Lectures on Electroweak Theory

Dominik Stöckinger, TU Dresden

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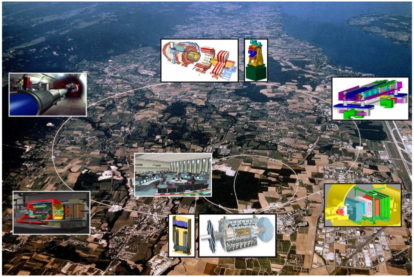
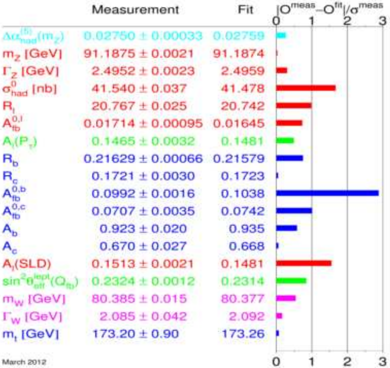
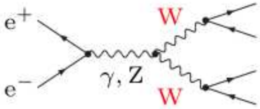
Higgs discovery at LHC



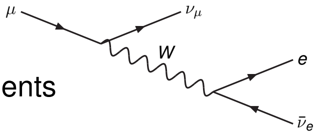
Precision measurements at LEP, SLC, Tevatron



	Measurement	Fit	$\lvert O^{meas}-O^{fit}\rvert/\sigma^{meas}$
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759	
m_Z [GeV]	91.1875 ± 0.0021	91.1874	
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959	
σ_{had}^0 [nb]	41.540 ± 0.037	41.478	
R_l	20.767 ± 0.025	20.742	
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645	
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481	
R_b	0.21629 ± 0.00066	0.21579	
R_c	0.1721 ± 0.0030	0.1723	
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038	
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742	
A_b	0.923 ± 0.020	0.935	
A_c	0.670 ± 0.027	0.668	
A_l(SLD)	0.1513 ± 0.0021	0.1481	
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314	
m_W [GeV]	80.385 ± 0.015	80.377	
Γ_W [GeV]	2.085 ± 0.042	2.092	
m_t [GeV]	173.20 ± 0.90	173.26	



Even more precise low-energy measurements



MuLan at PSI, $G_F = 1.1663787(6) \times 10^{-5}/\mathrm{GeV}^2$

QED tests

$a_e^{\mathrm{exp}} = 0.001\,159\,652\,180\,73(28)$ Gabrielse et al. 2008

Connected by nonperturbative QCD: $\Delta\alpha_{\mathrm{had}} = 0.02764(14)$
(SND, CMD2, KLOE, Babar, Belle, ...)

Reminder: structure of Yang-Mills theories

$$\mathcal{L}_{\text{YM}} = \bar{\psi} i\gamma^\mu D_\mu \psi + |D_\mu \phi|^2 - \frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \mathcal{L}_{\text{only matter}}$$

$$D^\mu = \partial^\mu + igT^a A_a^\mu$$

Specified by choice of:

- Lie group ($\leftrightarrow f^{abc}$) with N_{tot} generators $\rightarrow$ N_{tot} gauge fields
- matter representation(s) T^a $\qquad [T^a, T^b] = if^{abc} T^c$

Gauge transformations

$$U(\theta) = e^{-i\theta^a T^a} = 1 - i\theta^a T^a + \ldots$$

$$\delta\phi = -i\theta^a T^a \phi$$

$$\delta A_\mu^a = \partial_\mu \theta^a + g f^{abc} \theta^b A_\mu^c$$

SM = excellent description of data!

Plan:

1. What is the general theory which agrees with EW observables?
   - Which observables depend on which property of the theory?
   - What changes in deformations of the SM?
2. What is the fundamental theory description?
   - quantized, renormalized
   - gauge field theories
   - spontaneous symmetry breaking
3. Discussion of precise observables/quantities
   - Renormalization of the SM
   - Observables and calculations

Outline

1. Tree-level considerations
2. All-order considerations
3. Precise observables and calculations

Outline

1. Tree-level considerations
   - Observable facts and goal of EW theory
   - General $SU(2)_L \times U(1)_Y$ theories
   - One scalar Higgs doublet
   - Extensions of scalar sector
   - Extensions of gauge sector

Observable facts

Fermions (left-/right-handed): quantum numbers T_{3f}, $Q_f = T_{3f} + Y_f$

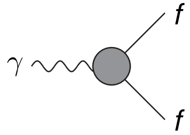
(define also $I_{3f} \equiv T_{3f_L}$ for every fermion)

Fermion	T_{3f}	Q_f	Y_f
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$-\frac{1}{2}$
e_R	0	-1	-1
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$	$\frac{1}{6}$
u_R	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	0	$-\frac{1}{3}$	$-\frac{1}{3}$

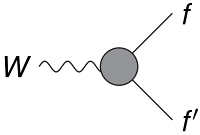
Existence of vector bosons γ, $W^{\pm}$, Z; masses = 0, M_W, M_Z

$$\text{Define } s_W^2 \equiv 1 - c_W^2 \equiv 1 - \frac{M_W^2}{M_Z^2}$$

Couplings of vector bosons to fermions, well-defined at on-shell momenta (Thomson limit, Z-resonance, muon decay)

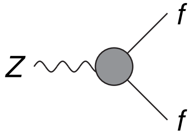


$$= -ieQ_f\gamma^\mu$$

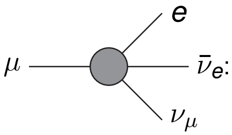


$$\approx -i\frac{e}{\sqrt{2}\sin\theta_{Wf}}\gamma^\mu\frac{1-\gamma^5}{2}$$

[expect $\left(\frac{e}{\sin\theta_{Wf}}\right)^2 \approx G_F M_W^2 4\sqrt{2}$]

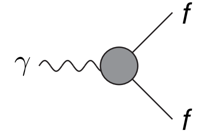


$$= -\frac{i\sqrt{G_F M_W^2 4\sqrt{2}}}{2\cos\theta_{Zf}}\gamma^\mu\left[\left(I_{3f} - Q_f\sin^2\theta^f_{\rm eff}\right) - I_{3f}\gamma_5\right]$$

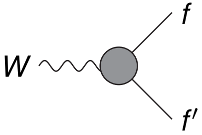


G_F

→ exp. definition of quantities e, G_F, s_W^2 $\sin^2\theta^f_{\rm eff}$, $\cos\theta_{Zf}$

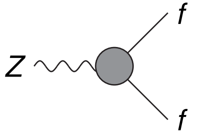


$$= -ieQ_f\gamma^\mu$$

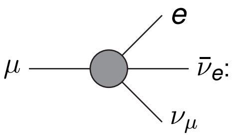


$$\approx -i\frac{e}{\sqrt{2}\sin\theta_{Wf}}\gamma^\mu\frac{1-\gamma^5}{2}$$

[expect $\left(\frac{e}{\sin\theta_{Wf}}\right)^2 \approx G_F M_W^2 4\sqrt{2}$]

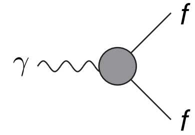


$$= -\frac{i\sqrt{G_F M_W^2 4\sqrt{2}}}{2\cos\theta_{Zf}}\gamma^\mu\left[\left(I_{3f} - Q_f\sin^2\theta^f_{\text{eff}}\right) - I_{3f}\gamma_5\right]$$

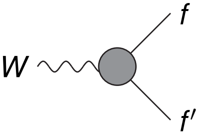


G_F

rewrite $\cos\theta_{Zf}^2 =: c_W^2/\rho_f = \frac{M_W^2}{\rho_f M_Z^2}$

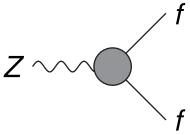


$$= -ieQ_f\gamma^\mu$$

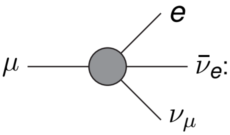


$$\approx -i\frac{e}{\sqrt{2}\sin\theta_{Wf}}\gamma^\mu\frac{1-\gamma^5}{2}$$

[expect $\left(\frac{e}{\sin\theta_{Wf}}\right)^2 \approx G_F M_W^2 4\sqrt{2}$]



$$= -\frac{i\sqrt{G_F\rho_f M_Z^2 4\sqrt{2}}}{2}\gamma^\mu\left[\left(I_{3f} - Q_f\sin^2\theta^f_{\text{eff}}\right) - I_{3f}\gamma_5\right]$$



G_F

$\rightarrow$ precise exp. definition of e, G_F, s_W^2 $\sin^2\theta^f_{\text{eff}}$, ρ_f

Observable facts — results

$M_Z = 91.1876(\ldots)$, $M_W = 80.404(\ldots)$, $s_W^2 = 0.2226$ [PDG]

Parameter	Average
ρ_ν	1.0030 ± 0.0031
ρ_ℓ	1.0050 ± 0.0010
ρ_b	1.059 ± 0.021
ρ_c	1.013 ± 0.021
$\sin^2\theta_\mathrm{eff}^\mathrm{lept}$	0.23128 ± 0.00019
$\sin^2\theta_\mathrm{eff}^\mathrm{b}$	0.281 ± 0.016
$\sin^2\theta_\mathrm{eff}^\mathrm{c}$	0.2355 ± 0.0059

[LEPEWWG Phys.Rept.]

Consequences for theory

Need explanation at two levels

- almost universality: $s_W \approx \sin\theta^f_{\text{eff}} \approx \cos\theta_{Zf}$, $\rho^f \approx 1$
- but there are non-negligible differences

In the following: derive theory statements valid at tree-level only!

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Statement for general $SU(2)_L \times U(1)_Y$ theories

Gauge theory $SU(2)_L \times U(1)_Y$ (fermion quantum numbers as in table above, $Q_f = T^3 + Y$)

- Can define gauge fields for γ, Z, $W^\pm$
- Tree-level couplings to fermions given in terms of $\sin\theta \equiv \frac{g'}{\sqrt{g^2+g'^2}}$

$$\sin\theta^f_{\text{eff}} = \sin\theta_{Wf} = \sin\theta_{Zf} = \sin\theta = \text{universal}$$

$$\rho^f = \frac{c_W^2}{\cos^2\theta} \equiv \rho = \text{universal}$$

(but $s_W \neq \sin\theta$ and $\rho \neq 1$ possible!)

rough agreement with these aspects of observations!

