Causality of Holographic Entanglement Entropy

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Workshop on Quantum Fields and Strings

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Based on: M. Headrick, VH, A. Lawrence, & M. Rangamani: 1408.6300;

& previous works w/ {H. Maxfield, M. Rangamani, & E. Tonni}: 1306.4004, 1306.4324, & 1312.6887

Motivation

- AdS/CFT correspondence:
  - Can provide invaluable insight into strongly coupled QFT & QG
  - To realize its full potential, need to further develop the dictionary…
- Natural expectation:
  - Physically important / natural constructs one side will have correspondingly important / natural duals on the other side…
  - We can then use these to probe bulk via boundary quantities
- Recent progress in QI vs. QG
  - Fundamental quantum information constructs (e.g. entanglement) seem to be intimately related to geometry!
- Hence study natural geometrical / causal constructs in bulk
- Useful tool in defining new quantities: general covariance…

OUTLINE

- Covariant holographic entanglement entropy respects CFT causality [Headrick, VH, Lawrence & Rangamani, '14]

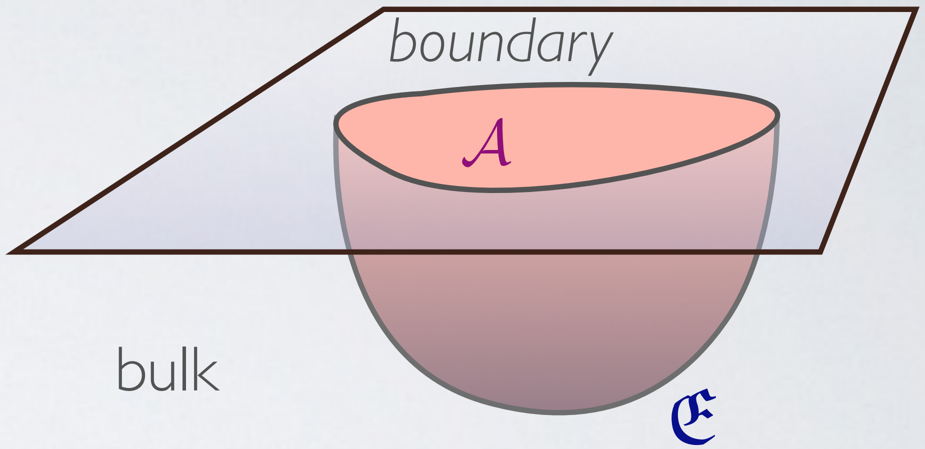
  Entanglement wedge, Causal shadow,
  & Causal wedge [VH&Rangamani '12; VH,MR,Tonni, '13]

- Probing inside black holes using EE [VH, Maxfield '13]

Holographic Entanglement Entropy

Proposal [RT=Ryu & Takayanagi, '06] for *static* configurations:

In the bulk EE $S_{\mathcal{A}}$ is captured by the area of minimal co-dimension-2 bulk surface $\mathfrak{E}$ *at constant t* anchored on $\partial\mathcal{A}$ & homol. to $\mathcal{A}$.



$$S_{\mathcal{A}} = \min_{\partial\mathfrak{E}=\partial\mathcal{A}} \frac{\text{Area}(\mathfrak{E})}{4\,G_N}$$

In *time-dependent* situations, covariantize:

- ✳ minimal surface → extremal surface [HRT=VH, Rangamani, Takayanagi '07]

This gives a well-defined quantity in any (arbitrarily time-dependent asymptotically AdS) spacetime ⇒ equally robust as in CFT

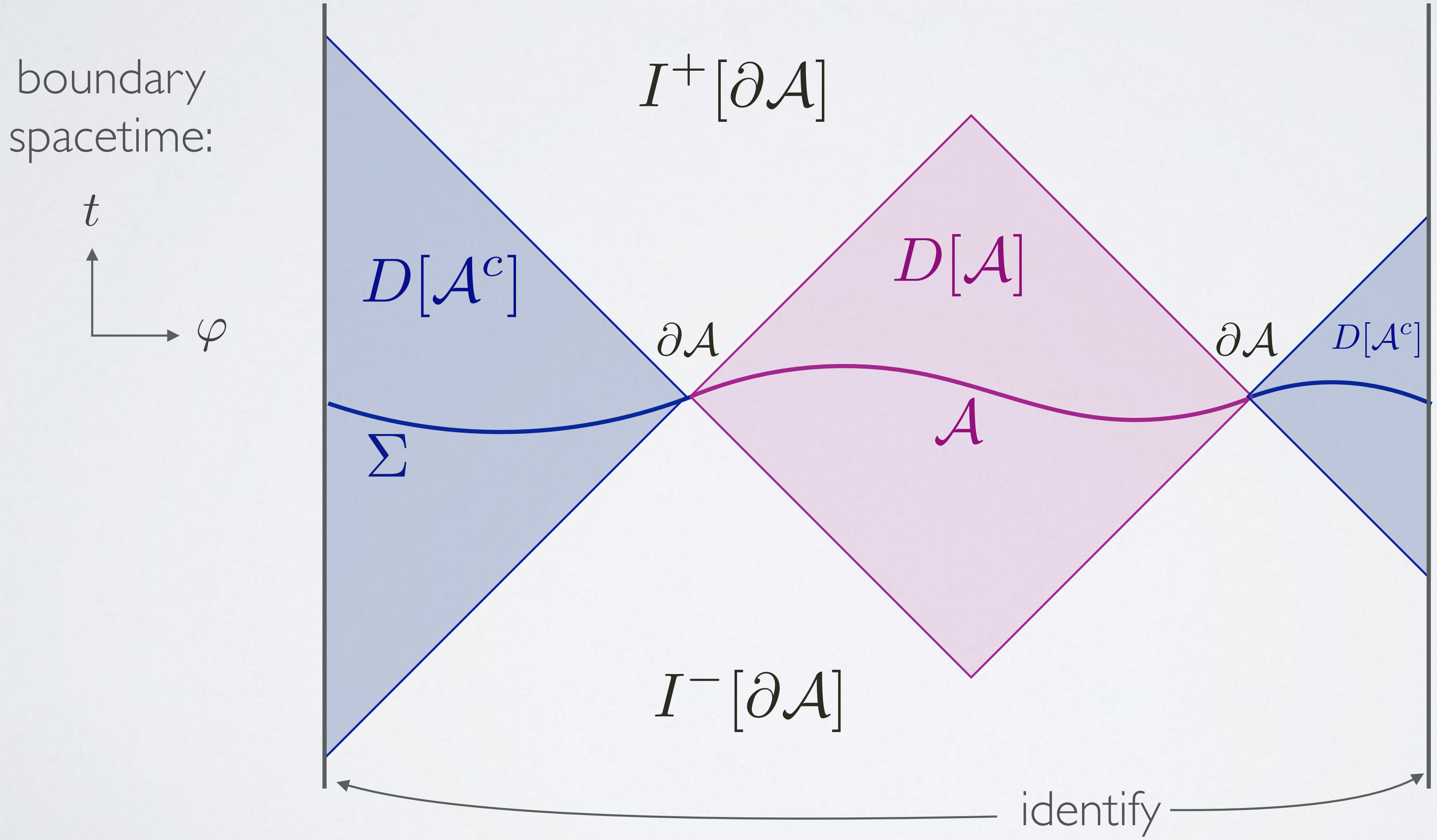
But we can't use Euclidean techniques for proof…

?: Is HRT prescription consistent with CFT constraints, e.g. causality?

CFT causal restriction

- Entanglement entropy $S_{\mathcal{A}}$ only depends on $D[\mathcal{A}]$ and not on Σ.
- Natural separation of boundary spacetime into 4 regions:

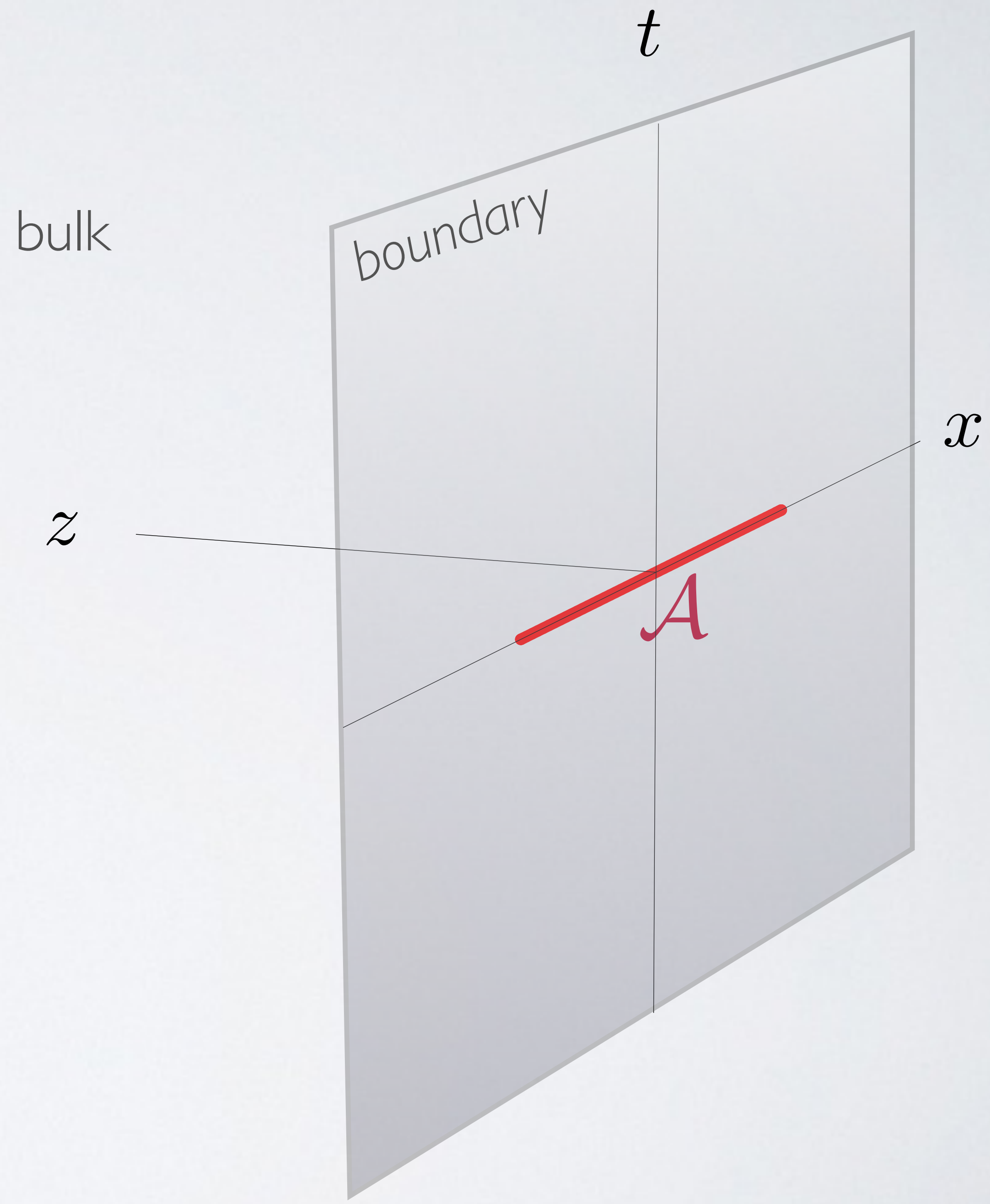
$$\partial\mathcal{M} = D[\mathcal{A}] \cup D[\mathcal{A}^c] \cup I^-[\partial\mathcal{A}] \cup I^+[\partial\mathcal{A}]$$



- EE should not be influenced by any change to state within $D[\mathcal{A}]$ or $D[\mathcal{A}^c]$.

Causal Wedge construction

- Consider a bdy region $\mathcal{A}$



sketch for planar AdS:

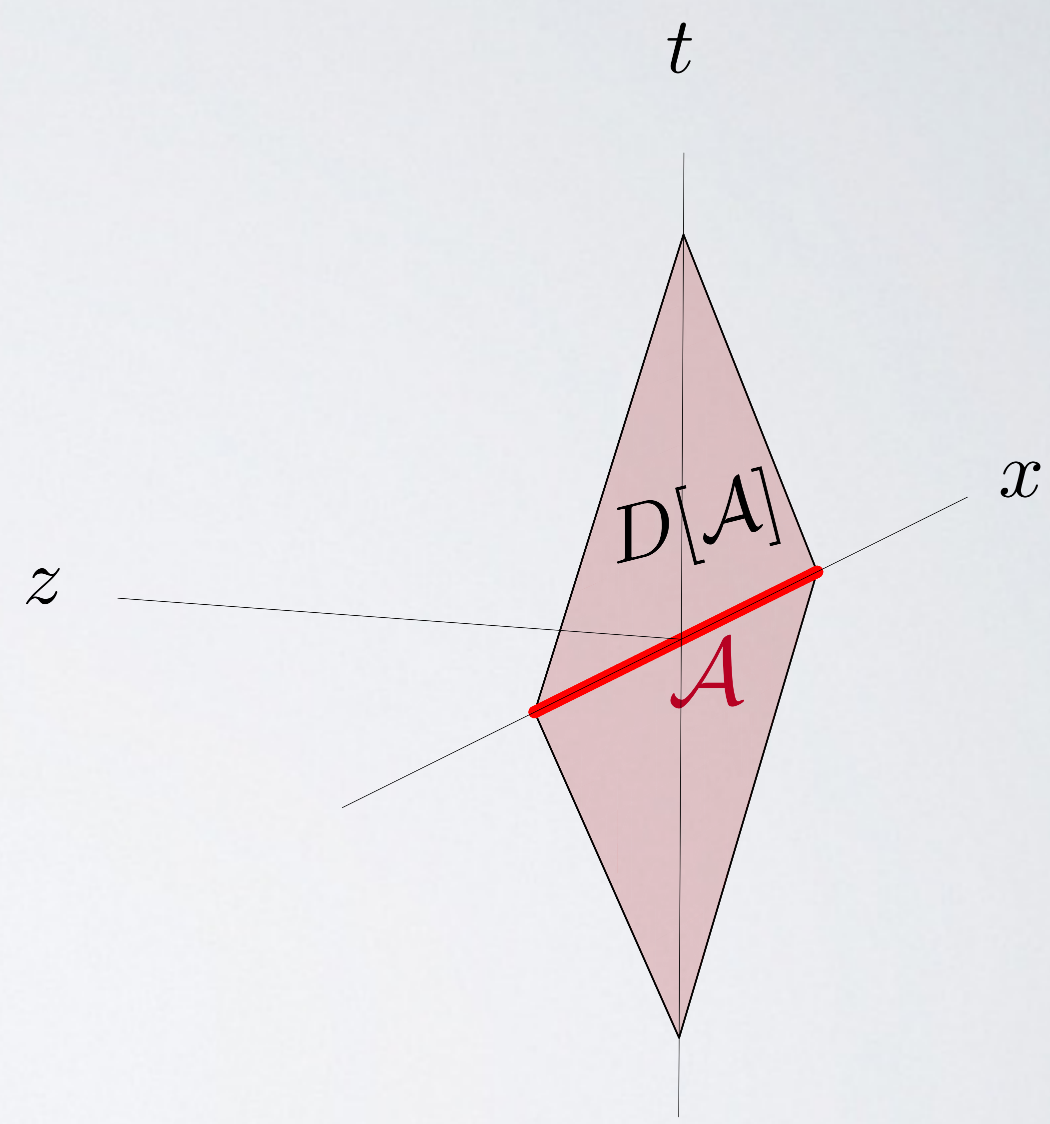
$$ds^2 = \frac{-dt^2 + dx^2 + dz^2}{z^2}$$

(boundary at $z = 0$)

