^2_{\tilde{t}_L} - M^2_{\tilde{t}_R})^2 + 4 m_t^2 |X_t|^2}\right)$$

$\Rightarrow$ independent of ϕ_{X_t}

but $\theta_{\tilde{t}}$ is now complex

$SU(2)$ relation $\Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L} \Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

More on complex phases: Neutralinos and charginos:

Higgsinos and electroweak gauginos mix

charged:

$$\tilde{W}^+, \tilde{h}_u^+ \to \tilde{\chi}_1^+, \tilde{\chi}_2^+, \qquad \tilde{W}^-, \tilde{h}_d^- \to \tilde{\chi}_1^-, \tilde{\chi}_2^-$$

$\Rightarrow$ charginos: mass eigenstates

mass matrix given in terms of M_2, μ, $\tan\beta$

neutral:

$$\underbrace{\tilde{\gamma}, \tilde{Z},}_{\tilde{W}^0, \tilde{B}^0} \tilde{h}_u^0, \tilde{h}_d^0 \to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$$

$\Rightarrow$ neutralinos: mass eigenstates

mass matrix given in terms of M_1, M_2, μ, $\tan\beta$

$\Rightarrow$ only one new parameter

$\Rightarrow$ MSSM predicts mass relations between neutralinos and charginos

Mixing of the $\mathcal{CP}$-even Higgs bosons:

Propagator/Mass matrix at tree-level:

$$\begin{pmatrix} q^2 - m_H^2 & 0 \\ 0 & q^2 - m_h^2 \end{pmatrix}$$

Propagator / mass matrix with higher-order corrections
($\to$ Feynman-diagrammatic approach):

$$M_{hH}^2(q^2) = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i,j = h,H)$: renormalized Higgs self-energies

$\mathcal{CP}$-even fields can mix

$\Rightarrow$ complex roots of $\det(M_{hH}^2(q^2))$: $\mathcal{M}_{h_i}^2 (i = 1,2)$: $\mathcal{M}^2 = M^2 - iM\Gamma$

Propagator/Mass matrix at tree-level with $\mathcal{CP}$ violation:

$$\begin{pmatrix} q^2 - m_A^2 & 0 & 0 \\ 0 & q^2 - m_H^2 & 0 \\ 0 & 0 & q^2 - m_h^2 \end{pmatrix}$$

Propagator / mass matrix with higher-order corrections
(→ Feynman-diagrammatic approach):

$$M^2_{hHA}(q^2) = \begin{pmatrix} q^2 - m_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

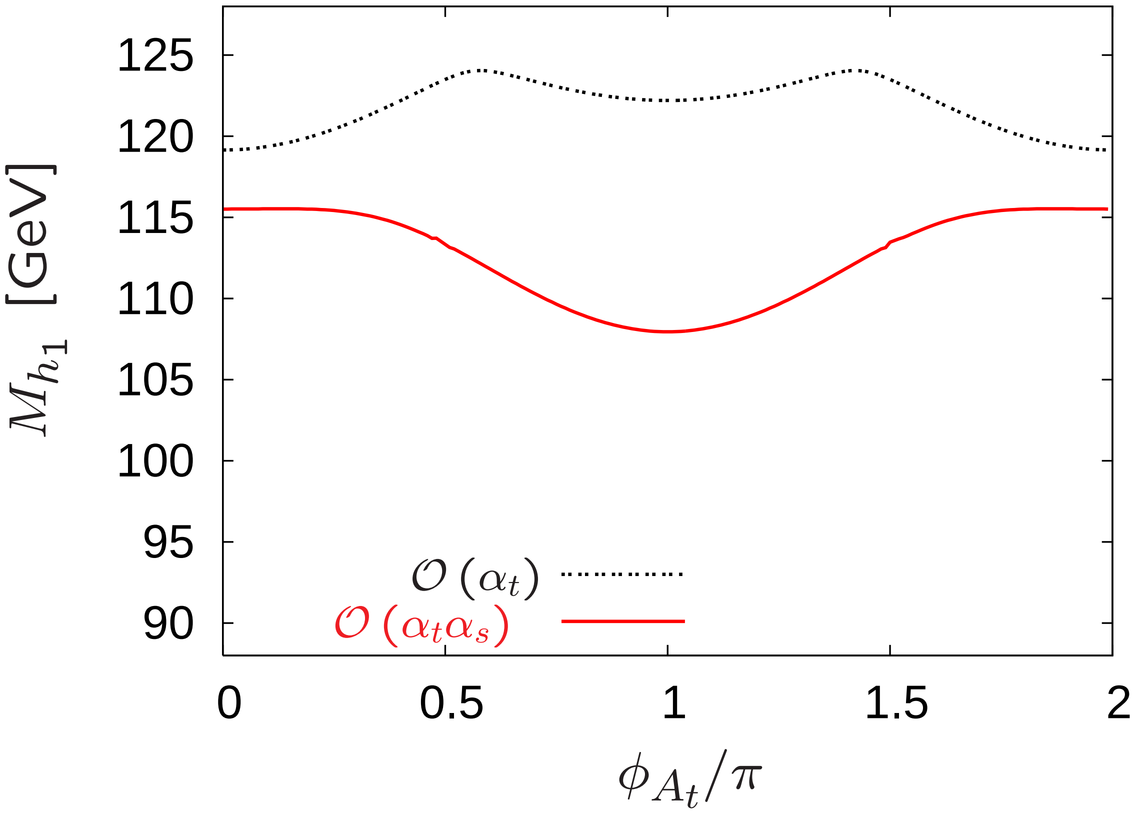
$\hat{\Sigma}_{ij}(q^2)$ $(i, j = h, H, A)$: renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0$ ⇒ $\mathcal{CP}$V, $\mathcal{CP}$-even and $\mathcal{CP}$-odd fields can mix

⇒ complex roots of $\det(M^2_{hHA}(q^2))$: $\mathcal{M}^2_{h_i} (i = 1, 2, 3)$: $\mathcal{M}^2 = M^2 - iM\Gamma$

M_{h_1} as a function of ϕ_{A_t}:

[S.H., W. Hollik, H. Rzehak, G. Weiglein '07]