Details/proof

SU(2)×U(1)⇒ four gauge fields

$$(W_\mu^1, W_\mu^2, W_\mu^3), B_\mu$$

gauge covariant derivative ($T^a = \frac{\sigma^2}{2}$ for doublets, =0 for singlets)

$$D_\mu = \partial_\mu + igT^a W_\mu^a + ig' YB^\mu$$

Lagrangian, gauge invariant kinetic term for all fermions

$$\mathcal{L}_{\text{kin}} = (\bar{\nu}_e \; \bar{e})\, i\gamma^\mu D_\mu \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L + \ldots$$

rearrange coupling part in D_μ

Details/proof

rearrange coupling part in D_μ: first define

$$\tan\theta = \frac{g'}{g}, e = \frac{gg'}{\sqrt{g^2+g'^2}} = gs_\theta = g'c_\theta$$

$$\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} B \\ W^3 \end{pmatrix}$$

then neutral part becomes, using $Q = T^3 + Y$,

$$\begin{aligned} & gT^3 W^3_\mu + g' Y B_\mu \\ =& (gs_\theta T^3 + g' c_\theta Y) A_\mu + (gc_\theta T^3 - g' s_\theta Y) Z_\mu \\ =& eQA_\mu + \frac{g}{c_\theta}\left(T^3 - s_\theta^2 Q\right) Z_\mu \end{aligned}$$

This yields the desired γ, Z couplings!

Details/proof

For charged part, define $W^\pm = \frac{W^1 \mp iW^2}{\sqrt{2}}$. For doublets, then:

$$
\begin{aligned}
& gT^1 W^1_\mu + gT^2 W^2_\mu \\
= & \frac{g}{2} \begin{pmatrix} 0 & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & 0 \end{pmatrix} \\
= & \frac{e}{\sqrt{2} s_\theta} \begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix}
\end{aligned}
$$

This yields the desired $W^\pm$ couplings!

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Statement for one scalar Higgs doublet (=SM)

Gauge theory as before, plus one scalar (Higgs) doublet Φ, $Y_\Phi = +\frac{1}{2}$ which gets vacuum expecation value $\langle\Phi\rangle \neq 0$

- Spontaneous symmetry breaking pattern

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \longrightarrow \mathrm{U}(1)_{\mathrm{e.m.}}$$

- Masses satisfy (at tree-level)

$$\frac{M_W}{M_Z} \equiv c_W = c_\theta, \quad \rho^f = 1,$$

rough agreement with these aspects of observations!

Computation of masses and ρ-parameter

Gauge invariance allows to choose $\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$

Then vacuum is invariant under $T^3 + Y = Q \to$ explains SSB pattern:

$$(T^3 + Y)\begin{pmatrix}0\\ v\end{pmatrix} = 0$$

Covariant derivative in vacuum (photon drops out)

$$D_\mu\Phi \longrightarrow \frac{1}{\sqrt{2}}\left[\frac{g}{c_\theta}\left(T^3 - s_\theta^2 Q\right) Z_\mu \begin{pmatrix}0\\ v\end{pmatrix} + \frac{g}{\sqrt{2}}\begin{pmatrix}0 & W_\mu^+\\ W_\mu^- & 0\end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix}\right]$$

Kinetic Φ term yields W, Z mass terms

$$\mathcal{L}_{\text{kin},\Phi} = |D_\mu\Phi|^2 \longrightarrow \frac{1}{2}M_Z^2 Z^\mu Z_\mu + M_W^2 W^{+\mu} W_\mu^-$$

with

$$M_Z^2 = \frac{g^2 v^2}{4 c_\theta^2}, \qquad M_W^2 = \frac{g^2 v^2}{4}$$

Useful way to write mass matrix

Write generators/couplings/gauge fields in unified way,

$$D_\mu = \partial_\mu + ig^A T^A V^A_\mu, \qquad \begin{aligned} g^A &= (g, g, g, g') \\ T^A &= (T^a, Y) \\ V^A_\mu &= (W^a_\mu, B_\mu) \end{aligned}$$

Then

$$\mathcal{L}_{\text{kin},\Phi} = |D_\mu \Phi|^2 \longrightarrow |V^A_\mu g^A T^A \langle\Phi\rangle|^2 = \frac{1}{2} V^A_\mu V^{B\mu} \mathcal{M}^2_{AB}$$

Can be written with symmetrized mass matrix

$$\mathcal{M}^2_{AB} = \langle\Phi\rangle^\dagger g^A g^B \{T^A, T^B\} \langle\Phi\rangle^\dagger$$

Here,

$$\mathcal{M}^2_{AB} = \frac{1}{4} \begin{pmatrix} g^2 v^2 & & & \\ & g^2 v^2 & & \\ & & g^2 v^2 & -g' g v^2 \\ & & -g' g v^2 & g'^2 v^2 \end{pmatrix}$$

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1st extension: example of new scalar fields in MSSM

Minimal supersymmetric SM contains many new scalar doublets and singlets, e.g. sleptons

- they can all get VEVs, signalled by tachyonic masses

$$M_{\tilde{e}}^2 = \begin{pmatrix} m_e^2 + m_{\tilde{e}}^2 + M_Z^2 c_{2\beta}(T_3 - Q s_W^2) & m_e\,(-\mu \tan\beta + A_e^*) \\ m_e\,(-\mu^* \tan\beta + A_e) & m_e^2 + m_{\tilde{e}}^2 + M_Z^2 c_{2\beta} Q s_W^2 \end{pmatrix}$$

- danger of charge-breaking (or colour-breaking) minima
- photon can get massive, no remnant $U(1)_{\text{e.m.}}$ symmetry

Such theories only viable if CCB minima are avoided $\rightarrow$ parameters must satisfy certain inequalities, e.g. $m_{\tilde{e}}^2 > 0$, $A_e < \ldots$

General scalar sector — gauge boson masses

Assume breaking structure

$$SU(2)_L \times U(1)_Y \to U(1)_{\text{e.m.}}$$

but many scalar fields Φ, each in some irreducible representation

$$\mathcal{M}^2_{AB} = \sum_\Phi \langle\Phi\rangle^\dagger \{g^A T^A, g^B T^B\} \langle\Phi\rangle$$

$$= \begin{pmatrix} g^2 v_W^2 & & & \\ & g^2 v_W^2 & & \\ & & g^2 v_Z^2 & -g' g v_Z^2 \\ & & -g' g v_Z^2 & g'^2 v_Z^2 \end{pmatrix}$$

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$$\frac{M_W^2}{M_Z^2 c_\theta^2} = \rho = \frac{v_W^2}{v_Z^2}$$

General scalar sector — gauge boson masses

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$$\rho = \frac{\sum_\Phi [T(T+1) - (T^3)^2]_\Phi v_\Phi^2}{\sum_\Phi 2[(T^3)^2]_\Phi v_\Phi^2}$$

General scalar sector — gauge boson masses

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- In theories with only doublets and singlets: $\rho = 1$
- More general cases: $\rho \neq 1 \Rightarrow$ Triplet VEVs must be small!

General scalar sector — gauge boson masses

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- mass matrix has $\mathrm{U}(1)_{\text{e.m.}}$ invariance $\leftrightarrow$ O(2) invariance:
- $v_W = v_Z$ would mean an additional O(3) custodial symmetry!

Example: Pure Higgs Triplet

- Triplet Φ, Y=0, SU(2)⇔O(3)-rotations
- but in vacuum: $\langle\Phi\rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$, no remnant SU(2) or O(3)
- mass term

$$\mathcal{M}^2_{ab} = \langle\Phi\rangle^\dagger\{T^a, T^b\}\langle\Phi\rangle = \begin{pmatrix} g_2^2 v^2 & & & \\ & g_2^2 v^2 & & \\ & & 0 & 0 \\ & & 0 & 0 \end{pmatrix}$$

$$M_W^2 = g_2^2 v^2, \qquad M_Z^2 = 0, \qquad \rho = \frac{M_W^2}{M_Z^2 c_\theta^2} = \infty$$

Proof: general form of the mass matrix $\mathcal{M}^2_{AB}$

$$\sum_\Phi \langle\Phi\rangle^\dagger\{g^A T^A, g^B T^B\}\langle\Phi\rangle = \begin{pmatrix} g^2 v_W^2 & & & \\ & g^2 v_W^2 & & \\ & & g^2 v_Z^2 & -g' g v_Z^2 \\ & & -g' g v_Z^2 & g'^2 v_Z^2 \end{pmatrix}$$

Use $\mathrm{U}(1)_{Q=T^3+Y}$ invariance of vacuum: $(T^3+Y)\langle\Phi\rangle = 0$

- $\mathcal{M}^2_{A3} = -\frac{g}{g'}\mathcal{M}^2_{A4}$ etc $\Rightarrow$ lower right block
- $0 = \langle\Phi\rangle^\dagger[T^3+Y, g^A T^A g^B T^B]\langle\Phi\rangle$ leads to

$$(A=1, B=2): \mathcal{M}^2_{11} = \mathcal{M}^2_{22}$$
$$(A=B=1,2): \mathcal{M}^2_{12} = \mathcal{M}^2_{21} = 0$$
$$(A=1, B=3): \mathcal{M}^2_{13} = \mathcal{M}^2_{14} = 0 \text{ etc}$$

- this proves the block structure

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- $\mathcal{M}^2_{A3} = -\frac{g}{g'}\mathcal{M}^2_{A4}$ etc $\Rightarrow$ lower right block
- $0 = \langle\Phi\rangle^\dagger [T^3+Y, g^A T^A g^B T^B] \langle\Phi\rangle$ leads to

$$\begin{aligned} (A=1, B=2) &: \mathcal{M}^2_{11} = \mathcal{M}^2_{22} \\ (A=B=1,2) &: \mathcal{M}^2_{12} = \mathcal{M}^2_{21} = 0 \\ (A=1, B=3) &: \mathcal{M}^2_{13} = \mathcal{M}^2_{14} = 0 \text{ etc} \end{aligned}$$