Causal Wedge construction

- Consider a bdy region $\mathcal{A}$
- Construct the bdy domain of dependence of $\mathcal{A}$, denoted $D[\mathcal{A}]$

(observables in the entire region $D[\mathcal{A}]$ can be determined solely from the initial conditions specified on $\mathcal{A}$)



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- The bulk causal future $J^+[D[\mathcal{A}]]$ and causal past $J^-[D[\mathcal{A}]]$ of $D[\mathcal{A}]$ characterize bulk points which can be influenced by, or influence $D[\mathcal{A}]$

t

$J^+[D[\mathcal{A}]]$

x

z

$\mathcal{A}$

$J^-[D[\mathcal{A}]]$

Causal Wedge construction

Bulk causal region naturally corresponding to $D[\mathcal{A}]$:

- Bulk causal wedge $\blacklozenge_{\mathcal{A}}$

  $$\blacklozenge_{\mathcal{A}} \equiv J^{-}[D[\mathcal{A}]] \cap J^{+}[D[\mathcal{A}]]$$

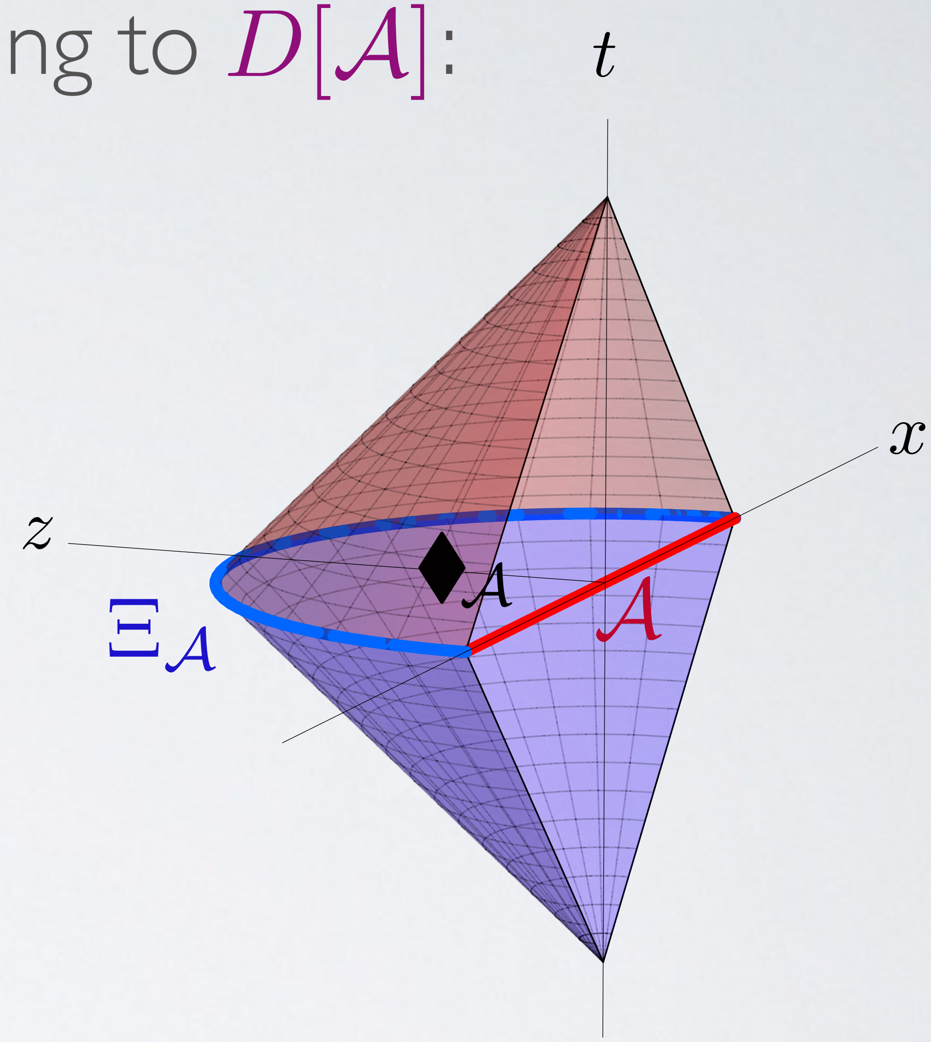
  = { bulk causal curves which begin and end on $D[\mathcal{A}]$ }

- Causal information surface $\Xi_{\mathcal{A}}$

  $$\Xi_{\mathcal{A}} \equiv \partial J^{-}[D[\mathcal{A}]] \cap \partial J^{+}[D[\mathcal{A}]]$$

- Causal holographic information $\chi_{\mathcal{A}}$

  $$\chi_{\mathcal{A}} \equiv \frac{\mathrm{Area}(\Xi_{\mathcal{A}})}{4\,G_N}$$



[VH&Rangamani '12]

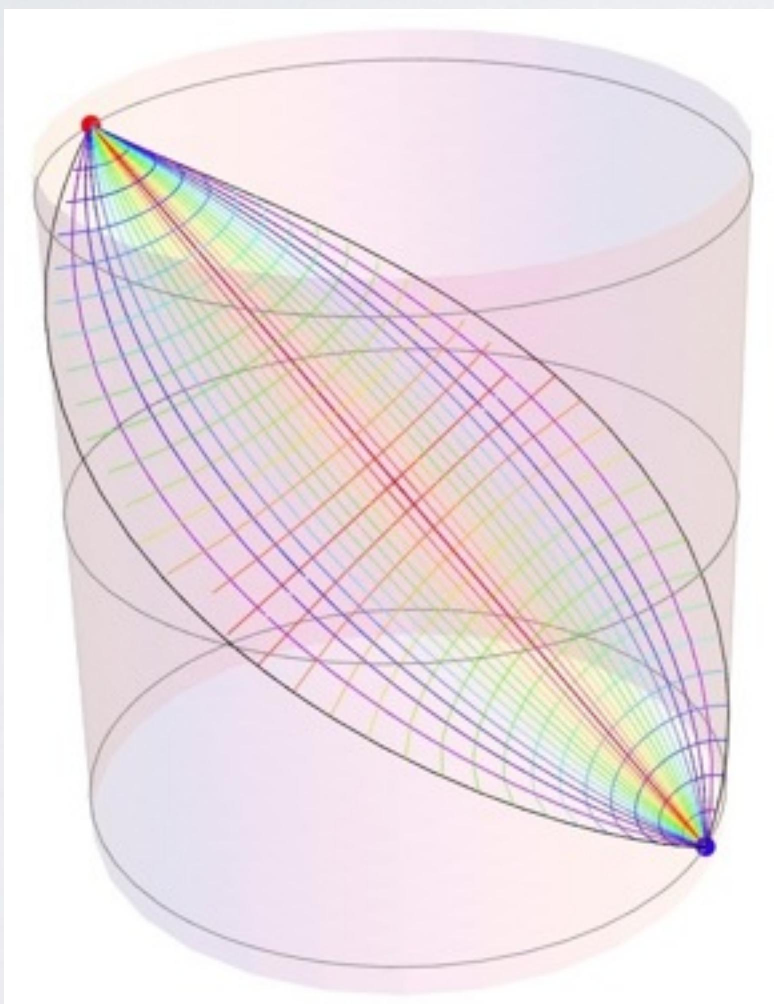
- In special cases, $\Xi_{\mathcal{A}} = \mathfrak{E}_{\mathcal{A}} \quad \Rightarrow \quad \chi = S_{\mathcal{A}}$, but in general they differ.

- Important Q: what is their interpretation within the dual CFT ?

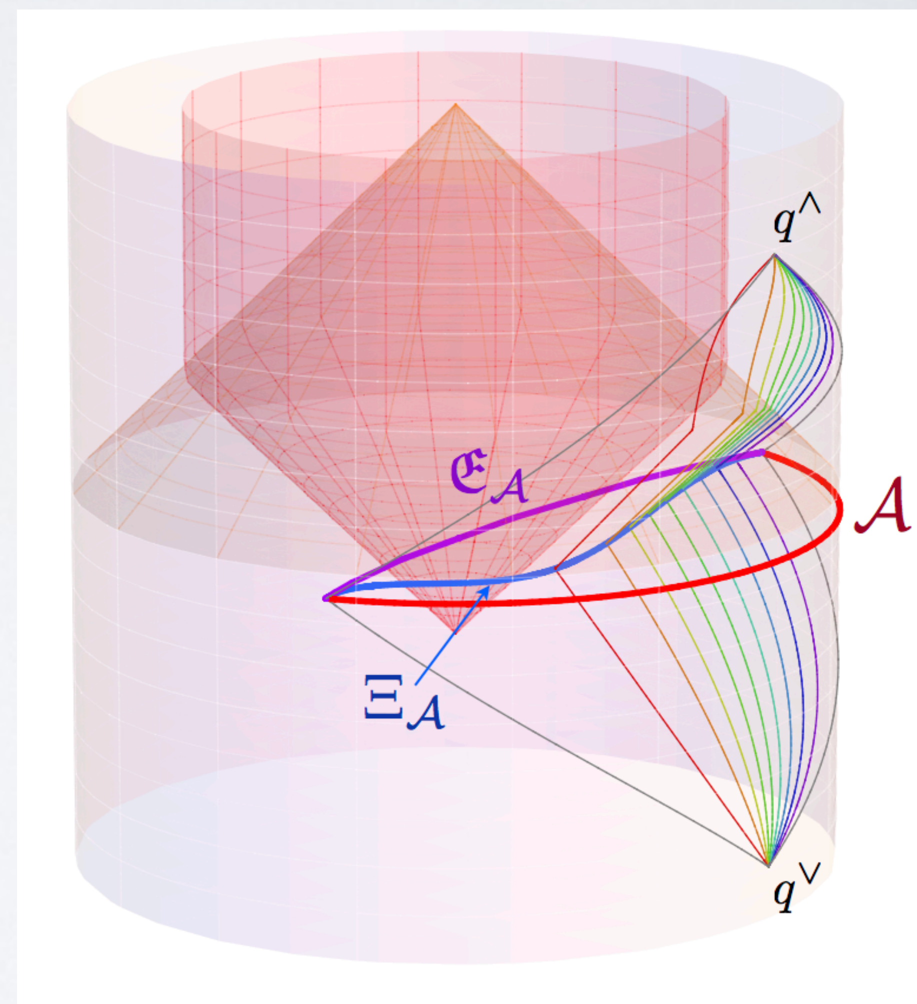
Causality upheld marginally

- Extremal surface cannot lie inside the causal wedge [VH&MR; Wall]
  - But in special cases $\mathfrak{E}_{\mathcal{A}}$ can be null related to $\Xi_{\mathcal{A}}$, e.g.:

pure AdS:



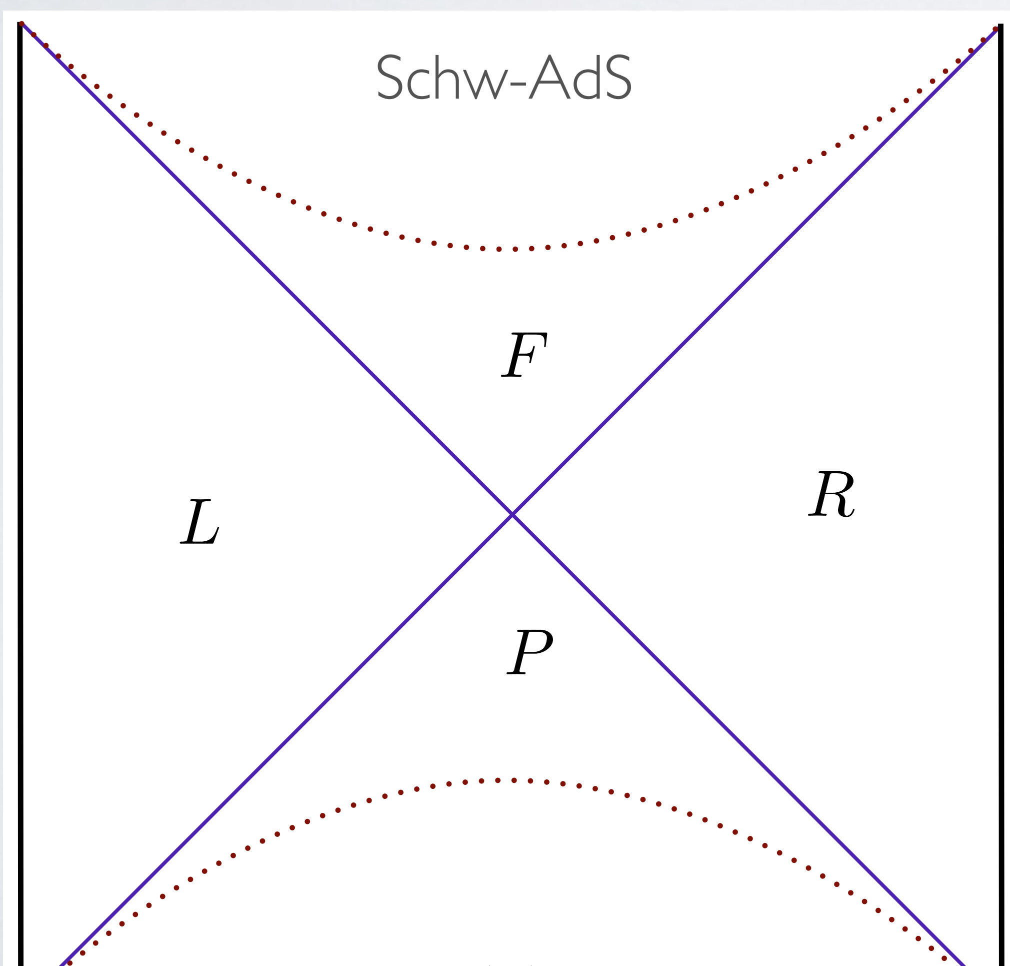
Vaidya-AdS:



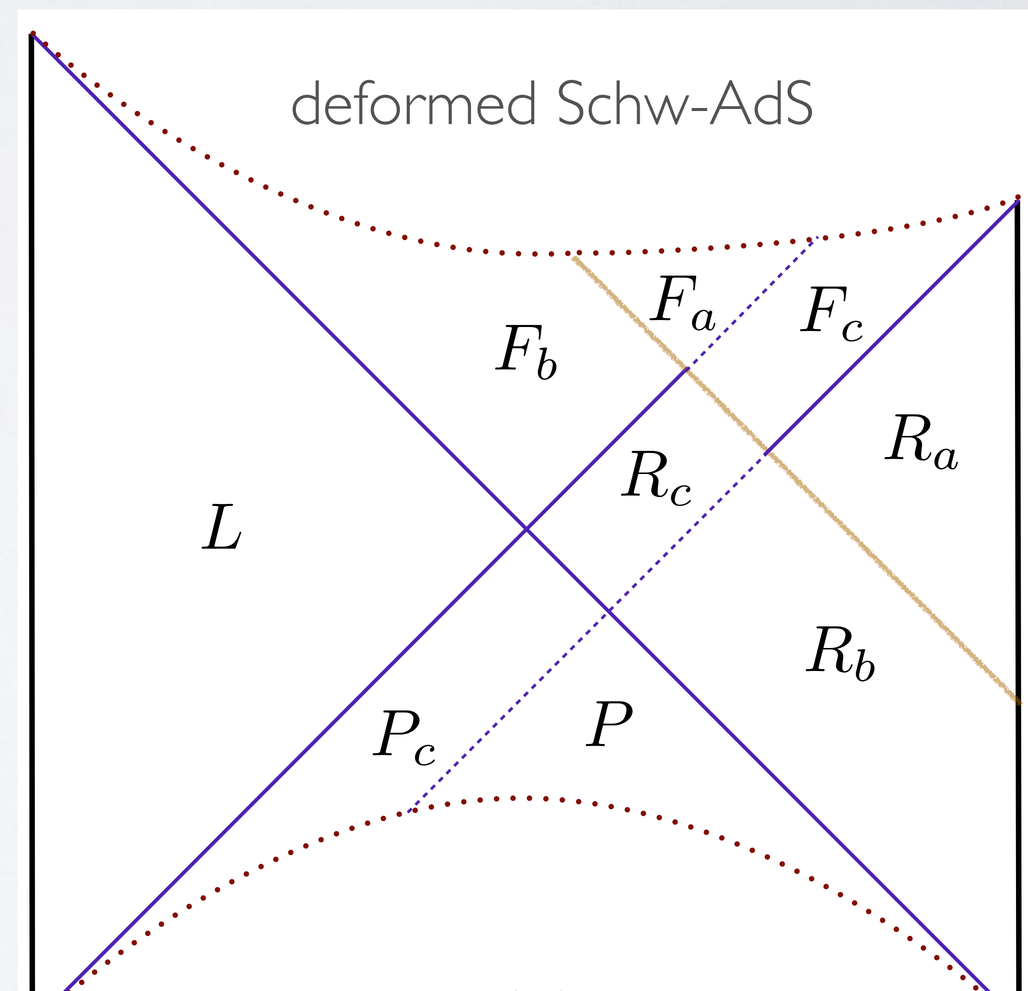
- Danger: is it possible to deform $\mathfrak{E}_{\mathcal{A}}$ s.t. timelike-separated from $\Xi_{\mathcal{A}}$?