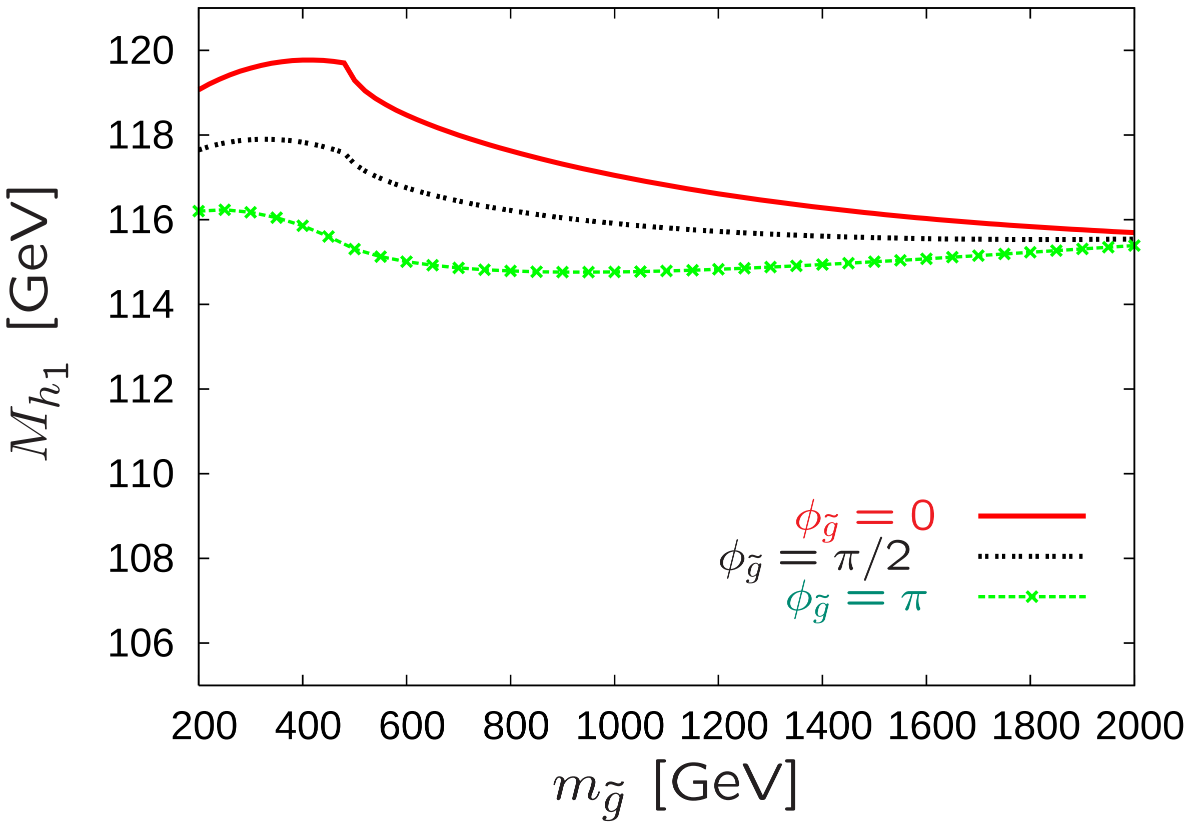
$M_{\text{SUSY}} = 1000$ GeV
$|A_t| = 2000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 150$ GeV

OS renormalization

$\Rightarrow$ modified dependence on ϕ_{A_t} at the 2-loop level

M_{h_1} as a function of $\phi_{\tilde{g}}$:

[S.H., W. Hollik, H. Rzehak, G. Weigelin '07]



$M_{\text{SUSY}} = 500$ GeV
$A_t = 1000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 500$ GeV

OS renormalization

$\Rightarrow$ threshold at $m_{\tilde{g}} = m_{\tilde{t}} + m_t$

$\Rightarrow$ large effects around threshold

$\Rightarrow$ phase dependence has to be taken into account

Back-up

6. New results for MSSM Higgs boson mass predictions

Predictions for m_h, m_H from diagonalization of tree-level mass matrix:

$\phi_1 - \phi_2$ basis:

$$M^{2,\text{tree}}_{\text{Higgs}} = \begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix}$$

$$\Big\Downarrow \leftarrow \text{Diagonalization}, \alpha$$

$$\begin{pmatrix} m_H^{2,\text{tree}} & 0 \\ 0 & m_h^{2,\text{tree}} \end{pmatrix}$$

$$m^{2,\text{tree}}_{H,h} = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$\Rightarrow m_h \leq M_Z$ at tree level

Method I:
Higher-order corrections in the Feynman diagrammatic method:

Propagator/Mass matrix at tree-level:

$$\begin{pmatrix} q^2 - m_H^2 & 0 \\ 0 & q^2 - m_h^2 \end{pmatrix}$$

Propagator / mass matrix with higher-order corrections
($\to$ Feynman-diagrammatic approach):

$$M_{hH}^2(q^2) = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i, j = h, H)$: renormalized Higgs self-energies

$\mathcal{CP}$-even fields can mix

$\Rightarrow$ complex roots of $\det(M_{hH}^2(q^2))$: $\mathcal{M}_{h_i}^2 (i = 1, 2)$: $\mathcal{M}^2 = M^2 - iM\Gamma$

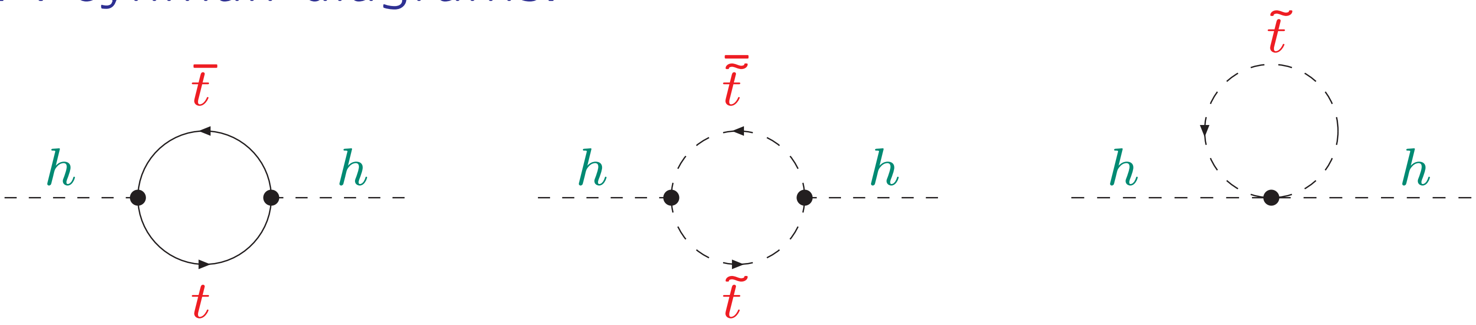
Calculation of renormalized Higgs boson self-energies:

$\hat{\Sigma}(q^2) = \hat{\Sigma}^{(1)}(q^2) + \hat{\Sigma}^{(2)}(q^2) + \ldots$

all MSSM particles contribute

main contribution: $t/\tilde{t}$ sector ($\tilde{t}$: scalar top, SUSY partner of the t)

1-Loop: Feynman diagrams:



Dominant 1-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

size of the corrections: $\mathcal{O}$(50 GeV)

$\Rightarrow$ 2-Loop calculation necessary!

