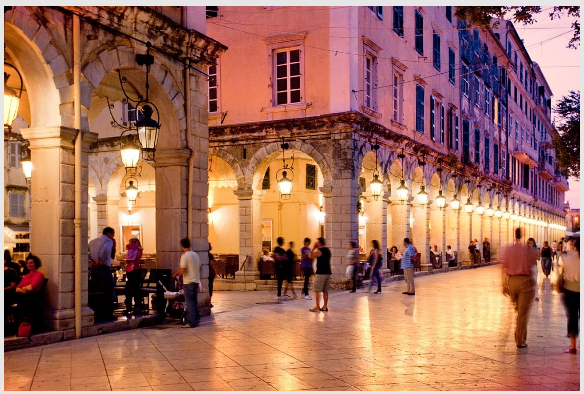
Precision phenomenology and differential equations approach to master integrals



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In collaboration with
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September 3 - 14, 2014

Outline

- First part (introduction for non-specialists)

  - Motivation for precision phenomenology

  - Loop calculations and OPP

  - Master Integrals (MI)

  References: “One-loop calculations in quantum field theory” Ellis,Kunszt, Melnikov, Zanderighi 2012 (arXiv:1105.4319); “Scattering Amplitudes in Gauge Theories” J.M. Henn, J.C. Plefka; “Feynman Integral Calculus” V.A. Smirnov

- Second part (more advanced)

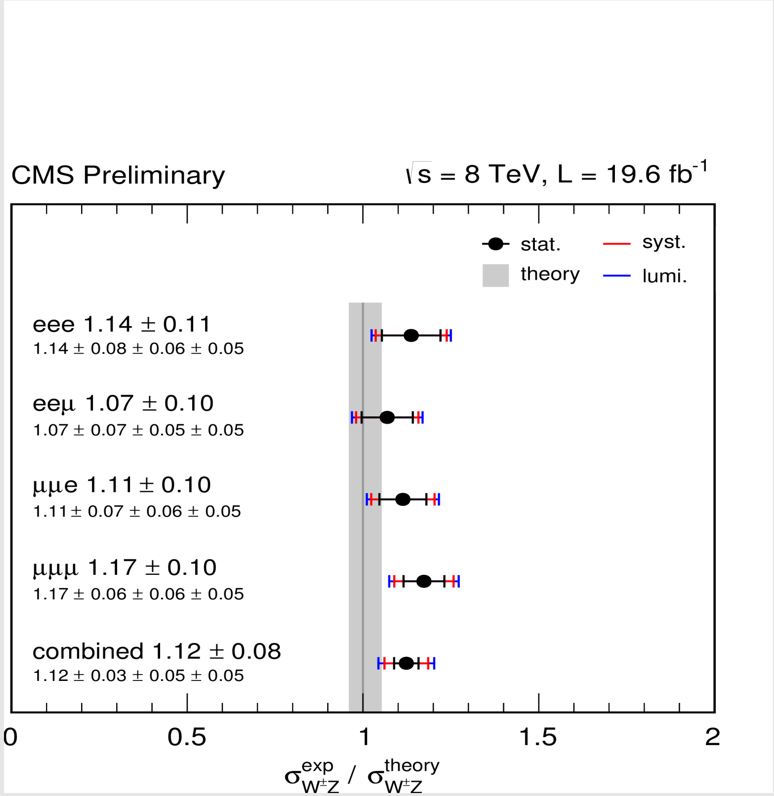
  - Differential Equations (DE) approach to MI

  - The recent x-DE method by C. Papadopoulos

  - Results and summary

  References: arXiv (1401.6057, 1407.0046)

Motivation (1)



https://twiki.cern.ch/twiki/pub/CMSPublic/PhysicsResultsSMP12006/ratioNLO_MET30_WInclusive.pdf

- Proton-proton collision into WZ
- There is some tension between theory and data: all measured cross sections are bigger than the predicted ones
- Possible explanations:

  1) Theoretical side: standard model not ok? New physics? ...

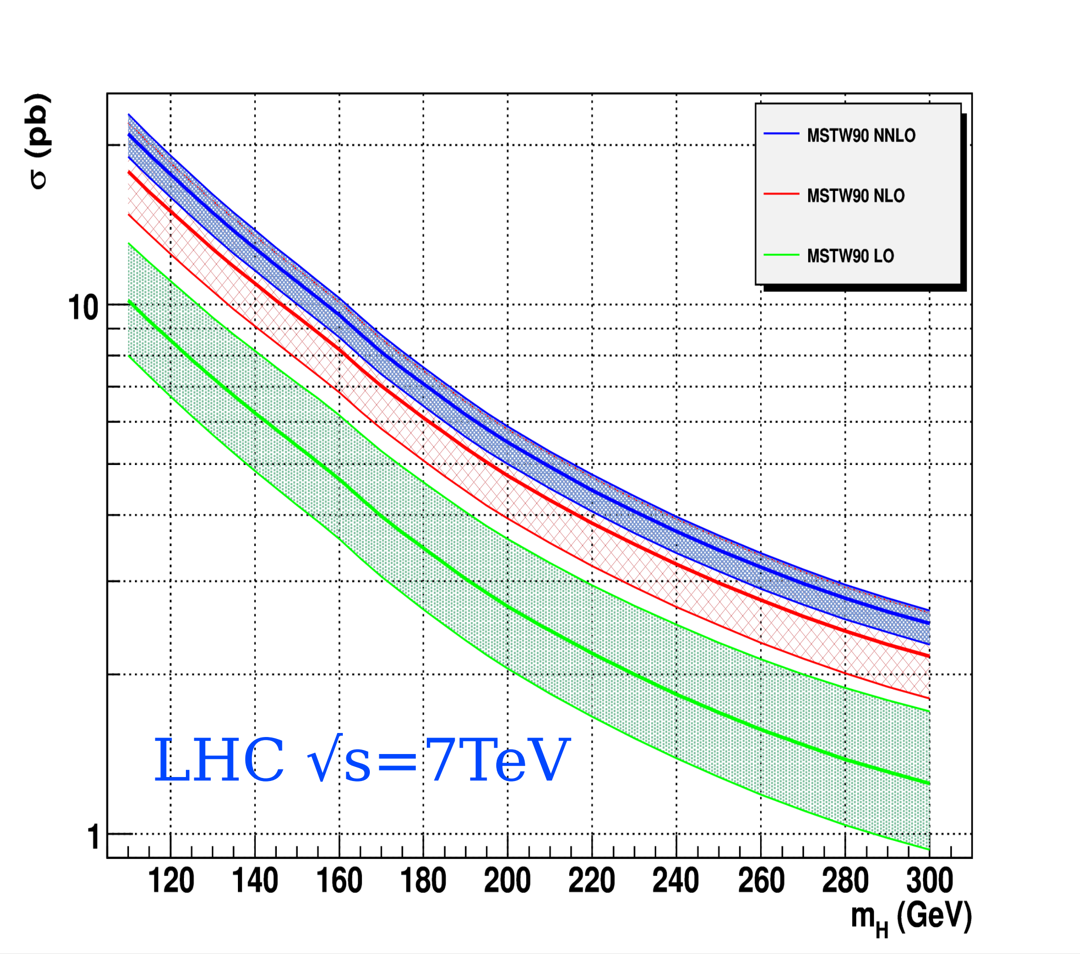
  2) Experimental side: Statistical fluctuations? Inadequate detector simulations? Errors in the analysis? ...