Dynamical eternal BH geometry

- Extremal surfaces cannot penetrate static BH event horizon [VH, '12]
- But they can penetrate dynamical BH event horizon [cf. Vaidya-AdS]
- Danger: can surface from on R bdy reach to causal communication w/ L bdy?



(a)



(b)

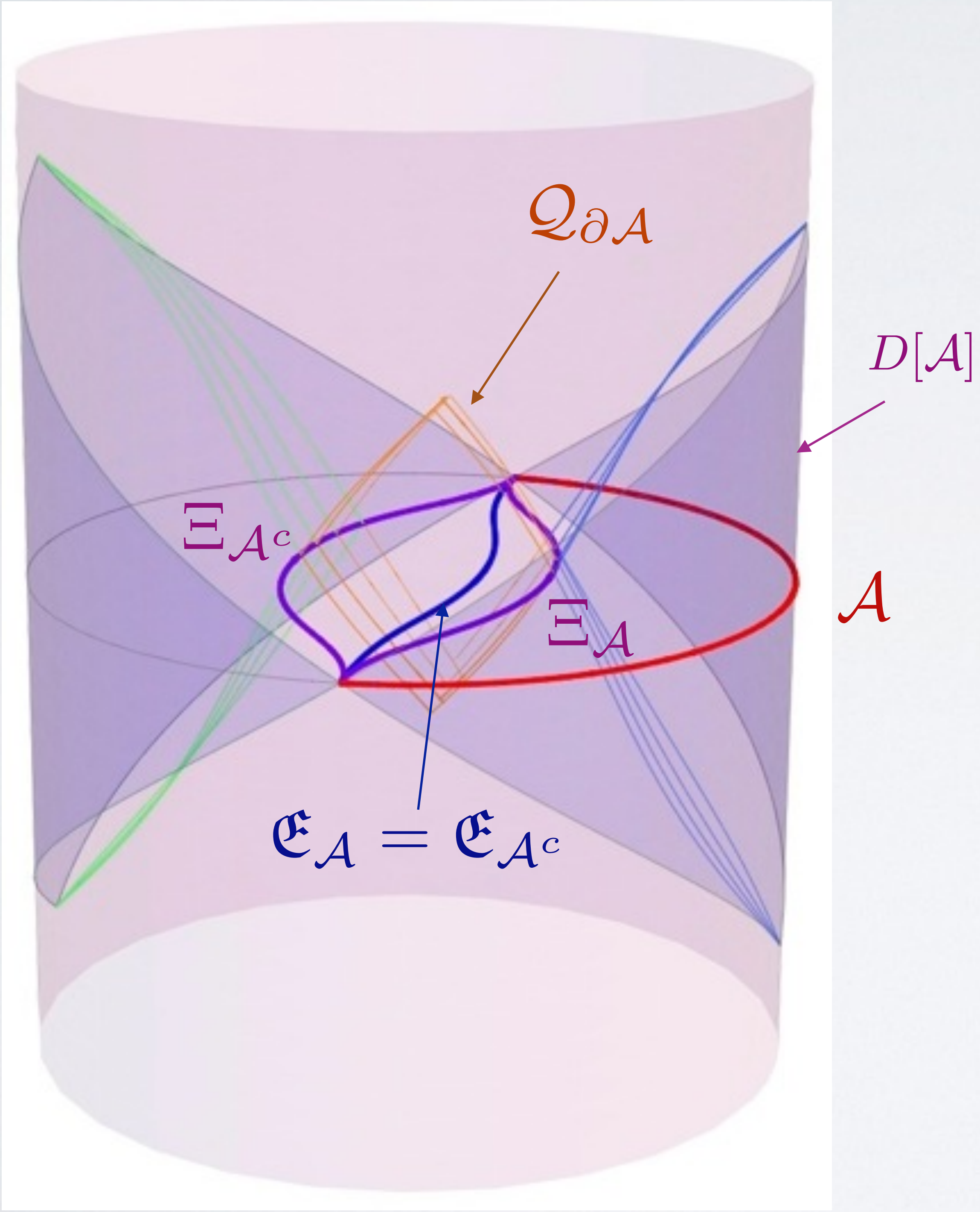
Bulk causal restriction

- A-priori, boundary causality of EE is not manifest in the bulk:
  - Need: extremal surface to be spacelike from the causal wedge… ✔
  - In eternal BH geometry, w/ 2 boundaries, need extremal surface anchored on R bdy to not reach into causal contact w/ L bdy… ✔

- We can show that both are satisfied robustly.
  - Generically, $\mathfrak{E}_{\mathcal{A}}$ is spacelike-separated from $\Xi_{\mathcal{A}}$
  - (otherwise violates Raychaudhuri equation)
  - Hence the extremal surface must lie within the causal shadow $\mathcal{Q}_{\partial\mathcal{A}}$

[Headrick, VH, Lawrence, & Rangamani, '14; cf. Wall '12]

Causal Shadow



$\mathcal{Q}_{\partial\mathcal{A}}$ = causal shadow =

bulk region which is causally disconnected from both $\mathcal{A}$ and $\mathcal{A}^c$

The extremal surface $\mathfrak{E}_{\mathcal{A}}$ necessarily lies inside $\mathcal{Q}_{\partial\mathcal{A}}$

Entanglement wedge

- Boundary spacetime separation:

$$\partial\mathcal{M} = D[\mathcal{A}] \cup D[\mathcal{A}^c] \cup I^-[\partial\mathcal{A}] \cup I^+[\partial\mathcal{A}]$$

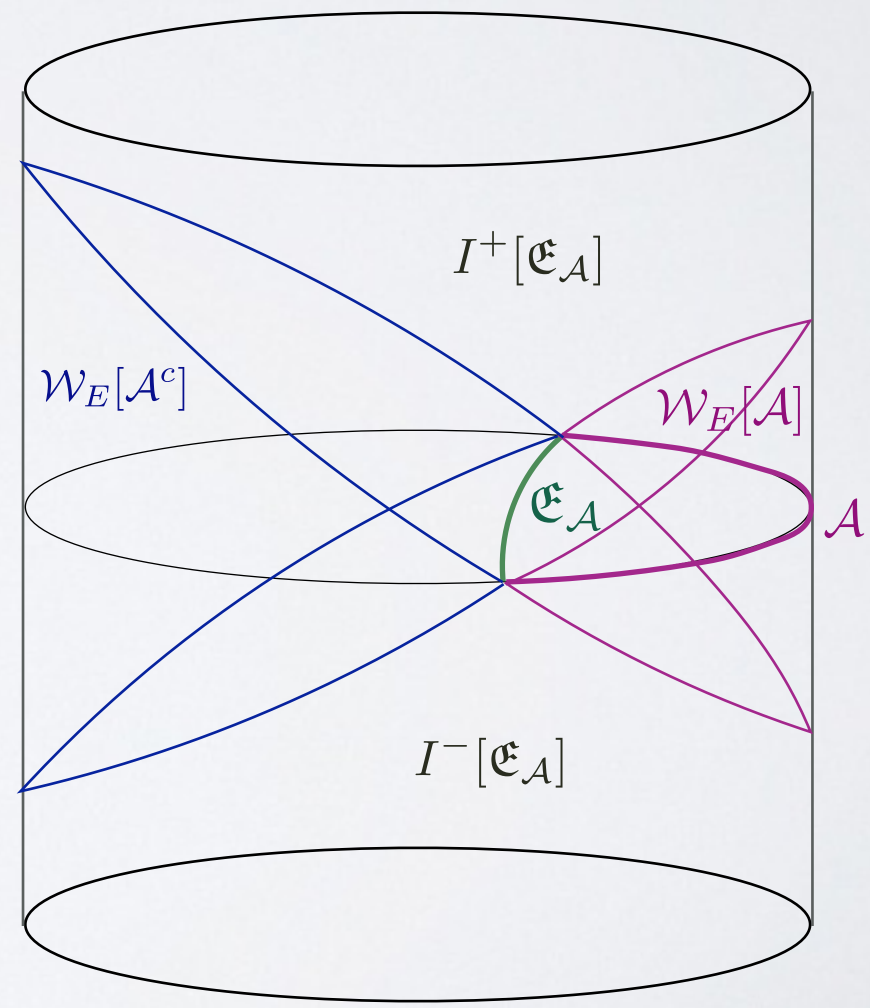
- This naturally induces a corresponding separation into 4 bulk regions:

$$\mathcal{M} = \mathcal{W}_E[\mathcal{A}] \cup \mathcal{W}_E[\mathcal{A}^c] \cup I^-[\mathfrak{E}_\mathcal{A}] \cup I^+[\mathfrak{E}_\mathcal{A}]$$

(for pure state)

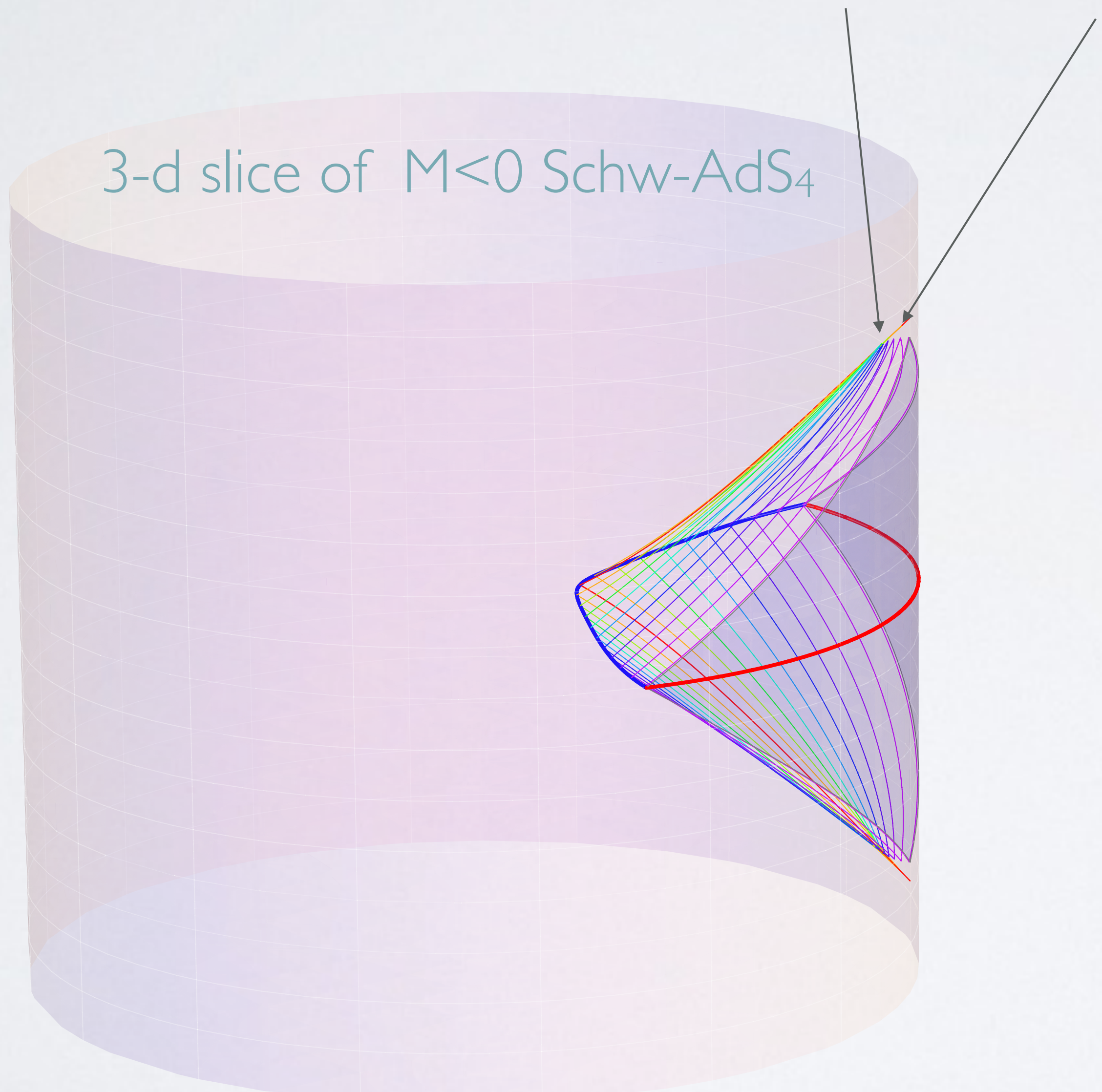
entanglement wedge of $\mathcal{A}$

- $\mathcal{W}_E[\mathcal{A}]$ ends on $D[\mathcal{A}]$
- contains the causal wedge $\blacklozenge_\mathcal{A}$
- generated by null geodesics normal to $\mathfrak{E}_\mathcal{A}$

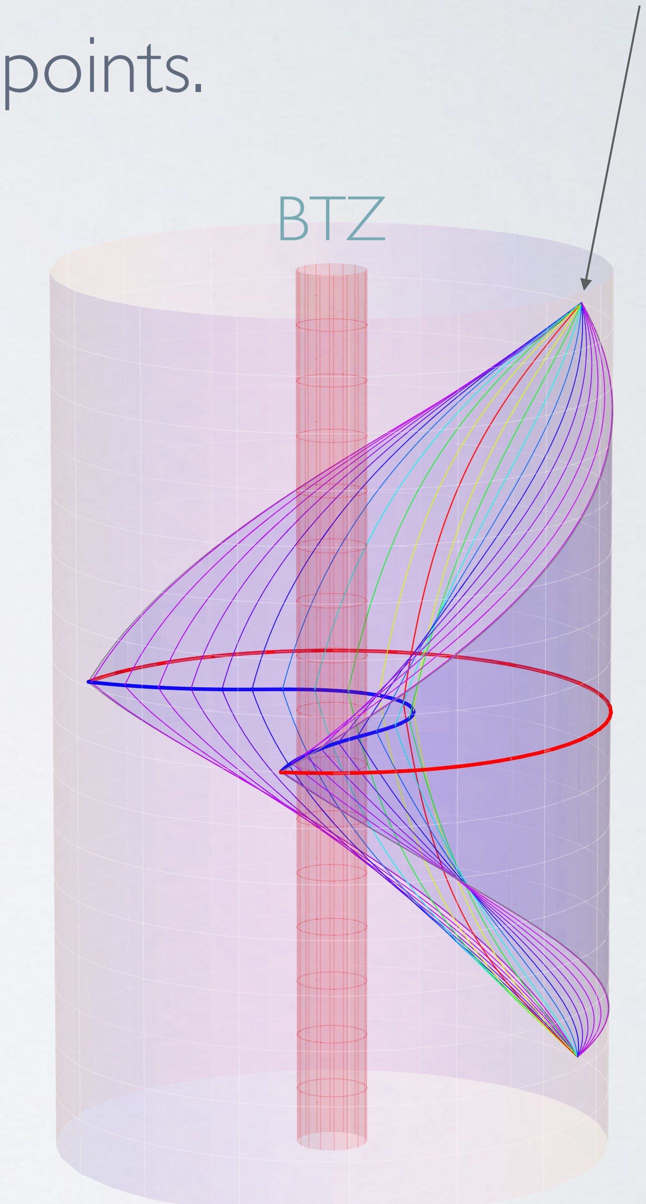


Entanglement wedge

- Only for special cases such as BTZ do generators of $\partial \mathcal{W}_E[\mathcal{A}]$ reach boundary.
- In general, the generators end at caustic / crossover points.



entanglement wedge $\supset$ causal wedge



entanglement wedge = causal wedge

Bulk dual of reduced density matrix?