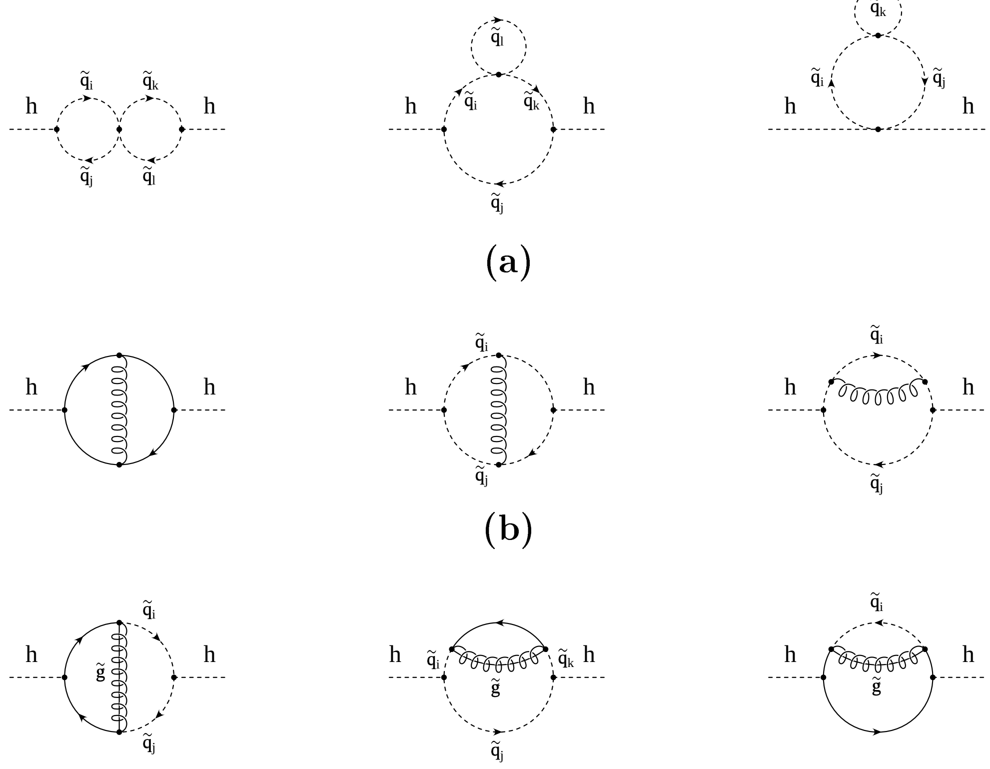
Example for two-loop: $\hat{\Sigma}^{(2)}(0)$

[*S. H., W. Hollik, G. Weiglein '98*]

dominant contributions of $\mathcal{O}(\alpha_t \alpha_s)$:

(a) pure scalar diagrams
(b) diagrams with gluonexchange
(c) diagrams with gluinoexchange

Quite complicated calculation ...
$\Rightarrow$ Need for computer algebra programs



(a) (b) (c)

To avoid large corrections:
On-shell renormalization of the scalar top sector $\Rightarrow X_t^{\text{OS}}$

$$\sim m_t^4 \left[\log^2 \left(\frac{m_{\tilde{t}}}{m_t} \right) + \log \left(\frac{m_{\tilde{t}}}{m_t} \right) \right]$$

['98 - '13:] $\Rightarrow$ many more corrections calculated!

Structure of higher-order corrections:

One-loop:

$$\Delta M_h^2 \sim m_t^2 \alpha_t \left[L + L^0\right] \ , \quad L := \log\left(\frac{m_{\tilde{t}}}{m_t}\right)$$

Two-loop:

$$\Delta M_h^2 \sim m_t^2 \left\{\alpha_t \alpha_s \left[L^2 + L + L^0\right] + \alpha_t^2 \left[L^2 + L + L^0\right]\right\}$$

Three-loop:

$$\begin{aligned}\Delta M_h^2 \sim m_t^2 \{ \quad & \alpha_t \alpha_s^2 \left[L^3 + L^2 + L + L^0\right] \\ & + \alpha_t^2 \alpha_s \left[L^3 + L^2 + L + L^0\right] \\ & + \alpha_t^3 \left[L^3 + L^2 + L + L^0\right] \}\end{aligned}$$

Partial results: [*S. Martin '07*]

[*R. Harlander, P. Kant, L. Mihaila, M. Steinhauser '08*] $\Rightarrow$ *H3m*

H3m adds $\mathcal{O}\left(\alpha_t \alpha_s^2\right)$ corrections to *FeynHiggs*

Large $m_{\tilde{t}}$ $\Rightarrow$ large L $\Rightarrow$ resummation of logs necessary $\Rightarrow$ Method **II**

Advantages of Feynman-diagrammatic method:

– all contributions at fixed order are captured

– trivial to include many SUSY scales

– full control over Higgs boson self-energies
$\rightarrow$ needed for other quantities (production and decay)

Problems of Feynman-diagrammatic method:

– always only fixed order

– large logs not captured beyond the calculated order

Method II: Log resummation via RGE's:

Simple example for log resummation:

SUSY mass scale: $M_{\text{SUSY}} = M_{\text{S}} \sim m_{\tilde{t}}$

Above M_{SUSY}: MSSM
Below M_{SUSY}: SM

Relevant SM parameters:
- quartic coupling λ
- top Yukawa coupling h_t $(\alpha_t = h_t^2/(4\,\pi))$
- strong coupling constant g_s $(\alpha_s = g_s^2/(4\,\pi))$

Procedure:

1. Take: $h_t(m_t), g_s(m_t)$

   SM RGEs for h_t, g_s: $h_t, g_s(m_t) \to h_t, g_s(M_{\text{S}})$

2. Take $\lambda(M_{\text{S}}), h_t(M_{\text{S}}), g_s(M_{\text{S}})$

   SM RGEs for λ, h_t, g_s: $\lambda, h_t, g_s(M_{\text{S}}) \to \lambda, h_t, g_s(m_t)$

3. Evaluate M_h^2

$$M_h^2 \sim 2\lambda(m_t)v^2$$

Advantages of RGE log resummation:

– large logs taken into account to all orders

– calculation can easily be extended to very large scales

Problems of RGE log resummation:

– not all contributions at fixed order are captured
  $\to$ sub-leading logs more difficult
  $\to$ momentum dependence

– difficult (impossible?): include many different SUSY scales

– difficult (impossible?): control over Higgs boson self-energies
  $\to$ needed for other quantities (production and decay)

Status

Our code (until 11/13):

FeynHiggs

`www.feynhiggs.de`

[*T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '98 – '14*]

$\rightarrow$ all Higgs masses, couplings, BRs, XSs (easy to link, easy to use :-)

– full one-loop (also for complex parameters)
– leading and subleading two-loop: $\mathcal{O}\left(\alpha_t\alpha_s, \alpha_b\alpha_s, \alpha_t^2, \alpha_t\alpha_b, \alpha_b^2\right)$
– running top mass (to minimize three-loop)

Remaining theoretical uncertainties in prediction for M_h in the MSSM:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

