CP violation in the scalar sector

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Per Osland
University of Bergen

Work with D. Emmanuel-Costa, O. M. Ogreid, M. N. Rebelo

Footnote

- The first derivative of a potential (when set to zero) defines the vacuum expectation value(s)
- The second derivatives of a potential define a mass-squared matrix

“Problems”

- Vacuum: When many fields, get many coupled equations (cubic and trigonometric)
- Mass matrices: When many fields, get large matrices to diagonalize
- Degeneracies

Simpler approach

- Pick a vacuum of interest (must identify possibilities)
- Pick a mass spectrum of interest
- Construct potential
- Check consistency (positivity etc)

Advantages

- Control of physical content
- Linear equations!

End of footnote

Motivation for three Higgs doublets

Three fermion generations may suggest three doublets

Interesting scenario for dark matter

Possibility of having a discrete symmetry and still having spontaneous CP violation

Rich phenomenology

Motivation for imposing discrete symmetries

Symmetries reduce the number of free parameters leading to (testable) predictions

Symmetries help to control HFCNC

Symmetries are needed to stabilise dark matter

Three $SU(2) \times U(1)$-symmetric doublets

Most general potential has 46 parameters (counted by Olaussen et al, 2011)

Consider S_3 symmetric potential

Basic papers:

(a) Pakvasa & Sugawara, 1978
(b) Derman, 1979
(c) Kubo, Okada, Sakamaki, 2004

(a,c): irreducible reps, (b): reducible rep

Two “Frameworks”

May work with the
reducible representation (Derman) or the
irreducible representations (Pakvasa & Sugawara, Das & Dey)

There is a linear map from one framework to the other

Reducible representation

$$\phi_1, \quad \phi_2, \quad \phi_3$$

$$\phi_i = \begin{pmatrix} \varphi_i^+ \\ (\rho_i + \eta_i + i\chi_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, 3$$

$$V = V_2 + V_4$$

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \tfrac{1}{2}\gamma \sum_{i<j} [\phi_i^\dagger \phi_j + \text{h.c.}],$$

$$V_4 = A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i<j} \{C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \overline{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \tfrac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \text{h.c.}]\}$$
$$+ \tfrac{1}{2} E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \text{h.c.}] + \sum_{i\neq j\neq k\neq i, j<k} \{\tfrac{1}{2} E_2 [(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \text{h.c.}]$$
$$+ \tfrac{1}{2} E_3 [(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \text{h.c.}] + \tfrac{1}{2} E_4 [(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \text{h.c.}]\}$$

10 parameters

Irreducible representations

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 - \phi_2) \\ \frac{1}{\sqrt{6}}(\phi_1 + \phi_2 - 2\phi_3) \end{pmatrix} \qquad h_S = \frac{1}{\sqrt{3}}(\phi_1 + \phi_2 + \phi_3)$$

$$h_i = \begin{pmatrix} h_i^+ \\ (w_i + \tilde{\eta}_i + i\tilde{\chi}_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, \quad h_S = \begin{pmatrix} h_S^+ \\ (w_S + \tilde{\eta}_S + i\tilde{\chi}_S)/\sqrt{2} \end{pmatrix}$$

$$V_2 = \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2)$$

$$\begin{aligned} V_4 = {} & \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ & + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) \\ & + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\ & + \lambda_8 (h_S^\dagger h_S)^2 \end{aligned}$$

10 parameters

Decomposition into these two irreducible representations

$$\begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

This definition does not treat equally ϕ_1, ϕ_2, ϕ_3**, they could be interchanged**

Notice similarity with tribimaximal mixing: **Harrison, Perkins and Scott, 1999**

$$\begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

This potential exhibits

$h_1 \to -h_1$ symmetry

but not $h_2 \to -h_2$

Equivalent doublet representation

$$\begin{pmatrix} \tilde{\chi}_1 \\ \tilde{\chi}_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

above symmetry becomes

$$\tilde{\chi}_1 \leftrightarrow \tilde{\chi}_2$$

In the irreducible-rep framework

the case $$\lambda_4 = 0$$

or, in the reducible-rep framework

$$4A - 2(C + \overline{C} + D) - E_1 + E_2 + E_3 + E_4 = 0$$

leads to a continuous SO(2) symmetry

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Massless states!