  3) Phenomenological side: predictions from standard model not accurate enough? “Only” NLO results?...

NLO calculations already involves very complicated calculations!!!
Very recently NNLO results came for ZZ and WW production, ZW expected...
[Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs; Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi]

Motivation (2)

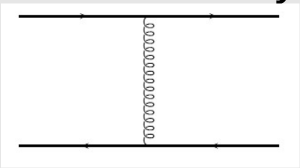
- Higgs boson production in the gluon-gluon fusion channel
- LO prediction is providing only order of magnitude estimation
- LO to NLO ~80-100% and no overlapping
- NLO to NNLO ~25% and overlapping
- NNNLO work in progress...



Accurate phenomenological calculations are mandatory in LHC era!!!

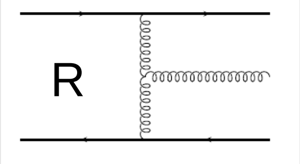
Definition of LO, NLO NNLO

- LO: mostly tree level Feynman diagrams (algebraic calculations)



Some processes are 1-loop already at LO, e.g. gg→H

- NLO (two contributions): the real part with one extra emitted parton (“more” algebra) and the virtual part involving one loop integration



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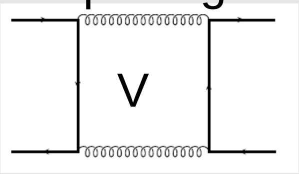
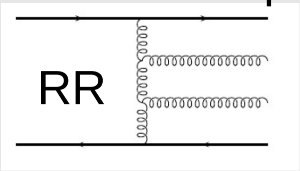


- NNLO (three contributions): double virtual corrections with two extra partons (“much more” algebra), the real-virtual contribution involving one extra parton and one loop integral and the two loop contribution (double integration)



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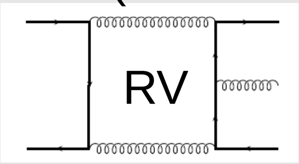

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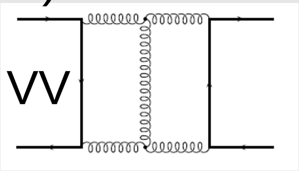


- Real corrections are in principle doable for any number of final state particles if enough CPU power and good algorithms are available (the algebraic complication ~ factorial of the number of particles).
- One and two loop contributions (subject of this talk) need the calculation of difficult integrals. One loop is now fully solved, but two loop is still an open problem

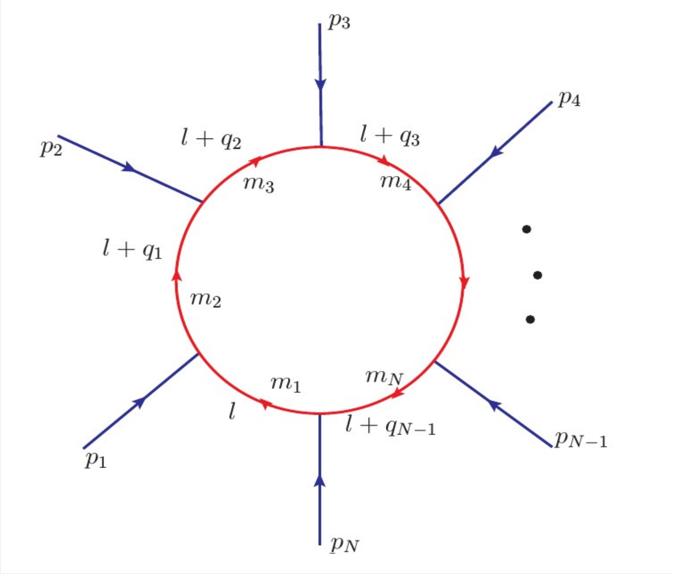
One loop integrals

- In renormalizable gauge theories (not only Standard Model!) we always have:

  - propagators $\text{scalar} \sim \frac{1}{q^2 - m^2} \quad \text{fermion} \sim \frac{\not{q} + m}{q^2 - m^2} \quad \text{vector} \sim \frac{g^{\mu\nu} + \ldots}{q^2 - m^2}$

  - couplings $\text{triple-gauge} \sim f^{abc}(g^{\mu\nu}(p_1 - p_2)^\rho + \ldots) \quad \text{fermion-gauge} \sim \gamma^{\mu\nu} T^a_{ij} \ldots$

- In any loop integral the complicated tensor structure is contained in the numerator $\mathcal{N}(l)$ (Dirac matrices, color structure, helicities ...). The denominator is “just” a product of scalars.