– From unknown higher-order corrections: $\Rightarrow \Delta M_h \approx 3$ GeV

$\rightarrow$ point-by-point calculation available in *FeynHiggs*

– From uncertainties in input parameters $\Delta m_t \approx 1$ GeV $\Rightarrow \Delta M_h \approx 1$ GeV

Our code (from 12/13 on): [arXiv:1312.4937]

FeynHiggs 2.10.0

`www.feynhiggs.de`

First and only code that provides:

Combination of

1.) Best available Feynman-diagrammatic result

and

2.) Leading and subleading logs from the top/stop sector

Supplemented by
Improved calulation of theory uncertainty: $\Delta M_h \lesssim 1.5$ GeV
(for the points analyzed so far)

RGE log resummation:

SM one-loop RGEs:

$$16\pi^2\beta_\lambda = 6(\lambda^2 + \lambda - h_t^4)\ ,$$
$$16\pi^2\beta_{h_t^2} = h_t^2(9/2h_t^2 - 8g_s^2)\ ,$$
$$16\pi^2\beta_{g_s^2} = -7g_s^4$$

$\Rightarrow$ at n-loop order: L^n

Our procedure:

– SM two-loop RGEs

– one-loop threshold correction for $\lambda(M_{\text{SUSY}})$:

$$\lambda(M_S) = \frac{3\,h_t^4}{8\,\pi^2} x_t^2 \left[1 - 1/12\,x_t^2\right]\ , \quad x_t = X_t^{\overline{\text{MS}}}/M_S$$

$\Rightarrow$ at n-loop order: $L^n + L^{n-1}$

– add correction ($\times 1/\sin^2\beta$) to $\hat{\Sigma}_{\phi_2\phi_2}$

– subtract leading and subleading logs at one- and two-loop (with X_t^{OS}) to avoid double counting

$\Rightarrow$ combination of best FD result with resummed LL, NLL corrections for large $m_{\tilde{t}}$

$\Rightarrow$ most precise M_h prediction for large $m_{\tilde{t}}$ $\Rightarrow$ FeynHiggs 2.10.0

Some numerical results

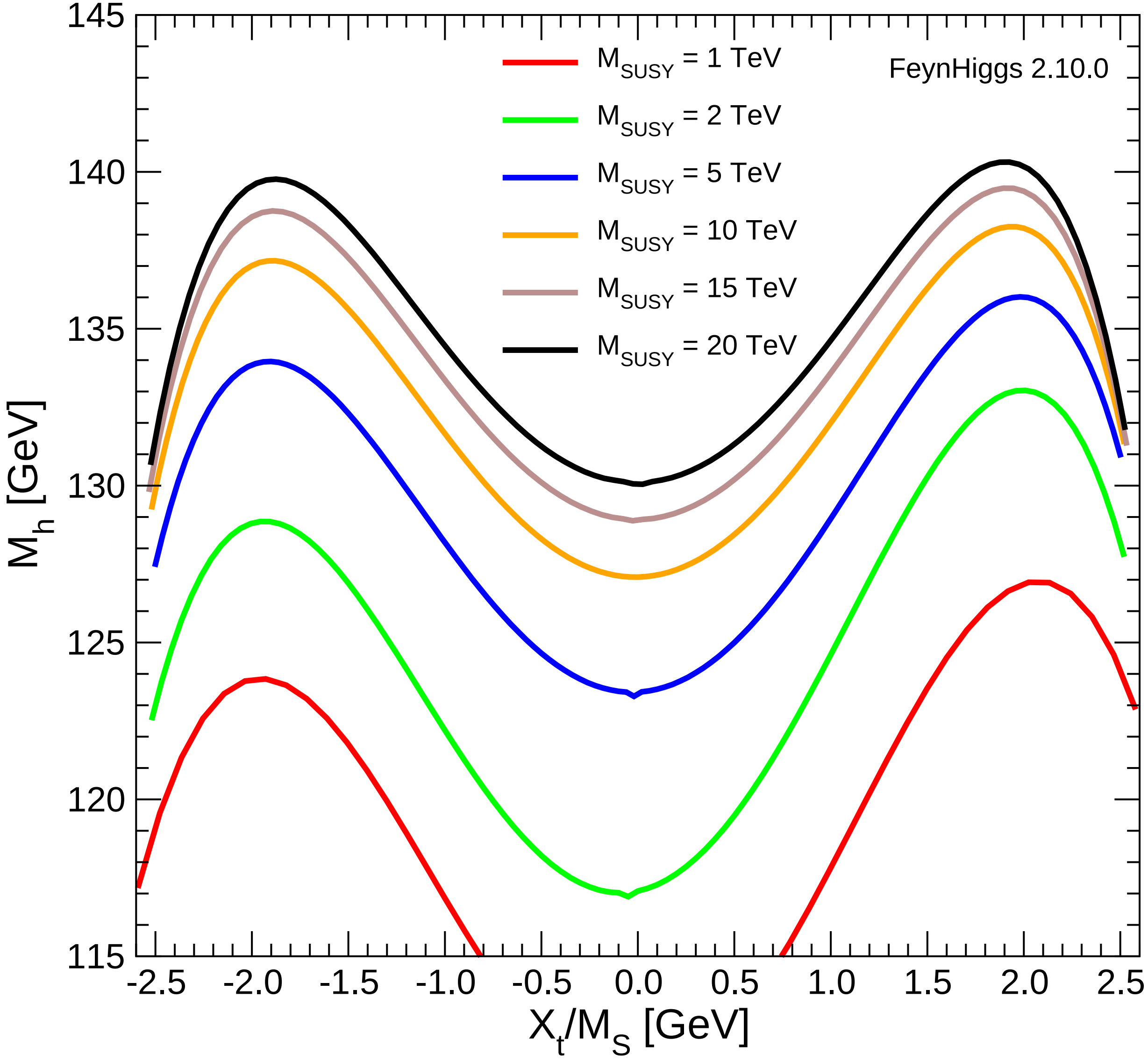
[*FeynHiggs 2.10.0*]

Parameters:

$$
\begin{aligned}
M_{\mathsf{S}} &= \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \\
M_A &= 1000 \text{ GeV} \\
\mu &= 1000 \text{ GeV} \\
M_2 &= 1000 \text{ GeV} \\
m_{\tilde{g}} &= 1600 \text{ GeV} \\
\tan\beta &= 10
\end{aligned}
$$