- this proves the block structure

Proof: ρ-parameter

Use:

$$\rho = \mathcal{M}^2_{11}/\mathcal{M}^2_{33} = \mathcal{M}^2_{22}/\mathcal{M}^2_{33}$$

$$\vec{T}^2 = (T^1)^2 + (T^2)^2 + (T^3)^2 \text{ Casimir operator of SU(2)}_L$$

$$\vec{T}^2\Phi = T(T+1)\Phi \text{ on irrep.}$$

Hence

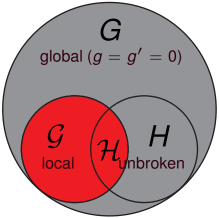
$$2\mathcal{M}^2_{11} + \mathcal{M}^2_{33} = 2g^2 \sum_\Phi \langle\Phi\rangle^\dagger [T(T+1)]_\Phi \langle\Phi\rangle$$

$$\mathcal{M}^2_{33} = 2g^2 \sum_\Phi \langle\Phi\rangle^\dagger [(T^3)^2]_\Phi \langle\Phi\rangle$$

and

$$\mathcal{M}^2_{11}/\mathcal{M}^2_{33} = \frac{\sum_\Phi [T(T+1) - (T^3)^2]_\Phi v_\Phi^2}{\sum_\Phi 2[(T^3)^2]_\Phi v_\Phi^2}$$

General scalar sector — Custodial symmetry

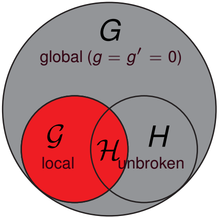


General scalar sector — Custodial symmetry

SM case: rewrite SM Higgs doublet and Higgs potential using

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \mathbf{\Phi} = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$

$$V(\mathbf{\Phi}) = -\frac{\mu^2}{2}\mathrm{Tr}(\mathbf{\Phi}^\dagger\mathbf{\Phi}) + \frac{\lambda}{16}\mathrm{Tr}(\mathbf{\Phi}^\dagger\mathbf{\Phi})^2$$



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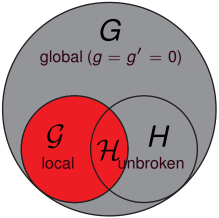

full SM global symmetry ($g = g'$ =Yukawas= 0, Note: $U(1)_Y$ and $U(1)_Q$ are subgroups):

$$\mathbf{\Phi} \to L\mathbf{\Phi}R^\dagger \qquad G = SU(2)_L \times SU(2)_R$$

full vacuum invariance

$$\langle\mathbf{\Phi}\rangle_{\text{vac}} = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \qquad H = SU(2)_{L=R}$$

This explains the O(3) symmetry of mass matrix, $\mathcal{M}^2_{AB} = \frac{1}{4}\begin{pmatrix} g^2v^2 & & & \\ & g^2v^2 & & \\ & & g^2v^2 & -g'gv^2 \\ & & -g'gv^2 & g'^2v^2 \end{pmatrix}$

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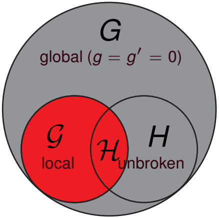

General case:

$$G = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$$
$$\mathcal{G} = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$
$$H = \mathrm{SU}(2)_{L=R}$$
$$\mathcal{H} = \mathrm{U}(1)_{\mathrm{e.m.}}$$

leads to $\rho = 1$ in the limit $g = g' = 0$, even nonperturbatively

[Sikivie,Susskind,Voloshin,Zakharov '80; Weinberg QFT, 21.4]

General scalar sector — Custodial symmetry

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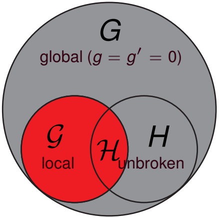


In SM, $SU(2)_L \times SU(2)_R$ exact $y^u = y^d$ etc, while $y^u \neq y^d$ lead to loop corrections to ρ

$$\mathcal{L}_{\text{Yukawa}} = y\left[\bar{u}_L\phi^c u_R + \bar{d}_L\phi d_R\right] = y\left[(\bar{u}_L\ \bar{d}_L)\mathbf{\Phi}\begin{pmatrix} u_R \\ d_R \end{pmatrix}\right]$$

Outline

1. Tree-level considerations
   - Observable facts and goal of EW theory
   - General $SU(2)_L \times U(1)_Y$ theories
   - One scalar Higgs doublet
   - Extensions of scalar sector
   - Extensions of gauge sector

General gauge sector — possibilities

SM-group = part of bigger group

- extra gauge bosons, uncharged Z' or charged W'
- can mix with Z and W (mass or kinetic mixing)

Constraints:

- On SM-like mass eigenstates Z_1, W_1 (masses and couplings)
- On new mass eigenstates (either heavy or weak couplings)
- Theory consistency: absence of gauge anomalies

SU(2)$_L$×U(1)$_Y$×X — particles and masses

extra Z' vector bosons, uncharged under $Q_{\text{e.m.}}$, SU(2)$_L$×U(1)$_Y$

$$(\underbrace{V^A}_{\text{SM}}, \underbrace{V^M}_{\text{X}})$$

vector boson mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} (\mathcal{M}^2_{\text{SM}})_{AB} & (\mathcal{M}^2_{\text{SM–X}})_{AN} \\ (\mathcal{M}^2_{\text{SM–X}})^T_{MB} & (\mathcal{M}^2_{\text{X}})_{MN} \end{pmatrix}$$

mass mixing only if $\exists\Phi$ charged under both SM and X

$$(\mathcal{M}^2_{\text{SM–X}})_{AN} = \sum_{\Phi} \langle\Phi\rangle^\dagger \{g^A T^A_{\text{SM}}, g^N T^N_{\text{X}}\} \langle\Phi\rangle$$

mass mixing only possible for Z boson. Photon/$W^\pm$ unaffected

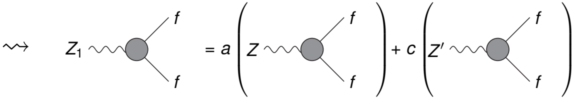
(proof as before, assuming remnant U(1)$_{\text{e.m.}}$ symmetry)

Example for $SU(2)_L \times U(1)_Y \times X$ — couplings

Z' model with extra $U(1)_{Y'}$ and singlet charged under $U(1)_Y$ and $U(1)_{Y'}$

$\Rightarrow$ mass eigenstates

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$$

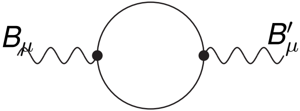


Constraints on such theories:

- Z_1 identified with observed Z: mass mixing must be small
  - deviations from SM-couplings
  - $\sin\theta^f_{\text{eff}}$ gets admixture, could be q, l, generation-dependent
- Z_2 either heavy or weak couplings

SU(2)$_L$×U(1)$_Y$×U(1)' — kinetic mixing

Three neutral gauge bosons B, B', W^3 which mix



Special possibility since abelian:

$$\mathcal{L} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}B'^{\mu\nu}B'_{\mu\nu} - \frac{\epsilon}{2}B^{\mu\nu}B'_{\mu\nu}$$

Unmix: $B \to B - \epsilon B'$ (Unmixing in full SM equivalent to: $\begin{pmatrix} A \\ Z \end{pmatrix} \to \begin{pmatrix} A - c_\theta \epsilon B' \\ Z + s_\theta \epsilon B' \end{pmatrix}$)

$$\mathcal{L} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1-\epsilon^2}{4}B'^{\mu\nu}B'_{\mu\nu}$$

Now, proceed as in Z–Z' mixing before (photon unaffected)

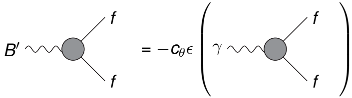
SU(2)$_L$×U(1)$_Y$×U(1)' — kinetic mixing

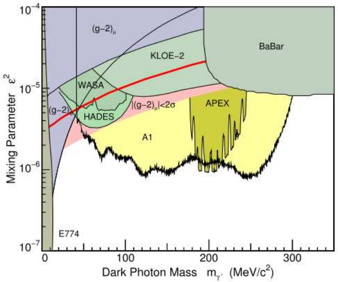
But new effect in couplings of new neutral gauge boson:

$$\mathcal{L} = j_{\text{e.m.}} A + J_Z Z + J' B'$$

$$\overset{\text{unmix}}{\rightarrow} j_{\text{e.m.}} A + J_Z Z + (J' - c_\theta \epsilon J_{\text{e.m.}} + s_\theta \epsilon J_Z) B'$$

Interesting: even if no mass mixing and $J' = 0$: “dark photon”





[A1/Mainz ’14]

More general: SM group = subgroup of some group

E.g.

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

Find hypercharge:

$$T^3_R + \frac{1}{2}(B-L) = Y$$

Fermion	Y_f
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$-\frac{1}{2}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_R$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\frac{1}{6}$
$\begin{pmatrix} u \\ d \end{pmatrix}_R$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$

Results and constraints:

- Two extra charged $W'^{\pm}$, one extra neutral Z'
- Constraints from mixing and from new particles as for Z'

Summary of tree-level results

- SU(2)$_L\times$U(1)$_Y$

  $$\gamma,\, Z,\, W^\pm,\, \sin\theta^f_{\text{eff}} = \sin\theta_{Wf} = \sin\theta_{Zf} = \sin\theta = \text{universal}$$

- one Φ doublet

  $$\rho = 1,\, \sin\theta = s_W$$

- more scalars

  $$\text{U(1)}_{\text{e.m.}} \text{ symmetry? } \rho \neq 1 \text{ possible}$$

- more gauge fields

  $$\text{mixing possible, } \sin\theta_i \text{ not universal}$$

Outlook: extra matter ($\rightarrow$ SUSY-lecture), non-renormalizable operators ($\rightarrow$ EFT-lecture)

Outline

1. Tree-level considerations
2. All-order considerations
3. Precise observables and calculations

Outline

2 All-order considerations

- General quantum gauge theories
- Slavnov-Taylor, BRS, gauge invariance
- Summary, examples, equivalence theorem
- Renormalization transformation of the SM
- Properties of v and tadpole renormalization
- On-shell and $\overline{\text{MS}}$ schemes

What is the essence of gauge theories?

[Note: gauge invariant $\mathcal{L}$ cannot be quantized]

What is the essence of quantum gauge theories?

[Note: gauge invariant $\mathcal{L}$ cannot be quantized]

Physics $\leftrightarrow$ equivalence classes!

[Note: gauge invariant $\mathcal{L}$ cannot be quantized]

Ansatz: structure of quantum gauge theory

$$|A\rangle \cong |A\rangle + Q|\chi\rangle$$ equivalent states differ by "Q-transformation"

$$Q|\text{phys}\rangle = 0$$ physical states invariant under "Q-transformation"

$$Q^2 = 0, \quad Q = \text{ linear operator}$$ for consistency

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States:

- Hilbert space of physical states = quotient space $\frac{\ker(Q)}{\mathrm{im}(Q)}$
- scalar product $\langle \text{phys}_1 | \text{phys}_2 \rangle$ independent of representative
- unphysical states: either $Q|\text{gh}\rangle \neq 0$ or $|\text{gh}\rangle = Q|\chi\rangle$

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$$Q^2 = 0, \quad Q = \text{ linear operator}$$ for consistency

Observables:

- Hamiltonian, P^μ, S-matrix etc should commute with $Q \rightsquigarrow$

$$S|\text{phys}\rangle = \text{ physical, rep-independent}$$

- proves unitarity of physical S-matrix (difficulty: prove that Hilbert space has positive definite norm)

What is the point of this?