?: What bulk region is reconstructable from $\rho_{\mathcal{A}}$?

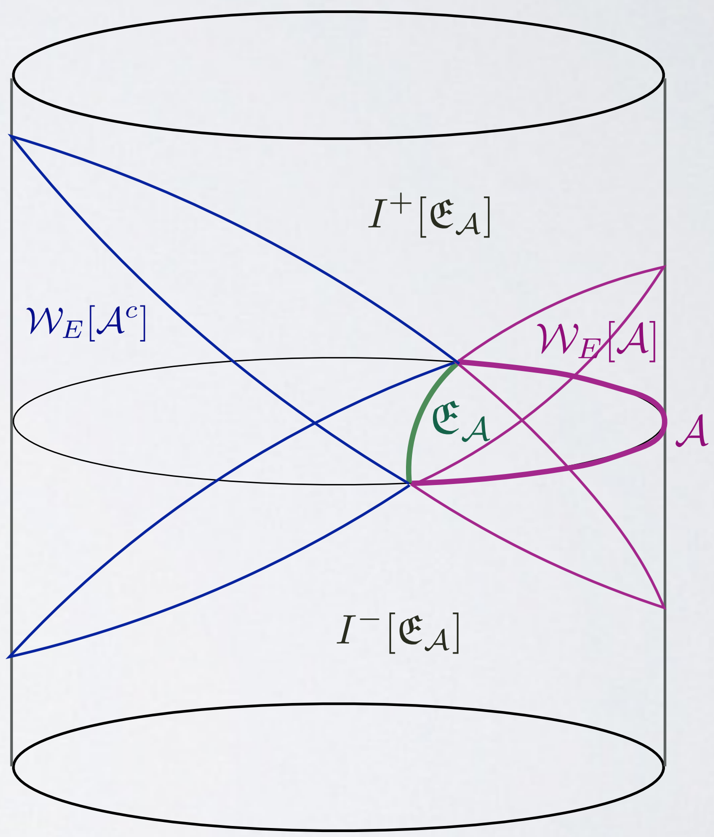
- Causal wedge $\blacklozenge_{\mathcal{A}}$?

  [Bousso, Leichenauer, & Rosenhaus, '12]

- Entanglement wedge $\mathcal{W}_E[\mathcal{A}]$?

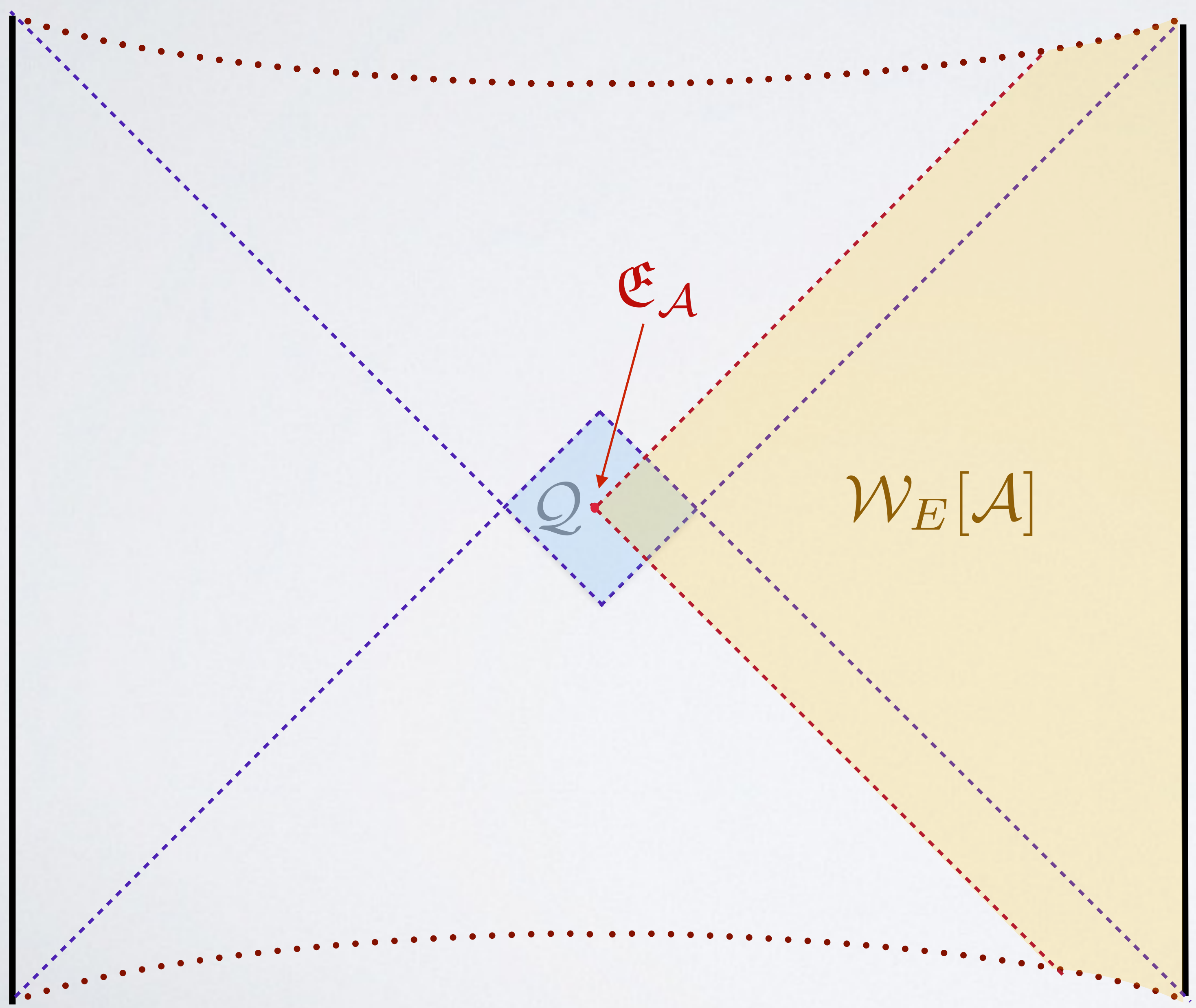
  our conjecture [HHLR].

  cf. also: [Czech, Karczmarek, Nogueira, Van Raamsdonk, '12; Wall, '12]



Entanglement wedge in deformed SAdS

In deformed eternal Schw-AdS, (compact) extremal surface corresponding to $\mathcal{A} = \Sigma_L$ or $\mathcal{A} = \Sigma_R$ must lie in the 'shadow region' $\mathcal{Q}$

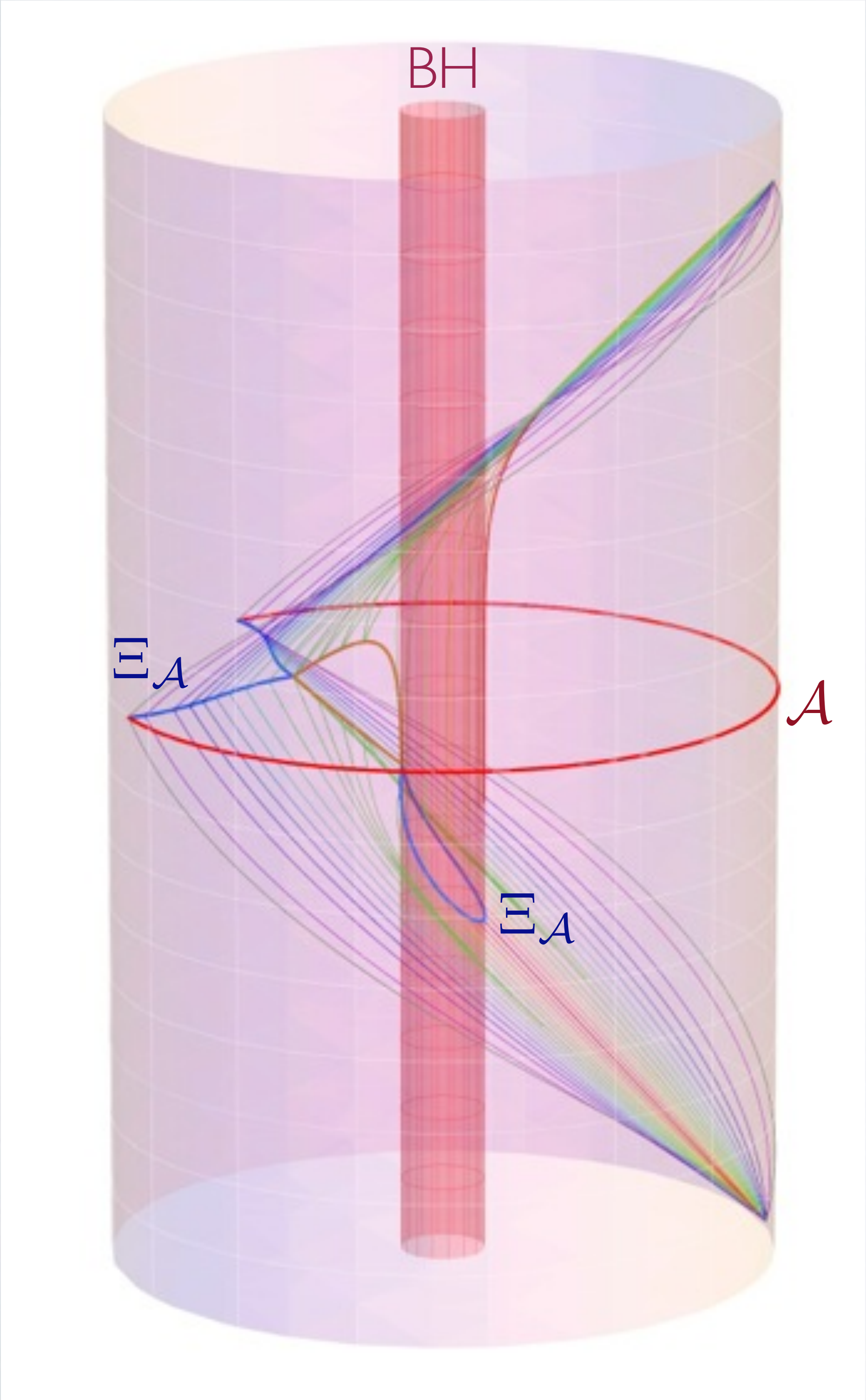


i.e. causally disconnected from both boundaries…

(for static Schw-AdS, shadow region = bifurcation surface)

$\Rightarrow$ Entanglement wedge extends past event horizon

Aside: one use of causal wedge



- Causal wedge can have holes…
- Important implication for entanglement:
  - whenever $\mathcal{A}$ is large enough for $\Xi_{\mathcal{A}}$ to have two disconnected pieces, there cannot exist a single connected extremal (minimal) surface $\mathfrak{E}_{\mathcal{A}}$ homologous to $\mathcal{A}$!
  - in such cases, $\Rightarrow \; S_{\mathcal{A}} = S_{\mathcal{A}^c} + S_{\text{BH}}$

    (saturates Araki-Lieb inequality)

    → *entanglement plateau*

    [VH, Maxfield, Rangamani, Tonni, '13]

    → two components to entanglement
  - Causal wedge argument guarantees this even for generic time-dependent BHs.

OUTLINE

- Covariant holographic entanglement entropy respects CFT causality

- Probing inside black holes using EE

Motivation

Gravity side:

- Black holes provide a window into quantum gravity
  - e.g. what resolves the curvature singularity?
- Study in AdS/CFT by considering a black hole in the bulk
- Can we probe it by extremal surfaces?
  - Not for static BH [VH '12]
  - Certainly for dynamically evolving BH (since horizon is teleological) [VH '02, Abajo-Arrastia,et.al. '06]

⇒ use rapidly-collapsing black hole in AdS → Vaidya-AdS

& ask how close to the singularity can extremal surfaces penetrate?

CFT side:

- Important question in physics: thermalization (e.g. after global quantum quench)

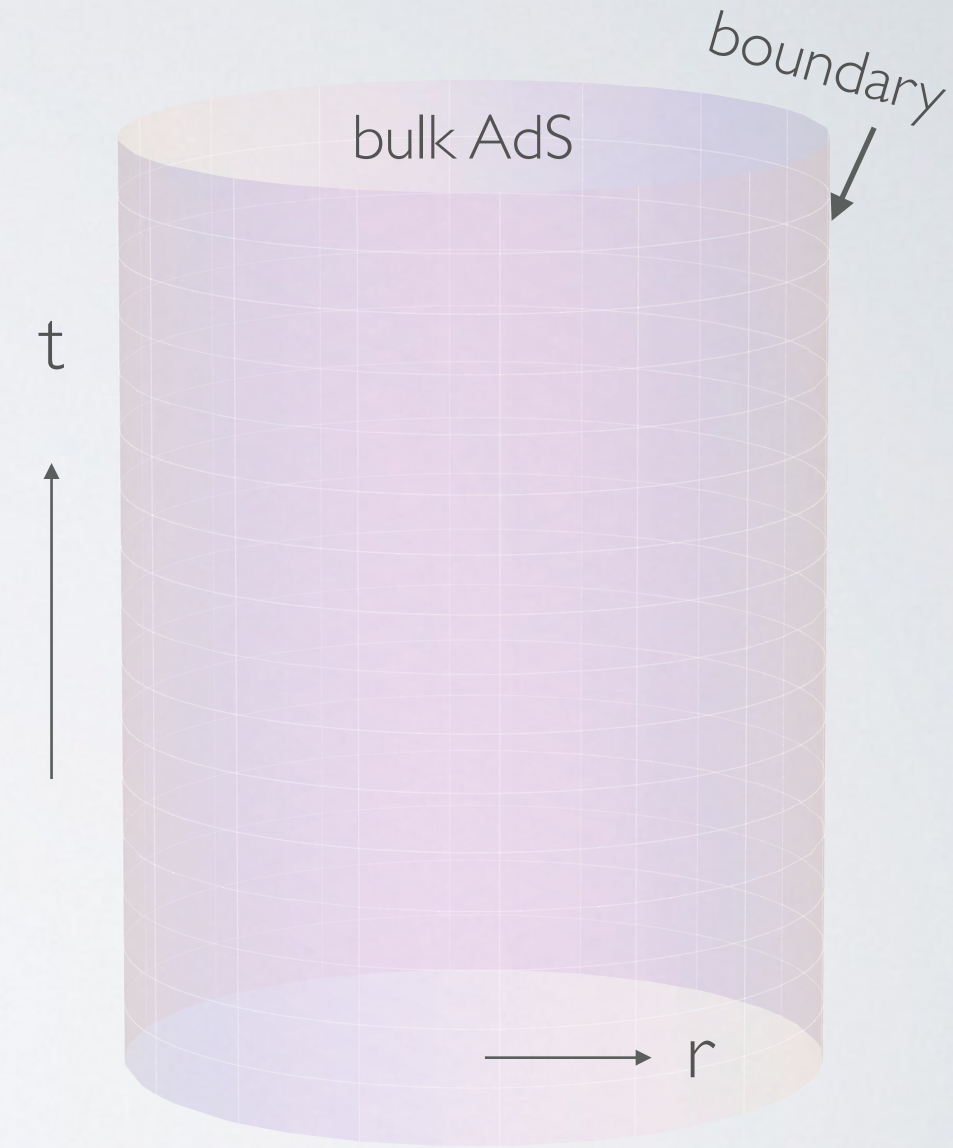
⇒ use AdS/CFT…

(recall: BH = thermal state)