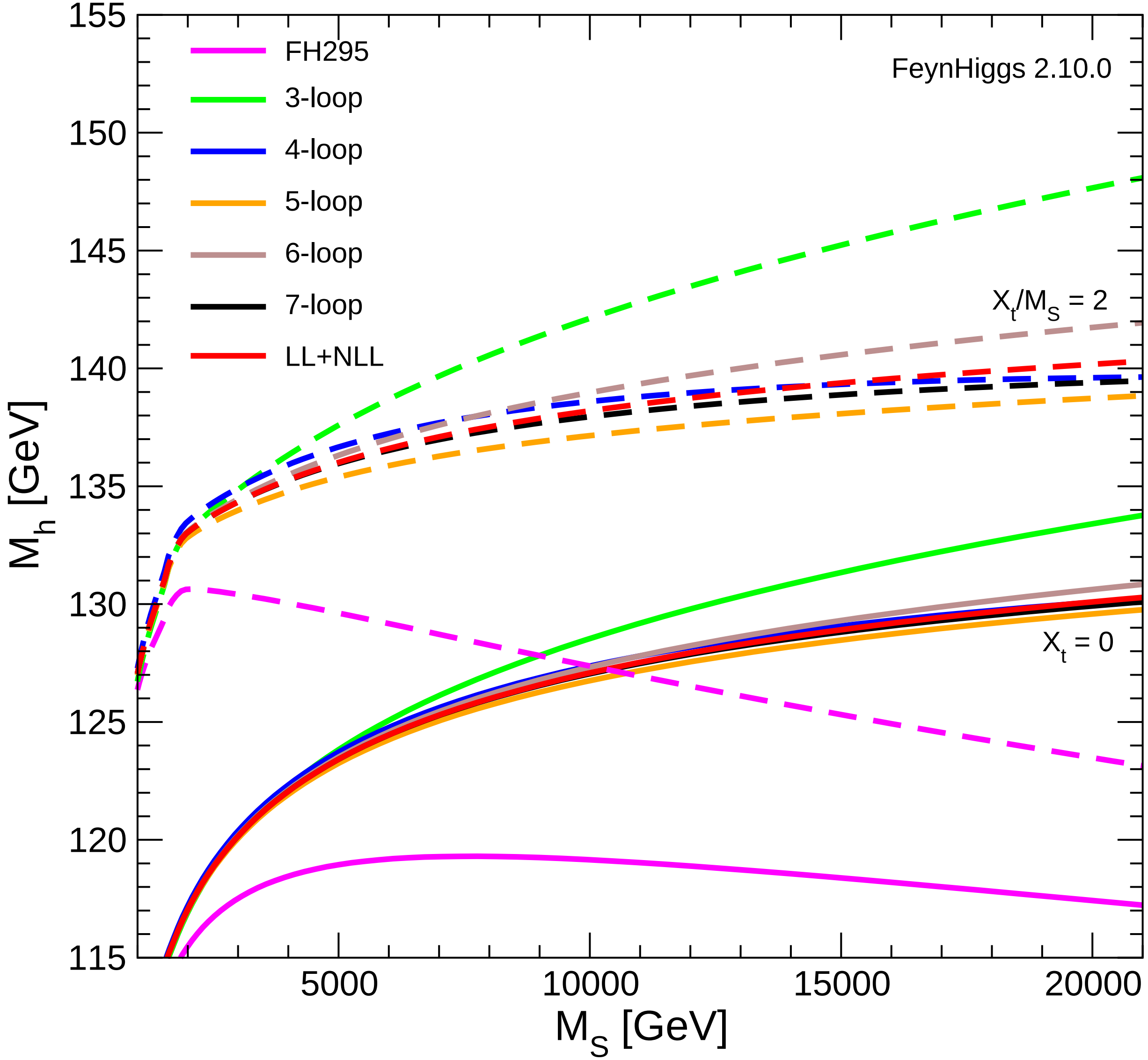
Vary M_{S}, X_t to analyze effects

$M_h(X_t/M_S)$: [FeynHiggs 2.10.0]



$\Rightarrow$ increase with M_S, maxima at $X_t/M_S = \pm 2$

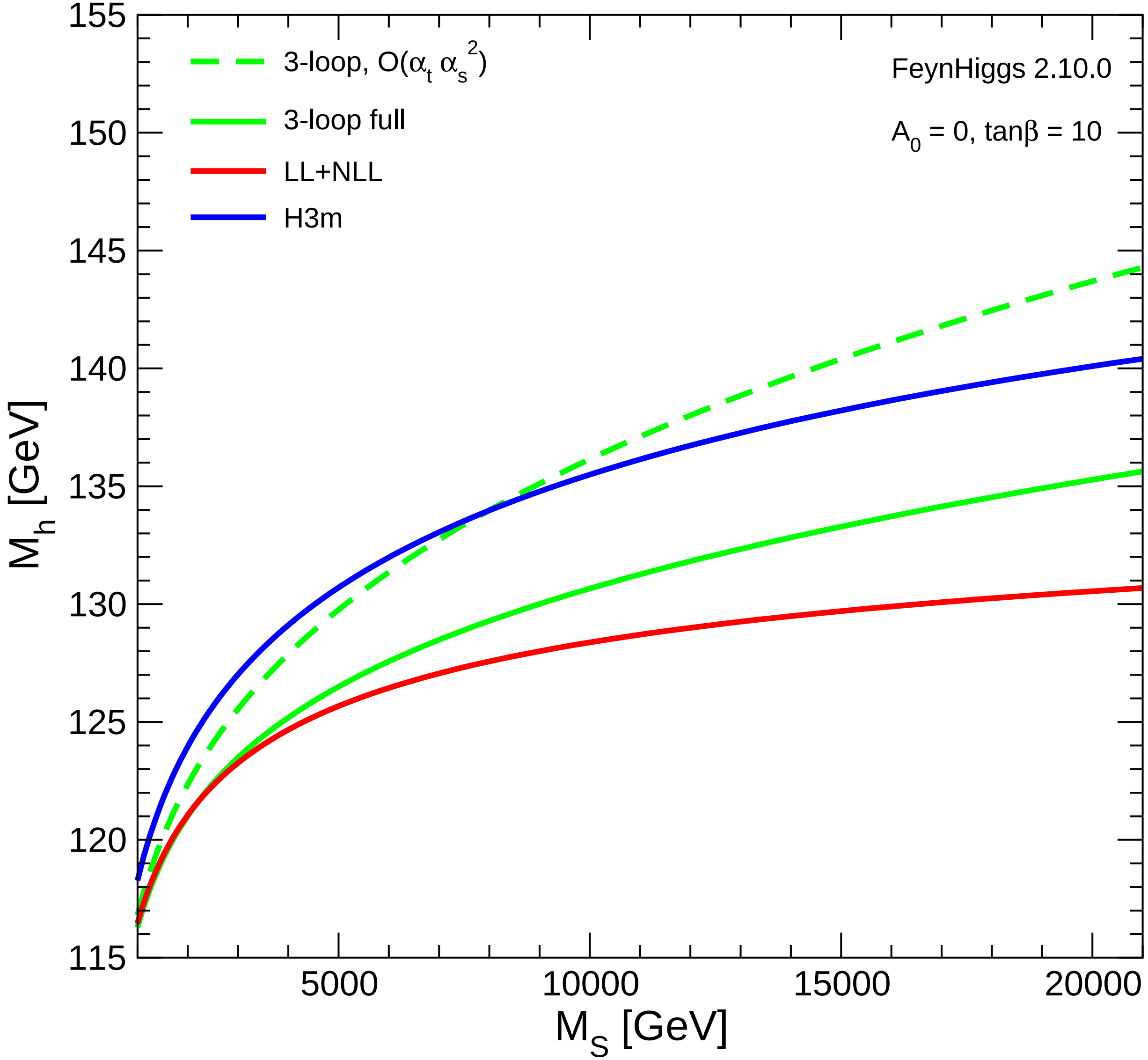
$M_h(M_S)$ for various approximations: [*FeynHiggs 2.10.0*]