Lorentz covariant field theory with vector field operator!

$$\langle 0| T\ A^\mu\ A^\nu |0\rangle^{\text{F.T.}} = \frac{A g^{\mu\nu} + B k^\mu k^\nu}{k^2 - m^2} + \ldots$$

problems:

- negative norm states typical; A^μ describes 4 d.o.f.; not all physical

solution by equivalence classes: here: indicate massless, free case

What is the point of this?

Lorentz covariant field theory with vector field operator!

$$\langle 0|T\ A^\mu\ A^\nu|0\rangle^{\text{F.T.}} = \frac{Ag^{\mu\nu} + Bk^\mu k^\nu}{k^2 - m^2} + \dots$$

problems:

- negative norm states typical; A^μ describes 4 d.o.f.; not all physical

solution by equivalence classes: here: indicate massless, free case

$$\begin{array}{rcccccccc} A^\mu|0\rangle = & \epsilon^\mu_{\text{T1}}|T1\rangle & + & \epsilon^\mu_{\text{T2}}|T2\rangle & + & k^\mu|S\rangle & + & \epsilon^\mu_{\text{L}}|L\rangle \\ QA^\mu|0\rangle = & 0 & + & 0 & + & k^\mu|\text{gh}\rangle & + & 0 \\ & & & & & & & \uparrow \\ & & & & & & & |L\rangle = Q|\overline{\text{gh}}\rangle \end{array}$$

- 2 physical d.o.f., these have positive norm (general proof difficult)

Note: this theory is not within the framework of Wightman axioms ($\Rightarrow$ Goldstone theorem not applicable)

Outline

2. All-order considerations
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Green functions in presence of Q

Fundamental field operators φ_i. Define

$$s\varphi_i = i[Q, \varphi_i]_{\mp} = \text{ renormalized local composite operator}$$

Vacuum is physical: $Q|0\rangle = 0$. Hence, Slavnov-Taylor identities

$$\begin{aligned} 0 &= \langle 0|[iQ, \varphi_1 \ldots \varphi_n]_{\mp}|0\rangle \\ &= \langle 0|(s\varphi_1)\ldots\varphi_n|0\rangle \pm \ldots \pm \langle 0|\varphi_1 \ldots (s\varphi_n)|0\rangle \end{aligned}$$

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Useful: STI on functional level (take $\frac{\delta^n}{\delta J_1 \dots J_n}$ to generate Green functions/identities)

$$Z(J, Y) = \int \mathcal{D}\varphi \; e^{i\int (\mathcal{L} + J_i\varphi_i + Y_i s\varphi_i)}$$

$$0 = \int_x J_i(x) \frac{\delta Z(J, Y)}{\delta Y_i(x)}$$

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Also useful: STI for 1PI functional (Legendre, $J \to -\frac{\delta\Gamma}{\delta\varphi}$) (aka Lee- or Zinn-Justin identities)

$$0 = \int_x \frac{\delta\Gamma(\varphi, Y)}{\delta Y_i(x)} \frac{\delta\Gamma(\varphi, Y)}{\delta\varphi_i(x)} \equiv \frac{1}{2}\Gamma \star \Gamma$$

Must be valid also in classical limit $\Gamma \to \Gamma_{\rm cl} \equiv \int \mathcal{L}_{\rm full}$

What are the Lagrangians of such theories?

- must depend on $Y_i \rightsquigarrow$ def. of classical BRS-transf.:

(also quantum numbers (ghost number, dimensionality) restricts Lagrangian → only possibility:)

$$\Gamma_{\text{cl}}(\varphi, Y) = \int \mathcal{L}_{\text{full}} = \int Y_i \underbrace{\left(\begin{array}{c} \text{some field} \\ \text{polynomial} \end{array} \right)}_{=: s\varphi_i} + \Gamma_{\text{cl,no-}Y}$$

- Then, STI requires class. action to be BRS invariant!

$$0 = \int_x \underbrace{\frac{\delta \Gamma_{\text{cl}}(\varphi, Y)}{\delta Y_i(x)}}_{s\varphi_i(x)} \frac{\delta \Gamma_{\text{cl}}(\varphi, Y)}{\delta \varphi_i(x)} = s\Gamma_{\text{cl}}$$

This is equivalent to three properties $\rightsquigarrow$ Answer:

$\mathcal{L}_{\text{full}} = Y_i s\varphi_i + \mathcal{L}_{\text{no-}Y}$	Y- and non-Y-parts
$0 = s^2\varphi_i$	Nilpotency
$0 = s\mathcal{L}_{\text{no-}Y}$	BRS invariance

Construction of Lagrangian with this structure

Now, concrete choice: Yang-Mills theories with vector fields A_μ^a

- postulate ghost field c^a for each vector field (require: ghost number conserved; BRS increases ghost number)
- most general ansatz for BRS transformations and Lagrangian:

$$sc^a = \frac{1}{2} g f^{abc} c^b c^c$$

$s^2 c = 0$ requires: f^{abc}=structure constants of some Lie algebra

recover Lie algebra structure

$$sA^a_\mu = \partial_\mu c^a + g f^{abc} c^b A^c_\mu$$

$s^2 A^a_\mu = 0$ requires universality of g and f^{abc}

recover gauge transformations

$$s\Phi_i = -igc^a T^a_{ij}\Phi_j$$

$s^2\Phi = 0$ requires: T^a=representation of Lie algebra

recover gauge transformations

$$\mathcal{L}_{\text{no-}Y,\text{so far}} = \mathcal{L}_{\text{gauge-inv}}(A_\mu, \Phi)$$

from ghost number zero: no c^a appears
and from BRS invariance

recover gauge invariance

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recover gauge invariance

Idea for gauge fixing

add any term $\mathcal{L}_{\text{trivial}} = sX$:

- doesn't change physics, but $s\mathcal{L}_{\text{trivial}} = 0$
- but X must have ghost number (-1)

motivates antighosts, auxiliary fields $(\bar{c}^a, B^a)$

$$s\bar{c}^a = B^a$$

$$sB^a = 0$$

theorem: $\frac{\ker(s)}{\text{im}(s)}$ doesn't change by enlarging field space in this way [Piguet,Sorella]

Now we can write gauge fixing term with any functional $F^a(A_\mu, \Phi)$:

$$\mathcal{L}_{\text{gauge-fix,ghost}} = sX = s\left(\frac{\xi}{2}\bar{c}^a B^a + \bar{c}^a F^a\right)$$

$$= \frac{\xi}{2}B^a B^a + B^a F^a \underbrace{-\bar{c}^a sF^a}_{\mathcal{L}_{\text{ghost}}}$$

Eliminate B^a by eq. of motion $\rightsquigarrow -\frac{1}{2\xi}(F^a)^2$

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Summary of resulting theory

$$\mathcal{L}_{\text{full}} = \underbrace{\mathcal{L}_{\text{gauge-inv}}(A_\mu, \Phi)}_{\text{physical, } \leftrightarrow \frac{\ker(s)}{\text{im}(s)}} + \underbrace{\underbrace{sX}_{\text{gauge fixing, ghosts}} + \underbrace{Y_i s\varphi_i}_{\text{BRS}\to\text{Lie algebra, gauge transf.}}}_{\text{unphysical, } \in \text{ im}(s)}$$

Theorem: Such theories are renormalizable:

1. Finiteness at all orders:
   - multiplicative renormalization of coupling and fields possible
   - phys. parameters=parameters of $\mathcal{L}_{\text{gauge-inv}}$
2. Phys. meaning of theory:
   - operator Q with all desired properties exists
   - physical states have positive norm
   - phys. S-matrix: unitary and gauge independent

['t Hooft, Veltman; Lee; Zinn-Justin;. . . ; Becchi, Rouet, Stora; Kugo, Ojima;. . . ; Kraus, Sibold, Grassi (general gauges, EWSM)]

Gauge independence, physical and unphysical parameters

Theorem:

$$\frac{\partial}{\partial\xi}(\text{phys. S-matrix}) = 0 \text{ if parameters in } \mathcal{L}_{\text{gauge-inv}} \leftrightarrow \frac{\text{ker}(s)}{\text{im}(s)} \text{ kept fixed}$$

(proof: study change of Green functions → LSZ reduction → no change of S-matrix)

Unphysical parameters in "trivial terms" ($Q^2 = 0$)

Gauge independence, physical and unphysical parameters

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(proof: study change of Green functions → LSZ reduction → no change of S-matrix)

Unphysical parameters in "trivial terms" ($Q^2 = 0$)

$$s^2 = 0 \qquad \longrightarrow \qquad \Delta\mathcal{L}_{\text{trivial}} = sX$$

$\rightsquigarrow$ $\Delta\mathcal{L}_{\text{trivial}}$ preserves STI, doesn't change physics

- e.g. gauge fixing parameters unphysical

Gauge independence, physical and unphysical parameters

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(proof: study change of Green functions → LSZ reduction → no change of S-matrix)

Unphysical parameters in "trivial terms" ($Q^2 = 0$)

$$\Gamma_{\text{cl}} \star (\Gamma_{\text{cl}} \star \mathcal{X}) = 0 \qquad \longrightarrow \qquad \Delta\Gamma_{\text{trivial}} = (\Gamma_{\text{cl}} \star \mathcal{X})$$

$\rightsquigarrow \Delta\mathcal{L}_{\text{trivial}}$ preserves STI, doesn't change physics

- field renormalization: $\Gamma_{\text{cl}} \star \int Y\varphi = \int \varphi \frac{\delta\Gamma_{\text{cl}}}{\delta\varphi} - \int Y \frac{\delta\Gamma_{\text{cl}}}{\delta Y}$
- VEV shift: $\Gamma_{\text{cl}} \star \int Y\delta v = \int \delta v \frac{\delta\Gamma_{\text{cl}}}{\delta\varphi}$

also direct diagrammatic proof possible: New Feynman rules from δv: $\delta v \frac{\partial}{\partial\varphi}\mathcal{L}_{\text{kin}} + \delta v \frac{\partial}{\partial\varphi}\mathcal{L}_{\text{int}}$, drop out in Green functions