The one-loop integral with *N* external particles can be written as *N*-propagator tensor integral:

$$I_N \sim \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{\mathcal{N}(l)}{((l+q_0)^2 - m_1^2)((l+q_1)^2 - m_2^2)\ldots.((l+q_{N-1})^2 - m_N^2)}$$

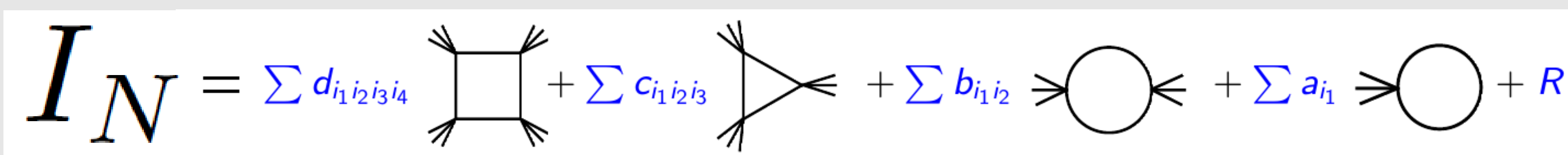
We work in D=4-2ε space-time dimensions. The limit ε→0 is taken at the end of the calculation, after infrared divergences are cancelled

OPP method for amplitudes at NLO

- Any one loop (tensor) integral can be reduced to sum of easier one loop integrals

  [G. Passarino and M. J. G. Veltman, Nucl. Phys. B 160 (1979) 151.; Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, Nucl. Phys. B 425 (1994) 217]

- In four dimensions there are at most 4 linearly independent external momenta



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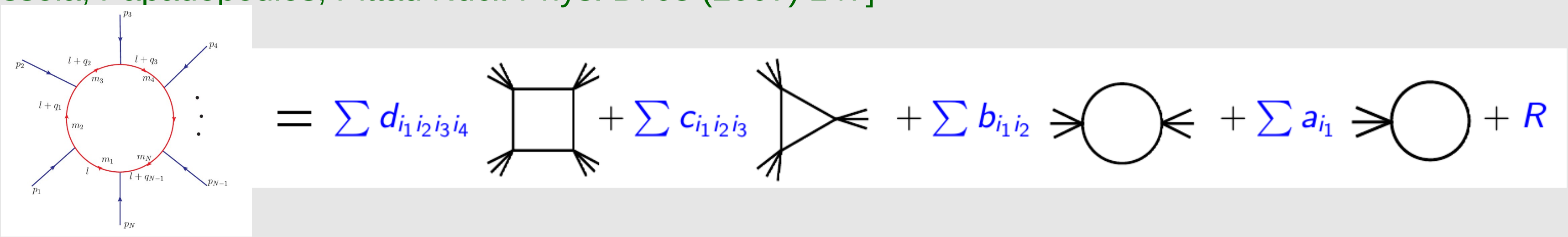
- In four dimensions there are at most 4 linearly independent external momenta
- The computation can be done directly for one-loop amplitudes (they are sum of all Feynman integrals)
- OPP method: any one-loop amplitude can be decomposed as sum of box, triangle, bubble and tadpole scalar MI

  [Ossola, Papadopoulos, Pittau Nucl. Phys. B763 (2007) 147]



- The MIs have to be calculated only once. They are valid for all processes and any gauge theory!
- The full information of the collision process (tensor structure and momentum dependence) is now in the coefficients d,c,b,a,R. Coefficients are all rational functions, calculated algebraically from products of trees using (generalized) unitarity

NLO revolution: example of two to n jets calculation

- Few years ago: draw all the Feynman diagrams, calculate all the integrals (n+3 denominators and tensor numerators), sum all the results...
- Today: work directly at the amplitude level and calculate only the coefficients d,c,b,a,R
- pp→#jets @NLO

  -2 jet production [K.Ellis, Z.Kunszt, D.Soper (1992); Giele, Glover, Kosower (1993)]

  -3 jet production [Trocsanyi (1996); Giele, Kilgore (1997) (gluon only); Nagy (2002)]

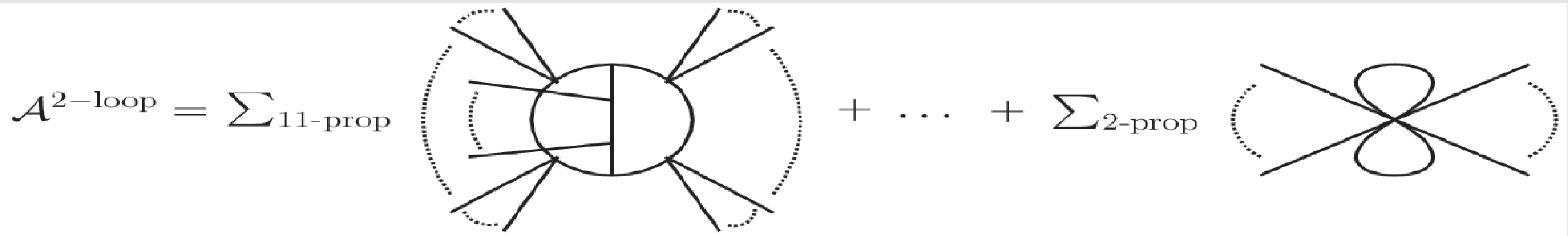
  -4 jet production [Z.Bern et al. (2011); Badger et al (2012)]

  -5 jet production [Badger et al (2013)]

Many numerical NLO tools: Formcalc [Hahn ’99], Golem (PV) [Binoth, Cullen et al ’08], Rocket [Ellis, Giele et al ’09], NJet [Badger, Biederman, Uwer & Yundin ’12], Blackhat [Berger, Bern, Dixon et al ’12], Helac-NLO [Bevilacqua, Czakon et al ’12], MCFM, GoSam, OpenLoops, Recola, MadGolem, MadLoop, MadFKS, aMC@NLO, ...

NNLO revolution: coming soon!

- Amplitude decomposition at two loops



- Finite basis of MI exists, but now MI may contain loop-dependent numerators as well (tensor integrals)
- By now reduction is substantially understood for 2-loop integrals (see Mastrolia's talk)

  In N=4 SYM [Bern, Carrasco, Johansson et al. ’09-’12], Maximal unitarity cuts in general QFT’s [Johansson, Kosower, Larsen et al. ’12-’13], Integrand reduction with polynomial division in general QFT’s [Ossola & Mastrolia ’11, Zhang ’12, Badger, Frellesvig & Zhang ’12-’13, Mastrolia, Mirabella, Ossola & Peraro ’12-’13, Kleis, Malamos, Papadopoulos & Verheynen ’12]

- Missing ingredient: library of Master integrals

Methods for calculating MI

- Rewriting integrals in different representations:

  - Parametric: Feynman/Schwinger/alpha parameters

  - Mellin-Barnes and nested sums [Bergere & Lam ’74, Ussyukina ’75,V. Smirnov ’99, Tausk ’99, Vermaseren ’99, Blumlein et al ’99,...]