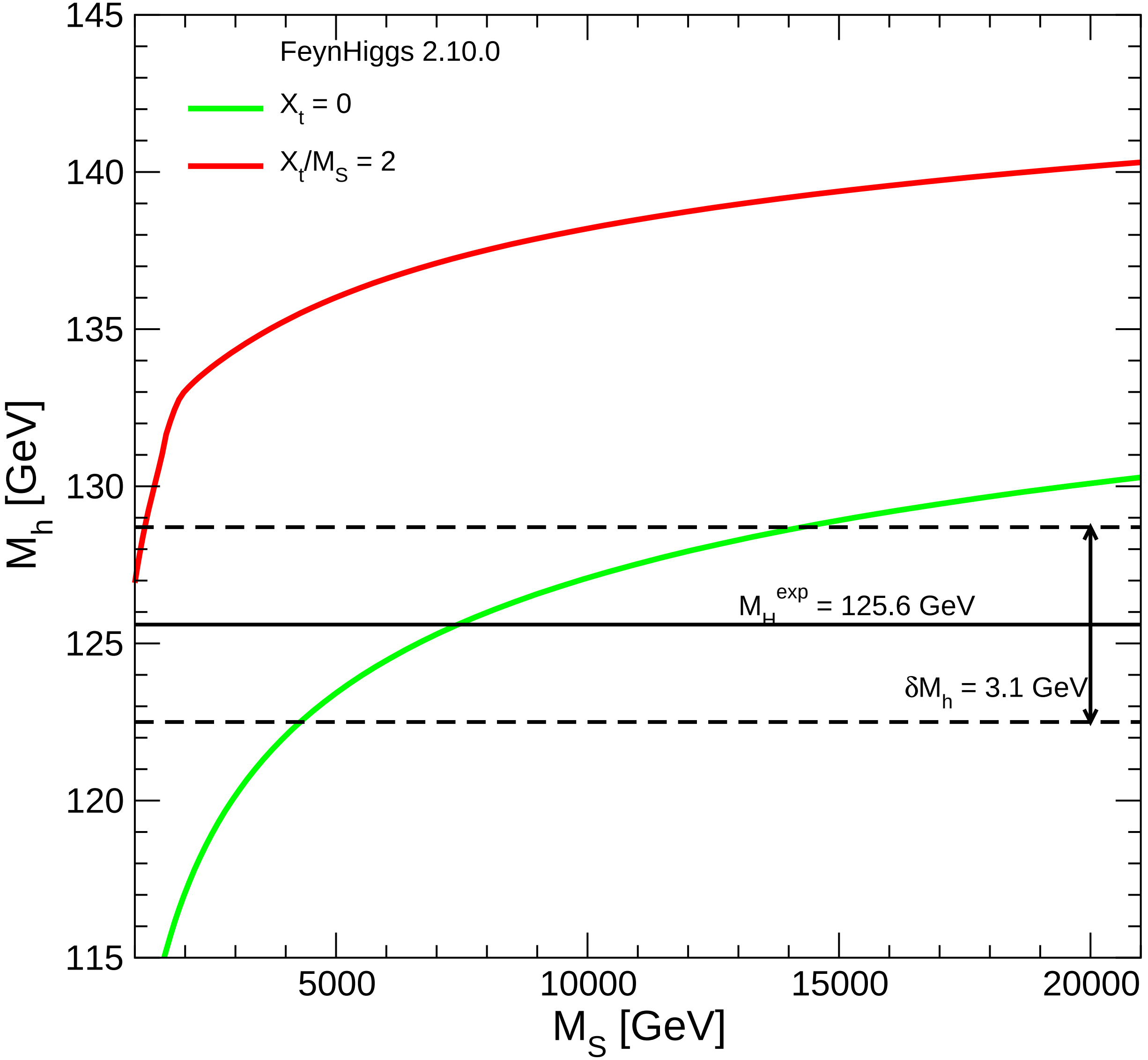
$\Rightarrow$ 3-loop good for $M_S \lesssim 2$ TeV, 7-loop: $\Delta \sim 1$ GeV for $M_S = 20$ TeV

$M_h(M_S)$ compared with H3m:

[*FeynHiggs 2.10.0*]



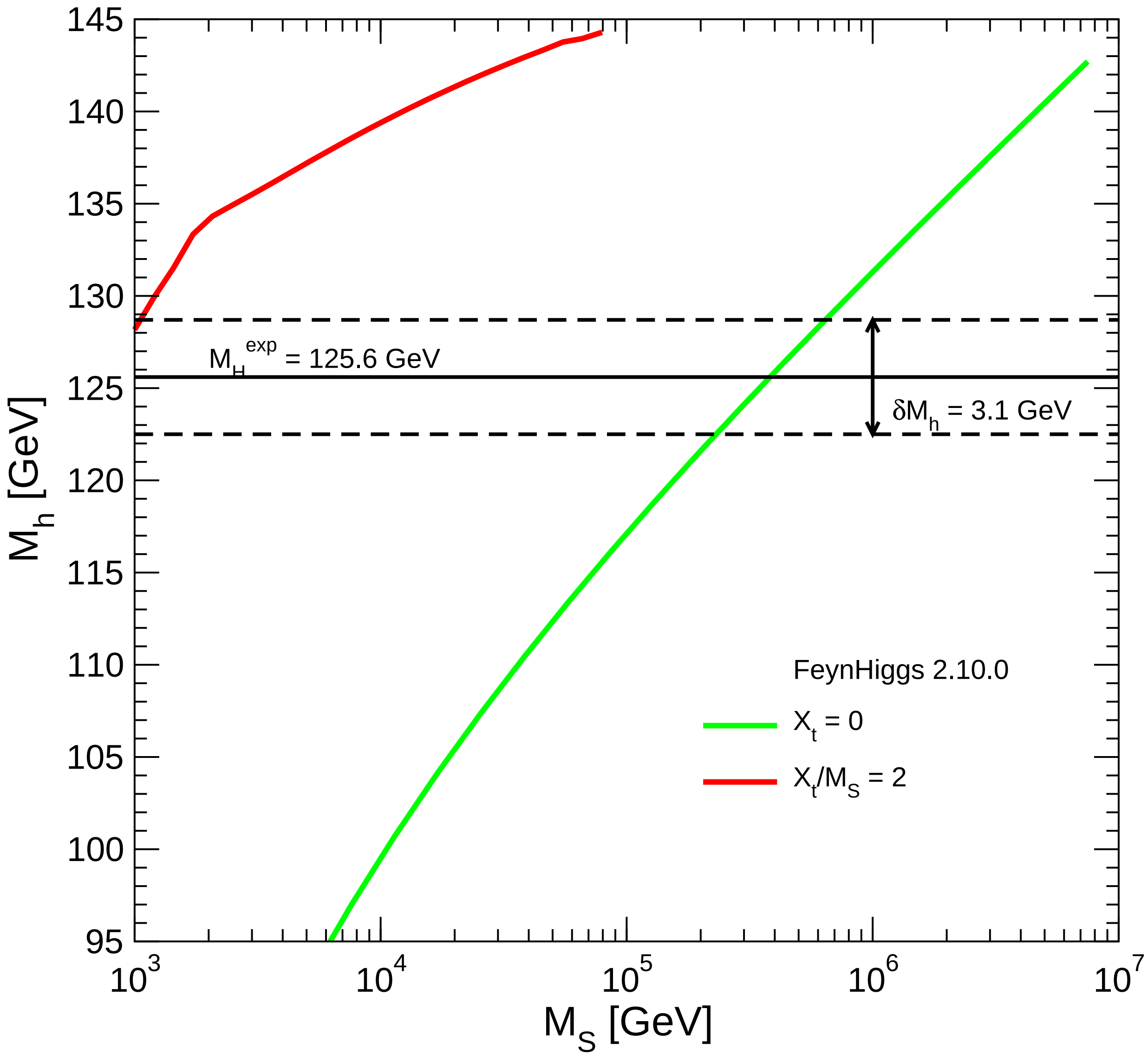
$\Rightarrow$ 3-loop $\mathcal{O}\left(\alpha_t^2\alpha_s, \alpha_t^3\right) \oplus$ beyond 3-loop important for precise M_h prediction!