[VH,Rangamani,Takayanagi; Abajo-Arrastia,Aparacio,Lopez '06; Balasubramanian et.al.; Albash et.al.; Liu&Suh; …]

Building up Vaidya-AdS

- start with vacuum state in CFT
  = pure AdS in bulk



Building up Vaidya-AdS

- start with vacuum state in CFT
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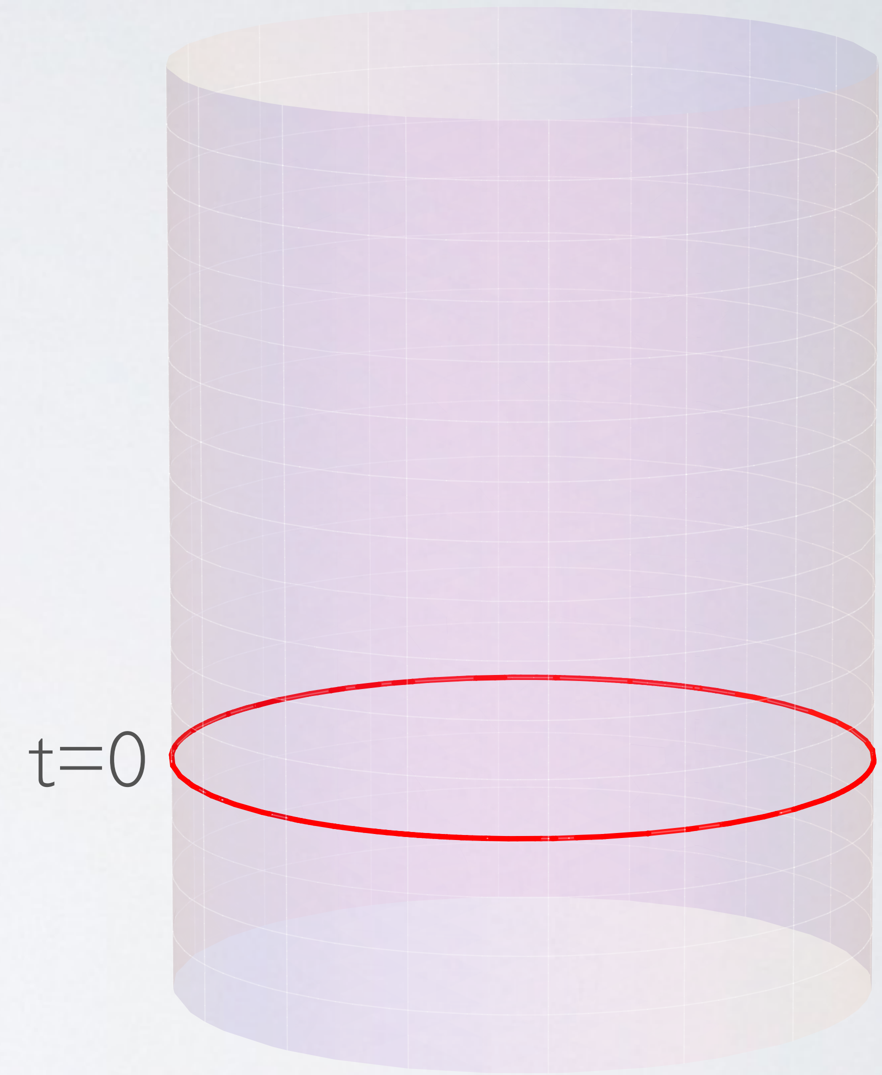
- at t=0, create a short-duration disturbance in the CFT (global quench)



Building up Vaidya-AdS

- start with vacuum state in CFT
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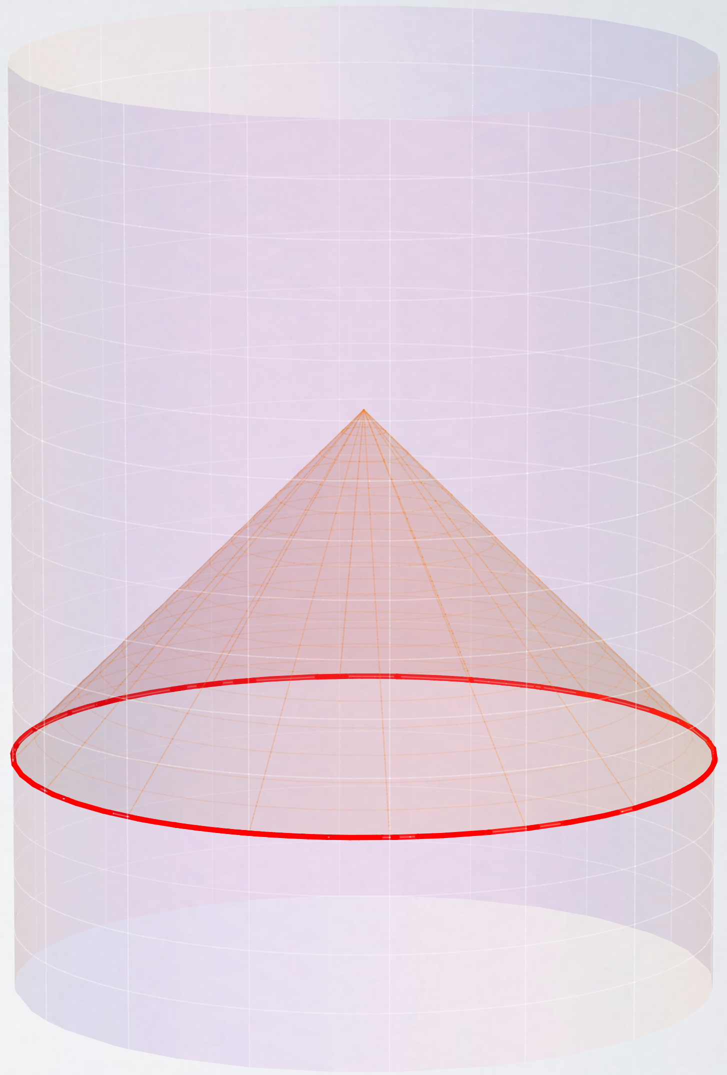
- at t=0, create a short-duration disturbance in the CFT (global quench)
- this will excite a pulse of matter (shell) in AdS which implodes under evolution



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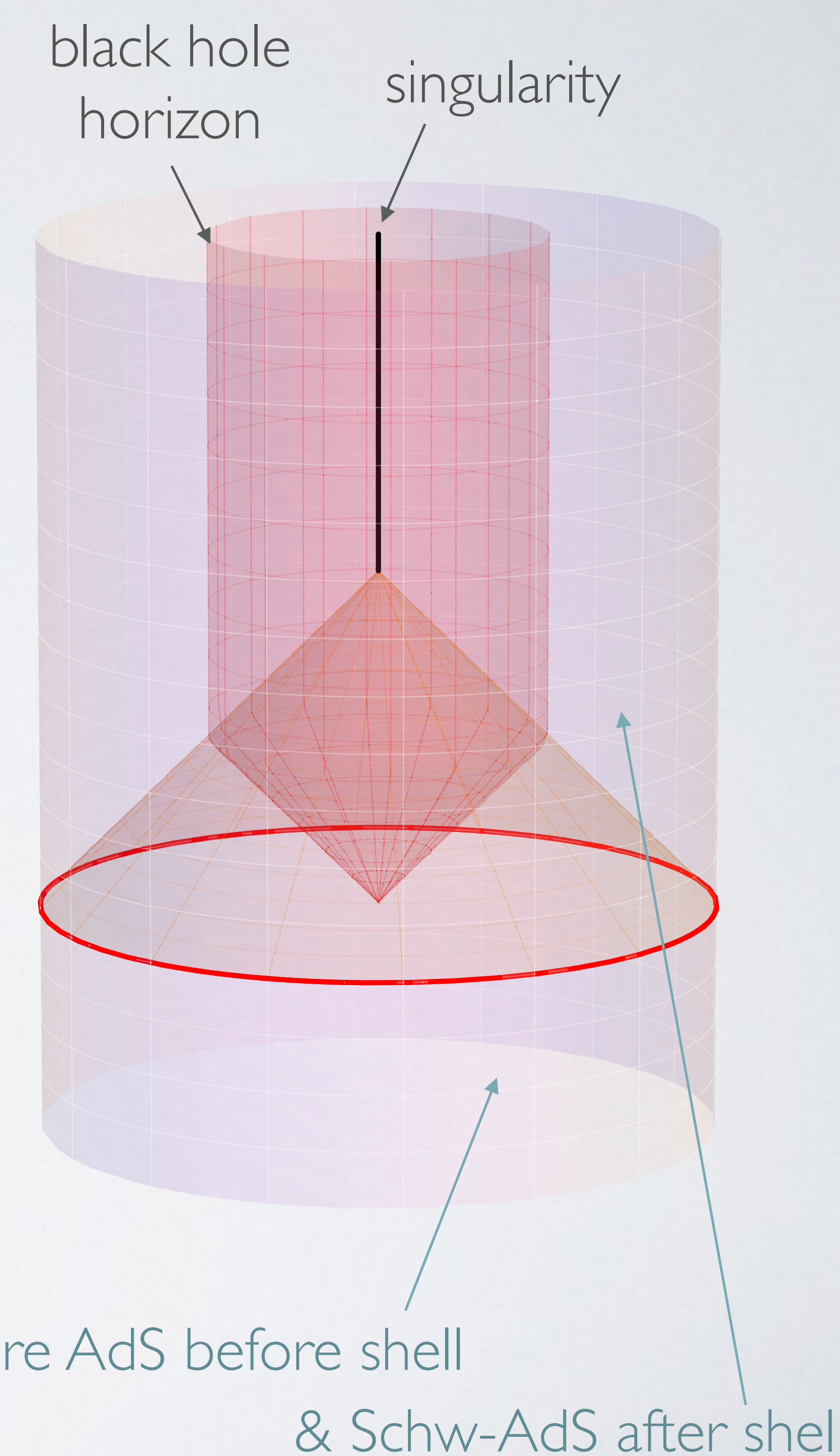
- gravitational backreaction: collapse to a black hole ⇒ CFT 'thermalizes'



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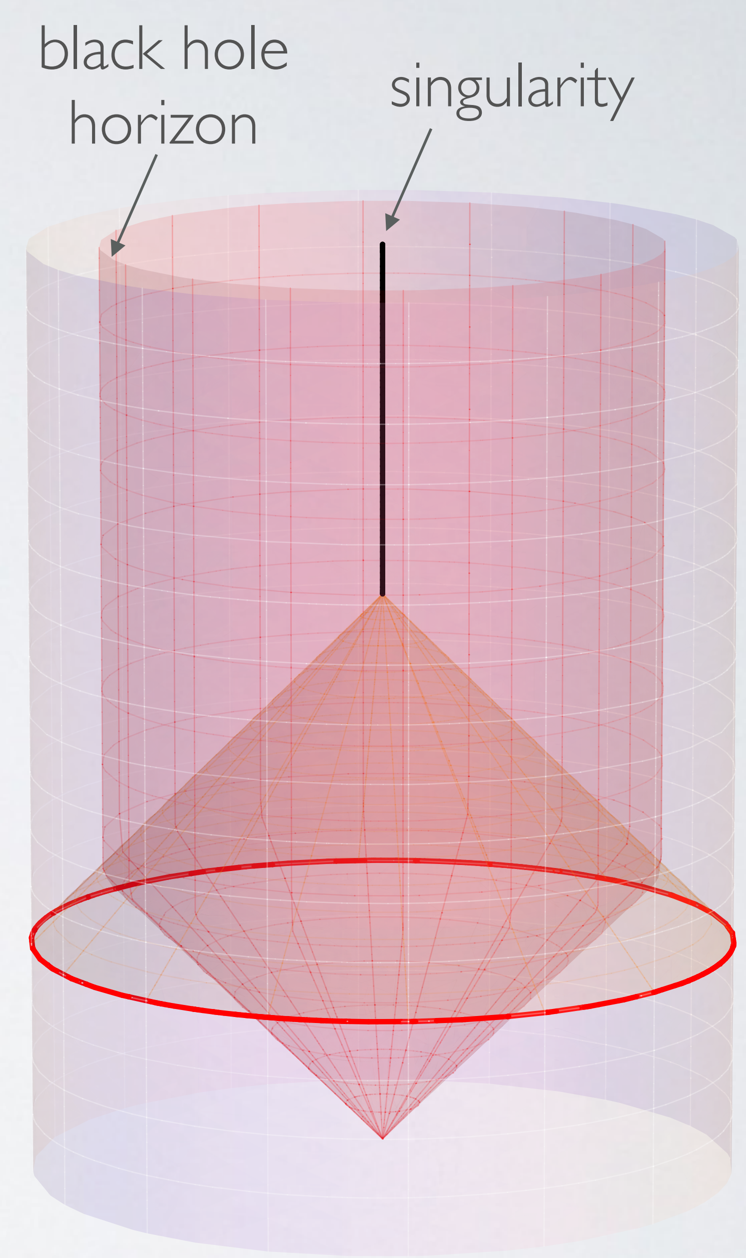
- large CFT energy ⇒ large BH



Building up Vaidya-AdS

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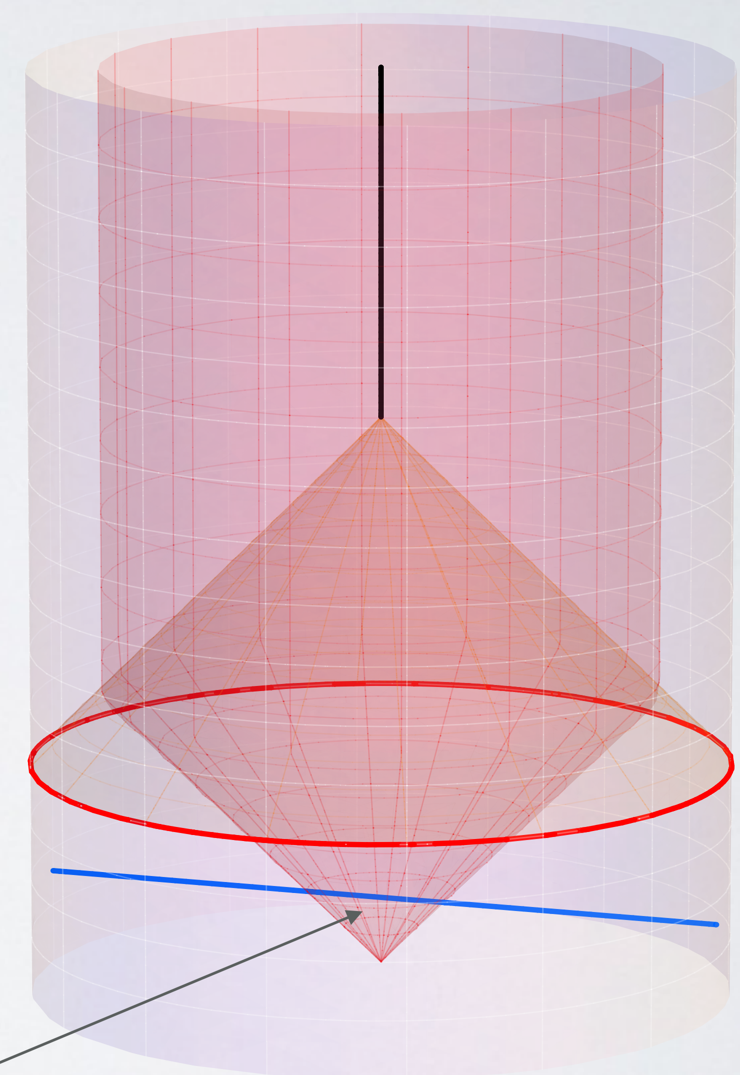
- gravitational backreaction: collapse to a black hole ⇒ CFT 'thermalizes'
- large CFT energy ⇒ large BH
- causality ⇒ geodesics (& extremal surfaces) can penetrate event horizon [VH '02]