Gauge independence, physical and unphysical parameters

Theorem:

$$\frac{\partial}{\partial\xi}(\text{phys. S-matrix}) = 0 \text{ if parameters in } \mathcal{L}_{\text{gauge-inv}} \leftrightarrow \frac{\ker(s)}{\operatorname{im}(s)} \text{ kept fixed}$$

(proof: study change of Green functions → LSZ reduction → no change of S-matrix)

Unphysical parameters in "trivial terms" ($Q^2 = 0$)

Hence, unphysical parameters (shifts don't change observables):

$$\xi \to \xi + \delta\xi, \quad \varphi \to \sqrt{Z}\varphi, Y \to \sqrt{Z}^{-1} Y, \quad \varphi \to \varphi + \delta v$$

Physical parameters are the ones in $\mathcal{L}_{\text{gauge-inv}}$

Simple examples: photon + scalar field ϕ, $Q_\phi = 1$

Scalar QED

Abelian Higgs model

[A. Pich]

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Scalar QED

Abelian Higgs model

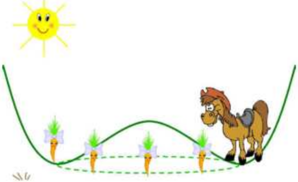
$$\mathcal{L}_{\text{full}} = \underbrace{\mathcal{L}_{\text{gauge-inv}}(A_\mu, \phi)}_{\text{the same in both cases}} + \underbrace{sX}_{\rightsquigarrow -\frac{1}{2\xi}(F^a)^2\text{: choose/adapt!}} + Y_i s\varphi_i$$

$$\phi = \frac{1}{\sqrt{2}}(H + v + iG)$$

$$\text{BRS:} \quad s(H + v + iG) = -ie(H + v + iG)$$

Simple examples: photon + scalar field ϕ, $Q_\phi = 1$

Scalar QED



Abelian Higgs model

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g.fix, unbroken case:

$F^a = \partial^\mu A^a_\mu$ usual QED/QCD-like g.fix

$F^a = n^\mu A^a_\mu$ axial gauge (physical, no ghost interactions even in QCD)

Simple examples: photon + scalar field ϕ, $Q_\phi = 1$

Scalar QED

Abelian Higgs model

$$\mathcal{L}_{\text{full}} = \underbrace{\mathcal{L}_{\text{gauge-inv}}(A_\mu, \phi)}_{\text{the same in both cases}} + \underbrace{sX}_{\rightsquigarrow -\frac{1}{2\xi}(F^a)^2\text{: choose/adapt!}} + Y_i s\varphi_i$$

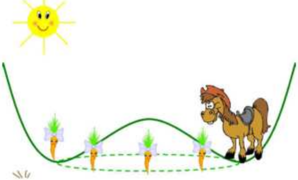
Broken case: R_ξ gauge cancels mixing $\partial^\mu A_\mu$ – G

$$F^a = \partial^\mu A^a_\mu - \xi e v G$$

$$|D^\mu \phi|^2 = \ldots + \frac{1}{2}\underbrace{e^2 v^2}_{M_A^2} A^\mu A_\mu + e v A^\mu \partial_\mu G$$

Simple examples: photon + scalar field ϕ, $Q_\phi = 1$

Scalar QED



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Now choose R_ξ gauge, $\xi = 1$, use $B = -F$

$$\mathcal{L}_{\text{full}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |D_\mu \frac{H + v + iG}{\sqrt{2}}|^2 - V(\phi) - \frac{1}{2}F^2 + (\text{gh}, Y),$$

$$F = \partial A - evG$$

Example cont'd — physical states

BRS transformations:

$$sA_\mu = \partial_\mu c$$
$$sc = 0$$
$$s\bar{c} = -\partial A + evG$$
$$sH = -eGc$$
$$sG = evc + eHc$$

Example cont'd — physical states

BRS transformations:

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To find physical states (tree-level, free theory): linearize→F.T.→on-shell

$$A_\mu = \partial_\mu S + L_\mu + T^1_\mu + T^2_\mu$$

unbroken:

$$k^2 = 0$$
$$\partial A \to -ikL$$

broken:

$$k^2 = e^2 v^2$$
$$\partial A \to -e^2 v^2 S$$

Example cont'd — physical states

both:

$$sT_{1,2} = 0 \qquad sL = 0 \qquad sS = c \qquad sc = 0$$

unbroken:

$$s\bar{c} = ikL$$
$$sH = 0$$
$$sG = 0$$

physical "BRS singlets":

$$Q|T^{1,2}, H, G\rangle = 0$$

unphysical "BRS quartet":

$$Q|\bar{c}\rangle \rightarrow |L\rangle$$
$$Q|S\rangle \rightarrow |c\rangle$$

broken:

$$s\bar{c} = e^2 v^2 S + evG$$
$$sH = 0$$
$$sG = evc$$

physical "BRS singlets":

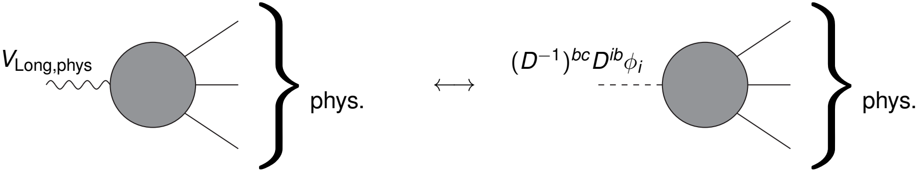
$$Q|T^{1,2}, L, H\rangle = 0$$

unphysical "BRS quartet":

$$Q|\bar{c}\rangle \rightarrow |evS + G\rangle$$
$$Q|evS - G\rangle \rightarrow |c\rangle$$

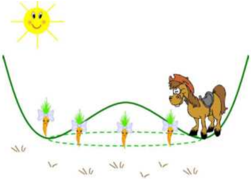
Towards Goldstone-boson equivalence theorem

D^{ia} — characterizes SSB (broken symmetries, masses, Goldstone modes)



SSB in general gauge theory with N_{tot} gauge fields and scalar fields $\phi_i + v_i$

$$s\phi_i = -igc^a T^a_{ij}(\phi_j + v_j)$$



Gauge transformation of vacuum:

$$\frac{\delta}{\delta c^a} s\phi_i = -igT^a_{ij} v_j = -ig \left(\begin{array}{c|c|c} T^1 v & T^2 v & \cdots \end{array} \right)_{ia}$$

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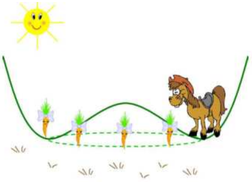

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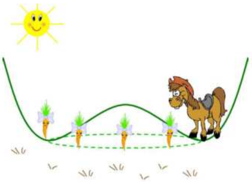
Rank of matrix $=$ # broken symmetries

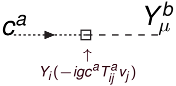
$N_{\text{tot}} -$Rank $=$ # unbroken symm.

This was tree-level, now in full theory



$$\Gamma_{c^a Y_i} = \frac{\delta}{\delta c^a} s\phi_i + \ldots$$

$$i\Gamma_{c^a Y_i}(k,-k) = $$



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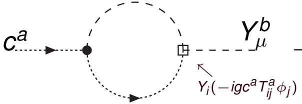

$+ \ldots$

$$= gT^a_{ij} v_j + \ldots$$

$$=: D^{ia}(k^2)$$

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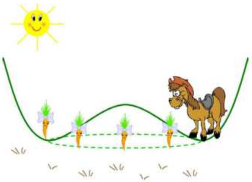

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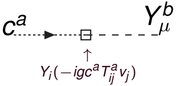

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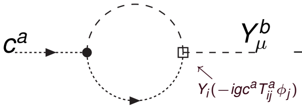

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$$= gT^a_{ij} v_j + \ldots$$

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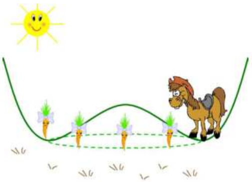
Rank of D^{ia} = # broken symmetries

$N_{\text{tot}} - \text{Rank}$ = # unbroken symm.

This was tree-level, now in full theory

$$\Gamma_{c^a Y_i} = \frac{\delta}{\delta c^a} s\phi_i + \dots$$



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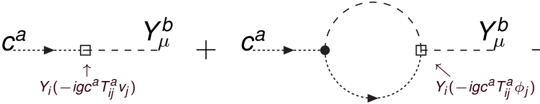
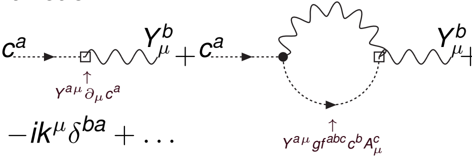


$$= gT^a_{ij} v_j + \dots$$

$$=: D^{ia}(k^2)$$

Another, similar vertex function:

$$i\Gamma_{c^a Y^b_\mu}(k,-k) =$$



$$= -ik^\mu \delta^{ba} + \dots$$

$$=: -ik^\mu D^{ba}(k^2)$$

Study full two-point functions

$$\text{using } 0 = \int_x \underbrace{\frac{\delta\Gamma(\varphi, Y)}{\delta Y_i(x)}}_{s\varphi_i + \text{loops}} \frac{\delta\Gamma(\varphi, Y)}{\delta\varphi_i(x)}$$

$$0 = \Gamma_{c^a Y^b_\mu} \Gamma_{A^\nu_c A^\mu_b} + \Gamma_{c^a Y_i} \Gamma_{A^\nu_c \phi_i}$$

Study full two-point functions

$$\text{using } 0 = \int_x \underbrace{\frac{\delta\Gamma(\varphi, Y)}{\delta Y_i(x)}}_{s\varphi_i + \text{loops}} \frac{\delta\Gamma(\varphi, Y)}{\delta\varphi_i(x)}$$

$$0 = -i \underbrace{D^{ab}(k^2)}_{\text{invertible}} \; k^\mu \Gamma_{A_c^\nu A_b^\mu} + \underbrace{D^{ai}(k^2)}_{\text{Rank}=N_{\text{broken}}} \; \Gamma_{A_c^\nu \phi_i}$$

Hence

$\exists \geq N_{\text{total}} - N_{\text{broken}}$ gauge fields with $k^\mu \Gamma_{A^\mu A^\nu} = 0 \Rightarrow$ massless!
Up to N_{broken} gauge fields can be massive!