⇒ upper bound on M_S?

$M_h(M_S)$ for $\tan\beta = 1\,(X_t = 0)$ or $\tan\beta = 40\,(X_t/M_S = 2)$:

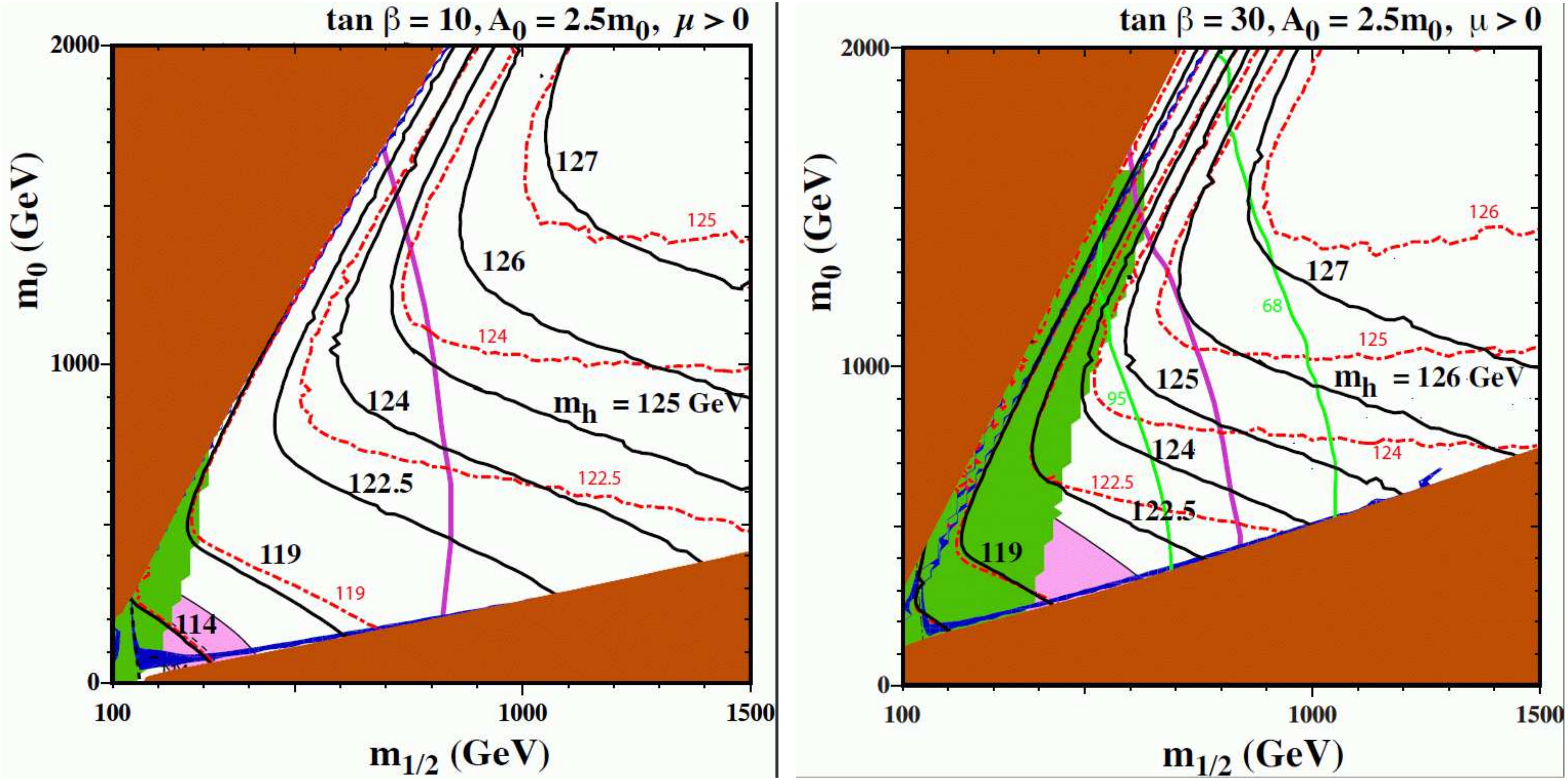
[FeynHiggs 2.10.0 - PRELIMINARY]



$\Rightarrow$ "upper bound": $M_S \lesssim 650$ TeV $\Rightarrow$ needs refinement!