Vaidya-AdS

Vaidya-AdS$_{d+1}$ spacetime, describing a null shell in AdS:

$$ds^2 = -f(r,v)\, dv^2 + 2\, dv\, dr + r^2 \left(d\theta^2 + \sin^2\theta\, d\Omega_{d-2}^2\right)$$

where $\quad f(r,v) = r^2 + 1 - \vartheta(v)\, m(r)$

with $\quad m(r) = \begin{cases} r_+^2 + 1 \quad , & \text{in AdS}_3 \\ \frac{r_+^2}{r^2}\,(r_+^2 + 1) \quad , & \text{in AdS}_5 \end{cases}$ $\quad$ i.e. d=2, i.e. d=4

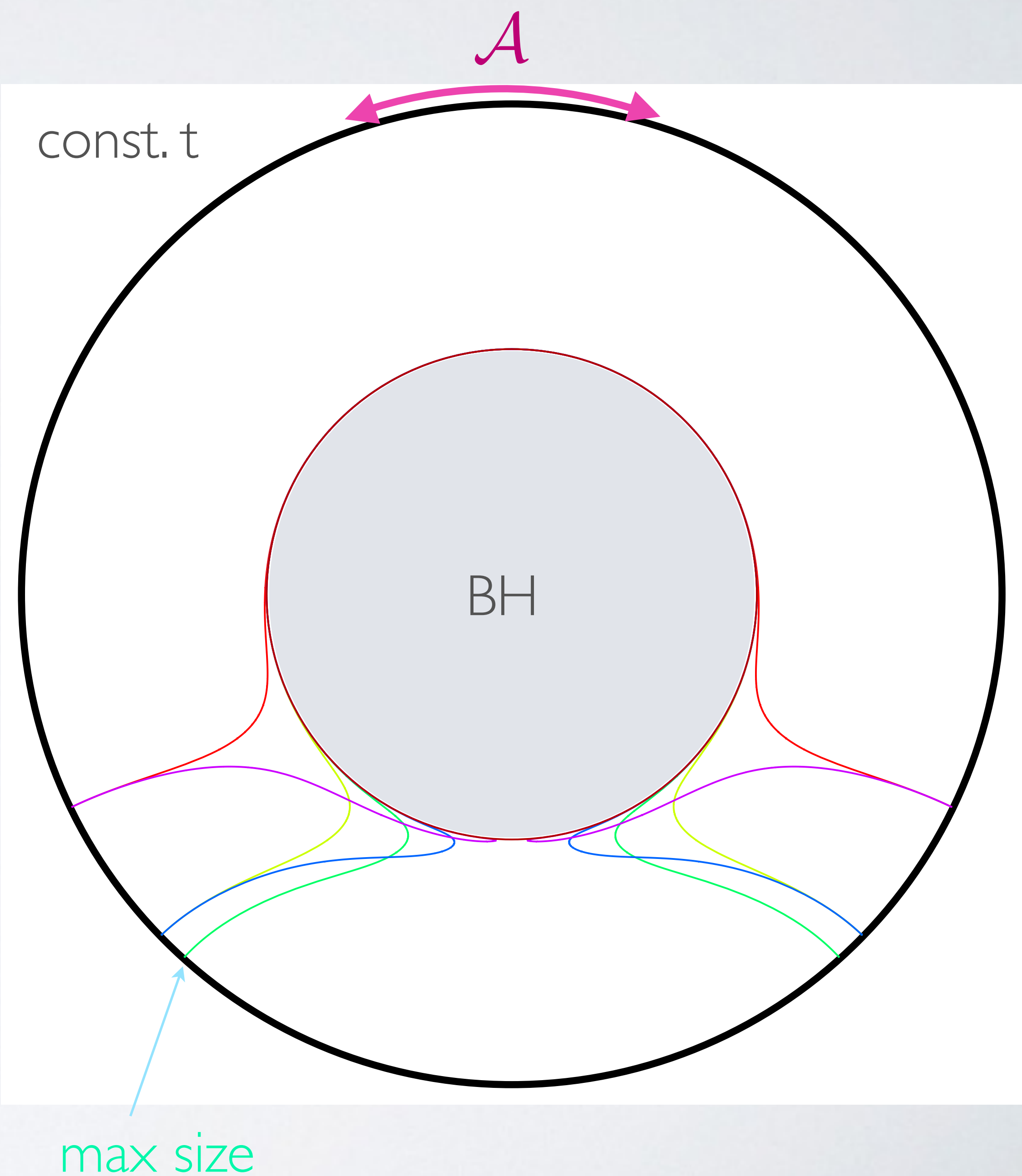
and $\quad \vartheta(v) = \begin{cases} 0 \quad , & \text{for } v < 0 \\ 1 \quad , & \text{for } v \geq 0 \end{cases}$ $\quad \rightarrow$ pure AdS, $\rightarrow$ Schw-AdS (or BTZ)

we can think of this as $\delta \to 0$ limit of smooth shell with thickness δ :

$$\vartheta(v) = \frac{1}{2}\left(\tanh\frac{v}{\delta} + 1\right)$$

Multitudes of surfaces

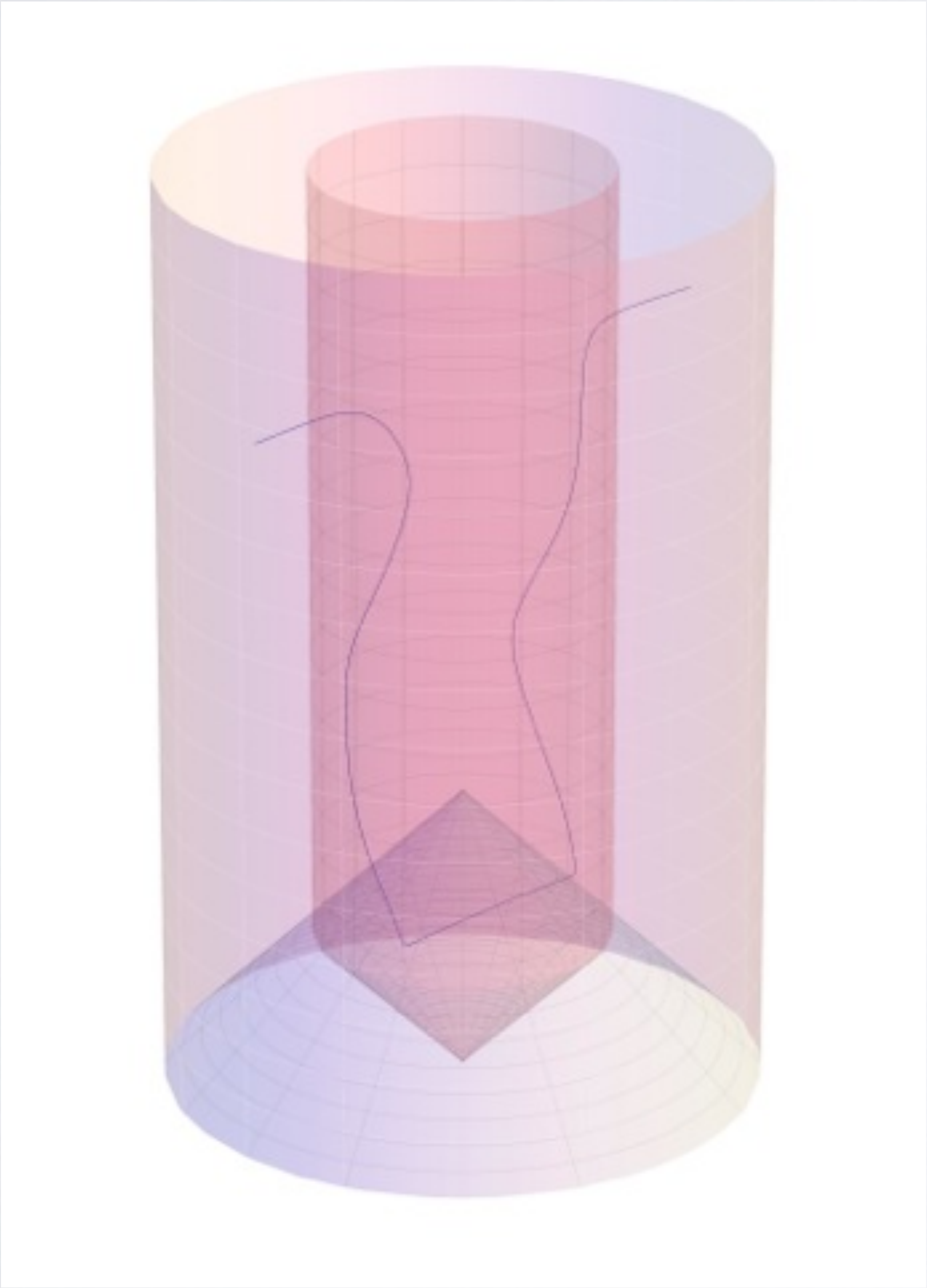
- Already for the static Schw-AdS$_{d+1}$, there is surprisingly rich structure of extremal surfaces: [VH,Maxfield,Rangamani,Tonni]



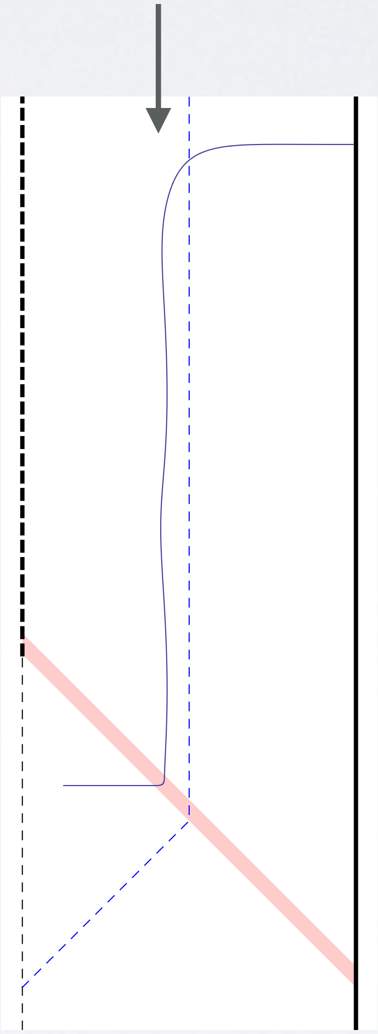
- For sufficiently small (or sufficiently large) region $\mathcal{A}$, only a single surface exists.
- For intermediate regions (shown), there exists infinite family of surfaces
- These have increasingly more intricate structure (with many folds), exhibiting a self-similar behavior.
- Recall: the nonexistence of extremal & homologous surface for large $\mathcal{A}$ is robust to deforming the state, and follows directly from causal wedge arguments.

Static surface inside BH

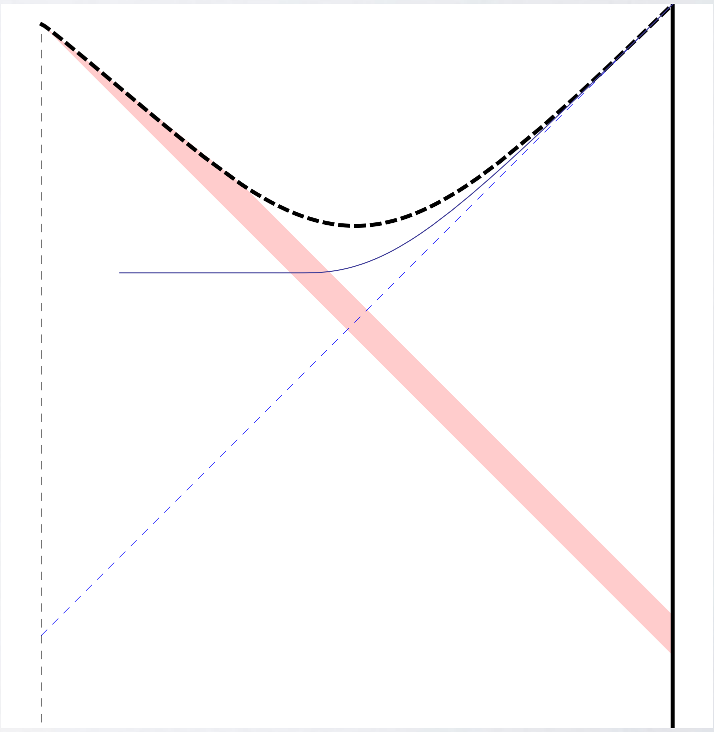
- surface can remain inside the horizon for arb. long



critical radius at which static Schw-AdS admits
a const-r extremal surface, extended in t.
[cf. Hartman & Maldacena, Liu & Suh]