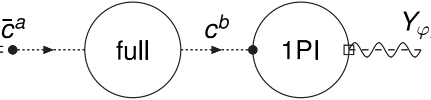
Another, auxiliary identity

With auxiliary field B, consider full Green functions:

$$0 = \langle\{Q, \bar{c}^a \varphi_k\}\rangle = \langle\underbrace{(s\bar{c}^a)}_{B^a}\varphi_k\rangle - \langle\bar{c}^a(s\varphi_k)\rangle$$

Last term, decomposed

$\langle\bar{c}^a(s\varphi_k)\rangle =$



$= \langle\bar{c}^a c^b\rangle i\Gamma_{c^b Y_{\varphi_k}}$

Hence

$$\langle B^a A^c_\mu\rangle = \langle\bar{c}^a c^b\rangle(-ik_\mu D^{cb})$$

$$\langle B^a \phi_i\rangle = \langle\bar{c}^a c^b\rangle D^{ib}$$

Goldstone-boson equivalence theorem

Physical states, matrix element of $B^a = \{Q, \bar{c}^a\}$ vanishes:

$$\begin{aligned} 0 &= \langle\, \text{phy,out} | B^a | \,\text{phy,in} \rangle \\ &= \langle 0 | B^a \varphi_k | 0 \rangle \langle\, \text{phy,out} | \varphi_k | \,\text{phy,in} \rangle |_{\varphi_k \text{ amputated}} \end{aligned}$$

insert last result, cancel ghost propagator

$$\begin{aligned} 0 = (-ik_\mu D^{cb}) &\langle\, \text{phy,out} | A^c_\mu | \,\text{phy,in} \rangle |_{A^c_\mu \text{ amputated}} \\ &+ D^{ib} \langle\, \text{phy,out} | \phi_i | \,\text{phy,in} \rangle |_{\phi_i \text{ amputated}} \end{aligned}$$

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These are building blocks for S-matrix elements

$$\underbrace{\epsilon_\mu^{\text{Long,phys}} \langle \text{phy,out} | A^c_\mu | \text{phy,in} \rangle |_{A^c_\mu \text{ amputated}}}_{S_{\text{Long,phys}}}$$

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$$\xrightarrow{\text{up to } \mathcal{O}(M_A/E)} \frac{k_\mu}{M_A} \langle \text{phy,out} | A^c_\mu | \text{phy,in} \rangle |_{A^c_\mu \text{ amputated}}$$

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Outline

2. All-order considerations
   - General quantum gauge theories
   - Slavnov-Taylor, BRS, gauge invariance
   - Summary, examples, equivalence theorem
   - Renormalization transformation of the SM
   - Properties of v and tadpole renormalization
   - On-shell and $\overline{\text{MS}}$ schemes

EW Standard Model

$$\mathcal{L} = -\frac{1}{4} V^{A\mu\nu} V^{A}_{\mu\nu} + \bar{\Psi} i\gamma^{\mu} D_{\mu} \Psi + |D_{\mu}\Phi|^2 - V(\Phi) + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{gauge-fix,gh}}$$

Covariant derivative

$$D_{\mu} = \partial_{\mu} + igT^{a} W^{a}_{\mu} + ig' Y B^{\mu}$$

Higgs potential

$$V(\Phi) = -\mu^2 |\Phi|^2 + \frac{\lambda}{4} |\Phi|^4$$

Parametrization of Higgs doublet with "VEV" (v is just a constant to be adjusted later)

$$\Phi = \begin{pmatrix} G^{+} \\ \frac{1}{\sqrt{2}}(H + v + iG^0) \end{pmatrix}$$

Free parameters of the EW Standard Model

Original (ignore gauge-fixing, field renormalization, and Yukawas)

$$g, g', \mu^2, \lambda; v$$

Equivalent set of parameters: $e = \frac{gg'}{\sqrt{g^2+g'^2}}$, masses, tadpole

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$$\mathcal{L}_{\text{lin,bil}} = \underbrace{\frac{g^2 v^2}{4}}_{=:M_W^2} W^+ W^- + \underbrace{\frac{(g^2+g'^2)v^2}{8}}_{=:\frac{1}{2}M_Z^2} Z^2$$

$$+ \underbrace{\left(\mu^2 v - \frac{1}{4}\lambda v^3\right)}_{=:t} H - \underbrace{(\frac{\lambda v^2}{4} + \frac{t}{v})}_{=:\frac{1}{2}M_H^2} H^2 + \frac{t}{2v}(G^0)^2 + \dots$$

Free parameters of the EW Standard Model

Original (ignore gauge-fixing, field renormalization, and Yukawas)

$$g, g', \mu^2, \lambda; v$$

Equivalent set of parameters: $e = \frac{gg'}{\sqrt{g^2+g'^2}}$, masses, tadpole

$$e, M_Z, M_W, M_H; t$$

Renormalization transformation

Apply

$$c \to c_{\text{bare}} = c + \delta c$$

on each parameter — either on original set

$$g, g', \mu^2, \lambda; v$$

or on equivalent set

$$e, M_Z, M_W, M_H; t$$

Cancels all divergences. (If gauge fixing, field renormalization, Yukawas are also treated)
Result = physical theory with 4 physical input parameters

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Remarks on unphysical renormalizations

From general theorem: gauge-independent relation

$$(\text{phys. S-matrix}) \leftrightarrow (g, g', \mu^2, \lambda)_{\text{bare}}$$

But δv is unphysical (needed for finite Green functions but don't change observables; similar for δZ, $\delta\xi$)

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But δv is unphysical (needed for finite Green functions but don't change observables; similar for δZ, $\delta\xi$)

Hence, two common choices for δv and δt

- Renormalize tadpole $T + \delta t = 0 \rightsquigarrow$ all Green functions finite, but δv is gauge-dependent ($\rightsquigarrow \delta M_{W,Z,H}$ gauge-dependent)
- don't renormalize tadpole $\delta t = 0 \Leftrightarrow v_{\text{bare}} = \left(\frac{4\mu^2}{\lambda}\right)_{\text{bare}}$ is gauge-independent [Bednyakov,Pikelner,Velizhanin'13]

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$$\text{(phys. S-matrix)} \leftrightarrow (g, g', \mu^2, \lambda)_{\text{bare}}$$

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VEV and Higgs field — combined renormalization transformation

$$(H+v) \to \sqrt{Z_H}H + v + \delta v$$

$$\text{or} \quad (H+v) \to \sqrt{Z_H}(H + v + \delta\bar{v})$$

Could $\delta\bar{v}$ be zero if we require $T + \delta t = 0$?
Yes, in Landau gauge ($\xi = 0$). No, in general. [Sperling,DS,Voigt '13]

Outline

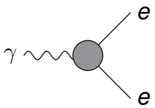
Charge renormalization: On-shell and $\overline{\text{MS}}$ schemes

Renormalization scheme: define split c_{bare} into $c + \delta c$

$\Leftrightarrow$

Define physical meaning and numerical value of c

On-shell:



$$\underset{\text{Thomson-limit}}{=} -ieQ_e\gamma^\mu$$

$\overline{\text{MS}}$-scheme:

$$\bar{e} + \delta\bar{e} = e_{\text{OS}} + \delta e_{\text{OS}}$$

$$\delta\bar{e} = \text{pure div.}$$

Result: (Universality! Difficult to prove at higher orders)

$$\frac{\delta e}{e} \overset{\text{1-loop}}{=} -\frac{1}{2}\delta Z_{AA} - \frac{s_W}{2c_W}\delta Z_{ZA}$$

$$\delta Z_{AA} = \Pi^\gamma(0)$$

Charge renormalization: On-shell and $\overline{\text{MS}}$ schemes

On-shell scheme, large contribution:

$$\Pi^\gamma(0) = \underbrace{\Pi^\gamma(0) - \Pi^\gamma_{\text{quark,lepton}}(M_Z^2)}_{\text{large logs, non-perturbative QCD}} + \underbrace{\Pi^\gamma_{\text{quark,lepton}}(M_Z^2)}_{\text{well-behaved}}$$

$$=: -\Delta\alpha(M_Z) + \Pi^\gamma_{\text{quark,lepton}}(M_Z^2)$$

Determine $\Delta\alpha$ from low-energy hadronic measurements.

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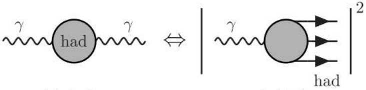


$\Pi'^{\,\text{had}}_{\gamma}(q^2)$ $\sim \sigma^{\text{had}}_{\text{tot}}(q^2)$

Reference	Result	Comment
Geshkenbein, Morgunov [24]	0.02780 ± 0.00006	$\mathcal{O}(\alpha_s)$ resonance model
Swartz [25]	0.02754 ± 0.00046	use of fitting function
Krasnikov, Rodenberg [26]	0.02737 ± 0.00039	PQCD for $\sqrt{s} > 2.3$ GeV
Kühn & Steinhauser [27]	0.02778 ± 0.00016	full $\mathcal{O}(\alpha_s^2)$ for $\sqrt{s} > 1.8$ GeV
Erler [19]	0.02779 ± 0.00020	conv. from $\overline{\text{MS}}$ scheme
Groote *et al.* [28]	0.02787 ± 0.00032	use of QCD sum rules
Martin *et al.* [29]	0.02741 ± 0.00019	incl. new BES data
de Troconiz, Yndurain [30]	0.02754 ± 0.00010	PQCD for $s > 2$ GeV^2
Jegerlehner [31]	0.02755 ± 0.00013	Adler function approach
Davier *et al.* [20]	0.02750 ± 0.00010	incl. new e^+e^- data, PQCD for $\sqrt{s} > 1.8$ GeV
Davier *et al.* [20]	0.02762 ± 0.00011	incl. τ decay data
Burkhardt, Pietrzyk [32]	0.02750 ± 0.00033	incl. BES/BABAR data, PQCD for $\sqrt{s} > 12$ GeV
Hagiwara *et al.* [33]	0.02764 ± 0.00014	incl. new e^+e^- data, PQCD for $\sqrt{s} = 2.6$–3.7, >11.1 GeV

$$\Delta\alpha_{\text{had}}(s) = -\frac{\alpha(0)s}{3\pi} P \int_{s_{\text{th}}}^{\infty} \frac{R_{\text{had}}(s')ds'}{s'(s'-s)}$$