Calculating integrals by parameterizations

- Relatively simple MI can be calculated by transforming the integrand
- Examples:

  - Schwinger parameterization

$$\frac{1}{A} = -i\int_0^\infty du\; e^{iuA} \qquad \frac{1}{A^n} = \frac{1}{(n-1)!}\int_0^\infty du\; u^{n-1}e^{-uA}$$

$$\int \frac{dp}{A(p)^n} = \frac{1}{\Gamma(n)}\int dp\int_0^\infty du\; u^{n-1}e^{-uA(p)} = \frac{1}{\Gamma(n)}\int_0^\infty du\; u^{n-1}\int dp\; e^{-uA(p)}$$

  - Feynman parameterization

$$\frac{1}{AB} = \frac{1}{A-B}\left(\frac{1}{B}-\frac{1}{A}\right) = \frac{1}{A-B}\int_B^A \frac{dz}{z^2}$$

$$u = \frac{z-B}{A-B} \quad \Rightarrow \quad \frac{1}{AB} = \int_0^1 \frac{du}{\left(uA+(1-u)B\right)^2}$$

$$\int \frac{dp}{A(p)B(p)} = \int dp\int_0^1 \frac{du}{\left(uA(p)+(1-u)B(p)\right)^2} = \int_0^1 du\int \frac{dp}{\left(uA(p)+(1-u)B(p)\right)^2}$$

$$\int \frac{dp}{A_1^{\alpha_1}(p)\dots A_n^{\alpha_n}(p)} = \frac{\Gamma(\alpha_1+\cdots+\alpha_n)}{\Gamma(\alpha_1)\dots\Gamma(\alpha_n)}\int_0^1 du_1\dots\int_0^1 du_n\int dp\frac{\delta\left(\sum u_i - 1\right)u_1^{\alpha_1-1}\dots u_n^{\alpha_n}}{\left(\sum u_i A_i(p)\right)^{\sum \alpha_i}}$$

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- Using relations and/or (cut) identities:

  - Integration by parts relations (IBP) [F. V. Tkachov, Physics Letters B100, 65 (1981); K. Chetyrkin and F. Tkachev, Nucl. Phys. B192, 159 (1981)]

  - Dimensional shifting relations [Tarasov ’96, Lee ’10, Lee,V. Smirnov & A. Smirnov ’10]

  - Loop-tree duality [Catani, Gleisberg, Krauss, Rodrigo and Winter ’08, Bierenbaum, Catani, Draggiotis, Rodrigo et al ’10-’14]

  - Integral reconstruction with cuts and co-products [Abreu, Britto, Duhr & Gardi ’14]

Integrations by part (IBP) identities
~ derivatives wrt internal momenta

- n-loop Feynman integral with m external momenta:

$$I(p_1,\ldots,p_m) \equiv \int_{k_1,\ldots,k_n} d^d k_1 \ldots d^d k_n \prod_i \frac{(p\cdot k)_i^{b_i}}{(q_i^2+m_i^2)^{a_i}}$$

- Integration by parts identities are generated by derivatives of internal momenta

$$\int_{k_1,\ldots,k_n} d^d k_1 \ldots d^d k_n \frac{\partial}{\partial k_j^\mu}\left(v^\mu \prod_i \frac{(p\cdot k)_i^{b_i}}{(q_i^2+m_i^2)^{a_i}}\right) = 0$$

- Simple example

$$I(a) \equiv \int d^d k \frac{1}{(k^2+m^2)^a}$$

If $I(a)$ is known, then any $I(a+n)$ is easily calculable

$$\begin{aligned} 0 &= \int d^d k \partial_k k \frac{1}{(k^2+m^2)^a} \\ &= \int d^d k \left\{ d\frac{1}{(k^2+m^2)^a} - 2a\frac{k^2+m^2-m^2}{(k^2+m^2)^{a+1}} \right\} \\ &= I(a)(d-2a) + 2am^2 I(a+1) \end{aligned}$$

$$\Rightarrow I(a+1) = -\frac{d-2a}{2am^2} I(a), \quad a \geq 1.$$

- Another example

$$I_{a,b,c,d,e} \equiv \int_{p,q} \frac{1}{(p^2)^a((p-k)^2)^b(q^2)^c((q-k)^2)^d((p-q)^2)^e} \Rightarrow \frac{1}{2}(4-d)I_{1,1,1,1,1} = I_{1,1,2,1,0} - I_{1,1,2,0,1}$$

Methods for calculating MI

- Rewriting integrals in different representations:

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- As solutions of differential equations (DE):

  - Differentiation w.r.t. invariants [Kotikov ’91, Remiddi ’97, Caffo, Czyz & Remiddi ’98, Gehrmann & Remiddi ’00, Henn ’13, Henn, Smirnov et al ’13-’14]

  - Differentiation w.r.t. externally introduced parameter (method of current talk) [Papadopoulos ’14]

- Many more: dispersion relations, dualities, ...

Differential equation method for MI

~ derivatives wrt external momenta

- Example: integration of the following MI: $I_{a,b}(p) = \int d^d k \frac{1}{(k^2)^a((k-p)^2)^b}$
- Construct the diff. operator $$p^2 \frac{\partial^2}{\partial p^2} I_{1,1}(p) = \frac{1}{2} p^\mu \frac{\partial}{\partial p^\mu} I_{1,1}(p)$$
- Apply the derivative on r.h.s. $$= \frac{1}{2}\left(-I_{2,0}(p) + p^2 I_{1,2}(p) + I_{1,1}(p)\right)$$
- Reduce back to MI with IBP identities $$= \frac{d-4}{2} I_{1,1}(p)$$
- We obtain the differential equation:

$$\frac{\partial^2}{\partial p^2} I_{1,1}(p) = \frac{d-4}{2p^2} I_{1,1}(p)$$

We map the problem from an integral to a differential equation

[Kotikov ’91, Remiddi ’97,Caffo, Czyz & Remiddi ’98,Gehrmann & Remiddi ’00]

Recent development of DE method

- In general we can calculate derivatives wrt external Lorentz invariant s

  (In general) differential equations are non homogeneous, they involve several MI

$$\frac{\partial}{\partial s_k} I_i = a_i I_i + \sum_{j \neq i} a_j I_j$$

- System of DE to be solved → matrix form (ε dependent, where d=4-2ε)

$$\frac{\partial}{\partial s_k} \overrightarrow{I} = M_k(s_k, \epsilon) \overrightarrow{I}$$