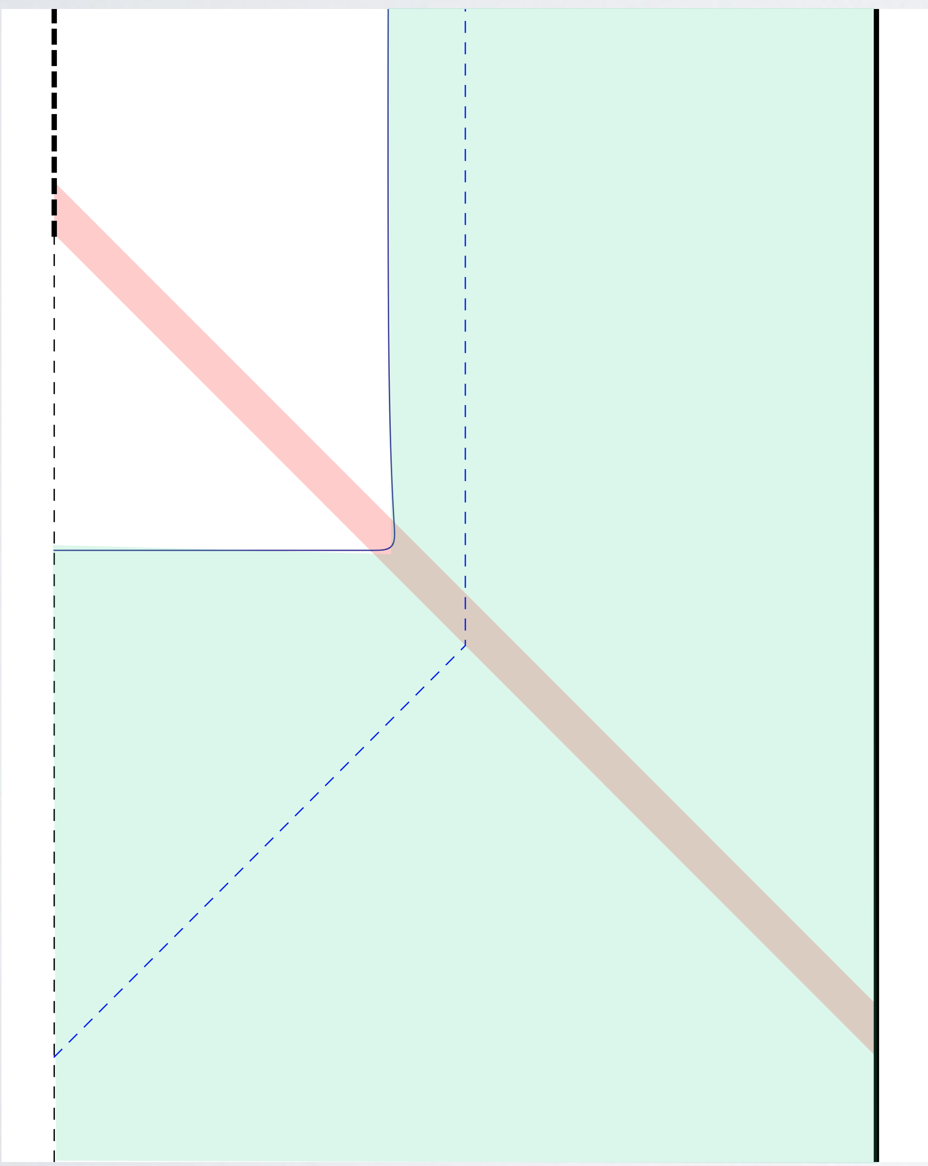


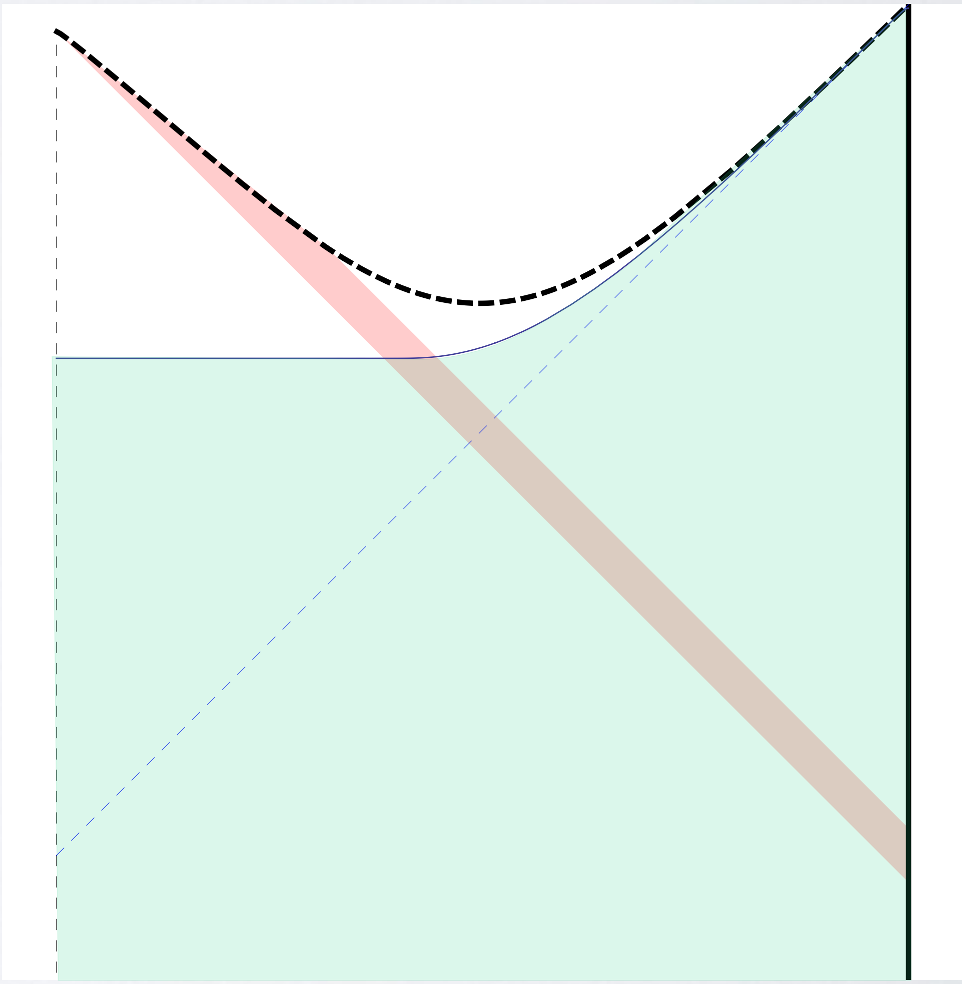
on Penrose diagram:



Region probed by such surfaces

- Any extremal surface anchored at t cannot penetrate past the critical-r surface inside the BH.
- Hence these necessarily remain bounded away from the singularity.

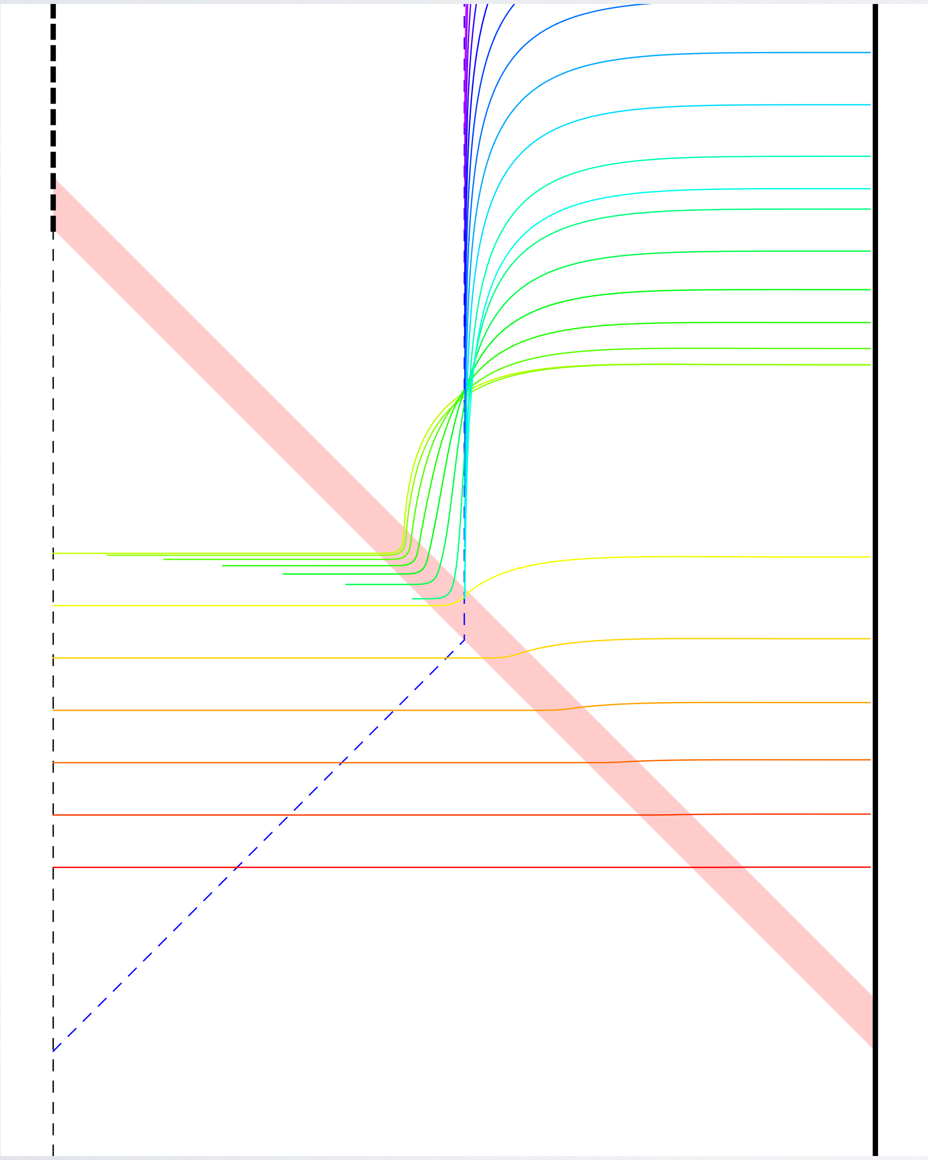


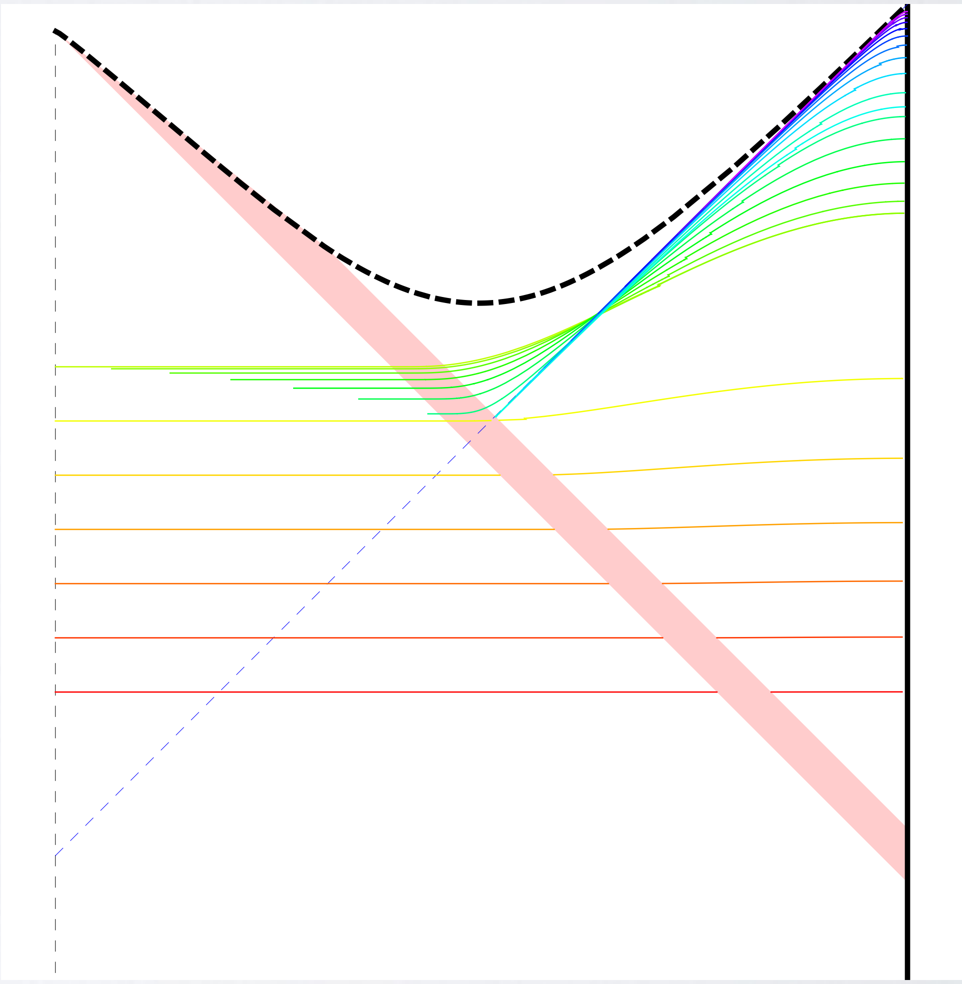


Region probed by smallest surfaces

smallest area 3-d extremal surfaces in Vaidya-AdS5 ($r_+ = 1$)

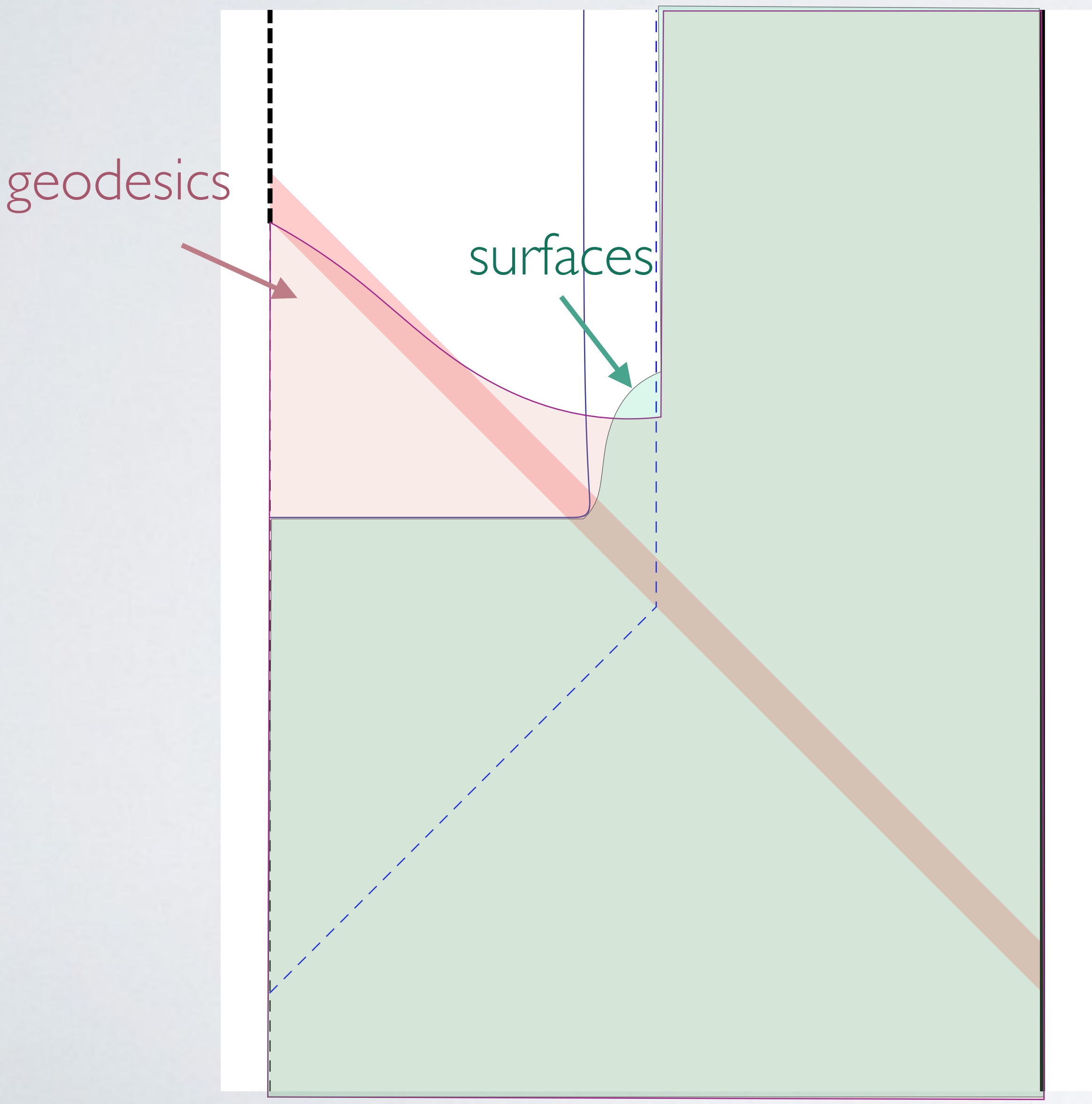
penetrate the black hole only for finite time after the shell