Effects in the CMSSM

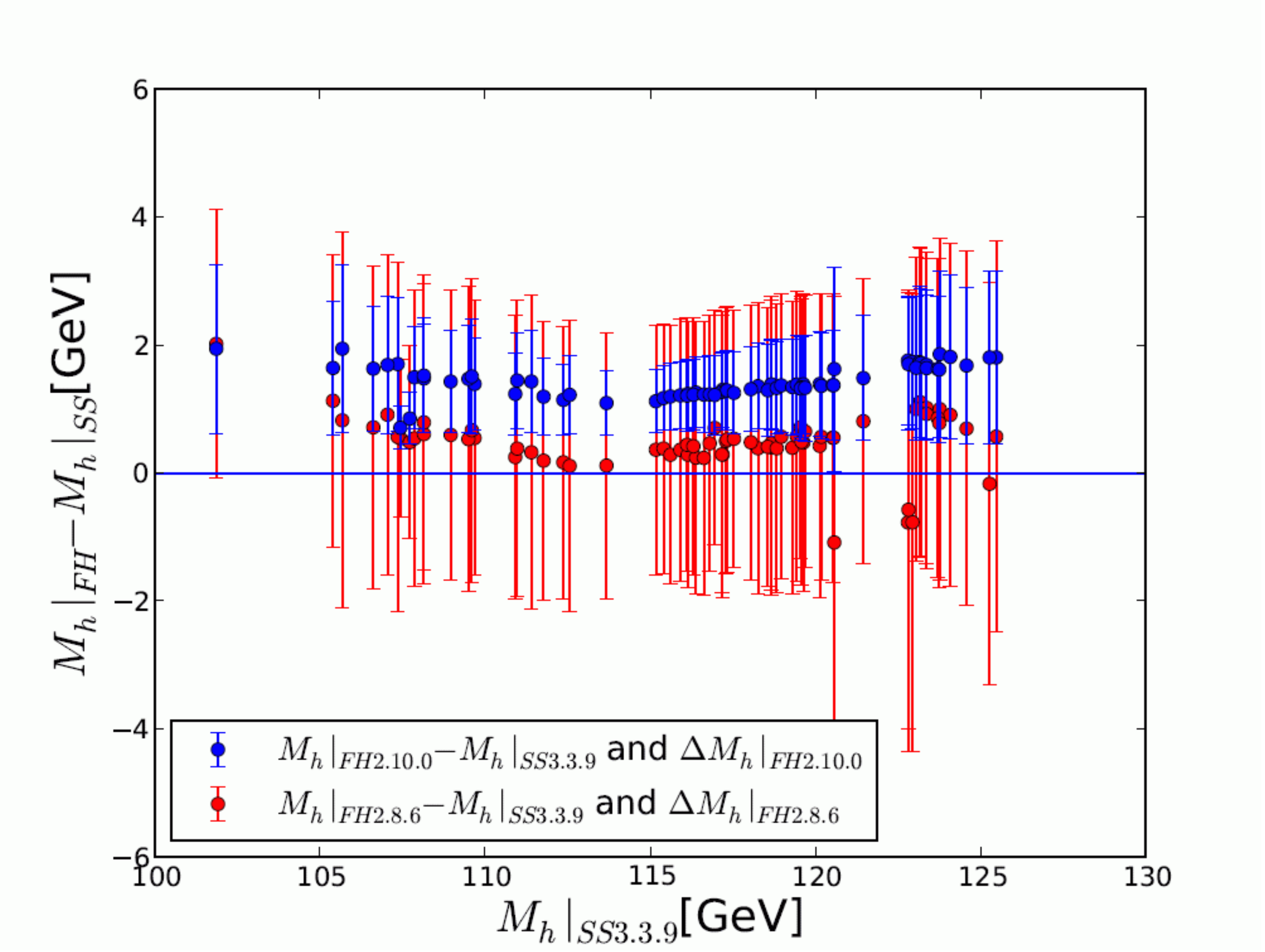
[*O. Buchmüller et al. '13*]



red-dashed: FeynHiggs 2.8.5 black: FeynHiggs 2.10.0

$\Rightarrow$ shift to larger masses, $M_h \sim 125.5$ GeV "easier"

Effects in the CMSSM: *FeynHiggs* vs. *SoftSusy* [*O. Buchmüller et al. '13*]



red: FeynHiggs 2.8.5 (incl. unc.) blue: FeynHiggs 2.10.0 (incl. updated unc.)

⇒ shift to larger masses, not captured by other codes

Perspectives

Can the theory precision meet the experimental precision?

A) Intrinsic uncertainty in the Feynman-diagrammatic method

B) Intrinsic uncertainty in the RGE method

C) Parametric uncertainties from SM input

A) Intrinsic uncertainty in the Feynman-diagrammatic method

$\mathcal{O}\left(\alpha_t\alpha_s^2\right)$ exists in *H3m*

$\to$ expansion in many mass scales necessary

$\Rightarrow$ progress possible, but difficult and slow

$\mathcal{O}\left(\alpha_t^2\alpha_s, \alpha_t^3\right)$ probably possible

Inclusion of $b/\tilde{b}$ very difficult (more and very different scales)

Corrections beyond 3-loop???

$\Rightarrow$ dedicated effort necessary!

B) Intrinsic uncertainty in the RGE method

Good recent overview paper: [*P. Draper, G. Lee, C. Wagner, arXiv:1312.5743*]

Missing in *FeynHiggs*:
– 3-loop RGE's
– 2-loop threshold corrections
– inclusion of more scales: EW scale, M_A

⇒ inclusion in *FeynHiggs* probably possible, but far from trivial
⇒ combination of FD and RGE method crucial!

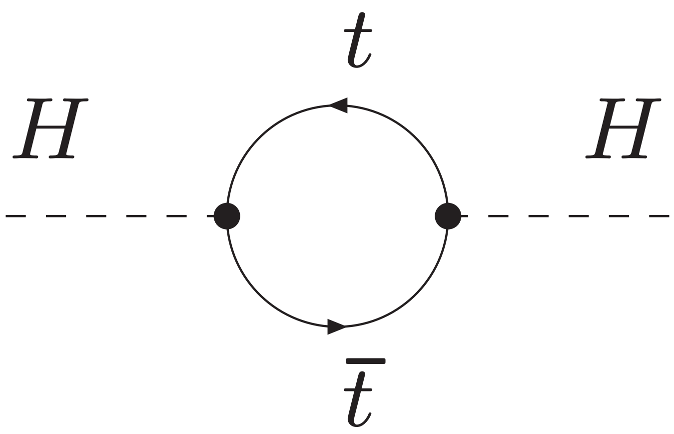
⇒ dedicated effort necessary!

Goal for future *FeynHiggs* version (5-10 years from now): $\Delta M_h^{\text{intr.}} \lesssim$ 500 MeV

⇒ knowledge of SUSY mass scales would be extremely helpful . . .

C) Intrinsic uncertainty from m_t:

Nearly any model: large coupling of the Higgs to the top quark:



$\Rightarrow$ one-loop corrections $\Delta M_H^2 \sim G_\mu m_t^4$

$\Rightarrow$ M_H depends sensitively on m_t in all models where M_H can be predicted (SM: M_H is free parameter)

SUSY as an example: $\Delta m_t \approx \pm 1$ GeV $\Rightarrow$ $\Delta M_h \approx \pm 1$ GeV

$\Rightarrow$ Precision Higgs physics needs e^+e^- precision of $\Delta m_t \sim 100$ MeV
$\Rightarrow$ $\Delta M_h \sim 100$ MeV cannot be surpassed (soon)