- Conjecture: the ε dependence can be extracted from the matrix by rotation [Henn '13]

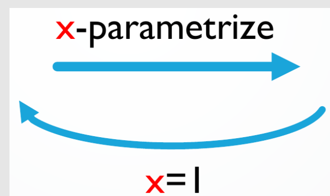
$$\frac{\partial}{\partial s_k} \overrightarrow{I_i'} = \epsilon M_k'(s_k) \overrightarrow{I_i'}$$

- If set of invariants s is correct, the system can be solved by Goncharov Polylogarithm. Boundary condition found (among other ways) by solving DE's in other invariants
- Several new results at two loops [Henn, Smirnov, Smirnov '13; Caola et al '14]
- Further recent developments on the topic (see Mastrolia's talk [Di Vita et al '14])

x-DE parameterization approach

[Papadopoulos '14]

- Introduce extra parameter x in the denominators of loop integral
- x-parameter describes off-shellness of (some) external legs
- External legs p_i , p_j



$$p_i + (1-x)q_i$$
$$x p_j$$

- Now the MI are dependent on the external parameter x
- New system of differential equations

$$\frac{\partial}{\partial x}\overrightarrow{I}(x) = M(x,\epsilon)\overrightarrow{I}(x)$$

Bottom-up approach

- Since the DE system is triangular: derivative of MI with m propagators ($I^{(m)}$) is always sum of MI having m or less propagators → we can adopt a bottom-up strategy to integrate the DEs

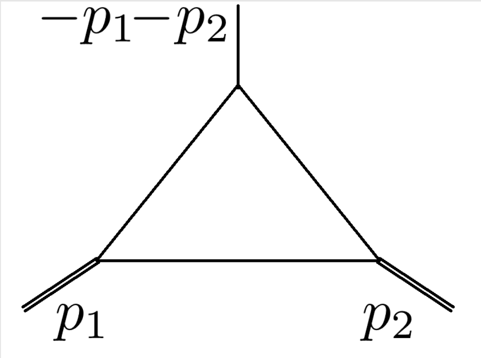
$$\frac{\partial}{\partial x} I_i^{(m)}(x) = \sum_{j;\, n \leq m} c_j I_j^{(n)}(x)$$

- Proper x-parametrization provides coefficients c_i that are rational functions → solution naturally lead to Goncharov Polylogaritms (GP)

$$GP(\underbrace{\alpha_1, \cdots, \alpha_n}_{\text{weight n}}; x) := \int_0^x dx' \frac{GP(\alpha_2, \cdots, \alpha_n; x')}{x' - \alpha_1}$$

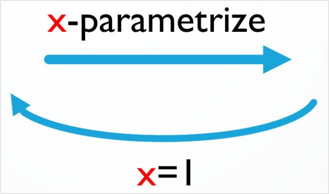
$$GP(;x) = 1, \qquad GP(\underbrace{0, \cdots, 0}_{\text{n times}}; x) = \frac{1}{n!} \log^n(x),$$

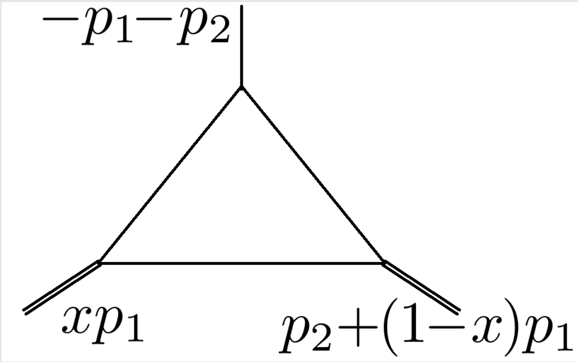
Example: 1-loop triangle



$$G_{111}(m_1,m_2) \propto \int \frac{d^dk}{k^2(k+p_1)^2(k+p_1+p_2)^2}$$

$$p_1^2 = m1, p_2^2 = m2, (p_1+p_2)^2 = 0$$





$$G_{111}(m_1,x) \propto \int \frac{d^dk}{k^2(k+xp_1)^2(k+p_1+p_2)^2}$$

$$p_1^2=m1, p_2^2=0, (p_1(1-x)+p_2)^2 \neq 0, (p_1+p_2)^2=0$$

- Differentiate to x and use IBP to reduce:

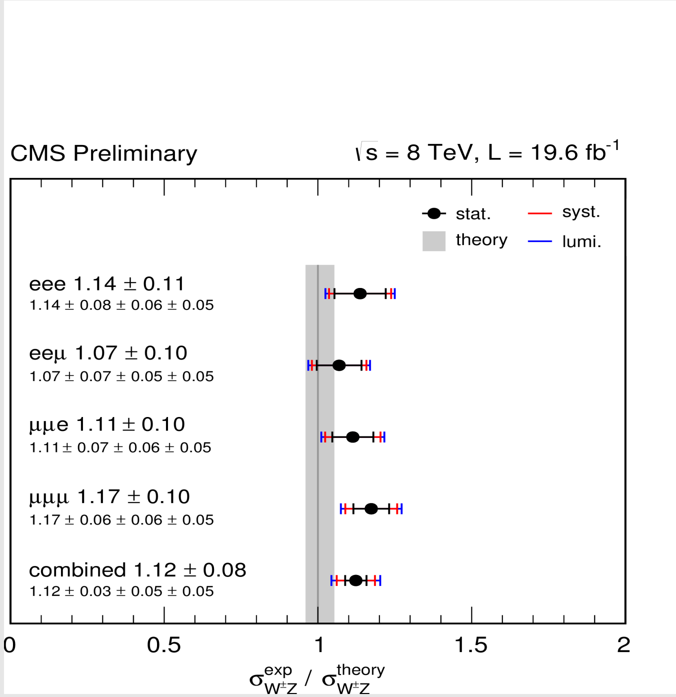
$$\frac{\partial}{\partial x}G_{111}(x) = \frac{-x^{-2-\epsilon}}{\epsilon^2 m_1}\left(\left((-m1-i0)^{-\epsilon}(1+2\epsilon)x^{-\epsilon}(-m1-i0)^{-\epsilon}(1-x)^{-1-\epsilon}(1+\epsilon-x(1+2\epsilon)\right)\right)\right)$$