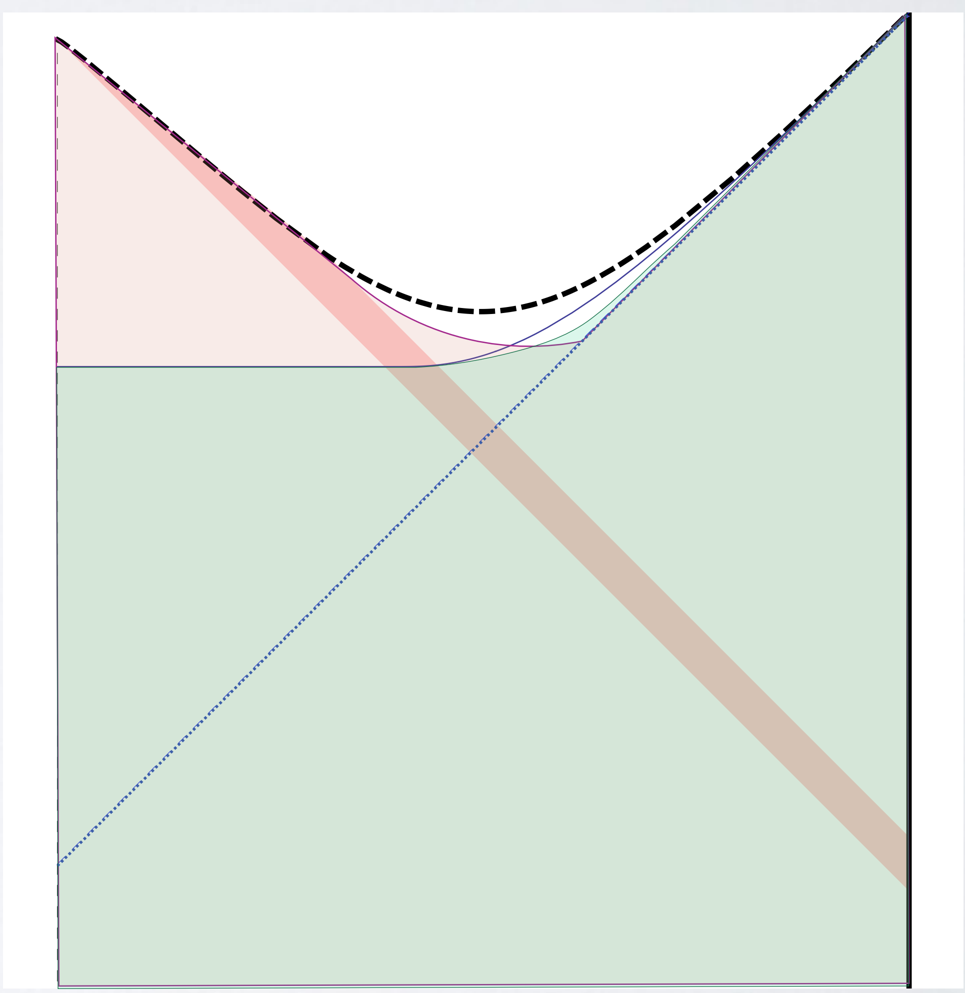




Cf. reach of 'dominant' geods vs. surfaces

shortest geodesics get closer to singularity, but
smallest area surfaces get inside BH till slightly later time.

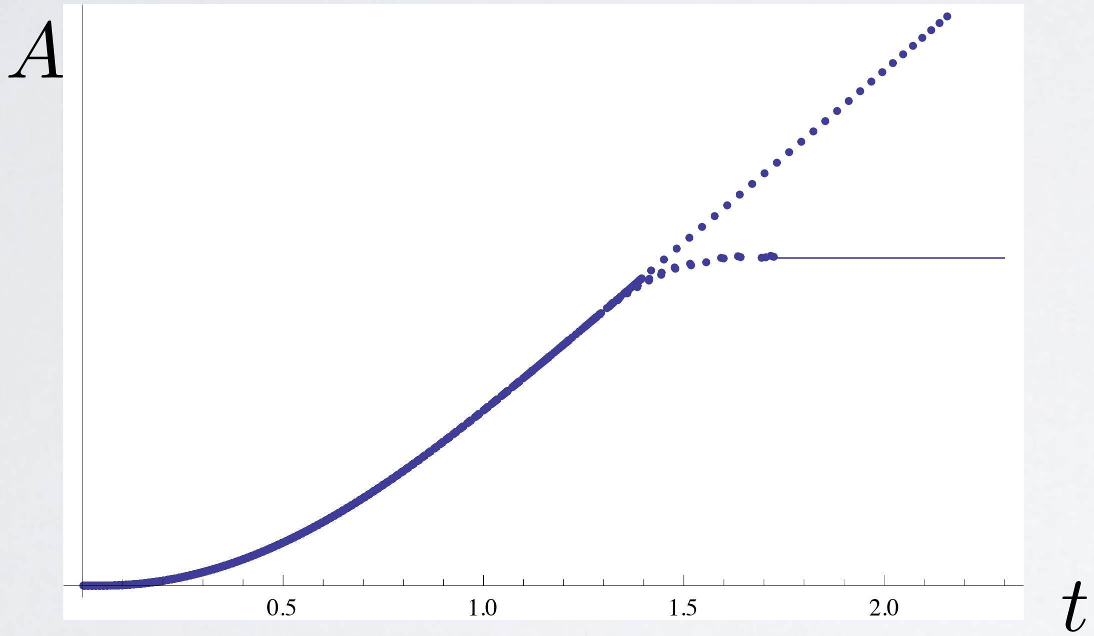




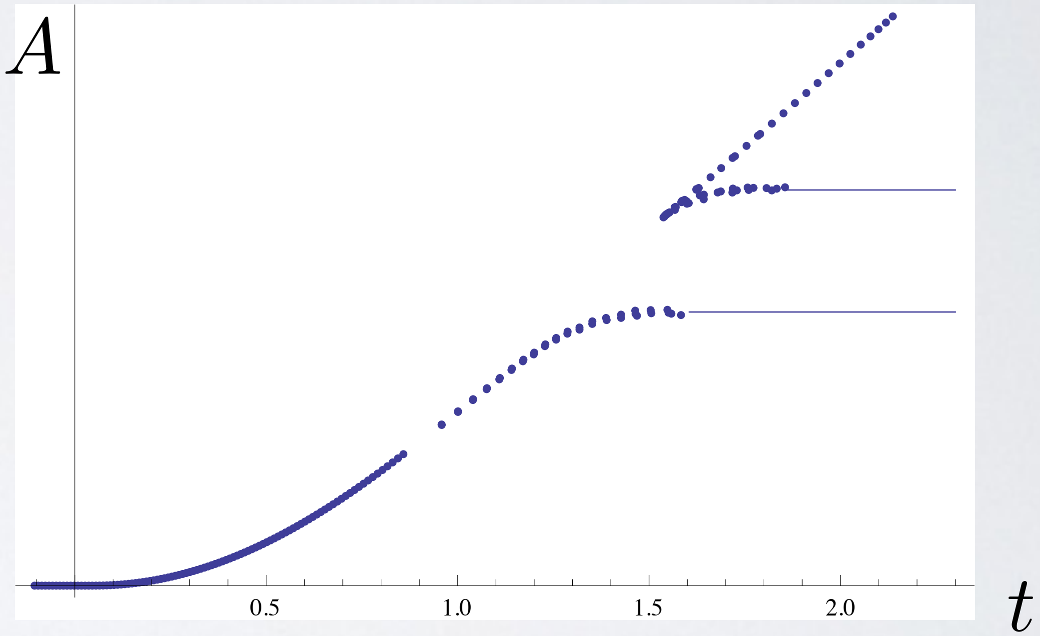
Thermalization in Vaidya-AdS5

- Thermalization via $S_{\mathcal{A}}(t)$ appears continuous and monotonic

hemispherical region



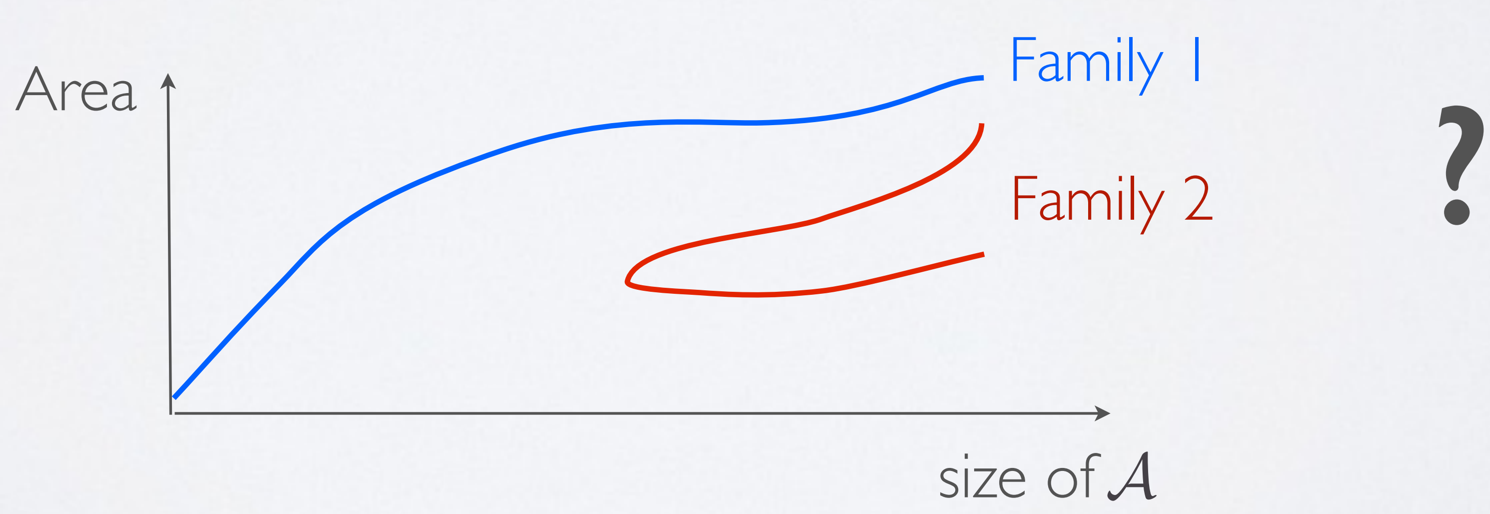
sub-hemispherical region



- Aside: Puzzle: Was this guaranteed?

Continuity of entanglement entropy?

- RT prescription (EE given by area of *minimal* surface) naturally implies continuity [VH, Maxfield, Rangamani, Tonni; Headrick]
- However, open question whether continuity is upheld by HRT (EE given by area of *extremal* surface).
- New families of extremal surfaces can appear, but is the following situation possible:



Summary

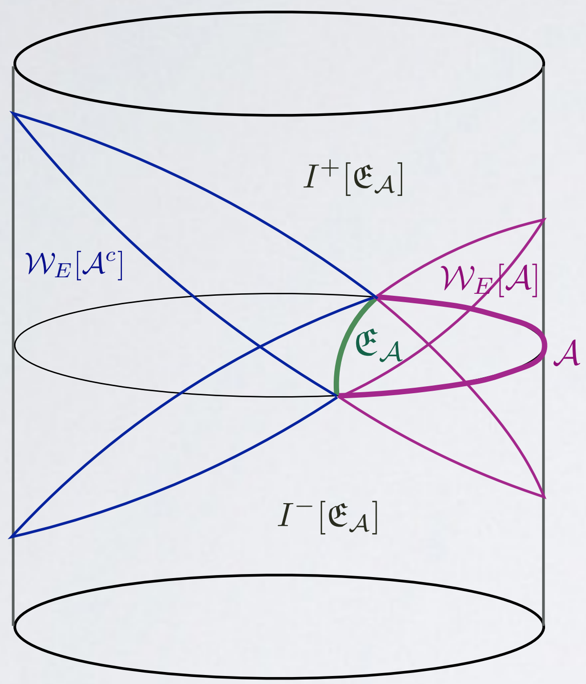
- General covariance is a powerful guiding principle for constructing physically interesting quantities.
- We have seen several distinct causal sets:
  - Causal wedge, Entanglement wedge, Causal shadow

- HRT is consistent with causality
- Entanglement wedge is most natural bulk dual of $\rho_{\mathcal{A}}$

- Looking inside black holes:
  - Extremal surfaces can penetrate into time-evolving (e.g. collapsing) BH, but they stay away from the curvature singularity…
  - Nevertheless, the entanglement wedge can reach up to the singularity.

Open Questions

- (How) can we reconstruct the spacetime metric inside the entire entanglement wedge from $\rho_{\mathcal{A}}$?
  ("easy" in regions reached by co-dim.2 extremal surfaces anchored in $D[\mathcal{A}]$ but these don't span the full $\mathcal{W}_E[\mathcal{A}]\dots$)
- Precise formulation of homology constraint for HRT?
- Proof of HRT…
- CFT dual of causal wedge & causal holographic information?


$I^+[\mathfrak{E}_\mathcal{A}]$
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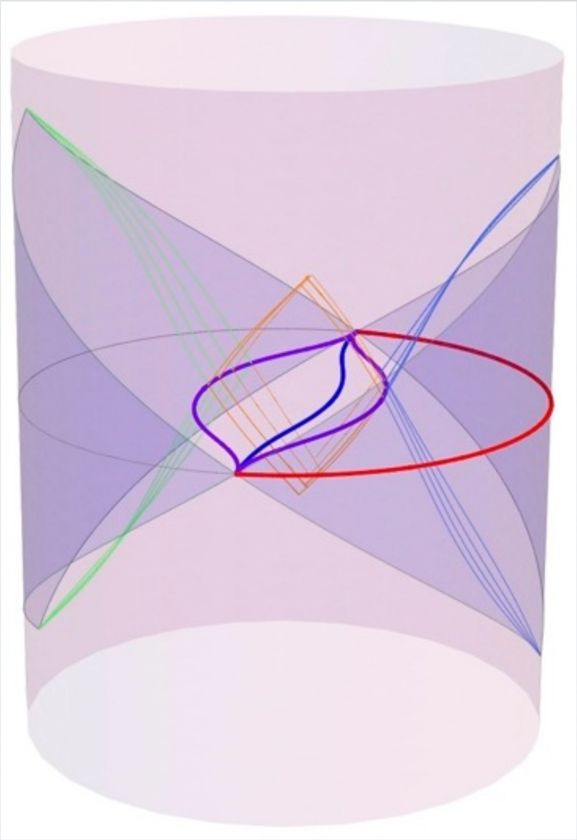
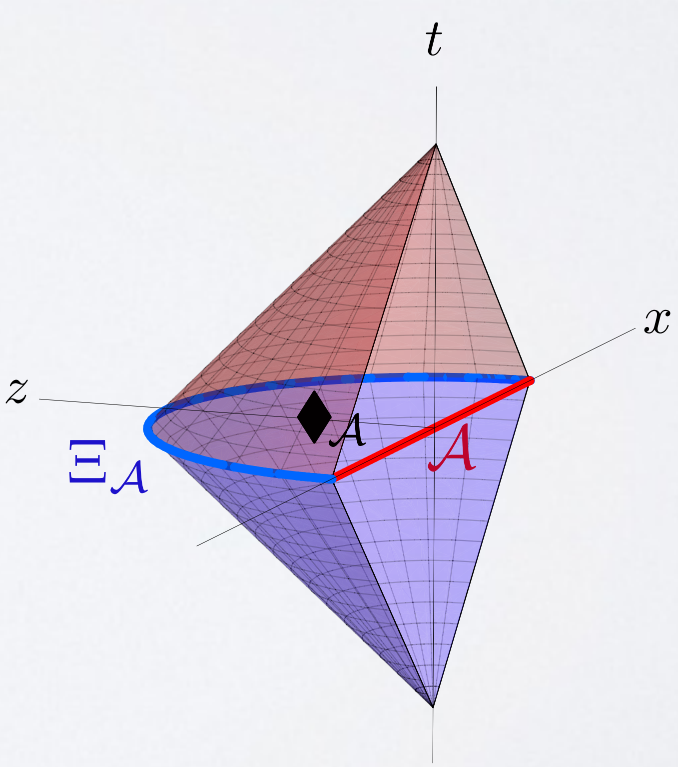
$\mathcal{A}$
$I^-[\mathfrak{E}_\mathcal{A}]$



Thank you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