Charge renormalization: On-shell and $\overline{\text{MS}}$ schemes

On-shell scheme, large contribution:

$$\Pi^\gamma(0) = \underbrace{\Pi^\gamma(0) - \Pi^\gamma_{\text{quark,lepton}}(M_Z^2)}_{\text{large logs, non-perturbative QCD}} + \underbrace{\Pi^\gamma_{\text{quark,lepton}}(M_Z^2)}_{\text{well-behaved}}$$

$$=: -\Delta\alpha(M_Z) + \Pi^\gamma_{\text{quark,lepton}}(M_Z^2)$$

Determine $\Delta\alpha$ from low-energy hadronic measurements.

$$\alpha(M_Z) := \frac{\alpha(0)}{1 - \Delta\alpha(M_Z)}$$

Correlation $\alpha(M_Z)$–$\bar{\alpha}(M_Z)$ is well-behaved

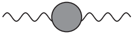
$$\alpha(0) = 1/137.035999074(44)$$

$$\alpha(M_Z) = 1/128.944(19) \qquad \text{[Hagiwara,Liao,Martin,Nomura,Teubner]}$$

$$\bar{\alpha}(M_Z) = 1/127.940(14) \qquad \text{[PDG]}$$

$M_{W,Z}$ renormalization: On-shell and $\overline{\text{MS}}$ schemes

On-shell:

W_T  $\Big|_{p^2=\mathcal{M}_W^2} = 0$

Subtlety: width!

$$\mathcal{M}_W^2 = M_W^2 - iM_W\Gamma_W$$

proper treatment necessary at $\geq$ 2-loop

$\overline{\text{MS}}$-scheme:

$$\bar{M}_W^2 + \delta\bar{M}_W^2 = M_{W\text{OS}}^2 + \delta M_{W\text{OS}}^2$$

Result depends on how tadpoles are treated!

If tadpoles=0, then $\bar{M}_W^2$=gauge-dependent (same for top-quark!)

$\overline{\text{MS}}$ Weak mixing angle:

$$\bar{s}_W^2 = \frac{\bar{g}'}{\sqrt{\bar{g}^2 + \bar{g}'^2}}$$

$$= s_{W\text{OS}}^2 + (\delta s_{W\text{OS}}^2)\big|_{\text{fin-part}}$$

$$\hat{\rho} := \frac{c_W^2}{\bar{c}_W^2}$$

$M_{W,Z}$ renormalization: On-shell and $\overline{\text{MS}}$ schemes

On-shell scheme, another large contribution: (consider 1-loop)

$$\delta s_W^2 = \delta\left(1 - \frac{M_W^2}{M_Z^2}\right) = -\frac{M_W^2}{M_Z^2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right)$$

$$\approx +\frac{M_W^2}{M_Z^2}\Delta\rho \sim \frac{m_t^2}{M_W^2}$$

Violation of custodial symmetry by mass splitting $m_t \neq m_b$.

Outline

1. Tree-level considerations
2. All-order considerations
3. Precise observables and calculations

Some Numbers

$M_W^{\mathrm{exp}} = 80.385(15)$ GeV		0.02%
SM	1Loop (top, Higgs)	$> 30\sigma$
MSSM	1Loop	$\mathcal{O}(0\ldots 10\sigma)$
$M_h^{\mathrm{exp}} = 125.09(0.24)$ GeV		0.2%
MSSM	1Loop	$> 30\sigma$

Some Numbers

$a_\mu^{\text{exp}} = 11\,659\,208.9(6.3)\ \times 10^{-10}$ 0.5ppm

SM	QED $\geq$ 2L	7000σ
	had	100σ
	weak	2.5σ
SUSY	loop effects	$\mathcal{O}(0 \ldots \pm 10\sigma)$

Outline

3. Precise observables and calculations
   - Vacuum structure
   - Precision observables and calculational techniques

Questions about the vacuum structure

$\underline{T = 0}$:

True vacuum groundstate?

- Evaluate effective potential

$$-\int V_{\text{eff}}(v) = \Gamma(H \to v = \text{const})$$

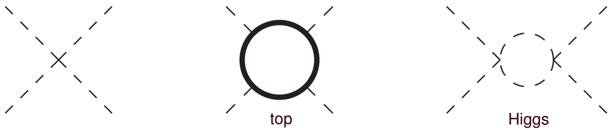
$\underline{T \neq 0}$:

Phase transitions at finite T?

- Evaluate free energy $-k_B T \ln Z$ with

$$Z = \text{Tr}\left(e^{-\frac{1}{k_B T}H}\right)$$

SM effective potential — leading terms



$$V_{\text{eff}}(v) = -\frac{\mu^2}{2}v^2 + \frac{\lambda}{16}v^4 - \frac{1}{16\pi^2}\left[\frac{1}{4}M_h^4\left(L_h - \frac{3}{2}\right) - 3M_t^4\left(L_t - \frac{3}{2}\right)\right] + \ldots$$

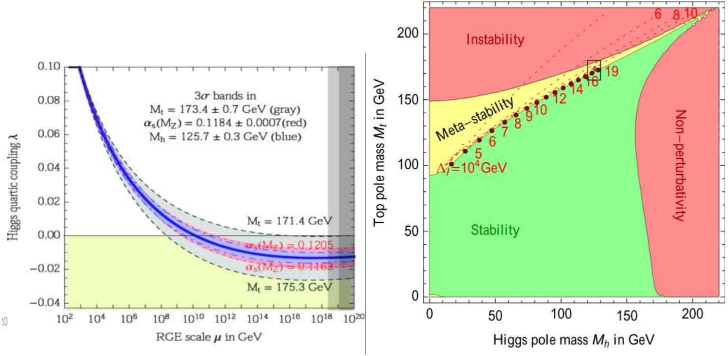
auxiliary v-dependent quantities $M_h^2 \equiv -\mu^2 + \frac{3}{4}\lambda v^2$, $M_t^2 \equiv \frac{1}{\sqrt{2}}y_t v$ and $L_i = \ln(M_i^2/\mu^2_{\overline{\text{MS}}})$

$$\overset{\text{for } v\to\infty}{\longrightarrow} v^4\left[\frac{\lambda}{16} + \frac{1}{64\pi^2}\left(\frac{9}{16}\lambda^2 - 3y_t^4\right)\ln\frac{v^2}{\mu^2_{\overline{\text{MS}}}}\right]$$

Can be positive ($\rightsquigarrow$ stable minimum) or negative (if $\frac{9}{16}\lambda^2 < 3y_t^4$ or $\sqrt{3}m_h < m_t$)

SM effective potential — precisely

- large logs $\ln \frac{v^2}{\mu^2_{\overline{\mathrm{MS}}}}$, can be resummed using RGE
- strategy: precise $\overline{\mathrm{MS}}$ input parameters at weak scale
- use precise β functions to run λ up to high scales

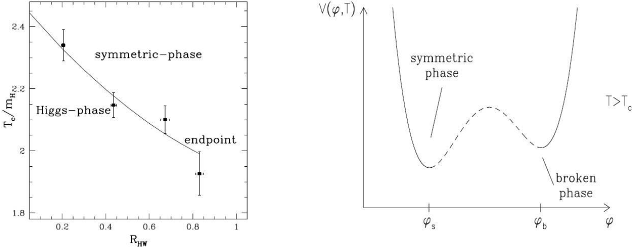


[Degrassi et al]

Just at the border between stability/metastability ⇝ no new physics?

The other SM vacuum question — Baryogenesis

- baryon–antibaryon asymmetry cannot be explained in SM alone (EW phase transition not first order, no deviation from equilibrium)



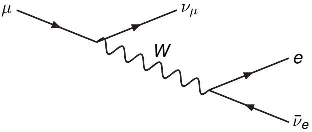
[Buchmüller]

- Hence, new physics needed! (But could be leptogenesis; doesn't have to modify Higgs potential)

Outline

3. Precise observables and calculations
   - Vacuum structure
   - Precision observables and calculational techniques

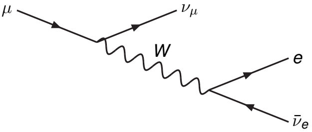
Muon decay



$$G_F = \frac{\pi\alpha(0)}{\sqrt{2}M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}(1+\Delta r)$$

Used for $G_F, \alpha(0), M_Z \overset{\text{further parameters}}{\Longrightarrow} M_W^2$ prediction

Muon decay



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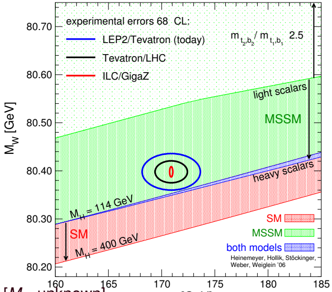
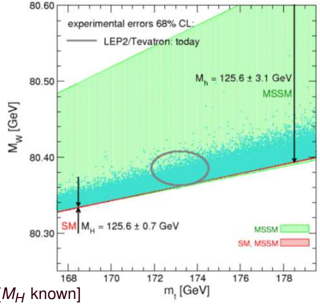
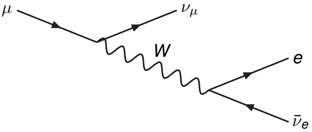


[M_H unknown]



[M_H known]

Muon decay



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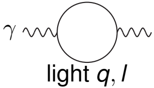
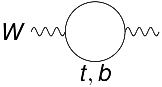
$$\text{Used for } G_F, \alpha(0), M_Z \overset{\text{further parameters}}{\Longrightarrow} M_W^2 \text{ prediction}$$

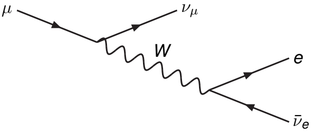
- Leading corrections from finite renormalizations:





$$\alpha(0) \to \alpha(0)(1+\Delta\alpha)\ , \qquad \frac{1}{s_W^2} \to \frac{1}{s_W^2}\left(1-\frac{c_W^2}{s_W^2}\Delta\rho\right)$$

Muon decay



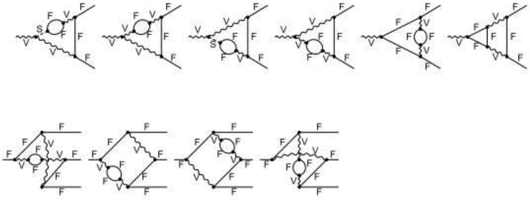
$$G_F = \frac{\pi\alpha(0)}{\sqrt{2}M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}(1+\Delta r)$$