- Subtracting the singularities, expanding the finite part and integrating in x leads to:

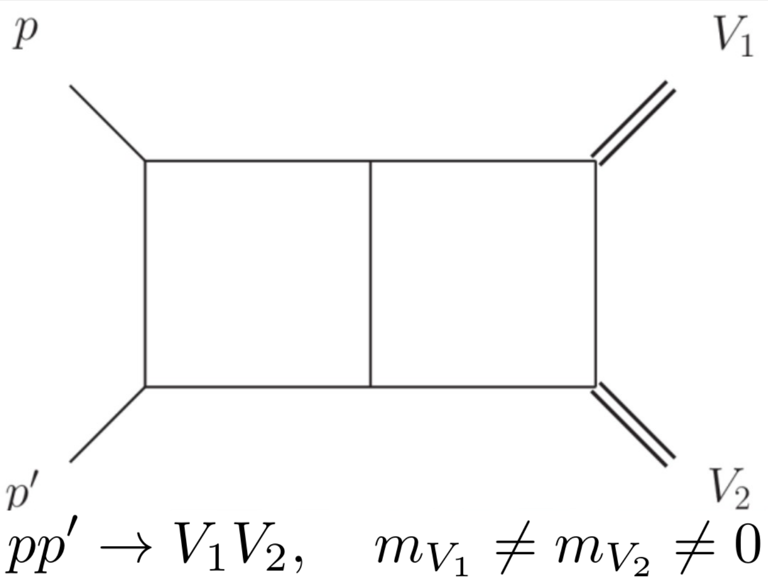
$$G_{111}(x) = \underbrace{G_{111}(0)}_{=const} + \frac{-(m_1-i0)^{-\epsilon}x^{-\epsilon}+(-m_1-i0)^{-\epsilon}x^{-2\epsilon}}{m_1 x\epsilon^2} + \frac{(m_1-i0)^{-\epsilon}(-x^{-\epsilon}+x+GP(1;x))}{m_1 x\epsilon} + O(\epsilon^0)$$

- This result agrees with expansion of exact solution

Two-loop double-box



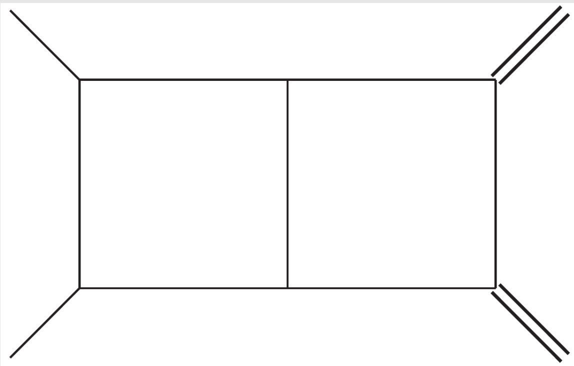
Example of planar diagrams:

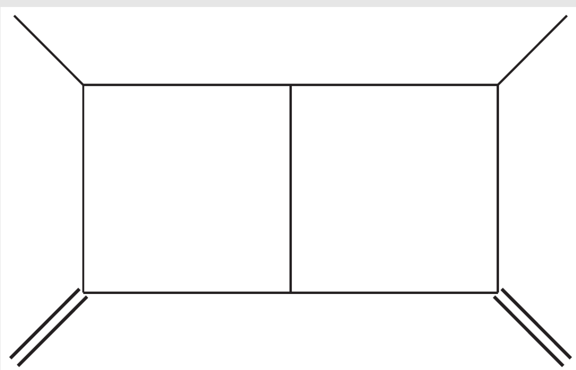


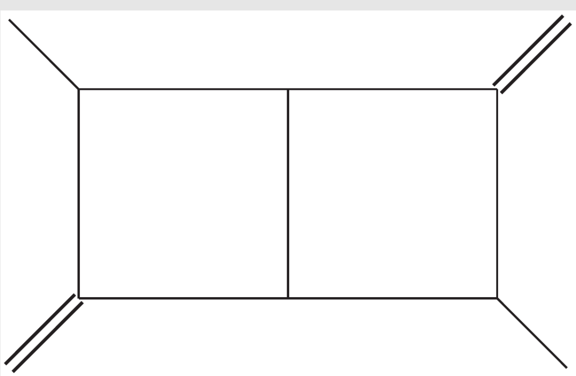
- Require 4-point two-loop MI with 2 off-shell legs and massless internal legs (at LHC light-flavor quarks are massless to good degree)
- Lots of efforts already done for diboson production [On-shell legs planar: V. Smirnov '99, V. Smirnov & Veretin '99; non-planar:Tausk '99, Anastasiou, Gehrmann, Oleari, Remiddi & Tausk '00; One off-shell leg (pl. +non-pl.) Gehrmann & Remiddi '00-'01; two off-shell legs with same masses planar: Gehrmann, Tancredi & Weihs '13; non-planar: Gehrman, Manteuffel, Tancredi & Weihs '14; two off-shell legs with different masses planar: Henn, Melnikov & Smirnov '14; non-planar: Caola, Henn, Melnikov & Smirnov '14]