At this stage, the two frameworks are equivalent

However, introducing Yukawa couplings, for example, in terms of

$$\phi_1, \quad \phi_2, \quad \phi_3$$

or

$$h_1, \quad h_2, \quad h_S$$

they would naturally be different

The vevs are related

$$w_1 = \frac{1}{\sqrt{2}}(\rho_1 - \rho_2)$$

$$w_2 = \frac{1}{\sqrt{6}}(\rho_1 + \rho_2 - 2\rho_3)$$

$$w_S = \frac{1}{\sqrt{3}}(\rho_1 + \rho_2 + \rho_3)$$

$$\rho_1 = \frac{1}{\sqrt{3}}w_S + \frac{1}{\sqrt{2}}w_1 + \frac{1}{\sqrt{6}}w_2$$

$$\rho_2 = \frac{1}{\sqrt{3}}w_S - \frac{1}{\sqrt{2}}w_1 + \frac{1}{\sqrt{6}}w_2$$

$$\rho_3 = \frac{1}{\sqrt{3}}w_S - \frac{\sqrt{2}}{\sqrt{3}}w_2$$

Vacua

Derivatives of potential wrt (complex) fields must vanish

Three complex derivatives = 0   or

Five real derivatives (3 moduli, 2 relative phases) = 0

The minimisation conditions must be consistent.
This is an important constraint on the potential.

May work in either framework

But a particular vacuum may look simpler in one framework than in the other.

Classical (real) vacua

The early literature focused on fermion masses and real vacua (no CPV):

Examples:

$\rho_2 = \rho_3$ Derman 1979

$w_1 = \sqrt{3} w_2$ Das & Dey 2014

Complex vacua

Complex vacua may allow CP violation

Examples:

C-0 $(\rho_1, \rho_2, \rho_3) = x(1, e^{2i\pi/3}, e^{-2i\pi/3}) \quad \Rightarrow w_S = 0$

C-I-a $(\rho_1, \rho_2, \rho_3) = x(1, e^{i\tau}, e^{i\tau}) \quad \Rightarrow w_1 = \sqrt{3} w_2$

C-I-a1 $(\rho_1, \rho_2, \rho_3) = x(e^{-i\tau}, 1, 1) \quad \Rightarrow w_1 = \sqrt{3} w_2$

C-I-a2 $(\rho_1, \rho_2, \rho_3) = x(1, e^{-i\tau}, 1) \quad \Rightarrow w_1 = -\sqrt{3} w_2$

C-I-a3 $(\rho_1, \rho_2, \rho_3) = x(1, 1, e^{-i\tau}) \quad \Rightarrow w_1 = 0$

C-I-a violates CP, C-0 does not

Complex vacua

Here, an overall phase rotation brings us from vacuum C-I-a to C-I-a1

$$\text{C-I-a} \xrightarrow{\exp(-i\tau)} \text{C-I-a1}$$

Next:

$$\text{C-I-a1} \xrightarrow{\rho_1 \leftrightarrow \rho_2} \text{C-I-a2} \xrightarrow{\rho_2 \leftrightarrow \rho_3} \text{C-I-a3}$$

These are all different names for one and the same vacuum

Complex vacua

Spontaneous CP violation

C-0	$(\rho_1, \rho_2, \rho_3) = x(1, e^{2i\pi/3}, e^{-2i\pi/3})$	$\Rightarrow w_S = 0$
C-I-a	$(\rho_1, \rho_2, \rho_3) = x(1, e^{i\tau}, e^{i\tau})$	$\Rightarrow w_1 = \sqrt{3} w_2$
C-I-a1	$(\rho_1, \rho_2, \rho_3) = x(e^{-i\tau}, 1, 1)$	$\Rightarrow w_1 = \sqrt{3} w_2$
C-I-a2	$(\rho_1, \rho_2, \rho_3) = x(1, e^{-i\tau}, 1)$	$\Rightarrow w_1 = -\sqrt{3} w_2$
C-I-a3	$(\rho_1, \rho_2, \rho_3) = x(1, 1, e^{-i\tau})$	$\Rightarrow w_1 = 0$

C-I-a violates CP, C-0 does not

- Complex vevs are no guarantee for SCPV
- The symmetry of the Lagrangian could "hide" the complex conjugation

Example: $\text{C-0:} \quad (\rho_1, \rho_2, \rho_3) = x(1, e^{2i\pi/3}, e^{-2i\pi/3})$

Complex conjugation:

$$x(1, e^{2i\pi/3}, e^{-2i\pi/3}) \Rightarrow x(1, e^{-2i\pi/3}, e^{2i\pi/3})$$

But the Lagrangian has a symmetry:

$$\phi_2 \leftrightarrow \phi_3 \quad \text{and} \quad \rho_2 \leftrightarrow \rho_3$$

which will undo the complex conjugation

Complex vacua

Complex vacua may allow CP violation

More:

C-II-a $\quad (\rho_1, \rho_2, \rho_3) = \hat{\rho}, \hat{\rho}, \hat{\rho}' e^{i\tau} \quad \Rightarrow w_S = 0$

C-II-b $\quad (\rho_1, \rho_2, \rho_3) = \hat{\rho}, \hat{\rho}' e^{i\tau}, \hat{\rho} \quad \Rightarrow w_1 = -\sqrt{3} w_2$

C-II-c $\quad (\rho_1, \rho_2, \rho_3) = \hat{\rho}' e^{i\tau}, \hat{\rho}, \hat{\rho} \quad \Rightarrow w_1 = \sqrt{3} w_2$