Used for $G_F, \alpha(0), M_Z \stackrel{\text{further parameters}}{\Longrightarrow} M_W^2$ prediction

$$\Delta r = \underbrace{\Delta\alpha}_{0.06} \underbrace{-\frac{c_W^2}{s_W^2}\Delta\rho}_{-0.03} + \underbrace{\Delta r^{\text{rem}}}_{0.01}$$

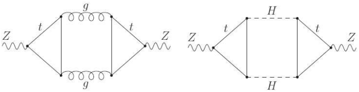
- Status: 1-loop, 2-loop [Awramik,Czakon,Freitas,Hollik,Walter,Weiglein;Degrassi,Gambino,Giardino], partial $>$ 2-loop known

Muon decay — precisely



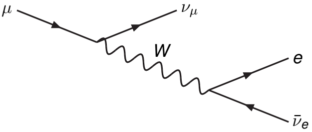
2-loop examples

[Hollik]



[vd Bij,Chetyrkin,Faisst,Jikia,Seidensticker'00]

Muon decay



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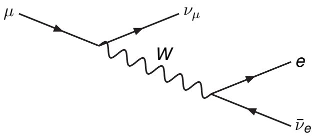
Muon decay

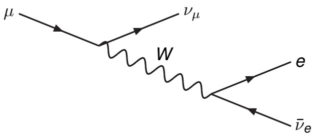


$$G_F = \frac{\pi\bar{\alpha}(M_Z)}{\sqrt{2}M_W^2\bar{s}_W^2}(1+\Delta\hat{r}_W)$$

where $M_W^2 = \hat{\rho}\bar{c}_W^2 M_Z^2$

Alternative $\overline{\text{MS}}$ setup $G_F, \bar{\alpha}, M_Z \overset{\text{further parameters}}{\Longrightarrow} \bar{s}_W^2 \Longrightarrow M_W^2$

Muon decay



$$G_F = \frac{\pi \bar{\alpha}(M_Z)}{\sqrt{2} M_W^2 \bar{s}_W^2}(1 + \Delta \hat{r}_W)$$

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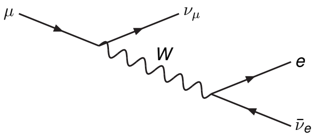
- current predictions: (theory uncertainty $\approx$ 3–4 MeV)

$$M_W^{\text{SM-th}} = \begin{cases} 80.3639 \text{ GeV} & [\text{OS}] \\ 80.3578 \text{ GeV} & [\overline{\text{MS}}] \end{cases}$$

Muon decay



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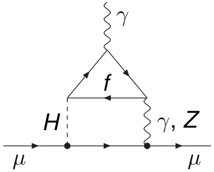
$$M_W^{\text{SM-th}} = \begin{cases} 80.3639\ \text{GeV} & [\text{OS}] \\ 80.3578\ \text{GeV} & [\overline{\text{MS}}] \end{cases}$$

Most important parametric uncertainties: m_{top}, $\Delta\alpha_{\text{had}}$

(changing them $\sim 1\sigma$ $\Delta M_W \sim$ 15MeV)

Subtlety: γ_5-problem in EW multiloop calculations



Naive anticommuting γ_5: $\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 0$

(mathematically not fully consistent,

but preserves appropriate Ward id.)

Math. consistent: $\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) \propto \epsilon^{\mu\nu\rho\sigma}$

['t Hooft, Veltman; Breitenlohner, Maison]

distinguishes 4-dim and $D - 4$-dim objects and breaks Ward identities

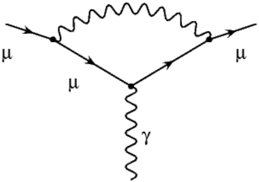
Naive treatment here (e.g. a_μ) correct since:

- difference to consistent treatment has the form of a local counterterm
  (as opposed to non-local or imaginary)
- this follows from power-counting after using anomaly cancellation

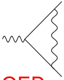
  [Heinemeyer,DS,Weiglein'04]

But this must be studied case by case

Muon $(g-2)$

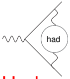


$$a_\mu = \frac{1}{2}(g-2)_\mu = \frac{\alpha(0)}{2\pi} + \ldots$$

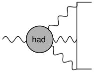


[in units 10^{-10}]

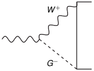
QED: 11 658 471.8 (0.0)



Had vp: 682.5 (4.2)

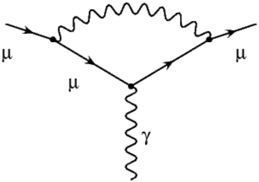


Had lbl: 10.5 (2.6)



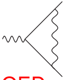
Weak: 15.36 (0.10)

Muon $(g-2)$



$$a_\mu = \frac{1}{2}(g-2)_\mu = \frac{\alpha(0)}{2\pi} + \dots$$

[in units 10^{-10}]



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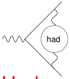

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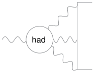

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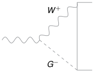
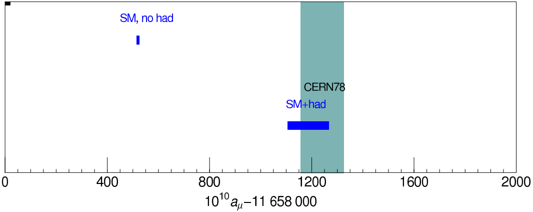


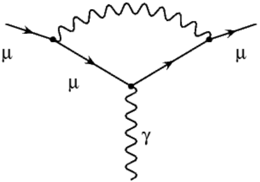
Weak: 15.36 (0.10)

Legacy of the old CERN experiments:



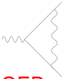
verified had contributions

Muon $(g-2)$

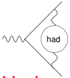


$$a_\mu = \frac{1}{2}(g-2)_\mu = \frac{\alpha(0)}{2\pi} + \ldots$$

[in units 10^{-10}]



QED: 11 658 471.8 (0.0)



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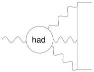

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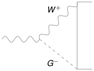
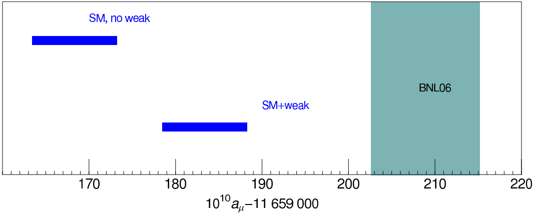


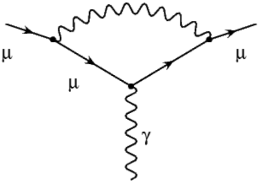
Weak: 15.36 (0.10)

Legacy of the Brookhaven experiments:



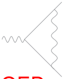
3σ deviation, larger than a_μ^{EW}!

Muon $(g-2)$

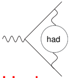


$$a_\mu = \frac{1}{2}(g-2)_\mu = \frac{\alpha(0)}{2\pi} + \dots$$

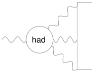
[in units 10^{-10}]



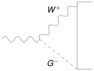
QED: 11 658 471.8 (0.0)



Had vp: 682.5 (4.2)

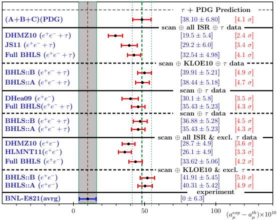


Had lbl: 10.5 (2.6)



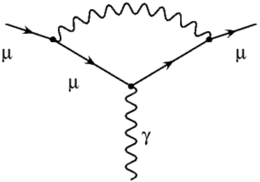
Weak: 15.36 (0.10)

Hadronic vp contributions from e^+e^- →had data [SND, CMD, KLOE, Babar, Belle, BES. . .]



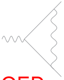
[Benayoun, David, DelBuono, Jegerlehner ’12]

Muon $(g-2)$

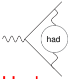


$$a_\mu = \frac{1}{2}(g-2)_\mu = \frac{\alpha(0)}{2\pi} + \ldots$$

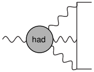
[in units 10^{-10}]



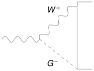
QED: 11 658 471.8 (0.0)



Had vp: 682.5 (4.2)



Had lbl: 10.5 (2.6)



Weak: 15.36 (0.10)

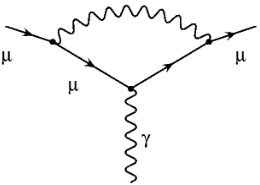
Weak contributions

- Leading:

$$a_\mu^{\mathrm{EW}(1)} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3}(1 - 4 s_W^2)^2 \right]$$
$$= 19.48 \times 10^{-10}$$

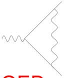
- 2-loop exactly known, very large $\mathcal{O}(20\%)$

Muon $(g-2)$

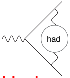


$$a_\mu = \frac{1}{2}(g-2)_\mu = \frac{\alpha(0)}{2\pi} + \ldots$$

[in units 10^{-10}]



QED: 11 658 471.8 (0.0)



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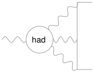

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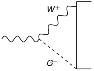
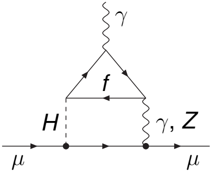
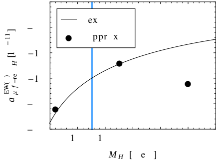


Weak: 15.36 (0.10)





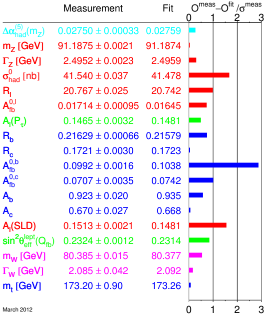
- exact evaluation of M_H-dependent parts
- consistent parametrization of 1-, 2-, 3-loop $\propto G_F \alpha^{n-1}$
- final result: $(15.36 \pm 0.10) \times 10^{-10}$ [Gnendiger,DS,Stöckinger-Kim'13]

Progress expected: Fermilab experiment (soon)

- low-energy measurements (μ-decay, a_μ, $e^+e^- \to$had, $\sin\theta$) important to test EW physics

Summary

- Very good agreement SM–data



Summary

- very deep QFT understanding of SM and similar theories

  $Q|phys\rangle = 0$ ⇒BRS⇒gauge invariance

- Renormalization in practice involves large contributions

  $\Delta\alpha$, $\Delta\rho$, $\overline{\text{MS}}$–OS schemes

  and subtleties

  $\partial_\xi \delta v \neq 0$, γ_5-problem

- Precision computations at $\geq$ 2-loop level done and needed
- Extensions of SM possible and motivated

What explains the existence/structure of scalar fields?
Supersymmetry? Compositeness? Something else? Nothing?