Double planar box: condition for choosing x parameterization

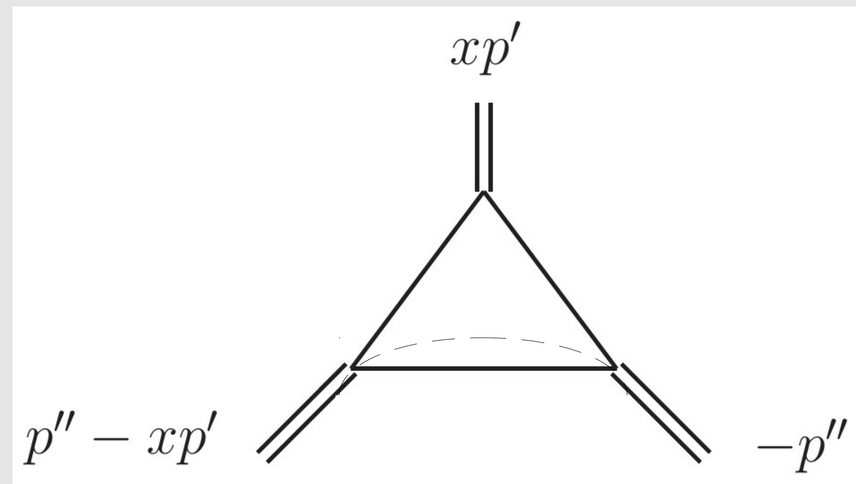
Papadopoulos arXiv:1401.6057





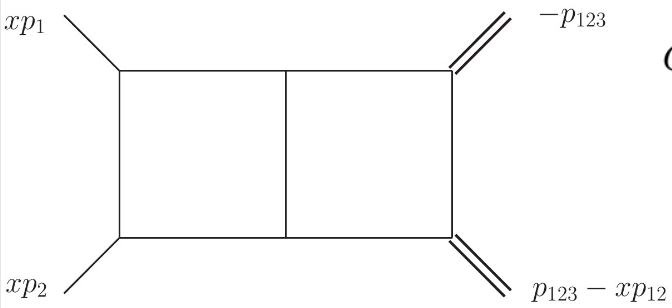


pinched massive triangle



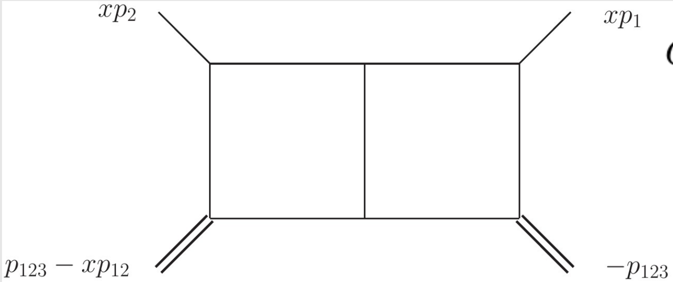
Double planar box: x parameterization

Wever, Papadopoulos, DT arXiv:1407.0046; work in progress



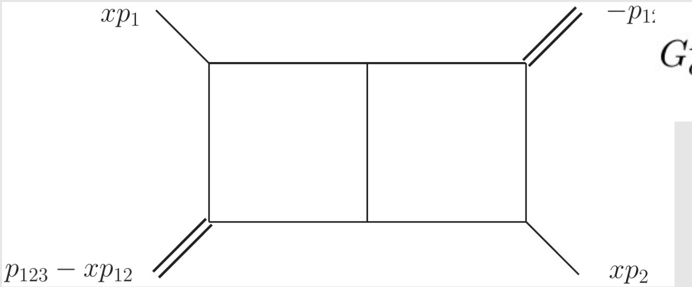
$$G^{P_{12}}_{a_1 \cdots a_9}(x, s, \epsilon) := e^{2\gamma_E \epsilon} \int \frac{d^d k_1}{i\pi^{d/2}} \frac{d^d k_2}{i\pi^{d/2}} \frac{1}{k_1^{2a_1}(k_1 + xp_1)^{2a_2}(k_1 + xp_{12})^{2a_3}(k_1 + p_{123})^{2a_4}} \times \frac{1}{k_2^{2a_5}(k_2 - xp_1)^{2a_6}(k_2 - xp_{12})^{2a_7}(k_2 - p_{123})^{2a_8}(k_1 + k_2)^{2a_9}},$$

Topology 1: system of 31 MI



$$G^{P_{13}}_{a_1 \cdots a_9}(x, s, \epsilon) := e^{2\gamma_E \epsilon} \int \frac{d^d k_1}{i\pi^{d/2}} \frac{d^d k_2}{i\pi^{d/2}} \frac{1}{k_1^{2a_1}(k_1 + xp_1)^{2a_2}(k_1 + xp_{12})^{2a_3}(k_1 + p_{123})^{2a_4}} \times \frac{1}{k_2^{2a_5}(k_2 - xp_1)^{2a_6}(k_2 - p_{12})^{2a_7}(k_2 - p_{123})^{2a_8}(k_1 + k_2)^{2a_9}},$$

Topology 2: system of 29 MI

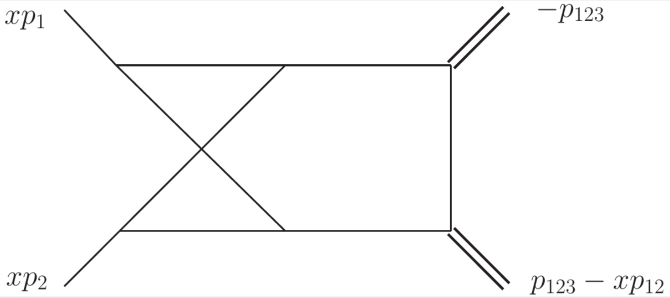


$$G^{P_{23}}_{a_1 \cdots a_9}(x, s, \epsilon) := e^{2\gamma_E \epsilon} \int \frac{d^d k_1}{i\pi^{d/2}} \frac{d^d k_2}{i\pi^{d/2}} \frac{1}{k_1^{2a_1}(k_1 + xp_1)^{2a_2}(k_1 + p_{123} - xp_2)^{2a_3}(k_1 + p_{123})^{2a_4}} \times \frac{1}{k_2^{2a_5}(k_2 - p_1)^{2a_6}(k_2 + xp_2 - p_{123})^{2a_7}(k_2 - p_{123})^{2a_8}(k_1 + k_2)^{2a_9}}$$

Topology 3: system of 28 MI

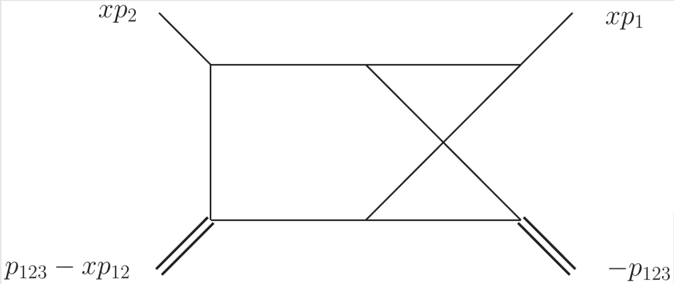
Double non-planar box: x parameterization