C-III $\quad (\rho_1, \rho_2, \rho_3) = \hat{\rho}_1, \hat{\rho}_2 e^{i\tau_2}, \hat{\rho}_3 e^{i\tau_3} \quad \Rightarrow w_1, w_2, w_S$

Complex vacua

Spontaneous CP violation

C-II-a	$(\rho_1, \rho_2, \rho_3) = \hat{\rho}, \hat{\rho}, \hat{\rho}' e^{i\tau}$	$\Rightarrow w_S = 0$
C-II-b	$(\rho_1, \rho_2, \rho_3) = \hat{\rho}, \hat{\rho}' e^{i\tau}, \hat{\rho}$	$\Rightarrow w_1 = -\sqrt{3} w_2$
C-II-c	$(\rho_1, \rho_2, \rho_3) = \hat{\rho}' e^{i\tau}, \hat{\rho}, \hat{\rho}$	$\Rightarrow w_1 = \sqrt{3} w_2$
C-III	$(\rho_1, \rho_2, \rho_3) = \hat{\rho}_1, \hat{\rho}_2 e^{i\tau_2}, \hat{\rho}_3 e^{i\tau_3}$	$\Rightarrow w_1, w_2, w_S$

Complex vacua $\lambda_4 = 0$

C-II-a	$(w_1, w_2, w_S) = (0, \hat{w}e^{i\sigma}, \hat{w}_S)$	$\Rightarrow (\rho, \rho, \rho')$
C-II-b	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, -\hat{w}e^{i\sigma}/\sqrt{3}, \hat{w}_S)$	$\Rightarrow (\rho, \rho', \rho)$
C-II-c	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{i\sigma}/\sqrt{3}, \hat{w}_S)$	$\Rightarrow (\rho', \rho, \rho)$
C-II-d	$(w_1, w_2, w_S) = (\hat{w}_1e^{i\sigma_1}, \hat{w}_2e^{i\sigma_2}, 0)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$
C-II-PS	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{-i\sigma}, \hat{w}_S)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$
C-II-IN	$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{i\sigma}, \hat{w}_S)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$
C-III	$(w_1, w_2, w_S) = (\hat{w}_1e^{i\sigma_1}, \hat{w}_2e^{i\sigma_2}, \hat{w}_S)$	$\Rightarrow (\rho_1, \rho_2, \rho_3)$

Spontaneous CP violation

C-II-a $\quad (w_1, w_2, w_S) = (0, \hat{w}e^{i\sigma}, \hat{w}_S) \quad \Rightarrow (\rho, \rho, \rho')$

C-II-b $\quad (w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, -\hat{w}e^{i\sigma}/\sqrt{3}, \hat{w}_S) \quad \Rightarrow (\rho, \rho', \rho)$

C-II-c $\quad (w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{i\sigma}/\sqrt{3}, \hat{w}_S) \quad \Rightarrow (\rho', \rho, \rho)$

C-II-d $\quad (w_1, w_2, w_S) = (\hat{w}_1e^{i\sigma_1}, \hat{w}_2e^{i\sigma_2}, 0) \quad \Rightarrow (\rho_1, \rho_2, \rho_3)$

C-II-PS $\quad (w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{-i\sigma}, \hat{w}_S) \quad \Rightarrow (\rho_1, \rho_2, \rho_3)$

C-II-IN $\quad (w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{i\sigma}, \hat{w}_S) \quad \Rightarrow (\rho_1, \rho_2, \rho_3)$

C-III $\quad (w_1, w_2, w_S) = (\hat{w}_1e^{i\sigma_1}, \hat{w}_2e^{i\sigma_2}, \hat{w}_S) \quad \Rightarrow (\rho_1, \rho_2, \rho_3)$

Note that C-II-PS does not violate CP

$$(w_1, w_2, w_S) = (\hat{w}e^{i\sigma}, \hat{w}e^{-i\sigma}, \hat{w}_S) \quad \xrightarrow{\text{c. c.}} \quad (\hat{w}e^{-i\sigma}, \hat{w}e^{i\sigma}, \hat{w}_S)$$

When $\lambda_4 = 0$ have symmetry

$$h_1 \leftrightarrow h_2 \quad \text{and} \quad w_1 \leftrightarrow w_2$$

When $\lambda_4 = 0$

there are massless states

Add a soft SO(2)-breaking term:

$$V \to V + \frac{1}{2}\nu^2(h_2^\dagger h_1 + h_1^\dagger h_2)$$

Vacuum conditions are changed

Our Aims

Determine whether Spontaneous CP violation in S_3 is compatible with a good inert dark matter candidate and what are the properties

Challenges include:

Determine necessary and sufficient vacuum stability conditions

Obey unitarity constraints for the potential

Obtain correct dark matter density

Concluding comments

- The S_3-symmetric scalar sector is very rich
- Two different (equivalent) frameworks
- Spontaneous CP violation can take place
- Room for Dark Matter
- Next: Yukawa couplings