Wever, Papadopoulos, DT work in progress



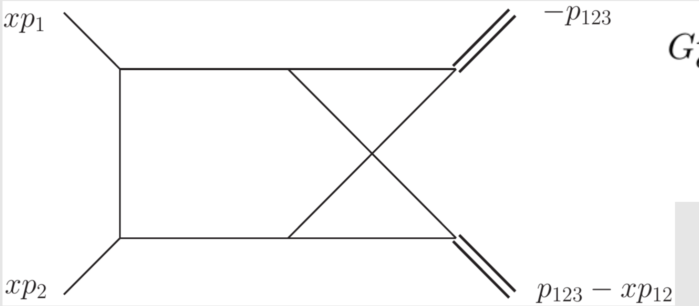
$$G^{N_{12}}_{a_1\cdots a_9}(x,s,\epsilon) := e^{2\gamma_E\epsilon}\int \frac{d^dk_1}{i\pi^{d/2}}\frac{d^dk_2}{i\pi^{d/2}}\frac{1}{k_1^{2a_1}(k_1+xp_1)^{2a_2}(k_1+xp_{12})^{2a_3}(k_1+p_{123})^{2a_4}} \times \frac{1}{k_2^{2a_5}(k_2-xp_1)^{2a_6}(k_2-p_{123})^{2a_7}(k_1+k_2+xp_2)^{2a_8}(k_1+k_2)^{2a_9}}$$

Topology 1: system of 35 MI



$$G^{N_{13}}_{a_1\cdots a_9}(x,s,\epsilon) := e^{2\gamma_E\epsilon}\int \frac{d^dk_1}{i\pi^{d/2}}\frac{d^dk_2}{i\pi^{d/2}}\frac{1}{k_1^{2a_1}(k_1+xp_1)^{2a_2}(k_1+xp_{12})^{2a_3}(k_1+p_{123})^{2a_4}} \times \frac{1}{k_2^{2a_5}(k_2-xp_{12})^{2a_6}(k_2-p_{123})^{2a_7}(k_1+k_2+xp_1)^{2a_8}(k_1+k_2)^{2a_9}}$$

Topology 2: system of 43 MI



$$G^{N_{34}}_{a_1\cdots a_9}(x,s,\epsilon) := e^{2\gamma_E\epsilon}\int \frac{d^dk_1}{i\pi^{d/2}}\frac{d^dk_2}{i\pi^{d/2}}\frac{1}{k_1^{2a_1}(k_1+xp_1)^{2a_2}(k_1+xp_{12})^{2a_3}(k_1+p_{123})^{2a_4}} \times \frac{1}{k_2^{2a_5}(k_2-xp_1)^{2a_6}(k_2-p_{123})^{2a_7}(k_1+k_2+xp_{12}-p_{123})^{2a_8}(k_1+k_2)^{2a_9}}$$

Topology 3: system of 51 MI

Example: solution in GP

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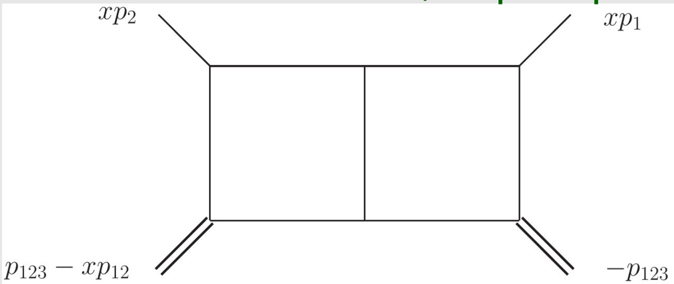
$$G^{P_{13}}_{011111011}(x,s,\epsilon) =$$



$$s_{12} := p_{12}^2, \qquad s_{23} := p_{23}^2, \qquad q := p_{123}^2$$

Solution as ε expansion:

$$A_3(\epsilon) = -e^{2\gamma_E \epsilon}\frac{\Gamma(1-\epsilon)^3\Gamma(1+2\epsilon)}{\Gamma(3-3\epsilon)}$$

$$\begin{aligned}
G^{P_{13}}_{011111011}(x,s,\epsilon) = \frac{A_3(\epsilon)}{x^2 s_{12}(-q+x(q-s_{23}))^2}\Bigg\{ & \frac{-1}{2\epsilon^4} + \frac{1}{\epsilon^3}\Bigg(-GP\left(\frac{q}{s_{12}};x\right) + 2\,GP\left(\frac{q}{q-s_{23}};x\right) \\
& +2\;GP(0;x) - GP(1;x) + \log\left(-s_{12}\right) + \frac{9}{4}\Bigg) + \frac{1}{4\epsilon^2}\Bigg(18\;GP\left(\frac{q}{s_{12}};x\right) - 36\;GP\left(\frac{q}{q-s_{23}};x\right) \\
& -8\;GP\left(0,\frac{q}{s_{12}};x\right) + 16\;GP\left(0,\frac{q}{q-s_{23}};x\right) + 8\;GP\left(\frac{s_{23}}{s_{12}}+1,\frac{q}{q-s_{23}};x\right) + \cdots\Bigg) \\
& +\frac{1}{\epsilon}\Bigg(9\left(GP\left(0,\frac{q}{s_{12}};x\right) + GP(0,1;x)\right) - 4\left(GP\left(0,0,\frac{q}{s_{12}};x\right) + GP(0,0,1;x)\right) + \cdots\Bigg) \\
& +6\left(GP\left(0,0,1,\xi_-;x\right) + GP\left(0,0,1,\xi_+;x\right)\right) - 2\,GP\left(0,0,\frac{q}{q-s_{23}},\frac{q\left(q-s_{23}\right)}{q^2-s_{23}\left(q+s_{12}\right)};x\right) + \cdots\Bigg\}
\end{aligned}$$

- Solution valid in the Euclidean region (numerical agreement found with SecDec) [Borowka, Carter & Heinrich]
- Solution can be extended to any kinematic region by analytic continuation, by adding infinitesimal imaginary part to all variables (numerical agreement found with known analytic results) [Henn, Melnikov & Smirnov '14]

Summary

- In LHC era multi-loop calculations are compulsory
- Two-loop automation is the next step: reduction substantially understood, library of MI mandatory but still missing
- Functional basis for large class of MI: Goncharov polylogarithms
- DE method is very fruitful for deriving MI in terms of GP
- x-DE method [Papadopoulos ’14] (often) captures GP solution, very algorithmic
- The x-DE approach naturally incorporates the boundary constraints
- Recent application: planar and non-planar double box (to appear soon on arXiv...)

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Ευχαριστώ!